Environmental regulation in economy with price signalling

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Abstract

The article studies the impact of a price signal of environmental quality on the optimal policy choice of environmental regulation. A monopoly uses price to signal that the production process doesn’t generate pollution. In order to prevent the conventional type to post the same price to mislead consumers, the clean type distorts the price relative to the level of complete information. The analysis distinguishes two cases for which the provision of high environmental quality requires extra marginal cost or extra fixed investment. The regulator sets a tax on the pollution. At a certain tax level, the cost of conventional product exceeds that of the green one, the price of the clean variety is distorted downwards. The regulator must consider price distortions due to signalling behavior when he chooses to maximize social welfare. Environmental externality, imperfect competition and information asymmetry suggest that in most cases the optimal regulation should be subsidy to polluting variety.

Keywords: Signaling, Environmental Regulation, Information and Environmental Product Quality

JEL Code: D82, L15, Q5
1 Motivation

It has been shown in the literature (Sengupta (2012)) that in a market with asymmetric information between the producer and consumers regarding product’s environmental quality, pollution permits can play a decisive role in overcoming the distortion of information asymmetry. It is argued that a strong environmental regulation can reverse the direction of price signal. A ‘strong’ tax on a polluting variety makes production of the clean variety less expensive than that of the dirty variety which permits the clean type to signal high environmental quality by downward price distortion.

The informational issue does not arise since a polluting firm has an incentive to purchase pollution permits to limit liability in case of an environmental catastrophe. The analysis regards the permit price as an exogenous parameter. In the present study I want to formalize the ideas from the social welfare point of view.

Let us follow the argument and evaluate the impact of pollution permits of the social welfare. Indeed, on the one hand the purchase of pollution permits increases the effective production costs of the dirty variety. The price of which is always set at the full information level. The monopoly supplier of the dirty variety adjusts the monopoly price for the dirty variety by the price of pollution permits thus passing a part of the cost to consumers. The demand decreases and the pollution level/intensity falls. On the other hand, for a clean variety the permits affects the signalling price. Hence, the pollution permits have at least two effects on the overall welfare. First, they control pollution and second, they affect the signalling price, i.e. the cost of information to consumers.

2 Introduction

In this paper we abstract from environmental quality certification for several reasons. First, for a range of goods certified by organic label the true organic quality may be compromised by the following. First, the excessive use of organic fertilizers on agricultural products creates a high nitrate concentration in organic goods degrading their environmental quality to the level of conventional products. Second, the cross pollination between organic and GM crops may confound environmental quality (this possibility has become feasible under current EU regulation, see for example The Ecologist). Finally, as we have learned from the example of flavescence doree, there are circumstances under which a
government may find it necessary to impose the use of a non-organic treatment on organically certified fields without withdrawing organic label. It was necessary to overcome the resistance to treatment in order to stop the epidemic that threatens to extinction wine yards (Berdah (2012)).

**Literature review.** Antelo and Loureiro (2009) analyse the effect of signalling and asymmetric information on environmental taxation. They show in a two-period Cournot oligopoly model where the regulator acts as Stackelberg leader that optimal taxes must be set below marginal damage and below Buchanan level.

They show that information asymmetry generates two additional effects: informational and signalling one. The former deprives the regulator the capacity/ability to distinguish between types, requiring him to expand environmental regulation also on clean types. The second, signalling effect both types of producer to distort output from profit-maximizing level. The attain separation the clean type must suppress and the dirty type must expand their output. Therefore, the pollution coming exclusively from dirty types will inevitable grow with asymmetric information. The environmental tax must target not only the trade-off between output and pollution but also signalling distortion. They demonstrate that whenever the regulator has a weak preference for pollution, the tax under asymmetric information is lower than in a symmetric case. In the intermediate range of pollution preferences, the low probability of dirty firm generates the same outcome. However, the overall result in the context of severity of environmental regulation, the discrepancy between the tax and the marginal damage from pollution is greater under information asymmetry than in a symmetric case (Proposition 3). This is a similar finding that the optimal tax must be further softened due to the distortion from information symmetry.

### 3 The model

**Quality.** Assume that a monopoly produces a good of environmental quality $e$ which is unobservable to consumers before and after purchase as for credence goods. Environmental quality can be high, $e$, or low, $e$, and has characteristics of/embraces/contains information on the mode of production in terms of environmental foot print such as the use of clean electricity or organic versus chemical fertilizers or recycled/recyclable materials. The value of $e$ describes
thus the production technology/abatement intensity. We assume that an environmental label is not informative as discussed in introduction. We assume further that the environmental damage is proportional to emissions and is summarized:

\[ d(q, e) = \delta (\bar{e} - e) q \]  

(1)

Environmental damage per unit of output is captured by \( \delta \) which is an objective/scientific measure of harm to the natural environmental, health, living quality, air or water quality etc. The quantity of purchased units is denoted with \( q \). The higher the value of environmental quality \( e \), the less is damage per unit of output. Thus, high environmental quality is free from environmental damage, while the conventional environmental quality (dirty type of production) ejects polluting emissions. Apriori, the only information consumers have about environmental quality is its possible realization/distribution:

\[ e = \{ \underline{e}; \bar{e} \} \]  

(2)

with \( \underline{e} < \bar{e} \) or \( 0 = \underline{e} < \bar{e} = 1 \).

Consumers. Any type of product provides positive utility to consumers but the one with higher environmental quality generates a higher utility. Consumers have green preferences as they experience disutility \( \gamma \) from low/insufficient/conventional environmental quality. As environmental quality has characteristics of semi-public good, beyond its cleanliness for the environment, it also has features of a private good in the sense that its qualities can be appropriated by consumers (low content of nitrates in vegetables, low content of sulfites in wines) which has a direct positive effect on health (no headache the following day). Consumers appreciate the virtue of environmental quality and have a greater net willingness to pay for it.

