

Horizontal Coordination and Transparency of Information

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Abstract

Rational agents might choose to invest in a certain kind of capital in a period in the hope of making higher returns from their investments made in consecutive periods. We examine the impact of such an interaction on the incidence of coordination failure and accordingly social welfare. In our set-up, investment complementarities are present both within periods (call vertical complementarity) and between periods (call horizontal complementarity). In particular, other than the underlying economic fundamental, the return on investment depends on its aggregate level in that period as well as the aggregate investment made in the previous period. The results suggest that full transparency is optimal at the social level as long as agents have an access to relatively more precise private information and complementarities are sufficiently low. More transparency otherwise reduces social welfare as the gain from better vertical coordination is outweighed by the loss resulted from lesser horizontal coordination.

Keywords Investment Complementarity · Coordination Failure · Social Welfare · Transparency · Global Games

JEL Codes D62, D79, D81, D82

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1 Introduction

The content and source of information is always a matter of subject for a decision maker to find an optimal course of action under uncertainty. The decision maker needs to balance his own private information against the one that is publicly known and observed by other market participants. However, it's not that obvious how to combine the two sources of information when making decisions, because both are noisy. The issue becomes more and more complicated in the presence of strategic complementarities (or substitutes). Strategic complementarities make it necessary for agents to coordinate their actions in order to acquire a desirable outcome. In such cases, greater access to information might be enticing for a decision maker to better predict others' actions and take appropriate

actions accordingly.

However, coordination failure might arise in these settings when agents fail to take those set of actions that are in their collective interest. In order to prevent such failures, central authorities such as central banks or governments choose to disseminate information about economic fundamentals through direct policy announcements or via media. This in turn shapes people's expectations about the future of the economy and affects their decisions. Besides, certain kinds of public devices, such as asset prices, help people learn about others' expectations and beliefs about economic fundamentals. However, it is something of a curate's egg. On the one hand, agents become more informed about economic fundamentals as well as others' beliefs and therefore are presumed to make more accurate decisions. On the other hand, they might over-react to the dissemination and choose to ignore their private signals. In such a case, the effectiveness of public information as a coordination device has become an issue and is of importance from a social point of view.

Morris and Shin (2002) has constructed a model to explore the impact of public and private information on social welfare in which individuals are rational utility maximizers and have two different objectives, given exogenous public and private information about the economic fundamental. First, they try to take actions as close as possible to the underlying fundamental. Second, they try not to take extreme actions compared to the others. The latter provides agents with coordination motive and is incorporated into the model by introducing Keynesian beauty contest. Their results suggest that the impact of public information is unambiguous when there is imperfect information and depends on whether agents have an access to socially valuable private information.

In particular, increased provision of public information tends to be more detrimental to social welfare as private information becomes more and more precise. It is resulted from the fact that the coordination motive entails placing too much weight on the public signal relative to the weights that would be used by the social planner (Morris and Shin, 2002). In this sense, being transparent or not depends on the information structure of a particular economy and full transparency might not be advisable to an economy in which agents do have independent information sources. It is also found that social welfare is positively correlated with the precision of private information irrespective of other parameter values.

In a different setting, Angeletos and Pavan (2004) has found contrasting results with the ones in Morris and Shin (2002). When the complementarities are weak it is always optimal to be transparent as much as possible because it helps investors internalize the positive externality and entails more effective coordination. Besides, more precise private information creates more heterogeneous expectations and deteriorates social coordination and accordingly social welfare under certain circumstances.

The difference in the results comes from the environments in which agents coordinate their actions. In the former model, the complementarity has a considerable impact on individual returns whereas it is socially wasteful to exert extra effort in the hope of providing more effective coordination. In contrast, the complementarity is present at the social level in the latter model so that more transparent public information, by permitting more effective coordination in the market, necessarily increases welfare (Angeletos and Pavan,

2004).

