Decentralisation in the Disposal of Waste:

A Welfare Approach

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Abstract

In this article we analyse incentives, equilibria and implications of the governance framework for the disposal of municipal solid waste. The key decisions revolve around the flow of waste among regions and the externalities (mainly pollution) associated with its final disposal, be it via incineration or landfill. When the regions are characterised by different levels of efficiency in the final treatment of waste, a certain degree of mobility might allow to reap the benefits of higher efficiency. On the other hand, as transportation and environmental costs implied by mobility and concentration become significant, a trade-off emerges. Our model evaluates the implications of this trade-off for the optimal degree of decentralisation in waste management.

Keywords: waste disposal, decentralisation, welfare.

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1 Introduction

Since the seminal work of Oates (1972) on fiscal federalism, a central question of public finance has been which level of a federation should be assigned the provision of public goods. Local jurisdictions, either municipalities or regions, are more likely to internalise local conditions and costs, but ignore inter-jurisdictional spillovers. On the other hand, central governments may internalize those spillovers, but are likely to neglect local conditions. Both dimensions are empirically relevant in environmental applications.\(^1\) Globalisation has added new scenarios: the concept of decentralisation could in fact be applied also to the regulation of waste disposal and environmental protection at super-national level. Waste generation and disposal are key areas of interest in this debate, where, according to the European Environmental Agency, waste volumes in the European Union are shifting (EEA, 2009, 2013), driven by changing production and consumption patterns (Andersen et al., 2007), whereas environmental costs associated with waste disposal essentially depend on regulation. Waste prevention is the top aim of European policy’s ‘waste hierarchy’, which lists municipal waste management (MWM) objectives in order of descending priority. In the context of the Circular Economy strategy, if waste cannot be prevented, it should be reused (or prepared for reuse), recycled, incinerated with energy recovery, or disposed of in landfill, if no other option is available (EUCOMM, 2017).

From an empirical point of view, there is some evidence of a Kuznets effect for the GDP-waste volumes relationship (Mazzanti et al., 2008, 2012; Mazzanti and Zoboli, 2008, 2009), although little evidence of a permanent decoupling. With reference to the so-called “NIMBY” attitude on the location of waste disposal sites (Fredriksson, 2000), the theoretical literature is less developed. The few existing contributions exploit the standard assumptions of the theory of fiscal federalism to explain the prevalence of decentralised decisions in waste management, but there is almost no agreement on which level of centralisation is more efficient. While Ogawa and Wildasin (2009)

\(^{1}\)For reviews see Banzhaf and Chupp (2011) and Buchholz et al. (2011).
argue that decentralisation might reach a more efficient allocation than centralisation, other studies claim that such a framework might spur undesirable and distorting effects, such as fiscal competition and “race to the bottom” (Oates and Schwab, 1988; Oates, 1999).

The aim of this paper is to develop a simple theoretical model to investigate key policy questions about the effects of decentralisation on waste flows, environment damages and, ultimately, welfare. Our theoretical interest into the key governance features of the problem at hand stems from the fact that across countries MWM is operated through a variety of decentralised solutions and regulations. The key decisions revolve around the cross-regional mobility of waste and the externalities (pollution) associated with its disposal, be it via incineration or landfill dumping. When the regions are characterised by different levels of efficiency in the processes they apply to the final treatment, a certain degree of mobility across regions might allow to reap the benefits of a higher efficiency. On the other hand, as transportation and other environmental costs implied by waste mobility and the concentration of its disposal become significant, a trade-off emerges. In a First Best environment benefits and costs are duly taken into account and an optimal solution can be found. Two are the essential features of this solution: 1) the investment in damage reducing activities takes into account the spillovers waste disposal produces: the stronger the spillover, the larger the investment; 2) the indirect effects that flows of waste across regions have on the environmental quality of all the other regions are accounted for. These spillovers may not be fully perceived at local level and in our model we show that the latter have an effect on: 1) the investment to mitigate pollution which is unambiguously suboptimal in the decentralised solution; 2) the size of the flow of waste; 3) its direction. These inefficiencies allow to conclude that decentralisation is, from the point of view of the entire community, a second best solution. However, this does not mean that all the regions are worse off: some of them may favour decentralisation, but their behaviour may create high welfare losses to other regions. In this case, upper tiers of government (national or super-national) may have to mitigate these problems with
specific actions to reduce waste mobility.

The paper is organised as follows. In Section 2 we briefly describe some of the systems of governance for MWM across Europe. In Section sec:The-model we present the model, the two regulatory frameworks and derive the optimal flow and investment levels. These results are then analysed and compared in 4 and discussed in Section ??, where we also derive the main policy implications of our analysis.

2 Municipal Waste Management Practises across Europe

In the European Union, the MWM governance system typically involves three institutional levels, sometimes with overlapping responsibilities:

- the national level, framed by the EU, mostly is in charge of economic, technical and environmental regulation;
- the regional level: focuses on planning of disposal capacity, enforcement of the self-sufficiency principle, authorization of facilities and overview of MWM practices;
- the local level: administers the organization of MWM services, within general rules concerning management and finance of local services, competition laws, etc.

The German and the Dutch frameworks embed several features adopted by other member states. In Germany, the responsibility for waste management is shared between the national government, the federal states and local authorities. The national Ministry of the Environment sets priorities, participates in the enactment of laws, oversees strategic planning, information and public relations and defines requirements for waste facilities. Each Federal State adopts its own waste management act containing supplementary regulations to the national law, e.g. concerning regional management concepts and rules on requirements for disposal. Each Federal State develops
a waste management plan for its area. In the Netherlands, the Environmental Management Act stipulates that the Ministry for Housing, Spatial Planning and the Environment must draw up a Waste Management Plan every six years. Obligations at the provincial level mostly concern the licensing and monitoring of treatment facilities, as well as the environmental rehabilitation of closed landfills sites. Municipalities are responsible for the collection of household waste in their own territory.

In France a new legislative framework has been set since 2007 with specific targets for waste management at the national level, although the implementation of waste prevention plans is decided at the municipality level. Until recently, Italy broadly followed the German model, but with sub-regional authorities (provinces) responsible for the planning, regulation of access to facilities and overview of MWM services. Access to landfill sites and incinerators was broadly restricted to provincial waste. New national laws have afterwards introduced the possibility for incinerators to receive municipal waste from any region in the country, subject to some restrictions \(^2\). Moreover, regional laws have fostered mobility of waste across provinces within the same region. Finally, in the UK waste policy is a devolved matter: the administrations of Scotland, Wales and Northern Ireland are fully responsible for their strategy and policy-related MWM. From the above, it is then clear that, at least across the EU, the issue of municipal and industrial waste mobility is at the centre of the regulatory debate. In practice, however, the choice on the optimal degree of mobility is a discussion about the optimal degree of decentralisation.

