The Good, The Bad, and The Not So Ugly: Unanimity Voting with Ambiguous Information*

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Abstract

Seeking unanimous consensus in collective decision-making situations creates the tendency for individuals within a group to vote strategically against their private information especially as the size of the group gets larger. In jury trials, this leads to the paradox that the more demanding the hurdle for conviction is, the more likely it is that a jury will convict an innocent defendant. We challenge these established results, by exploring voting behaviour when collective decision-making occurs based on information, the reliability of which is ambiguous. With ambiguity-averse voters, who are MaxMin Expected Utility Maximizers, we demonstrate that unanimity voting is compatible with instances of informative voting, outperforming other voting rules, such as majority voting.

Keywords: ambiguity; ambiguity aversion; Maxmin EU; unanimity vs. majority voting rule; jury paradox; simulations

JEL codes: C11, D7, D81

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1 Introduction

> "...two factors [are] commonly used to determine a choice situation, the relative desirability of the possible pay-offs and the relative likelihood of the events affecting them, but in a third dimension of the problem of choice: the nature of one's information concerning the relative likelihood of events. What is at issue might be called the ambiguity of this information, a quality depending on the amount, type, reliability and 'unanimity' of information, and giving rise to one's degree of confidence in an estimate of relative likelihoods."
> Daniel Ellsberg, *The Quarterly Journal of Economics*, pp. 657-659.

Much real world negotiation and decision-making take place in small groups. Their deliberations determine outcomes that matter to a multitude of agents, from single individuals, households, businesses and organisations, to communities and wider society. They impact, for instance, the lives of organ recipients, the fates of defendants in jury trials, and the allocation of research funding. Collective deliberation is thus an important research area in the fields of social choice, political economy, as well as political science. A fundamental question in this area is to identify which deliberation processes and rules achieve the best collective decisions. Existing studies concur that majority voting should be preferred in many collective decision settings. This consensus is rooted in two major findings, the Condorcet Jury Theorem (CJT)[1], and what we could refer to as the Jury Paradox. The CJT establishes that collective decisions generated by majority voting have a higher probability of selecting the correct alternative than the decision made by a single expert, especially as the size of the group grows. When making decisions on imperfect information, the "Wisdom of the Crowds" leads to better outcomes than individual decision-making. The Jury Paradox posits the superiority of the majority rule over the unanimity rule when well-intentioned voters are strategic in their voting. Out of all voting rules, unanimity gives individuals the strongest incentives to strategically vote against their private information, leading to suboptimal decisions. The CJT does not contemplate such strategic voting.

These results were famously established in the seminal papers of Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998), which investigate the CJT within Bayesian-Nash Equilibrium (BNE) settings. They generalize the CJT by analysing it in game theoretical frameworks, in which voters share common preferences and receive imperfectly informative private signals to base their votes upon. In these settings, the result of the CJT that collective decisions generated by majority voting have a higher probability of selecting the correct alternative than the decision made by a single expert, especially as the size of the group grows, still holds true. This is trivially so, if each juror's voting strategy prescribes voters to vote in accordance with their signals. Put differently, if informative voting constitutes a pure strategy equilibrium for their voting behaviour the resulting collective decision is more reliable than the decision made by any single individual due to virtues of (truthful) information aggregation. Less trivially, when informative voting is likely not to prevail as a Bayesian-Nash Equilibrium, i.e., when strategic vot-

[1] Marquis de Condorcet, *Essai sur l'Application de l'Analyse à la Probabilité des Décisions Rendues à la Pluralité des Voix*, L'Imprimerie Royale, Paris, 1785.

ing equilibria with randomization become pervasive, with voters discarding their private information, the CJT is somewhat reinforced: Majority voting still leads to information aggregation of superior quality to that obtainable under unanimity voting as the size of the group grows larger, although majority voting does not aggregate information all that well anymore compared to when voters vote sincerely.

Beyond its being robust to the lack of voting sincerely in many real world situations, the result that the majority rule is a superior voting criterion hinges upon the highly unrealistic assumption that individuals cast their votes based on information whose reliability is commonly known and can be precisely assessed. For instance, in models of jury trials, jurors are assumed to know the exact probability that any piece of evidence is correct. In such a setting, it is conventional for voters to update their beliefs about the likelihood of a given state of the world, using the precisely measurable prior probabilities available to them to form posterior probabilities that incorporate any new information using Bayes' rule. Contrary to the assumptions made in the established literature, voters, in reality, need to reach a decision based on information whose accuracy cannot be perfectly assessed. Indeed, ambiguity exists not only in the inability to assign well-defined, numerical probabilities to specific events, but is also embedded in the language, signals, and social norms used by agents to communicate with one another in decision-making contexts. In spite of this, there are virtually no models of voting under ambiguity, for us to predict the likely consequences of facing such an ambiguous world.[2]

The goal of this study is to fill this gap, by investigating the validity of the CJT for BNE settings characterized by *ambiguous information* in a similar fashion as in Ellsberg (1961), i.e., when the distribution of the reliability of the information given to voters is not precisely known. To explore how the ambiguity of information in such settings affects voters' behavior, we assume voters to be ambiguity-averse and MaxMin Expected Utility (MMEU) maximizers, à la Gilboa and Schmeidler (1989). Therefore, in our model ambiguity-averse voters assign their priors in an act-contingent manner: they assess each of their actions by its associated minimum expected utility. Furthermore, to capture voters' belief formation and revision in the face of ambiguity, we allow for Full-Bayesian Updating, as in Pires (2002). Also, this is in line with Eichberger, Grant, and Kelsey (2007), which provides an axiomatic proof for updating non-additive capacities by using the Full-Bayesian Updating rule.

Our results demonstrate that in spite of adhering to unanimous voting there exist instances in which voters facing ambiguous information would revert to adopting informative voting equilibrium strategies, where they would have behaved strategically by voting against their private information, otherwise. The intuition for this result lies in the observation that when information becomes ambiguous, voters are more reluctant to rely on the collective information of others but their own, changing their 'perceived' pivotality, hence their optimal strategies as to whether to vote informatively or against their private information when their votes contribute to the final collective decision. This intuition is

[2]Ellis (2016) studies information aggregation exclusively under majority rule by assuming that there is ambiguity regarding the payoff-relevant state. The findings of this study indicate that the CJT fails to hold as voters who exhibit Ellsberg's type preferences strictly prefer randomizing as compared to adopting informative voting strategies, especially if the precision of their information is too low to overcome the uncertainty of the prior.

consistent with other settings analyzing the effect of ambiguity on the private provision of public goods. In those settings, it has been shown that in large societies a unique equilibrium exists characterized by less free-riding than in the absence of ambiguity[3].

This is compatible, for example, with unanimous voting in a 12-person jury trial in which ambiguous information is allowed for with respect to the reliability of the private signal voters receive before casting their votes.

Let us recall the canonical 12-person jury example from Feddersen and Pesendorfer (1998). This is a clear illustration of the failure for unanimity to deliver high quality collective decisions: it demonstrates how unanimity voting is prone to strategic voting with randomization to the detriment of information aggregation. It does so by considering a scenario in which the reliability of the information voters possess is precisely measured and exactly equal to $p = 0.8$. Figure 6 illustrates what the cut-off value of the level of reasonable doubt, denoted by q, should be for informative voting to be an equilibrium if the unanimity rule were to be chosen. The figure shows that only when such reasonable doubt level is set to, or above, $q = 99\%$ there is a chance for informative voting to prevail as an equilibrium of this voting scenario. Informative voting is represented by the equilibrium strategy for each juror choosing the probabilities to vote to convict, respectively contingent on having received an innocent signal i, $\sigma(i)$, or a guilty signal g, $\sigma(g)$, to be equal to $\sigma = (\sigma(i), \sigma(g)) = (0, 1)$: jurors vote to acquit if they receive an innocent signal, and only vote to convict if they receive a guilty signal. This canonical 12-person jury example implies that it is virtually impossible for unanimity voting to prevent jurors from engaging in strategic behavior involving some degree of randomization, i.e., leading them to choose to convict with some positive probability although they received an innocent signal. In turn, Feddersen and Pesendorfer (1998) show that strategic voting leads to higher type I errors (probability of convicting the innocent) under unanimity voting than when adhering to majority voting.

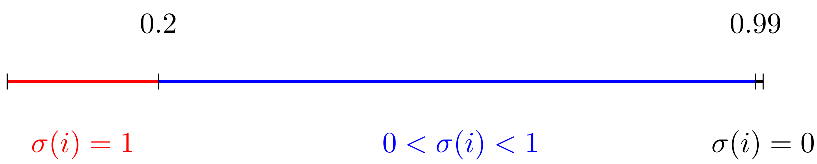

Figure 1: 12-person jury: Threshold values of q for different voting equilibria when $p = 0.8$

However, this famous example which counters the virtues of adhering to unanimity voting, namely that of allowing for inclusion and consensus-building to matter in collective decisions, reflects a scenario in which jurors know precisely the reliability of the information provided to them, before casting their votes. What if that reliability was not possible to be precisely measured? How would jurors, in the face of ambiguous information behave, when forming and revising their beliefs leading to their casting of votes? How would ambiguous information thereby contribute to the quality of the collective decision? Our analysis represents a first attempt in addressing these questions. It demonstrates that, for instance, under unanimity voting, if voters were confronted with information whose reliability is ambiguous, for example, taking values in the interval $[0.55, 0.8]$, then

[3] See, for example, Bailey, Eichberger, and Kelsey (2005).

informative voting would be observed in equilibrium for a much larger range of thresholds of reasonable doubts. In particular, Figure 7 illustrates what the cut-off value for informative voting being an equilibrium would be in this alternative ambiguous setting: $q = 89.4\%$. This is much smaller than the level of reasonable doubt required to be imposed for informative voting to occur in the absence of ambiguous information, which was $q = 99\%$, making it now more likely for unanimity to lead to informative voting for a 12-person jury case.