The consumer’s utility function is quasi-linear in all other goods and is given by:

\[ U(p, q, \bar{e}) = (\alpha - \gamma (\bar{e} - \hat{e})) q - \frac{1}{2} q^2 + I - pq \]  

(3)

where \( \alpha \) stands for the gross willingness to pay for one unit of product, \( \gamma \) measures the deficit in environmental quality, \( \hat{e} \) is perceived environmental quality which is determined as part of perfect Bayesian equilibrium where firm’s strategy is its price, \( I \) is income and \( p \) is the price. Consumers may purchase multiple units of the good but sampling doesn’t guarantee the satisfaction of future consumption as with restaurant visits.
The consumers’ demand for the good of perceived quality $\hat{e}$ is given by:

$$D(p, \hat{e}) = \alpha - \gamma (\bar{e} - \hat{e}) - p$$

(4)

Note that price elasticity $\eta (p, \hat{e}) \equiv \frac{\partial D}{\partial p} \bigg|_{p=\hat{e}}$ is a decreasing function of perceived environmental quality. An increase in $\hat{e}$ makes consumers more captive from the monopolists point of view (?). As consumers have an incomplete knowledge about firm’s environmental quality, they must form their beliefs about environmental performance on the basis of observable characteristics. Observing the price, consumers update their beliefs and infer environmental quality of the product. Let $\mu (p) : \mathbb{R}^+ \rightarrow [0, 1]$ denote consumers’ posterior beliefs that the good is of high environmental quality when the price is $p$. Then, the perceived quality $\hat{e}$ is the expected probability that the good is clean, that is $\hat{e} \equiv e (\mu) = \mu \bar{e} + (1 - \mu) \underline{e}$. Rearranging (4) gives the demand function under asymmetric information:

$$D(p, \mu) = \alpha - \gamma (\bar{e} - e (\mu)) - p$$

(5)

Note that $\frac{\partial D(p, \mu)}{\partial \mu} = \gamma e' (\mu) = \gamma (\bar{e} - \underline{e})$ is increasing in $\mu$, i.e. optimistic expectations of environmental quality raises demand. To exclude trivial solutions, the following parameter restriction is necessary:

$$\alpha - \gamma > c (\bar{e})$$

(A.1)

Assumption (A.1) implies that any firm type faces positive demand despite possibly biased perception of environmental quality/ regardless its correct identification by consumers. It guarantees that even when the true type is not correctly identified, the firm generates positive profit. Note that $\gamma > e' (\bar{e})$ implies that the net social value of the clean good is greater then the dirty one, and vise versa.

The firm. In the market there is a single firm of environmental type $e$ which value is determined by nature\(^1\). The firm produces the good at a marginal cost $c (e)$ and has a fixed investment cost $F$. In what follows we will separately analyze two cases\(^2\). First, marginal costs between high and low environmental qualities differ with no technology investment. As it is conventional in the literature (Kurtyka and Mahenc (2011)), high environmental quality is more

\(^1\)We abstract from producer’s technology choice.

\(^2\)André, González, and Porteiro (2009) analyse a similar profit structure to investigate...
costly than the low one. For simplicity assume later that \( c \equiv c(e) > c(e) = 0 \).
Without loss of generality, the dirty type manufactures at a zero marginal cost and hence, \( c \) measures the difference in marginal cost between clean and dirty production technologies. The marginal cost is increasing in \( e \). The setting describes a production process that requires more manpower to provide high environmental quality. In agriculture, for example, one can substitute application of herbicide (dirty type) by repetitive mechanical weeding which preserves soil quality (clean type).

Second, marginal production costs are equal and set to zero but the clean technology requires an initial capital investment. In the setting the fixed cost \( F \) can represent an installation of clean-up filters or such.

The gross profit for the firm depends on its true price-cost margin and on the perceived environmental quality which defines the demand for the good:

\[
\pi(p, e, \mu) = (p - c(e))(\alpha - \gamma(e - \mu)) - eF \tag{6}
\]

Substituting \( \hat{\mu} \) with \( e(\mu) \) and using (5) in (6) we obtain the functional form of profit under asymmetric information:

\[
\pi(p, e, \mu) = (p - c(e))(\alpha - \gamma(e - e(\mu))) - eF \tag{7}
\]

The profit function is strictly concave in \( p \), it attains its maximum at profit-maximizing price \( p^*(e) = \frac{\alpha - \gamma(e - \mu) + c(e)}{2} \). The maximized profit is \( \frac{(e - e(\mu) - c(e))^2}{4} \).
This provides a benchmark for full information outcome.

**Lemma 1** Given full information about environmental quality the producer charges the price

\[
p^*(e) = \frac{\alpha - \gamma(e - \mu) + c(e)}{2} \tag{8}
\]

that satisfies \( \frac{p^*(e) - c(e)}{p^*(e)} = \frac{1}{\eta(p, e)} \)

This is a standard result that a monopoly equalizes the Lerner index to the inverse price elasticity of demand. As it can be verified, \( \eta(p^*(e)) \) is increasing in \( e \) when evaluated at profit-maximizing price implying that a higher environmental quality has a higher price elasticity than the conventional quality.

Denote \( \mu_0 \) consumers’ prior belief of high environmental quality. This parameter describes all the public information available to consumers regarding environmental quality. When \( e(\mu_0) > e \) implies that quality expectations are overestimated.
Since every producer type prefers high expectations of environmental quality regardless of true performance, the firm of type \( e \) can affect its profit when revealing its type. The producer’s objective is to maximize profit with respect to \( p \) given consumers beliefs about environmental quality. The firm’s strategy must form perfect Bayesian equilibrium wherein the price choice will inform consumers’ perception of environmental quality. Denote \( g(\tau) \) and \( g(\varepsilon) \) the equilibrium prices for clean and dirty producers respectively. Then, \((g(\tau), g(\varepsilon), \mu(p))\) is the equilibrium strategy given the conditions:

1. For \( e = \tau, \varepsilon \quad g(e) = \arg \max \pi(p, e, \mu) \).
2. If \( g(\tau) \neq g(\varepsilon) \), then \( \mu(g(\varepsilon)) = 0 \) and \( \mu(g(\tau)) = 1 \). If \( g(\tau) = g(\varepsilon) \), then \( \mu(g(\varepsilon)) = \mu_0 \).

The first condition states that for each type the price strategy must be profit-maximizing given consumers’ beliefs. The second condition imposes the Bayes rule for belief updating. When the price is informative, then consumers correctly identify producers’ type, when the price is uninformative, the posterior and prior beliefs coincide.

