Mutli-tier tax competition on Gasoline

Marie-Laure Breuillé* Emmanuelle Taugourdeau †

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Abstract

This paper aims to analyze the fiscal interactions arising from gasoline taxation in a federation. We adopt a general theoretical model for studying simultaneous vertical and horizontal tax competition by i) introducing a specific monetary cost of refueling ii) assuming that the price of the gasoline is affected by either excise taxes (local and federal) and the VAT rate, ii) considering elastic demand for gasoline. We show that horizontal taxes are strategic complements but vertical taxes are strategic substitutes. Moreover, horizontal excise taxes are strategic substitutes with VAT whereas the result is unclear for the reaction between local and federal excise taxes. Finally, we show that the reaction functions of the different taxes crucially differ according to the pattern of decision making (Social planner, Nash or decentralized leadership).

Keywords: Fiscal Federalism, Gasoline Taxation, Nash equilibrium

JEL Codes: E62, H7, Q48

1 Introduction

Taxes represent 61.9% of unleaded gasoline price and 56.2% of gasoline price in 2015 for the French consumer. The central government levied 14,9 billions of euros from the domestic consumption tax on energy products (the fourth source of central tax revenue), plus VAT tax revenues. Regional governments levied 5,3 billions of euros from a regional modulation of the domestic consumption tax on energy products and the "départements" received 6,5 billions of euros from the central government to compensate some transfers of competencies (DGCE (2016)). All in all, tax revenues from the domestic consumption tax on energy products amount to 27,4 billions of euros in 2015 in France. Currently, the differences among prices at the pump essentially depend on the cost of distribution, as all regions except two set the upper limit of the regional modulation. However, what would be the consequences of removing the upper limit of the regional modulation, which is

*CAESAR, INRA
†CREST, University of Paris-Saclay

an option for ensuring more revenue to newly reinforced regions? This question is of particularly high importance in a context of decreasing transfers granted by the central government to local ones together with the devolution of powers from the Central Government to local authorities which obliges local governments to raise more revenues. Nowadays, gasoline taxation represents around 30% of the revenue from indirect taxation of local authorities (regions and dpartements) and less than 10% of the whole local revenues. In this context we can wonder if gasoline taxation could be an efficient source of additional local tax revenue.

To address this question, we assume that two gas stations located on both sides of a regional border could set two very different prices, implied by two different regional tax policies, giving rise to a strategic behaviour of the French driver. The driver, and therefore the tax base (i.e., the number of liters of gasoline purchased), is indeed mobile, but partially as the cost in terms of gasoline consumed of not fueling in the closest gas station must at least be covered by the price difference between two gas stations. This architecture of the taxation of oil products would give rise to horizontal tax interactions among regions –through the regional modulation of the rate of the domestic consumption tax on energy products– and vertical tax interactions –as the domestic consumption tax on energy products is also levied at the national level– as well as tax interdependence between two national instruments, i.e. the domestic consumption tax on energy products and the VAT.

Our paper aims at modelling this two-tier tax competition game, with a specific good which is gasoline, and two tax instruments at the top tier. We thus contribute to the seminal paper of horizontal cross-border shopping by Kanbur and Keen (1993) through i) an endogenous demand for the good whereas it is exogenous in Kanbur and Keen (1993); ii) the specification of two costs, i.e. a psychological cost proportional to the distance traveled to purchase the gasoline and a monetary cost that characterizes the fact that traveling to buy gasoline implies the consumption of gasoline, rather than a cost of crossing the border in Kanbur and Keen (1993), iii) the existence of vertical tax interactions and two tax instruments at the top tier, in addition to horizontal interactions. Price elasticity of gasoline demand is a key issue in our model. Recent meta-analysis as the one by Brons et al. (2008) have shown that price elasticity demand for gasoline is higher in long run due to the lag of adjustment of the consumer behaviors. Nevertheless, even if the price elasticity of demand is quite small in the short term, we argue that consumers are quite sensitive to a difference in gasoline prices for a quite small distance. The idea that consumers react to a change in gasoline taxes is supported by the paper by Coglianese et al. (2016) who show that the demand for gasoline crucially increases just before a tax increase and is delayed before a tax decrease, rendering the tax instrument endogenous.

Devereux et al. (2007) already considered horizontal and vertical competition in excise taxes, with endogenous demand and cross-border shopping. However, our structure of cost is more sophisticated for gasoline than for their good (e.g. cigarettes). Furthermore, we introduce also the existence of VAT in our model (in addition to excise taxes). A strand of the literature has tried to identify the tax responses of excise taxes in a context of vertical competition (Keen, 1998) or horizontal competition with cross-border shopping(Kanbur and Keen, 1993). In line with the theoretical model by Devereux et al. (2007), we extend the cross boarding shopping models by allowing an individual demand for gasoline to be price-elastic and so tax elastic. Our paper is also related with the literature on tax rate interactions and tax reaction function that has been mainly developed in capital tax competition models (see Vrijburg and de Mooij (2016)). We still adopt a general theoretical model for analyzing simultaneous vertical and horizontal competition that we adapt to the peculiar case of gasoline. Our results show that regional excise taxes are strategic complements, which is currently standard in a model of tax competition with Leviathans. The impact of the local taxes on the federal ones are more interesting : local taxes are strategic substitutes with VAT and may be strategic complements or strategic substitutes with the federal excise tax, depending on the curvature of the demand for gasoline. Our simulation show that the excises taxes are more likely to be strategic complements with standard demand function. Finally, we show that the tax reaction functions of a social planner or a central government with leadership are crucially modified compared with the ones from the Nash game. As a result, we can argue that the type of decision-making crucially matters in the system of gasoline taxation.

The paper is organized as follow: section 2 describes the model and exhibits the fiscal interaction functions. Section 3 compare the central planner program to the Nash equilibrium and Section 4 exhibits the comparison of the Nash program with a sequential game (decentralized leadership). The last section concludes.

2 The model

We consider a federal country, which is modeled as a segment –in line with Hotelling (1929)– of length 2 with 2 regions i $(i = 1, 2)$ of equal size. Region 1 belongs to the interval $[-1, 0]$ and region 2 to the interval $[0, 1]$, the geographical border being 0. We assume that a gasoline station is located in each region, respectively at $S_1 = -1$ in region 1 and $S_2 = 1$ in region 2.

Each region i is populated by N identical agents uniformly distributed over the territory. For the sake of simplicity, we assume that $N = 1$. An agent k who lives in region i benefits from the consumption of two goods: a numeraire private good c_k^i consumed in her location and a quantity of gasoline x_k^j purshased in the station j of her choice, with $j = 1, 2$.

A special feature of the good "gasoline" is that the act of purchasing itself implies the consumption of part of the purchased amount because driving to the gas station consumes gasoline. We denote by α the gasoline consumption per unit of distance, whose monetary cost depends on the after-tax gasoline price per unit P_j in station j. In addition to this monetary cost, the agent bears a psychological cost δ (or opportunity cost of not devoting time to another activity), which is measured in terms of time per unit of distance. The distance between the location s_k^i of an agent k and the location S_j of the gas station j is measured by $|s_k^i - S_j|$.

For each unit of gasoline sold by the station S_j at a pre-tax price p_j, the federal government levies a federal gasoline excise tax T and the government of the region of the station $(i = j)$ levies a regional origine-based gasoline excise tax t_j. In addition, the federal government applies a rate θ of VAT on the purchase of a quantity x_k^j of gasoline at an after-excise-tax price $q_j \equiv p_j + t_j + T$ and on the private good consumption c_k^i. The overall after-tax gasoline price per quantity demanded for gasoline in station j is thus:

$$P_j \equiv q_j (1 + \theta) \equiv (p_j + t_j + T) (1 + \theta) .$$

Each agent is endowed with $\overline{y}$. The budget constraint of an agent k located in s_k^i, who chooses the gas station S_j, is

$$\overline{y} = c_k^i(1+\theta) + x_k^j P_j + (\delta + \alpha P_j)\left|s_k^i - S_j\right|.$$

We assume that the utility function of an agent k located in region i and consuming in station j takes a quasi-linear form, i.e., $c_k^i + u(x_k^j)$, where $u(.)$ is an increasing and concave function.

2.1 The agent's behaviour

The demand for gasoline The demand for gasoline is derived from the maximisation of the agent's utility function under her budget constraint, i.e.,

$$\widehat{x}_k^j = \arg\max\{-x_k^j \frac{P_j}{(1+\theta)} + u(x_k^j)\}$$

and therefore equalizes the marginal utility to the gasoline price net of VAT

$$u'(x_k^j) = \frac{P_j}{(1+\theta)} = q_j. \quad (1)$$

The demand for gasoline does not depend on the location of the agent once she has chosen her station. Thereafter, we will denote $\widehat{x}_k^j = x^j$. Using (1) to replace $\frac{P_j}{(1+\theta)}$ by $u'(x^j)$ into the utility function, and differentiating with respect to x^j, we show that the indirect utility increases w.r.t. x^j, i.e. $\frac{\partial\left(u(x^j)-x^j q_j\right)}{\partial x^j} = u'(x^j) - u'(x^j) - x^j u''(x^j) > 0$, and that:

$$u(x^j) - x^j q_j > u(x^i) - x^i q_i \Longleftrightarrow q_j < q_i \quad (2)$$

Differenciating the FOC (1) with respect to each tax, we obtain:

$$\frac{dx^j}{dt_j} = \frac{dx^j}{dT} = \frac{1}{u''(x^j)} < 0 \text{ and } \frac{dx^j}{d\theta} = 0.$$

The demand for gasoline decreases with respect to both excise taxes while it does not react to the VAT because VAT affects both the demand for the numeraire good and the gasoline demand.

The choice of the station Agent k will purchase the gasoline in station S_j located in region j if and only if $V_k^1 > V_k^2$ with the indirect utility function $V_k^j \equiv \frac{\overline{y}}{(1+\theta)} - x^j \frac{P_j}{(1+\theta)} - \frac{(\delta+\alpha P_j)}{(1+\theta)} |s_k - S_j| + u(x^j)$. Let denote by $\widetilde{s}$ the location of the agent indifferent between purchasing gasoline in region 1 or in region 2, i.e., for which $V_k^1 = V_k^2$:

$$\widetilde{s} = \frac{u(x^1) - (x^1 + \alpha)\, q_1 - (u(x^2) - (x^2 + \alpha)\, q_2)}{(\frac{2\delta}{(1+\theta)} + \alpha\,(q_1 + q_2))}. \quad (3)$$

From (2), the numerator of (3) (and therefore $\widetilde{s}$ as the denominator is positive) is always positive for $q_1 < q_2$, then:

$$\widetilde{s} > 0 \Longleftrightarrow q_1 < q_2. \tag{4}$$

Only the price of gasoline, net of VAT, influences the choice of the station. For $q_1 = q_2$, each agent consumes in her region of residence. For $q_1 < q_2$, the threshold $\widetilde{s}$ will be located in region 2, and agents located in $[0, \widetilde{s}]$ will cross the border to fuel in S_1 in region 1 (and *vice-versa*).