In both settings, there exists only one investment tool on which agents try to align their actions in order to maximize their expected payoffs. However, there are some cases in which agents need to choose among different investment options (or tools) whose returns are possibly dependent upon each other. One such example would be the case where agents choose to invest in a certain kind of capital in a period in the hope of making higher returns from their investments made in consecutive periods. The optimal bundle of investment in this case depends on individual preference, information about returns on investment in different periods as well as whether individual investments are strategic complements or substitutes of each other.

The main goal in this study is to examine the impact of such an interaction on the the incidence of coordination failure and accordingly social welfare in the presence of investment complementarities both within periods (call vertical complementarity) and between periods (call horizontal complementarity). In particular, we construct a two-period model in what follows. The return on investment in period one depends both on the economic fundamental and aggregate level of investment in that period. However, other than those two, the return of investment in period two also depends on the aggregate investment made in period one.

In this context, we first focus on the impact of horizontal complementarity on the incidence of coordination failure and accordingly social welfare. Coordination failures waste socially valuable investments and hence might hamper the growth of the economy as a whole. Second, we examine how the precision of public and private information affects social welfare and analyze to what extent more transparent (public) information would be desirable from a social perspective.

The results suggest that the introduction of horizontal complementarity makes coordination failures more likely and hence might deteriorate social welfare in the sense that it provides the economy with extra source of uncertainty. An investor can choose extreme bundles, such as high investment in period one with a lower one in period two or vice versa, or prefer more balanced bundles. In this sense, investors need to decide how much weight they give on public and private information as well as need to predict inter-temporal composition of others' investment bundles to form his own bundle.

We also find that more transparency is not always optimal from a social point of view. Whether or not it is socially desirable depends on both complementary levels and the precision of public information relative to the private one. When the public information is relatively less precise, increase in transparency is beneficial to the society as a whole. However, it reduces social welfare in the opposite case. The adverse impact of public information results from the fact that agents begin to ignore their private signals and give more and more weight on the public signal, as it becomes more precise, relative to weights that would be used by the social planner (Morris and Shin, 2002).

The impact of the precision of private information is also ambiguous in the sense that whether or not it improves social welfare depends on the degree of complementarities,

relative precision of private information, and absolute precision of public information. When the precision of public information is too low, the increase in private precision has a positive impact on social welfare. However, as the precision of public information becomes greater in absolute terms the impact turns out to be negative since the benefit of lower uncertainty, due to higher private precision, is outweighed by the cost of lower coordination (Angeletos and Pavan, 2004).

The rest of the paper is organized as follows. The literature review is given in Section 2, the model is presented in Section 3, equilibrium analysis is conducted and the results are presented in Section 4, and Section 5 concludes.

2 Literature Review

In addition to Morris and Shin (2002) and Angeletos and Pavan (2004), the effect of public information has also been examined in the literature within the framework of global games. Morris et al. (2006) have shown that public information is still harmful to social welfare even if we allow for correlated signals. In a different setting, Cornand and Heinemann (2008) have demonstrated that public information is socially beneficial and hence should always be provided with maximum precision but, under certain conditions, not to all agents.

Chan and Chiu (1999) and Metz (2002) have analyzed the consequences of information dissemination on currency crisis. In contrast with the results of the former, Metz (2002) has found that more transparency reduces the likelihood of a currency crisis when the economic conditions are strong and the opposite occurs in case of weaker fundamentals. In a similar fashion, Heinemann and Illing (2002) has found that higher transparency might reduce strategic uncertainty and accordingly helps to reduce incentives for speculative attacks.

Coordination problems, its ramifications and possible amelioration have also been widely investigated in the literature. Cooper and Andrew (1988) is an early instance of how coordination problems arises in the existence of strategic complementarities. Hellwig (2005) has established a model based on Woodford (2002) to examine monetary economies in which complementarities arise in pricing decisions and their results are in line with the findings of Angeletos and Pavan (2004).

Angeletos, Hellwig and Pavan (2003) has shown, in a global coordination game, that the possibility that policy choices convey information might lead to policy traps where both the optimal policy and the coordination outcome are dictated by self-fulfilling market expectations. In this regard, it has been argued that the impact of higher transparency does depend on policy choices as well as the interpretations of them by market participants. Morris and Shin (2004) has constructed a model in which creditors of a distressed borrower face a coordination problem and found that more transparency does not generally mitigate the coordination problem.