3 The model

We study the effects of waste disposal on environmental protection and welfare in a model of a country divided into \(N\) equally-sized jurisdictions, or regions. decisions on environmental

\(^{2}\)The practical application of this principle is still uncertain because there is jurisdictional conflict between regions and the national government as to which waste should be allowed to flow.
Figure 1: Example of a graph of the connections among regions

Protection and waste disposal can be taken locally or by a higher Government level (Central Government CG). Each region is endowed with fixed income $Y_i$ and an environmental good (e.g., clean air, unspoiled land) amounting to $z$. Income generates an amount of waste equal to $q_i$, which can be disposed of in the same Region $i$, or can be exported to the others. We denote by $w_i$ the total quantity of waste disposed of in Region $i$. Waste treatment is costly, as it depends on the technology used and the policy actions each region undertakes to reduce the related environmental damage and the final treatment of waste is harmful to the environment. Each unit of waste lowers the available quantity of the environmental good by an amount $v$. Waste can be reduced by investing in a technology that mitigates its environmental impact by a quantity-equivalent $r_i$. We assume that the net environmental damage is proportional to the difference between the quantity of waste and the investment each region undertakes. Analytically,

$$v(w_i - r_i) \quad i = 1, \ldots, N. \quad (1)$$

Pollution from the final disposal of waste spills over boundaries: treatment in Region $i$ causes pollution in Region $j$ at rate $k_{ij}$, which mainly depends on distance, and it is inversely correlated with the latter. For $k_{ij} = 0$ there is no spillover: the damage produced by waste disposal
activities does not spread to the neighbouring region, therefore those activities can be considered a local public bad. If \( k_{ij} = 1 \), waste disposal becomes a public bad; for \( 0 < k_{ij} < 1 \) it is a local public bad with spillovers. We assume symmetry in the external effects, i.e. \( k_{ij} = k_{ji} \). In Figure 1 the system of relations among the regions due to the spillover effect is described using a graph, where regions are represented by nodes and an undirected edge between Region \( i \) and Region \( j \) exists if \( k_{ij} \neq 0 \). If Region 1 is equidistant from Region 2 and Region 3, we can expect \( k_{12} \) to be similar to \( k_{13} \); on the other hand, if Region 5 is nearer to Region 3 than to Region 6, we can expect to have \( k_{35} > k_{56} \).

The quantity \( w_i \) of waste disposed in each region depends on the local production \( (q_i) \) and on the net sum of waste flows from/to other regions. We will denote \( \Delta_{ij} \) the net waste flow from Region \( j \) to Region \( i \). If this flow is positive, waste travels from Region \( j \) to \( i \) and the latter has to dispose of \( \Delta_{ij} \) locally. On the other hand, if the flow is negative waste travels from Region \( i \) to \( j \) and the latter sends locally produced waste for disposal outside its boundaries. Taking into account all the \( \frac{N(N−1)}{2} \) possible flows between regions, the environmental damage for Region \( i \) is:

\[
v(q_i - r_i + \sum_{j \neq i} \Delta_{ij}) \quad i = 1, \ldots, N. \tag{2}
\]

In each region the stock of the environmental good, net of the damage produced by intra- and extra-regional waste disposal activities, can be written as:

\[
z - v(q_i - r_i + \sum_{j \neq i} \Delta_{ij} + \sum_{j \neq i} k_{ij}(q_j - r_j + \sum_{\ell \neq j} \Delta_{\ell j}))
\]

We assume that one unit of waste has a region-specific treatment cost equal to \( p_i \) which depends on several factors, such as the technology chosen to reduce the environmental damage and local factors affecting productivity. The region-specific investment produces a cost equal to \( \frac{\gamma_i}{2} r_i^2 \), i.e. the cost is quadratic in the quantity of reduced damage; \( \frac{\gamma_i}{2} \) is a measure of local efficiency in
reducing the cost related to environmental protection. The transport for moving waste from Region \( j \) to Region \( i \) is equal to \( t_{ij} \) and is related to distance. Again, we assume symmetry in costs, i.e. \( t_{ij} = t_{ji} \). In general, these increase with the distance and we can account for the well-known proximity principle by setting \( t_{ij} = +\infty \) if the distance between the involved regions is beyond the limits set by national laws to allow the transfer of waste\(^3\).

The welfare function for each region is a linear combination of disposable income and the net utility that can be derived from the environmental good and can therefore be written as:

\[
W_i = Y_i + \beta_i \left[ z - v \left( q_i - r_i + \sum_{j \neq i} \Delta_{ij} + \sum_{j \neq i} k_{ij} (q_j - r_j + \sum_{\ell \neq j} \Delta_{j\ell}) \right) \right] \\
- \frac{\gamma_i}{2} r_i^2 - p_i \left( q_i + \sum_{j \neq i} m_{ij} \Delta_{ij} - \frac{1}{2} \sum_{j \neq i} t_{ij} \Delta_{ij}^2 \right) \\
\Delta_{ij} = -\Delta_{ji} \quad \forall i, j = 1, \ldots, N, \ i \neq j,
\]

where \( \beta_i \) represents the preferences (specific to each community) for the environmental good. The term \( m_{ij} \) is the exchange price for each unit of waste flowing between Region \( i \) and Region \( j \); it also represents the unit compensation regions receive to treat waste that has not been produced within their boundaries. While in a centralised setting this price is set by the government, in the decentralised model it will be the outcome of bargaining between the regions.

This framework allows us to study the effects of decentralisation, i.e. the discretion each region enjoys in determining its level of waste-reducing activities and the implications of treating imported waste. The preference parameter is region-specific, so that we also allow for the possibility that communities value waste production and treatment differently in relation to the environmental situation of their region. In contexts similar to the present one, where information is symmetric, the centralised solution is always to be preferred to any decentralised one.

However, while this is true for total welfare, the distribution of benefits among the regions may

\(^3\)For example, in Australia there is a limit of 150km, unless this is the closest landfill.
vary significantly and it may have important impacts on the overall outcome and the regulatory choices.