From comparative statics, we derive the following Lemma:

Lemma 1 *According to property (2), we have*

$$\frac{\partial \widetilde{s}}{\partial q_1} = \frac{\partial \widetilde{s}}{\partial p_1} = \frac{\partial \widetilde{s}}{\partial t_1} < 0$$

$$\frac{\partial \widetilde{s}}{\partial q_2} = \frac{\partial \widetilde{s}}{\partial p_2} = \frac{\partial \widetilde{s}}{\partial t_2} > 0$$

$$\frac{\partial \widetilde{s}}{\partial \theta} > 0 \iff q_1 < q_2$$

$$\frac{\partial \widetilde{s}}{\partial T} > 0 \Longleftrightarrow q_1 > q_2$$

Proof. The results are derived from the expression of the derivatives:

$\frac{\partial \widetilde{s}}{\partial p_1} = \frac{\partial \widetilde{s}}{\partial t_1} = \frac{-\left(x^1+\alpha\right)\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)-\alpha\left(\left(u(x^1)-\left(x^1+\alpha\right)q_1\right)-\left(u(x^2)-\left(x^2+\alpha\right)q_2\right)\right)}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)^2} = \frac{-x^1-\alpha(1+\widetilde{s})}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)}$ with $\widetilde{s} \in [-1, 1]$

$\frac{\partial \widetilde{s}}{\partial p_2} = \frac{\partial \widetilde{s}}{\partial t_2} = \frac{\left(x^2+\alpha\right)\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)-\alpha\left(\left(u(x^1)-\left(x^1+\alpha\right)q_1\right)-\left(u(x^2)-\left(x^2+\alpha\right)q_2\right)\right)}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)^2} = \frac{x^2+\alpha(1-\widetilde{s})}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)}$ with $\widetilde{s} \in [-1, 1]$

$\frac{\partial \widetilde{s}}{\partial \theta} = \frac{2\delta}{(1+\theta)^2}\frac{\left(u(x^1)-\left(x^1+\alpha\right)q_1\right)-\left(u(x^2)-\left(x^2+\alpha\right)q_2\right)}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)^2} = \frac{2\delta}{(1+\theta)^2}\frac{\widetilde{s}}{\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)}$

$\frac{\partial \widetilde{s}}{\partial T} = \frac{\frac{2\delta}{(1+\theta)}\left(x^2-x^1\right)-2\alpha\left(\left(u(x^1)-\left(x^1+\alpha\right)q_1\right)-\left(u(x^2)-\left(x^2+\alpha\right)q_2\right)\right)}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)^2} = \frac{\frac{2\delta}{(1+\theta)}\left(x^2-x^1\right)-2\alpha\left(\widetilde{s}\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)\right)}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)^2}$

and taking into account the properties (2) and (4). ■

Non surprisingly, the threshold consumer moves towards region 2 (resp. 1) when the tax rate in region 1(resp. 2) increases. The impact of the federal taxes depends on the asymmetry between regions. For a lower net of VAT price in region 1, a rise in VAT makes the threshold moving towards region 2 while a rise in the lump sum tax T makes the threshold moving in the opposite way. These opposite effects are due to the fact that the VAT rate does not modify the demand for gasoline because it affects the two types of consumption (numeraire good and gasoline) contrary to the federal excise tax that relies only on gasoline. Even if the excise tax affects similarly the price net of VAT in both regions, Expression (3) shows that a rise in VAT, by concerning also the numeraire good, decreases the distance cost which tends to move the threshold towards the external borders for a given asymmetry between prices p_i. The effect of the excise tax T is quite different since if affects both the demand for gasoline (negatively) and the net of VAT price (positively). Expression

(3) shows that the excise tax increases the distance cost which tends to move the threshold towards the central border (border between regions) for a given asymmetry of p_i and diminishes the utilities net of monetary costs in both regions. Due to the properties of the utility function, this decrease is stronger in the country which applies a smaller p_i. These two effects working in the same way tend to move the threshold agent towards the central border.

2.2 Local taxes reaction functions

In this section, we assume that the federal taxes are given (θ, T) and we look at the tax competition that arises between regions. The local governments act as Leviathan and aim at maximizing their revenue from taxes, i.e. for the region 1:

$$\begin{aligned} r_1(t_1, t_2, T, \theta) &= t_1 X_1 \\ \text{with } X_1(t_1, t_2, T, \theta) &= x^1 s_1(q_1, q_2, \theta) \\ \text{where } s_1(q_1, q_2, \theta) &= 1 + \widetilde{s} \end{aligned}$$

and for region 2

$$\begin{aligned} r_2(t_1, t_2, T, \theta) &= t_2 X_2 \\ \text{with } X_2(t_1, t_2, T, \theta) &= x^2 s_2(q_1, q_2, \theta) \\ \text{where } s_2(q_1, q_2, \theta) &= 1 - \widetilde{s} \end{aligned}$$

In the above conditions, s_i represents the number of agents that will purchase gasoline in region i and X_i stands for the total demand for gasoline in region i and therefore constitutes the tax base on which the local tax rate can rely on.

Combining the first order conditions for region i gives

$$\frac{\partial r_i}{\partial t_i} = 0 \Longleftrightarrow 1 + \varepsilon_{x_i} + \varepsilon_{s_i} = 0 => \Omega^i(t_1, t_2; \theta, T) = 0 \text{ for } X_i > 0 \tag{5}$$

with $\varepsilon_{x_i} = \frac{t_i}{x^i}\frac{\partial x^i}{\partial t_i}$ and $\varepsilon_{s_i} = \frac{t_i}{s_i}\frac{\partial s_i}{\partial t_i}$.

Here, ε_{x_i} stands for the tax elasticity of the individual demand for gasoline that we call the intensive elasticity and ε_{s_i} stands for the tax elasticity of the number of shoppers or extensive elasticity[1]. The intensive elasticity can also be decomposed as the ratio of the tax over price q_i multiplied by the price elasticity of gasoline $\left(\varepsilon_{x_i} = \frac{t_i}{q_i}\varepsilon_{q_i}\right)$. The extensive elasticity depending on the number of agents that will purchase the gasoline in the region (tax base), it depends on the consumer threshold $\widetilde{s}$.

[1] We use the usual vocabulary for labor taxation that apply here.

Lemma 2 *We have:*
i) $\varepsilon_{x_i} < 0$
ii) $\frac{\partial s_1}{\partial t_1} \leqslant 0$ *and* $\frac{\partial s_2}{\partial t_2} \leqslant 0$
iii) $\varepsilon_{s_i} \leqslant 0$

Proof. The first result is straighforward since the intensive elasticity is of the sign of the price elasticity. The second result is derived from the reaction of the threshold agent to a change in the local taxes ($\frac{\partial s_2}{\partial t_2} = -\frac{\partial \tilde{s}}{\partial t_2}$ and $\frac{\partial s_1}{\partial t_1} = \frac{\partial \tilde{s}}{\partial t_2}$); the third result follows directly from ii). ■

The analysis of the comparative statics gives the following results.

Proposition 3 *For symmetric countries, at the symmetric Nash equilibrium, the slope of horizontal and vertical tax reaction functions of the local taxes are:*

$$\frac{\partial t_i}{\partial t_j} = t\frac{(\varepsilon_s)^2}{\Omega^1_{t_i}} > 0 \ \forall i;$$

$$\frac{\partial t_i}{\partial \theta} = -\frac{\frac{\varepsilon_s}{\rho}\frac{2\delta}{(1+\theta)^2}}{\Omega^1_{t_i}} < 0 \ \forall i;$$

$$\frac{\partial t_i}{\partial T} = -\frac{-\frac{1}{\rho}\left(2\alpha\varepsilon_s + tx'\right) + \varepsilon_x\left[-\frac{\varepsilon_x}{t} + \frac{x''}{x'}\right]}{\Omega^1_{t_i}} \ \forall i.$$

with $\Omega^1_{t_i} = \frac{2}{t}\left(-1+\varepsilon_x\varepsilon_s\right) + \varepsilon_x\frac{x''}{x'} + \frac{1}{\rho}\left(-\alpha\varepsilon_s - x't\right) < 0$ *from the concavity condition and* $\rho = \frac{2\delta}{(1+\theta)} + \alpha\left(q_1+q_2\right) > 0$ *is the distance cost.*

Proof. See Appendix 1 ■

The following proposition highlights that local taxes are strategic complements, which is standard in the literature on capital tax competition. The VAT rate levied by the central government decreases the local excise taxes and this effect goes through the distance cost. The VAT decreases the distance cost so that the extensive elasticity is lower in absolute value when the VAT is large. The decrease of the distance cost tends to move the threshold agent, who is located at $\tilde{s} = 0$ at the symmetric equilibrium, towards the external borders. This is supposed to benefit to the local governments by increasing the tax base. Then, local governments are able to increase their local excise taxes which tends to move back the threshold agent without loosing revenue.
In line with Devereux et al. (2007), we are not able to sign the local tax reaction function to the central excise tax. The reaction depends on different effects : the first term of the numerator characterizes how the extensive elasticity i.e the elasticity of the number of shoppers with respect to the local tax reacts to a change in the federal excise tax. This effect is clearly positive and works in the opposite way compared to the VAT effect: an increase in T diminishes the distance cost and increases the difference in utilities so that a high T tends to leave the threshold to the center and the extensive elasticity to the local tax is higher. The second term represents the response of the elasticity of gasoline with respect to the local tax to an increase in the federal excise tax. This effect is ambiguous and depends on the form of the utility function u. For particular properties of the demand for gasoline, we are able to state clear-cut effects:

