

Government expenditure, external and domestic public debt, and economic growth*

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Abstract

This paper analyzes the relationship between government expenditure, tax on returns to assets, public debt, and economic growth. Public debt is composed of two components, domestic debt and external debt. We show that an increase in the tax rate on returns to assets leads to an increase in government expenditure, consumption, and domestic debt. However, the impact of tax rate on external debt is ambiguous. In particular when the productivity of capital on production is low, the impact is negative. However, when the productivity is high enough, the relation between external debt and the tax rate exhibits a bell-shaped form, i.e. external debt firstly rises then decreases with the tax rate.

Keywords: Tax on returns to assets; Public expenditure; Domestic debt; External debt; Growth

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1 Introduction

In recent years, many studies have focused on the impact of external debt and public investment on growth (see, for example, Clements et al. 2003, Ejigayehu 2013, Zaman and Arslan 2014, and Bedir and Soydan 2015). Indeed, the question of the impact of public investment on growth and its financing has long divided economists. Following the neoclassical growth theory, the growth rate is determined by capital formulation and, consequently, fiscal policy has a major role (see Peacock and Shaw 1971, Peacock and Wiseman 1979). The neoclassical authors indicated that an increase of tax will raise economic growth. They stated that a lower growth rate may imply a greater consumption net of external diseconomies if the latter (as a share of aggregate production) increases with growth. They also underlined that investment may cause more externalities than current expenditure, in particular, if the latter is related to personal services. Different literatures converge on the same conclusion, i.e. public expenditure promotes economic growth in the short term.

Barro (1990) distinguished two types of government expenditure, productive and unproductive expenditure. He stated that the economy's growth is negatively correlated with the ratio of government spending to GDP and there is a positive relationship between public investment and output growth. In the same vein, Aschauer (1989) found that the government productive expenditure can stimulate output expansion. While Devarajan et al. (1996) agreed that government expenditure has a relationship with economic growth, each component of it has a different effects on growth. Particularly, current expenditure of government is associated with a higher growth whereas government productive expenditures in capital, transport, communication, health, and education have a negative impact on growth. In addition, Devarajan et al. (1996) and Angelopoulos et al. (2007) obtained that economic growth depends not only on the physical production of

typical components of public spending, but also on the ratio of government expenditure allocated on them. On the contrary, Mundle (1999) and Glomm and Ravikumar (1997) stated that government spending in infrastructure and social services have a significant impact on the long-run growth rate. Hence, these governments need to shift away from taxes on production and trade to taxes on income, consumption, and value added. In their study about fiscal decentralization, government spending, and economic growth in China, Zhang and Zou (1998) showed that the central government's spending positively impacts economic growth. However, local government spending negatively affects growth. The same finding was also reached by Xie et al. (1999) and Thornton (2007) when the authors studied about the decentralization and economic growth in the United States and in OECD countries, respectively. In contrast to previous studies, using cross-section data for the United States, Akai and Sakata (2002) got a different result following which fiscal decentralization contributes to economic growth.

In a research on growth effects of government expenditure and taxation in developed countries by using panel data of rich countries for the period 1970-1995, Folster and Henrekson (2001) found a negative relation between public expenditure and economic growth. Easterly and Rebelo (1993) stressed that taxes on international trade have a strong association with economic growth in poor countries whereas income taxes are a main determinant of growth in industrial countries. Furthermore, Gupta et al. (2005) indicated that in low-income countries, the overall composition of public expenditure toward productive uses is particularly important for fostering growth, and that reducing current expenditures tend to trigger higher growth rates than adjustments based on revenue increases and cuts in productive spending. Moreover, reductions in the public sector wage bill are not harmful for economic growth.

Taxation affects not only individuals and firms but also economic growth. Cebula (1995) highlighted that higher maximum levels of federal government personal income tax