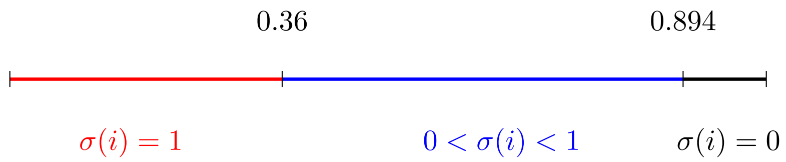

Figure 2: 12-person jury: Threshold values of q for different voting equilibria when $p = [0.55, 0.8]$

This surprising result is not just peculiar to the selected 12-person jury example and the particular spread considered for the interval within which the reliability of the information given to voters lies within. Our analysis provides a much wider support for when informative voting equilibrium prevails over mixed strategy voting equilibria under the unanimity rule. This, in turn, validates the claim that unanimity voting can preserve the efficient aggregation of information in some collective decision-making scenarios, thereby providing the basis as to when to restore it as a desirable voting rule. Therefore, our results are relevant for many real-world decision-making situations, involving medium-to-large size groups.

The remainder of this study is organised as follows. Section 2 contains a review of the related literature on other forms of ambiguity. Section 3 presents the canonical jury trial model and its main findings as studied in the seminal paper by Feddersen and Pesendorfer (1998). Section 4 introduces a theoretical modelling for the cases of the unanimity voting rule and majority voting rule, under ambiguity. Section 6 provides some comparative statics results. Finally, section 7 concludes. Appendix A contains some technical proofs.

2 Related Literature on Other Forms of Ambiguity

The Expected Utility (EU) theory of Neumann and Morgenstern (1947) assumes that the outcomes of the events under examination have objectively known probabilities. They define the preferences over acts by a real-valued utility function of the choices weighted by the objective probabilities of the outcomes of the states.

However, cases when the probability measure of the events is known to all decision makers hardly exist in real life. Decision makers are not able to form purely objective beliefs regarding the states unless they are confronted with a fair coin, a perfect die, or a well-made roulette wheel. Knight (1921) is the first person to distinguish 'risk' from 'uncertainty' by referring to the existence/absence of objective probabilities. 'Risk' is defined by (associated with) events the objective probability measure of which could either (i) be theoretically deduced, which means that individuals are able to form priori probabilities; or (ii) be determined by empirical frequencies, which means individuals can generate statistical probabilities. Knight uses the notion of 'uncertainty,' when referring to events that do not fall within these two categories, that is, if either of the previous methods are not available for measuring the objective probabilities of such events. He also suggests that even in the uncertain cases, individuals can form estimates, which represent the concept of subjective probabilities, when making decisions based on them.

Savage (1954) suggested that probabilities are not necessarily something objectively known. Instead, decision makers have their subjective beliefs regarding the probability measure of the states. For example, unlike the roulette lottery, the horse lottery does not associate a known chance with each observation of the lottery. In other words, the decision maker cannot assign a specific probability to the outcomes of a horse lottery.

Thus, in Subjective Expected Utility (SEU) theory, preference relations over acts are represented by some real-valued utility function on the set of the consequences weighted by the subjective probabilities of the states; whereas the individual's choice behaviour in situations of risk is predictable under certain postulates, such as complete ordering and the sure-thing principle.

Anscombe and Aumann (1963) established the theory of State-Dependent Expected Utility by combining EU and SEU. They started by redefining the word 'probability'. They separated 'probability' into two very different concepts. When it is interpreted within the 'logical' sense, it means the plausibility of some events or reasonableness of some expectations, whereas if it is interpreted within the sense of 'physics', it is roughly identical to the word 'chances', which refers to the proportion of successes in some events in the statistical way. This allows to transform a choice under uncertainty into a two-stage lottery-act framework.

Although SEU gives a rather accurate prediction of a decision maker's gambling choice and his/her reflective choice behaviour, Ellsberg (1961) points out that Savage's normative rules are not applicable whenever there is an unmeasurable uncertainty in the relative likelihood of the events. In his paper, ambiguity exists whenever there is inadequate information regarding the relative likelihood of the events. For example, ambiguity could be caused purely by lack of information. It could also be due to the fact that the decision maker receives contradicting information or/and the source of information is not credible. He provided a famous thought experiment and proved that there is a non-negligible minority of decision makers who violate Savage's axioms, who are not able to reduce the unmeasurable uncertainty to risk, or to apply the von Neumann–Morgenstern Expected Utility Theory.

In the Ellsberg two-colour urn experiment, decision makers are faced with two urns containing 100 balls each of either red or black colour, from which one ball will be randomly drawn. Let us suppose that in Urn A, the composition of red and black balls is not known to the decision maker. However, in Urn B, there are 50 red balls and 50 black ones. Decision makers are asked which one they prefer, (1) to bet on Red_A or to bet on $Black_A$? (2) to bet on Red_B or $Black_B$? (3) to bet on Red_A or Red_B? (4) to bet on $Black_A$ or $Black_B$? To 'bet on Red_A' means that the decision maker chooses to draw a ball from Urn A; and that he/she will receive a prize a if the drawn ball is red, which means Red_A occurs. If the drawn ball is black, then the decision maker receives the prize b, which means not–Red_A occurs; and the amount of prize a is bigger than b. Also, Red_A, $Black_A$, Red_B and $Black_B$ are mutually exclusive.

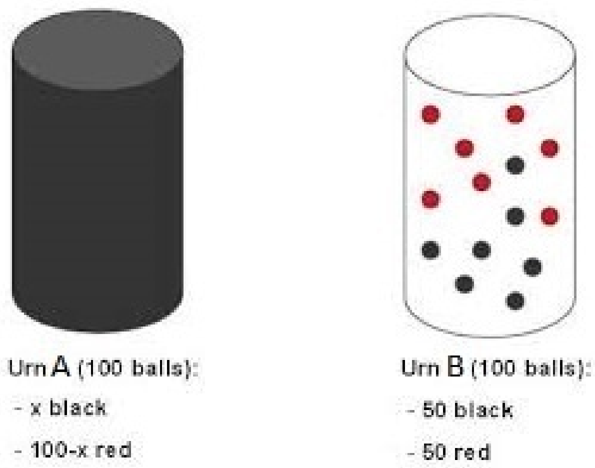

Figure 3: Ellsberg Two-colour Urn Experiment

According to Savage's theorem, the individuals should be indifferent to either of the options for these four questions. This means that individuals should be indifferent with respect to the colour they bet on. Moreover, they should also be indifferent with respect to the urn they choose to bet on. A number of people, including Savage himself, although being indifferent between the options of questions (1) and (2), and those of questions (3) and (4), nevertheless prefer betting on Red_B to Red_A, and $Black_B$ to $Black_A$. This preference obviously violates the Savage Axioms. Thus, the preferences elicited from the Ellsberg Urn game cannot be explained by the Savage Axioms. This contradiction between ambiguity and SEU theory becomes a major challenge to game theory and rational choice theory.

As stated in Table 1 below, let a and b be the payoffs, $a > b$, such that, for example, in gamble I, if 'R_A' occurs, the payoff of betting on 'R_A' is a. According to Savage's theorem, individual should be indifferent between gamble I and gamble II. Also, they should be indifferent between gamble III and gamble IV. Following Savage's postulates, complete ordering and the sure-thing principle, individuals are indifferent between gamble V and gamble VI, which means that decision makers are not only indifferent to bet on either of the colour from each urn, but also are indifferent to bet on either of the urns.

Then, starting with the assumption that the individual prefers gamble III to gamble I, we could make certain transformations toward gamble I and gamble III on the basis of complete ordering and the sure-thing principle and keep the preference unchanged, that is individuals always prefer the second gamble in the five pairs listed in Table 2. If the

Table 1: Ellsberg Two-Colour Urn Game

	R_A	B_A	R_B	B_B
I	a	b	b	b
II	b	a	b	b
III	b	b	a	b
IV	b	b	b	a
V	a	a	b	b
VI	b	b	a	a

payoff of betting on $Black_B$ changes from b to a, we have the payoffs as gamble I' and III' in Table 2. According to the sure-thing principle, preference regarding a pair of gambles will not change by the payoff values of events, for which both gambles have the same payoffs. Thus, gamble III' is preferred to gamble I'. As gamble III' is equivalent to gamble VI in Table 1 and gamble VI is indifferent to gamble V, we can transform III' to III". Gamble III"' is preferred to gamble I"' after we apply the sure-thing principle by changing the value of the payoffs of the event Red_A from a to b under both gambles. Then, we get that gamble III"" is preferred to gamble I"" as gamble III"' is equivalent to gamble II in Table 1 and individuals are indifferent between II and I. However, gamble III"" (equivalent to I) is preferred to gamble I"" (equivalent to III), which contradicts the assumption that gamble III is preferred to I. Thus, the Savage Axioms cannot explain these preference relations, opening up the door for alternative explanations.

Table 2: Transformed Ellsberg Two-Colour Urn Game

	R_A	B_A	R_B	B_B
I	a	b	b	b
III	b	b	a	b
I'	a	b	b	a
III'	b	b	a	a
I"	a	b	b	a
III"	a	a	b	b
I"'	b	b	b	a
III"'	b	a	b	b
I""	b	b	a	b
III""	a	b	b	b

In Ellsberg's three-colour urn game, the participants exhibit the same pattern as they do in the previous two-colour experiment. The participants are given an urn containing

90 balls, of which 30 are red and the remaining 60 are either black or yellow. In this alternative experiment, participants prefer betting on events for which they know more about the probability measure over the states—the event that a red (black or yellow) ball will be picked out of this urn to betting on black (red) one. As shown in Table 3, betting on red has a winning probability of 1/3 and betting on either black or yellow has a winning probability of 2/3; thus, the decision maker prefers X to Y and Y' to X'. Since these two pairs of acts are identical without taking yellow into consideration, if $X \succ Y$, then $X' \succ Y'$. However, such a pattern also violates the sure-thing principle. To be consistent with the sure-thing principle, X is preferred to Y and X' is preferred to Y', since the sure-thing principle requires decision makers to ignore the states in which the act leads to the same payoff. This means that the state yellow will not influence the individuals' choice when comparing acts of X and Y; and the same applies to X' and Y'.

