# Profit-maximizing Wages under Duopoly 

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#### Abstract

Using a duopoly model with endogenous order of moves, this study provides a potential explanation for why firms might pay their employees higher wages than rival firms or the market-clearing rate: Setting higher wages can serve as a commitment device to obtain the preferred order of moves in subsequent price competition. This holds even if the wage increase does not enhance worker productivity or efficiency. Simultaneous wage setting admits no pure-strategy Nash equilibrium, as their best responses form a cycle wherein firms repeatedly overbid in wages. Sequential wage setting leads to wage dispersion even among homogeneous workers and firms: the wage-setting leader offers high wages such that the rival firm would not want to overbid in equilibrium. In contrast, in quantity competition, duopolists have no such incentives because a firm that pays a wage higher than the competitor will be unsuccessful in obtaining first-mover advantages in subsequent quantity competition.


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Keywords: Endogenous timing; price leadership; wage setting; heterogeneous duopoly; wage commitment.

[^0]
## 1 Introduction

There are several theories explaining why firms pay more than the market-clearing wage rate. The efficiency-wage hypothesis suggests that firms do so to elicit greater worker effort (e.g., Shapiro and Stiglitz 1984) or to reduce costly labor turnover (e.g., Stiglitz 1974). The giftexchange hypothesis suggests that firms may offer wages above the market-clearing rate as a gift to induce reciprocal worker effort (e.g., Akerlof 1982). Union-firm wage bargaining may also raise a wage rate above the market-clearing level (e.g., McDonald and Solow 1981).

In this study, we provide another theoretical rationale for why firms might pay their employees a higher wage than the market-clearing rate or more than their rival firm does: increasing wage rates can give the firm the advantageous order of moves in future competition. This holds even when the wage increase is just a transfer from employers to employees, meaning that it does not enhance worker productivity.

We adopt a simple model of duopoly with endogenous order of moves by including a pre-stage, in which firms set and announce their wage rate. The model consists of two firms producing horizontally differentiated products using material (or capital) and labor. The firms can flexibly hire any number of workers in a perfectly competitive labor market at a market-clearing wage rate or above. In the first stage, firms simultaneously or sequentially publicize their wage rates. In the second stage, firms simultaneously or sequentially compete on price or quantity, in which the timing of moves in price or quantity setting is endogenously determined. We consider two different approaches to determine which firm, efficient or inefficient, will act as a leader in the market competition, namely, endogenous-timing or auction-like approaches. The first approach is to employ the "observable delay" game of Hamilton and Slutsky (1990) and the second one is to compare the two firms' maximum willingness to pay for the preferred timing of moves. We first show that in either case, a firm committing to a higher wage will take the advantageous second-mover position in subsequent price competition. In particular, in the endogenous-timing
approach, the situation where a more efficient firm leads and a less efficient firm follows is a risk-dominant Nash equilibrium. Additionally, in the auction-like approach, a less efficient firm's maximum willingness to pay for the position of the price-setting follower is greater than that of a more efficient firm. Therefore, in a duopoly price competition, firms have an incentive to pay more than the rival firm in order to commit to their high marginal cost and to profit from second-mover advantage, given the rival firm's wage rate. The result that higher costs leads to higher profits suggests the possibility that firms strategically offer higher wage rates even if these do not increase worker productivity or efficiency.

Second, we show that a simultaneous-move wage setting prior to price competition has a cyclical best-response structure in which firms keep overbidding each other in wages until the wage rate nears the point where it would be more profitable to set the market-clearing (or minimum) wages. Such overbidding can be seen as wage races to the top. We show that there is no pure-strategy subgame-perfect equilibrium of the game. Sequential-move wage setting yields a unique Nash equilibrium where a wage-setting leader offers wages that are way above the market-clearing rate and a wage-setting follower offers market-clearing wage rates. Finally, considering a quantity competition model with endogenous timing of moves, we show that a firm offering a higher wage will not have first-mover advantages, implying that firms never have an incentive to set a wage rate that is higher than the market-clearing one or a rival firm's one.

The main contributions of this study are twofold: First, this study is, as far as we know, the first attempt to model strategic pre-commitment to affect not only future competition but also the endogenously determined order of moves in the competition. A large volume of literature has investigated the endogenous timing of moves in various settings with price- and quantity-setting duopoly (Hamilton and Slutsky 1990; Deneckere and Kovenock 1992; van Damme and Hurkens 1999, 2004; Amir and Stepanova 2006). Hamilton and Slutsky (1990) provide a novel analysis of games of endogenous timing in duopoly games. Amir and Stepanova (2006) and van Damme and Hurkens (2004) consider a price-setting duopoly model with endogenous timing and show that
the risk-dominant Nash equilibrium of the game is the outcome of sequential play with the more efficient firm as price-setting leader and the less efficient firm as follower. However, these previous studies do not consider the strategic behavior of firms to preempt the preferred order of moves in price or quantity competition. Our study extends previous models by introducing a pre-stage of wage setting into price and quantity competition and investigates the firms strategic incentive to affect the endogenously determined order of moves in subsequent competition. Furthermore, our study also considers a case in which the order of moves is determined by an auction-like mechanism that compares the firms' willingness to pay for the preferred timing.

The second contribution of this study is that it introduces a mechanism of wage determination into a duopoly model and provides a new theoretical rationale for why firms might pay their employees a higher wage than the market-clearing rate. Even if higher wage rates do not enhance worker productivity and even if the labor market is competitive, firms may have an incentive to give their employees higher wages. Furthermore, our result can explain why wage dispersion arises even if firms and workers are homogeneous. The wage dispersion between firms is the result of strategic motives to win the preferred order of moves in competition. In particular, our sequential wage-setting game yields an especially large wage dispersion in which one firm chooses the high wage rate and the other, does the lowest one. ${ }^{1}$ We also show that the wage premium that a wage-setting leader pays is greater when their products are more differentiated and/or when the firm has smaller marginal material costs than does its rival.

[^1]This article is organized as follows. Section 2 describes the basic framework of our model and characterizes the Nash equilibrium of the simultaneous-move and sequential-move price competition. We then consider two approaches to determine the endogenous timing of moves in price competition and characterize the equilibrium in the simultaneous-move and sequential-move wage setting. Section 3 considers the case of quantity competition instead of price competition. Section 4 briefly mentions potential applications of our model to other contexts and draws lessons for management strategy from the results. Section 5 concludes the paper.