rate and corporate income tax rate have a negative impact on economic growth, based on an empirical investigation for the period 1955-1972 in the United States. Angelopoulos et al. (2007) found that the average tax rate (as measured by tax revenue over GDP) and the associated fiscal size of the government (as measured by total expenditure over GDP) are significantly and negatively correlated with growth. By using disaggregated taxes, their results indicated (but this is not robust) that the growth effect of effective labor income tax is negative. Similarly, Easterly and Rebelo (1993) addressed that the ratio of tax revenues to GDP has a negative impact on growth, using data on OECD countries for the period 1960-1988. Lastly, the growth effect of effective capital income tax is positive although not significant. However, there is evidence that even through the mix of direct and indirect taxes is an important determinant of long-run growth and investment rates, but in practice, Mendoza and Asea (1997) underlined that plausible changes in tax rates seem to be unlikely to affect growth. Using Harberger model with panel data regression for the period 1965-1991 in 11 OECD countries, the authors found that the effects of 10 percentage point tax cuts on the investment rate are about 0.5 and 1.5 percentage points but growth effects are very small, approximately 0.1 to 0.2 of a percentage point. Mullen and Williams (1994) obtained that higher marginal tax rates are associated with a slower output growth and that lower marginal tax rates are able to have a positive impact on economic growth. The results of Mullen and Williams (1994) mean that changes in effective tax rates have an important effect on economic growth and that average tax rates and growth constitutes a significant relationship. In a non-stochastic model, Lee et al. (1997) showed that tax significantly affects growth and a tax cut rises the economy's growth rate. However, if consumers are risk averse enough, the growth rate might be decreased with a tax cut. Furthermore, Kim (1998) supposed that tax systems across countries have a significant relation with growth in which differences in taxes can explain growth discrepancy. The author also stated that tax reform may influence economic

growth and that the hypothetical elimination of all taxes in the US raises approximately 0.85 percentage points of growth rate in the calibrated model. Lin and Russo (1999) found different figures with Kim (1998). For instance, there would be an increase in the growth rate by 0.63 percentage points if all the income taxes were eliminated and US debt-to-capital ratio was about 33%. When the corporate tax for innovative companies is eliminated, the growth rate will decrease by 0.20 percentage points.

By analyzing taxation and growth in an overlapping generations model, Yakita (2003) showed that the flat-rate wage tax elevates the growth rate and the flat-rate income tax does not stimulate economic growth. These results are different with Lucas' (1986) findings that labor income taxes stimulate economic growth while capital taxes do not. In their research, Lee and Gordon (2005) concluded that corporate tax rates have a negative impact on economic growth (i.e. a cut in corporate tax rate by 10% will raise economic growth from 1% to 2%) whereas the personal tax rates have no clear evidence. Angelopoulos et al. (2007) recognized that some kinds of taxes such as labor income tax are negatively related to growth, meanwhile capital income and corporate income taxes are positively related to growth.

Regarding public debt, Greiner (2007) assumed that the ratio of primary surplus to gross domestic income is a positive linear function of the debt to gross domestic debt ratio. The author also stated that a sustainable balanced growth path exists if the government uses a certain part of the tax revenue for the debt services. In other researches, Reinhart and Rogoff (2010) and Herndon et al. (2014) showed that public debt has a positive impact on economic growth and there is a higher ratio of public debt to GDP leads to a lower GDP growth rate. For instance, if the ratio of public debt to GDP is lower than 30%, the average GDP growth rate is about 4.1%. On the contrary, the growth rate is reduced to 2.2% if the ratio of public debt to GDP becomes larger than 90%. In a study on the role of government debt on economic growth across twelve Euro-area countries, Checherita

and Rother (2010) found that public debt and economic growth have a nonlinear relation and that a higher public debt-to-GDP ratio is on average associated with a lower long-term growth rate when debt is above the range of 90-100% of GDP. In practice, the ratio of public debt to GDP in each country is different, for example, in European countries where it is regulated at the level of 60% of GDP following the Maastricht criteria. In the case of developing countries such as Vietnam, the figure is 65%.

Clements et al. (2003) stressed that high levels of public debt can depress economic growth in low-income countries and the corresponding threshold level of external debt is estimated around 50% of GDP in their simulation exercise. In the same vein, according to Ejigayehu (2013), Zaman and Arslan (2014), and Soydan and Bedir (2015), the empirical results generally reveal that the accumulation of external debt is associated with an increase in economic growth up to an optimal level, and an additional increase of external indebtedness beyond the level has inversely contributed to the economy. In other words, there exists a threshold above which a too high level of external debt has a negative effect on growth.