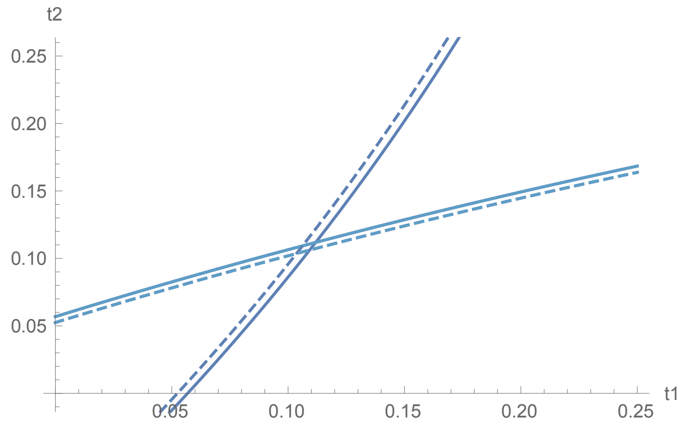

Figure 1: Effect of a rise in θ from $\theta = 0.2$ to $\theta = 0.35$

Corollary 4 *i) For inelastic demand, we have* $\frac{\partial t_i}{\partial T} = \frac{\frac{1}{\rho}(2\alpha\varepsilon_s)}{\Omega^1_{t_i}} > 0$ *with* $\Omega^1_{t_i} = -\frac{2}{t} - \alpha\frac{\varepsilon_s}{\rho}$.

ii) For an iso-elastic demand we have $\frac{\partial t_i}{\partial T} = \frac{\frac{1}{\rho}(2\alpha\varepsilon_s + tx') + \frac{\varepsilon_x}{q}}{\Omega^1_{t_i}} > 0$

Proof. An inelastic demand function implies $x' = 0$ and then $\varepsilon_x = 0$. For an iso elastic demand we have $q\frac{x''}{x'} = (\varepsilon_x\frac{q}{t} - 1)$ ■ Corrolary 4 supports the idea that for p

Proposition 1 applies for symmetric levels of the pre-tax prices $p_1 = p_2$. Figures 1 and 2 illustrate the reaction functions of the local taxes in an asymmetric case ($p_1 = 0.55$ and $p_2 = 0.5$) for a rise in T (Figure 1) and a rise in θ (Figure 2)[2]. The dashed curves correspond to the reaction functions resulting from a rise in the federal tax. The figures confirm the effects that has been highlighted in the symmetric case: the local taxes are strategic complements and a rise in VAT implies a decrease of the local tax. The new equilibrium resulting from the rise in θ implies lower local taxes. Finally, a rise in the federal excise tax has an opposite effect: it tends to increase the local tax reaction function and the Nash equilibrium results in higher local taxes. Note that the utility function that has been chosen to draw the figures implies $x'' < 0$ which reinforces the probability for the excise taxes to be complements.

Corollary 4 supports the idea that for peculiar demand functions, the local and federal excise taxes are strategic complements. For an inelastic demand, only the extensive elasticity matters whereas for an iso-elastic demand, the response of the elasticity is identical along the demand curve. Then, in addition to the extensive margin, the intensive margin matters but both affects

[2]The model is calibrated for a utility function of the form $u(x_i) = \beta x_i^{1/2}$ with $\overline{y} = 120$, β is calibrated so as to correspond to a refuel of 60 euros, α corresponds to a gasoline consumption of 7 liters per 100 kilometers, and $\delta = 2$ is calibrated to correspond to the cost of time evaluated by Pisani-Ferry, 2013 for sales. The initial federal taxes are fixed at their effective level, i.e. $T = 0.63$ and $\theta = 0.20$. Prices p_1 and p_2 are also chosen to correspond to the closest effective prices

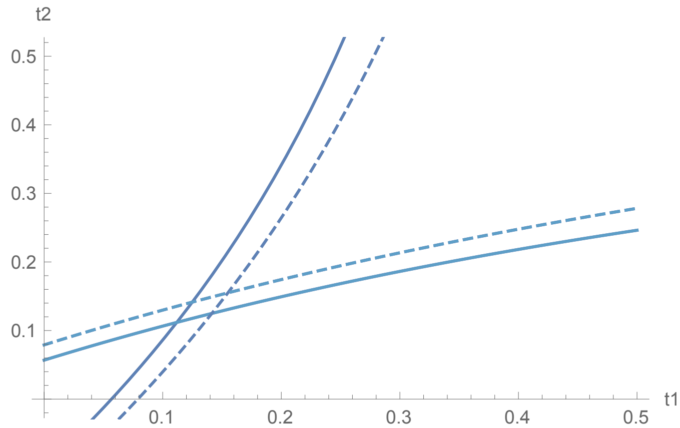

Figure 2: Effect of a rise in T from $T = 0.63$ to $T = 0.8$

play in the same way.

2.3 Federal tax reaction functions

We assume that the central government also acts as a Leviathan and maximizes his revenue with respect to the national taxes (θ, T) with

$$R(t_1, t_2, T, \theta) = \theta C + \sum_{i=1}^{2} (\theta q_i + T) X_i$$

and

$$C(t_1, t_2, T, \theta) = \int_{-1}^{\widetilde{s}} c^1 ds + \int_{\widetilde{s}}^{1} c^2 ds$$

with $c^1 = \frac{\overline{y} - x^1 P_1 - (\delta + \alpha P_1)(s+1)}{(1+\theta)}$ and $c^2 = \frac{\overline{y} - x^2 P_2 - (\delta + \alpha P_2)(1-s)}{(1+\theta)}$

The first order conditions with respect to the two federal fiscal tools give

$$\frac{\partial R}{\partial T} = \theta \frac{\partial C}{\partial T} + \sum_{i=1}^{2} \left((\theta + 1) X_i + \left(\frac{\theta}{1+\theta} P_i + T \right) \frac{\partial X_i}{\partial T} \right) = 0 \Longleftrightarrow \Theta^T (\theta, T; t_1, t_2) = 0$$

$$\frac{\partial R}{\partial \theta} = C + \theta \frac{\partial C}{\partial \theta} + \sum_{i=1}^{2} \left(q_i X_i + \left(\frac{\theta}{1+\theta} P_i + T \right) \frac{\partial X_i}{\partial \theta} \right) = 0 \Longleftrightarrow \Theta^\theta (\theta, T; t_1, t_2) = 0$$

From the first order conditions, we are able to derive the tax reaction functions:

Proposition 5 *For symmetric countries, at symmetric Nash equilibrium, the slope of horizontal and vertical tax reaction functions of the federal taxes are:*

$$\begin{aligned}
\frac{\partial T}{\partial t_i} &= -\frac{x' + Tx''}{\Theta_T^T} < 0 \text{ if } T\frac{x''}{x'} < -1; \\
\frac{\partial \theta}{\partial t_i} &= \frac{\frac{\alpha}{2}}{\Theta_\theta^\theta} < 0; \\
\frac{\partial \theta}{\partial T} &= -\frac{-2\left(x'q + x\right) - \alpha}{\Theta_\theta^\theta} < 0 \text{ for } -1 < \varepsilon < 0 \text{ with } \varepsilon = x'\frac{q}{x} \\
\frac{\partial T}{\partial \theta} &= \frac{\alpha}{\Theta_T^T} < 0
\end{aligned}$$

with $\Theta_\theta^\theta = 2\left[\left(\frac{2\overline{y}-\delta}{(1+\theta)^2}\right)(2\theta - 1)\right] < 0$ *and* $\Theta_T^T = 2\left[2x' + Tx''\right] < 0$ *from the concavity condition*

Proof. see Appendix 2 ■

From the federal tax reaction functions we can deduce that the VAT rate is a decreasing function of the local taxes. At the symmetric equilibrium, most of the effects collapse at the federal level mainly because VAT does not affect the demand for gasoline. The only effect of the local taxes that remains effective goes through the monetary cost. An increase in the local tax t_i increases the monetary costs of traveling to refuel (even for agents that buy gasoline in their region) that diminishes the consumption of the numeraire good. In order to compensate this diminution of the demand for the numeraire good, the federal government is inclined to decrease the VAT.

The impact of the local tax on the federal excise tax is still ambiguous. Contrary to the VAT rate, the federal and local excise taxes affect the demand for gasoline. Then, on one hand, the increase of the local tax modifies the reaction of the gasoline demand to the federal excise tax (Tx'') in the numerator. The sign of this effect depends on the form of the utility function u. On the other hand, the local tax affects directly the regional demand for gasoline. This effect (first term) is clearly negative and pushes the federal government to diminish its excise tax to limit the decrease in gasoline consumption.

Finally, the vertical interactions between the two federal taxes are also negative with reasonable assumptions on the price elasticity of demand [3]. At the symmetric equilibrium, the effect of the federal excise tax on the number of shoppers collapse at the federal level but the effect on the demand of gasoline still exists (first term of the numerator) which is reasonably negative assuming that the price elasticity of the demand for gasoline is smaller that 1 in absolute value. This effect is augmented by the effect of the excise tax on the monetary traveling cost that also diminishes the gasoline demand. Following an increase in the federal excise tax, the central government is then inclined to reduce the VAT in order to compensate the diminution of consumption.

[3] In empirical analysis, let us recall that the price elasticity is never higher than 1 in absolute value (included long run elasticities).

The Nash equilibrium being the result of the combination of both the horizontal and vertical tax reaction functions, even in the symmetric case, we are not able to derive expressions that could give us useful information about the level of the taxes. As an illustration, we simulate the Nash equilibrium with the parameters already used to graph the horizontal reaction functions. As a benchmark, we also simulate the local taxes for given federal taxes (the rates that are applied in France):

	τ_i	τ_j	T	θ
T=0.63 and $\theta = 0.20$	0.112	0.112	fixed	fixed
Nash equilibrium	0.149	0.149	0.91	0.816

The interesting result from these simulations are not so far the levels of the taxes but the low level of the local taxes compared to the national ones that could explain the existence of a ceiling for the local taxes. However, for the effective values of the Federal taxes ($T = 0.63$ and$\theta = 0.20$, the efficient local taxes are far from the effective one ($t = 0.025$)).