## 2 The Model of Price Competition

Consider a continuum of consumers to be distributed uniformly on a Hotelling line segment $[0,1]$ with mass $1 .{ }^{2}$ The location of an arbitrary consumer indexed by $x \in[0,1]$ is associated with his/her preferences. There are only two competing firms indexed by $i=1,2$ in this market. The firms are located at either end of the unit interval, reflecting horizontal product differentiation: Firm 1 is located at 0 and Firm 2 is located at 1 . Each firm $i$ sells a good $i$ at a uniform price $p_{i}$. The utility of each consumer $x$ is defined by

$$
U(x)= \begin{cases}v-p_{1}-t x & \text { if bought from Firm 1 } \\ v-p_{2}-t(1-x) & \text { if bought from Firm 2 }\end{cases}
$$

where $v$ is the value of consumer $x$ on his/her ideal product, and $t x$ and $t(1-x)$ are the costs of buying a brand that is different from the consumer's ideal choice. Let $\hat{x}$ denote the marginal consumer who is indifferent between purchasing goods 1 and 2 . Hence, we have

$$
\hat{x}=\frac{-p_{1}+p_{2}+t}{2 t}
$$

Therefore, the demand for goods 1 and 2 are $y_{1}=\hat{x}$ and $y_{2}=1-\hat{x}$.

[^2]Firm $i$ 's profits are given by $\pi_{i}\left(p_{i}, p_{j}\right)=\left[p_{i}-\gamma_{i}\left(c_{i}, w_{i}\right)\right] y_{i}$, where $\gamma_{i}\left(c_{i}, w_{i}\right)$ is the marginal cost for firm $i$ and $c_{i} \geq 0$ is the marginal material (or capital) cost and $w_{i} \in\left[w^{*}, \infty\right.$ ) is the endogenously determined wage rate (or marginal labor cost) of firm $i$. The lower bound of wage rate $w^{*}$ corresponds to the market-clearing wage rate or the legal minimum wage rate. We assume that firms have some market power in the product market, but not in the labor market. Therefore, the firms can flexibly hire any number of homogeneous workers in a perfectly competitive labor market at $w^{*}$ or above. For simplicity, we assume $\gamma_{i}\left(c_{i}, w_{i}\right) \equiv c_{i}+w_{i}$, implying that one unit of material (or capital) and labor is needed to product one unit of product. ${ }^{3}$ We treat the material costs as exogenous because they tend to be less flexible in the short run compared to the wage rates. The material costs can also be interpreted as the productivity or production efficiency of the firm. Here we call a firm that has smaller (larger) $c_{i}$ as a low-cost (high-cost) firm. The wage rate is an endogenous variable that can be chosen freely by each firm, unless the rate is below the market-clearing or minimum wage rate. We assume that high wages do not improve worker productivity or efficiency in production: wages are only transfers between the employer and employees. However, the choice of wage rate can serve as a commitment; firms can obtain information about changes in the wage rates offered by other firms from, for example, job magazines.

The timing of the game is as follows: in the first stage of the game, each firm simultaneously or sequentially sets/announces its wage rate. In the second stage, each firm simultaneously or sequentially chooses its price. The timing of moves at the price-competition stage is determined in the interim between the first and second stages.

[^3]
### 2.1 Simultaneous and sequential price-setting

First, we derive a second-stage Nash equilibrium in the case where prices are chosen simultaneously by both firms. Profit maximization yields the reaction functions given by

$$
\begin{equation*}
p_{i}=R_{i}\left(p_{j}\right) \equiv\left(t+\gamma_{i}+p_{j}\right) / 2, \tag{1}
\end{equation*}
$$

which indicates that the price choices are strategic complements since $R_{i}^{\prime}=1 / 2>0$. Solving (1) characterizes the second-stage Nash equilibrium:

$$
\begin{equation*}
p_{i}^{N}=\frac{3 t+2 \gamma_{i}+\gamma_{j}}{3}, \quad y_{i}^{N}=\frac{3 t-\gamma_{i}+\gamma_{j}}{6 t}, \quad \pi_{i}^{N}=\frac{\left(3 t-\gamma_{i}+\gamma_{j}\right)^{2}}{18 t} . \tag{2}
\end{equation*}
$$

The superscript $N$ refers to the equilibrium variable in the simultaneous-move price competition stage.

Second, we derive a second-stage equilibrium in the case where prices are chosen sequentially. Let us assume that Firm 1 is a leader $(L)$ and Firm 2 is a follower $(F)$ in choosing prices, which we call Case LF. In this case, the reaction function of the follower firm (Firm 2) is the same as (1), and the maximization problem for the leader firm (Firm 1) is $\max _{p_{1}} \pi_{1}\left(p_{1}, R_{2}\left(p_{1}\right)\right)$. We derive the subgame-perfect Nash equilibrium for Case LF as

$$
\begin{array}{ll}
p_{1}^{L}=\frac{3 t+\gamma_{1}+\gamma_{2}}{2}, & p_{2}^{F}=\frac{5 t+\gamma_{1}+3 \gamma_{2}}{4}, \\
y_{1}^{L}=\frac{3 t-\gamma_{1}+\gamma_{2}}{8 t}, & y_{2}^{F}=\frac{5 t+\gamma_{1}-\gamma_{2}}{8 t}, \\
\pi_{1}^{L}=\frac{\left(3 t-\gamma_{1}+\gamma_{2}\right)^{2}}{16 t}, & \pi_{2}^{F}=\frac{\left(5 t+\gamma_{1}-\gamma_{2}\right)^{2}}{32 t}, \tag{3}
\end{array}
$$

where superscripts $L$ and $F$ are used to denote equilibrium variables in the second stage of leader and follower, respectively. We find that when $\left|\gamma_{1}-\gamma_{2}\right|<3 t, y_{i}^{k}>0$ holds for all $i \in\{1,2\}$ and $k \in\{N, L, F\}$, that is, the market is not monopolized.

The second-stage Nash equilibrium for Case FL, where Firm 1 is a follower and Firm 2 is a leader in price setting, can be derived similarly.

### 2.2 Coordinating the timing of moves in price setting

Between the first and second stages of the game, the order of moves in the subsequent pricecompetition stage is determined endogenously. We consider two approaches to determine the order of moves: one is to employ the "observable delay" game developed by Hamilton and Slutsky (1990) and the other is what we call the auction-like approach, which compares the maximum willingness to pay for the preferred timing of moves between firms. We then show that the two different approaches yield qualitatively the same conclusion on the strategic nature of wage contracts.