### 3.1 Emission permit market

Suppose that there is an environmental agency that is concerned with polluting emissions. As the agency is a public authority its main concern is however social welfare. The regulator is aware of the distortions in the economy which are pollution externality, monopoly price and information asymmetry. As proposed by Sengupta (2012) the regulator arranges a market of emission permits. From the point of view of dirty producer, the purchase of emission permits enables him to limit liability for the environmental damage from polluting emissions and at the same time it increases the effective marginal cost of the output of the dirty type. If the production process causes significant environmental that turns into significant damage in the future/potentially, emission permits limit liability to pay a penalty or damage compensation by a court of law. A polluting firm has an incentive to voluntarily purchase emission permits. Therefore, information asymmetry doesn’t arise between the regulator and the monopolist. From the point of view of the regulator, the trade of emission permits allows to introduce pollution control simultaneously correcting other distortionary effects in the economy. The sole role of the regulator is to choose an permit price \( t \) that maximizes social welfare given the probability of the
clear and the dirty type. Besides we assume that the funds collected through the regulation are transferred back to the economy.

Clearly, permit price must be non-negative requiring that the domain of interest is

\[ t > 0 \quad (A.2) \]

In the economy where two types may be present, there is an environmental authority that issues

Let \( \delta \) denote the environmental damage from emission per unit of output, then assumption (A.3) ensures that regulation is desirable.

\[ \gamma < \delta \quad (A.3) \]

As consumers have green preferences, they internalize some environmental damage through the utility loss and a weaker demand. However, when consumers underestimate environmental damage from emission, the regulator arranges the market of emission permits. When consumers correctly estimate environmental damage, i.e. \( \gamma = \delta \), the pollution externality is internalized and any environmental regulation is superfluous.

Throughout the article we maintain the assumption that

\[ \alpha - \gamma > t \quad (A.4) \]

Assumption (A.4) ensures that the social value of the dirty variety is greater than emission permit price.

Under environmental regulation the producer of type \( e \) earns profit:

\[ \pi (p, e, \mu, t) = (p - c(e) - t (1 - e)) (\alpha - \gamma (\bar{e} - \mu(e)) - p) - eF \quad (9) \]

The timing of the game is as follows: at stage 1 the regulator commits to a policy by setting a tax. At stage 2 the firm learns the realization of his type and chooses the price given the tax. At stage 3 consumers make their purchasing decision.

4 Asymmetric information

Now let us consider producer strategies under asymmetric information. The producer must take into account how the choice of price influences consumers’
Separating equilibrium. Following Mailath (1987), let us examine the necessary and sufficient conditions for an existence of the separating equilibrium.

1. \( \pi_\mu > 0 \) (belief monotonicity)
2. \( \pi_{ep} = c_e' - t > (\leq) 0 \) (type monotonicity)
3. \( S = -\frac{\pi_{e,p}(p,e,\mu)}{\pi_{p}(p,e,\mu)} \) is strictly monotonic function of \( e \) (single crossing)

Condition 1 states that profits are increasing in the perceived environmental quality. Thus, the worst perception of a producer the consumers may draw is \( \mu = 0 \) regardless of an actual environmental performance. Condition 2 describes how an increase in environmental quality affects the profitability of changing price. Recall that \( c'(e) = c \). Then, when weak regulation is at place, i.e. \( c > t \), an increase in environmental quality rises the profitability of price variation and vise versa for strong regulation with \( c < t \) as the clean type is effectively more efficient in providing high environmental quality. Condition 3 states the single crossing property which measures the marginal rate of substitution between price and perceived quality. The derivation of condition 3 with respect to environmental quality gives:

\[
\frac{-\partial S}{\partial e} = \frac{(c'_e - t) \gamma (\alpha - \gamma (1 - \mu) - p)}{(\alpha - \gamma (1 - \mu) - 2p + c(e) + t (1 - e))^2} \tag{10}
\]

The expression in (10) is positive when \( c > t \). When environmental quality rises the producer is willing to charge a higher price for the increase in the perceived environmental quality. On the contrary, when \( c < t \), the producer is willing to charge a lower price for the same increase. Note that the single crossing property holds only when \( c'(e) - t \neq 0 \) or \( D_{\mu} (p, \mu) \neq 0 \). Indeed, the difference in the effective production marginal costs among producers characterizes price margin earned per unit of output, this defines the direction of signalling price distortion. When effective production marginal costs are higher for clean variety, an upward price distortion is less painful/costly for clean than for dirty type as the net loss is smaller. When the effective marginal costs are reversed and the clean variety is effectively less expensive to produce, the clean type has an interest to distort the price downwards hindering mimic behavior of a dirty type. The price deviations from the profit-maximizing level is partly compensated by
the demand sensibility to environmental quality as the consumers are willing to pay the premium for higher environmental quality and the demand is greater per assumption when the type is correctly identified.

Lemma 2 Under conditions 1-3, separating equilibrium prices are such that 
\[ g(e) = p^*(e) \] and (i) \( g(\bar{e}) > p^*(\bar{e}) \) when \( c(\bar{e}) > \max\{c(e), t\} \) or (ii) \( g(\bar{e}) < p^*(\bar{e}) \) when \( c(\bar{e}) < \max\{c(e), t\} \).

Producers can signal to consumers superior environmental quality through price distortion. The direction of the signal is related to the difference in effective production marginal costs. Moreover, price signal has been shown to be socially costly. A clean type looses a lower profit margin per consumer from an increase or decrease in price than does the dirty type.

Lemma 3 Ceteris paribus, upward signal has a greater social cost than downward signal (???).

An upward signal is welfare decreasing as the price distortion reduces both producer and consumers’ surpluses creating a pure deadweight loss from the reduced amount of trade. However, a downwards price signal increases consumers’ surplus as they benefit from a lower (relatively to the full information case) equilibrium price\(^3\). Although producer surplus decreases with downward signal contrary to the full information case, the net effect of a downward signal has a less drastic welfare-reducing effect than does the upward signal.

Separating prices. In a separating equilibrium the producer’s type is correctly identified by consumers. Given the revealed information and environmental regulation, the highest profit the dirty type can earn is at the full information price, \( p^*(e, t) = \frac{\alpha-c(e)+t}{2} \). Then, his separating price strategy is \( g(e) = p^*(e, t) \) which secures him the maximized full information profit \( \pi(g(e), e, 0) = \frac{\alpha-c(e)-t}{2} \). However, when the dirty type is thought to be clean upon choosing a price \( p \), he faces the demand for the clean variety, \( \alpha-p \), which generates him a mimicking profit \( \pi(p, e, 1) = (p-c(e)-t) (\alpha-p) \).