In our paper, we consider a growth model that includes the issues underlined above, i.e. we investigate the relation between growth, public investment, tax on returns to assets, and public debt. Our study distinguishes two types of public debt, domestic debt and external debt, whereas most of existing theoretical works only considered domestic debt (e.g. Battaglini and Coate 2008, Greiner, 2007, Elmendorf and Mankiw 1999, among others). We study the balanced growth path of the model and focus on the impact of the tax rate on returns to private assets on the macroeconomic equilibrium.¹

The remaining of the paper is organized as follows. The theoretical model, based

¹We only focus on the effect of tax on returns to assets on the decentralized equilibrium in the presence of two types of public debt (domestic debt and external debt). We do not discuss the welfare aspect and, in particular, how the tax rate can be set in order to maximize welfare. This issue as well as the optimal growth (from the central planner's viewpoint) are obviously very important and deserve to be investigated in a further study.

on Barro (1990) and Greiner (2007), is introduced in Section 2. Section 3 presents the equilibrium of the model while Section 4 characterizes the balanced growth path (BGP) of the economy. The effects of tax on returns to assets on the steady state of the economy is analyzed in Section 5. The last section concludes the study and gives some perspectives for further research.

2 Model

The growth model presented in this section is based on the models developed by Barro (1990) and Greiner (2007). Our economy comprises three sectors, namely government, firms, and consumers.

2.1 Government

We assume that at each period t the government can collect tax on returns to assets held by private agents. It can also borrow from the domestic and international financial markets, which correspond to two types of public debt, domestic debt D_t with interest rate r_t^D and external debt B_t with interest rate r_t^B . As the country has no power on the international financial market, $\{r_t^B\}_{t=0}^{\infty}$ is a sequence of exogenous external interest rates. On the spending side, the government can share its resources between public expenditure devoted to production of final goods and reimbursement of interests and capital of domestic and external debts.²

The government budget constraint can be expressed as follows:³

$$G_t + (r_t^B + 1)B_t + (r_t^D + 1)D_t = \tau_t r_t^A A_t + B_{t+1} + D_{t+1}. \quad (1)$$

²Recall that we distinguish two types of public debt, domestic debt and external debt, whereas most of existing theoretical studies only considered domestic debt (e.g. Battaglini and Coate 2008, Greiner, 2007, Elmendorf and Mankiw 1999, among others).

³All variables are expressed in terms of real values.

where A_t is the stock of assets held by private agents, τ_t is the tax rate on returns to assets, r_t^A is the interest rate of asset, and G_t is the flow of government expenditure.

Following Greiner (2007), we assume that public debt is not over a certain proportion of total output in order to guarantee sustainability of public debt:

$$G_t + \eta(B_t + D_t) \leq \phi Y_t + \tau_t r_t^A A_t, \quad (2)$$

with ϕ and $\eta \in \mathbb{R}_+$ are constants. Parameter ϕ determines whether the level of the primary surplus rises or falls with an increase in gross domestic income, η determines how strong the primary surplus reacts to changes in domestic debt and external debt, η may be considered as a feedback parameter of domestic debt and external debt.

Inequality (2) means that total government expenditure and government's borrowing are not exceeded government's revenue which comes from tax collection and a certain proportion of total output. Equation (2) can be also rewritten as

$$G_t - \tau_t r_t^A A_t + \eta(B_t + D_t) \leq \phi Y_t.$$

This condition means that budget deficit ($G_t - \tau_t r_t^A A_t$) can be financed by domestic and external debt, which can be covered by a proportion of production. This condition is motivated by some empirical facts through the Maastricht criteria (public debt lower than 60% of GDP, budget deficit is lower than 3% of GDP), threshold of public debt set in some developing countries (such as in Vietnam where the threshold is 65% of GDP), and the discussion about the relation between public debt and growth since the seminal paper of Reinhart and Rogoff (2010).

Let r_{t-1}^{BD} denote the interest rate which satisfies:

$$B_{t-1} r_{t-1}^B + D_{t-1} r_{t-1}^D = (B_{t-1} + D_{t-1}) r_{t-1}^{BD} \quad (3)$$

or equivalently,

$$r_t^{BD} = \frac{B_t}{B_t + D_t} r_t^B + \frac{D_t}{B_t + D_t} r_t^D. \quad (4)$$

Equation (4) indicates that r_{t-1}^{BD} is an average interest rate of r_{t-1}^B and r_{t-1}^D . There always exists an interest rate r_{t-1}^{BD} with given $r_{t-1}^B, r_{t-1}^D, B_{t-1}$ and D_{t-1} . Equation (3) can be rewritten as follows:

$$B_{t-1}(1 + r_{t-1}^B - \eta) + D_{t-1}(1 + r_{t-1}^D - \eta) = (B_{t-1} + D_{t-1})(1 + r_{t-1}^{BD} - \eta). \quad (5)$$

At equilibrium, condition (2) must bind. Together (1), (2) and (3) lead to

$$B_t + D_t = (B_{t-1} + D_{t-1})(1 + r_{t-1}^{BD} - \eta) + \phi Y_{t-1}. \quad (6)$$

We now look at the sustainability of public debt. Following Greiner's (2007) terms, sustainability of public debt states that the current value of public debt must equal the sum of discounted future non-interest surpluses. The sufficient condition for the sustainability of public debt is summarized in the following proposition.