### 2.2.1 Risk dominance in an endogenous timing game

First, we derive the plausible timing of moves in price competition by applying the endogenous timing (observable delay) game of Hamilton and Slutsky (1990). ${ }^{4}$ Firms are assumed to noncooperatively choose their preference between moving early or late in the subsequent stage: if both firms choose to move early (strategy Leads) or to move late (strategy Follows), a simultaneous-move price competition will be enforced in the second stage. If one firm chooses Leads and the other chooses Follows, a sequential-move price competition will be enforced in the second stage. Table 1 shows the normal form representation of the timing game.

Comparing $\pi_{i}^{F}$ and $\pi_{i}^{L}$ with $\pi_{i}^{N}$, we have

$$
\begin{align*}
& \pi_{i}^{F}-\pi_{i}^{N}=\frac{\left(\gamma_{i}-\gamma_{j}+3 t\right)\left[27 t-7\left(\gamma_{i}-\gamma_{j}\right)\right]}{288 t}>0,  \tag{4}\\
& \pi_{i}^{L}-\pi_{i}^{N}=\frac{\left(3 t-\gamma_{i}+\gamma_{j}\right)^{2}}{144 t}>0, \tag{5}
\end{align*}
$$

where the inequality comes from the assumption of $\left|\gamma_{i}-\gamma_{j}\right|<3 t$. The best responses are underlined in the payoff matrix of Table 1. This proves that the (pure strategy) Nash equilibria

[^4]| F1 $\backslash$ F2 | Leads | Follows |
| :--- | :--- | :--- |
| Leads | $\pi_{1}^{N}, \pi_{2}^{N}$ | $\underline{\pi_{1}^{L}}, \underline{\pi_{2}^{F}}$ |
| Follows | $\underline{\pi_{1}^{F}}, \underline{\pi_{2}^{L}}$ | $\pi_{1}^{N}, \pi_{2}^{N}$ |

Table 1: Payoff matrix of endogenous timing in the second stage
are $\{$ Leads, Follows $\}$ (Case LF) and $\{$ Follows, Leads $\}$ (Case FL) in this endogenous timing game. ${ }^{5}$

We then move on to the issue of coordination, i.e., the issue of how to select one of the two possible Nash equilibria. Here we apply the concept of risk dominance criterion (Harsanyi and Selten 1988). ${ }^{6}$ The equilibrium LF (Firm 1 leads and Firm 2 follows) risk dominates equilibrium FL if the former is associated with the larger product of deviation losses $\Theta .{ }^{7}$ Specifically, the condition is:

$$
\Theta \equiv\left(\pi_{1}^{L}-\pi_{1}^{N}\right)\left(\pi_{2}^{F}-\pi_{2}^{N}\right)-\left(\pi_{1}^{F}-\pi_{1}^{N}\right)\left(\pi_{2}^{L}-\pi_{2}^{N}\right)>0 .
$$

[^5]Using (2) and (3), we have

$$
\Theta=\frac{\left[15 t^{2}-\left(\gamma_{1}-\gamma_{2}\right)^{2}\right]\left(\gamma_{2}-\gamma_{1}\right)}{576 t}>0 \Leftrightarrow \gamma_{1}<\gamma_{2},
$$

which indicates that the equilibrium LF is a risk-dominant Nash equilibrium if and only if $\gamma_{1}<\gamma_{2}$ : the situation where a low-cost firm leads and a high-cost firm follows is a risk-dominant Nash equilibrium. ${ }^{8}$

### 2.2.2 Comparison of the maximum willingness to pay for moving late

Here, we take another approach, an auction-like approach, to determine the timing of moves in price competition. When each firm prefers to determine its own price after the rival firm has determined its price, the problem is which of the two firms can wait longer to set its price. Consider the situation where a delay in pricing causes both firms the same amount of losses (or opportunity costs) per unit of time. The firm whose benefits from being the second mover outweigh the losses from waiting, can wait longer. If both firms know that the benefits of becoming the second-mover for Firm 1 outweigh those for Firm 2, both firms can expect that, at a future date, Firm 2 will be the first to get impatient with waiting. In this case, Firm 2 does not have an incentive to wait and, thus, sets prices first without any delay (at time 0 ). Therefore, by comparing the firms' maximum willingness to pay for pricing late, we can identify which firm can wait more on price setting. That is why we call this as an auction-like approach.

We already know from (4) and (5) that each firm strictly prefers the sequential-move rather than the simultaneous-move price setting. In addition, from (2) and (3), we have

$$
\pi_{i}^{F}-\pi_{i}^{L}=\frac{7 t^{2}+2 t\left(\gamma_{i}-\gamma_{j}\right)-\left(\gamma_{i}-\gamma_{j}\right)^{2}}{32 t} \gtreqless 0 \Leftrightarrow \gamma_{i}-\gamma_{j} \gtreqless-(2 \sqrt{2}-1) t,
$$

which implies that if Firm $i$ 's marginal costs are much smaller than Firm $j$, then Firm $i$ prefers to be a leader rather than a follower and Firm $j$ prefers to be a follower. We immediately find

[^6]that, in this case, a low-cost firm chooses to be a leader whereas a high-cost firm chooses to be a follower. On the other hand, if the difference between $\gamma_{1}$ and $\gamma_{2}$ is smaller than $(2 \sqrt{2}-1) t$, then both firms prefer to be a follower. Then, the amount that Firm $i$ would be willing to pay for the position of price-setting follower is $b_{i} \equiv \pi_{i}^{F}-\pi_{i}^{L}$. We have
$$
b_{1}-b_{2}=\frac{\gamma_{1}-\gamma_{2}}{8}>0 \Leftrightarrow \gamma_{1}>\gamma_{2}
$$
which implies that the high-cost firm can wait more than the low-cost firm to become a secondmover (follower) in price competition: if both firms know $b_{1}>b_{2}$, which means that Firm 1 can wait more than Firm 2, then Firm 2 plunges into moving first because waiting only causes losses. ${ }^{9}$ As a result, the situation where a low-cost firm becomes a leader and a high-cost firm becomes a follower is a plausible equilibrium. ${ }^{10}$ Incidentally, the result that a more impatient (weaker) player immediately concedes also corresponds to the unique equilibrium of an asymmetric "war of attrition" game. ${ }^{11}$

Then we have the following result for the coordination of the timing of moves.

## Result 1

In our Hotelling price competition model, the situation in which a low-cost firm becomes a price-

[^7]setting leader and a high-cost firm becomes a price-setting follower is (i) a risk-dominant Nash equilibrium of the endogenous timing game, and (ii) a plausible equilibrium when the delay of pricing can cause losses for both firms.