3.1 Decentralisation

We define decentralisation an arrangement in which each region sets its own level of investment and waste disposal according only to its preferences and resources. Each region chooses the level of investment \( r_i \) and the waste flow \( \Delta_{ij}, j = 1, \ldots, N, i \neq j \), that maximises the following welfare function

\[
W_i = Y_i + \beta_i \left[ z - v \left( q_i + \sum_{j \neq i} k_{ij} \left( q_j + \sum_{\ell \neq j} \Delta_{\ell j} - r_j \right) \right) \right] - \frac{7_i}{2} r_i^2 - p_i \left( q_i + \sum_{j \neq i} \Delta_{ij} \right) + \sum_{j \neq i} m_{ij} \Delta_{ij} - \frac{1}{2} \sum_{j \neq i} t_{ij} \Delta_{ij}^2 \tag{4}
\]

Each region optimises without taking into account the condition \( \Delta_{ij} = -\Delta_{ji} \) and the spillovers effects, as an upper government level would do. To reconcile decentralisation with market-clearing conditions, it is also necessary to find a price \( m_{ij}^d \) that allows waste imports to match waste exports. The derivation of the optimal solution is presented in Appendix A. An internal solution exists if:

\[
q_i + \sum_{j \in \mathcal{J}_i^d} \frac{p_j - p_i + v(1 - k_{ij}) (\beta_j - \beta_i)}{2 t_{ij}} > 0, \quad i = 1, \ldots, N, \quad \mathcal{J}_i^d = \{1 \leq j \leq N : j \neq i, \quad p_j - p_i + v(1 - k_{ij}) (\beta_j - \beta_i) < 0\}
\]

which can be interpreted as a condition on the relative importance of the transport costs \( t_{ij} \). The latter should be “sufficiently low” to make waste flows convenient; however they should also be high enough to avoid the existence of regions wishing to move all their waste outside their region. For all \( i,j = 1, \ldots, N \), with \( i \neq j \) the optimal values for the flow and the investment,
along with the exchange price are:

\[ \Delta_{ij}^d = \frac{p_j - p_i}{2t_{ij}} + \frac{v(1 - k_{ij})(\beta_j - \beta_i)}{2t_{ij}}, \]  

(6a)

\[ r_i^d = \frac{\beta_i v}{\gamma_i}, \]  

(6b)

\[ m_{ij}^d = \frac{p_i + p_j}{2} + \frac{v(1 - k_{ij})(\beta_i + \beta_j)}{2}. \]  

(6c)

The flow of waste is determined by the sign of \( p_j - p_i + v(1 - k_{ij})(\beta_j - \beta_i) \). Let us assume \( p_j \geq p_i \); then waste flows from Region \( j \) to Region \( i \) if the following condition is satisfied:

\[ p_j - p_i \geq v(1 - k_{ij})(\beta_i - \beta_j). \]  

(7)

Moving waste from a region with a higher price is optimal if the price difference is higher than the difference in environmental damage caused by mobility. No mobility is optimal if the following condition holds:

\[ p_j - p_i = v(1 - k_{ij})(\beta_j - \beta_i). \]

This obviously happens if \( p_j = p_i \) and \( \beta_j = \beta_i \), that is if the regions are symmetric, since there is not a comparative advantage in treating waste elsewhere, but in general, it is sufficient that the price difference offsets the difference in environmental damage. Apart from these cases, since no mobility \( (\Delta_{ij} = 0) \) is a possible outcome of the bargaining solution, from a welfare point of view the decentralised solution with mobility is always preferred to the solution without mobility. In fact, it represents a Pareto improvement, since both regions are better off.

### 3.2 Centralisation

In the presence of spillovers across regions, when there are no advantages in terms of productivity differentials in decentralisation nor asymmetry of information about local preferences exist, wel-
fare is optimised by a central planner that jointly maximises utility for all regions (Oates, 2008; Tresch, 2002). Let us then assume that waste management decisions on flows and investment on environmental protection are taken by a central planner. The problem for the regulator is to find the optimal values of waste flows ($\Delta_{ij}$), the optimal investment in environment protection technology ($r_i$) and the transfer price $m_{ij}$ that maximise total welfare, i.e. to solve

$$\max_{\Delta_{ij}, r_i, m_{ij}} \sum_{i=1}^{N} W_i, \quad \text{s.t. } \Delta_{ij} = -\Delta_{ji} \forall i, j = 1, \ldots, N, i \neq j,$$

where $W_i$ is the function defined in (4).

As in the case of decentralisation the existence of an internal solution depends on the level of transport prices (see equation (17) in Appendix B). The following optimal values of waste disposal and investment are derived in Appendix B:

$$\Delta_{ij}^c = \frac{p_j - p_i}{2t_{ij}} + \frac{v}{2t_{ij}} (1 - k_{ij}) (\beta_j - \beta_i) + \frac{v}{2t_{ij}} \sum_{s \neq i,j} \beta_s (k_{js} - k_{is})$$ (8a)

$$r_i^c = \frac{v}{\gamma_i} \left( \beta_i + \sum_{j \neq i} \beta_j k_{ij} \right)$$ (8b)

The optimal investment in environmental protection activities depends on the value each region attaches to the environment and to the sum of the negative externalities that its depletion causes to neighbouring regions (the term $\sum_{j \neq i} \beta_j k_{ij}$ accounts for this aspect). It is therefore higher than under decentralisation, as one might expect. Productivity also affects investment: the lower the parameter $\gamma_i$, the higher the investment. It is important to note that $r_i^c$ is also unambiguously correlated to the presence of the spillovers $k_{ij}$: the stronger the spillover, the larger the investment.

The flow of waste between any two regions depends on the effects that such flow has on the other $N - 2$ regions through the spillovers. In fact the optimal flow in (8a) is the sum of the
optimal flow in decentralisation, that is also equal to the optimal flow in a 2-region model, while
the third term relates to the presence of the other \( N - 2 \) regions. As expected, in this setting the
central authority has to correct both the investment and the flow to compensate for the presence
of spillovers. To better analyse this effect on mobility, let us compare the flow in a \( N \)-region
setting with optimal quantities \( \Delta_{c}^{N} \) with a new one where a further region is added. Then the
new optimal values \( \Delta_{c}^{N+1} \) for the “old” regions become:

\[
\Delta_{ij}^{c,N+1} = \Delta_{ij}^{c,N} + \frac{v}{2t_{ij}} \beta_{N+1} (k_{ij,N+1} - k_{i,N+1}), \quad i,j = 1,\ldots,N, \ i \neq j.
\]

Let us assume that \( \Delta_{ij}^{c,N} > 0 \), that is, Region \( j \) sends waste to Region \( i \) in the \( N \)-region case. In
the new setting Region \( j \) sends even more waste to Region \( i \) if its spillover with respect to the new
region is higher than the spillover effect between Region \( i \) and the newcomer. When the spillover
effect caused by Region \( N \) is the same for Region \( i \) and Region \( j \), the flow is unchanged, but the
investment to reduce damage will anyway increase. With reference to Figure 1, let us assume
that Region 6 is added to a previously defined network comprising the other 5 regions. In this
particular case, Region 6 is rather detached from the rest of the network and since \( k_{i6} = 0 \) for all
\( i \neq 5 \) the flow among Regions 1 to 4 is unchanged, while for Region 5 \( \Delta_{i5}^{c,6} = \Delta_{i5}^{c,5} + \frac{v}{2t_{i5}} \beta_{6}k_{56} \) for
all \( i = 1,\ldots,4 \). Thus Region 5 either receives less or sends out more to Region 1 to 4 in the new
setting: the new region is a sort of outlier, to which other regions can (and probably will) send
waste. Since waste disposal affects Region 5 through the spillover \( k_{56} > 0 \), the total quantity of
waste treated in Region 5 has to be lowered.