Proposition 1 *Define that γ_t is growth rate of gross domestic income Y_t , and r_t^{BD} is determined by equation (4). The sufficient condition for the sustainability of public debt is $\max\{\sup_t \gamma_t, 0\} < \inf_t r_t^{BD} - \eta$.*

Proof. Equation (6) can be expressed as follows (using equation (5)):

$$B_t + D_t = (B_0 + D_0) \prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta) + \sum_{s=1}^t \phi Y_{t-s} \prod_{j=1}^{s-1} (1 + r_{t-j}^{BD} - \eta), \quad (7)$$

which is equivalent to

$$B_0 + D_0 = \frac{B_t + D_t}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)} - \frac{\sum_{s=1}^t \phi Y_{t-s} \prod_{j=1}^{s-1} (1 + r_{t-j}^{BD} - \eta)}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)}. \quad (8)$$

Sustainability of public debt is characterized by

$$B_0 + D_0 = \lim_{t \rightarrow \infty} \left(\frac{B_t + D_t}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)} \right). \quad (9)$$

Condition (9) is verified if

$$\lim_{t \rightarrow \infty} \frac{\sum_{s=1}^t \phi Y_{t-s} \prod_{j=1}^{s-1} (1 + r_{t-j}^{BD} - \eta)}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)} = 0. \quad (10)$$

Denote that γ_t is the growth rate of total production income Y_t . Hence, $Y_{t-s} = \prod_{j=0}^{t-s} (1 + \gamma_j) Y_0$. We then get

$$\begin{aligned} \frac{\sum_{s=1}^t \phi Y_{t-s} \prod_{j=1}^{s-1} (1 + r_{t-j}^{BD} - \eta)}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)} &= \phi Y_0 \frac{\sum_{s=1}^t \prod_{j=0}^{t-s} (1 + \gamma_j) \prod_{j=1}^{s-1} (1 + r_{t-j}^{BD} - \eta)}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)} \\ &= \phi Y_0 \sum_{j=1}^t \prod_{s=j}^t \left(\frac{1 + \gamma_{t-s}}{1 + r_{t-s}^{BD} - \eta} \right) \end{aligned}$$

Hence, if $\max\{\sup_t \gamma_t, 0\} < \inf_t r_t^{BD} - \eta$ then condition (9) is verified . ■

As our model has domestic debt and external debt, sustainability of debt means that in the long run the discounted value of the sum of two debts cannot exceed the initial total debt (or in other words, current value of public debt must equal the sum of discounted future non-interest surpluses) given in equation (9), which holds if equation (10) is satisfied. This corresponds to the No-Ponzi-Game (NPG) condition for our model. For the Ramsey growth model with (only domestic) public debt, the NPG condition can be found in Heijdra and Van Der Ploeg (2002).

For our model, the NPG condition is satisfied if $\max\{\sup_t \gamma_t, 0\} < \inf_t r_t^{BD} - \eta$. In

other words, output growth rate γ_t should be sufficiently lower than the average interest rate r_t^{BD} . If output growth rate is higher than $r_t^{BD} - \eta$, in this case the expression in (10) will tend to infinity and, consequently, the right-hand side term of equation (8) will converge to minus infinity, implying that the initial total debt cannot be covered (i.e. debt is not sustainable).

2.2 Firms

We assume that the production of the final good depends on the stock of private capital and government spending:

$$Y_t = F(K_t, G_t) = HK_t^\alpha G_t^{1-\alpha} \quad (11)$$

where $0 < \alpha < 1$ is output elasticity with respect to capital (and $1 - \alpha$ is the elasticity corresponding to public spending), H is total factor productivity or technological level. The production function F is strictly increasing in both variables, strictly concave in K . The production function also verifies (i) $F(0, G) = 0$ and (ii) $F(K, 0) > 0$ if $K > 0$. Here, G may be considered as a positive externality for the production. The profit is given by $\pi_t = F(K_t, G_t) - r_t^K K_t$ (r_t^K is the interest rate of capital). The first-order condition (FOC) for profit maximization is

$$F'_K(K_t, G_t) = r_t^K. \quad (12)$$

By substituting equation (11) into equation (12), the interest rate of capital can be written as

$$r_t^K = \alpha H K_t^{\alpha-1} G_t^{1-\alpha} = \alpha H \left(\frac{G_t}{K_t} \right)^{1-\alpha}, \quad (13)$$

or, equivalently,

$$r_t^K = \alpha H g_t^{1-\alpha}, \quad (14)$$

where $g_t \equiv G_t/K_t$. Equation (13) implies that interest rate of private capital is determined by total factor productivity, output elasticity with respect to public spending, and the ratio of government expenditure and private capital.