When the clean type set such a price that doesn’t inform the consumers leading to take him for the dirty one, the clean type makes a profit \( \pi(p, \bar{e}, 0) = (p-c(\bar{e})) (\alpha-\gamma-p) \). The maximum is achieved at price \( p = \frac{\alpha-c(\bar{e})}{4} \) yielding \( \frac{\alpha-\gamma-c(\bar{e})}{4} \).

\(^3\)Signalling price never attains the socially optimal level of output which equilizes price to marginal costs because the monopolist through signal reveals his true type which allows exercising monopoly power.
To attain separation, the clean producer type must set the price \( g(\pi) \) that verifies the following conditions:

- \[
\pi (g(\pi), \bar{\pi}, 1) > \max_p \pi (p, \bar{\pi}, 0) \quad \text{(IR)}
\]
- \[
\max_p \pi (p, \bar{\pi}, 0) \geq \pi (g(\pi), \bar{\pi}, 1) \quad \text{(IC)}
\]

Condition (IR) requires that the clean type’s profit at separation is greater than any other profit he may achieve when he is not identified as clean type. Condition (IC) states that the dirty type obtains a higher profit by truth-telling and revealing his type than by simulating/mimicking the clean type. Denote \( P_{IR} \) and \( P_{IC} \) the set of prices that verify conditions (IR) and (IC) respectively (see 7 for calculations).

**Proposition 1** There is a set of separating equilibrium prices such that \( g(\pi) = \frac{\alpha - \gamma + c_{\pi} + t}{2} \) and \( g(\bar{\pi}) \in P_{IR} \cap P_{IC} \).

The continuum of separating equilibrium prices that signal high environmental quality contains an infinite number of prices. According to Cho and Kreps (1987), the criterion for equilibrium selection requires that the price distortion is minimal relative to the full information price. It follows that there is a unique price that signals the high environmental quality and belongs to the set of separating equilibrium prices.

**Corollary 2** The unique price that signals high environmental quality is (i) the lower bound \( p^* \in P_{IR} \cap P_{IC} \) when \( c(\pi) > \max \{ c(\pi), t \} \) and (ii) the upper bound \( \bar{p}^* \in P_{IR} \cap P_{IC} \) when \( c(\pi) < \max \{ c(\pi), t \} \). The explicit expression for \( p^* \) is

\[
\frac{1}{2} \left( \alpha + t \pm \sqrt{2\alpha - \gamma - 2t} \right) \quad (11)
\]

Refer to (7.1) for explicit solution of (11). To reveal the true environmental type the clean producer must distort the price from the optimal full information level. In equilibrium consumers observe the price and infer the true environmental type. When the clean type is more costly to produce, i.e. weak environmental regulation, \( t < c(\pi) \), the signalling price is distorted upwards and the consumers must incur the signalling cost. However, when environmental regulation is strong, \( t > c(\pi) \), the effective marginal cost reverse making the
clean type more 'cost' efficient. Then, the clean producer signals his type by a downward price distortion below the full information level. Downward price signal increases consumers’ surplus.

**Pooling equilibrium.** Let us consider if a pooling price can form equilibria. Let $g$ denote an uninformative price from which consumers cannot update prior beliefs, the price that is transmit no information about/ independent of environmental quality. Then, consumers’ posterior beliefs are equal to prior, $\mu_0$. A pooling equilibrium can exist if profit that the monopolist earns posting a pooling price, $\pi (g, e, \mu_0)$, verifies the condition:

$$\pi (g, e, \mu_0) \geq \frac{(\alpha - \gamma - c(e) - t(1-e))^2}{4}, e = \bar{e}, \bar{e}$$

That is, the profit earned under pooling equilibrium is greater than the profit whatever type of producer would generate if he is considered to be of dirty type with certainty.

5 Optimal environmental regulation

A benevolent environmental regulator who is concerned with social welfare and environmental damage must take into account the effects that a purchase of emission permits might have on the economy when selecting an socially optimal permit price. The regulator is unable to offset the monopoly power of the producer, neither he can interfere in the information asymmetry being an inherent feature of the product. However, the environmental regulator is aware that in the economy the information is revealed as a means of price signal. Optimal environmental regulation must thus the social cost of the signal affected by the environmental regulation.

5.1 Case 1: $c = c(\bar{e}) > c(e) = 0$, $F = 0$

Assuming that the society is indifferent to redistributional effects (and abstracting from the shadow price of public funds), the regulator’s uses producer’s and consumers’ surpluses as welfare measures net the damage from pollution. The regulator’s objective function $W(t)$ is

$$\max_t \mu W_c(t) + (1 - \mu) W_d(t)$$

(13)
where \( \mu \) measures the probability of high environmental quality and \( 1 - \mu \) indicates the probability of dirty type. The subscripts \( c \) and \( d \) indicate clean and dirty welfare respectively. The probabilistic welfare is similar to Freixas, Guesnerie, and Tirole (1985), p.178, in the present context, the probability of high environmental quality describes the percentage of clean to dirty markets in the economy as a whole.

If the producer is of clean type, the production process is emission free and in equilibrium the producer sets the separating price \( g(\bar{e}) = p^s(t) \). As shown in (7.1) the equilibrium separating price of clean type closely relates to the profit function of the dirty type, and in particular to the cost structure of the mimicking type. While condition (IR) defines the direction of signalling distortion, condition (IC) defines the value/bound of the signalling price. At \( p^s(t) \) dirty type is exactly indifferent between charging the full information price and earning the full information maximized profit and the mimicking clean type. Hence, the variation of \( t \) affects directly the separating price of clean type.

Social welfare when the clean firm is active is characterized by the signalling distortion with the following functional form:

\[
W_c(t) = \int_0^{q^s} (D(x, 1) - p^s) \, dx + \pi_c(p^s, c, 1) \tag{14}
\]

\[
= \left( \alpha - \frac{1}{2} q^s(t) - c \right) q^s(t) \tag{15}
\]

with \( q^s(t) \) the amount of trade for clean product at the signalling price.

When the dirty firm is active, social welfare is characterized by polluting emissions and pollution control, tax yields and undistorted prices:

\[
W_d(t) = \int_0^{q^f} \left( D(x, 0) - p^f_d \right) \, dx + \pi_d(p^f_d, t, 0) + t q_d(t) - \delta q^f_d(t) \tag{16}
\]

\[
= \left( \alpha - \gamma - \frac{1}{2} q^f_d(t) - \delta \right) q^f_d(t) \tag{17}
\]

with \( q^f_d(t) \) the amount of trade for dirty product at the full information price.

