Pooling natural catastrophe risks in a community

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Abstract

We analyze the design of insurance contracts when individual risks are correlated across risk-averse agents in a community, which generates collective risk. The community is equipped with a public insurer that supplies insurance contracts to its members and has access to costly reinsurance outside the community. When reserves are costless, risk-averse agents fully insure their individual risk and share a fraction of the collective risk by setting insurer's reserves, from which they receive dividends when collective losses are low. When reserves are costly, they only partially insure their individual risk, hence obtaining a lower indemnity in case of high collective losses than in case of low collective losses. In addition, they may receive dividends in the latter case, depending on the trade-off between reserve costs and reinsurance costs. We illustrate the emergence of these mutual insurance contracts with indemnities and dividends contingent on collective losses for the community of the Caribbean countries exposed to natural catastrophe risks.

Keywords: individual risk, collective risk, insurance contracts, mutual insurance, natural catastrophe.

JEL classification: D86, G22, G28, Q54.

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1 Introduction

The recent Hurricane Matthew has reminded, if necessary, that the Caribbean countries are located in a region of the world widely exposed to large natural catastrophe risks. Even though aggregate damages per year in this region are usually below 1 billion dollars, much higher losses are expected in 2016 with Category 5 Hurricane Matthew striking Barbados, Saint Vincent and the Grenadines, Saint Lucia and Haiti.[1] In the last decade, year 2010 was also dramatic with more than 8 billion dollars of damages due to a devastating earthquake in Haiti and a highly above average hurricane season in the Caribbean.[2] In this context, the non-for-profit Caribbean Catastrophe Risk Insurance Facility (CCRIF) is designed to supply insurance contracts to the Caribbean countries (CCRIF SPC (2014),Cummins & Mahul (2009)). To deal with potential high collective losses due to the spatial risk correlation, the CCRIF purchases reinsurance outside the community and builds financial reserves. Since both reinsurance and reserves are costly because of reinsurance and financial market imperfections (Froot (2001), Gollier (2005), Ibragimov et al. (2009), Jaffee & Russell (1997), Kousky & Cooke (2012) and Zanjani (2002)), the CCRIF supplies to its members insurance contracts which are mutual in the sense that they depend on collective losses. Individual indemnities are lower when collective losses are high than when collective losses are low (which enables to limit reinsurance purchase and reserve building). Moreover, dividends[3] are redistributed to the insureds when collective losses are low and reserves remain after the payment of indemnities. The CCRIF, created in 2007, is one example of such facilities that have emerged in different regions of the world exposed to natural catastrophes in the last twenty years. The Florida Hurricane Catastrophe Fund (FHCF) and the California Earthquake Authority (CEA) respectively created in 1993 and 1996 are other examples (Kousky (2010) and Kunreuther & Michel-Kerjan (2009)).

The present paper analyzes the optimal design of insurance contracts by a pooling insurance facility when the collective risks are not negligible, reinsurance is above fair prices and reserves are costly. We consider a community of identical risk-averse agents (representing for instance the Caribbean countries) in a one-period model. Each agent faces two individual states: it can either suffer a loss or not. At the collective level,

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[1]Recent information on Hurricane Matthew can be found on the Weather Company website (https://weather.com/).

[2]Information on natural disaster losses in the Caribbean countries can be found on the EMDAT International Disaster Database (http://www.emdat.be/).

[3]Dividends are given through premium discounts after a year with low collective losses.

there are two states of nature, the normal one and the catastrophic one, respectively characterized by low and high fraction of the agents affected.[4] We consider a non-for-profit pooling insurance facility for the community (representing for instance the CCRIF for the Caribbean countries). The insurance facility supplies mutual insurance contracts to the agents in the community. For one contract, it charges a premium and pays an indemnity to the insured if affected. For an individual loss, the indemnity level in the normal state may differ from the indemnity level in the catastrophic state. The insurance contract may also include a dividend if the normal state occurs. Besides, the insurance facility has access to reinsurance outside the community. We analyze the characteristics of the optimal insurance and reinsurance contracts for the community, when reinsurance is above fair prices.

With costless reserves, the optimal insurance contract consists in full coverage for individual losses in both the normal state and the catastrophic state. Moreover, it includes a strictly positive dividend in the normal state to retain some of the collective risk inside the community. This retention of risk enables to lower the reinsurance bill.[5] The higher the cost of reinsurance, the higher the premium and the dividend because the insurer substitutes reinsurance by a higher reserve from the agents to pay the high total indemnities of the catastrophic state. In practice, requiring premiums ex-ante[6] generates a reserve cost for the agents in the community. Indeed, the immobilized capital cannot be used for other purpose (i.e consumption or investment).[7] In this case, it is Pareto improving to implement a contract with a lower indemnity for an individual loss in the catastrophic state than in the normal state. Moreover, the optimal contract still features full coverage and dividend in the normal state if and only if the marginal reserve cost is low enough relative to the marginal reinsurance cost.

Relative to the economics literature on optimal insurance contract in the presence of collective risks (Bernard (2013)), we relax simultaneously two assumptions to further

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[4]We consider only two individual states to keep the model tractable. At the collective level, we consider two and only two states of nature respectively to model collective risks and to keep the model tractable.

[5]Notice that the absence of reserve costs does not imply the absence of reinsurance purchase. Indeed, reinsurance allows to cover individual risks in the catastrophic state while handling collective risk whereas reserves do without handling collective risk.

[6]Premiums are required ex-ante to pay reinsurance premiums and to secure a reserve which avoids participation default. Moreover, raising the premiums ex-ante enables to transfer indemnities quickly to affected agents.

[7]For this other purpose, the agents would have to borrow more costly external capital instead of using the immobilized capital.

understand the issue of insurer financial capacity and its impact on optimal insurance contract. First, we relax the assumption of insured weak participation in the insurer reserve by allowing dividends in the insurance contract while considering the associated reserve costs. Second, we relax the assumption of non-contingent financial capacity by allowing reinsurance purchase outside the community while considering reinsurance costs. These two features are critical to understand the role of insurance facilities such as the CCRIF and their insurance contracts.

Without explicitly modeling the insurer financial capacity, Doherty & Schlesinger (1990) and Cummins & Mahul (2004) consider the case where the indemnity level increases with the premium level in normal states but not in catastrophic states.[8] In this framework, the optimal contract with actuarially fair insurance consists in partial coverage for individual losses in normal states in order to preserve the welfare level in catastrophic states.[9] In the same spirit, Bernard & Ludkovski (2012) and Mahul & Wright (2004, 2007) show that it can be optimal to have full coverage in normal states when the indemnity level increases with the premium level less rapidly in catastrophic states than in normal states. Besides, Mahul & Wright (2004, 2007) show that insurance with more-than-full coverage in normal states can be Pareto-improving. This result suggests that setting dividends in the contract can be further Pareto-improving because it is a better way to redistribute reserves remaining beyond full coverage in normal states.

To model the insurer financial capacity and explain why indemnities cannot be fully paid in catastrophic states, Biffis & Millossovich (2012), Charpentier & Le Maux (2014) and Zhou et al. (2010) consider an insurer with a reserve composed by the premiums raised plus a given amount of assets. In this framework, the optimal insurance contract with actuarially fair insurance consists in full coverage for individual losses in normal states but not in catastrophic states because of insurer limited reserve. Malinvaud (1973) and Penalva-Zuasti (2008) show that fair insurance with dividends within the insurance contract or through a separated ownership contract leads to full coverage for individual

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[8]This contingency can be seen as contractual or as a "default risk" with similar perception by insureds and insurer. In the quoted theoretical papers in the present literature review, "default risk" is used in this sense. Other papers such as Cummins & Mahul (2003) analyze the impact of divergent beliefs about default risk.

[9]When the individual risk has multiple loss levels in normal states (Cummins & Mahul (2004)), the optimal partial coverage contract is an excess-of-loss insurance policy (i.e. with a strictly positive deductible). When the individual risk has only one loss level (Doherty & Schlesinger (1990) and the present paper), we cannot distinguish for partial insurance between excess-of-loss insurance and proportional insurance.

losses in both normal and catastrophic states, in line with Borch mutuality principle.[10]

Rather than considering no participation or fair participation of insureds in the reserve[11], we consider that it is costly for insureds to participate in the reserve. On the other hand, we assume that the insurer has access to costly reinsurance outside the community. With costly reinsurance and costless reserves, Doherty & Dionne (1993) and Doherty & Schlesinger (2002) show that the optimal contract consists in the full elimination of individual risks, plus partial coverage of the collective risks. By considering reserve costs, we analyze both the impact of reserve cost and reinsurance cost on the optimal contract as well as the trade-off between reserve and reinsurance.