There are also other studies examining the role of public information from a social perspective: See, for instance, Hellwig (2002), Woodford (2005), Svensson (2006), Angeletos

and Pavan (2007), Anctil et al. (2010), and Bouvard et al. (2015).

3 The Model

There is a continuum of risk neutral agents indexed by i over the unit interval $[0, 1]$. Agents (or investors) choose how much capital they invest in period one and two in order to maximize their life-time utility:

$$v_i(k_{i,1}, k_{i,2}) = u_i(k_{i,1}) + \frac{1}{1+r_f} u_i(k_{i,2}) \quad (1)$$

$$\text{with } u_i(k_{i,t}) = A_t k_{i,t} - \frac{1}{2} k_{i,t}^2 \quad \text{for } t = 1, 2 \quad (2)$$

where $k_{i,t} \in \mathbb{R}$, $\frac{1}{2} k_{i,t}^2$ is the cost of the investment in period t and r_f is the risk-free rate. A_t represents the gross return on the investment in period t given by

$$A_t = (1 - \lambda)\theta + \lambda K_t + \mathbb{1}_{(t=2)}[\Lambda(K_{t-1} - \theta)] \quad (3)$$

$$\text{with } K_t = \int_0^1 k_{j,t} dj. \quad (4)$$

where $\mathbb{1}$ is an indicator function. The return of investment in both periods depends on an exogenous state variable θ that summarizes economic fundamentals. It could be thought as an aggregate variable containing all the relevant information about the general economic environment. Individuals have a common prior about the fundamental which is uninformative in the sense that it follows a uniform distribution over entire the real line. The return on investment in period t also depends on the aggregate level of investment K_t which makes the actions strategic complements of each other within a particular period where the degree of complementarity is parametrized by λ . That is, λ parametrizes the degree of vertical complementarity which is assumed to be the same across periods ¹.

The true value of the fundamental θ is unknown at the time when investors need to decide their investment levels since its true value will be realized at the end of the second period. However, each investor receives an exogenous private signal x_i with a certain precision rate β :

$$x_i = \theta + \frac{1}{\sqrt{\beta}} \varepsilon_i \quad (5)$$

where ε_i is standard normal, independent of θ , and independent and identically distributed across agents. All investors also observes a public signal with precision α :

$$z = \theta + \frac{1}{\sqrt{\alpha}} \varepsilon \quad (6)$$

where ε is standard normal and independent of θ and ε_i .

¹Nothing substantial hinges on this assumption, which are made for the purposes of simplifying the statement of our results.

Other than the economic fundamental and intra-period complementarity, return on investment in period two (i.e. A_2) also depends on the aggregate level of investment made in period one (i.e. K_1). The latter introduces complementarity between periods which is captured by the parameter Λ . That is, Λ parametrizes the degree of horizontal complementarity.

We have formulated social welfare function as a utilitarian aggregator:

$$W(\theta) = \int_0^1 v_i di. \quad (7)$$

As in Angeletos and Pavan (2004), we are also interested in the region where the social welfare function is concave in K_t in order for lotteries not to be desirable from a social point of view. To this end, we assume $\lambda \in [0, 1/2)$. Observe that there is no need to put an extra restriction on Λ , which means it could take any value between zero and one.