### 2.3 Wage setting in the first stage

In the first stage, each firm non-cooperatively chooses $w_{i} \in\left[w^{*}, \infty\right)$, where the lower limit $w^{*}$ is the market-clearing wage or minimum wage rate. We first show that there is no pure-strategy Nash equilibrium of the simultaneous wage-setting stage. Then, we investigate the equilibrium of sequential wage setting and show that either firm offers wages that exceed the market-clearing level to preempt the second-mover position in price competition.

### 2.3.1 Simultaneous wage setting

We first consider the case in which each firm simultaneously chooses its wage rate in the first stage of the game. Hereafter, we denote the second-stage equilibrium profits of firm $i$ for $k \in\{N, L, F\}$ as $\pi_{i}^{k}\left(w_{i} ; w_{j}\right)$, in which the first element is the firm's own wage rate and the second, the rival's wage rate.

Let us derive each firm's best responses to its rival's wage. As we already know from Result 1, firms expect that a relatively high-cost firm will become a follower in the price competition stage. When Firm $j$ chooses $w_{j}$, Firm $i$ can become a follower by setting $w_{i}^{\prime} \equiv w_{j}+c_{j}-c_{i}+\epsilon$ such that $\gamma_{i}>\gamma_{j}$, where $\epsilon$ is an arbitrary small positive value. Hereafter, we call the strategy for choosing $w_{i}^{\prime}$ the wage overbidding. From (3), the associated profits are given by

$$
\begin{equation*}
\lim _{\epsilon \rightarrow 0} \pi_{i}^{F}\left(w_{i}^{\prime} ; w_{j}\right)=\frac{25 t}{32} \tag{6}
\end{equation*}
$$

for any $w_{j} \geq w^{*}$. When Firm $i$ sets its wage rates such that $w_{i}<w_{j}+c_{j}-c_{i}$, then Firm $i$ becomes a leader in the price competition stage. In that case, Firm $i$ 's best strategy is choosing $w^{*}$, the lowest possible wage rate, because setting $w_{i} \in\left(w^{*}, w_{j}+c_{j}-c_{i}\right)$ simply increases the marginal production costs and provides no benefits to the firm. Therefore, the associated profits,


Figure 1: Reaction functions in wage setting: the cases of $c_{1}=c_{2}$ (left) and $c_{1}>c_{2}$ (right)
for any $w_{j} \geq w^{*}$, are given by

$$
\begin{equation*}
\pi_{i}^{L}\left(w^{*} ; w_{j}\right)=\frac{\left[3 t-w^{*}+w_{j}-\left(c_{i}-c_{j}\right)\right]^{2}}{16 t} . \tag{7}
\end{equation*}
$$

Comparing (6) with (7), we find that Firm $i$ 's best response is to choose

$$
w_{i}= \begin{cases}w_{j}+c_{j}-c_{i}+\epsilon & \text { for } w_{j}<\bar{w}_{j}  \tag{8}\\ w^{*} & \text { for } w_{j} \geq \bar{w}_{j}\end{cases}
$$

where

$$
\begin{equation*}
\bar{w}_{j} \equiv w^{*}+(5 \sqrt{2}-6) t / 2+\left(c_{i}-c_{j}\right) . \tag{9}
\end{equation*}
$$

Equation (8) implies that Firm $i$ chooses wage overbidding when the rival's wages are lower than the threshold rate $\bar{w}_{j}$ and chooses the market-clearing wage rate when the rival's wages are higher than $\bar{w}_{j}$.

Now we have the following result:

## Result 2

In the simultaneous wage-setting stage, there does not exist a Nash equilibrium in pure strategies.

Proof: We prove the case in which marginal material costs are the same $\left(c_{1}=c_{2}\right)$. From the best response functions (8), if firms start with the market clearing wage $w^{*}$, then each firm initially has an incentive to set its wage rate that exceeds its rival's to have the second-mover position in
subsequent price competition. However, (8) and (9) indicate that once one firm's (say Firm 1's) wage rate reaches a threshold wage $\bar{w}_{1}$, it is more profitable for Firm 2 to set the lower-bound wage rate (a market clearing wage $w^{*}$ ) than to overbid it. Then, Firm 1 responds by overbidding Firm 2. This completes the cycle. Therefore, there is no pure-strategy Nash equilibrium. We can immediately see that there is no pure strategy Nash equilibrium in the case of asymmetric marginal material costs $\left(c_{1} \neq c_{2}\right)$, as depicted in the right panel of Figure 1.

Figure 1 illustrates the reaction functions in the case of $c_{1}=c_{2}$ and $c_{1}>c_{2}$. We can see from the figure that the reaction function of Firm $i$ is upward-sloping over the rival's wage $w_{j} \in\left[w^{*}, \bar{w}_{j}\right)$ and flat over $w_{j} \in\left[\bar{w}_{j}, \infty\right)$. For $w_{1} \in\left[w^{*}, \bar{w}_{1}\right]$ and $w_{2} \in\left[w^{*}, \bar{w}_{2}\right]$, each firm has a strategic incentive to set its wage rate above that of the rival's to gain the advantageous order of moves in price competition, implying the wage race to the top. However, once firm $j$ sets the threshold wage rate $\bar{w}_{j}$, another firm $i$ chooses to $w^{*}$, so the wage rates evolve in cycles like Edgeworth cycles (Edgeworth 1925)..$^{12}$ Because their reaction functions have no intersections, this indicates there is no pure-strategy Nash equilibrium in this game. ${ }^{13}$

### 2.3.2 Sequential wage setting

As shown above, there is no pure strategy Nash equilibrium in the case of simultaneous wage setting. Here, we consider a sequential wage setting in the first stage in order to investigate

[^8]firms' incentives to offer wages that exceed a market-clearing level. Without loss of generality, we assume $c_{1}>c_{2}$, that is, Firm 1 has higher marginal material costs than Firm 2.