2.3 Consumers

The representative consumer's instantaneous utility function is assumed to have the isoelastic form

$$U(C_t) = \begin{cases} \frac{C_t^{1-\rho}-1}{1-\rho} & \text{if } \rho \neq 1 \\ \ln C_t & \text{if } \rho = 1 \end{cases} \quad (15)$$

where $\rho > 0$ is the intertemporal elasticity of substitution. The representative consumer chooses her consumption, her stock of assets, and her government bonds to maximize her inter-temporal utility $\sum_{t=0}^{+\infty} \beta^t U(C_t)$, where $\beta > 0$ is the discount rate, under the budget constraint

$$C_t + A_{t+1} + D_{t+1} \leq [r_t^A(1 - \tau_t) + 1] A_t + (r_t^D + 1)D_t + \pi_t \quad (16)$$

and positivity constraints $C_t \geq 0$ and $A_t \geq 0, \forall t$. Note that C_t, A_t, D_t , and π_t are respectively consumption, private assets, domestic debt hold by the consumer, and the profit she receives as the firm owner.⁴

⁴We assume that there is no tax on government bond interest. Indeed, when such a tax exists, the consumer's budget constraint will include the term $r_t^D(1 - \tau_t^D)D_t$ instead of $r_t^D D_t$. In this case, the non-arbitrage condition between private assets and government bonds is $r_t^A(1 - \tau_t^A) = r_t^D(1 - \tau_t^D)$, which implies $r_t^A = r_t^D$ and $\tau_t^A = \tau_t^D$. For simplification purpose, we do not impose any tax on government bonds and consequently the implied non-arbitrage condition (see also below) will become equation (21).

The Lagrangian is

$$L = \sum_{t=0}^{\infty} \beta^t U(C_t) - \sum_{t=0}^{\infty} \lambda_t \{ [r_t^A(1 - \tau_t) + 1]A_t + (r_t^D + 1)D_t + \pi_t - C_t - A_{t+1} - D_{t+1} \} + \sum_{t=1}^{\infty} \mu_t A_t.$$

The FOCs are given as follows, $\forall t$,

$$\beta^t U'(C_t) + \lambda_t = 0, \quad (17)$$

$$\lambda_t [(1 + r_t^A(1 - \tau_t))] - \lambda_{t-1} - \mu_t = 0, \quad (18)$$

$$\lambda_t(1 + r_t^D) - \lambda_{t-1} = 0, \quad (19)$$

$$\mu_t A_t = 0. \quad (20)$$

The slackness condition in (20) means that $A_t > 0, \mu_t = 0$ or $A_t = 0, \mu_t > 0$. These FOCs and the budget constraint will provide a solution of the consumer's optimization program.

Solving for an interior solution ($A_t > 0$), conditions (18)-(20) give:

$$r_t^D = r_t^A(1 - \tau_t). \quad (21)$$

The equality between the interest rate of domestic debt and the net interest rate of private asset given in (21) represents the non-arbitrage condition between holding domestic debt and holding private capital. Furthermore, conditions (17) and (19) give

$$\frac{U'(C_{t-1})}{U'(C_t)} = \beta(1 + r_t^D), \quad (22)$$

which is the usual Keynes-Ramsey rule which states that the marginal utility of past consumption is equal to the discounted marginal utility of current consumption times the interest rate.