3 Nash versus Social Planner

In this section we aim to determine how the program of a social planner who cares about the whole revenue of the governments (local and central) may modify the tax reaction functions compared with the Nash game. The whole revenue (SP) writes:

$$\begin{aligned} SP &= R(t_1, t_2, T, \theta) + r_1(t_1, t_2, T, \theta) + r_2(t_1, t_2, T, \theta) \\ &= t_1 X_1 + t_2 X_2 + \theta C + \sum_{i=1}^{2} (\theta q_i + T) X_i \end{aligned} \tag{6}$$

To compare the tax reaction functions, we first have to focus on the tax externalities which are presented in the following lemma:

Lemma 6 *In the symmetric case:*
i) the local taxes exhibit negative externalities both at the horizontal and vertical levels : $\frac{\partial r_j}{\partial t_i} < 0$ *and* $\frac{\partial R}{\partial t_i} < 0$
ii) the federal excise tax exhibits negative externalities on local governments' revenue : $\frac{\partial r_1}{\partial T} + \frac{\partial r_2}{\partial T} < 0$.
iii) the federal sales tax exhibits no externality on the sum of the local governments' revenues : $\frac{\partial r_1}{\partial \theta} + \frac{\partial r_2}{\partial \theta} = 0$.

Proof.
With $r_1 = t_1 (1 + \widetilde{s}) x_1$ and $r_2 = t_2 (1 - \widetilde{s}) x_2$

$$\frac{\partial r_1}{\partial t_2} = t_1 x_1 \frac{\partial \widetilde{s}}{\partial t_1} < 0 \text{ and } \frac{\partial r_2}{\partial t_1} = -t_2 x_2 \frac{\partial \widetilde{s}}{\partial t_2} > 0$$

$$\frac{\partial r_1}{\partial T}+\frac{\partial r_2}{\partial T}=t_1x_1\frac{\partial \widetilde{s}}{\partial T}+(1+\widetilde{s})\,t_1\frac{\partial x_1}{\partial T}+(1-\widetilde{s})\,t_2\frac{\partial x_2}{\partial T}-t_2x_2\frac{\partial \widetilde{s}}{\partial T}=t_1\frac{\partial x_1}{\partial T}+t_2\frac{\partial x_2}{\partial T}<0 \text{ in the symmetric case}$$

$$\frac{\partial r_1}{\partial \theta}+\frac{\partial r_2}{\partial \theta}=t_1x_1\frac{\partial \widetilde{s}}{\partial \theta}-t_2x_2\frac{\partial \widetilde{s}}{\partial \theta}=0 \text{ in the symmetric case}$$

With $R=\theta C+\sum_{i=1}^{2}\left(\theta q_i+T\right)X_i$

$$\begin{aligned}\frac{\partial R}{\partial t_1} &= \theta\frac{\partial C}{\partial t_1}+\theta X_1+\left(\theta q_1+T\right)\left(\frac{\partial x_1}{\partial t_1}\left(1+\widetilde{s}\right)+\frac{\partial \widetilde{s}}{\partial t_1}x_1\right)+\left(\theta q_2+T\right)\left(-\frac{\partial \widetilde{s}}{\partial t_1}x_2\right)\\ &= \theta\frac{\partial C}{\partial t_1}+\theta x_1+\left(\theta q+T\right)\frac{\partial x_1}{\partial t_1}\end{aligned}$$

From (9) we know that for the symmetric case we have $\frac{\partial C}{\partial t_1}=-\left(\frac{\partial x_1}{\partial t_1}q_1+x_1\right)-\frac{\alpha}{2}$ so that

$$\frac{\partial R}{\partial t_1}=-\frac{\alpha}{2}\theta+Tx'<0$$

and symmetrically for t_2

$$\frac{\partial R}{\partial t_2}=-\frac{\alpha}{2}\theta+Tx'<0$$

■

The explanation of the spillover effects are the following: a rise a one of the local tax (let us say t_1) makes the threshold agent, initially in $\tilde{s}=0$, moving on the left side of the line i.e. between -1 and 0. This implies lower number of agents who choose to refuel in region 1 and a following decrease in the local tax revenue. A rise in t_1 also diminishes the demand for gasoline which affects the central government revenue through both the excise tax revenue and the sale tax revenue. A rise in the federal excise tax does not modify the threshold agent (remains in $\tilde{s}=0$) but affects the demand for gasoline that diminishes the local government revenue. Finally, an increase in θ does not affect the sum of the local revenues because the VAT rate does not affect the gasoline demand and does not modify the threshold agent at the symmetric equilibrium because both the regions are symmetrically affected.

From the previous lemma, we can compare the tax reaction functions derived from the symmetric Social Planner program and the symmetric Nash game:

Proposition 7 *In the symmetric case, compared to the Nash game, the reaction functions from the Social Planner program are:*
i) driven downward for both the local and federal excise taxes
ii) unchanged for the sales tax.

Proof. Evaluated at the symmetric Nash equilibrium, the first order conditions rewrite:

$$\frac{\partial SP}{\partial t_i} = \frac{\partial R}{\partial t_i} + \frac{\partial r_j}{\partial t_i} < 0$$
$$\frac{\partial SP}{\partial T} = \sum \frac{\partial r_i}{\partial T} < 0$$
$$\frac{\partial SP}{\partial \theta} = \sum \frac{\partial r_i}{\partial \theta} = 0$$

from the lemma above.

■

Since we have four different taxes in our social planner program it is quite difficult to compare the equilibria resulting from the Nash game and the Social planner solution. We are only able to compare the equilibria, fixing two of the instruments.

Let us first assume that t_1 and t_2 are fixed. Due to the fact that the reaction function resulting from CPO with respect to the excise tax is shifted downward, we can deduce that the federal taxes are lower when decided by a Social planner. Similarly, with fixed federal taxes (θ and T) we can state that the local taxes are lower under the social planner solution than under the Nash Solution. These results are directly explained by the negative externalities that follow an increase in taxes through the decrease of gasoline demand.

4 Sequential vertical interactions

Now if we quite realistically assume that the setting of the federal taxes is more rigid than the decision about the local excise taxes, we can consider the choice of the taxes as a sequential game in which the federal taxes are decided in a first stage and the local governments adjust their choice in a second stage. Once again, our aim is to determine how the tax reaction functions are modified in this setting compared to a symmetric Nash game.

Solving this program backward, the first stage of the game corresponds to the local government first order condition (5). Now, the first order conditions with respect to the two fiscal tools reduce to:

$$\frac{\partial R}{\partial T} + \sum \frac{\partial R}{\partial t_i}\frac{\partial t_i}{\partial T} = 0$$
$$\frac{\partial R}{\partial \theta} + \sum \frac{\partial R}{\partial t_i}\frac{\partial t_i}{\partial \theta} = 0$$

Proposition 8 *In the symmetric case, compared to the Nash game, the federal reaction functions with the sequential game are:*
i) driven downward (resp. upward) for the excise tax T if excise taxes (t_i and t) are strategic complements (resp. strategic substitutes)
ii) driven upward for the sales tax.

Proof. Evaluated at the Nash equilibrium, the first order conditions with respect to federal taxes rewrite:

$$\frac{\partial R}{\partial t_i}\frac{\partial t_i}{\partial \theta} > 0$$

$$\frac{\partial R}{\partial t_i}\frac{\partial t_i}{\partial T} > 0 \Longleftrightarrow \frac{\partial t_i}{\partial T} < 0.$$

Since at the symmetric Nash equilibrium we have:

$$\frac{\partial R}{\partial t_i} = -\frac{\alpha\theta}{2} + T\frac{\partial x}{\partial t_i} < 0$$

■

Here again, we are only able to compare the Nash solution with the central leadership solution by fixing one of the federal tax. Let us assume that the federal excise tax is fixed. Then the sequential game will exhibit a lower VAT rate than at the Nash game but higher local excise taxes due to the negative interactions between the VAT rate and the local excise taxes.

Now, if we fix the VAT rate and analyze the choice of the excise taxes (game which is more realistic), the sequential game exhibits a higher federal excise tax if excise taxes are strategic complements and a lower federal excise tax if excise taxes are strategic substitutes. In both cases, the local taxes are higher at the decentralized leadership equilibrium than at the Nash equilibrium. Indeed, the decentralized leadership equilibrium gives a leadership power to the central government that is able to internalize the tax reaction functions and the local externalities on the federal revenue.

5 Conclusion

This paper aims to analyze the multiple strategic interactions that take place in the complex system of gasoline taxation as it is implemented in countries like France. The complexity of the system leads its evaluation in terms of fiscal efficiency difficult. By disentangling the different mechanisms, we are able to present some key results that drive the main forces of the gasoline taxation.

Even if our theoretical results are presented in a symmetric case, important elements can be extracted for this peculiar case. One of the first important result is that VAT and excise taxes (both at the local and federal levels) appear to be strategic substitutes. This results is mainly explained by the fact that the VAT rate affects also the numeraire good. Moreover, while the local excise taxes are strategic complements, which is standard in the horizontal tax competition literature and more particularly when governments maximize tax revenues rather than welfare, the interactions between the local and federal excise taxes are not clear-cut. They definitely depend on the slope of the demand function. For a ”classical” demand style, excise taxes are more likely to be strategic complements. However, the demand for gasoline is quite complicated to grasp as shown by the empirical literature on the price elasticity that has not reached yet a clear consensus on the size of the price elasticity, a key feature of our analysis.