The first contribution of the present paper is to develop a simple and tractable model to analyze the optimal design of insurance contracts by a pooling insurance facility to manage risks that are correlated across individuals. The second contribution is to study the impact of reinsurance cost and risk correlation on the optimal insurance contract further than previous works. The third contribution is to consider the reserve cost potentially generated by raising premiums ex-ante. The paper is organized as follows. Section 2 sets up the model of a community with correlated individual risks along with the insurance and reinsurance contracts. Section 3 provides an analysis of the optimal insurance and reinsurance contracts with comparative statics analysis. Section 4 presents the example of the Caribbean countries and their insurance facility. Section 5 concludes.

2 The model

2.1 The community of agents

We consider a community of N agents identical in terms of preferences, initial wealth and exposure to risk.[12] The preferences of the representative agent satisfy the von Neumann-Morgenstern axioms, with $u(.)$ the corresponding utility function which is strictly increasing, globally concave and twice continuously differentiable. The representative agent has an initial wealth w and is exposed to a potential loss l. The individual risks can generate

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[10] Borch (1962) demonstrates that in a community where agents are exposed to individual risks with collective components, it is Pareto optimal to eliminate individual risks and to share collective risks.

[11]No participation (Biffis & Millossovich (2012), Charpentier & Le Maux (2014) and Zhou et al. (2010)) is equivalent to extremely costly reserve for insureds. Fair participation (Malinvaud (1973) and Penalva-Zuasti (2008)) is equivalent to costless reserve for insureds.

[12]Heterogeneity of individuals raises questions related to asymmetric information that are out of the scope of our analysis.

a significant collective risk either because N is not large enough or because individual risks are correlated. To model the collective risks, we consider two states of nature, one catastrophic and one normal. With a probability p (such that $0 < p < 1$), a catastrophe occurs and the fraction of agents enduring a loss of size l is q_c. Otherwise, the normal state occurs and the fraction of agents enduring the same loss l is $q_n < q_c$.[13] In this template, the individual probability of enduring a loss l is q_c in the catastrophic state and q_n in the normal state, and the unconditional individual probability of enduring a loss l is: $\overline{q} = (1-p)q_n + pq_c$. The individual random wealth without risk sharing scheme is characterized in figure 1. Besides, the collective random wealth of the N agents is characterized in figure 2. With N large, the coefficient δ of correlation between individual risks is well approximated by: $\delta = \frac{p(1-p)}{\overline{q}(1-\overline{q})}(q_c - q_n)^2$ (proof in appendix A). The higher the difference between the fraction q_c of affected agents in the catastrophic state and the fraction q_n of affected agents in the normal state, the higher the risk correlation between agents.[14] Note that q_n and q_c can be expressed as functions of the individual probability $\overline{q}$ of being affected and the correlation δ between individual risks, plus the probability p of catastrophe: $q_n = \overline{q} - p(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$ and $q_c = \overline{q} + (1-p)(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$.
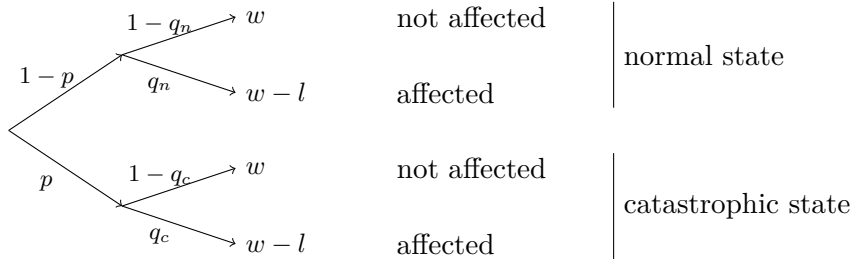

Figure 1: individual random wealth of the representative agent
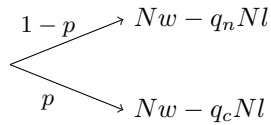

Figure 2: collective random wealth of the N agents

In this template, average individual loss depends on the state of nature, its value is

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[13]As pointed out by Malinvaud (1973), considering two different individual loss levels in the normal and catastrophic states could be considered as two different risks.

[14]The fully correlated case ($\delta = 1$) is characterized by $q_n = 0$, $q_c = 1$ and $0 < p < 1$, in which everyone endures a loss or no one. The no-correlated case ($\delta = 0$) would correspond to $q_n = q_c$, $p = 1$ or $p = 0$, in which there is only one collective state.

$q_n l$ in the normal state and $q_c l$ in the catastrophic state. Thus, the expected value of the average individual loss is $\bar{q}l$ and its variance is $\bar{q}(1 - \bar{q})\delta l^2$ (proof in appendix A). The higher the individual probability $\bar{q}$ of being affected, the higher the expected average loss. The more correlated the individual risks, the more volatile the average loss.[15] Figure 3 illustrates for two different sets of parameters the cumulative distribution functions for the individual loss (thick bars) and for the average individual loss (thin bars). The spread between q_n and q_c is smaller in 3a than in 3b, while in both cases $p = 0.2$ and $\bar{q} = 0.3$. The individual probability of being affected $\bar{q}$ is similar for the two sets of parameters, whereas the correlation across individual risks δ is smaller in 3a than in 3b, which makes a difference for risk sharing mechanism as detailed in the paper.

(a) $p = 0.20$, $q_n = 0.25$, $q_c = 0.50$, $\bar{q} = 0.3$, $\delta = 0.048$

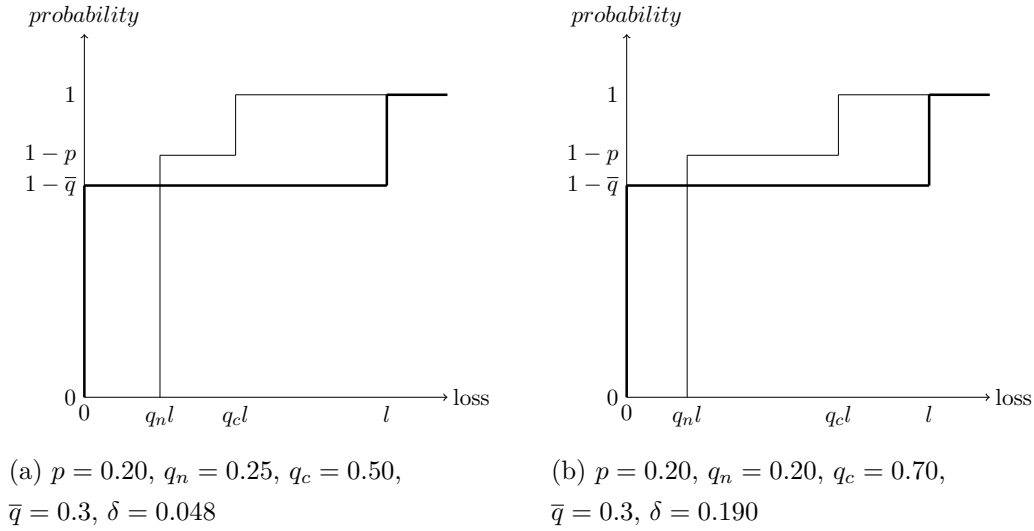(b) $p = 0.20$, $q_n = 0.20$, $q_c = 0.70$, $\bar{q} = 0.3$, $\delta = 0.190$

Figure 3: Cumulative distribution functions for individual loss (thick bars) and average individual loss (thin bars) for two different sets of parameters in 3a and 3b.

2.2 Insurance and reinsurance contracts

We consider that the community is equipped with a pooling insurance facility, also called the insurer. The insurer faces two states of nature, the normal one and the catastrophic one, in which it respectively has a fraction q_n and a fraction q_c of its insureds that have to be indemnified. A standard-type insurance contract is a couple (α, τ). In this case, α is the premium paid by the agent and $\tau \geq 0$ is the indemnity received by the agent if the latter endures a loss l. A mutual-type insurance contract is a quadruple $(\alpha, \tau, \epsilon, \pi)$. In this

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[15]Cummins (2006) and Cummins & Trainar (2009) have more insights on the relation between the risk correlation and the average loss volatility.

case, α is the premium paid by the agent, $\tau \geq 0$ is the indemnity received by the agent in the normal state if the latter endures a loss l, $\tau - \epsilon \geq 0$ is the indemnity received by the agent in the catastrophic state if the latter endures a loss l and $\pi \geq 0$ is the dividend received by the agent in the normal state. This contract is called mutual-type contract because each agent shares a fraction of the collective risk of the community. Indeed, ϵ and π make the insurance contract directly depend on the collective losses, contrary to the standard contract. The standard contract is a specific case of the mutual contract with $\epsilon = 0$ and $\pi = 0$.[16] Besides, we consider that the contract can generate an opportunity or reserve cost for the insured. Indeed, when premiums are raised ex-ante while indemnities and dividends are given ex-post, the secured capital cannot be used for other purpose (i.e. consumption or investment). Thus, the agents may have to raise more costly external capital instead of using this capital[17], which generates a reserve cost for the agents. The higher the required premium α, the costlier should be the marginal external capital and thus the higher should be the marginal reserve cost. This means that the reserve cost function denoted $\lambda^r(\alpha)$ should be increasing and convex relative to the premium α. The agent wealth profile with a mutual-type contract is represented in figure 4.