Using (15) and (15) in (13) we obtain:

\[
W(t) = \mu \left( \alpha - \frac{1}{2} q^s(t) - c \right) q^s(t) + (1 - \mu) \left( \alpha - \gamma - \frac{1}{2} q^f_d(t) - \delta \right) q^f_d(t) \tag{18}
\]
The first order condition to (18) requires:

$$\mu q''(t)(\alpha - q^*(t) - c) + (1 - \mu) q''_d(t) \left(\alpha - \gamma - q^*_d(t) - \delta\right) = 0$$

To ensure the global maximum, the second order condition must verify:

$$\frac{\partial^2 W(t)}{\partial t^2} < 0$$

To constraint social welfare to a concave function for all $\mu$, (20) must hold for each firm type: $\frac{\partial^2 W(t)}{\partial t^2} = q''(t) [\alpha - c - q(t)] - q'(t)^2 < 0$. Recall that $q^*_d(t) = \frac{\alpha - \gamma - t}{2}$. It is straightforward that the condition holds for the dirty type. For the clean type, the concavity is given when $q''(t) < 0$. Using (11) in (4) we obtain $q^*(t) = \frac{1}{2} (\alpha - t \mp \sqrt{2\alpha - \gamma - 2t})$. Note the discriminant is positive given Assumption (A.4). An upward signal shrinks the output of the clean variety while a downward price signal expands output beyond the full information level. Denote the subscript on $t$ the severity of environmental regulation with $s$ being a strong and $w$ being a weak environmental regulation. It is easy to check that $q''(t_s) < 0$; and $q''(t_w) < 0$ given Assumption (A.4).

Substituting $q^*(t) = \alpha - p^*(t)$ and $q^*_d(t) = \frac{\alpha - \gamma - t}{2}$ and rewriting, we obtain an implicit expression for the optimal tax:

$$t^*_e = \delta - (\alpha - \gamma - \delta) - 2 \frac{\mu}{1 - \mu} \frac{q''(t)}{q''_d(t)} (p^*(t) - c)$$

The optimal price of emission permits is based on three elements. First, the level of $t$ is defined by environmental damage $\delta$ from emissions, it is the conventional Pigou principal equalizing unit emission price to marginal external diseconomy. Second, the next term is negative, it reduces the Pigouvian level of tax to take into account the monopoly price choice. Let us denote the first two terms in (21) Buchanan effect, $t^B$, after the author who first introduced the issue in Buchanan (1969). The last term is in the centre of interest of the present paper. This term measures the social cost of information asymmetry and of price signal in particular. In the following we refer to this term the 'signalling price effect'.

First note that the signalling price effect is negative, it mitigates the severity of environmental regulation. It measures the distortion than the price signal creates, from the point of view of social welfare, when clean product’s price deviate from the marginal cost. Note that this term is greater for an upward
price signal since \( p^s(t_w) > p^s(\pi) > c \). For the downward price signal the order is reversed: \( p^s(\pi) > p^s(t_s) > c \); this implies that under strong regulation the signalling price effect would have a smaller impact on the optimal level of permit price than would be under weak regulation.

Also \( \mu \) affects the importance of signalling price effect. The term \( \frac{\mu}{1-\mu} \) measures the prevalence of clean to dirty markets in the economy, it is a weight that the regulator places on the price signalling effect. For \( \mu \to 0 \), the optimal permit price approaches the Buchanan level; for high \( \mu \), to the contrary, the price signalling effect is significant. The fraction \( \frac{\mu}{1-\mu} \) makes the difference/orrelates optimal permit price to Buchanan level of regulation, hence \( t_c^* \in [0, t^{B}] \).

The expression (21) is not explicit, however it can be shown that

**Lemma 4** The optimal emission permit price sinks in \( \mu \):

\[
\frac{\partial t_c^*}{\partial \mu} < 0
\]  

(22)

See Proof (7.2).

Another term that weighs the price signalling effect is \( \frac{\mu}{q'_s(t)} \). Contrary to fraction \( \frac{\mu}{1-\mu} \), it makes the difference opposing the upward versus the downward price signal, measuring the marginal rate of change in \( t \) of the demand for clean and dirty product.

**Claim 3** When the clean type reveals his type by an upward signalling, \( \frac{q''_s(t)}{q'_s(t)} < 1 \) while for a downward signal \( \frac{q''_s(t)}{q'_s(t)} > 1 \).

See Proof (7.3) for details. This implies that the marginal rate of change of demand lowers the level of permit price under upward signal to the less extent than under upward signal, in other words, \( \frac{q''_s(t)}{q'_s(t)} \) mitigates/constraints the level of \( t_s \), assigns a greater weight to the price signalling effect. This implies that \( t_s \) lowers its level more rapidly relative to \( t_w \) (however it is likely that for \( t_w \), the difference \( p^s(t_w) - c \) is greater relative to \( t_s \)). Why? The quantity traded under strong regulation is decreasing and concave in \( t \), while under weak regulation it is decreasing and convex. In both regimes the producer of clean type generate the same level of profit. Consumers benefit from reduced price which is surplus increasing. Thus, the only reason must be the social cost that is imposed by the regulation on the producer of dirty type creating almost classical deadweight loss by the contraction of traded output.
Proposition 4. For any $\mu \neq 0$ the optimal emission permit price, $t^*_e$, must be set below the optimal level of regulation of polluting monopoly $t^B$.

The proof is straightforward. This implies that an additional distortion in the economy - information asymmetry, requires to further reduce 'the price of emissions'. Price distortion unambiguously reduces producer surplus. Although downward price signal relatively improves consumers’ surplus, the market price never attains its first-best level that requires the equality to marginal social cost.

5.1.1 When is emission permit trading desirable?

Let us investigate if emission permit trading is always a right/feasible/optimal device to improve welfare. In this subsection we analyze the conditions under which in the economy where the producer of high environmental quality signals his type, emission permits are capable to effectively correct market distortions/let us characterize 'a signalling economy' where emission permit market is socially desirable?

It is convenient to separately analyze parameter maps for each regulation type. Let us first examine the case of weak regulation, i.e. $t < c(\bar{t})$.