6 Appendices

6.1 Appendix 1

- For all i the first order condition writes

$$\frac{\partial r_i}{\partial t_i} = 0 \qquad \Longleftrightarrow X_i \left(1 + \frac{t_i}{x^i}\frac{\partial x^i}{\partial t_i} + \frac{t_i}{s_i}\frac{\partial s_i}{\partial t_i}\right) = 0$$

$$\Longleftrightarrow \Omega^i\left(t_1, t_2; \theta, T\right) = \left(1 + \frac{t_i}{x^i}\frac{\partial x^i}{\partial t_i} + \frac{t_i}{s_i}\frac{\partial s_i}{\partial t_i}\right) = 1 + \varepsilon_{x_i} + \varepsilon_{s_i} = 0 \qquad (7)$$

At the equilibrium, the concavity condition from the local government program yields $\Omega^1_{t_1} < 0$ and $\Omega^2_{t_2} < 0$ with

$$\Omega^1_{t_1} = \underbrace{\left(\frac{1}{x^i}\frac{\partial x^i}{\partial t_i} + \frac{1}{s_i}\frac{\partial s_i}{\partial t_i}\right)}_{<0 \text{ at the equilibrium } \left(-\frac{1}{t_i}\right)} + t_i\left(\underbrace{-\frac{1}{\left(x^i\right)^2}\left(\frac{\partial x^i}{\partial t_i}\right)^2}_{<0} + \frac{1}{x^i}\frac{\partial^2 x^i}{\partial t_i^2}\underbrace{-\frac{1}{s_i^2}\left(\frac{\partial s_i}{\partial t_i}\right)^2}_{<0} + \frac{1}{s_i}\frac{\partial^2 s_i}{\partial t_i^2}\right) < 0$$

- For the particular case of symmetric countries ($p_1 = p_2 = p$), we have

$$\Omega^1_{t_1} = \frac{2}{t}\left(-1 + \varepsilon_x \varepsilon_s\right) + \varepsilon_x \frac{x''}{x'} + \frac{t}{\rho}\left(-x' + \alpha\frac{(x+\alpha)}{\rho}\right)$$

We proceed to the comparative statics :

We first start analyzing the vertical tax competition
1- with respect to θ
By differentiating (7), we obtain:

$$\frac{\partial t_1}{\partial \theta} = -\frac{\Omega^1_\theta}{\Omega^1_{t_1}} \text{ and } \frac{\partial t_2}{\partial \theta} = -\frac{\Omega^2_\theta}{\Omega^2_{t_2}}$$

The concavity condition from the local government program yields $\Omega^1_{t_1} < 0$ and $\Omega^2_{t_2} < 0$. The sign of Ω^i_θ gives the sign of $\frac{\partial t_i}{\partial \theta}$

$$\Omega^i_\theta = \frac{\partial \varepsilon_{s_i}}{\partial \theta} \text{ since } \frac{\partial \varepsilon_{x_i}}{\partial \theta} = 0 \text{ from } \frac{\partial x^i}{\partial \theta} = 0$$

we have

$$\frac{\partial \varepsilon_{s_i}}{\partial \theta} = t_i\left[-\frac{1}{s_i^2}\frac{\partial s_i}{\partial t_i}\frac{\partial s_i}{\partial \theta} + \frac{1}{s_i}\frac{\partial^2 s_i}{\partial t_i \partial \theta}\right]$$

with

$$s_1 = (1 + \widetilde{s}) \text{ and } s_2 = (1 - \widetilde{s})$$

then

$$\begin{aligned}\frac{\partial s_1}{\partial t_1} &= \frac{\partial \widetilde{s}}{\partial t_1} \text{ and } \frac{\partial s_2}{\partial t_2} = -\frac{\partial \widetilde{s}}{\partial t_2} \\ \frac{\partial s_1}{\partial \theta} &= -\frac{\partial s_2}{\partial \theta} = \frac{\partial \widetilde{s}}{\partial \theta} \text{ and } \frac{\partial^2 s_1}{\partial t_1 \partial \theta} = \frac{\partial^2 \widetilde{s}}{\partial t_1 \partial \theta} \text{ and } \frac{\partial^2 s_2}{\partial t_2 \partial \theta} = -\frac{\partial^2 \widetilde{s}}{\partial t_2 \partial \theta}\end{aligned}$$

with

$$\begin{aligned}\frac{\partial^2 \widetilde{s}}{\partial t_1 \partial \theta} &= \frac{-\alpha \frac{\partial \widetilde{s}}{\partial \theta} + \frac{2\delta}{(1+\theta)^2} \frac{\partial \widetilde{s}}{\partial t_1}}{\frac{2\delta}{(1+\theta)} + \alpha (q_1 + q_2)} \\ &\text{and} \\ \frac{\partial^2 \widetilde{s}}{\partial t_2 \partial \theta} &= -\frac{-\alpha \frac{\partial \widetilde{s}}{\partial \theta} + \frac{2\delta}{(1+\theta)^2} \frac{\partial \widetilde{s}}{\partial t_2}}{\frac{2\delta}{(1+\theta)} + \alpha (q_1 + q_2)}\end{aligned}$$

At the symmetric equilibrium ($p_1 = p_2$ so that $t_1 = t_2$) we have $\widetilde{s} = 0$, $\frac{\partial \widetilde{s}}{\partial \theta} = 0$ and $s_i = 1 > 0$ so that

$$\begin{aligned}\frac{\partial^2 s_1}{\partial t_1 \partial \theta} &= \frac{\frac{2\delta}{(1+\theta)^2} \frac{\partial \widetilde{s}}{\partial t_1}}{\frac{2\delta}{(1+\theta)} + \alpha (q_1 + q_2)} < 0 \\ &\text{and} \\ \frac{\partial^2 s_2}{\partial t_2 \partial \theta} &= -\frac{\frac{2\delta}{(1+\theta)^2} \frac{\partial \widetilde{s}}{\partial t_2}}{\frac{2\delta}{(1+\theta)} + \alpha (q_1 + q_2)} < 0\end{aligned}$$

and

$$\frac{\partial \varepsilon_{s_i}}{\partial \theta} = t_i \frac{\partial^2 s_i}{\partial t_i \partial \theta} < 0 \text{ so that } \frac{\partial t_i}{\partial \theta} < 0$$

2- with respect to T

$$\frac{\partial t_1}{\partial T} = -\frac{\Omega_T^1}{\Omega_{t_1}^1} \text{ and } \frac{\partial t_2}{\partial T} = -\frac{\Omega_T^2}{\Omega_{t_2}^2}$$

the sign of Ω_T^i gives the sign of $\frac{\partial t_i}{\partial T}$

$$\begin{aligned}\Omega_T^i &= \left(\frac{\partial \varepsilon_{x_i}}{\partial T} + \frac{\partial \varepsilon_{s_i}}{\partial T} \right) \\ &\text{with} \\ \frac{\partial \varepsilon_{x_i}}{\partial T} &= t_i \left[\underbrace{-\frac{1}{(x^i)^2} \frac{\partial x^i}{\partial t_i} \frac{\partial x^i}{\partial T}}_{<0} + \frac{1}{x^i} \frac{\partial^2 x^i}{\partial t_i \partial T} \right] \\ \frac{\partial \varepsilon_{s_i}}{\partial T} &= t_i \left[-\frac{1}{s_i^2} \frac{\partial s_i}{\partial t_i} \frac{\partial s_i}{\partial T} + \frac{1}{s_i} \frac{\partial^2 s_i}{\partial t_i \partial T} \right]\end{aligned}$$

Let us start by analysing $\frac{\partial \varepsilon_{s_i}}{\partial T}$ with $s_1 = (1+\widetilde{s})$ and $s_2 = (1-\widetilde{s})$

$$\frac{\partial s_1}{\partial T} = -\frac{\partial s_2}{\partial T} = \frac{\partial \widetilde{s}}{\partial T} \text{ and } \frac{\partial^2 s_1}{\partial t_1 \partial T} = \frac{\partial^2 \widetilde{s}}{\partial t_1 \partial T} \text{ and } \frac{\partial^2 s_2}{\partial t_2 \partial T} = -\frac{\partial^2 \widetilde{s}}{\partial t_2 \partial T}$$

$$\frac{\partial^2 \widetilde{s}}{\partial t_1 \partial T} = \frac{-\alpha \frac{\partial \widetilde{s}}{\partial T} - \frac{\partial x_k^1}{\partial T} - 2\alpha \frac{\partial \widetilde{s}}{\partial t_1}}{\frac{2\delta}{(1+\theta)} + \alpha (q_1 + q_2)}$$

and

$$\frac{\partial^2 \widetilde{s}}{\partial t_2 \partial T} = \frac{-\alpha \frac{\partial \widetilde{s}}{\partial T} + \frac{\partial x_k^2}{\partial T} - 2\alpha \frac{\partial \widetilde{s}}{\partial t_2}}{\frac{2\delta}{(1+\theta)} + \alpha (q_1 + q_2)}$$

At the symmetric equilibrium ($p_1 = p_2$ so that $t_1 = t_2$) we have $\widetilde{s} = 0$, $\frac{\partial \widetilde{s}}{\partial T} = 0$, $\frac{\partial \widetilde{s}}{\partial t_1} < 0$, $\frac{\partial \widetilde{s}}{\partial t_2} > 0$ and $s_i = 1 > 0$ so that

$$\frac{\partial^2 \widetilde{s}}{\partial t_1 \partial T} = \frac{-\frac{\partial x^1}{\partial T} - 2\alpha \frac{\partial \widetilde{s}}{\partial t_1}}{\frac{2\delta}{(1+\theta)} + \alpha (q_1 + q_2)} = \frac{\partial^2 \widetilde{s}}{\partial t_1^2} > 0$$

and

$$\frac{\partial^2 \widetilde{s}}{\partial t_2 \partial T} = \frac{\frac{\partial x^2}{\partial T} - 2\alpha \frac{\partial \widetilde{s}}{\partial t_2}}{\frac{2\delta}{(1+\theta)} + \alpha (q_1 + q_2)} = \frac{\partial^2 \widetilde{s}}{\partial t_2^2} = -\frac{\partial^2 s_2}{\partial t_2 \partial T} < 0$$

so that

$$\frac{\partial \varepsilon_{s_i}}{\partial T} = \frac{t_i}{s_i} \frac{\partial^2 s_i}{\partial t_i \partial T} = -\frac{1}{\rho} \left(2\alpha \varepsilon_s + t x'\right) > 0$$

Let us now analyse $\frac{\partial \varepsilon_{x_i}}{\partial T}$

$$\frac{\partial \varepsilon_{x_i}}{\partial T} = t_i \left[-\frac{1}{(x^i)^2} \frac{\partial x^i}{\partial t_i} \frac{\partial x^i}{\partial T} + \frac{1}{x^i} \frac{\partial^2 x^i}{\partial t_i \partial T} \right] = \varepsilon_x \left[-\frac{\varepsilon_x}{t} + \frac{x''}{x'} \right]$$

Then

$$\Omega_T^i = \left(\frac{\partial \varepsilon_{x_i}}{\partial T} + \frac{\partial \varepsilon_{s_i}}{\partial T} \right) = \frac{1}{\rho} \left(2\alpha \varepsilon_s - t x'\right) + \varepsilon_x \left[-\frac{\varepsilon_x}{t} + \frac{x''}{x'} \right]$$