$$1 - q_n \rightarrow w_1 = w - \alpha - \lambda^r(\alpha) + \pi$$
$$1 - p$$
$$q_n \rightarrow w_2 = w - \alpha - \lambda^r(\alpha) - l + \tau + \pi$$
$$1 - q_c \rightarrow w_3 = w - \alpha - \lambda^r(\alpha)$$
$$p$$
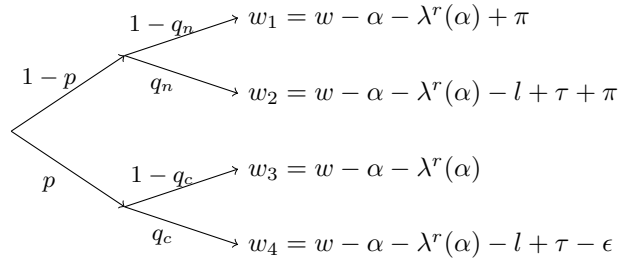$$q_c \rightarrow w_4 = w - \alpha - \lambda^r(\alpha) - l + \tau - \epsilon$$

Figure 4: agent wealth profile with insurance contract

The insurer has to manage the collective risks generated by the aggregation of the insured individual risks. It can purchase reinsurance outside the community, to be able to pay the higher total claims of the catastrophic state. Purchasing a reinsurance contract, with an indemnity $\tau^R \geq 0$ in the catastrophic state occuring with a probability p, costs

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[16]The mutual insurance contracts defined here are in the spirit of the economics literature detailed in introduction with indemnities and dividends contingent on collective losses. In practice, it is for example in the spirit of the contracts supplied by the CCRIF to the Caribbean countries. The premium α corresponds to the regular premium plus the up-front participation fee in the contracts supplied by the CCRIF. The dividend π corresponds to the premium discount of the following year if losses are not too catastrophic. The indemnity gap ϵ between normal state and catastrophic state is also acknowledged by the CCRIF. See section 4 for more details on the CCRIF insurance contracts.

[17]Froot (2001) for instance brings up the higher cost of external capital.

$(1 + \lambda^R)p\tau^R$, in which $\lambda^R \geq 0$ is the reinsurance loading factor.[18] For reinsurance to be relevant, we need to have $(1 + \lambda^R)p < 1$.[19] With the insurance contracts supplied to the agents and the reinsurance contract purchased outside the community, the insurer wealth profile is detailed in figure 5.
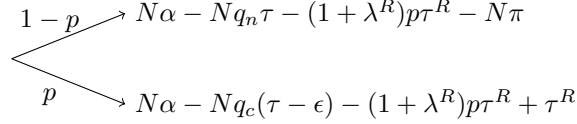
$$
\begin{array}{l}
\nearrow^{\;1-p} \quad N\alpha - Nq_n\tau - (1+\lambda^R)p\tau^R - N\pi \\[2ex]
\searrow_{\;p} \quad N\alpha - Nq_c(\tau - \epsilon) - (1+\lambda^R)p\tau^R + \tau^R
\end{array}
$$

Figure 5: insurer profit profile

3 Optimal insurance and reinsurance

The optimal insurance and reinsurance contracts for the community consist in maximizing the expected utility of the representative agent under the insurer budget constraints.

3.1 Budget constraints

The mutual insurance facility cannot pay claims unless it has secured the funds either through raised premiums or purchased reinsurance. With the budget constraints in both the normal state and the catastrophic state (budget expressions in figure 5), the optimal insurance and reinsurance contracts are thus the solution of the following maximization problem:

$$
\begin{aligned}
\max_{\alpha,\tau,\epsilon,\pi,\tau^R} \quad & \mathbb{E}(u(\tilde{w})) \\
\text{s.t. } & N\alpha - Nq_n\tau - (1+\lambda^R)p\tau^R - N\pi \geq 0 \\
& N\alpha - Nq_c(\tau - \epsilon) - (1+\lambda^R)p\tau^R + \tau^R \geq 0 \\
& \tau \geq 0, \ \tau - \epsilon \geq 0, \ \pi \geq 0, \ \tau^R \geq 0.
\end{aligned}
\tag{1}
$$

Because utility is increasing with wealth, the budget constraints are binding in the two states of nature, the catastrophic one and the normal one. The subtraction of the two

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[18]λ^R corresponds to reinsurance frictional costs as detailed in Froot (2001). Reinsurance contracts can be either standard reinsurance contracts supplied by reinsurance companies or insurance-linked securities supplied by financial investors. The latter have emerged in the nineties because financial markets have larger financial capacities than standard reinsurance markets to supply contracts for very large risks.

[19]If $(1 + \lambda^R)p \geq 1$, purchasing reinsurance would have no sense for the CCRIF because it would lose money in both the normal state and the catastrophic state with the reinsurance contract.

binding budget constraints gives the purchased reinsurance indemnity τ^R:

$$\tau^R = Nq_c(\tau - \epsilon) - Nq_n\tau - N\pi \geq 0. \tag{2}$$

The insurance facility has to purchase a reinsurance indemnity in order to cover the difference between the amount due in the catastrophic state $(Nq_c(\tau - \epsilon))$ and the amount due in the normal state $(Nq_n\tau + N\pi)$. With (2), the binding budget constraints give the required insurance premium α:

$$\alpha = \left(1 + \frac{p(q_c - q_n)}{\overline{q}}\lambda^R\right)\overline{q}\tau + \left(1 - \frac{p}{1-p}\lambda^R\right)(1-p)\pi - \left(1 + \lambda^R\right)pq_c\epsilon, \tag{3}$$

which simplifies, if reinsurance is binding $(\tau^R = 0)$, to:

$$\alpha = q_c(\tau - \epsilon). \tag{4}$$

To be able to pay indemnities and dividends, the insurance facility requires the premium (3) (if $\tau^R > 0$) or (4) (if $\tau^R = 0$) for the contract $(\alpha, \tau, \epsilon, \pi)$. If it is not valuable to purchase reinsurance, the insurance facility has to raise premiums (4) in order to be able to pay all the indemnities in the catastrophic state. If it is valuable to purchase reinsurance, it is not necessary to raise such levels of premiums but the insurance facility has to pass on the cost of reinsurance to insureds, which explains the loading factor $\frac{p(q_c-q_n)}{\overline{q}}\lambda^R$ in front of τ in (3). As shown by (2), allowing a dividend in the normal state $(\pi > 0)$ or a lower indemnity in the catastrophic state $(\epsilon > 0)$ enables to lower the purchase of reinsurance. With a dividend in the normal state $(\pi > 0)$, the premium is affected in two opposite directions. The first effect is straightforward: a higher dividend implies a higher premium. The second opposite effect is due to the fact that the insurer has to purchase less reinsurance thanks to the reserve from the insureds and appears through λ^R in the coefficient in front of π in (3). Note that the factor in front of π in (3) is globally positive because $(1 + \lambda^R)p < 1$. With a lower indemnity in the catastrophic state $(\epsilon > 0)$, the premium is reduced through two channels. The first effect is straightforward: a lower indemnity implies a lower premium. The second effect is due to the fact that the insurer has to purchase less reinsurance and appears through λ^R in the coefficient in front of ϵ in (3). If the agents in the community can have direct access to reinsurance with the same loading factor λ^R, it is more valuable to insure through the insurance facility because: $1 + \frac{p(q_c-q_n)}{\overline{q}}\lambda^R < 1 + \lambda^R$, thanks to partial diversification done by the insurance facility. This is true with standard insurance contracts and thus also true with mutual contracts. The higher the cost of reinsurance, the more valuable the facility. The lower the correlation between participants, the more efficient the pooling and thus the more valuable the facility.[20]

[20]This could explain why the program has been extended to Central American Countries since 2015. However, we have assumed that there are no management costs for the insurance facility. If there are, the

3.2 Insurance and reinsurance contracts

With the binding budget constraints, the maximization problem (1) for the optimal contracts boils down to:[21]

$$\max_{\tau,\epsilon,\pi} \mathbb{E}(u(\tilde{w}))$$

$$\text{s.t. } \alpha = \left(1 + \frac{p(q_c - q_n)}{\overline{q}}\lambda^R\right)\overline{q}\tau + \left(1 - \frac{p}{1-p}\lambda^R\right)(1-p)\pi - \left(1 + \lambda^R\right)pq_c\epsilon \quad (5)$$

$$\tau \geq 0, \ \pi \geq 0, \ (q_c - q_n)\tau - \pi - q_c\epsilon \geq 0.$$

Note that, with standard insurance contracts ($\pi = 0$ and $\epsilon = 0$), the maximization problem (5) corresponds to the standard Mossin problem, in which the optimal coverage level is obtained by the marginal tradeoff between the aversion to risk and the cost due to reinsurance ($\frac{p(q_c - q_n)}{\overline{q}}\lambda^R$).