4 Equilibrium Analysis, and Results and Findings

4.1 Equilibrium Analysis

Each agent chooses $k_{i,1}$ and $k_{i,2}$ in order to maximize his expected life-time utility conditioned on the available information, $E_i [v_i(k_{i,1}, k_{i,2}) | x_i, z]$, and assuming the risk-free rate is zero ², it results in

$$k_{i,1} = (1 - \lambda)E_i [\theta | x_i, z] + \lambda E_i [K_1 | x_i, z]. \quad (8)$$

Using Lemma 1 given in Appendix 1, an agent i has the following posterior belief about θ . Conditional on x_i and z , it is normally distributed with mean $\frac{\alpha z + \beta x_i}{\alpha + \beta}$ and variance $\frac{1}{\alpha + \beta}$. Morris and Shin (2002) has proved that equilibrium investment strategies are linear in private and public signal and moreover this equilibrium is unique. The optimal amount of period-one investment in the equilibrium could then be found as (See Appendix 2 for further details):

$$k_{i,1} = \eta_1 x_i + (1 - \eta_1)z \quad \text{where } \eta_1 = \frac{\beta(1 - \lambda)}{\alpha + \beta(1 - \lambda)}. \quad (9)$$

An agent also chooses how much investment he should make in period two whose optimality condition is given as follows:

$$k_{i,2} = (1 - \lambda - \Lambda)E_i [\theta | x_i, z] + \Lambda E_i [K_1 | x_i, z] + \lambda E_i [K_2 | x_i, z] \quad (10)$$

In a similar fashion, we here also assume that agents follow a linear strategy when choosing their period-two investments. Based on this argument, we have found the following optimal strategy (See Appendix 2 for further details).

$$k_{i,2} = \eta_2 x_i + (1 - \eta_2)z \quad \text{where } \eta_2 = \frac{\beta(1 - \lambda)}{[\alpha + \beta(1 - \lambda)]} - \Lambda \frac{\alpha\beta}{[\alpha + \beta(1 - \lambda)]^2} \quad (11)$$

²The results continue to hold for plausible values of risk-free rate.

Given the optimum investment strategies, expected social welfare conditional on the realization of the fundamental is equal to

$$E[W(\theta)] = \theta^2 + \gamma - \frac{1}{2} \sum_{t=1}^2 \varkappa_t \quad (12)$$

$$\text{where } \gamma = \Lambda(1 - \eta_1)(1 - \eta_2) \frac{1}{\alpha} \quad \text{and} \quad \varkappa_t = \eta_t^2 \frac{1}{\beta} + (1 - 2\lambda)(1 - \eta_t)^2 \frac{1}{\alpha}.$$

Equation (12) suggests that expected social welfare depends on three things. The first term parametrizes the effect of the underlying fundamental. The second term γ is due to the existence of horizontal complementarity and is a function of the covariance between aggregate investment levels in period one and two (See Appendix 3 for further details). Finally, the last term \varkappa measures welfare consequences of the heterogeneity in individual investment levels and captures the effect of volatility in aggregate investment.

4.2 Results and Findings

In our setting, an agent could make a certain return from his investment irrespective of the complementarity levels because his total return depends on the underlying economic fundamental as well. This in turn affects his investment decisions depending upon his expectation about the fundamental. In this context, he needs to choose how much weight he puts on his private information and public information. When making this decision, he also needs to choose an optimal bundle of investment within and between periods due to the presence of two kinds of complementarity. Vertical complementarity has been examined in Angeletos and Pavan (2004) and shown that it makes coordination failures more likely, and hence we focus more on horizontal complementarity.

Horizontal complementarity also increases strategic uncertainty and makes coordination failures more likely in the following sense. Period-two investment level of an agent depends on his expectation about the aggregate level of investment in that period as well as the aggregate level of the previous period. If he expects other agents to put more weight on period-one or -two investment, he tends to choose to do so. However, it might be socially wasteful in such a case that the complementarity of investments between periods is high. The results might be opposite when the complementarity between periods is sufficiently low in the sense that choosing a more balanced bundle would not provide enough gain at the social level.

In light of these observations, we can now examine the impact of private and public information on social welfare. Since it is analytically hard in our setting to analyze the expected social welfare, we simulate the results under different specifications³. Let's focus first on the social consequences of transparency, which is measured by absolute or relative precision of public information as in Angeletos and Pavan (2004). When horizontal complementarity is sufficiently low, the results are basically the same as Angeletos and Pavan (2004). In particular, increase in the precision of public information α improves social welfare irrespective of the precision of private information β because it helps agents

³The results are robust in different simulation settings and hence we have presented only the results for uniformly distributed parameters.

better coordinate their actions. In this regard, full transparency is always optimal.