![Figure 1: Parameters for weak regulation](image)

Figure 1 represents the region plot for gross valuation of the good, $\alpha$, on the
horizontal axes and environmental damage from emissions, $\delta$, on the vertical axes. By Assumptions (A.2) the domain of $\delta$ is constrained from below by $\gamma$, per definition of weak regulation. It is constrained by $c$ from above. First, let us observe where price signalling is taking place and where it is unnecessary. Recall that separating price of high environmental quality is $g(\tau) \in P_{IR} \cap P_{IC}$. Whenever $P_{IR} \subset P_{IC}$ or $p^s(t_w) \leq p^r(\tau)$, it suffice to post full information price to reveal the true type, otherwise he must distort the price. The zone where price distortion is unnecessary is highlighted yellow, elsewhere the quality is revealed by price signal. The two fields are separated with a green line (signalling border) underneath of which the optimal emission permit price must be equal to $t^B$ as there is no distortion from/not affected by information asymmetry/informational distortion. Hence, above the signalling border the optimal price of emission permit must be set according to (21).

The dashed blue line indicate the border for product desirability: whenever damage exceeds the valuation of the good, the production is not socially desirable. The dashed line in yellow traces $t^B$ being equal to zero. For a monopoly by pollution externality this border indicates where it is socially optimal to subsidize output. As has been shown in Proposition 4 optimal permit price is always less stringent that $t^B$. Indeed, the red continuous line that depicts values where $t^c = 0$ lies in the area where under Buchanan regulation a positive tax is required. The continuous blue line marks $t^* \rightarrow c$. The green area in between is where optimal permit price must be set to positive values. The resting area in purple is where dirty production must optimally receive subsidy.

Finally, it is worth empathizing that the area of weak regulation (the green triangle in the top left corner) depends directly on $\mu$. The value used to produce Figure 1 is $\mu = .1$, a relatively low probability of clean monopoly. As it has stated in (4) the optimal permit price decrease in $\mu$. Hence, it can be shown that there is a critical level of $\mu \equiv \bar{\mu}$ beyond which a positive permit price in never socially optimal (my rough estimation: $\bar{\mu} \cong .15$).

A fairly similar parameter map plots the regions of strong regulation, i.e. $t > c(\tau)$. The triangle filled with colors is the area in question where permit price exceeds the production cost of the clean type. For strong regulation the top right triangle in yellow is the regions where price signal isn’t necessary and both producer types post full information price. The rectangular in blue corresponds to the area where true type is revealed by price signal. Again, the fields are separated by the green continuous line that relates the parameter equalizing signalling and full information prices. The red continuous line marks/depicts
permit price that would be equal to production cost of the clean variety. Consequently, the region constraint between ‘strong’ permit price (the red line), signalling border (the green line) and product desirability (the blue line) qualifies for a positive permit prices of strong regulation. As in the case of weak regulation, the condition for the ‘strong’ permit price depends on $\mu$. Therefore, an increase in $\mu$ move the red line upwards until, ultimately, the region of strong regulation is empty. The region of strong regulation is a non-empty set when $\mu$ is low ($\bar{\mu} < .15$), in Figure 2 $\mu = .1$. The crucial condition for non-empty strong regulation is that environmental damage must be significant enough that would have required $\bar{t}^B > 0$, then together with low prevalence of clean production, it is indeed socially desirable to impose strong regulation.

Generally, we have shown that emission permit trading can be socially desirable and can have the potential for welfare improvement. However, it is not automatic. To be such the economy must be mainly composed of markets with dirty types, the production of dirty type must have high environmental footprint relative to social value of the product and consumers’ must have fairly ‘light’ disutility from pollution damage (not too green). Otherwise, monopoly and informational distortions would outweigh pollution externality and would optimally require a subsidy to dirty producer type.
5.1.2 Consumers entirely internalize the pollution externality.

Let us briefly examine the outcome of optimal environmental regulation when consumers entirely internalize the pollution externality. Suppose that consumers’ disutility from pollution correspond exactly to the objective (scientific) level of damage from pollution, i.e. $\delta = \gamma$. Hence, the regulator doesn’t need to take into account the negative externality from pollution because consumers by their behavior have already accounted for it. Otherwise it would lead to a double/redundant according of the environmental damage . Then, we can rewrite (18) to get:

$$W(t) = \mu \left( \alpha - \frac{1}{2} q^e(t) - c \right) q^e(t) + \left( \alpha - \gamma - \frac{1}{2} q^f(t) \right) q^f(t)$$

(23)

The corresponding first order condition, when replacing $q^f(t)$ by $\frac{\alpha - \gamma - t}{2} \frac{1}{1 - \mu}$ and $q^f(t)$ by $-\frac{1}{2}$, is given by $\mu q^e(t) (p^e(t) - c) - \frac{1}{2} (1 - \mu) q^f(t) (\alpha - \gamma + t) = 0$.

If the regulator knows that the negative externality from pollution is internalized, he must use permit market to correct the monopoly distortion and the informational distortion. Substituting $-\frac{1}{2} \left( 1 \mp \sqrt{\frac{\gamma}{2\alpha - \gamma - 2t}} \right)$ for $q^e(t)$ for weak and strong regulation respectively and rewriting the above first order condition we obtain the implicit form of the optimal permit price:

$$t^*_e = - (\alpha - \gamma) - 2 \frac{\mu}{1 - \mu} \left( 1 \mp \sqrt{\frac{\gamma}{2\alpha - \gamma - 2t}} \right) (p^e(t) - c)$$

(24)

The expression in (24) is negative for weak and strong regulation. When $\delta = \gamma$, the regulator would need to subsidy the producer to correct the distortions in the economy. It is almost intuitive that when negative externality is not given, the emission market is neither necessary nor capable/suitable to improve/as a means of raising welfare. Hence, the assumption about consumers underestimating environmental damage is crucial for the analysis of environmental regulation through emission permit trading in the economy with price signalling.