3.2.1 Without reserve cost ($\lambda^r(\alpha) = 0$)

We first consider the case in which raising premiums ex-ante does not generate a reserve cost for the insured ($\lambda^r(\alpha) = 0$). The first order conditions of (5) are derived in appendix B.1. If it is valuable to purchase reinsurance (i.e. $\frac{\tau^R}{N} = (q_c - q_n)\tau - \pi - q_c\epsilon \geq 0$ is not binding), the optimal contract has indemnities τ and $\tau - \epsilon$ and dividend π such that:

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w - \alpha - l + \tau + \pi)}{u'(w - \alpha + \pi)} = 1, \quad (6)$$

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w - \alpha - l + \tau - \epsilon)}{u'(w - \alpha)} = 1, \quad (7)$$

$$\frac{u'(w_3)}{u'(w_1)} = \frac{u'(w - \alpha)}{u'(w - \alpha + \pi)} = \frac{1 + \lambda^R}{1 - \frac{p}{1-p}\lambda^R}. \quad (8)$$

If it is not valuable to purchase reinsurance (i.e. $\frac{\tau^R}{N} = (q_c - q_n)\tau - \pi - q_c\epsilon \geq 0$ is binding), the optimal contract has indemnities τ and $\tau - \epsilon$ plus dividend π such that:

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w - \alpha - l + \tau + \pi)}{u'(w - \alpha + \pi)} = 1, \quad (9)$$

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pooling insurance facility is valuable if the cost of implementing the facility generates a loading factor λ^i such that: $1 + \lambda^i + \frac{p(q_c - q_n)}{\overline{q}}\lambda^R < 1 + \lambda^R$. In the case of the Caribbean countries, extending the insurance facility to Central American Countries is valuable if it does not add too much management costs.

[21]The last inequality constraint in (5) corresponds to $\tau^R \geq 0$. The inequality constraint $\tau - \epsilon \geq 0$ is not written because it is necessarily verified with the other inequality constraints. Indeed, we have at least as much money for indemnities in the catastrophic state as the amount of money for indemnities and dividends in the normal state.

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w - \alpha - l + \tau - \epsilon)}{u'(w - \alpha)} = 1, \tag{10}$$

$$\pi = (q_c - q_n)\tau - q_c\epsilon. \tag{11}$$

Whether reinsurance is purchased or not, the optimal insurance contract is such that: $w_1 = w_2$ and $w_3 = w_4$ (thanks to (6) and (7) or (9) and (10)), which means that $\tau = l$ and $\epsilon = 0$. Besides, when $\lambda^R = 0$, (8) tells that $\pi = 0$, (3) gives $\alpha = \bar{q}l$ and (2) gives $\tau^R = N(q_c - q_n)l$. When $0 < \lambda^R < \lambda^{R*}$, (8) tells that $\pi > 0$, (3) gives $\alpha = (\bar{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi$ and (2) gives $\tau^R = N(q_c - q_n)l - N\pi > 0$. λ^{R*} is determined with (8) and the additional constraint $\frac{\tau^R}{N} = (q_c - q_n)l - \pi = 0$, which tells that $\pi = (q_c - q_n)l$ and $\alpha = q_c l$. When $\lambda^{R*} \leq \lambda^R$, (11) tells that $\pi = (q_c - q_n)l$, (4) gives $\alpha = q_c l$ and reinsurance is not purchased $\tau^R = 0$.

Proposition 1 *The optimal insurance and reinsurance contracts are such that:*

(i) when $\lambda^R = 0$: $\tau = l$, $\epsilon = 0$, $\pi = 0$, $\alpha = \bar{q}l$, $\tau^R = N(q_c - q_n)l$;

(ii) when $0 < \lambda^R < \lambda^{R*}$: $\tau = l$, $\epsilon = 0$, $\pi > 0$, $\alpha = (\bar{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi$, $\tau^R = N(q_c - q_n)l - N\pi > 0$;

(iii) when $\lambda^{R*} \leq \lambda^R$: $\tau = l$, $\epsilon = 0$, $\pi = (q_c - q_n)l$, $\alpha = q_c l$, $\tau^R = 0$;

in which λ^{R*} is such that $\frac{u'(w - q_c l)}{u'(w - q_n l)} = \frac{1 + \lambda^{R*}}{1 - \frac{p}{1-p}\lambda^{R*}}$.

Proposition 1 states that the optimal insurance contract has full coverage for a given individual loss in both normal and catastrophic states ($\tau = l$ and $\epsilon = 0$) whatever the cost of reinsurance (λ^R). The optimal contract eliminates individual risks, which is in line with Borch mutuality principle. Besides, proposition 1 states that the optimal contract has dividend ($\pi > 0$) in the normal state if and only if reinsurance is supplied above fair prices ($\lambda^R > 0$). If reinsurance is fair ($\lambda^R = 0$), the insurance facility fully reinsures the collective risk ($\tau^R = N(q_c - q_n)l$) and the optimal insurance contract is standard, i.e. without any dividend in the normal state ($\pi = 0$). If reinsurance is not fair ($\lambda^R > 0$), a mutual contract (i.e. with $\pi > 0$) is better than a standard contract because it enables the risk-averse agent to bear a part of the collective risk contrary to the standard contract, which is valuable because reinsurance is costly. If reinsurance is excessively above fair prices ($\lambda^{R*} \leq \lambda^R$), the insurance facility does not purchase reinsurance ($\tau^R = 0$) and the optimal insurance contract is with dividend in the normal state corresponding to the indemnity difference between the catastrophic state and the normal state ($\pi = (q_c - q_n)l$). If reinsurance is

reasonably above fair prices ($0 < \lambda^R < \lambda^{R*}$), the insurance facility partially reinsures the collective risk ($\tau^R > 0$) and the optimal insurance contract is with dividend in the normal state ($\pi > 0$).

Proposition 2 *With $0 < \lambda^R < \lambda^{R*}$ (and a CARA utility function[22]), we have for the optimal insurance and reinsurance contracts: $\frac{d\pi}{d\lambda^R} > 0$, $\frac{d\alpha}{d\lambda^R} > 0$, $\frac{d\tau^R}{d\lambda^R} < 0$.*

Proposition 2 is proved in appendix B.2. It states that the higher the reinsurance cost (i.e. λ^R), the lower the reinsurance purchase and the higher the premium and the dividend in the normal state. Indeed, to be able to cover individual losses in the catastrophic state when reinsurance purchase is decreased, the insurance facility has to increase the reserve financed by the insureds through higher premiums. Moreover, it has higher dividends to give to the insureds if the catastrophic state does not occur. In the extreme case where λ^R reaches λ^{R*}, reinsurance is not purchased ($\tau^R = 0$) and the premium and the dividend respectively reach the highest levels $\alpha = q_c l$ and $\pi = (q_c - q_n)l$.

Proposition 3 *We have for the optimal insurance and reinsurance contracts:*

(i) when $\lambda^R = 0$: $\frac{d\pi}{d\delta} = 0$, $\frac{d\alpha}{d\delta} = 0$, $\frac{d\tau^R}{d\delta} > 0$;

(ii) when $0 < \lambda^R < \lambda^{R*}$ (with a CARA utility function): $\frac{d\pi}{d\delta} = 0$, $\frac{d\alpha}{d\delta} > 0$, $\frac{d\tau^R}{d\delta} > 0$;

(iii) when $\lambda^{R*} \leq \lambda^R$: $\frac{d\pi}{d\delta} > 0$, $\frac{d\alpha}{d\delta} > 0$, $\frac{d\tau^R}{d\delta} = 0$.

Proposition 3 is obtained thanks to proposition 1, recalling that $q_n = \overline{q} - p(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$ and $q_c = \overline{q} + (1-p)(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$ ((i) and (iii) are obvious and (ii) is proved in appendix B.2). Firstly, it states that the insurance contract is affected by a change of correlation δ if and only if reinsurance is not fair ($\lambda^R > 0$). If reinsurance is fair, only the average probability $\overline{q}$ and the loss l affects the insurance contract because the collective risk is fully reinsured without any cost. If reinsurance is not fair, the higher the correlation δ, the larger the collective risk and the more expensive its coverage. If reinsurance is not

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[22]The coefficient of absolute risk aversion of a utility function $u(.)$ is by definition $A(.) = -\frac{u''(.)}{u'(.)}$. We consider here a utility function with a constant absolute risk aversion A (also called CARA utility function). If the utility function is not CARA, there is an additional wealth effect. However, as long as this effect is of secondary order, it does not change the results. Note that if this effect was not of secondary order, it would have been observed that insurance can be a Giffen good (i.e. a higher premium leading to a higher purchase of insurance). To our knowledge, empirical analysis on the purchase of natural disaster insurance have not observed such behaviors. For instance, Browne & Hoyt (2000) and Grace et al. (2004) observe that when insurance price increases, the demand for insurance decreases.

too costly $(0 < \lambda^R < \lambda^{R*})$, an increase of δ is managed by an increase of reinsurance purchase to be able to cover the higher total indemnities in the catastrophic state and the insurance facility has to translate the cost of reinsurance to insureds through higher premiums. If reinsurance is too costly $(\lambda^{R*} \leq \lambda^R)$, an increase of δ is managed by an increase of the reserve through higher premiums and the insurance facility has higher dividends to distribute if the catastrophe does not occur. In both cases, higher correlation δ leads to higher premiums.