The impact of α on social welfare however becomes unambiguous when horizontal complementarity is sufficiently high. Relative precision of public information and the degree of vertical complementarity do matter in this case. The interval over which the relative precision α/β is less than one half, the γ in equation (12) is an increasing function of α . On this interval, welfare increases in transparency regardless of the degree of vertical complementarity. That is, more precise public information is socially beneficial in this case since it provides better horizontal coordination as well as better vertical coordination. The full transparency is then still optimal under these circumstances.

If α/β is greater than one half, an increase in α has two opposing effects on social welfare. As in the former case, it provides better coordination in a vertical manner. However, it magnifies the effectiveness of public information when making investment allocation between periods. It could be seen from equations (8) and (11) as the weight of public information in period-two investment is greater compared to the weight of period-one. This might result in an overreaction of agents to public information and partial ignorance of their privately valuable information. Although public information is extremely effective in influencing actions in this case, the danger arises from the fact that it is too effective at doing so (Morris and Shin, 2002).

The degree of vertical complementarity matters in this case from a social point of view. That is, whether the total effect of an increase in α on social welfare is positive or negative depends on λ . If it is sufficiently low, social welfare increases in transparency because the gain obtained from better vertical coordination outweighs the loss resulted from the deteriorated horizontal coordination. However, as the degree of vertical complementarity gets larger and larger, the impact of worsened horizontal coordination on social welfare magnifies and increase in α deteriorates social welfare.

As in Angeletos and Pavan (2004), the impact of the precision of private information on welfare is unambiguous. Aside from the degrees of complementarities and relative precision, the impact also depends on absolute precision. There are four possible scenarios in terms of complementarity levels. In the first case, both vertical and horizontal complementarities are low in which case social welfare increases in β regardless of α . In the opposite case, when both are high, higher private precision is detrimental to horizontal coordination and in most cases to vertical coordination. Therefore, it results in lower social welfare in most cases unless public information has unrealistically low precision.

To illustrate more, observe that given α the term γ in equation (12) is negatively related with β since γ is positively related with relative precision. In this setting if the precision of public information is (unrealistically) too low in absolute terms, the loss made resulted from worsening horizontal coordination is outweighed by the gain from better vertical coordination. However, if α exceeds a certain threshold, the last term in equation (12) also decreases in private precision. In this case, an increase in private precision deteriorates both vertical and horizontal coordination and results in a significant drop in social welfare. This effect gets much worse when absolute public precision gets higher.

In either of the remaining two cases, agents can better coordinate as long as the relative precision α/β is not too high. They tend to invest more in period one if horizontal complementarity is too low and vice versa which in turn makes coordination failures less likely. In the first case, when vertical complementarity is low and horizontal complementarity is high, increases in private precision improves social welfare even though it might cause a slight decrease at the first place. This argument is in fact more valid for lower levels of horizontal complementarity. In the other case, the impact of private information becomes more unambiguous and depends on the very parameter values of other variables. However, in general, social welfare tends to decrease to a certain point, and then improves slightly as the loss from worsened horizontal coordination begins to be compensated by the gain from better vertical coordination.

5 Conclusion

We construct a two-period model in the presence of investment complementarities both within periods (call vertical complementarity) and between periods (call horizontal complementarity) in order to examine the impact of intra- and inter-temporal investment complementarities on the the incidence of coordination failure and accordingly social welfare. In particular, the return on investment in a particular period depends both on the economic fundamental and aggregate level of investment in that period. However, other than those two, the return of investment in period two also depends on the aggregate investment made in period one.

The existence of horizontal complementarity increases strategic uncertainty and makes coordination failures more likely. The results suggest that full transparency is optimal at the social level as long as agents have an access to relatively more precise private information and complementarities are sufficiently low. More transparency otherwise reduces social welfare as the gain from better vertical coordination is outweighed by the loss resulted from lesser horizontal coordination. We have also found that the impact of the precision of private information on welfare is unambiguous. Aside from the degrees of complementarities and relative precision, the effect of it also depends on absolute precision of public information.