Table 3: Ellsberg Three-Colour Urn Game

Number of balls			
	30	60	
Act	Red	Black	Yellow
X	W	0	0
Y	0	W	0
X'	W	0	W
Y'	0	W	W

In addition, the first order stochastic dominance axiom is violated by Ellsberg-type preferences. In the two-colour urn game, the probability of winning by betting on Red_A is higher than from betting on Red_B if the composition of red and black in Urn A is $(60, 40)$, for instance. However, $Red_B \succ Red_A$ to the Ellsberg type. The explanation of the three-colour urn game is fairly similar to that of the two-colour case.

Moreover, not only Savage's theorem but also other subjective utility theories with additive probabilities are proved to be implausible, as they fail to infer the probabilities from the decision maker's choice for Ellsberg's ambiguous urn game. In the two-colour case, the individual prefers to bet on Red_B rather than Red_A. This means the same as that the decision maker believes that $P(Red_B) > P(Red_A)$, which indicates that $1 - P(Red_B) < 1 - P(Red_A)$. However, this contradicts the preference of the individual, $P(Black_B) > P(Black_A)$. Analogously, in the three-colour urn game, denote the subjective probabilities of drawing a red, black and yellow by $P(Red)$, $P(Black)$ and $P(Yellow)$, respectively. $Y' \succ X'$ indicates $P(Black \cup Yellow) > P(Red \cup Yellow)$. Thus, when probabilities are additive, $P(Black \cup Yellow) = P(Black) + P(Yellow)$ and $P(Red \cup Yellow) = P(Red) + P(Yellow)$. However, given the preference $Y' \succ X'$, $P(Red) < P(Black)$. Thus, this contradicts the preference $X \succ Y$.

A series of empirical studies have been conducted following Ellsberg's thought experiment so as to test the existence of ambiguity and ambiguity aversion. The aversion to ambiguous choices have been well demonstrated in the replications of the Ellsberg urn experiments in Becker and Brownson (1964), Larson (1980), Hogarth and Einhorn (1990), Bernasconi and Loomes (1992), Seidenfeld and Wasserman (1993), Keren and Gerritsen (1999), Ivanov (2011), among others. In these experiments, as in Ellsberg's Urn experiment, objective probabilities exist; nevertheless, individuals can only partially access such measurements. If individuals were allowed to access the whole objective probabilities measurement, for example, by looking into Ellsberg's Urn A, and by seeing every ball in it, then, they would know the exact measurement of the objective probabilities of each ball to be drawn from that urn. Thus, Urn A would be no longer ambiguous, rather a risky urn and each individual would be able to settle on the same explicit probability measure of the event of a particular ball being drawn from it and, hence, form an identical prior/belief from such well-defined probability measure. However, there is another type of ambiguity, where the underlying objective probabilities measure is intrinsically unknown/unmeasurable. Unlike Ellsberg's design of the game, some experiments have taken natural events, such as betting on future stock prices, or GNP, that is, events for which there exists conflicting advice regarding their probability distributions. Those are instances of events with ambiguous probabilities, as objective probability measures can neither be deduced theoretically nor generated by obtaining sufficiently empirical frequencies for them, to test decision makers' attitudes towards them, as in MacCrimmon (1968), Goldsmith and Sahlin (1983), and Einhorn and Hogarth (1985).

Other experiments have found that individuals exhibit an ambiguity seeking attitude when the probability for gain is low and when the probability for loss is high (see Kahn and Sarin (1988), Curley, Young, and Yates (1989)).

Besides decision analysis, the concepts of ambiguity and ambiguity aversion also prevail in other realistic applications. Kellner (2010) argues that a tournament contract is preferred to an independent contract in an ambiguous situation, where the relationship between effort and output is opaque. Tournament contracts will not be favourable when agents are risk averse but ambiguity neutral. However, as long as ambiguity aversion occurs, rank-dependent tournaments will most often be attractive for agents over effort-dependent contracts, although they may not be optimal. When the relation between effort and output is opaque, agents prefer a tournament contract, where the wage is based on the ranks of the agents' contributions, to an independent contract, where the wage is solely based on the agent's own performance, because a tournament contract removes this ambiguity from the unknown distribution of output in the principal–agent problem (Kellner and Riener (2011)).

Dickhaut, Lunawat, Pronin, and Stecher (2011) study investment behaviour both under uncertainty and ambiguous probabilities. They find that only one-third of investors act consistently with SEU in a first-price sealed bid auction when deciding on their investment in a financial asset. This shows that bidders tend to bid higher than the expected return of the assets, given the range of the expected return when not informed of the specific probabilities of each asset. The experimental work of Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010) also displays substantial heterogeneity in attitudes towards ambiguity when choosing a portfolio with ambiguous Arrow securities. This generates different financial market equilibria than with the traditional approach, which assumes that agents are ambiguity neutral.

Also, in the auction market, the effect of ambiguity is amplified by its mechanism (Malmendier and Szeidl, 2008). It is claimed that bidders with the most market experience overbid more frequently than inexperienced ones. Bidders face ambiguous information as to all alternative auction goods and their winning bid. Bidders may over-value the good, because of the anchoring effect. In addition, unlike commodity-type goods, bidders are constrained by differentiated private values for goods like antiques, paintings, and collectibles, as the private values of the other competing bidders are unknown.

Similarly, when compared to individuals, firms in the market face an even more perplexing strategic environment. Armstrong and Huck (2010) found that entrepreneurs are over-optimistic, caring more about satisfactory results than optimisation, and resorting to rules of thumb when making strategic decisions for firms. In comparing the present or future outcomes with the historical figures of the firm itself or with the previous outcomes of their peers, entrepreneurs feel happy as long as the outcome remains as high as previous outcomes, when the probability of reaching the target set by the optimal strategy is rather ambiguous. Thus, entrepreneurs do not spend time on calculating the optimal equilibrium strategies even when there is no search cost. They adopt rules of thumb, resorting to imitating the successful strategies of their rivals or peers, against other strategies which would have been optimal instead.

Given the abundant evidence for ambiguity averse attitudes in well controlled laboratory experiments and in real life, other theories than SEU have been explored in order to solve/overcome the Ellsberg paradox, which incorporate ambiguity aversion. Inspired by this idea, Gilboa and Schmeidler (1989) assume that the decision makers formulate a set of possible additive probabilities when faced with ambiguity. They redefine the independence axiom for a non-unique prior. Moreover, they define uncertainty aversion. Thus, in the ambiguous urn game, the uncertainty averse decision maker takes the minimal expected utility over the prior set as his/her utility, for all priors in this set. In other words, preferences are represented by the minimum expected utilities over the set of possible probability measures. In Ellsberg's two-colour urn game, if the decision maker forms a prior set with all possible probability distributions of red and black balls in Urn A, the minimal expected utility of betting on Urn A will be zero for a utility maximising decision maker. With the probability of a red (black) ball picked from the risky Urn B as 1/2, the decision maker will prefer betting on the risky urn as long as the payoff of betting on Urn B is bigger than zero. Similarly, in the three-colour problem, the probability measure of $P(Black)$ could be $[0, 2/3]$. We assume the reward of winning the bet is W. Then, the expected utility is $(1/3)W$ and the minimum expected utility is 0 if the individual bets on the ambiguous urn. However, both the expected utility and the minimum expected utility is $(1/3)W$ if the individual bets on the unambiguous urn with an equal number of red, black and yellow balls in it. This explains why individuals prefer the unambiguous urn to the ambiguous one.

Schmeidler (1989) restates Ellsberg's point that the probability assigned to an uncertain event is not only based on the information the decision maker receives when forming such a probability; the missing information reflects a heuristic part, which the decision maker takes into consideration to assess the uncertainty probability component. He axiomatises SEU in an Anscombe and Aumann framework. Schmeidler replaces the classical independence axiom with a weaker condition: co-monotonic independence; this allows SEU to be generalised to allow non-additive probability measures. A non-additive probability describes the probabilities of two equally likely events as being equal but not necessarily 1/2, unless the information set for assigning the probabilities is rich enough. In the Ellsberg experiment, the probability measure on the set of states need not be additive, due to the fact that the decision maker receives little information. For example, in the two-colour urn, $P(Black_A) = P(Red_A)$. However, the sum of $P(Black_A)$ and $P(Red_A)$ need not be 1. $1 - P(Black_A) - P(Red_A)$ measures the decision maker's confidence in the probability. Thus, the capacity of a red (black) ball to be picked from Urn B is 0.5 and the capacity of getting a red (black) ball picked from Urn A is smaller than 1/2. We might assign 0 capacities to the events of Red_A and $Black_A$, so that betting on either of the colours from urn A gives the decision maker a zero utility. Thus, SEU with non-additive probability also gives a conceivable explanation of the observed preferences from the ambiguous urn game.

Although ambiguity aversion has been observed through well controlled laboratory experiments and in real life, studies only focus on comparing decision-makings under different scenarios, without and with ambiguity. This means individuals are given both the risky environment and the ambiguous one, and they are asked to make decisions as if they were confronted with the Ellsberg Urn game. In reality, decision makers do not always have both risky and ambiguous scenarios to choose from. The risky world and the ambiguous world are mutually exclusive, which means individuals could start with being in the ambiguous world, with the risky world never becoming available. In the remainder of this study, we examine voters confronted with ambiguity: voters cannot choose to switch to a non-ambiguous world.

2.1 The Collective Voting Game Under Ambiguity: The MMEU Approach

As in Feddersen and Pesendorfer (1998) a group of n jurors, $j = 1, \cdots, n$, have to reach a verdict on a defendant, who could be either "guilty"-G, or "innocent"-I with ex-ante equal probability, i.e., $Pr(G) = Pr(I) = 1/2$. Each juror is expected to cast a vote $\{C, A\}$ either to 'C=convict' or to 'A=acquit' the defendant based on the evidence received, with precision p, where $p = Pr(g|G) = Pr(i|I)$. The individual votes then contribute towards the collective verdict. As before, we assume that all jurors have the same preferences with respect to the outcome of the verdict, that is, they all want to reach the correct judgment. Their preferences are defined as follows:

$$u(A, I) = u(C, G) = 0,$$

$$u(C, I) = -q,$$

$$u(A, G) = -(1 - q),$$

with $q \in (0, 1)$ representing once again the threshold of reasonable doubt for conviction.