To sum up, the premium α increases from $\overline{q}l$ to $q_c l$ when λ^R increases from 0 to high values and it also increases when risk correlation δ increases. Thus, with costly reinsurance and significant risk correlation, the required premiums can reach high levels for insureds if the individual loss l is significant. In this case, which is relevant for natural disaster risks, raising such levels of premiums ex-ante can generate a reserve cost for insureds, which are considered in the following section.

3.2.2 With reserve cost $(\lambda^r(\alpha) \geq 0)$

We now consider the case in which raising premiums ex-ante generate a reserve cost for the insureds $(\lambda^r(\alpha) \geq 0)$. As explained in section 2.2, we assume that the reserve cost function $\lambda^r(.)$ is increasing and convex relative to the premium α. We analyze how the marginal reserve cost $\lambda^{r\prime}(\alpha)$ affects the optimal insurance and reinsurance contracts. We consider $\lambda^R \leq \lambda^{R*}$, which means that purchasing some reinsurance is valuable. The first order conditions of (5) are derived in appendix C.1. If it is valuable to have dividend in the normal state $\pi \geq 0$, the optimal contract has indemnities τ and $\tau - \epsilon$ and dividend π such that:

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w - \alpha - \lambda^r(\alpha) - l + \tau + \pi)}{u'(w - \alpha - \lambda^r(\alpha) + \pi)} = 1, \tag{12}$$

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w - \alpha - \lambda^r(\alpha) - l + \tau - \epsilon)}{u'(w - \alpha - \lambda^r(\alpha))} = \frac{(1 + \lambda^{r\prime}(\alpha))(1 + \lambda^R)}{(1 + \lambda^{r\prime}(\alpha))(1 + \lambda^R) - \frac{\lambda^{r\prime}(\alpha)}{p(1-q_c)}}, \tag{13}$$

$$\frac{u'(w_3)}{u'(w_1)} = \frac{u'(w - \alpha - \lambda^r(\alpha))}{u'(w - \alpha - \lambda^r(\alpha) + \pi)} = \frac{(1 + \lambda^{r\prime}(\alpha))(1 + \lambda^R) - \frac{\lambda^{r\prime}(\alpha)}{p(1-q_c)}}{(1 + \lambda^{r\prime}(\alpha))(1 - \frac{p}{1-p}\lambda^R)}. \tag{14}$$

If it is not valuable to have dividend in the normal state (i.e. $\pi \geq 0$ is binding), the optimal contract has indemnities τ and $\tau - \epsilon$ and dividend π such that:

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w - \alpha - \lambda^r(\alpha) - l + \tau)}{u'(w - \alpha - \lambda^r(\alpha))} = \frac{(1 + \lambda^{r\prime}(\alpha))(1 - \frac{p}{1-p}\lambda^R)}{(1 + \lambda^{r\prime}(\alpha))(1 - \frac{p(q_c-q_n)}{1-\overline{q}}\lambda^R) - \frac{\lambda^{r\prime}(\alpha)}{1-\overline{q}}}, \tag{15}$$

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w - \alpha - \lambda^r(\alpha) - l + \tau - \epsilon)}{u'(w - \alpha - \lambda^r(\alpha))} = \frac{(1 + \lambda^{r'}(\alpha))(1 + \lambda^R)}{(1 + \lambda^{r'}(\alpha))(1 - \frac{p(q_c - q_n)}{1 - \overline{q}}\lambda^R) - \frac{\lambda^{r'}(\alpha)}{1 - \overline{q}}}. \quad (16)$$

$$\pi = 0, \quad (17)$$

Proposition 4 *The optimal insurance and reinsurance contracts are such that:*

(i) when $0 < \lambda^{r'}(\alpha) < \lambda^{r*}$: $\tau = l$, $\epsilon > 0$, $\pi > 0$, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi - (1 + \lambda^R)pq_c\epsilon$, $\tau^R = N(q_c - q_n)l - N\pi - Nq_c\epsilon > 0$;

(ii) when $\lambda^{r'}(\alpha) = \lambda^{r*}$: $\tau = l$, $\epsilon > 0$, $\pi = 0$, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l - (1 + \lambda^R)pq_c\epsilon$, $\tau^R = N(q_c - q_n)l - Nq_c\epsilon > 0$;

(iii) when $\lambda^{r'}(\alpha) > \lambda^{r*}$: $\tau < l$, $\epsilon > 0$, $\pi = 0$, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)\tau - (1 + \lambda^R)pq_c\epsilon$, $\tau^R = N(q_c - q_n)\tau - Nq_c\epsilon > 0$;

in which $\lambda^{r*} = \frac{p(1 - q_c)}{1 - p - p(1 - q_c)\lambda^R}\lambda^R$.

Proposition 4 is derived from the first order conditions of (5) written above plus (2), (3) and (4).[23] Firstly, it states that the optimal insurance contract has lower coverage for a given individual loss in the catastrophic state than in the normal state ($\epsilon > 0$) when increasing the premium α generates a reserve cost ($\lambda^{r'}(\alpha) > 0$). In this case, it is not valuable to cover fully individual losses in the catastrophic state, which means that the optimal contract does not fully eliminate individual risks and does not fulfill the Borch mutuality principle. This is a second-best insurance contract when insurance premiums have to be raised ex-ante and generate a reserve cost. Besides, relative to the case without a reserve cost, proposition 4 states that the optimal contract may not always have dividend in the normal state. If the marginal reserve cost is too high relative to the reinsurance cost ($\lambda^{r'}(\alpha) \geq \frac{p(1 - q_c)}{1 - p - p(1 - q_c)\lambda^R}\lambda^R$), it is not valuable to have dividend in the normal state, which means that it is more valuable to spend all the premiums to reinsure rather than to keep some reserves which would be given back through dividends in the normal state. In this case, it is even valuable to lower the indemnity in the normal state relative to full coverage ($\tau < l$) to increase reinsurance and the indemnity in the catastrophic state. We consider in the following a constant marginal reserve cost $\lambda^{r'}(\alpha) = \lambda^r$.

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[23]λ^{r*} is obtained with (14) equal to 1. Besides, $\lambda^{r'}(\alpha) > \frac{p(1 - q_c)}{1 - p - p(1 - q_c)\lambda^R}\lambda^R$ tells that (15) is strictly greater than 1 and $\tau < l$.

Proposition 5 *With $0 < \lambda^r < \lambda^{r*}$ (and a CARA utility function), we have for the optimal insurance and reinsurance contracts: $\frac{d\epsilon}{d\lambda^r} > 0$, $\frac{d\pi}{d\lambda^r} < 0$, $\frac{d\alpha}{d\lambda^r} < 0$, $\frac{d\tau^R}{d\lambda^r} > 0$.*

Proposition 5 is proved in appendix C.2. It states that an increase of the marginal reserve cost (λ^r) leads to a decrease of the premium to limit the reserve cost for the insured. Thus, it leads to a decrease of the indemnity in the catastrophic state and a decrease of the dividend in the normal state. However, to limit the indemnity decrease in the catastrophic state, reinsurance purchase is increased in this case.

Proposition 6 *With $0 < \lambda^r < \lambda^{r*}$ (and a CARA utility function), we have for the optimal insurance and reinsurance contracts: $\frac{d\epsilon}{d\delta} > 0$, $\frac{d\pi}{d\delta} < 0$, $\frac{d\alpha}{d\delta}$ is ambiguous and $\frac{d\tau^R}{d\delta} > 0$.*

Proposition 6 is proved in appendix C.2. It states that an increase of the correlation δ leads to an increase of reinsurance purchase because the collective risk increases with δ. On the one hand, the premium has to increase because reinsurance is costly. On the other hand, increasing the premium generates an additional reserve cost. That is why the indemnity in the catastrophic state and the dividend in the normal state are lowered and finally the variation of the premium is ambiguous.

4 Caribbean countries and natural catastrophe insurance

The Caribbean countries are located in a region of the world exposed to large natural catastrophe risks. Figure 6 exhibits natural disaster losses in this region from 1965 to 2014.[24] Collective losses are widely variable from one year to another because natural disasters in the region can have large spatial impacts. Year 2010 corresponds to the highest losses with more than 8 billion dollars of damages due to a dramatic earthquake hitting Haiti in January and a highly above average hurricane season in the Caribbean during the second part of the year. Year 2004 corresponds to the second highest losses with nearly 7 billion dollars of damages also due to a highly above average hurricane season affecting many countries such as the Bahamas, the Cayman Islands, Grenada and Jamaica.