5.2 Case 2: $c(e) = c(e) = 0$, $F_c > 0$, $F_d = 0$

Let us now consider the economy in which dirty and clean type do not differ in production marginal cost but do differ in environmental technology. The clean type has a positive fixed cost that is spent on pollution abatement which

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4The expression $-\frac{1}{2} \left( 1 - \sqrt{\frac{\gamma}{2\alpha - \gamma - 2t}} \right) < 0$ given Assumption (A.4).
allows to maintain environmental quality. The dirty type doesn’t make such an investment, thus we set \( c(\bar{e}) = c(e) = 0 \), \( F_c > 0 \), \( F_d = 0 \). Note that when unregulated such an economy under full information would allow the clean type to produce output when \( F < \frac{\alpha - \gamma}{2} \). Further more, when \( F < \frac{\alpha - \gamma}{4} \), the clean type generates a greater profit than the clean type. Under information asymmetry, such an economy doesn’t facilitate the clean producer to reveal his true type because the separating equilibrium condition (2) doesn’t hold, indeed, \( \pi_{ep} = 0 \) (? ? ?). The unregulated economy must produce/result in a pooling equilibrium.

**Pooling equilibrium.** The pooling equilibrium is characterized by \( g(\bar{e}) = g(e) \) and \( \mu(g(e)) = \mu_0 \). Consumers anticipate that if the producer is clean he would charge \( \frac{\alpha - \gamma}{2} \) and if he is of dirty type, he would charge \( \frac{\alpha - \gamma}{2} \), hence the pooling price is \( p = \frac{\alpha - \gamma(1 - \mu_0)}{2} \) when the clean type find is able to secure a non-negative profit, i.e. when \( F \leq \frac{\alpha - \gamma(1 - \mu_0)}{4} \), with clean and dirty types earning respectively \( \pi(\bar{e}) = \frac{\alpha - \gamma(1 - \mu_0)^2}{4} - F \) and \( \pi(e) = \frac{\alpha - \gamma(1 - \mu_0)^2}{4} \). Note that when quality is unobservable, the clean type always generates a lower profit than a dirty type. When fixed cost exceeds \( \frac{\alpha - \gamma(1 - \mu_0)^2}{4} \), the market unravels with the clean variety not being offered. Solely dirty type is active posting the full information price and earning full information profit.

**Environmental regulation.** When the dirty type purchases emission permits, separating equilibrium becomes feasible. The separating prices are determined accordingly to Proposition 1 and Corollary 2. Note that because of separating equilibrium condition (1), i.e. \( \pi_{e} > 0 \), the upper bound of the admissible interval for \( F \) increases.

The regulator’s objective function \( W(t) \) is now:

\[
\mu \left[ \left( \alpha - \frac{1}{2} q_s(t) \right) q_s(t) - F \right] + (1 - \mu) \left( \alpha - \gamma - \frac{1}{2} q_d(t) - \delta \right) q_d(t) \tag{25}
\]

Similarly, the fist order condition becomes:

\[
\mu q^{st}(t) (\alpha - q^s(t)) + (1 - \mu) q^{f^t}_d(t) \left( \alpha - \gamma - q^f_d(t) - \delta \right) = 0 \tag{26}
\]

The optimal permit price are determined according to:

\[
t^*_c = \delta - (\alpha - \gamma - \delta) - 2 \frac{\mu}{1 - \mu} \frac{q^{st}(t)}{q^{f^t}_d(t)} p^s(t) \tag{27}
\]
The zones of optimal regulatory policy are depicted in Figure 3. The information is revealed by a downward price signal as in the strong regulation. The domain of $\delta$ is constrained from below by $\gamma$. The interpretation of the region plot is the same as for the case of strong regulation. The only difference with the above mentioned case is that there is no requirement now that the permit price must exceed the tax. Therefore, the welfare loss associated with the purchase of permits must be lower than in the strong regulation.

In Case 1, low permit prices induce a supplementary (to signalling) upward increase in price distortion. Clearly, it is welfare reducing as it shrinks consumers’ surplus. Only permit prices above $c(\bar{e})$ permit a downward signal. However, in Case 2, the introduction of permit trading directly lead to downward price distortion... Nevertheless, the positive permit price is socially desirable for low probability of high environmental quality.

To maintain a non-negative price of emission permits the following parameters are essential: $\delta$, environmental damage, can interpret it, following Antelo and Loureiro (2009), the regulator’s taste for the environmental, $\mu$, the probability of a firm to be of clean type, $\gamma$, consumers’ dislike of pollution. Noteworthy, the area for positive permit price is located close to the $\alpha - \gamma = \delta$ border. This implies that the net social valuation must be marginal. All else equal, an increase in clean variety’s production cost ‘moves’ the area of positive price
upwards, while an increase in $\gamma$ leads to an increase in the area of positive taxation. This means that a higher disutility from pollution induces a more stringent regulation.

6 Discussion/Conclusion

$\delta > \gamma$ is an essential assumption to make this type of environmental regulation effective.

What is the effect of signalling on emissions?

Robustness?

Antelo and Loureiro (2009) come to a similar conclusion that informational deficiency on part of the regulator leads/drives him to soften the level/severity of taxation relative to marginal damage/Pigou principle and to polluting monopoly/Buchanan level./.

7 Appendix

7.1 Derivation of signalling price

Case 5 1: signalling high environmental quality without regulation

The clean variety will signal the high environmental quality (HEQ) when its profit with correctly identified quality is higher than with a misidentified quality. The rationality incentive (RI) for the clean variety, $RI_{HEQ}$, is:

$$\pi_v(p_v, 1) \geq \pi_v(p)$$

$$(p - c)(\alpha - p) \geq \pi_v(p)$$

Note that in the worst case when consumers misidentify HEQ the highest profit the clean variety would obtain is $\pi_v(p) = \max(p - c)(\alpha - \gamma - p)$. The optimal profit under the worst beliefs is thus $\pi_v(p) = \frac{(\alpha - \gamma - c)^2}{4}$.

Rewriting the $RI_{HEQ}$ we obtain the condition:

$$(p - c)(\alpha - p) \geq \frac{(\alpha - \gamma - c)^2}{4}$$

Similarly, the $RI_{LEQ}$ is $p(\alpha - \gamma - p) \geq \frac{(\alpha - \gamma)^2}{4}$ which is the identity. Hence, 30 is the only bounding condition among RI.
The incentive compatibility constraints ensures that the profit generated from truth-telling is higher than mimicing the other type. The $IC_{HEQ}$ is:

\[
\pi_v(p_v, 1) \geq \pi_v(p, 0) \tag{31}
\]

\[
\frac{(\alpha - c)^2}{4} \geq (p - c)(\alpha - \gamma - p) \tag{32}
\]

This constraint is verified if the $RI_{HEQ}$ holds.