By using the utility function in (15), equation (22) becomes

$$\frac{C_t}{C_{t-1}} = [\beta(1 + r_t^D)]^{1/\rho}. \quad (23)$$

3 Equilibrium

Equilibrium of model is a solution of the following equations:

Balancedness of the government budget:

$$G_t + r_t^B B_t + r_t^D D_t + B_t + D_t = r_t^A \tau_t K_t + B_{t+1} + D_{t+1}$$

Sustainability of debt condition:

$$G_t + \eta(B_t + D_t) = \phi Y_t + \tau_t r_t^A A_t.$$

Balancedness of consumer budget:

$$C_t + A_{t+1} + D_{t+1} = [r_t^A(1 - \tau_t) + 1] A_t + (r_t^D + 1)D_t + \pi_t.$$

Keynes-Ramsey rule:

$$\frac{U'(C_{t-1})}{U'(C_t)} = \beta(1 + r_t^D).$$

Market clearing for the capital:

$$K_t = A_t$$

Market clearing for the aggregate good:

$$C_t + K_{t+1} = F(K_t, G_t) + K_t.$$

Market clearing for the domestic debt:

$$D_{t+1} + \tau_t r_t^K K_t = (1 + r_t^D) D_t.$$

And interest rates of capital and domestic debt:

$$r_t^K = F'_K(K_t, G_t),$$

$$r_t^D = r_t^A(1 - \tau_t).$$

The equilibrium must also satisfy the NPG condition (for the sustainability of public debt) in Proposition 1 (i.e. $\max\{\sup_t \gamma_t, 0\} < \inf_t r_t^{BD} - \eta$) and the transversality condition $\lim_{t \rightarrow \infty} \beta^t K_t = 0$.⁵

4 Balanced growth path

Let us define $g_t \equiv \frac{G_t}{K_t}$, $b_t \equiv \frac{B_t}{K_t}$, $d_t \equiv \frac{D_t}{K_t}$, $c_t \equiv \frac{C_t}{K_t}$, $\xi_c \equiv \frac{C_{t+1}}{C_t}$, $\xi_b \equiv \frac{B_{t+1}}{B_t}$, $\xi_d \equiv \frac{D_{t+1}}{D_t}$, and $\xi_k \equiv \frac{K_{t+1}}{K_t}$. The solution for the model with the variables G_t , C_t , B_t , D_t , and K_t is equivalent to the solution with new variables g_t , c_t , b_t , and d_t . Equations (1), (2), (16), (23), and the good market clearing condition become

$$g_t + (1 + r_t^B)b_t + (1 + r_t^D)d_t = r_t^K \tau_t + (b_{t+1} + d_{t+1})\xi_k, \quad (24)$$

$$\tau_t r_t^K - g_t = \phi H g_t^{1-\alpha} + \eta(b_t + d_t), \quad (25)$$

$$\frac{c_{t+1}}{c_t} \xi_k = [\beta(1 + r_t^D)]^{1/\rho}, \quad (26)$$

$$c_t + \xi_k + d_{t+1}\xi_k = (1 - \tau_t)r_t^K + 1 + (1 + r_t^D)d_t, \quad (27)$$

⁵It should be noted that the model includes three predetermined variables (K, B, D) and one non-predetermined variable (C).

with $\xi_k = Hg_t^{1-\alpha} + 1 - c_t$.

By substituting equation (13) into equations (24)-(27) and by using the non arbitrage condition (21), we get the following system

$$g_t + (1 + r_t^B)b_t + [1 + (1 - \tau_t)\alpha Hg_t^{1-\alpha}] d_t = \alpha Hg_t^{1-\alpha}\tau_t + (b_{t+1} + d_{t+1})(Hg_t^{1-\alpha} + 1 - c_t) \quad (28)$$

$$\tau_t \alpha Hg_t^{1-\alpha} - g_t = \phi Hg_t^{1-\alpha} + \eta(b_t + d_t), \quad (29)$$

$$\frac{c_{t+1}}{c_t} (Hg_t^{1-\alpha} + 1 - c_t) = [\beta(1 + (1 - \tau_t)\alpha Hg_t^{1-\alpha})]^{1/\rho}, \quad (30)$$

$$c_t + (1 + d_{t+1})(Hg_t^{1-\alpha} + 1 - c_t) = [1 + (1 - \tau_t)\alpha Hg_t^{1-\alpha}] (1 + d_t). \quad (31)$$