First, we consider a case where Firm 1 (a high-cost firm) chooses its wage rate first, and then Firm 2 chooses its wage rate after observing Firm 1's choice. If Firm 2 chooses $w^{*}$ after observing $w_{1}$, then Firm 2 must become a first-mover in subsequent price competition for any $w_{1} \in\left[w^{*}, \infty\right)$ because $c_{1}>c_{2}$. The associated profits are

$$
\pi_{2}^{L}\left(w^{*} ; w_{1}\right)=\frac{\left(3 t+w_{1}-w^{*}+c_{1}-c_{2}\right)^{2}}{16}
$$

If Firm 2 chooses wage overbidding, $w_{2}^{\prime}=w_{1}+c_{1}-c_{2}+\epsilon$, then $\gamma_{1}<\gamma_{2}$ holds and Firm 2 will preempt a second-mover position in the subsequent price competition. The associated profits are

$$
\lim _{\epsilon \rightarrow 0} \pi_{2}^{F}\left(w_{2}^{\prime} ; w_{1}\right)=\frac{25 t}{32}
$$

Comparing them yields

$$
\pi_{2}^{L}\left(w^{*} ; w_{1}\right)-\lim _{\epsilon \rightarrow 0} \pi_{2}^{F}\left(w_{2}^{\prime} ; w_{1}\right)>0 \Leftrightarrow w_{1}>w^{*}+(5 \sqrt{2}-6) t / 2-\left(c_{1}-c_{2}\right)=\bar{w}_{1}
$$

If Firm 1 sets its wage rate $w_{1}$ higher than $\bar{w}_{1}$, then Firm 2 has no incentives to overbid and sets $w_{2}$ at the market-clearing rate $w^{*}$. Notice that if $\left(c_{1}-c_{2}\right)>(5 \sqrt{2}-6) t / 2$, then $\bar{w}_{1}<w^{*}$ necessarily holds, indicating that Firm 2 has no incentive to overbid even when Firm 1 offers the lowest wage $w^{*}$.

Now we investigate Firm 1's choice. If Firm 1 sets $w_{1}=\bar{w}_{1}+\epsilon$, then Firm 2 will give up overbidding the wage and set $w_{2}=w^{*}$. In this case, Firm 1 will become a follower in the price competition stage. This yields the payoff

$$
\begin{equation*}
\lim _{\epsilon \rightarrow 0} \pi_{1}^{F}\left(\bar{w}_{1}+\epsilon ; w^{*}\right)=\frac{(16-5 \sqrt{2})^{2} t}{128} \tag{10}
\end{equation*}
$$

If Firm 1 sets its wage at the lowest possible rate, $w^{*}$, Firm 2 will overbid it. In this case, Firm 1's profits are

$$
\begin{equation*}
\lim _{\epsilon \rightarrow 0} \pi_{1}^{L}\left(w^{*} ; w_{2}^{\prime}\right)=\frac{9 t}{16} \tag{11}
\end{equation*}
$$

Comparing (10) with (11) yields

$$
\frac{(16-5 \sqrt{2})^{2} t}{128}-\frac{9 t}{16}=\frac{(117-80 \sqrt{2}) t}{64}>0
$$

which indicates that by setting $w_{1}=\bar{w}_{1}+\epsilon$, which discourages Firm 2 from overbidding, it is beneficial for Firm 1 if $\left(c_{1}-c_{2}\right)<(5 \sqrt{2}-6) t / 2$. In the case of $\left(c_{1}-c_{2}\right) \geq(5 \sqrt{2}-6) t / 2$, it holds that $\bar{w}_{1}<w^{*}$, implying that Firm 1 chooses $w^{*}$. Therefore, we have the following proposition on the equilibrium of this sequential-move wage setting stage.

## Result 3

Suppose $c_{1}>c_{2}$. If Firm 1 can set its wage rate first, the equilibrium is characterized by

$$
\left(w_{1}, w_{2}\right)= \begin{cases}\left(\bar{w}_{1}+\epsilon, w^{*}\right) & \text { for }\left(c_{1}-c_{2}\right)<(5 \sqrt{2}-6) t / 2 \\ \left(w^{*}, w^{*}\right) & \text { for }\left(c_{1}-c_{2}\right) \geq(5 \sqrt{2}-6) t / 2\end{cases}
$$

where $\bar{w}_{1}=w^{*}+(5 \sqrt{2}-6) t / 2-\left(c_{1}-c_{2}\right)$.

Second, we consider a case in which Firm 2 (a more efficient firm) is a wage-setting leader. The best response of Firm 1 (the wage-setting follower) against $w_{2}$ is to overbid $\gamma_{2}$ by setting $w_{1}^{\prime}=w_{2}+c_{2}-c_{1}+\epsilon$ or to set the market-clearing wage rate $w^{*}$. Comparing them yields

$$
\begin{aligned}
\pi_{1}^{F}\left(w_{1}^{\prime} ; w_{2}\right)-\pi_{1}^{L}\left(w^{*} ; w_{2}\right) & =\frac{25 t}{32}-\frac{\left(3 t+w_{2}-w^{*}-c_{1}+c_{2}\right)^{2}}{16 t} \gtreqless 0 \\
& \Leftrightarrow \quad w_{2} \lesseqgtr w^{*}+(5 \sqrt{2}-6) t / 2+c_{1}-c_{2}=\bar{w}_{2}
\end{aligned}
$$

which indicates that if Firm 2 sets $w_{2}$ higher than $\bar{w}_{2}$, then Firm 1 chooses not to overbid it and sets $w_{1}=w^{*}$.

Next, we check the profitability of Firm 2. If Firm 2 sets $w_{2}=\bar{w}_{2}+\epsilon$, then Firm 1 chooses $w_{1}=w^{*}$. The associated profits for Firm 2 are

$$
\begin{equation*}
\lim _{\epsilon \rightarrow 0} \pi_{2}^{F}\left(\bar{w}_{2}+\epsilon ; w^{*}\right)=\frac{(16-5 \sqrt{2})^{2} t}{128} \tag{12}
\end{equation*}
$$

If Firm 2 sets $w_{2}=w^{*}$, then Firm 1 just has to choose $w_{1}=w^{*}$ to preempt the price-setting follower's position and, therefore, Firm 2 becomes a price-setting leader in price competition.

The associated profits are

$$
\begin{equation*}
\pi_{2}^{L}\left(w^{*} ; w^{*}\right)=\frac{\left(3 t+c_{1}-c_{2}\right)^{2}}{16 t} \tag{13}
\end{equation*}
$$

Comparing (12) with (13) yields

$$
\begin{aligned}
\frac{(16-5 \sqrt{2})^{2} t}{128}-\frac{\left(3 t+c_{1}-c_{2}\right)^{2}}{16 t} & =\frac{(153-80 \sqrt{2}) t^{2}-4\left(3 t+c_{1}-c_{2}\right)^{2}}{64 t} \gtreqless 0 \\
& \Leftrightarrow \quad\left(c_{1}-c_{2}\right) \lesseqgtr(8 \sqrt{2}-11) t / 2
\end{aligned}
$$

## Result 4

Suppose $c_{1}>c_{2}$. If Firm 2 can set its wage rate first, the equilibrium is characterized by

$$
\left(w_{1}, w_{2}\right)= \begin{cases}\left(w^{*}, \bar{w}_{2}+\epsilon\right) & \text { for }\left(c_{1}-c_{2}\right)<(8 \sqrt{2}-11) t / 2 \\ \left(w^{*}, w^{*}\right) & \text { for }\left(c_{1}-c_{2}\right) \geq(8 \sqrt{2}-11) t / 2\end{cases}
$$

where $\bar{w}_{2}=w^{*}+(5 \sqrt{2}-6) t / 2+c_{1}-c_{2}$.