From (8a) we note that the direction of the flow depends on the sign of:

\[
p_{j} - p_{i} + v (1 - k_{ij}) (\beta_{j} - \beta_{i}) + v \sum_{s \neq i,j} \beta_{s} (k_{js} - k_{is}),
\]

so that Region \( j \) sends waste to Region \( i \) (\( \Delta_{ij}^{c} > 0 \)) if the price difference for waste disposal
between \( j \) and \( i \) is higher than the difference in the damage caused by this flow.

No mobility is the optimal choice if:

\[
p_i - p_j = v \left( (1 - k_{ij}) (\beta_j - \beta_i) + \sum_{s \neq i, j} \beta_s (k_{js} - k_{is}) \right).
\]

In fact the difference in price offsets the difference in the environmental damage and for this reason waste mobility is not desirable. As opposed to the decentralised option, in centralisation it is not sufficient to have symmetry in prices \( p_i = p_j \) and in environmental concern \( \beta_i = \beta_j \) in order to have no mobility. The symmetry involves also the condition:

\[
\sum_{s \neq i, j} \beta_s k_{js} = \sum_{s \neq i, j} \beta_s k_{is}
\]

that is it is also necessary that disposal either Region \( i \) or Region \( j \) causes the same overall damage to the local environment, so there is no comparative advantage in moving waste.

The maximisation process does not allow to determine the prices \( m_{ij}^c \) for the waste flow among regions, since total welfare does not depend on them. Central Government can choose them arbitrarily, but this choice will affect the welfare level of each region. This may cause decentralised, second best solutions which imply a sub-optimal level of environmental protection and a higher cost in terms of waste disposal to be preferred by some regions.

Setting \( m_{ij}^c \) is one of the most important tasks for CG. If it is too low it will reduce welfare of the more efficient regions in terms of disposal. In our model the productivity parameters are fixed, but in the long run this may not be the case. For example, a transfer price equal to the marginal cost may lead to a race to the bottom effect, as suggested by the traditional literature (Oates and Schwab, 1988). This distortion is to be considered by the regulators in setting the transfer price, especially in extended regulatory horizons.
4 A comparison of the centralised and decentralised solutions

The model presented in the previous section shows that the choice of the optimal level of governance for MWM does not yield a straightforward solution. From the point of view of total welfare, a centralised system with waste trading is a more desirable outcome, but obviously the transfer price plays a key role. This means that there are conditions under which it might be difficult to attain an efficient and equitable equilibrium. The distortions that arise are even more significant if we consider the choice of which level - whether the Central Government or the Regions - should be responsible for MWM. There does not seem to be a unique solution and much depends on the starting point. In a centralised system waste exchange maximises total welfare, but the some regions may oppose this mechanism unless the price for disposal across the border is sufficiently high.

In what follows the choice between a centralised system and decentralisation will be analysed. The comparison will be made on different levels: the level of investment in waste treatment ($r_i$), the flows of traded waste ($\Delta_{ij}$), the environmental damage and welfare. To better understand the role of spillovers in the subsequent analysis some special cases will also be considered: an
equidistant distribution of regions (i.e. $k_{ij} = k$ for all $i, j$) and the two simple cases with $N = 3$ depicted in Figure 2, where $k_{12} = k_{13} = k$, while $k_{23} \neq k$. That is, Region 1 is equidistant from Region 2 and Region 3, while Region 2 and Region 3 can be further away (graph on the left in 2) or close together (graph on the right in 2).

**Investment** The optimal level of investment with centralisation is higher than in the decentralised system, as one might expect. In a decentralised environment regions do not take into account the spillovers created by their own activities and total investment is thus set at a sub-optimal level. From Table 1 the difference in investments in the two settings is equal to:

$$\Delta r_i := r^d_i - r^c_i = -\frac{\nu}{\gamma_i} \sum_{j \neq i} \beta_j k_{ij}.$$  

(9)

Because of spillovers, a centralised scheme imposes higher costs for investment in environmental damage reducing activities to all regions. The level of investment is suboptimal in decentralisation even when regions share the same environmental concern, are equally efficient and spillovers do not vary across regions (i.e. $\beta_i = \beta$, $\gamma_i = \gamma$ and $k_{ij} = k$ for all $i, j$). The difference is larger for regions with a higher number of connections (as for Region 3 in Figure 1) or close to regions...
having a strong concern for environmental damage. For the homogeneous case \( k_{ij} = k \) for all \( i, j \) the difference is higher for efficient regions, but also for those with low environmental concern. In the three-regions cases of Figure 2 the difference in investment for Regions 2 and 3 is either lower or higher than in the homogeneous case, according to the sign of \( k_{23} - k \). It is lower if it is negative (triangle on the left), higher if it is positive (triangle on the right).

In this respect, our model confirms the results of the literature showing that centralisation provides a better environmental protection.

**Flow** The waste flow in the two settings may be quite different. From equation (6a) we note that the waste flow between Regions \( i \) and \( j \) in decentralisation is determined by the ratio of the difference in price and environmental preferences between the two regions as well as the transport cost. Since the regulator’s objective is to maximise social welfare, the flow under centralisation has an additional term, which depends on the indirect effects that moved waste has on the remaining \( N - 2 \) regions. From Table 1t this is given by:

\[
\Delta \Delta_{ij} := \Delta^d_{ij} - \Delta^c_{ij} = \frac{v}{2t_{ij}} \sum_{s \neq i,j} \beta_s (k_{is} - k_{js}).
\]

The term on the right is the difference of the indirect (through spillovers) effects caused by treating \( \Delta_{ij} \) in Region \( i \) or in Region \( j \) on the rest of the community. This aspect is neglected in decentralisation and corrected by the regulator in the centralised solution. Suppose for example that in decentralisation waste flows from Region \( j \) to Region \( i \) (i.e. \( \Delta^d_{ij} > 0 \)). If the effect of disposing waste in Region \( i \) causes more damage to the other \( N - 2 \) regions compared to treating it in Region \( j \) (i.e. the right-hand side in (10) is positive), in centralisation this flow is either cut or can even change direction, as shown in Figure 3. Obviously, the flow in the two regimes is the same if the spillovers do not change across regions, i.e. \( k_{ij} = k \) for all \( i \) and \( j \). In this case the waste flow among the regions is already optimal in decentralisation and the regulator’s
The waste flow in the centralised and decentralised models.
The flow in decentralisation is positive (from Region \( j \) to Region \( i \)) above the red line and it has the same sign above the blue line in centralisation. In the area between the two, the flow direction is inverted; on the green dotted line the moved quantity is the same, i.e. \( |\Delta c_{ij}| = |\Delta d_{ij}| \).