3- Horizontal tax competition $\frac{\partial t_i}{\partial t_j}$

$$\frac{\partial t_i}{\partial t_j} = -\frac{\Omega_{t_j}^i}{\Omega_{t_i}^i}$$

then

$$sign \frac{\partial t_i}{\partial t_j} = sign \Omega_{t_j}^i$$

with

$$\Omega_{t_j}^i = \left(\frac{\partial \varepsilon_{x_i}}{\partial t_j} + \frac{\partial \varepsilon_{s_i}}{\partial t_j} \right) = t_i \left[-\frac{1}{s_i^2} \frac{\partial s_i}{\partial t_i} \frac{\partial s_i}{\partial t_j} + \frac{1}{s_i} \frac{\partial^2 s_i}{\partial t_i \partial t_j} \right] \text{ since } \frac{\partial \varepsilon_{x_i}}{\partial t_j} = 0$$

$$\frac{\partial s_1}{\partial t_i}=\frac{\partial \widetilde{s}}{\partial t_i};\ \frac{\partial s_2}{\partial t_i}=-\frac{\partial \widetilde{s}}{\partial t_i} \text{ and } \frac{\partial^2 s_1}{\partial t_1\partial t_2}=\frac{\partial^2 \widetilde{s}}{\partial t_1\partial t_2} \text{ and } \frac{\partial^2 s_2}{\partial t_2\partial t_1}=-\frac{\partial^2 \widetilde{s}}{\partial t_2\partial t_1}$$

with

$$\frac{\partial^2 \widetilde{s}}{\partial t_1\partial t_2} = \frac{-\alpha\frac{\partial \widetilde{s}}{\partial t_2}-\alpha\frac{\partial \widetilde{s}}{\partial t_1}}{\frac{2\delta}{(1+\theta)}+\alpha\left(q_1+q_2\right)}=\frac{-\alpha\frac{\partial \widetilde{s}}{\partial T}}{\frac{2\delta}{(1+\theta)}+\alpha\left(q_1+q_2\right)}$$

and

$$\frac{\partial^2 \widetilde{s}}{\partial t_2\partial t_1} = \frac{-\alpha\frac{\partial \widetilde{s}}{\partial t_1}-\alpha\frac{\partial \widetilde{s}}{\partial t_2}}{\frac{2\delta}{(1+\theta)}+\alpha\left(q_1+q_2\right)}=\frac{-\alpha\frac{\partial \widetilde{s}}{\partial T}}{\frac{2\delta}{(1+\theta)}+\alpha\left(q_1+q_2\right)}$$

At the symmetric equilibrium ($p_1=p_2$ so that $t_1=t_2$) we have $\widetilde{s}=0$, $\frac{\partial \widetilde{s}}{\partial T}=0$, $\frac{\partial \widetilde{s}}{\partial t_1}<0$, $\frac{\partial \widetilde{s}}{\partial t_2}>0$ and $s_i=N>0$ so that

$$\Omega^i_{t_j}=t_i\left[-\frac{1}{s_i^2}\frac{\partial s_i}{\partial t_i}\frac{\partial s_i}{\partial t_j}\right]=-\frac{1}{t}\left(\varepsilon_s\right)^2<0$$

From Lemma 1.

6.2 Appendix 2

1/ Lump sum tax T

$$\frac{\partial R}{\partial T}=\theta\frac{\partial C}{\partial T}+\sum_{i=1}^{2}\left(1+\theta\right)s_i x^i+\theta\sum_{i=1}^{2}q_i\frac{\partial\left(s_i x^i\right)}{\partial T}+T\sum_{i=1}^{2}\frac{\partial\left(s_i x^i\right)}{\partial T}0\Longleftrightarrow\Theta^T\left(\theta,T;t_1,t_2\right)=0$$

The existence of an equilibrium requires

$$\Theta^T_T=\left[\theta\frac{\partial^2 C}{\partial T^2}+\sum_{i=1}^{2}\left(1+\theta\right)\frac{\partial s_i x_i}{\partial T}+\theta\sum_{i=1}^{2}\left(\frac{\partial q_i}{\partial T}\frac{\partial\left(s_i x_i\right)}{\partial T}+q_i\frac{\partial^2\left(s_i x_i\right)}{\partial T^2}\right)+\sum_{i=1}^{2}\frac{\partial\left(s_i x_i\right)}{\partial T}+T\sum_{i=1}^{2}\frac{\partial^2\left(s_i x_i\right)}{\partial T^2}\right]$$

which gives at the symmetric equilibrium

$$\Theta^T_T=2\left[2x'+Tx''\right]<0$$

We have

$$\frac{\partial T}{\partial t_i}=-\frac{\Theta^T_{t_i}}{\Theta^T_T}$$

Then the sign of $\Theta^T_{t_i}$ gives the sign of $\frac{\partial T}{\partial t_i}$

$$\Theta^T_{t_k}=\theta\frac{\partial C}{\partial T\partial t_k}+\sum_{i=1}^{2}\left(1+\theta\right)\frac{\partial\left(s_i x^i\right)}{\partial t_k}+\theta\sum_{i=1}^{2}q_i\frac{\partial\left(s_i x^i\right)}{\partial T\partial t_k}+T\sum_{i=1}^{2}\frac{\partial\left(s_i x^i\right)}{\partial T\partial t_k}+\theta\frac{\partial\left(s_k x^k\right)}{\partial T}$$

Since $s_1 = 1 + \widetilde{s}$ and $s_2 = 1 - \widetilde{s}$ we have

$$\begin{aligned}
\frac{\partial\left(s_1 x^1\right)}{\partial t_1} &= \frac{\partial \widetilde{s}}{\partial t_1} x_1 + \frac{\partial x_1}{\partial t_1}\left(\widetilde{s}+1\right) \\
\frac{\partial\left(s_2 x^2\right)}{\partial t_1} &= -\frac{\partial \widetilde{s}}{\partial t_1} x_2 \\
\frac{\partial\left(s_1 x^1\right)}{\partial T} &= \frac{\partial \widetilde{s}}{\partial T} x_1 + \frac{\partial x_1}{\partial T}\left(\widetilde{s}+1\right) \\
\frac{\partial\left(s_2 x^2\right)}{\partial T} &= -\frac{\partial \widetilde{s}}{\partial T} x_2 + \frac{\partial x_2}{\partial T}\left(1-\widetilde{s}\right) \\
\frac{\partial^2\left(s_1 x^1\right)}{\partial T \partial t_1} &= \frac{\partial^2 \widetilde{s}}{\partial T \partial t_1} x_1 + \frac{\partial \widetilde{s}}{\partial T}\frac{\partial x_1}{\partial t_1} + \frac{\partial^2 x_1}{\partial T \partial t_1}\left(\widetilde{s}+1\right) + \frac{\partial x_1}{\partial T}\frac{\partial \widetilde{s}}{\partial t_1} \\
\frac{\partial^2\left(s_2 x^2\right)}{\partial T \partial t_1} &= -\frac{\partial^2 \widetilde{s}}{\partial T \partial t_1} x_2 - \frac{\partial \widetilde{s}}{\partial t_1}\frac{\partial x_2}{\partial T} \\
\frac{\partial^2\left(s_1 x^1\right)}{\partial T^2} &= \frac{\partial^2 \widetilde{s}}{\partial T^2} x_1 + 2\frac{\partial \widetilde{s}}{\partial T}\frac{\partial x_1}{\partial T} + \frac{\partial^2 x_1}{\partial T^2}\left(\widetilde{s}+1\right) \\
\frac{\partial^2\left(s_2 x^2\right)}{\partial T^2} &= -\frac{\partial^2 \widetilde{s}}{\partial T^2} x_2 - 2\frac{\partial \widetilde{s}}{\partial T}\frac{\partial x_2}{\partial T} + \frac{\partial^2 x_2}{\partial T^2}\left(1-\widetilde{s}\right)
\end{aligned}$$

$$C\left(t_1, t_2, T, \theta\right) = \int_{-1}^{\widetilde{s}} c^1 ds + \int_{\widetilde{s}}^{1} c^2 ds$$

Since $c^1 = \frac{\overline{y} - x^1 P_1 - (\delta + \alpha P_1)(s+1)}{(1+\theta)}$ we have $\int_{-1}^{\widetilde{s}} c^1 ds = \int_{-1}^{\widetilde{s}} \frac{\overline{y} - x^1 P_1 - (\delta + \alpha P_1)}{(1+\theta)} ds - \int_{-1}^{\widetilde{s}} \frac{(\delta + \alpha P_1) s}{(1+\theta)} ds = \frac{\overline{y} - x^1 P_1 - (\delta + \alpha P_1)}{(1+\theta)}\left(\widetilde{s}+1\right) - \frac{(\delta + \alpha P_1)}{2(1+\theta)}\left(\widetilde{s}^2 - 1\right)$

and similarly for c^2 we have $c^2 = \frac{\overline{y} - x^2 P_2 - (\delta + \alpha P_2)(1-s)}{(1+\theta)}$ so that $\int_{\widetilde{s}}^{1} c^2 ds = \int_{\widetilde{s}}^{1} \frac{\overline{y} - x^2 P_2 - (\delta + \alpha P_2)}{(1+\theta)} ds + \int_{\widetilde{s}}^{1} \frac{(\delta + \alpha P_2) s}{(1+\theta)} ds = \frac{\overline{y} - x^2 P_2 - (\delta + \alpha P_2)}{(1+\theta)}\left(1-\widetilde{s}\right) + \frac{(\delta + \alpha P_2)}{2(1+\theta)}\left(1-\widetilde{s}^2\right)$

Then we can rewrite C as

$$C = \frac{\overline{y} - x^1 P_1 - (\delta + \alpha P_1)}{(1+\theta)}\left(\widetilde{s}+1\right) - \frac{(\delta + \alpha P_1)}{2(1+\theta)}\left(\widetilde{s}^2 - 1\right) + \frac{\overline{y} - x^2 P_2 - (\delta + \alpha P_2)}{(1+\theta)}\left(1-\widetilde{s}\right) + \frac{(\delta + \alpha P_2)}{2(1+\theta)}\left(1-\widetilde{s}^2\right)$$

with simplifications we obtain

$$C = \frac{\overline{y} - x^1 P_1}{(1+\theta)}\left(\widetilde{s}+1\right) - \frac{(\delta + \alpha P_1)}{2(1+\theta)}\left(1+\widetilde{s}\right)^2 + \frac{\overline{y} - x^2 P_2}{(1+\theta)}\left(1-\widetilde{s}\right) - \frac{(\delta + \alpha P_2)}{2(1+\theta)}\left(1-\widetilde{s}\right)^2 \qquad (8)$$