References

- [1] Anctil, R. M., Dickhaut, J., Johnson, C., & Kanodia, C. (2010). Does information transparency decrease coordination failure? *Games and Economic Behavior*, 70(2), 228-241.
- [2] Angeletos, G.-M., Hellwig, C., & Pavan, A. (2003). Coordination and Policy Traps. National Bureau of Economic Research, Working Paper No. 9767.
- [3] Angeletos, G.-M., & Pavan, A. (2004). Transparency of Information and Coordination in Economies with Investment Complementarities. *American Economic Review*, 94(2), 91-98.
- [4] Angeletos, G.-M., & Pavan, A. (2007). Efficient Use of Information and Social Value of Information. *Econometrica*, 75(4), 1103-1142.

- [5] Bouvard, M., Chaigneau, P., & Motta, A. De. (2015). Transparency in the Financial System: Rollover Risk and Crises. *The Journal of Finance*, 70(4), 1805-1837.
- [6] Chan, K. S., & Chiu, Y. S. (2002). The role of (non-)transparency in a currency crisis model. *European Economic Review*, 46(2), 397-416.
- [7] Cooper, R., & John, A. (1988). Coordinating Coordination Failures in Keynesian Models. *The Quarterly Journal of Economics*, 103(3), 441-463.
- [8] Cornand, C., & Heinemann, F. (2008). Optimal degree of public information dissemination. *Economic Journal*, 118(528), 718-742.
- [9] Heinemann, F., & Illing, G. (2002). Speculative attacks: Unique equilibrium and transparency. *Journal of International Economics*, 58(2), 429-450.
- [10] Hellwig, C. (2002). Public Announcements, Adjustment Delays, and the Business Cycle. mimeo, UCLA Economics Online Papers.
- [11] Hellwig, C. (2005). Heterogeneous Information and the Welfare Effects of Public Information Disclosures. mimeo, UCLA Economics Online Papers.
- [12] Metz, C. E. (2002). Private and Public Information in Self-fulfilling Currency Crises. *Journal of Economics*, 76(1), 65-85.
- [13] Morris, S., & Shin, H. (2002). Social Value of Public Information. *American Economic Review*, 92(5), 1521-1534.
- [14] Morris, S., & Shin, H. S. (2004). Coordination Risk and the Pricing of Debt. *European Economic Review*, 17(1-2), 133-153.
- [15] Morris, S., Shin, H. S., & Tong, H. (2006). Social Value of Public Information: Morris and Shin (2002) Is Actually Pro-Transparency, Not Con: Reply. *The American Economic Review*, 96(1), 453-455.
- [16] Svensson, L. E. O. (2006). Social Value of Public Information: Comment: Morris and Shin (2002) Is Actually Pro-Transparency, Not Con. *The American Economic Review*, 96(1), 448-452.
- [17] Woodford, M. (2001). Imperfect Common Knowledge and the Effects of Monetary Policy in Knowledge. National Bureau of Economic Research, Working Paper No 8673.

APPENDICES

APPENDIX 1

Lemma 1 *Let y and x be represented as a bi-variate normal distribution*

$$\begin{bmatrix} y \\ x \end{bmatrix} \sim N \left(\begin{bmatrix} E_y \\ E_x \end{bmatrix}, \begin{bmatrix} V_y & V_{yx} \\ V_{xy} & V_x \end{bmatrix} \right).$$

The conditional expectation and variance of y are given then:

$$E[y | x] = E_y + \frac{V_{yx}}{V_x}(x - E_x) \quad \text{and} \quad \text{Var}[y | x] = V_y - \frac{V_{yx}^2}{V_x}$$