In this context, the Caribbean Catastrophe Risk Insurance Facility (CCRIF) was created in 2007 with the technical assistance of the World Bank. This non-for-profit multi-country insurance pool is designed to supply disaster-relief insurance policies to the governments of sixteen Caribbean countries for earthquake, hurricane and excess rainfall losses.

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[24]EMDAT International Disaster Database (http://www.emdat.be/)
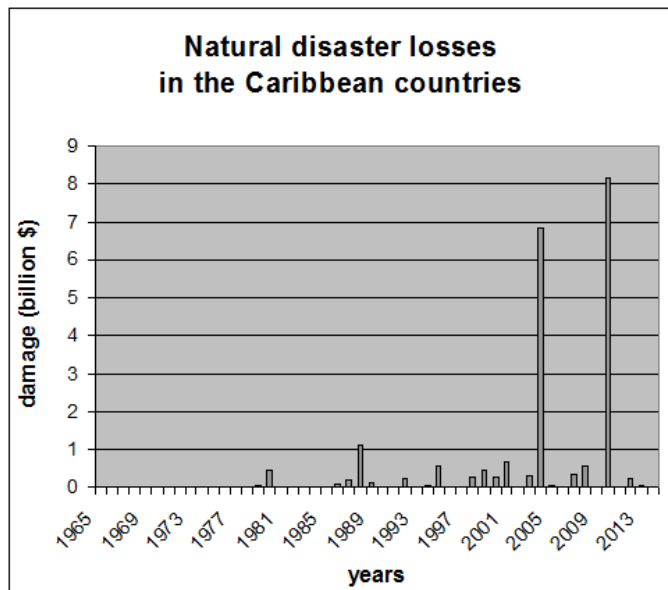
Figure 6: Natural disaster losses in the Caribbean countries.

The program has been extended to Central American countries since 2015. The facility aims at pooling the risks faced by its members and reduce the cost the members would individually face if they directly insured on the reinsurance market. The annual reports of the CCRIF are publicly available[25] and provide useful information about the proposed catastrophe insurance contracts. The insurance policies are parametric (i.e. with triggers based on geological and meteorological indexes), which enable to transfer payouts to affected countries within the fourteen days following a disaster. The CCRIF reports a stable number of 29 or 30 policies sold to the Caribbean countries each year since its inception.[26] The collective risk faced by the CCRIF has remained rather stable as well. Figure 7a displays the cumulative density function of the aggregation of the risks covered by the CCRIF and reported in its annual reports from year 2007-2008 to 2013-2014. The darker lines represent the cumulative distribution of this aggregate risk faced by the pool in the earlier periods of its existence.[27] Using the cumulative distribution functions with

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[25]http://www.ccrif.org/content/publications/reports/annual

[26]The CCRIF can sell more than one policy per country per year because insurance policies for the different types of natural disasters are separated.

[27]Insured losses in figure 7a are much lower than total losses related to natural disasters in figure 6. This is due to the fact that the CCRIF supplies insurance contracts to governments but not to households or businesses. Moreover, this type of contracts aim at covering short term liquidity needs related to natural disaster risks. Finally, it is due to the fact that the governments purchase from the CCRIF only partial insurance.

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the information about the structure of the reinsurance scheme bought by the CCRIF, we can compute an estimated loading factor paid by the organization as : $\lambda^R = \frac{\alpha^R}{\mathbb{E}(L)} - 1$, where α^R is the premium paid by the CCRIF to reinsurers and $\mathbb{E}(L)$ is the expected loss reinsured. Figure 7b displays its evolution through the years and shows that the CCRIF faces a significant loading factor on reinsurance, which can explain why it only partially reinsures the collective risk. The figure shows that reinsurers increased their prices a lot in 2010, following the large earthquake affecting Haiti.
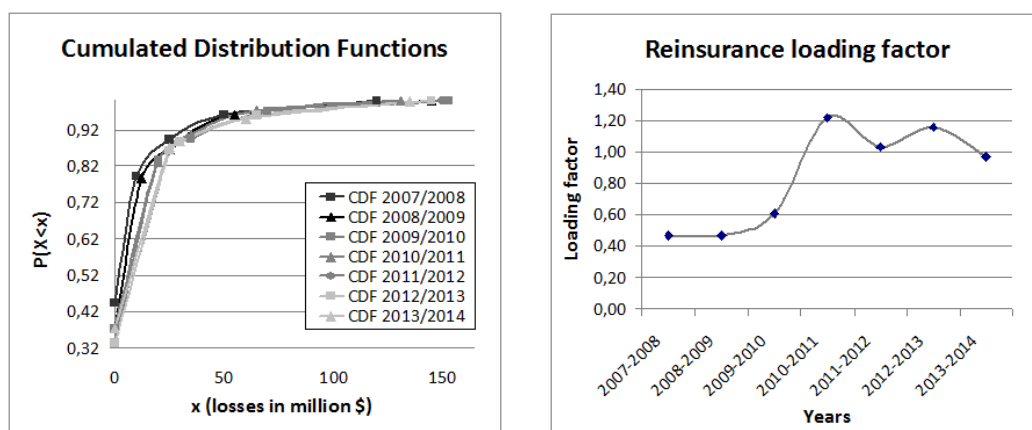
(a) Aggregate risks faced by the CCRIF   (b) Reinsurance loading factor

Figure 7: Aggregate risks resulting from risk pooling by the CCRIF and estimated reinsurance loading factor faced by the CCRIF.

As the CCRIF only partially reinsures the collective risk, it supplies to its members insurance contracts which are mutual in the sense that they depend on collective losses. In addition to the regular insurance premiums, the facility requires its members to pay an up-front participation fee. Audited financial statements report that "it is Managements intent that participation fee deposits are available to fund losses in the event that funds from retained earnings, reinsurers and the Donor Trust are insufficient. If deposits are used to fund losses, it is also Managements intent that any subsequent earnings generated by the Group will be used to reinstate the deposits to their original carrying value". Figure 8 shows that the total amount of premiums was effectively much higher the first year than the following years between 2007-2008 and 2013-2014. It has not been necessary to raise high premiums the following years because no extremely large claims had to be paid during these years. Since 2007, 21 payouts totaling 68 million dollars have been transfered to 10 affected member governments. In terms of claims to be paid, the worth year between

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2007-2008 and 2013-2014 is 2010-2011 with 17 million dollars payouts. However, it will be overpassed by 2016-2017, given that Hurricane Matthew has already led to payouts totaling 29 million dollars in October 2016. Additional catastrophic events in 2016-2017 could lead the CCRIF to use the deposits for payouts and then to raise high premiums to reinstate the deposits for 2017-2018. The yearly insurance contract is similar to a contract with a high premium requested at the beginning of the year in exchange for an indemnity if the insured is affected during the year and a dividend at the end of the year if collective losses are not too catastrophic. In the present case, the dividend is given through a premium discount at the beginning of the following year. Besides, the CCRIF acknowledges the possibility of lowered indemnities in catastrophic states: "The CCRIF can currently survive a series of loss events with a less than 1 in 10,000 chance of occurring in any given year. Due to planned premium reductions, the safety level drops somewhat through the course of our 10-year forward modeling. However, the lowest projected survivability for the CCRIF in the 10-year modeled period is about 1 in 3000 chance of claims exceeding capacity in any one year." In other words, the CCRIF acknowledges to supply contracts such that the indemnity for one individual loss level is lower in highly catastrophic states than in the other states of nature.
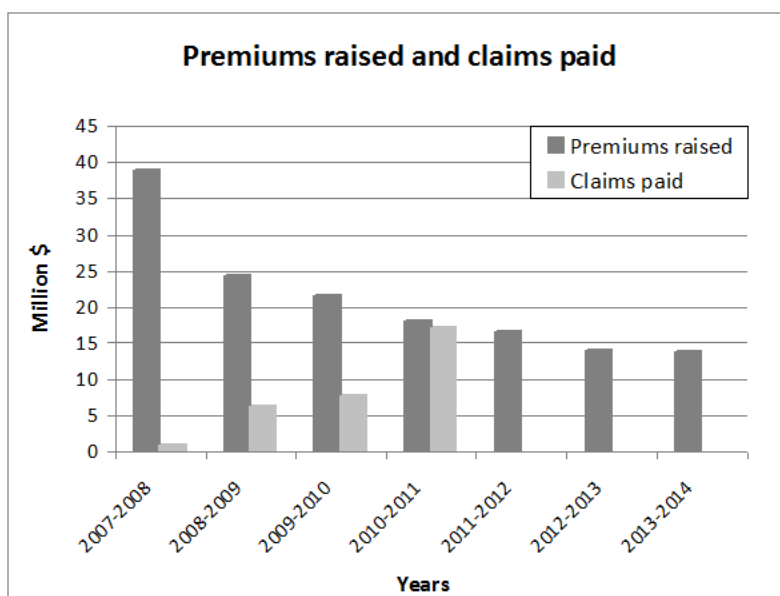

Figure 8: Premiums raised and claims paid by the CCRIF.