This has interesting consequences on CG’s policy in decentralising waste treatment: from the above analysis, a viable strategy to reduce the gap in the flow difference is to cluster regions and allow mobility only among groups where distance variability is low. In this respect, the EU proximity principle may be seen as a way to reduce the undesired effects arising from waste flow in a decentralised environment.

For more insight on the effects of the spillovers on the flow difference, let us examine the flows in the two examples of Figure 2. From (10) we have:

\[
\begin{align*}
\Delta \Delta_{12} &= \frac{v}{2t_{12}} \beta_3 (k - k_{23}), \\
\Delta \Delta_{13} &= \frac{v}{2t_{13}} \beta_2 (k - k_{23}), \\
\Delta \Delta_{23} &= 0.
\end{align*}
\]

The waste flow between Region 2 and Region 3 is the same in the two settings because for
Region 1 the spillover derived from waste disposal in Region 2 and 3 is the same. The sign of the other differences depends on the sign of $k - k_{23}$: if it is positive, outflows from Region 1 are higher in decentralisation.\(^4\) For the same reason, Region 1’s inflow is lower in a centralised setting - actually it can even become an outflow if the difference in the spillovers is sufficiently high. Interpreting the parameter $k_{ij}$ as distance between Region $i$ and Region $j$ (the shorter the distance, the higher the spillover), the above parameters describe the case to the left in Figure 2. Region 1 is closer to Region 2 and Region 3 than the distance within the other two, thus from a social welfare point of view it is better to treat less waste in Region 1. The situation is instead reversed in the case presented on the right in Figure 2. Since Region 1 is farther away, the flow in centralisation has the effect to concentrate more waste compared with decentralisation, either by increasing the inflow or by limiting its outflow.

**Environmental damage** Let us now turn to the examination of the difference in the environmental damage, defined as the region-specific valuation of the reduction in the environmental good brought about by waste disposal. As shown above, the level of investment in environmental protection is not optimal in decentralisation, hence total environmental damage is higher in this setting. However, this does not necessarily mean that all the regions are worse off, as shown below. For each Region $i$, the overall environmental damage (net of transport costs) is given by:

$$v\beta_i\left[\sum_{j\neq i} \Delta_{ij} - r_i + \sum_{j\neq i} k_{ij}(q_j + \sum_{l\neq j} \Delta_{jl} - r_j)\right],$$

\(^4\)Waste disposed either in Region 2 or in Region 3 gives rise to a lower overall damage (because of the smaller spillover $k_{23}$).
therefore from (9) the difference in environmental damage between decentralisation and centralisation amounts to:

$$v \beta_i \sum_{j \neq i} \left[ (1 - k_{ij}) \Delta \Delta_{ij} + \sum_{i \neq j, \ell \neq i} \Delta \Delta_{j\ell} (k_{ij} - k_{i\ell}) \right] + v^2 \beta_i \sum_{j \neq i} k_{ij} \left( \frac{\beta_j}{\gamma_i} + \frac{1}{\gamma_j} \sum_{\ell \neq j} \beta_{\ell} k_{j\ell} \right).$$

The first part ($DF_i$) depends on the flow difference, the second ($DI_i$) on the investment. Note that the latter is always positive because more money is invested for the reduction of the environmental damage in centralisation.

In the homogeneous case $k_{ij} = k$ for all $i, j$ there is no flow difference ($\Delta \Delta_{ij} = 0$ for all $i, j$) and the difference in the environmental damage only depends on the difference in investment. Therefore, the damage difference is positive. In this respect, centralisation always performs better. In fact:

$$DI_i = v^2 k \beta_i \left( \frac{1}{\gamma_i} \sum_{j \neq i} \beta_j + \frac{k}{\gamma_j} \sum_{\ell \neq j} \beta_{\ell} \right).$$

From the above equation, it is easy to show that if all regions have the same environmental preferences ($\beta_i = \beta$ for all $i$), the most efficient (lowest $\gamma_i$) incurs the highest damage difference. In the same way, when regions are all equally efficient, the damage difference is higher for regions with higher $\beta_i$.

Let us analyse the case $N = 3$ with $k_{12} = k_{13} = k$ and $k > k_{23}$ (graph on the left in Figure 2). The difference in the environmental damage caused by the flow difference is:

- **Region 1**: $DF_1 = v \beta_1 (1 - k)(\Delta \Delta_{12} + \Delta \Delta_{13}),$
- **Region 2**: $DF_2 = v \beta_2 \left[ (k - k_{23})\Delta \Delta_{13} - (1 - k)\Delta \Delta_{12} \right],$
- **Region 3**: $DF_3 = v \beta_3 \left[ (k - k_{23})\Delta \Delta_{12} - (1 - k)\Delta \Delta_{13} \right].$
From (11) the first is positive, thus Region 1 always enjoys a better environmental protection under centralisation. For the other two regions this summand depends on the size of $k$ and of the ratio of the flow differences $\frac{\Delta \Delta_{13}}{\Delta \Delta_{12}}$:

$$DF_2 > 0 \Leftrightarrow \frac{\Delta \Delta_{13}}{\Delta \Delta_{12}} > \frac{1 - k}{k - k_{23}}$$
$$DF_3 > 0 \Leftrightarrow \frac{\Delta \Delta_{13}}{\Delta \Delta_{12}} < \frac{k - k_{23}}{1 - k}.$$  

In both regions the flow related environmental damage is lower in centralisation if $k - k_{23} > 1 - k$, that is if $k > \hat{k} := \frac{1 + k_{23}}{2}$ and $\frac{1 - k}{k - k_{23}} < \frac{\Delta \Delta_{13}}{\Delta \Delta_{12}} < \frac{k - k_{23}}{1 - k}$. The situation is reversed for both regions if $k \in (k_{23}, \hat{k})$ and $\frac{k - k_{23}}{1 - k} < \frac{\Delta \Delta_{13}}{\Delta \Delta_{12}} < \frac{1 - k}{k - k_{23}}$. In the remaining cases it holds:

$$\begin{cases} \frac{\Delta \Delta_{13}}{\Delta \Delta_{12}} > \max \left\{ \frac{1 - k}{k - k_{23}}, \frac{k - k_{23}}{1 - k} \right\}, & DF_2 > 0, DF_3 < 0, \\ \frac{\Delta \Delta_{13}}{\Delta \Delta_{12}} < \min \left\{ \frac{1 - k}{k - k_{23}}, \frac{k - k_{23}}{1 - k} \right\}, & DF_2 < 0, DF_3 > 0. \end{cases}$$  