However, before jurors cast their votes, each voter j receives an independent randomly drawn private and imperfect signal $s_j \in \{g, i\}$ as the evidence, with random and ambiguous precision p with $p \in \mathcal{P} = [\underline{p}, \bar{p}]$ and $1/2 < \underline{p} < \bar{p} < 1$. No further probabilistic information about the signal precision is provided. This implies that, differing once more from the existing jury voting literature, the quality of the private signals is imprecisely measured. In particular, we allow such precision to fall within two levels, in the sense that the domain of the information quality is a closed interval, rather than a set including only two points as in Pan, Fabrizi, and Lippert (2016), or a singleton (Feddersen and Pesendorfer, 1998). The interval ambiguity with respect to the signal precision could be understood as the case in which a piece of evidence, say, hinting toward the defendant being guilty, tells us that the probability that the defendant is guilty is at least 60%, but at most 90%. However, except for this, there is no extra information provided regarding the probability measure of the underling true accuracy of the information (evidence) each voter receives.

Define the set of all possible priors as Π, $\Pi = [\underline{p}, \bar{p}]$. The ambiguity averse voter j will choose $\pi_j \in \Pi$ that provides the best among the worst expected utility for each possible actions, $\{C, A\}$.

With this in mind, we maintain the assumption that after observing the private signals, each juror casts her vote simultaneously, according to the strategy $\sigma_j(s_j, \pi_j)$, which is the probability that voter j votes for conviction conditional on her private signal s_j and her subjectively formed prior π_j. As before, the collective decision is determined by the voting rule $\hat{k}$, $\hat{k} \leq n$. The given voting rule is the simple majority rule when $\hat{k} = (n+1)/2$; and it is the unanimity rule when $\hat{k} = n$. And, as always, the verdict is either acquittal or conviction, depending on whether the threshold of necessary votes to convict is either not reached, or reached.

2.2 Informative Voting

A voter j's expected utility of voting for acquittal, conditional on being pivotal and receiving an innocent signal is

$$E[u_j(A, \cdot) \mid piv, s_j = i] = u_j(A|I)Pr(I|piv, s_j = i) + u_j(A|G)Pr(G|piv, s_j = i).$$

Because $u_j(A|I) = 0$ and $u_j(A|G) = -(1-q)$, we then have

$$E[u_j(A, \cdot) \mid piv, s_j = i] = -(1-q)Pr(G|piv, s_j = i).$$

Denote the posterior belief that the defendant is guilty conditional on the voter being pivotal and having received signal i, when all other voters vote informatively, as $\beta^i_G(\pi_j, \sigma(\cdot))$. Hence,

$$Pr(G|piv, s_j = i) = \beta^i_G(\pi_j, \sigma(\cdot)) = \frac{1}{1 + (\frac{\pi_j}{1-\pi_j})(\frac{1-\pi_j}{\pi_j})^{n-1}}.$$

For an ambiguity averse voter j, we can determine what is the selected belief or prior π_j, that corresponds to the action leading to the highest among the minimum expected utilities from choosing, say, to acquit a defendant.

To do so, we first determine the prior, among those one can hold, which leads to the lowest utility of acquitting a guilty when receiving the innocent signal:

$$min_{\pi_j \in \Pi} E[u_j(A, \cdot) \mid piv, s_j = i] = -(1-q) max_{\pi_j \in \Pi} \beta^i_G(\pi_j, \sigma(\cdot)).$$

Denote $max_{\pi_j \in \Pi} \beta^i_G(\pi_j, \sigma(\cdot))$ as $\bar{\beta}^i_G(\pi, \sigma(\cdot))$, so that

$$min_{\pi_j \in \Pi} E[u_j(A, \cdot) \mid piv, s_j = i] = -(1-q) \bar{\beta}^i_G(\pi_j, \sigma(\cdot)).$$

We can repeat the exercise for a voter j's expected utility of voting for conviction, conditional on being pivotal and receiving an innocent signal. This leads to:

$$E[u_j(C, \cdot) \mid piv, s_j = i] = u_j(C|I) Pr(I|piv, s_j = i) + u_j(C|G) Pr(G|piv, s_j = i),$$

which is equivalent to

$$E[u_j(C, \cdot) \mid piv, s_j = i] = -q Pr(I|piv, s_j = i).$$

An ambiguity averse voter assesses her action to vote to convict by the minimum expected utility of this action, conditional on being pivotal and receiving an innocent signal. That is,

$$min_{\pi_j \in \Pi} E[u_j(C, \cdot) \mid piv, s_j = i] = -q max_{\pi_j \in \Pi} \beta^i_I(\pi_j, \sigma(\cdot)).$$

We know that $\beta^i_I(\pi_j) = 1 - \beta^i_G(\pi_j)$. Hence, $max_{\pi_j \in \Pi} \beta^i_I(\pi_j, \sigma(\cdot)) = 1 - min_{\pi_j \in \Pi} \beta^i_G(\pi_j, \sigma(\cdot))$. Define $min_{\pi_j \in \Pi} \beta^i_G(\pi_j, \sigma(\cdot))$ as $\underline{\beta}^i_G(\pi_j, \sigma(\cdot))$, we then have

$$min_{\pi_j \in \Pi} E[u_j(C, \cdot) \mid piv, s_j = i] = -q(1 - \underline{\beta}^i_G(\pi_j, \sigma(\cdot))).$$

Thus, the ambiguity averse voter will vote for acquittal informatively if and only if her minimum expected utility of voting for acquittal is bigger than the minimum utility of voting for conviction, given her private signal is i, conditional on being pivotal and all other voters voting informatively, i.e.,

$$min_{\pi_j \in \Pi} E[u_j(A, \cdot) \mid s_j = i] > min_{\pi_j \in \Pi} E[u_j(C, \cdot) \mid s_j = i].$$

As the utility of convicting the guilty and acquitting the innocent is zero, we have

$$min_{\pi_j \in \Pi} E[u_j(A, G) \mid s_j = i] > min_{\pi_j \in \Pi} E[u_j(C, I) \mid s_j = i], \tag{1}$$

which is equivalent to:

$$-(1-q) max_{\pi_j \in \Pi} Pr(G|s_j = i) > -q max_{\pi_j \in \Pi} Pr(I|s_j = i).$$

Therefore, requiring condition (1) to be satisfied is equivalent to verifying that the following condition holds:

$$-(1-q) \bar{\beta}^i_G(\pi_j, \sigma(\cdot)) > -q \bar{\beta}^i_I(\pi_j, \sigma(\cdot)).$$

Because $\bar{\beta}^i_I(\pi_j, \sigma(\cdot)) = 1 - \underline{\beta}^i_G(\pi_j, \sigma(\cdot))$, we then have

$$\frac{\bar{\beta}^i_G(\pi_j, \sigma(\cdot))}{1 - \underline{\beta}^i_G(\pi_j, \sigma(\cdot))} < \frac{q}{1-q}.$$

Analogously, the ambiguity averse j voter will vote for conviction informatively if and only if

$$min_{\pi_j \in \Pi} E[u_j(C, \cdot) \mid piv, s_j = g] > min_{\pi_j \in \Pi} E[u_j(A, \cdot) \mid piv, s_j = g].$$

As the utility of convicting the guilty and acquitting the innocent is zero, we have

$$min_{\pi_j \in \Pi} E[u_j(C, I) \mid s_j = i] > min_{\pi_j \in \Pi} E[u_j(A, G) \mid s_j = i], \tag{2}$$

which is equivalent to:

$$-q(1 - \underline{\beta}^g_G(\pi_j, \sigma(\cdot))) > -(1-q)\bar{\beta}^g_G(\pi_j, \sigma(\cdot)),$$

that is

$$\frac{\bar{\beta}^g_G(\pi_j, \sigma(\cdot))}{1 - \underline{\beta}^g_G(\pi_j, \sigma(\cdot))} > \frac{q}{1-q}.$$

Given that in an informative equilibrium all jurors behave the same and that $\sigma(i) = 0$ and $\sigma(g) = 1$, in the remainder of this study, we omit the index j, when referring to the equilibrium beliefs and strategies of a specific juror, and, for the rest of this subsection, we also omit to specify the equilibrium strategy when describing the belief function.

Thus, the condition for informative voting being a Nash equilibrium, can simply be written as $\frac{\bar{\beta}^i_G(\pi)}{1-\underline{\beta}^i_G(\pi)} < q < \frac{\bar{\beta}^g_G(\pi)}{1-\underline{\beta}^g_G(\pi)}$.

Notice that

$$\beta^i_G(\pi) = \frac{(1-\pi)\pi^{\hat{k}-1}(1-\pi)^{n-\hat{k}}}{(1-\pi)\pi^{\hat{k}-1}(1-\pi)^{n-\hat{k}} + \pi(1-\pi)^{\hat{k}-1}\pi^{n-\hat{k}}} = \frac{1}{1 + (\frac{1-\pi}{\pi})^{2\hat{k}-n-2}}$$

is strictly increasing with π when $k \geq \frac{n+2}{2}$, for example, when $\hat{k} = n$, it reaches its maximum when $\pi = \bar{p}$; whereas, if $k < \frac{n+2}{2}$, for example, when $\hat{k} = \frac{n+1}{2}$, β^i_G reaches its maximum when $\pi = \underline{p}$; and,

$$\beta^g_G(\pi) = \frac{\pi\pi^{\hat{k}-1}(1-\pi)^{n-\hat{k}}}{\pi\pi^{\hat{k}-1}(1-\pi)^{n-\hat{k}} + (1-\pi)(1-\pi)^{\hat{k}-1}\pi^{n-\hat{k}}} = \frac{1}{1 + (\frac{1-\pi}{\pi})^{2\hat{k}-n}}$$

is strictly increasing with p when $k \geq \frac{n}{2}$, when $\pi = \underline{p}$, it reaches its minimum.