APPENDIX 2

Assuming investment strategies are linear in private and public signal such that

$$k_{i,1} = \eta_1 x_i + (1 - \eta_1)z \quad \text{and} \quad k_{i,2} = \eta_2 x_i + (1 - \eta_2)z$$

the optimum capital strategies could be found as follows:

$$\begin{aligned} & \arg \max_{k_{i,1}} E_i [v_i(k_{i,1}, k_{i,2}) | x_i, z] = u_i(k_{i,1}) + \frac{1}{1+r} u_i(k_{i,2}) \\ \text{F.O.C.} \quad & k_{i,1} = E_i [A_1 | x_i, z] = (1 - \lambda) E_i [\theta | x_i, z] + \lambda E_i [K_1 | x_i, z] \\ E_i [K_1 | x_i, z] &= E_i \left[\int_0^1 k_{j,1} dj | x_i, z \right] = E_i \left[\int_0^1 (\eta_1 x_j + (1 - \eta_1)z) dj | x_i, z \right] \\ &= \eta_1 \int_0^1 E [x_j | x_i, z] dj + (1 - \eta_1)z = \eta_1 \frac{\alpha z + \beta x_i}{\alpha + \beta} + (1 - \eta_1)z \\ &\Rightarrow k_{i,1} = (1 - \lambda) \frac{\alpha z + \beta x_i}{\alpha + \beta} + \lambda \left(\eta_1 \frac{\alpha z + \beta x_i}{\alpha + \beta} + (1 - \eta_1)z \right) \\ &\Rightarrow \eta_1 = (1 - \lambda) \frac{\beta}{\alpha + \beta} + \lambda \frac{\beta}{\alpha + \beta} \eta_1 \Rightarrow \eta_1 = \frac{\beta(1 - \lambda)}{\alpha + \beta(1 - \lambda)} \\ &\Rightarrow k_{i,1} = \frac{\beta(1 - \lambda)}{\alpha + \beta(1 - \lambda)} x_i + \left(\frac{\alpha}{\alpha + \beta(1 - \lambda)} \right) z \end{aligned}$$

$$\begin{aligned}
& \arg \max_{k_{i,2}} E_i [v_i(k_{i,1}, k_{i,2}) | x_i, z] = u_i(k_{i,1}) + \frac{1}{1+r} u_i(k_{i,2}) \\
F.O.C. \quad & k_{i,2} = E_i [A_2 | x_i, z] = E_i [(1 - \lambda - \Lambda)\theta + \Lambda K_1 + \lambda K_2 | x_i, z] \\
& = (1 - \lambda - \Lambda)E_i [\theta | x_i, z] + \Lambda E_i [K_1 | x_i, z] + \lambda E_i [K_2 | x_i, z] \\
E_i [K_2 | x_i, z] & = E_i \left[\int_0^1 k_{j,2} dj | x_i, z \right] = E_i \left[\int_0^1 (\eta_2 x_j + (1 - \eta_2)z) dj | x_i, z \right] \\
& = \eta_2 \int_0^1 E_i [x_j | x_i, z] dj + (1 - \eta_2)z = \eta_2 \frac{\alpha z + \beta x_i}{\alpha + \beta} + (1 - \eta_2)z \\
\Rightarrow k_{i,2} & = (1 - \lambda - \Lambda) \frac{\alpha z + \beta x_i}{\alpha + \beta} + \Lambda \left(\eta_1 \frac{\alpha z + \beta x_i}{\alpha + \beta} + (1 - \eta_1)z \right) \\
& \quad + \lambda \left(\eta_2 \frac{\alpha z + \beta x_i}{\alpha + \beta} + (1 - \eta_2)z \right) \\
\Rightarrow \eta_2 & = (1 - \lambda - \Lambda) \frac{\beta}{\alpha + \beta} + \Lambda \frac{\beta}{\alpha + \beta} \eta_1 + \lambda \frac{\beta}{\alpha + \beta} \eta_2 \\
\Rightarrow \eta_2 & = \frac{\beta(1 - \lambda)}{[\alpha + \beta(1 - \lambda)]} - \Lambda \frac{\alpha \beta}{[\alpha + \beta(1 - \lambda)]^2}
\end{aligned}$$