To sum up, in centralisation less waste is disposed of in Region 1 compared to decentralisation, either because inflows are lower, or because outflows are stronger. The flow change has a countervailing effect on the environmental damage in Regions 2 and 3: it increases because the waste disposed locally and the part producing spillovers $k_{23}$ increase, but the part with spillover $k$ decreases. Both Region 2 and 3 can benefit from the flow change if $k$ is sufficiently high and the ratio $\frac{\Delta \Delta_{13}}{\Delta \Delta_{12}} = \frac{\beta_2}{\beta_3} \frac{\Delta_{13}}{\Delta_{12}}$ is within a specific interval. Otherwise, at most one of the regions may have an advantage, depending on the size of the proportion between the increase in waste disposed off locally and the decrease in waste treated in Region 1. This result characterises in our model the well known NIMBY (Not In My BackYard) effect. In particular, it shows that in some cases regions may prefer a decentralised solution to centralisation to reduce their environmental damage. However, given that the total damage is higher in this setting, this means that some regions will suffer a considerable decrease in the quality of their environmental good. This effect is more likely to exist when the distance between the exchanging regions is not homogeneous and
the effect is positively related to the value of the preference for the environmental good. Once again, the proximity principle may reduce this risk.

If instead $k < k_{23}$ (corresponding to the scheme on the right of Figure 2) the analysis is easier: $DF_1 < 0$, while $DF_2$ and $DF_3$ are both positive and $DF_2 > DF_3$ iff $|\Delta \Delta_{12}| > |\Delta \Delta_{13}|$.

Since $DI_i > 0$ for all $i$, whenever $DF_i \geq 0$ the environmental damage for Region $i$ is lower in centralisation; if this is negative, the sign then depends on the relative magnitude of $DF_i$ and $DI_i$. The latter is:

\[
\begin{align*}
\text{Region 2 :} & \quad DI_2 = v^2 \beta_2 \left( \beta_1 \left( \frac{k}{\gamma_2} + \frac{k_{23}}{\gamma_3} \right) + \beta_2 \left( \frac{k^2}{\gamma_1} + \frac{k_{23}^2}{\gamma_3} \right) + \beta_3 \left( \frac{k^2}{\gamma_1} + \frac{k_{23}^2}{\gamma_2} \right) \right) \\
\text{Region 3 :} & \quad DI_3 = v^2 \beta_3 \left( \beta_1 \left( \frac{k}{\gamma_3} + \frac{k_{23}}{\gamma_2} \right) + \beta_2 \left( \frac{k^2}{\gamma_1} + \frac{k_{23}^2}{\gamma_3} \right) + \beta_3 \left( \frac{k^2}{\gamma_1} + \frac{k_{23}^2}{\gamma_2} \right) \right).
\end{align*}
\]

If all regions have the same environmental concern, $DI_2 > DI_3$ if $\gamma_2 < \gamma_3$, while if all regions are equally efficient $DI_2 > DI_3$ if $\beta_2 > \beta_3$.

In general when $DF_i < 0$, if $t_{12}, t_{13}$ are sufficiently high, the environmental damage is lower in centralisation for all regions.

**Welfare** Total welfare is obviously lower in the decentralised solution, because the reduction in welfare due to a suboptimal investment in environmental protection offsets the increase in welfare due to the reduction in the investment cost, as shown by traditional literature (Oates, 2008; Tresch, 2002). In what follows the welfare difference for each region, i.e. $\Delta W_i = W_i^d - W_i^c$.
will be analysed. From the definition of \( W \) it is:

\[
\Delta W_i = -v_i \beta_i \left( \sum_{j \neq i} (1 - k_{ij}) \Delta \Delta_{ij} + \sum_{j \neq i} \sum_{\ell > j, \ell \neq i} \Delta \Delta_{j\ell} (k_{ij} - k_{i\ell}) \right) - p_i \sum_{j \neq i} \Delta \Delta_{ij} + \left( \sum_{j \neq i} (m_{ij}^d \Delta_{ij}^d - m_{ij}^c \Delta_{ij}^c) - \frac{1}{2} \sum_{j \neq i} t_{ij} \left( (\Delta_{ij}^d)^2 - (\Delta_{ij}^c)^2 \right) \right) + v_i \beta_i \left( \Delta r_i + \sum_{j \neq i} k_{ij} \Delta r_j \right) - \frac{1}{2} \gamma_i \left( (r_i^d)^2 - (r_i^c)^2 \right).
\]

As for the environmental damage, its terms can be separated in two parts by grouping those depending on the difference in the waste flow (first two lines of the above equation) and those related to the difference in investment in the two schemes (last line). Summing and subtracting \( \sum_{j \neq i} m_{ij}^c \Delta_{ij}^d \) and using the relation \( m_{ij}^d = p_i + v_i \beta_i (1 - k_{ij}) + t_{ij} \Delta_{ij}^d \), the flow-related part of the difference can be rewritten as:

\[
\frac{1}{2} \sum_{j \neq i} t_{ij} \Delta \Delta_{ij}^2 + \sum_{j \neq i} (m_{ij}^d - m_{ij}^c) \Delta \Delta_{ij}^c - v_i \beta_i \sum_{j \neq i} \sum_{\ell > j, \ell \neq i} \Delta \Delta_{j\ell} (k_{ij} - k_{i\ell}), \tag{12}
\]

while from (9) the remaining part equals:

\[
v^2 \left( \frac{1}{2} \gamma_i \left( \sum_{j \neq i} \beta_j k_{ij} \right)^2 - \beta_i \sum_{j \neq i} k_{ij} \sum_{\ell \neq j} \beta_{\ell j} \right). \tag{13}
\]

The difference in total welfare does not depend on the choice of \( m_{ij}^c \), but the latter influences its allocation among the regions and can be used as a policy instrument to reduce the welfare gain some regions may have in asking for decentralisation or in opposing the process of centralisation.

In the homogeneous case where \( k_{ij} = k \) for all \( i, j \), if \( m_{ij}^c = m_{ij}^d \) (note that in this case there is no flow difference, thus this is the “market price” also for the centralised case), the welfare difference
only depends on the investment difference and it is equal to:

$$
\Delta W_i = v^2 k^2 \left[ \frac{1}{2\gamma_i} \left( \sum_{j \neq i} \beta_j \right)^2 - \beta_i \left( \sum_{j \neq i, \ell \neq j} \frac{1}{\gamma_j} \sum_{\ell} \beta_\ell \right) \right].
$$

When $\beta_i = \beta$ for all $i$, the welfare difference is lower for regions with high $\gamma_i$, i.e. the least efficient region suffers the highest welfare loss. The same holds when all regions are equally efficient and the analysis is done on the parameters $\beta_i$; the higher the environmental concern, the larger the welfare loss.