Proposition 1 *Under the Maxmin approach and ambiguous information p, with $p \in [\underline{p}, \bar{p}]$, informative voting is an equilibrium for ambiguity averse voters if and only if* $\frac{\bar{\beta}^i_G(\pi)}{1-\underline{\beta}^i_G(\pi)} < \frac{q}{1-q} < \frac{\bar{\beta}^g_G(\pi)}{1-\underline{\beta}^g_G(\pi)}$.

2.3 Strategic Voting Under Unanimity

In this section, we study the Symmetric Responsive Nash Equilibrium under unanimous voting rule, when informative voting is not an equilibrium, that is, equation (1) and equation (2) are not satisfied at the same time. Under the Maxmin approach, voter strategic behaviour is still captured by considering the minimum level of utility of either votes one can cast, and each voter chooses the action, which gives the highest utility between the two.

A voter j's expected utility of voting for acquittal, conditional on being pivotal and receiving an innocent signal is

$$E[u_j(A,\cdot) \mid piv, s_j = i] = -(1-q)Pr(G|piv, s_j = i).$$

We denote $Pr(G|piv, s_j = i)$ as $\beta_G^i(\pi, \sigma(\cdot))$, which is the posterior belief that the defendant is guilty conditional on the voter being pivotal and receive signal i, that is

$$\beta_G^i(\pi, \sigma(\cdot)) = \frac{1}{1 + (\frac{\pi}{1-\pi})(\frac{\gamma_I}{\gamma_G})^{n-1}},$$

where

$$\gamma_I(\pi, \sigma(\cdot)) = \pi\sigma(i) + (1-\pi)\sigma(g);$$

and

$$\gamma_G(\pi, \sigma(\cdot)) = \pi\sigma(g) + (1-\pi)\sigma(i).$$

An ambiguity averse voter assesses her action to acquit by its minimum expected utility among all possible priors, that is,

$$min_{\pi\in\Pi} E[u_j(A,\cdot) \mid piv, s_j = i] = -(1-q) max_{\pi\in\Pi}\beta_G^i(\pi, \sigma(\cdot)).$$

Define $max_{\pi\in\Pi}\beta_G^i(\pi, \sigma(\cdot))$ as $\bar{\beta}_G^i(\pi, \sigma(\cdot))$, we have

$$min_{\pi\in\Pi} E[u_j(A,\cdot) \mid piv, s_j = i] = -(1-q)\bar{\beta}_G^i(\pi, \sigma(\cdot)). \tag{3}$$

An ambiguity averse voter accesses her action to convict by its minimum expected utility, conditional on being pivotal and receiving an innocent signal. That is,

$$min_{\pi\in\Pi} E[u_j(C,\cdot) \mid piv, s_j = i] = -q max_{\pi\in\Pi}\beta_I^i(\pi, \sigma(\cdot)).$$

We know $\beta_I^i(\pi, \sigma(\cdot)) = 1 - \beta_G^i(\pi, \sigma(\cdot))$. Hence, $max_{\pi\in\Pi}\beta_I^i(\pi, \sigma(\cdot)) = 1 - min_{\pi\in\Pi}\beta_G^i(\pi, \sigma(\cdot))$. Define $min_{\pi\in\Pi}\beta_G^i(\pi, \sigma(\cdot))$ as $\underline{\beta}_G^i(\pi, \sigma(\cdot))$, we then have

$$min_{\pi\in\Pi} E[u_j(C,\cdot) \mid piv, s_j = i] = -q(1 - \underline{\beta}_G^i(\pi, \sigma(\cdot))). \tag{4}$$

Similarly, an ambiguity averse voter's minimum expected utility of voting to acquit, conditional on being pivotal and receiving a guilty signal is

$$min_{\pi\in\Pi} E[u_j(A,\cdot) \mid piv, s_j = g] = -(1-q)\bar{\beta}_G^g(\pi, \sigma(\cdot)); \tag{5}$$

and her minimum expected utility of voting to convict, conditional on being pivotal and receiving a guilty signal is

$$min_{\pi\in\Pi}E[u_j(C,\cdot) \mid piv, s_j = g] = -q(1 - \underline{\beta}^g_G(\pi, \sigma(\cdot))), \tag{6}$$

where

$$\beta^g_G(\pi, \sigma(\cdot)) = \frac{1}{1 + (\frac{1-\pi}{\pi})(\frac{\gamma_I}{\gamma_G})^{n-1}},$$

where

$$\gamma_I(\pi, \sigma(\cdot)) = \pi\sigma(i) + (1 - \pi)\sigma(g);$$

and

$$\gamma_G(\pi, \sigma(\cdot)) = \pi\sigma(g) + (1 - \pi)\sigma(i).$$

Hence, $\bar{\beta}^g_G(\pi, \sigma(\cdot))$ and $\underline{\beta}^g_G(\pi, \sigma(\cdot))$ are respectively the maximum level of the posterior belief and the minimum level of the posterior belief that the defendant is guilty conditional on the voter being pivotal and receiving signal g.

Notice that because receiving a guilty signal can never be information in favour of the innocence of the defendant more than receiving an innocent signal can ever be, we know that $\beta^g_G(\pi, \sigma(\cdot)) > \beta^i_G(\pi, \sigma(\cdot))$. Therefore, $\bar{\beta}^g_G(\pi, \sigma(\cdot)) > \bar{\beta}^i_G(\pi, \sigma(\cdot))$; and $1-\underline{\beta}^g_G(\pi, \sigma(\cdot)) < 1 - \underline{\beta}^i_G(\pi, \sigma(\cdot))$. Thus:

$$\frac{\bar{\beta}^g_G(\pi, \sigma(\cdot))}{1 - \underline{\beta}^g_G(\pi, \sigma(\cdot))} > \frac{\bar{\beta}^i_G(\pi, \sigma(\cdot))}{1 - \underline{\beta}^i_G(\pi, \sigma(\cdot))}. \tag{7}$$

Lemma 1 *If* $\frac{\bar{\beta}^g_G(\pi,\sigma(\cdot))}{1-\underline{\beta}^g_G(\pi,\sigma(\cdot))} > \frac{\bar{\beta}^i_G(\pi,\sigma(\cdot))}{1-\underline{\beta}^i_G(\pi,\sigma(\cdot))}$, $(0 < \sigma(i) < 1, \sigma(g) = 1)$ *is the Symmetric Responsive Nash Equilibrium.*

Proof. Assume equation (1) does not hold, that is voter's minimum expected utility of voting for acquittal is no larger than the minimum expected utility of voting for conviction, conditional on being pivotal and receiving an innocent signal, that is

$$-(1 - q)\bar{\beta}^i_G(\pi, \sigma(\cdot)) \leq -q(1 - \underline{\beta}^i_G(\pi, \sigma(\cdot))).$$

It is equivalent to

$$\frac{\bar{\beta}^i_G(\pi, \sigma(\cdot))}{1 - \underline{\beta}^i_G(\pi, \sigma(\cdot))} \geq \frac{q}{1 - q}.$$

If inequality (7) holds, it must be

$$\frac{\bar{\beta}^g_G(\pi, \sigma(\cdot))}{1 - \underline{\beta}^g_G(\pi, \sigma(\cdot))} > \frac{q}{1 - q}.$$

And this proves that if $0 < \sigma(i) \leq 1$, then $\sigma(g) = 1$. Because the strategic voting equilibrium has to be responsive, then we have the equilibrium, such that $(0 < \sigma(i) <$

$1, \sigma(g) = 1)$. And $0 < \sigma(i) < 1$ simply means that voters randomise when receiving an innocent signal, which requires

$$\frac{\bar{\beta}^i_G(\pi, \sigma(i))}{1 - \underline{\beta}^i_G(\pi, \sigma(i))} = \frac{q}{1-q}.$$

If equation (2) fails to hold, that is voter's minimum expected utility of voting for conviction is no larger than the minimum expected utility of voting for acquittal, conditional on being pivotal and receiving a guilty signal, that is

$$-q(1 - \underline{\beta}^g_G(\pi, \sigma(\cdot))) \leq -(1-q)\bar{\beta}^g_G(\pi, \sigma(\cdot)),$$

we can also conclude that

$$\frac{\bar{\beta}^g_G(\pi, \sigma(\cdot))}{1 - \underline{\beta}^g_G(\pi, \sigma(\cdot))} \leq \frac{q}{1-q}.$$

And because of equation (7), we have

$$\frac{\bar{\beta}^i_G(\pi, \sigma(\cdot))}{1 - \underline{\beta}^i_G(\pi, \sigma(\cdot))} < \frac{q}{1-q},$$

leading to a contradiction, since it says that when $0 \leq \sigma(g) < 1$, $\sigma(i) = 0)$. Due to the requirement of being responsive, $\sigma(g)$ cannot equal to 0, however, $(0 < \sigma(g) < 1, \sigma(i) = 0)$ would not satisfy being a symmetric responsive Nash equilibrium. When $0 < \sigma(g) < 1$, being pivotal means that the other $n-1$ voters all received signal g, because if they received signal i, they would vote to acquit with probability 1. But if this is the case, then a pivotal voter with a guilty signal would always vote to convict. Thus, this contradicts the assumption $0 < \sigma(g) < 1$.

■

Lemma 2 *The function* $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))}$ *is continuous at every* $\sigma(i) \in [0, 1]$.