The two results indicate that if the difference in the marginal material costs between firms is small (or if the productivity gap between firms is small), the wage-setting leader strategically chooses a wage rate that is higher than the market-clearing one to preempt the second-mover position in the subsequent price competition, whereas the wage-setting follower chooses the market-clearing wage rate. In other words, substantial wage dispersion arises if firms have strategic motives in setting their wage rate. Notice that such wage dispersion does not stem from the sequential choices in wage rate: if the timing of moves in price competition are exogenously given, both the wage-setting leader and the follower choose $w^{*}$.

From Results 3 and 4 , we find $\bar{w}_{1}<\bar{w}_{2}$ for $c_{1}>c_{2}$ and $\partial \bar{w}_{i} / \partial t>0$ for $i=1,2$. Now we have the following results on wage premium that a wage-setting leader offers.

## Result 5

The wage premium that would be required for a more efficient firm to discourage the less efficient rival firm from overbidding is larger than that for a less efficient firm. Furthermore, the more differentiated the products are, the higher is the wage premium that a wage-setting leader offers.

Therefore, the wage dispersion between firms widens as products are differentiated. The intuition is simple. If the products are homogenous $(t=0)$, price competition leads to marginal cost pricing, implying that the second-mover advantages vanish. Because the second-mover advantage $\left(\pi_{i}^{F}-\pi_{i}^{L}\right)$ is an increasing function in $t$, the wage-setting leader can pay higher wages for its employees to preempt the advantageous position as $t$ increases. As is obvious, if $t$ is extremely large, the market would not be fully covered and both firms would be locally monopolistic, setting a minimum wage rate.

Two caveats are necessary. First, although our results indicate that a wage-setting leader has strategic incentives to offer wages that exceed a market-clearing level in order to preempt the position of the second mover in price competition, the profits of the wage-setting follower are necessarily greater than those of the wage-setting leader. Defining the wage-setting leader's and follower's profits in the subgame-perfect Nash equilibrium as $\pi_{i}^{W L}$ and $\pi_{i}^{W F}$, respectively, we can confirm that from

$$
\begin{equation*}
\pi_{1}^{W L}-\pi_{2}^{W F}=\left[\pi_{1}^{F}\left(\bar{w}_{1} ; w^{*}\right)-\pi_{2}^{L}\left(w^{*} ; \bar{w}_{1}\right)\right]=-\frac{(80 \sqrt{2}-103) t}{64}<0 \tag{14}
\end{equation*}
$$

where the first bracket represents the difference of profits between firms when Firm 1 is a wage-setting leader and sets its wage at $\bar{w}_{1}$ (in this case, Firm 1 becomes the follower in the subsequent price competition) and the second bracket represents the difference of profits when Firm 2 is a wage-setting leader and sets its wage at $\bar{w}_{2}$. This indicates that a firm that pays $w^{*}$ (a wage-setting follower) realizes greater profits than the rival firm that pays more than the market-clearing wage because of its smaller total marginal costs. Also, this result holds for asymmetric material costs between firms.

Second, we do not explicitly consider which firm would be a wage-setting leader or follower. As we can see from (14), the wage-setting leader who commits to higher wages loses, implying both firms want to be a wage-setting follower. Therefore, as in price competition, firms prefer to move late in their wage announcement. Which firm can wait more? Unfortunately, we cannot
answer that question in our model. The reasons are (i) the simultaneous-move wage setting has no pure-strategy Nash equilibrium, which implies that we cannot model the coordination in timing of moves in the wage-setting stage by employing the endogenous timing game; (ii) each firm's maximum willingness to pay for moving late for wage setting is the same even under asymmetric marginal material costs:

$$
\pi_{1}^{W F}-\pi_{1}^{W L}=\pi_{2}^{W F}-\pi_{2}^{W L}=(80 \sqrt{2}-103) t / 64
$$

## 3 The Model of Quantity Competition

This section investigates the case in which firms compete on quantity (not price) in the second stage, and shows that neither firm has an incentive to offer wages that exceed a market-clearing level under quantity competition.

Consider two firms $i \in\{1,2\}$ producing homogeneous products with a linear inverse demand $P=1-Y_{1}-Y_{2}$, where $Y_{i}$ is Firm $i$ 's output. Profits are $\Pi_{i}=\left(P-\gamma_{i}\right) Y_{i}$, where $\gamma_{i} \equiv c_{i}+w_{i}$ is the total marginal costs, $c_{i}$ is the marginal material costs, and $w_{i}$ is the wage rate for Firm $i$. The structure of the game is the same as before except for the second stage.

After some manipulations, we easily obtain the second-stage equilibrium in the simultaneousmove quantity competition as

$$
Y_{i}^{N}=\frac{1-2 \gamma_{i}+\gamma_{j}}{3}, \quad \Pi_{i}^{N}=\left(\frac{1-2 \gamma_{i}+\gamma_{j}}{3}\right)^{2}
$$

and that in the sequential-move quantity competition as

$$
Y_{i}^{L}=\frac{1-2 \gamma_{i}+\gamma_{j}}{2}, \quad Y_{i}^{F}=\frac{1-3 \gamma_{i}+2 \gamma_{j}}{4}, \quad \Pi_{i}^{L}=\frac{\left(1-2 \gamma_{i}+\gamma_{j}\right)^{2}}{8}, \quad \Pi_{i}^{F}=\frac{\left(1-3 \gamma_{i}+2 \gamma_{j}\right)^{2}}{16}
$$

where the superscript $N$ refers to the variable in the equilibrium of simultaneous-move quantity competition, the superscripts $L$ and $F$ refer to the leader's and follower's variable in the equilibrium of sequential-move quantity competition.