Then

$$\begin{aligned}
\frac{\partial C}{\partial T} &= \frac{\partial \widetilde{s}}{\partial T}\left(\frac{\overline{y}-x^1P_1}{(1+\theta)}\right)-\left(\frac{\partial x^1}{\partial T}q_1+x^1\right)(\widetilde{s}+1)-2\frac{\partial \widetilde{s}}{\partial T}\frac{(\delta+\alpha P_1)}{2(1+\theta)}(1+\widetilde{s})-\frac{\alpha}{2}(1+\widetilde{s})^2 \\
&\quad -\frac{\partial \widetilde{s}}{\partial T}\left(\frac{\overline{y}-x^2P_2}{(1+\theta)}\right)-\left(\frac{\partial x^2}{\partial T}q_2+x^2\right)(1-\widetilde{s})+2\frac{\partial \widetilde{s}}{\partial T}\frac{(\delta+\alpha P_2)}{2(1+\theta)}(1-\widetilde{s})-\frac{\alpha}{2}(1-\widetilde{s})^2 \\
&= \frac{\partial \widetilde{s}}{\partial T}\left(\frac{x^2P_2-x^1P_1}{(1+\theta)}\right)-\left(\frac{\partial x^1}{\partial T}q_1+x^1\right)(\widetilde{s}+1)-\frac{\partial \widetilde{s}}{\partial T}\frac{(\delta+\alpha P_1)}{(1+\theta)}(1+\widetilde{s})-\alpha\left(1+\widetilde{s}^2\right)- \\
&\quad \left(\frac{\partial x^2}{\partial T}q_2+x^2\right)(1-\widetilde{s})+\frac{\partial \widetilde{s}}{\partial T}\frac{(\delta+\alpha P_2)}{(1+\theta)}(1-\widetilde{s})
\end{aligned}$$

$$\begin{aligned}
\frac{\partial C^2}{\partial T^2} &= \frac{\partial^2 \widetilde{s}}{\partial T^2}\left(\frac{x^2P_2-x^1P_1}{(1+\theta)}\right)+\frac{\partial \widetilde{s}}{\partial T}\left(x^2-x^1\right)+\frac{\partial \widetilde{s}}{\partial T}\left(\frac{\frac{\partial x^2}{\partial T}P_2-\frac{\partial x^1}{\partial T}P_1}{(1+\theta)}\right)-\left(2\frac{\partial x^1}{\partial T}+\frac{\partial^2 x^1}{\partial T^2}q_1\right)(\widetilde{s}+1)- \\
&\quad \left(\frac{\partial x^1}{\partial T}q_1+x^1\right)\frac{\partial \widetilde{s}}{\partial T}-\frac{\partial^2 \widetilde{s}}{\partial T^2}\frac{(\delta+\alpha P_1)}{(1+\theta)}(1+\widetilde{s})-\left(\frac{\partial \widetilde{s}}{\partial T}\right)^2\frac{(\delta+\alpha P_1)}{(1+\theta)}-\frac{\partial \widetilde{s}}{\partial T}\alpha(1+\widetilde{s})-2\alpha\left(\frac{\partial \widetilde{s}}{\partial T}\right)\widetilde{s} \\
&\quad -\left(2\frac{\partial x^2}{\partial T}+\frac{\partial^2 x^2}{\partial T^2}q_2\right)(1-\widetilde{s})+\left(\frac{\partial x^2}{\partial T}q_2+x^2\right)\frac{\partial \widetilde{s}}{\partial T}+\frac{\partial^2 \widetilde{s}}{\partial T^2}\frac{(\delta+\alpha P_2)}{(1+\theta)}(1-\widetilde{s})+\frac{\partial \widetilde{s}}{\partial T}\alpha(1-\widetilde{s}) \\
&\quad -\left(\frac{\partial \widetilde{s}}{\partial T}\right)^2\frac{(\delta+\alpha P_2)}{(1+\theta)}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2 C}{\partial T\partial t_1} &= \frac{\partial^2 \widetilde{s}}{\partial T\partial t_1}\left(\frac{x^2P_2-x^1P_1}{(1+\theta)}\right)+\frac{\partial \widetilde{s}}{\partial T}\left(-\frac{\partial x^1}{\partial t_1}q_1-x^1\right)-\left(\frac{\partial x^1}{\partial T}q_1+x^1\right)\frac{\partial \widetilde{s}}{\partial t_1}-\left(\frac{\partial x^1}{\partial T\partial t_1}q_1+\frac{\partial x^1}{\partial T}+\frac{\partial x^1}{\partial t_1}\right)(\\
&\quad -\alpha\frac{\partial \widetilde{s}}{\partial T}(1+\widetilde{s})-2\alpha\widetilde{s}\frac{\partial \widetilde{s}}{\partial t_1}+\frac{\partial \widetilde{s}}{\partial t_1}\left(\frac{\partial x^2}{\partial T}q_2+x^2\right)+\frac{\partial^2 \widetilde{s}}{\partial T\partial t_1}\frac{(\delta+\alpha P_2)}{(1+\theta)}(1-\widetilde{s})-\frac{\partial \widetilde{s}}{\partial t_1}\frac{\partial \widetilde{s}}{\partial T}\frac{(\delta+\alpha P_2)}{(1+\theta)}
\end{aligned}$$

At the symmetric equilibrium we have $p_1 = p_2$ so that $t_1 = t_2$ and $\widetilde{s} = 0$, $\frac{\partial \widetilde{s}}{\partial T} = 0$,

$$\frac{\partial C}{\partial T} = -2\left(x'q + x\right) - \alpha$$

$$\begin{aligned}
\frac{\partial^2 C}{\partial T\partial t_1} &= -\left(\frac{\partial x^1}{\partial T\partial t_1}q_1+\frac{\partial x^1}{\partial T}+\frac{\partial x^1}{\partial t_1}\right) \\
&= -\left(x^{''}q+2x'\right)
\end{aligned}$$

and

$$\frac{\partial C^2}{\partial T^2} = -\left(2\frac{\partial x^1}{\partial T}+\frac{\partial^2 x^1}{\partial T^2}q_1\right)-\left(2\frac{\partial x^2}{\partial T}+\frac{\partial^2 x^2}{\partial T^2}q_2\right) = -2\left(2x'+qx^{''}\right)$$

Combining all the effects in $\Theta^T_{t_k}$, we obtain at the symmetric equilibrium

$$\begin{aligned}
\Theta^T_{t_k} &= -\theta\left(x^{''}q+2x'\right)+(1+\theta)\,x'+\theta\left(x^{''}q\right)+Tx^{''}+\theta x' \\
&= x'+Tx^{''}
\end{aligned}$$

2/ VAT θ

$$\frac{\partial R}{\partial \theta} = C + \theta \frac{\partial C}{\partial \theta} + \sum_{i=1}^{2} q_i s_i x^i + \theta \sum_{i=1}^{2} q_i x^i \frac{\partial s_i}{\partial \theta} + T \sum_{i=1}^{2} x^i \frac{\partial s_i}{\partial \theta} \Longleftrightarrow \Theta^{\theta}\left(\theta, T; t_1, t_2\right) = 0$$

The existence of an equilibrium requires

$$\Theta_{\theta}^{\theta} = \left[2\frac{\partial C}{\partial \theta} + \theta \frac{\partial^2 C}{\partial \theta^2} + 2\sum_{i=1}^{2} q_i \frac{\partial s_i}{\partial \theta} x_i + \theta \sum_{i=1}^{2} q_i x_i \frac{\partial^2 s_i}{\partial \theta^2} + T \sum_{i=1}^{2} x_i \frac{\partial^2 s_i}{\partial \theta^2} \right] \leq 0$$

which gives at the symmetric equilibrium

$$\Theta_{\theta}^{\theta} = 2\left[\left(\frac{2\overline{y} - \delta}{(1+\theta)^2} \right) (2\theta - 1) \right] \leq 0 \text{ implies that } \theta < \frac{1}{2} \text{ revoir}$$

We have

$$\frac{\partial \theta}{\partial t_i} = -\frac{\Theta_{t_i}^{\theta}}{\Theta_{\theta}^{\theta}}$$

Then the sign of $\Theta_{t_i}^{\theta}$ gives the sign of $\frac{\partial \theta}{\partial t_i}$

$$\Theta_{t_i}^{\theta} = \frac{\partial C}{\partial t_1} + \theta \frac{\partial^2 C}{\partial \theta \partial t_1} + \sum_{i=1}^{2} \left(\frac{\partial \left(q_i x_i s_i\right)}{\partial t_1} + \theta \frac{\partial \left(q_i x_i\right)}{\partial t_1} \frac{\partial s_i}{\partial \theta} + \theta q_i x^i \frac{\partial^2 s_i}{\partial \theta \partial t_1} + T \frac{\partial^2 s_i}{\partial \theta \partial t_1} x_i + T \frac{\partial s_i}{\partial \theta} \frac{\partial x^i}{\partial t_1} \right)$$

$$\begin{aligned}
\frac{\partial \left(q_1 s_1 x^1\right)}{\partial t_1} &= q_1 \frac{\partial \widetilde{s}}{\partial t_1} x_1 + q_1 \frac{\partial x_1}{\partial t_1} \left(\widetilde{s} + 1\right) + x_1 \left(1 + \widetilde{s}\right) \\
\frac{\partial \left(q_2 s_2 x^2\right)}{\partial t_1} &= -q_2 \frac{\partial \widetilde{s}}{\partial t_1} x_2 \\
\frac{\partial \left(q_1 x^1\right)}{\partial t_1} &= x_1 + q_1 \frac{\partial x_1}{\partial t_1} \\
\frac{\partial \left(q_2 x^2\right)}{\partial t_1} &= 0
\end{aligned}$$