Proof. Let $\Pi = (0.5, 1)$. Define $\phi : [0, 1] \to 2^{\Pi}$ by $\phi(\sigma(i)) = [\underline{\pi}, \bar{\pi}]$ for every $\sigma(i) \in [0, 1]$. Note that ϕ is nonempty, continuous and compact-valued and that $\beta^i_G(\pi, \sigma(i))$ is continuous in both π and $\sigma(i)$. Thus by Berge Maximum Theorem (Aliprantis and Border, 2006), both $\bar{\beta}^i_G(\pi, \sigma(i))$ and $\underline{\beta}^i_G(\pi, \sigma(i))$ are continuous, and so is $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))}$. ■

Therefore, Lemma 1 suggests that the symmetric responsive equilibrium exists when $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))} = \frac{q}{1-q}$. As shown in Figure 4,[4] if $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))} < \frac{q}{1-q}$, voters will vote informatively, which is the orange shaded area. If $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))} > \frac{q}{1-q}$, then voters vote for conviction regardless of the signals, that is $(\sigma(i) = 1, \sigma(g) = 1)$, which is the green shaded

[4] Figure 4 has been obtained by a numerical simulation of the function $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))}$, when fixing $n = 5$, $p \in [0.70, 0.80]$ and letting the variable $\sigma(i)$ vary between 0 and 1. Other simulations for other values of n and the interval of p led to similar qualitative behaviours for this function. Since we do not have a formal proof that the function is strictly monotonic, we provide this figure for purely illustrative purposes, and only claim existence of a strategic equilibrium for intermediate levels of q, leaving the proof of uniqueness of such equilibrium to further research.

area. In between is the are where voters can randomise their strategy when receiving an innocent signal. And there exists such strategy $0 < \sigma^*(i) < 1$ if and only if there is a horizontal line $\frac{q}{1-q}$ intercepting the continuous function $\frac{\bar{\beta}^i_G(\pi,\sigma^*(i))}{1-\underline{\beta}^i_G(\pi,\sigma^*(i))}$.

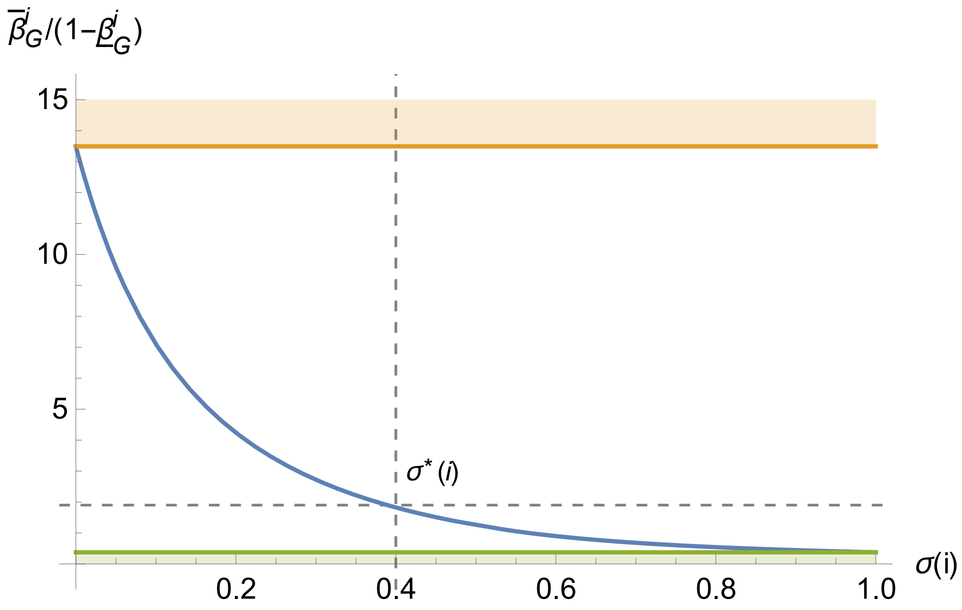

Figure 4: Symmetric Responsive Equilibrium $(0 < \sigma(i) < 1, \sigma(g) = 1)$

Proposition 2 *Under the Maxmin approach and ambiguous information p, with $p \in [\underline{p}, \bar{p}]$, there exists a Symmetric Responsive Nash Equilibrium for the unanimity rule, when $\frac{1-\underline{p}}{1-\underline{p}+\bar{p}} < q < \frac{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}}{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}(2+(\frac{1-\bar{p}}{\bar{p}})^{n-2})}$, such that $0 < \sigma^*(i) < 1$, and such that $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))} = \frac{q}{1-q}$.*

Proof. Given $\sigma(g) = 1$, then

$$\gamma_I(\pi, \sigma(i)) = \pi\sigma(i) + (1 - \pi);$$

and

$$\gamma_G(\pi, \sigma(i)) = \pi + (1 - \pi)\sigma(i),$$

with $0 < \sigma(i) < 1$. Then,

$$\beta^i_G(\pi, \sigma(i)) = \frac{1}{1 + (\frac{\pi}{1-\pi})(\frac{\pi\sigma(i)+(1-\pi)}{\pi+(1-\pi)\sigma(i)})^{n-1}}.$$

Because $\beta^i_G(\pi, \sigma(i))$ is continuous at $\sigma(i) = 0$, as shown in Figure 5, we have

$$\lim_{\sigma(i)\to 0^+} \beta^i_G(\pi, \sigma(i)) = \frac{1}{1 + (\frac{1-\pi}{\pi})^{n-2}}.$$

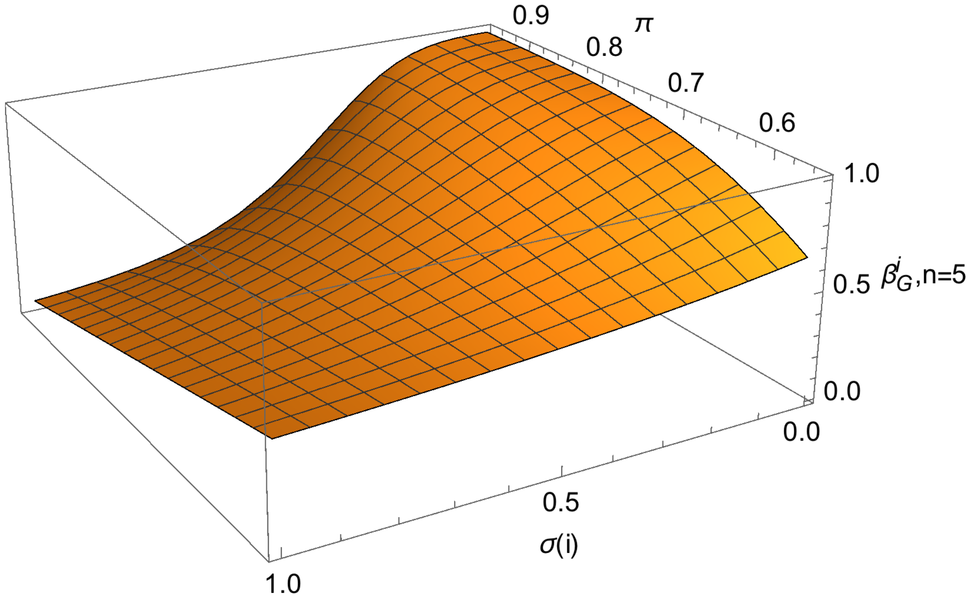

Figure 5: 3D Plot of $\beta^i_G(\pi, \sigma(i))$

Hence,

$$\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi^*, \sigma(i)) = \frac{1}{1+(\frac{1-\pi^*}{\pi^*})^{n-2}},$$

with $\pi^* = \bar{p}$, and

$$\lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi^*, \sigma(i)) = \frac{1}{1+(\frac{1-\pi^*}{\pi^*})^{n-2}},$$

with $\pi^* = \underline{p}$. Therefore, by continuity

$$\lim_{\sigma(i)\to 0^+} \frac{\bar{\beta}^i_G(\pi, \sigma(i))}{1-\underline{\beta}^i_G(\pi, \sigma(i))} = \frac{\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi, \sigma(i))}{1-\lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi, \sigma(i))} = \frac{\frac{1}{1+(\frac{1-\bar{p}}{\bar{p}})^{n-2}}}{1-\frac{1}{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}}}. \tag{8}$$

Similarly, $\beta^i_G(\pi, \sigma(i))$ is continues at $\sigma(i) = 1$, and then,

$$\lim_{\sigma(i)\to 1^-} \beta^i_G(\pi, \sigma(i)) = \frac{1}{1+(\frac{\pi}{1-\pi})}.$$

Hence,

$$\lim_{\sigma(i)\to 1^-} \bar{\beta}^i_G(\pi^*, \sigma(i)) = \frac{1}{1+(\frac{\pi^*}{1-\pi^*})},$$

with $\pi^* = \underline{p}$, and

$$\lim_{\sigma(i)\to 1^-} \underline{\beta}^i_G(\pi^*, \sigma(i)) = \frac{1}{1+(\frac{\pi^*}{1-\pi^*})},$$

with $\pi^* = \bar{p}$. Therefore, by continuity

$$\lim_{\sigma(i)\to 1^-} \frac{\bar{\beta}^i_G(\pi, \sigma(i))}{1-\underline{\beta}^i_G(\pi, \sigma(i))} = \frac{\lim_{\sigma(i)\to 1^-} \bar{\beta}^i_G(\pi, \sigma(i))}{1-\lim_{\sigma(i)\to 1^-} \underline{\beta}^i_G(\pi, \sigma(i))} = \frac{\frac{1}{1+(\frac{\underline{p}}{1-\underline{p}})}}{1-\frac{1}{1+(\frac{\bar{p}}{1-\bar{p}})}} = \frac{1-\underline{p}}{\bar{p}}. \tag{9}$$

Therefore, whenever $\frac{q}{1-q}$ is strictly between the values identified in equations (8) and (9), that is $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))} = \frac{q}{1-q}$, there exists $0 < \sigma(i) < 1$ as an equilibrium.

Note that this condition can also be split into two components, as follows:

$$q < \frac{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}}{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}(2+(\frac{1-\bar{p}}{\bar{p}})^{n-2})}; \tag{10}$$

and

$$q > \frac{1-\underline{p}}{1-\underline{p}+\bar{p}}. \tag{11}$$

Therefore, we proved that as long as $\frac{1-\underline{p}}{1-\underline{p}+\bar{p}} < q < \frac{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}}{1+(\frac{1-\underline{p}}{\underline{p}})^{n-2}(2+(\frac{1-\bar{p}}{\bar{p}})^{n-2})}$, there exists $0 < \sigma^*(i) < 1$, which is the equilibrium strategy when voter receives signal i.