As before, we consider two approaches to determine the timing of moves: endogenous timing (observable delay) and auction-like approaches. First, we immediately find that the simultaneousmove timing is the unique Nash equilibrium of the endogenous timing game because $\Pi_{i}^{L}>\Pi_{i}^{N}>$ $\Pi_{i}^{F}$ and therefore, the strategy Leads is the dominant strategy for both firms. Second, we consider an auction-like approach that compares bids for the right to preferred order of moves. Because the first mover has an advantage in the sequential quantity competition game, each firm bids for the right to first-mover position of the first mover. In that case, Firm $i$ 's maximum willingness to bid to move first is $B_{i}=\Pi_{L}-\Pi_{F}$. Then we have

$$
B_{1}-B_{2}=\frac{\left(2-\gamma_{1}-\gamma_{2}\right)\left(\gamma_{2}-\gamma_{1}\right)}{16}>0 \Leftrightarrow \gamma_{1}<\gamma_{2},
$$

which indicates that the maximum willingness to pay for moving first of a relatively low-cost firm is greater than that of a high-cost firm. In other words, a low-cost firm can bid off the right to move first and enjoy first-mover advantages.

In the first stage, each firm chooses its wage rate. We can immediately see that none of the firms has an incentive to raise its wage rate for both second-stage rationales of endogenoustiming and bidding for the preferred order of moves. If the order of moves is decided through endogenous timing, a simultaneous-move situation is a Nash equilibrium irrespective of the relative magnitudes of $\gamma_{1}$ and $\gamma_{2}$, Thus, neither firm has an incentive to raise its marginal costs because $\partial \Pi_{i}^{N} / \partial \gamma_{i}<0, \forall i \in\{1,2\}$. If the order of moves is decided by an auction-like mechanism, as we have already shown, a low-cost firm will have the advantageous leader's position in quantity competition. Therefore, in the first stage, both firms offer the market-clearing wage rates because raising wage rates never affects the order of moves in quantity competition.

## Result 6

In quantity competition under duopoly, none of the firms has strategic incentives to set wage rates that are higher than the market-clearing rates.

## 4 Discussion

In this section, we briefly discuss several commitment strategies, other than wage commitment, for duopoly firms. Then, we explain why we focus on the wage contract as a commitment strategy by stating our belief in the superiority of it. Finally, we draw lessons for management strategy from our research.

Our results raise the theoretical possibility of offering higher wages as a means of winning an advantageous position in future competition. The important point here is that wage increases can serve as a commitment to higher total marginal costs than the rival firm's. From this aspect, other commitment strategies that affect future marginal costs may work as well. The first possible strategy is to engage in costly corporate social responsibility (CSR) activities, such as contributing a certain ratio of the total sales to a charity or paying "fair" wages to employees. Our results indicate that firms may engage in such strategic CSR activities even when these activities do not enhance brand image, customer satisfaction, or employee morale.

The second possible strategy is for firms to locate their plants far from the market (consuming area) to commit to their future high shipping costs. The strategy could be superior to the costly CSR activities in its ability to commit because the location choices are long-term commitments and cannot be reversed easily. Consider a case in which a firm incurs shipping costs that are proportional to the distance between its plants and the market center. Then, we can apply our model by interpreting $w_{i}$ as firm $i$ 's shipping cost, instead of wage rate. Our results suggest that if the duopoly firms decide their plant locations sequentially and then compete on price, the firm setting up its plant first (the leader) chooses its plant location far from the market to commit to future high shipping costs and preempt the second-mover advantages in price competition. However, the firm setting up its plant second (the follower) chooses its plant location as close as possible to the market to save shipping costs, implying geographic dispersion of plant locations. On the contrary, if the duopoly firms compete in output (or capacity), both firms choose their
plant location as close as possible to the market, implying geographic agglomeration of plant locations.

The third possible strategy for firms revolved around the adoption of more efficient production technologies. As shown in this study, having lower marginal production costs than a rival firm may give the rival an advantageous position in future price competition. Therefore, firms may intentionally go for lower technologies than their rivals or not update their production equipment even if more efficient production technologies are available to firms without any additional costs.

Although there are several strategies for preempting the advantageous position in future competition, as listed above, the strategy of setting higher wages can be the most effective and credible. This is because the wage contract (or announcement) is (i) a legally binding agreement between employers and employees (the third-party players) and therefore cannot be rescinded unilaterally, and (ii) is usually publicly observable to all, including the rival firm, through job information magazines or websites.

Our results may have several managerial implications in duopoly. First, managements facing price competition in the product market should determine wages after the rival firm does so. If the rival pays fairly low wages, the management can enjoy second-mover advantages in subsequent price competition for a small sacrifice by paying slightly higher wages such that its marginal costs outweigh those of the rival. In contrast, the rival may pay significantly higher wages to preempt the advantageous position in subsequent price competition. In that case, the management should offer the lowest possible wages because of the large benefit of cost reduction. Second, the management facing quantity competition in the product market should determine the lowest possible wages because it cannot have the advantageous order of moves in the competition by offering a high-wage contract.

## 5 Concluding Remarks

There are many theoretical studies of endogenous timing that examine whether duopoly firms move simultaneously or sequentially in a quantity/price competition and what types of firms would be a leader or follower. Taking a further step, we examine what strategic actions the firms would take to preempt the advantageous order of moves in future competition if they understand the mechanisms of such studies. We have focused on wage contracts as commitment devices. We have shown that in price competition under duopoly, firms have an incentive to set higher wage rates than the market-clearing rates in order to have the advantageous order of moves in price competition. This arises when the difference in marginal production costs between firms is not too large. Our findings provide a theoretical rationale for why firms might pay their employees higher wages even if the wage increase does not enhance worker productivity and why wage dispersion arises even if firms and workers are homogeneous.

We have considered strategic actions by the player himself to guarantee an advantageous position in future competition. Future research should also consider such strategic actions by third-party players. For example, consider the strategic trade policy model of Brander and Spencer (1985) and Eaton and Grossman (1986), in which a domestic firm and a foreign firm compete on price in a third market; both the domestic government may have an incentive to impose higher export taxes on its domestic firm than the foreign government, not only for causing each firm to export less but also for preempting an advantageous timing of moves in price competition. For another example, consider a case in which a domestic firm and a foreign firm compete on price in the domestic market. In this case, the domestic government usually has an incentive to impose a tariff on imports (foreign firm's product) with exogenous order of moves in market competition. However, if the order of moves is endogenous, the domestic government may have an incentive to subsidize imports in order to give the domestic firm second-mover advantage in price competition.