From (8) we obtain

$$\begin{aligned}
\frac{\partial C}{\partial \theta} &= \frac{\partial \widetilde{s}}{\partial \theta} \left(\frac{\overline{y} - x^1 P_1}{(1+\theta)} \right) - \left(\frac{\overline{y}}{(1+\theta)^2} \right) \left(\widetilde{s} + 1\right) - 2\frac{\partial \widetilde{s}}{\partial \theta} \frac{(\delta + \alpha P_1)}{2(1+\theta)} \left(1 + \widetilde{s}\right) + \frac{\delta}{2\left(1+\theta\right)^2} \left(1 + \widetilde{s}\right)^2 - \\
&\quad \frac{\partial \widetilde{s}}{\partial \theta} \left(\frac{\overline{y} - x^2 P_2}{(1+\theta)} \right) - \left(\frac{\overline{y}}{(1+\theta)^2} \right) \left(1 - \widetilde{s}\right) + 2\frac{\partial \widetilde{s}}{\partial \theta} \frac{(\delta + \alpha P_2)}{2(1+\theta)} \left(1 - \widetilde{s}\right) + \frac{\delta}{2\left(1+\theta\right)^2} \left(1 - \widetilde{s}\right)^2 \\
&= \frac{\partial \widetilde{s}}{\partial \theta} \left(\frac{x^2 P_2 - x^1 P_1}{(1+\theta)} \right) - 2\left(\frac{\overline{y}}{(1+\theta)^2} \right) - \frac{\partial \widetilde{s}}{\partial \theta} \frac{(\delta + \alpha P_1)}{(1+\theta)} \left(1 + \widetilde{s}\right) + \frac{\delta}{(1+\theta)^2} \left(1 + \widetilde{s}^2\right) + \\
&\quad \frac{\partial \widetilde{s}}{\partial \theta} \frac{(\delta + \alpha P_2)}{(1+\theta)} \left(1 - \widetilde{s}\right)
\end{aligned}$$

and

$$\frac{\partial^2 C}{\partial\theta\partial t_1} = \frac{\partial^2 \widetilde{s}}{\partial\theta\partial t_1}\left(\frac{x^2 P_2 - x^1 P_1}{(1+\theta)}\right) + \frac{\partial \widetilde{s}}{\partial\theta}\left(-\frac{\partial x^1}{\partial t_1}q_1 - x^1\right) - \frac{\partial^2 \widetilde{s}}{\partial\theta\partial t_1}\frac{(\delta+\alpha P_1)}{(1+\theta)}(1+\widetilde{s}) - \frac{\partial \widetilde{s}}{\partial\theta}\frac{\partial \widetilde{s}}{\partial t_1}\frac{(\delta+\alpha P_1)}{(1+\theta)}$$
$$-\alpha\frac{\partial \widetilde{s}}{\partial\theta}(1+\widetilde{s}) + 2\widetilde{s}\frac{\delta}{(1+\theta)^2}\frac{\partial \widetilde{s}}{\partial t_1} + \frac{\partial^2 \widetilde{s}}{\partial\theta\partial t_1}\frac{(\delta+\alpha P_2)}{(1+\theta)}(1-\widetilde{s}) - \frac{\partial \widetilde{s}}{\partial t_1}\frac{\partial \widetilde{s}}{\partial\theta}\frac{(\delta+\alpha P_2)}{(1+\theta)}$$

Finally

$$\frac{\partial C}{\partial t_1} = \frac{\partial \widetilde{s}}{\partial t_1}\left(\frac{\overline{y} - x^1 P_1}{(1+\theta)}\right) - \left(\frac{\partial x^1}{\partial t_1}q_1 + x^1\right)(\widetilde{s}+1) - 2\frac{\partial \widetilde{s}}{\partial t_1}\frac{(\delta+\alpha P_1)}{2(1+\theta)}(1+\widetilde{s}) - \frac{\alpha}{2}(1+\widetilde{s})^2$$
$$-\frac{\partial \widetilde{s}}{\partial t_1}\left(\frac{\overline{y} - x^2 P_2}{(1+\theta)}\right) + 2\frac{\partial \widetilde{s}}{\partial t_1}\frac{(\delta+\alpha P_2)}{2(1+\theta)}(1-\widetilde{s})$$

At the symmetric equilibrium we have $p_1 = p_2$ so that $t_1 = t_2$ and $\widetilde{s} = 0$, $\frac{\partial \widetilde{s}}{\partial T} = 0$,
we have

$$\frac{\partial C}{\partial\theta} = \frac{\delta - 2\overline{y}}{(1+\theta)^2}$$

and

$$\frac{\partial^2 C}{\partial\theta\partial t_1} = 0$$

and

$$\frac{\partial C}{\partial t_1} = -\left(\frac{\partial x^1}{\partial t_1}q_1 + x^1\right) - \frac{\alpha}{2} \tag{9}$$

Combining all the effects in $\Theta_{t_k}^T$, we obtain at the symmetric equilibrium

$$\begin{aligned}\Theta_{t_i}^{\theta} &= \frac{\partial C}{\partial t_1} + \theta\frac{\partial^2 C}{\partial\theta\partial t_1} + \sum_{i=1}^{2}\left(\frac{\partial(q_i x_i s_i)}{\partial t_1} + \theta\frac{\partial(q_i x_i)}{\partial t_1}\frac{\partial s_i}{\partial\theta} + \theta q_i x^i\frac{\partial^2 s_i}{\partial\theta\partial t_1} + T\frac{\partial^2 s_i}{\partial\theta\partial t_1}x_i + T\frac{\partial s_i}{\partial\theta}\frac{\partial x^i}{\partial t_1}\right)\\ &= -\left(\frac{\partial x^1}{\partial t_1}q_1 + x^1\right) - \frac{\alpha}{2} + q_1\frac{\partial x_1}{\partial t_1} + x_1\\ &= -\frac{\alpha}{2}\end{aligned}$$

3/ Federal tax interactions

$$\frac{\partial R}{\partial\theta} = C + \theta\frac{\partial C}{\partial\theta} + \sum_{i=1}^{2}q_i s_i x^i + \theta\sum_{i=1}^{2}q_i x^i\frac{\partial s_i}{\partial\theta} + T\sum_{i=1}^{2}x^i\frac{\partial s_i}{\partial\theta} \Longleftrightarrow \Theta^{\theta}(\theta, T; t_1, t_2) = 0$$

We have

$$\frac{\partial\theta}{\partial T} = -\frac{\Theta_T^{\theta}}{\Theta_{\theta}^{\theta}}$$

Then the sign of $\Theta_{t_i}^{\theta}$ gives the sign of $\frac{\partial\theta}{\partial T}$

$$\Theta_T^{\theta}=\frac{\partial C}{\partial T}+\theta\frac{\partial^2 C}{\partial\theta\partial T}+\sum_{i=1}^{2}\left(\frac{\partial\left(q_i x_i s_i\right)}{\partial T}+\theta\frac{\partial\left(q_i x_i\right)}{\partial T}\frac{\partial s_i}{\partial\theta}+\theta q_i x^i\frac{\partial^2 s_i}{\partial\theta\partial T}+T\frac{\partial^2 s_i}{\partial\theta\partial T}x_i+T\frac{\partial s_i}{\partial\theta}\frac{\partial x^i}{\partial T}\right)$$

$$\begin{aligned}\frac{\partial C}{\partial T} &= \frac{\partial\widetilde{s}}{\partial T}\left(\frac{x^2P_2-x^1P_1}{(1+\theta)}\right)-\left(\frac{\partial x^1}{\partial T}q_1+x^1\right)(\widetilde{s}+1)-\frac{\partial\widetilde{s}}{\partial T}\frac{(\delta+\alpha P_1)}{(1+\theta)}(1+\widetilde{s})-\alpha\left(1+\widetilde{s}^2\right)\\ &\quad-\left(\frac{\partial x^2}{\partial T}q_2+x^2\right)(1-\widetilde{s})+\frac{\partial\widetilde{s}}{\partial T}\frac{(\delta+\alpha P_2)}{(1+\theta)}(1-\widetilde{s})\end{aligned}$$

$$\frac{\partial C}{\partial\theta}=\frac{\partial\widetilde{s}}{\partial\theta}\left(\frac{x^2P_2-x^1P_1}{(1+\theta)}\right)-2\left(\frac{\overline{y}}{(1+\theta)^2}\right)-\frac{\partial\widetilde{s}}{\partial\theta}\frac{(\delta+\alpha P_1)}{(1+\theta)}(1+\widetilde{s})+\frac{\delta}{(1+\theta)^2}\left(1+\widetilde{s}^2\right)+\frac{\partial\widetilde{s}}{\partial\theta}\frac{(\delta+\alpha P_2)}{(1+\theta)}(1-\widetilde{s})$$

$$\begin{aligned}\frac{\partial^2 C/N}{\partial\theta\partial T} &= \frac{\partial^2\widetilde{s}}{\partial\theta\partial T}\left(\frac{x^2P_2-x^1P_1}{(1+\theta)}\right)+\frac{\partial\widetilde{s}}{\partial\theta}\left(-\frac{\partial x^1}{\partial T}q_1-x^1+\frac{\partial x^2}{\partial T}q_2+x^2\right)-\frac{\partial^2\widetilde{s}}{\partial\theta\partial T}\frac{(\delta+\alpha P_1)}{(1+\theta)}(1+\widetilde{s})\\ &\quad-\frac{\partial\widetilde{s}}{\partial\theta}\frac{\partial\widetilde{s}}{\partial T}\frac{(\delta+\alpha P_1)}{(1+\theta)}-\alpha\frac{\partial\widetilde{s}}{\partial\theta}(1+\widetilde{s})+2\widetilde{s}\frac{\delta}{(1+\theta)^2}\frac{\partial\widetilde{s}}{\partial T}+\frac{\partial^2\widetilde{s}}{\partial\theta\partial T}\frac{(\delta+\alpha P_2)}{(1+\theta)}(1-\widetilde{s})-\\ &\quad\frac{\partial\widetilde{s}}{\partial T}\frac{\partial\widetilde{s}}{\partial\theta}\frac{(\delta+\alpha P_2)}{(1+\theta)}+\alpha\frac{\partial\widetilde{s}}{\partial\theta}(1-\widetilde{s})\end{aligned}$$

At the symmetric equilibrium we have

$$\frac{\partial C}{\partial T}=-2\left(x'q+x\right)-\alpha$$

$$\frac{\partial^2 C}{\partial\theta\partial T}=0$$

$$\begin{aligned}\Theta_T^{\theta} &= \frac{\partial C}{\partial T}+\theta\frac{\partial^2 C}{\partial\theta\partial T}+\sum_{i=1}^{2}\left(\frac{\partial\left(q_i x_i s_i\right)}{\partial T}+\theta\frac{\partial\left(q_i x_i\right)}{\partial T}\frac{\partial s_i}{\partial\theta}+\theta q_i x^i\frac{\partial^2 s_i}{\partial\theta\partial T}+T\frac{\partial^2 s_i}{\partial\theta\partial T}x_i+T\frac{\partial s_i}{\partial\theta}\frac{\partial x^i}{\partial T}\right)\\ &= -2\left(x'q+x\right)-\alpha\end{aligned}$$

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