Lastly, the compatibility constraint for the low type, $CI_{LEQ}$, is:

\[
\pi_b(p, 0) \geq \pi_b(p, 1) \tag{33}
\]

\[
\frac{(\alpha - \gamma)^2}{4} \geq p(\alpha - p) \tag{34}
\]

Note that $\pi_b(p, 0) = \max_p p(\alpha - \gamma - p) = \frac{(\alpha - \gamma)^2}{4}$.

The signalling price of the HEQ must verify the system of bounding conditions $RI_{HEQ}$ and $CI_{LEQ}$:

\[
\begin{cases}
(p - c)(\alpha - p) \geq \frac{(\alpha - \gamma - c)^2}{4} \\
\frac{(\alpha - \gamma)^2}{4} \geq p(\alpha - p)
\end{cases} \tag{35}
\]

Note that the two conditions can be rewritten as a function $f(p, c) = (p - c)(\alpha - p) - \frac{(\alpha - \gamma - c)^2}{4}$. To find the separating equilibrium the system must hold:

\[
f(p, c) \geq 0 \geq f(p, 0) \tag{36}
\]

The 35 represents two inequalities for polynomials, whose solution must the overlap of intervals defined by the two conditions. The price solving the equation $f(p, c) = 0$ is

\[
p_{1,2} = \frac{1}{2} \left( \alpha + c \pm \sqrt{\gamma (2\alpha - \gamma - 2c)} \right) \tag{37}
\]

To have a solution to the polynomial function $f(\cdot)$, the discriminant has to be positive:

\[
\gamma (2\alpha - \gamma - 2c) \geq 0 \tag{38}
\]

Rewriting 38 we obtain

\[
\frac{\gamma}{2} \leq \alpha - c \tag{39}
\]

This inequality holds always under Assumption (A.1).

\(^5 LEQ \) stands for low environmental quality.
To verify the conditions in 35, signalling price \( p^s \) must be element of sets of intervals, i.e. \( p^s \in [p_{v1}; p_{v2}] \cap ((-\infty; p_{b1}] \cup [p_{b2}; +\infty)) \). Because of the cost difference between the varieties, the separating price must belong to the interval \([p_{b2}; p_{v2}]\), which corresponds to \( \frac{1}{2} \left( \alpha + \sqrt{\gamma (2 \alpha - \gamma)} \right) \). At the lower bound, \( p^s = \frac{1}{2} \left( \alpha + \sqrt{\gamma (2 \alpha - \gamma)} \right) \), the profit distortion of the HEQ from signalling is minimal, which is the lowest upward distortion of the price necessary to discourage the LEQ from mimicking the clean variety. Note that the signalling price of the clean variety is set/bounded to the characteristics of the dirty variety, in particular, the signalling price of the HEQ depends on the effective marginal production cost of the dirty variety (in the equation above it is zero).

**Case 6 2: signalling high environmental quality under the environmental regulation**

Same as in case 1 except that in the interval \( P_{IC} = (-\infty; p_{b1}] \cup [p_{b2}; +\infty) \), the roots \( p_{b1}, p_{b2} = \frac{1}{2} \left( \alpha + t \pm \sqrt{\gamma (2 \alpha - \gamma)} \right) \) taking into account the emission permit price as marginal expenses per unit of dirty output.

### 7.2 Proof of Claim 4

**Proof.** Recall that (21) \( \frac{\partial W(t)}{\partial \mu} \equiv F(t, \mu; \alpha, \gamma, \delta, c) = 0 \) an implicit function which is define in the neighborhood of \( t^*_\mu \). To determine \( \frac{dt}{d\mu} \), take derivatives \( F_\mu \) and \( F_t \) (which correspond to \( W_{1,\mu} \) and \( W_{1,t} \)) and substitute \( p^s(t) = \alpha - q^d(t) \) to obtain:

\[
\frac{dt}{d\mu} = -\frac{\frac{1}{2} (\alpha - \gamma - 2\delta + t) + (p^s(t) - c) q^m(t)}{\mu (p^s(t) - c) q^m(t) - \frac{1}{2} (1 - \mu) - \mu q^m(t)^2}
\]

Let us first consider strong regulation. Note that the denominator is then always negative since \( q^m(t_s) < 0 \). For \( \mu \neq 0, \alpha - \gamma - 2\delta + t < 0 \) according to Corollary (?) and \( q^m(t_s) < 0 \) \( \forall t \). Hence \( \frac{dt}{d\mu} < 0 \).

Now consider weak regulation. The numerator is negative under Assumption (A.4). The first term of the denominator is now positive because \( q^m(t_w) > 0 \) \( \forall t \).

To demonstrate that the denominator in (40) is negative...??

### 7.3 Proof of Claim 3

**Proof.** Recall that \( q^d(t) = \frac{\alpha - \gamma - t}{2} \) with \( \frac{\partial q^d(t)}{\partial t} = -\frac{1}{2} \) and that \( q^s(t_w) = \)
Then, \( \frac{\partial q^*(t_w)}{\partial t_w} = -\frac{1}{2} \left( 1 - \sqrt{\frac{\gamma}{2\alpha - \gamma - 2t}} \right) < 0 \) given Assumption (A.4) and \( \frac{\partial^2 q^*(t_w)}{\partial t_w^2} = \sqrt{\frac{\gamma}{2(2\alpha - \gamma - 2t)}} \). For strong regulation, \( q^*(t_s) = \frac{1}{2} \left( \alpha - t + \sqrt{\gamma (2\alpha - \gamma)} \right), \frac{\partial q^*(t_s)}{\partial t_s} = -\frac{1}{2} \left( 1 + \sqrt{\frac{\gamma}{2\alpha - \gamma - 2t}} \right) < 0 \) \( \forall t \) and \( \frac{\partial^2 q^*(t_s)}{\partial t_s^2} = -\sqrt{\frac{\gamma}{2(2\alpha - \gamma - 2t)}} \). Hence, for the upward signal, \( \frac{q''(t_w)}{q'_w(t)} = 1 - \sqrt{\frac{\gamma}{2\alpha - \gamma - 2t}} < 1 \) and for the downward signal, \( \frac{q''(t_s)}{q'_s(t)} = 1 + \sqrt{\frac{\gamma}{2\alpha - \gamma - 2t}} > 1 \). □

References