## Appendix

In this Appendix, we derive the mixed strategy Nash equilibrium in the endogenous timing game studied in Section 2.2.1 (see Table 1). Let $\phi_{i}$ and $1-\phi_{i}$ be the probability that firm $i$ chooses Leads and Follows, respectively. In the mixed strategy Nash equilibrium, the probability is obtained as

$$
\phi_{i}^{*}=\frac{2\left(3 t+\gamma_{i}-\gamma_{j}\right)^{2}}{99 t^{2}+6 t\left(\gamma_{i}-\gamma_{j}\right)-5\left(\gamma_{i}-\gamma_{j}\right)^{2}} .
$$

Then, we have

$$
\phi_{1}^{*}-\phi_{2}^{*}=\frac{144 t\left[15 t^{2}-\left(\gamma_{1}-\gamma_{2}\right)^{2}\right]\left(\gamma_{1}-\gamma_{2}\right)}{\left[99 t^{2}+6 t\left(\gamma_{1}-\gamma_{2}\right)-5\left(\gamma_{1}-\gamma_{2}\right)\right]\left[99 t^{2}-6 t\left(\gamma_{1}-\gamma_{2}\right)-5\left(\gamma_{1}-\gamma_{2}\right)\right]},
$$

which implies $\phi_{1}^{*} \geq \phi_{2}^{*}$ for $\gamma_{1} \geq \gamma_{2}$. In the mixed-strategy Nash equilibrium of the endogenous timing stage, the relatively high-cost firm is more likely to choose strategy Leads than is the relatively low-cost firm. However, the equilibrium is unstable. Therefore, we use the riskdominance criterion for selecting a stable pure-strategy Nash equilibrium in the main body.

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[^1]:    ${ }^{1}$ Many theoretical studies on frictional labor markets predict that wages can diverge among ex-ante homogeneous workers (e.g., Mortensen 1970; Lucas and Prescott 1974; Pissarides 1985; Mortensen and Pissarides 1994, among many others). In these studies, the wage dispersion arises when workers do not know the wages offered by all firms. See Mortensen (2005) for reviews of the theoretical and empirical examinations on this topic. Furthermore, our mechanism also differs from that of Ohnishi $(2003,2012)$. In his mechanism, a wage contract is one where employees receive a wage premium if they can produce more than a certain output level. Such a contract actually works as a commitment to produce a certain level of output before market competition. In contrast, our wage contract is not conditional on production levels or worker productivities.

[^2]:    ${ }^{2}$ Although we use the simplest possible Hotelling model to examine price competition, the results obtained in this study hold qualitatively, under certain assumptions, for more general demand structures such as the linear demand with horizontal product differentiation.

[^3]:    ${ }^{3}$ All of our results hold qualitatively for more general continuous marginal cost function, as long as $\partial \gamma_{i} / \partial c_{i} \geq 0$ and $\partial \gamma_{i} / \partial w_{i}>0$.

[^4]:    ${ }^{4}$ By employing the "action commitment" game of Hamilton and Slutsky (1990), van Damme and Hurkens $(1999,2004)$ show that the low-cost firm will emerge as the endogenous leader in both quantity- and price-setting duopoly situations. The action commitment game is quite different from the observable delay game used in our study. See Hamilton and Slutsky (1990) on this point.

[^5]:    ${ }^{5}$ In the Appendix, we present our derivation of the mixed strategy Nash equilibrium of the endogenous timing stage and show that a relatively high-cost firm is more likely to choose the strategy Leads than the low-cost firm. This result is quite in contrast with the results obtained by using the risk dominance criterion and by comparing the maximum willingness to pay for moving late described below. However, this mixed-strategy Nash equilibrium is unstable and is very unlikely to arise in practice because any small perturbation will lead to the breakdown of the equilibrium. Therefore, we use a risk-dominance criterion to select a stable pure-strategy Nash equilibrium.
    ${ }^{6}$ There are some justifications for why firms select a risk dominant Nash equilibrium. First, as shown in the seminal paper by Kandori et al. (1993), evolutionary learning processes force equilibrium selection in accordance with risk dominance. Second, as shown by Carlsson and van Damme (1993), iterated dominance in a global game forces players to conform to the risk dominance criterion.
    ${ }^{7}$ Obviously, we cannot select one risk-dominant Nash equilibrium if the firms choose their wage rate such that $\gamma_{1}=\gamma_{2}$ (or $w_{1}+c_{1}=w_{2}+c_{2}$ ) in the first stage of the game. However, no firm will choose its wage rate such that $\gamma_{1}=\gamma_{2}$ in the first stage as shown in Section 2.3.

[^6]:    ${ }^{8}$ Similar results are obtained by van Damme and Hurkens (2004) and Amir and Stepanova (2006). They show that a risk-dominant Nash equilibrium of the timing game of Bertrand price competition (not a Hotelling model of price competition as in our study) is sequential play with the low-cost (or efficient) firm as a price-setting leader.

[^7]:    ${ }^{9}$ This coordination mode, for example, corresponds to a situation where two firms are considering the best timing to run advertising on their product prices.
    ${ }^{10}$ If the firms chooses their wage rate such that $\gamma_{1}=\gamma_{2}$ (i.e., $c_{1}+w_{1}=c_{2}+w_{2}$ ), then we have $b_{1}=b_{2}$, implying that we cannot identify which firm will be a price-setting leader or follower. However, such case never be an equilibrium in the first stage.
    ${ }^{11}$ The auction-like approach we consider here is a variant of the "War of Attrition" which has many economic applications such as the study of firm exit (Ghemawat and Nalebuff, 1985; Fudenberg and Tirole, 1986). In a war of attrition model with complete information, there are multiple equilibria with an instant exit of one player and equilibria involving constant hazard-rate exit. However, as shown by Kornhauser et al. (1989), an asymmetric war of attrition that includes a specific irrational strategy for each player yields a unique equilibrium in which a weaker player instantly concedes at the beginning of the game. Myatt (2005) also shows such a result of an instant exit of an "ex-ante" weaker player. In our model, a more impatient player who has a lower willingness to pay for waiting is a low-cost firm, implying the low-cost firm will instantly concede at the beginning of the price competition when we apply the model of Kornhauser et al. (1989) and Myatt (2005).

[^8]:    ${ }^{12}$ Edgeworth (1925) shows that there exists no price equilibrium in pure strategy when two firms face capacity constraints. He predicts a pattern of cycling in price over time. If firms start with a high price (e.g., monopoly price), then each firm has an incentive to undercut its rival's price. The undercutting continues until the war becomes too costly, at which point one firm increases its price. Then, the prices fall again. For more discussion on the Edgeworth cycle, see Maskin and Tirole (1988).
    ${ }^{13}$ Not surprisingly, there exists an equilibrium in mixed strategies in which firms randomize over wages: a mixed strategy equilibrium that consists of a pair of probability distributions over $w_{i} \in\left[w^{*}, \bar{w}_{i}\right]$ with the property that any strategy chosen with positive probability must be optimal against the rival's probability mixture. Derivation for the mixed strategy equilibrium is beyond our scope here.