For the case $N = 3$ with $k_{12} = k_{13} = k$ and $k \neq k_{23}$ as in Figure 2, for Region 1 the last term in (12) is zero. From previous calculations it also holds:

$$
\sum_{i \neq 1} \left( \frac{1}{2} \sum_{j \neq i} t_{ij} \Delta \Delta_{ij} - v \beta_i \sum_{j \neq i} \sum_{\ell > j, \ell \neq i} \Delta \Delta_{j\ell} (k_{ij} - k_{i\ell}) \right) < 0.
$$

Thus, if $m_{ij}^c = m_{ij}^d$, the flow-related welfare difference is positive for Region 1 and negative for the region among the remaining two for which the ratio $\beta_i^2 / t_{1k}$, $k \neq i$ is highest. For the other, the sign depends on the parameters. If $t_{12} = t_{13}$, the region with the lowest environmental concern may have a gain, but only if its parameter is less than half the other region’s. In all other cases, for both Region 2 and Region 3 the flow-related difference is negative.

On the side of the investment-dependent welfare difference, an analysis analogous to the homogeneous case can be performed, with similar results. As a rule of thumb, less efficient regions experience the highest welfare loss and in general the efficiency gap has to be quite relevant in order for the most efficient to capture some gain.
5 Discussion and Conclusions

Many environmental problems have implications that inherently go beyond regional borders. In principle, the level of pollution in a region depends on several factors: the level of economic activity, the investments made to reduce emissions, and the decisions taken by neighbouring regions. For this reason, the assignment of functions for environmental protection has received great attention in the literature. The allocation of this function to the central level may be more efficient because it allows to take into account the spillovers, but this may no longer be the case when we introduce distributional considerations. Ogawa and Wildasin (2009) study this trade-off and show that it may be possible for decentralised solutions to be more efficient, even in the presence of spillovers (Oates, 1972; Koethenbuerg, 2008). In this article we take a different point of view and show that for MWM the presence of externalities might not allow the most intuitive outcome to emerge. A First Best solution is more efficient in reducing the negative impact that waste disposal has on the quality of the environment. However, some regions may sometimes prefer a suboptimal solution, depending on 1) the price set for across-the-border disposal; 2) the relative productivity of the investment in waste-reducing technology; 3) the preferences for the environmental good; 4) the relative distance across regions and the number of neighbours. Second-best solutions usually imply that the impact on the environment is heavier than optimal, either because the level of investment in the technology that mitigates the impact on the environment is suboptimal, or because the allocation among the regions does not minimise the environmental externality. However, this result may not hold at the regional level. We showed for which combinations of parameters a region may be better off in a decentralised solution, either with respect to welfare, environmental damage, or both.

This result is quite interesting for its policy implications, since it shows that in this case the interests of one or more regions may diverge from those of the whole country. Those findings are also in line with empirical results that show that environmental protection is generally stronger
under centralisation. Our model adds possible theoretical explanations as to why decentralisation may prevail in actual MWM even when a centralised solution should be preferred from a total welfare point of view.

Our model offers a sound theoretical background to policies implemented by national and supra-national authorities. For example, the rationale for the Proximity Principle (SSP/PP), that forces local communities to dispose of waste in the same district, is to reduce the difference in the flow of waste between First-Best and the decentralised solution and in the long run to make communities develop common strategies for environmental protection. If we approximate the level of spillover with distance, the proximity principle implies that regions trade waste among clusters where interregional distances are rather similar. In this case the flow in decentralisation is very similar to the one in FB. In the same line, the prohibition to move waste beyond a specific distance responds to the problems highlighted in section 4 as concerns the welfare loss that some regions may suffer as a result of the behaviour of neighbour regions.

Our model can be extended in several ways. The first extensions can be made by considering oligopolistic games, where regions or clusters act as leaders and followers, with potential convergence effects. A second very interesting extension is to consider the quantities of waste \( q_i \) as a variable that partly depends on the level of income, as well as on the price that the community pays for waste disposal. Indeed, if the community had to pay waste disposal on its full marginal price rather than a tax, in a context where cross border waste disposal is allowed, further trade-offs would emerge. A centralised model with a transfer price equal to the marginal cost maximises the level of protection in terms of investment in pollution-reducing technologies, but might increase the production of waste since it is priced below its cost. On the other hand, under decentralisation the investment in pollution-reducing technologies is lower, but the cross border price is higher and this might significantly reduce the quantity of waste produced.
References


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A Optimal conditions for the decentralised model

Having defined $\Delta_{ij}$ the quantity of waste flowing between Region $i$ and Region $j$, with the convention that $\Delta_{ij} > 0$ when the direction of the flow is from $j$ to $i$, the welfare in Region $i$ is given by:

$$W_i = Y_i + \beta_i \left[ z - v \left( q_i + \sum_{j \neq i} \Delta_{ij} - r_i + \sum_{j \neq i} k_{ij} (q_j + \sum_{k \neq j} \Delta_{jk} - r_j) \right) - \frac{\gamma_i}{2} r_i^2 - p_i \left( q_i + \sum_{j \neq i} \Delta_{ij} \right) + \sum_{j \neq i} m_{ij} \Delta_{ij} - \frac{1}{2} \sum_{j \neq i} t_{ij} \Delta_{ij}^2 \right].$$

(14)

Region $i$ maximises $W_i$ over $r_i \geq 0$ and $\Delta_{ij}$, $j = 1, \ldots, N$, under the constraint that the outgoing flow does not exceed its total waste quantity, that is

$$q_i + \sum_{j \neq i} \min\{\Delta_{ij}, 0\} \geq 0, \quad i = 1, \ldots, N.$$

The optimal investment can be found directly from the FOC:

$$\frac{\partial W_i}{\partial r_i} = v \beta_i - \gamma_i r_i = 0$$

and gives the quantity in (6b).

As for the optimal flow between Region $i$ and Region $j$, taking into account the equality $\Delta_{ij} = -\Delta_{ji}$, the FOC

$$\frac{\partial W_i}{\partial \Delta_{ij}} = -v \beta_i (1 - k_{ij}) - p_i + m_{ij} - t_{ij} \Delta_{ij} = 0$$

gives the following demand function:

$$\Delta_{ij} = \frac{m_{ij} - p_i}{t_{ij}} - \frac{v \beta_i (1 - k_{ij})}{t_{ij}}.$$
To reconcile decentralisation with market-clearing conditions, it is necessary to find transfer prices $m_{ij} = m_{ji}$, satisfying the optimal choice of each region and the market-clearing condition $\Delta_{ij} = -\Delta_{ji}$. The optimal prices are:

$$m_{ij}^d = \frac{p_i + p_j}{2} + \frac{v(1 - k_{ij})(\beta_i + \beta_j)}{2}, \quad i, j = 1, \ldots, N, \ i \neq j$$

so that the optimal quantity of waste flow between Region $i$ and Region $j$ is:

$$\Delta_{ij}^d = \frac{p_j - p_i}{2t_{ij}} + \frac{v(1 - k_{ij})(\beta_j - \beta_i)}{2t_{ij}}.$$