■

In addition, if condition (11) is violated, that is $q < \frac{1-\underline{p}}{1-\underline{p}+\bar{p}}$, there exists a Symmetric Non-Responsive Strategic Nash Equilibrium, that is $(\sigma(i) = 1, \sigma(g) = 1)$, where voters vote for conviction regardless of their signals. This strategy leads to the highest type I error, $Pr(C|I) = 1$, and the lowest type II error, $Pr(A|G) = 0$.

Corollary 1 *Under the Maxmin approach and ambiguous information p, with $p \in [\underline{p}, \bar{p}]$, for unanimous voting, there exists Symmetric Non-Responsive Nash Equilibrium with $(\sigma(i) = 1, \sigma(g) = 1)$ if and only if $\frac{\bar{\beta}^i_G(\pi,\sigma(i))}{1-\underline{\beta}^i_G(\pi,\sigma(i))} > \frac{q}{1-q}$, that is as long as $q < \frac{1-\underline{p}}{1-\underline{p}+\bar{p}}$.*

2.4 Strategic Voting Under Non-Unanimity

For non-unanimous voting, $\hat{k} \neq n$, we have

$$\beta^i_G(\pi, \sigma(\cdot), \hat{k}) = \frac{1}{1+(\frac{\pi}{1-\pi})(\frac{\gamma_I}{\gamma_G})^{\hat{k}-1}(\frac{1-\gamma_I}{1-\gamma_G})^{n-\hat{k}}}$$

and

$$\beta^g_G(\pi, \sigma(\cdot), \hat{k}) = \frac{1}{1+(\frac{1-\pi}{\pi})(\frac{\gamma_I}{\gamma_G})^{\hat{k}-1}(\frac{1-\gamma_I}{1-\gamma_G})^{n-\hat{k}}},$$

where

$$\gamma_I(\pi, \sigma(\cdot)) = \pi\sigma(i) + (1-\pi)\sigma(g),$$

and

$$\gamma_G(\pi, \sigma(\cdot)) = \pi\sigma(g) + (1-\pi)\sigma(i).$$

Because $\beta^g_G(\pi, \sigma(\cdot), \hat{k}) > \beta^i_G(\pi, \sigma(\cdot), \hat{k})$, Lemma 1 also holds for the case where $\hat{k} \neq n$. Hence, we also have a Symmetric Responsive Nash Equilibrium for non-unanimous voting, $(0 < \sigma(i) < 1, \sigma(g) = 1)$. Using Lemma 2, we can also prove the existence of the symmetric responsive Nash equilibrium for non-unanimous voting rule. The formal proofs can be found in Appendix A, whereas the main results for this case are summarised below.

Proposition 3 *Under the Maxmin approach and ambiguous information p, with $p \in [\underline{p}, \bar{p}]$, for the non-unanimous voting rule, there exists Symmetric Responsive Nash Equilibria, $(0 < \sigma(i) < 1, \sigma(g) = 1)$, if and only if $\frac{\bar{\beta}^i_G(\pi,\sigma(i),\hat{k})}{1-\underline{\beta}^i_G(\pi,\sigma(i),\hat{k})} = \frac{q}{1-q}$, that is,*

1. *if $\hat{k} > \frac{n+2}{2}$ and $\frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})} < q < \frac{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}}{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}(2+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2})}$;*

2. *if $0 < \hat{k} \leq \frac{n+2}{2}$ and $\frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})} < q < \frac{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}}{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}(2+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2})}$.*

Corollary 2 *Under the Maxmin approach and ambiguous information p, with $p \in [\underline{p}, \bar{p}]$, for non-unanimous voting, if $q < \frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})}$, there exists Symmetric Non-Responsive Nash Equilibria, that is $(\sigma(i) = 1, \sigma(g) = 1)$, where voters vote for conviction regardless of their signals, which leads to the highest type I error, $Pr(C|I) = 1$, and the lowest type II error, $Pr(A|G) = 0$.*

3 Comparative Statics Results

Consider the 12-person jury example as in Feddersen and Pesendorfer (1998), that is when $n = 12$, $p = 0.8$ and $\hat{k} = 12$ or 7. We know that when signal precision is uniquely defined, voters behave symmetrically and responsibly if and only if $1-p < q < \frac{1}{1+(\frac{1-p}{p})^{n-2}}$. Especially, from Figure 6, we can see the cut-off value of q for informative voting being an equilibrium is very high, which is almost 1. This says that, it is almost impossible for voters to vote informatively in this scenario. For any level of q, which is exogenously given and set equal to $q = 0.9$, which is below the cut-off value for informative voting, strategic equilibria arise. We can compute the voter's strategy for this case, which is exactly equal to $\sigma(i) = 0.575$, $\sigma(g) = 1$.

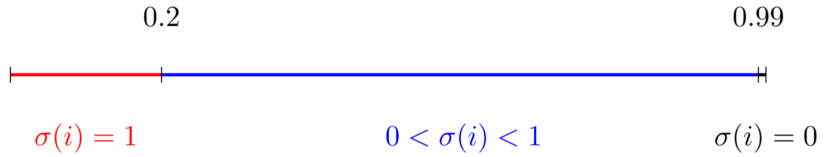

Figure 6: 12-person jury: Threshold values of q for different voting equilibria when $p = 0.8$

However, if we were in the presence of ambiguous information and voters were ambiguity averse and choosing their beliefs according to the Maxmin, we would be able to observe voters voting informatively under the unanimity rule, for a larger range of thresholds of reasonable doubts, below the level 0.9.

Take the signal precision to belong to $p \in [0.6, 0.8]$, from Figure 7, the cut-off value for informative voting being an equilibrium is 0.894, which is smaller than the given reasonable doubt level 0.9. Voters' strategy is $\sigma(i) = 0$, $\sigma(g) = 1$ in this case, since voters cast their votes according to the signals they receive.

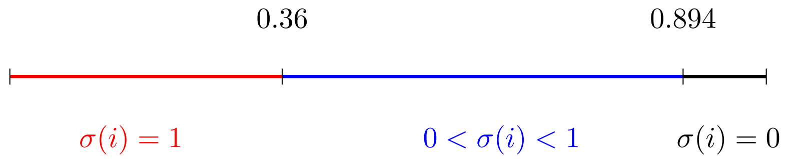

Figure 7: 12-person jury: Threshold values of q for different voting equilibria when $p = [0.55, 0.8]$

On the other hand, although the cut-off value for the strategy $\sigma(i) = 1$, $\sigma(g) = 1$ being an equilibrium is increased, it will never exceed 0.5.

Under majority voting rule, if the signal precision is ambiguous, we observe similar results as we do under the unanimous voting rule, that is, the threshold for informative voting being an equilibrium is lower given $p \in [0.6, 0.8]$ than that when $p = 0.8$. The results are summarised in Table 4.

In Table 4, we looked at the three different voting rules, unanimity, simple majority and super majority. We found that (1) when information is downward ambiguous, the threshold level of q for voting informatively is lower. Thus, (2) informative voting is the only equilibrium for these three voting rules. The unanimity rule is the least preferred one when $p = 0.8$. However, (3) when $p = [0.55, 0.8]$, it outperforms other voting rules as it leads to the smallest type I error as opposed to other rules.

In Pan et al. (2016), we present and discuss experiments for the two-point non-common prior model. Based on the same parameters as the ones used in those experiments, $n = 5$, $q = 0.5$, $p = \{0.6, 0.9\}$, we first check how the threshold level of the reasonable doubt required for informative voting to be an equilibrium is affected by the introduction of imprecise probabilities belonging to an interval, rather, as opposed to the case when the precision of the signal is known and unique. We do so, by conducting a simple simulation so as to find out what is the voting strategy under unanimous voting rule when interval ambiguous information is provided instead of two-point ambiguous information, that is when signal precision is at least 0.6, but at most 0.9.

Table 5 shows that if the signal precision is amplified from its initial value 0.6, the threshold level of q for voting informatively under unanimity voting is increased. Whereas, if the signal precision is undermined from its initial value 0.9, the threshold level of q under the unanimity rule for voting informatively is decreased. Conversely for the majority voting rule. Because in the experiments we conducted we set the $q = 0.5$, which is very low, we do not observe the switch of voting strategy.

However, the ambiguous information not only affects the threshold level of q, it also affects the symmetric responsive voting strategy $\sigma(i)$. From Table 6, we do observe the dramatic decrease of type I error for unanimous voting rule when the information is amplified from 0.6, which is caused by the decrease in $\sigma(i)$. This suggests that if q is set fairly low, we should amplify the information precision from its initial level as it will lower the probability of voting against the received private signals.

Table 4: 12-Person Jury Case under Different Information Structures, given $q = 0.9$

Signal Precision	Voting Rule	Threshold for $\sigma(i) = 1$	Threshold for $\sigma(i) = 0$	Informative Voting	$Pr(C\|I)$	$Pr(A\|G)$
$p = 0.8$	$\hat{k} = 12$	0.2	0.99	No	0.0069	0.6540
	$\hat{k} = 8$	0.00098	0.94	No	0.0011	0.0666
	$\hat{k} = 7$	0.00025	0.5	Yes	0.0039	0.0194
$p = [0.55, 0.8]$	$\hat{k} = 12$	0.36	0.894	Yes	0.0000	0.9313
	$\hat{k} = 8$	0.218	0.701	Yes	0.0006	0.0726
	$\hat{k} = 7$	0.188	0.5	Yes	0.0039	0.0194

Table 5: Group Decision under Different Information Structures, given $n = 5$, $q = 0.5$

Signal Precision	Voting Rule	Threshold for $\sigma(i) = 0$	Informative Voting
$p = 0.6$	$\hat{k} = 5$	0.7714	No
	$\hat{k} = 3$	0.4	Yes
$p = 0.9$	$\hat{k} = 5$	0.9986	No
	$\hat{k} = 3$	0.1	Yes
$p = [0.6, 0.9]$	$\hat{k} = 5$	0.8137	No
	$\hat{k} = 3$	0.3077	Yes

Table 6: Voting Strategies and Resulted Errors across Different Information Structures, given $n = 5$, $q = 0.5$

$n = 5$	$p = 0.6$	$p = 0.9$	$p = \{0.6, 0.9\}$		$p = (0.6, 0.9)$	
$\hat{k} = 5$			True $p = 0.6$	True $p = 0.9$	True $p = 0.6$	True $p = 0.9$
$\sigma(i)$	0.5959	0.4982	0.5618	0.5618	0.5248	0.5248
$Pr(C\|I)$	0.25	0.0496	0.2176	0.0815	0.1867	0.0614
$Pr(A\|G)$	0.59	0.2227	0.6185	0.2007	0.6515	0.2161
$\hat{k} = 3$			True $p = 0.6$	True $p = 0.9$	True $p = 0.6$	True $p = 0.9$
$\sigma(i)$	0	0	0	0	0	0
$Pr(C\|I)$	0.32	0.0086	0.32	0.0086	0.32	0.0086
$Pr(A\|G)$	0.32	0.0086	0.32	0.0086	0.32	0.0086

4 Conclusion

We explored the effect of ambiguity in collective decision-making, using a jury voting model à la Feddersen and Pesendorfer (1998) in which the reliability of the private information voters possess becomes ambiguous, and by allowing for ambiguity-averse voters who are MaxMin Expected Utility maximizers à la Gilboa and Schmeidler (1989) when forming subjective beliefs prior to casting their votes.