A balanced growth path equilibrium is defined by $x_{t+1} = x_t = x^*$, $x = c, b, d, g$. The BGP is hence given by the following quantities:

$$g^* = \left(\frac{1 - \beta}{\beta(1 - \tau)\alpha H} \right)^{\frac{1}{1-\alpha}}, \quad (32)$$

$$c^* = \frac{1 - \beta}{\beta(1 - \tau)\alpha}, \quad (33)$$

$$d^* = \frac{1 - \alpha(1 - \tau)}{\alpha(1 - \tau)}, \quad (34)$$

$$b^* = \frac{1}{\eta} \left[\frac{(1 - \beta)\tau}{\beta(1 - \tau)} - \left(\frac{1 - \beta}{\beta(1 - \tau)\alpha H} \right)^{\frac{1}{1-\alpha}} - \frac{(1 - \beta)\phi}{\beta(1 - \tau)\alpha} - \eta \frac{1 - \alpha(1 - \tau)}{\alpha(1 - \tau)} \right], \quad (35)$$

and the following interest rates

$$r^{D*} = \frac{1 - \beta}{\beta}, \quad (36)$$

$$r^{K*} = \frac{1 - \beta}{\beta(1 - \tau)}. \quad (37)$$

The results show that the ratios of government expenditure, consumption, and domestic debt over private capital at the BGP depend on parameters such as tax rate (τ), discount rate (β), output elasticities (α and $1 - \alpha$), and technological level (H). In ad-

dition to these parameters, the BGP value of the ratio of external debt to capital also depend on the slopes of the budget surplus function with respect to output (ϕ) and total public debt (η). Furthermore, we observe that the interest rates of domestic debt and private capital at the BGP are determined only by the consumer's discount rate and the tax rate on returns to assets. At the BGP, tax rate has an impact on almost of all macroeconomic variables while it does not affect interest rate of domestic debt which is determined only by discount rate (β). It is easy to find that the relationship between interest rate of domestic debt (r^{D*}) and discounted rate (β) is negative because of negative derivative of the interest rate of domestic debt with respect to discount rate. In the next section, we will investigate the impact of tax rate on the rest of macroeconomic variables.

5 Impact of taxation on economy

The impacts of the tax rate on returns to assets on the macroeconomic variables of the model (government expenditure, consumption, domestic debt, external debt, and interest rate of capital) can be summarized in the following proposition.

Proposition 2 *Other things being equal,*

(i) g^* , c^* , d^* , and r^{K*} increase with τ ,

(ii) a) If $\alpha \leq \frac{\eta\beta}{1-\beta} + \phi$, b^* decreases with τ ,

b) If $\alpha > \frac{\eta\beta}{1-\beta} + \phi$, b^* increases with τ if $\tau < \bar{\tau}$ and decreases with τ if $\tau \geq \bar{\tau}$,

where

$$\bar{\tau} \equiv 1 - \frac{\left(\frac{1-\beta}{\alpha\beta H}\right)^{\frac{1}{\alpha}}}{\left[\left(\frac{1-\beta}{\beta}(\alpha - \phi) - \eta\right) \frac{1-\alpha}{\alpha}\right]^{\frac{1-\alpha}{\alpha}}}. \quad (38)$$

Proof. The derivatives of g^* , c^* , d^* , b^* with respect to τ can be obtained from equations (32)- (35) and (37):

$$\frac{\partial g^*}{\partial \tau} = \frac{1}{(1-\alpha)(1-\tau)^2} \left(\frac{1-\beta}{\alpha\beta H} \right)^{\frac{1}{1-\alpha}} \left(\frac{1}{1-\tau} \right)^{\frac{\alpha}{1-\alpha}} \geq 0, \quad (39)$$

$$\frac{\partial c^*}{\partial \tau} = \frac{1-\beta}{\alpha\beta(1-\tau)^2} \geq 0, \quad (40)$$

$$\frac{\partial d^*}{\partial \tau} = \frac{1}{\alpha(1-\tau)^2} \geq 0, \quad (41)$$

$$\frac{\partial b^*}{\partial \tau} = \frac{1}{\eta(1-\tau)^2} \left[\frac{(1-\beta)(\alpha-\phi) - \eta\beta}{\alpha\beta} - \frac{1}{1-\alpha} \left(\frac{1-\beta}{\alpha\beta H} \right)^{\frac{1}{1-\alpha}} \left(\frac{1}{1-\tau} \right)^{\frac{\alpha}{1-\alpha}} \right], \quad (42)$$

$$\frac{\partial r^{K*}}{\partial \tau} = \frac{1-\beta}{\beta(1-\tau)^2} \geq 0. \quad (43)$$

We observe that the derivative of g^* , c^* , d^* , and r^{K*} with respect to τ as given in equations (39), (40) and (41) are positive because $0 < \alpha, \beta, \tau < 1$, which verify points (i) of the proposition.