For the existence of the above internal solution the following condition is required:

$$q_i + \sum_{j \in J_i^d} \frac{p_j - p_i + v(1 - k_{ij})(\beta_j - \beta_i)}{2t_{ij}} > 0, \quad i = 1, \ldots, N,$$

where $J_i^d$ is the set of indices

$$J_i^d = \{ 1 \leq j \leq N : j \neq i, \ p_j - p_i + v(1 - k_{ij})(\beta_j - \beta_i) < 0 \}.$$

B Optimal conditions for the centralised model

In centralisation the regulator has to find the optimal values $\Delta_{ij}$ that maximise total welfare, i.e. the function $W = \sum_{i=1}^n W_i$, with $W_i$ given by (14), under the constraint $\Delta_{ij} = -\Delta_{ji}$. The latter can be directly taken into account by using only the variables $\Delta_{ij}$, $i < j$, $i, j = 1, \ldots, N$. 
\( r_i, \ldots, r_N \) and writing:

\[
W = \sum_{i=1}^{n} \left[ Y_i + \beta_i \left( z - v \left( q_i + \sum_{j>i} \Delta_{ij} - \sum_{j<i} \Delta_{ji} - r_i - \sum_{j\neq i} k_{ij} \left( q_j + \sum_{l>j} \Delta_{jl} - \sum_{l<j} \Delta_{lj} - r_j \right) \right) \right) \\
- \frac{\gamma_i}{2} r_i^2 - p_i \left( q_i + \sum_{j>i} \Delta_{ij} - \sum_{j<i} \Delta_{ji} \right) + \sum_{j>i} m_{ij} \Delta_{ij} - \sum_{j<i} m_{ij} \Delta_{ji} - \frac{1}{2} \sum_{j\neq i} t_{ij} \Delta_{ij}^2 \right].
\]

Some feasibility constraints have also to be taken into account: obviously each region’s outgoing flow cannot exceed the amount of waste at its disposal. In our model we also assume that regions are not allowed to resell imported waste, therefore we impose as a constraint that the sum of all the outgoing flows from each region cannot exceed than the total amount of waste produced in the region itself. Collecting terms in the summation defining \( W \) we can write the optimisation problem as:

\[
\begin{aligned}
\text{Max} & \sum_{i=1}^{n} \left[ Y_i + \beta_i (z - v q_i) - v \sum_{j>i} \left( (\beta_i - \beta_j)(1 - k_{ij}) + \sum_{s\neq i,j} (\beta_s (k_{is} - k_{js})) \right) \Delta_{ij} \\
& + v \left( \beta_i + \sum_{j\neq i} \beta_j k_{ij} \right) r_i - \frac{\gamma_i}{2} r_i^2 - p_i q_i + \sum_{j>i} (p_j - p_i) \Delta_{ij} - \sum_{j>i} t_{ij} \Delta_{ij}^2 \right] \\
\text{s.t.} & \quad r_i \geq 0, \quad i = 1, \ldots, N, \\
& \quad q_i - \frac{1}{2} \sum_{j\neq i} |\Delta_{ij}| + \frac{1}{2} \sum_{j>i} \Delta_{ij} - \frac{1}{2} \sum_{j<i} \Delta_{ji} \geq 0, \quad i = 1, \ldots, N
\end{aligned}
\]

(the last formula is equal to \( q_i + \sum_{j: \Delta_{ij} < 0} \Delta_{ij} \) under the constraint \( \Delta_{ij} = -\Delta_{ji} \)). Note that the term \( \Delta_{ij} \left( v (\beta_i - \beta_j)(1 - k_{ij}) + v \sum_{s\neq i,j} \beta_s (k_{is} - k_{js}) \right) \) represents the total damage caused to the community by the exchange of \( \Delta_{ij} \) between Region \( i \) and Region \( j \), divided into the sum of a “direct” consequence involving the differential damage in the two regions and the indirect effect that this flow has on the other regions.
The optimal values for the variables $r_i$ reported in (8b) are simply found from the F.O.C.:

\[
\frac{\partial W}{\partial r_i} = v \left( \beta_i + \sum_{j \neq i} \beta_j k_{ij} \right) - \gamma_i r_i = 0, \quad i = 1, \ldots, N
\]

and are equal to

\[
r^c_i = \frac{v}{\gamma_i} \left( \beta_i + \sum_{j \neq i} \beta_j k_{ji} \right).
\]

Interior optimal values for $\Delta_{ij}$ are the solutions of the linear system:

\[
\frac{\partial W}{\partial \Delta_{ij}} = p_j - p_i + v \left[ (\beta_j - \beta_i) (1 - k_{ij}) + \sum_{s \neq i,j} \beta_s (k_{js} - k_{is}) \right] - 2t_{ij} \Delta_{ij} = 0, \quad i, j = 1, \ldots, N, \quad i < j
\]

and are given by:

\[
\Delta^c_{ij} = \frac{p_j - p_i + v \left[ (\beta_j - \beta_i) (1 - k_{ij}) + \sum_{s \neq i,j} \beta_s (k_{js} - k_{is}) \right]}{2t_{ij}}, \quad i, j = 1, \ldots, N, \quad i < j. \quad (16)
\]

Each of the above values is feasible whenever the constraints in (15) are all non-binding. This requirement can be expressed as a condition on the model’s parameters. In fact calling $J^c_i$ the set of indices

\[
J^c_i = \left\{ 1 \leq j \leq N : j \neq i, \quad p_j - p_i + v \left[ (\beta_j - \beta_i) (1 - k_{ij}) + \sum_{s \neq i,j} \beta_s (k_{js} - k_{is}) \right] < 0 \right\}
\]

the existence of an interior solution is guaranteed if:

\[
q_i + \sum_{j \in J^c_i} \frac{p_j - p_i + v \left[ (\beta_j - \beta_i) (1 - k_{ij}) + \sum_{s \neq i,j} \beta_s (k_{js} - k_{is}) \right]}{2t_{ij}} > 0, \quad i = 1, \ldots, N. \quad (17)
\]
i.e. for sufficiently high unit transportation costs $t_{ij}$.

Total welfare does not depend on the exchange price $m_{ij}$ and it is not possible to set it using a clearing market condition, since, apart from the case of equal spillovers, the set of equations

$$
\Delta c_{ij} = -\frac{v \beta_i (1 - k_{ij})}{t_{ij}} + \frac{\hat{m}_{ij} - p_i}{t_{ij}}
$$

gives rise to solutions with $\hat{m}_{ij} \neq \hat{m}_{ji}$. Note however that $m^d_{ij}$ is the average of $\hat{m}_{ij}$ and $\hat{m}_{ji}$.