APPENDIX 3

$$\begin{aligned}
W(\theta) & = \sum_{t=1}^2 \int_0^1 v_i di = \sum_{t=1}^2 \int_0^1 \left(A_t k_{i,t} - \frac{1}{2} k_{i,t}^2 \right) di = \sum_{t=1}^2 \left(A_t K_t - \frac{1}{2} \int_0^1 k_{i,t}^2 di \right) \\
\Rightarrow E[W(\theta)] & = \sum_{t=1}^2 E[A_t K_t | \theta] - \frac{1}{2} \sum_{t=1}^2 E \left[\int_0^1 k_{i,t}^2 di | \theta \right] \\
& = \sum_{t=1}^2 E \left[((1 - \lambda)\theta + \lambda K_t + \mathbb{1}_{(t=2)}[\Lambda(K_{t-1} - \theta)]) \cdot K_t | \theta \right] - \frac{1}{2} \sum_{t=1}^2 E \left[\int_0^1 k_{i,t}^2 di | \theta \right] \\
& = \sum_{t=1}^2 (1 - \lambda)\theta E[K_t | \cdot] + \lambda E[K_t^2 | \cdot] + \Lambda \mathbb{1}_{(t=2)} \{E[K_{t-1} K_t | \cdot] - \theta E[K_t | \cdot]\} - \frac{1}{2} \sum_{t=1}^2 E \left[\int_0^1 k_{i,t}^2 di | \cdot \right]
\end{aligned}$$

$$\begin{aligned}
E[K_t | \theta] & = E[\eta_t \theta + (1 - \eta_t)z | \theta] = \theta \\
E[K_t^2 | \theta] & = \text{var}(K_t | \theta) + E[K_t | \theta]^2 = (1 - \eta_t)^2 \frac{1}{\alpha} + \theta^2
\end{aligned}$$

$$\begin{aligned}
E[K_{t-1}K_t | \theta] &= Cov(K_{t-1}, K_t | \theta) + E[K_{t-1} | \theta] \cdot E[K_t | \theta] \\
&= Cov[\eta_1\theta + (1 - \eta_1)z, \eta_2\theta + (1 - \eta_2)z | \theta] + \theta^2 \\
&= (1 - \eta_1)(1 - \eta_2)Cov(z, z | \theta) + \theta^2 = (1 - \eta_1)(1 - \eta_2)\frac{1}{\alpha} + \theta^2
\end{aligned}$$

$$\begin{aligned}
-\frac{1}{2}\int_0^1 k_{i,t}^2 di &= \frac{1}{2}\int_0^1 [K_t^2 - 2k_{i,t}K_t] di - \frac{1}{2}\int_0^1 [k_{i,t}^2 + K_t^2 - 2k_{i,t}K_t] di \\
&= -\frac{1}{2}K_t^2 - \frac{1}{2}\int_0^1 (k_{i,t} - K_t)^2 di = -\frac{1}{2}K_t^2 - \frac{1}{2}\eta_t^2 \int_0^1 [x_i - \theta]^2 di \\
&= -\frac{1}{2}K_t^2 - \frac{1}{2}\eta_t^2 \frac{1}{\beta}
\end{aligned}$$

$$\Rightarrow E[W(\theta)] = \theta^2 + \gamma - \frac{1}{2}\sum_{t=1}^2 \varkappa_t$$

where $\gamma = \Lambda(1 - \eta_1)(1 - \eta_2)\frac{1}{\alpha}$ and $\varkappa_t = \eta_t^2 \frac{1}{\beta} + (1 - 2\lambda)(1 - \eta_t)^2 \frac{1}{\alpha}$

with $\eta_1 = \frac{(1 - \lambda)\beta}{\alpha + \beta(1 - \lambda)}$ and $\eta_2 = \frac{\beta(1 - \lambda)}{[\alpha + \beta(1 - \lambda)]} - \Lambda \frac{\alpha\beta}{[\alpha + \beta(1 - \lambda)]^2}$.