Finally, concerning the derivative of b^* with respect to τ , the result is ambiguous. Indeed, from equation (42), we can easily check that condition $\alpha \leq \frac{\eta\beta}{1-\beta} + \phi$ sufficiently implies that $\frac{\partial b^*}{\partial \tau} \leq 0$. However, when $\alpha > \frac{\eta\beta}{1-\beta} + \phi$, b^* can either increase or decrease depending on a threshold value of the tax rate. The latter is obtained after some arithmetic manipulation as

$$\bar{\tau} \equiv 1 - \frac{\left(\frac{1-\beta}{\alpha\beta H} \right)^{\frac{1}{\alpha}}}{\left[\left(\frac{1-\beta}{\beta} (\alpha - \phi) - \eta \right) \frac{1-\alpha}{\alpha} \right]^{\frac{1-\alpha}{\alpha}}}.$$

As a result, in the case of $\alpha > \frac{\eta\beta}{1-\beta} + \phi$, external debt increases with τ if τ is lower than this threshold and decreases if τ is higher. This verifies point (ii.b) of the proposition. ■

This result means that at the steady state if tax rate τ increases, government expenditure g^* , consumption c^* , domestic debt d^* , and interest rate on private assets r^{K*} increase. In other words, an increase in tax rate on returns to assets boosts government expenditure, consumption, domestic debt, and interest rate on capital. Indeed, an in-

crease of the tax rate on returns to private assets (or capital) implies a reallocation of the household income in favor of government bond (or domestic debt, d^*) to the detriment of private assets. However, this tax increase will raise interest rate on private assets (r^{K^*}), which in turn makes private assets more attractive than government bond. The positive effect on government bond dominates the negative one, leading to an increase of government bond at the steady state.

Regarding government expenditure, a tax increase raises interest rate r^{K^*} and then the quantity associated to tax revenue (τr^{K^*}), which fosters government expenditure (according to the government budget constraint).

Concerning consumption, an increase of tax rate on returns to private assets diminishes total available income. Consequently, the consumer reallocates her income in favor of consumption (c^*). However, this tax increase makes capital more attractive than consumption as a consequence of a rise of interest on private assets. It results in a higher output as both private capital and public expenditure rise after a tax increase. The positive effect on consumption dominates the negative one, resulting in an expansion of consumption with respect to tax rate at the steady-state.

In terms of impact of tax rate on external debt, if the productivity of physical capital (α) is too small or the ratio of debt to GDP (ϕ) and the feedback parameter of public debt (η) are large, external debt decreases with the tax rate. This result explains that when the productivity of capital is small enough (or the debt ratio and the feedback parameter are sufficiently large), a tax increase generates a higher difficulty to the government to borrow money from international financial markets (i.e. negative relation between b^* and τ). However, if the productivity of capital is sufficiently high (or the debt ratio and the feedback parameter are sufficiently small), external debt rises if the tax rate is lower than a certain threshold and it diminishes when tax rate is larger than this threshold. This gives rise to a bell-shaped form relation between external debt and tax rate in the case of

high productivity of capital. Obviously, in this situation, an increase of tax rate (when it is still low enough) is well supported by the economy and external debt rises. On the contrary, when the tax rate is at a too high level, it becomes harmful for the economy as the payment ability of the government becomes lower and then it is too hard to borrow from international financial markets.

6 Conclusion

In this paper, we investigate the relationship between government expenditure, tax on asset returns, economic growth, and public debt. The main message of the paper, which considers both domestic and external debt in a Barro growth model, resides in Proposition 2. In particular, the paper shows that an increase of tax on returns to assets can positively impact the steady-state values of main macroeconomic variables expressed in ratios of physical capital (consumption, public expenditure, domestic debt). This result is consistent with Greiner (2007) in the case of income tax and Angelopoulos et al. (2007) when capital income tax is considered.

Regarding external debt, the analysis underlines the key role of three parameters: the productivity of private capital, the size of public debt ratio, and the feedback parameter of public debt. The relation between external debt and tax rate at the steady state depends on the relative values of these three parameters. If the productivity of physical capital is small or the ratio of debt is large (or the feedback parameter of public debt is large), the effect of taxation is negative. As the situation of low productivity usually arises in poor countries where governments control almost of economic activities, we can expect that an increase in tax rate leads to a reduction in external debt (similarly to the finding of Greiner 2007 in the case of income tax). On the opposite, in case of high productivity of capital (often observed in developed countries), an increase in tax rate

can boost external debt as long as tax rate does not exceed a certain threshold, otherwise, the relation is decreasing.

In a further study, we will develop the welfare aspect of this model. It would also be worth addressing the question of optimal growth and investigating how the tax rate can be set in order to maximize welfare.

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