5 Conclusion

In the present paper, we have built a simple model to analyze the type of insurance contracts that emerge when risks are correlated across risk-averse agents in a community. For the sake of realism, we have considered that the community simultaneously chooses the type of contract sold to its members and the level of reinsurance it purchases, given that reinsurance is available at a cost higher than fair price. In this scheme, the insurer of the community supplies mutual contracts which are contingent on the state of nature. Without transaction costs in the community, risk-averse agents fully insure against their individual risk and share collective risk by getting some dividend in normal states of nature. With premiums raised ex-ante and generating a reserve cost, risk-averse agents only partially insure against their individual risk, getting a lower indemnity in catastrophic states than in normal states, and get some dividend in normal states if the marginal cost of the reserve is low relative to the marginal cost of reinsurance. Our model highlights the tradeoff between reinsurance and reserve: if reinsurance is costly and reserve is not too costly, the promise of dividends in normal states enables the community to raise high premiums that are used as reserves to better indemnify in catastrophic states. Our analysis helps to understand the limits that risk correlation, costly reinsurance and costly reserve represent for risk sharing and how the contracts in a community can be improved through higher flexibility. Indeed, contracts with contingent indemnity and dividend enable to share better individual risks and collective risks. We have illustrated these mechanisms with the example of the Caribbean Catastrophe Risk Insurance Facility (CCRIF) that combines reinsurance and mutual contracts with indemnity and dividend contingent on the collective state.

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A Risk correlation

With the loss represented by the random variable $\tilde{x}^i$ for individual i and the probability $\overline{q} = (1-p)q_n + pq_c$ of having a loss l, we have:

$$\tilde{x}^i = \begin{cases} -l & \text{with probability} \quad \overline{q} \\ 0 & \text{with probability} \quad 1-\overline{q} \end{cases}$$

In the normal state, the probability that individual i is affected is q_n. Besides, in the normal state, if individual i is affected, agent j is affected with a probability $\frac{q_n N - 1}{N}$, which is well approximated by q_n when N is large. In the catastrophic state, this is similar with q_c instead of q_n. Thus, when N is large, we have with a good approximation:

$$\tilde{x}^i \tilde{x}^j = \begin{cases} l^2 & \text{with probability} \quad (1-p)q_n^2 + pq_c^2 \\ 0 & \text{with probability} \quad 1-(1-p)q_n^2 - pq_c^2 \end{cases}$$

The correlation between individual risks is:

$$\delta = \frac{COV(\tilde{x}^i, \tilde{x}^j)}{(VAR(\tilde{x}^i)VAR(\tilde{x}^j))^{0.5}}.$$

We have:

$$\begin{aligned} COV(\tilde{x}^i, \tilde{x}^j) &= \mathbb{E}(\tilde{x}^i \tilde{x}^j) - \mathbb{E}(\tilde{x}^i)\mathbb{E}(\tilde{x}^j) \\ &= l^2((1-p)q_n^2 + pq_c^2) - (-l\overline{q})^2 \\ &= l^2((1-p)q_n^2 + pq_c^2 - \overline{q}^2), \end{aligned}$$

$$\begin{aligned} VAR(\tilde{x}^i) &= \mathbb{E}((\tilde{x}^i)^2) - \mathbb{E}(\tilde{x}^i)^2 \\ &= l^2\overline{q} - (-l\overline{q})^2 \\ &= l^2\overline{q}(1-\overline{q}). \end{aligned}$$

Then, when N is large, the coefficient of correlation is with a good approximation:

$$\begin{aligned} \delta &= \frac{(1-p)q_n^2 + pq_c^2 - \overline{q}^2}{\overline{q}(1-\overline{q})} \\ &= \frac{p(1-p)}{\overline{q}(1-\overline{q})}(q_c - q_n)^2. \end{aligned}$$

With the average individual loss represented by the random variable $\tilde{X}$, we have:

$$\tilde{X} = \begin{cases} q_c l & \text{with probability} \quad p \\ q_n l & \text{with probability} \quad 1-p \end{cases}$$

This can also be written as $\tilde{X} = \tilde{q}l$ where:

$$\tilde{q} = \begin{cases} q_c & \text{with probability} \quad p \\ q_n & \text{with probability} \quad 1-p \end{cases}$$

Hence, the variance of the average individual loss is: $\text{Var}(\tilde{X}) = \text{Var}(\tilde{q})l^2$, with:

$$\tilde{q}^2 = \begin{cases} q_c^2 & \text{with probability} \quad p \\ q_n^2 & \text{with probability} \quad 1-p \end{cases}$$

$$\begin{aligned} \text{Var}(\tilde{q}) &= \mathbb{E}(\tilde{q}^2) - (\mathbb{E}(\tilde{q}))^2 \\ &= (1-p)q_n^2 + pq_c^2 - \bar{q}^2. \end{aligned}$$

The variance of the average individual loss is then:

$$\text{Var}(\tilde{X}) = \delta\bar{q}(1-\bar{q})l^2.$$

B   Without reserve cost $\left(\lambda^r(\alpha) = 0\right)$

B.1   Derivation of the FOC of (5)

If the inequality constraints are not strictly binding in (5), the first order conditions of (5) relative to τ, ϵ and π are respectively:

$$-(1 + \frac{p(q_c - q_n)}{\bar{q}}\lambda^R)\bar{q}\mathbb{E}(u'(\tilde{w})) + (1-p)q_n u'(w_2) + pq_c u'(w_4) = 0, \tag{18}$$

$$(1 + \lambda^R)pq_c\mathbb{E}(u'(\tilde{w})) - pq_c u'(w_4) = 0, \tag{19}$$

$$-(1 - \frac{p}{1-p}\lambda^R)(1-p)\mathbb{E}(u'(\tilde{w})) + (1-p)(1-q_n)u'(w_1) + (1-p)q_n u'(w_2) = 0. \tag{20}$$

Firstly, (19) gives:
$$u'(w_4) = (1 + \lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{21}$$

Secondly, with $\bar{q} = (1-p)q_n + pq_c$, the combination of (18) and (19) gives:
$$u'(w_2) = (1 - \frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{22}$$

Thirdly, (20) gives with the latter equation:
$$u'(w_1) = (1 - \frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{23}$$

Fourthly, with (21), (22), (23) and the definition of $\mathbb{E}(u'(\tilde{w}))$, we get:

$$u'(w_3) = (1 + \lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{24}$$

If the inequality constraints are not strictly binding in (5) except $(q_c - q_n)\tau - \pi - q_c\epsilon \geq 0$, we have then $\pi = (q_c - q_n)\tau - q_c\epsilon$ and (5) boils down to:

$$\begin{aligned}
\max_{\tau,\epsilon} \ & \mathbb{E}(u(\tilde{w})) \\
\text{s.t. } & \alpha = q_c(\tau - \epsilon) \\
& \pi = (q_c - q_n)\tau - q_c\epsilon.
\end{aligned} \tag{25}$$

The first order conditions of (25) relative to τ and ϵ are respectively:

$$-q_c\mathbb{E}(u'(\tilde{w})) + (q_c - q_n)(1-p)((1-q_n)u'(w_1) + q_nu'(w_2)) + (1-p)q_nu'(w_2) + pq_cu'(w_4) = 0, \tag{26}$$

$$q_c\mathbb{E}(u'(\tilde{w})) - q_c(1-p)((1-q_n)u'(w_1) + q_nu'(w_2)) - pq_cu'(w_4) = 0. \tag{27}$$

Firstly, the sum of (26) and (27) gives:

$$u'(w_2) = u'(w_1). \tag{28}$$

Secondly, (27) gives with the latter equation:

$$u'(w_4) = u'(w_3). \tag{29}$$

B.2 Comparative statics

We consider a CARA utility function $u(.)$, i.e. with $A = -\frac{u''(.)}{u'(.)} > 0$ constant.

With $0 < \lambda^R < \lambda^{R*}$, (8) gives:

$$(1 - \frac{p}{1-p}\lambda^R)u''(w_3)dw_3 - \frac{p}{1-p}u'(w_3)d\lambda^R = (1 + \lambda^R)u''(w_1)dw_1 + u'(w_1)d\lambda^R. \tag{30}$$

With (8), (30) can be rewritten:

$$-A(dw_3 - dw_1) = (\frac{p}{1-p-p\lambda^R} + \frac{1}{1+\lambda^R})d\lambda^R, \tag{31}$$

which finally gives with $\pi = w_1 - w_3$:

$$\frac{d\pi}{d\lambda^R} = \frac{1}{A(1-p-p\lambda^R)(1+\lambda^R)}. \tag{32}$$

$\frac{d\pi}{d\lambda^R} > 0$ because $(1 + \lambda^R)p < 1$. Besides, $\alpha = (\bar{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi$ and $\tau^R = N(q_c - q_n)l - N\pi > 0$ respectively give with (32):

$$\frac{d\alpha}{d\lambda^R} = p((q_c - q_n)l - \pi) + \frac{1}{A(1 + \lambda^R)}, \tag{33}$$

$$\frac{d\tau^R}{d\lambda^R} = -\frac{N}{A(1 - p - p\lambda^R)(1 + \lambda^R)}. \tag{34}$$

$\frac{d\alpha}{d\lambda^R} > 0$ because $\frac{\tau^R}{N} = (q_c - q_n)l - \pi > 0$. $\frac{d\tau^R}{d\lambda^R} < 0$ because $(1 + \lambda^R)p < 1$.