In this new environment, we chose the Full-Bayesian updating rule to update the conditional probability of the defendant being guilty conditional on the pivotal voter getting a certain signal. Pires (2002) provides a decision-theoretic axiomatization of this updating rule. Also, Eichberger et al. (2007) provides an axiomatic proof for updating non-additive capacities by using the Full-Bayesian Updating rule. The reason for abandoning the standard Bayesian Updating rule, $Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$, is that the implicit assumption it requires, namely that both $A \cap B$ and B are measurable sets, is violated when ambiguity in those probability measures is introduced. Put differently, under ambiguity, there is no sufficient information for decision-makers to assign a precise probability for all relevant events, Therefore, the conditional probability $Pr(A|B)$ is not well-defined, making the standard Bayesian Updating rule not a proper updating rule to use under ambiguity.

In this ambiguous environment, and when adhering to unanimity voting, we proved the existence of both an informative voting equilibrium and of strategic voting equilibria. Unanimity voting leads to informative voting in cases in which strategic voting with randomization would have otherwise occurred in the absence of ambiguity.

This has allowed us to show instances in which unanimity voting outperforms non-unanimous voting and, therefore, to provide us with alternative ways to improve upon the type I errors induced by the adoption of the unanimity rule. This can be achieved by selecting the appropriate width of the interval within which the ambiguity lies, for any combinations of the given level of the threshold of reasonable doubt and jury size. For example, we found that if we can undermine the information precision from its initial level, we can allow for a wider range of the reasonable doubt for informative voting to prevail as an equilibrium. In some alternative collective decision-making scenarios, we might instead be restricted to satisfying a very low level of the reasonable doubt for a collective decision to be reached in a specific direction. In those scenarios, if unanimity voting were to be chosen, we would need to amplify the information precision from its initial level to induce the lowering of the probability of voting against the private signal, that is, to decrease the occurrence of type I errors.

The intuition for these results lies in the observation that when information becomes ambiguous, voters are more reluctant to rely on the collective information of others but their own, changing their 'perceived' pivotality, hence their optimal strategies as to whether to vote informatively or against their private information when their votes contribute to the final collective decision. Under unanimity, this helps restore incentives for voters to vote informatively in equilibrium, in spite of, and in accordance with, each of them voting strategically.

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A Proof of Proposition 3

Proof. Given $\sigma(g) = 1$, we have

$$\beta_G^i(\pi, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{\pi}{1-\pi})(\frac{\pi\sigma(i)+(1-\pi)}{\pi+(1-\pi)\sigma(i)})^{\hat{k}-1}(\frac{\pi}{1-\pi})^{n-\hat{k}}} = \frac{1}{1 + (\frac{\pi}{1-\pi})^{n-\hat{k}+1}(\frac{\pi\sigma(i)+(1-\pi)}{\pi+(1-\pi)\sigma(i)})^{\hat{k}-1}}.$$

Because $\beta^i_G(\pi, \sigma(i), \hat{k})$ is continuous at $\sigma(i) = 1$, and then,

$$\lim_{\sigma(i)\to 1^-} \beta^i_G(\pi, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{\pi}{1-\pi})^{n-\hat{k}+1}}.$$

Hence,

$$\lim_{\sigma(i)\to 1^-} \bar{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{\pi^*}{1-\pi^*})^{n-\hat{k}+1}},$$

with $\pi^* = \underline{p}$, and

$$\lim_{\sigma(i)\to 1^-} \underline{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{\pi^*}{1-\pi^*})^{n-\hat{k}+1}},$$

with $\pi^* = \bar{p}$. Therefore,

$$\lim_{\sigma(i)\to 1^-} \frac{\bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1 - \underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = \frac{\lim_{\sigma(i)\to 1^-} \bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1 - \lim_{\sigma(i)\to 1^-} \underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = \frac{\frac{1}{1+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1}}}{1 - \frac{1}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}}. \tag{12}$$

And $\beta^i_G(\pi, \sigma(i), \hat{k})$ is continuous at $\sigma(i) = 0$, then, we have

$$\lim_{\sigma(i)\to 0^+} \beta^i_G(\pi, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{1-\pi}{\pi})^{2\hat{k}-n-2}}.$$

We can see that the monotonicity of $\lim_{\sigma(i)\to 0^+} \beta^i_G(\pi, \sigma(i), \hat{k})$ depends on $\hat{k}$. When $\hat{k} > \frac{n+2}{2}$, $\lim_{\sigma(i)\to 0^+} \beta^i_G(\pi, \sigma(i)v)$ is an increasing function of π. When $\frac{n+1}{2} < \hat{k} \leq \frac{n+2}{2}$, $\lim_{\sigma(i)\to 0^+} \beta^i_G(\pi, \sigma(i), \hat{k})$ is an increasing function of π.

If $\hat{k} > \frac{n+2}{2}$,

$$\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}},$$

with $\pi^* = \bar{p}$, and

$$\lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}},$$

with $\pi^* = \underline{p}$. Therefore,

$$\lim_{\sigma(i)\to 0^+} \frac{\bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1 - \underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = \frac{\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1 - \lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = \frac{\frac{1}{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}}}{1 - \frac{1}{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}}}. \tag{13}$$

Therefore, whenever $\frac{q}{1-q}$ is strictly between the values identified in equations (12) and (13), there exists $\sigma^*(i) \in (0, 1)$ such that $\frac{\bar{\beta}^i_G(\pi, \sigma^*(i), \hat{k})}{1-\underline{\beta}^i_G(\pi, \sigma^*(i), \hat{k})} = \frac{q}{1-q}$.

This is equivalent to requiring that conditions (14) and (15) below are satisfied at the same time:

$$q > \frac{1 + (\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1 + (\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2 + (\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})}; \tag{14}$$

and

$$q < \frac{1 + (\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}}{1 + (\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}(2 + (\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2})}. \tag{15}$$

Thus, if $\hat{k} > \frac{n+2}{2}$, as long as $\frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})} < q < \frac{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}}{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}(2+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2})}$, we have $0 < \sigma(i) < 1$ as the equilibrium strategy, which indicates that voters are indifferent to vote for convicting and acquitting when receiving signal i.

If $0 < \hat{k} < \frac{n+2}{2}$,

$$\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}},$$

with $\pi^* = \underline{p}$, and

$$\lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}},$$

with $\pi^* = \bar{p}$. Therefore,

$$\lim_{\sigma(i)\to 0^+} \frac{\bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1 - \underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = \frac{\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1 - \lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = \frac{\frac{1}{1+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2}}}{1 - \frac{1}{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}}}. \tag{16}$$

Therefore, whenever $\frac{q}{1-q}$ is strictly between the limit identified in equation (12) and the one identified in equation (16), there exists $\sigma^*(i) \in (0,1)$ such that $\frac{\bar{\beta}^i_G(\pi,\sigma^*(i),\hat{k})}{1-\underline{\beta}^i_G(\pi,\sigma^*(i),\hat{k})} = \frac{q}{1-q}$. Notice that this condition can be broken down into two conditions, as follows:

$$q > \frac{1 + (\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1 + (\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2 + (\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})}; \tag{17}$$

and

$$q < \frac{1 + (\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}}{1 + (\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}(2 + (\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2})}. \tag{18}$$

If $\hat{k} = \frac{n+2}{2}$, we have

$$\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1 + (\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}} = \frac{1}{2},$$

and

$$\lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi^*, \sigma(i), \hat{k}) = \frac{1}{1+(\frac{1-\pi^*}{\pi^*})^{2\hat{k}-n-2}} = \frac{1}{2}.$$

Therefore,

$$\lim_{\sigma(i)\to 0^+} \frac{\bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1-\underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = \frac{\lim_{\sigma(i)\to 0^+} \bar{\beta}^i_G(\pi, \sigma(i), \hat{k})}{1-\lim_{\sigma(i)\to 0^+} \underline{\beta}^i_G(\pi, \sigma(i), \hat{k})} = 1. \tag{19}$$

And, we know that when $\hat{k} \leq \frac{n+2}{2}$, the limit value identified by condition (16) is larger than 1. Therefore, whenever $\frac{q}{1-q}$ is strictly between the limit identified in equation (12) and the one identified in equation (16), there exists $0 < \sigma(i) < 1$ as the symmetric responsive equilibrium strategy, that is, $\frac{n+1}{2} < \hat{k} \leq \frac{n+2}{2}$, as long as $\frac{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\hat{k}+1}(2+(\frac{\underline{p}}{1-\underline{p}})^{n-\hat{k}+1})} < q < \frac{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}}{1+(\frac{1-\bar{p}}{\bar{p}})^{2\hat{k}-n-2}(2+(\frac{1-\underline{p}}{\underline{p}})^{2\hat{k}-n-2})}$.

■