With $0 < \lambda^R < \lambda^{R*}$, (8) gives similarly:

$$\frac{d\pi}{d\delta} = 0. \tag{35}$$

Besides, $\alpha = (\bar{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi$ and $\tau^R = N(q_c - q_n)l - N\pi > 0$ respectively give with (35):

$$\frac{d\alpha}{d\delta} = p\frac{d(q_c - q_n)}{d\delta}\lambda^R l, \tag{36}$$

$$\frac{d\tau^R}{d\delta} = N\frac{d(q_c - q_n)}{d\delta}l. \tag{37}$$

Because $\frac{d(q_c - q_n)}{d\delta} > 0$, $\frac{d\alpha}{d\delta} > 0$ and $\frac{d\tau^R}{d\delta} > 0$.

C With reserve cost $(\lambda^r(\alpha) \geq 0)$

C.1 Derivation of the FOC of (5)

If the inequality constraints are not strictly binding in (5), the first order conditions of (5) relative to τ, ϵ and π are respectively:

$$-(1 + \lambda^{r\prime}(\alpha))(1 + \frac{p(q_c - q_n)}{\bar{q}}\lambda^R)\bar{q}\mathbb{E}(u'(\tilde{w})) + (1 - p)q_n u'(w_2) + pq_c u'(w_4) = 0, \tag{38}$$

$$(1 + \lambda^{r\prime}(\alpha))(1 + \lambda^R)pq_c\mathbb{E}(u'(\tilde{w})) - pq_c u'(w_4) = 0, \tag{39}$$

$$-(1 + \lambda^{r\prime}(\alpha))(1 - \frac{p}{1 - p}\lambda^R)(1 - p)\mathbb{E}(u'(\tilde{w})) + (1 - p)(1 - q_n)u'(w_1) + (1 - p)q_n u'(w_2) = 0. \tag{40}$$

Firstly, (39) gives:

$$u'(w_4) = (1 + \lambda^{r\prime}(\alpha))(1 + \lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{41}$$

Secondly, with $\bar{q} = (1-p)q_n + pq_c$, the combination of (38) and (39) gives:

$$u'(w_2) = (1 + \lambda^{r\prime}(\alpha))(1 - \frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{42}$$

Thirdly, (40) gives with the latter equation:

$$u'(w_1) = (1 + \lambda^{r\prime}(\alpha))(1 - \frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{43}$$

Fourthly, with (41), (42), (43) and the definition of $\mathbb{E}(u'(\tilde{w}))$, we get:

$$u'(w_3) = \left((1 + \lambda^{r\prime}(\alpha))(1 + \lambda^R) - \frac{\lambda^{r\prime}(\alpha)}{p(1-q_c)}\right)\mathbb{E}(u'(\tilde{w})). \tag{44}$$

If the inequality constraints are not strictly binding in (5) except $\pi \geq 0$, we have $\pi = 0$ (which states $u'(w_1) = u'(w_3)$) and the first order conditions of (5) relative to τ and ϵ are respectively:

$$-(1 + \lambda^{r\prime}(\alpha))(1 + \frac{p(q_c - q_n)}{\bar{q}}\lambda^R)\bar{q}\mathbb{E}(u'(\tilde{w})) + (1-p)q_n u'(w_2) + pq_c u'(w_4) = 0, \tag{45}$$

$$(1 + \lambda^{r\prime}(\alpha))(1 + \lambda^R)pq_c\mathbb{E}(u'(\tilde{w})) - pq_c u'(w_4) = 0. \tag{46}$$

Firstly, (46) gives:

$$u'(w_4) = (1 + \lambda^{r\prime}(\alpha))(1 + \lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{47}$$

Secondly, with $\bar{q} = (1-p)q_n + pq_c$, the combination of (45) and (46) gives:

$$u'(w_2) = (1 + \lambda^{r\prime}(\alpha))(1 - \frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{48}$$

Thirdly, with (47), (48) and the definition of $\mathbb{E}(u'(\tilde{w}))$, we get:

$$u'(w_1) = u'(w_3) = \left((1 + \lambda^{r\prime}(\alpha))(1 - \frac{p(q_c - q_n)}{1 - \bar{q}}\lambda^R) - \frac{\lambda^{r\prime}(\alpha)}{1 - \bar{q}}\right)\mathbb{E}(u'(\tilde{w})). \tag{49}$$

C.2   Comparative statics

We consider a CARA utility function function $u(.)$, i.e. with $A = -\frac{u''(.)}{u'(.)} > 0$ constant.

With $0 < \lambda^r < \lambda^{r*}$, (13) gives:

$$(1 - \frac{\lambda^r}{p(1-q_c)(1+\lambda^R)(1+\lambda^r)})u''(w_4)dw_4 - \frac{\frac{1}{(1+\lambda^r)^2}}{p(1-q_c)(1+\lambda^R)}u'(w_4)d\lambda^r = u''(w_3)dw_3. \tag{50}$$

With (13), (50) can be rewritten:

$$-A(dw_4 - dw_3) = \frac{1}{p(1-q_c)(1+\lambda^R)(1+\lambda^r)^2 - \lambda^r(1+\lambda^r)}d\lambda^r, \tag{51}$$

which finally gives with $\epsilon = w_3 - w_4$:

$$\frac{d\epsilon}{d\lambda^r} = \frac{1}{A(p(1-q_c)(1+\lambda^R)(1+\lambda^r)^2 - \lambda^r(1+\lambda^r))}. \tag{52}$$

Similarly, (14) gives:

$$\frac{d\pi}{d\lambda^r} = -\frac{1}{A(p(1-q_c)(1+\lambda^R)(1+\lambda^r)^2 - \lambda^r(1+\lambda^r))}. \tag{53}$$

Besides, $\alpha = (\bar{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi - (1+\lambda^R)pq_c\epsilon$ and $\tau^R = N(q_c - q_n)l - N\pi - Nq_c\epsilon > 0$ respectively give with (52) and (53):

$$\frac{d\alpha}{d\lambda^r} = -(1 - p(1-q_c)(1+\lambda^R))\frac{d\epsilon}{d\lambda^r}, \tag{54}$$

$$\frac{d\tau^R}{d\lambda^r} = N(1 - q_c)\frac{d\epsilon}{d\lambda^r}. \tag{55}$$

Note that $\lambda^r < \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$ and $(1+\lambda^R)p < 1$ give: $p(1-q_c)(1+\lambda^R)(1+\lambda^r) - \lambda^r > 0$, which tells the sign of the four latter equations.

With $0 < \lambda^r < \lambda^{r*}$, (13) and (14) give similarly:

$$\frac{d\epsilon}{dq_c} = \frac{\lambda^r}{A(p(1-q_c)^2(1+\lambda^R)(1+\lambda^r) - \lambda^r(1-q_c))}, \tag{56}$$

$$\frac{d\pi}{dq_c} = -\frac{\lambda^r}{A(p(1-q_c)^2(1+\lambda^R)(1+\lambda^r) - \lambda^r(1-q_c))}. \tag{57}$$

Note that $\lambda^r < \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$ and $(1+\lambda^R)p < 1$ give: $p(1-q_c)(1+\lambda^R)(1+\lambda^r) - \lambda^r > 0$. Thus, $\frac{d\epsilon}{dq_c} > 0$ and $\frac{d\pi}{dq_c} < 0$. Because $\frac{d\epsilon}{dq_n} = 0$ and $\frac{d\pi}{dq_n} = 0$, we thus have: $\frac{d\epsilon}{d\delta} > 0$ and $\frac{d\pi}{d\delta} < 0$. Besides, $\alpha = (\bar{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi - (1 + \lambda^R)pq_c\epsilon$ and $\tau^R = N(q_c - q_n)l - N\pi - Nq_c\epsilon > 0$ respectively give with (56) and (57):

$$\frac{d\alpha}{d\delta} = p\frac{d(q_c - q_n)}{d\delta}\lambda^R l - \left((1 - p(1-q_c)(1+\lambda^R))\frac{d\epsilon}{dq_c} + (1+\lambda^R)p\epsilon\right)\frac{dq_c}{d\delta}, \tag{58}$$

$$\frac{d\tau^R}{d\delta} = \left(N(l - \epsilon) + N(1 - q_c)\frac{d\epsilon}{dq_c}\right)\frac{dq_c}{d\delta} - Nl\frac{dq_n}{d\delta}. \tag{59}$$

Thus, $\frac{d\alpha}{d\delta}$ is ambiguous and $\frac{d\tau^R}{d\delta} > 0$.