Debt management reverses the treand of fertility decline

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Abstract

By introducing a debt management rule into an extended model of Lapan and Enders (1990), we analyze a dynamic relationship among public debt, fertility, and per capita income growth. The rule proposed is simple and intuitive: public debt newly issued is an increasing function of the government's primary balance. We show that the public debt-GDP ratio decreases monotonically toward zero, and both fertility rate and per capita income growth rate continue to increase in transition, if an initially indebted government adopts a tight management rule. Unfortunately, the long-run growth rate is low and the long-run fertility rate is high relative to the social optimal because of capital externality and a trade-off between fertility and bequests. The optimal policy requires an additional policy instrument, that is, a subsidy for bequests.

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1 Introduction

Public debt management is one of the most important policy issues in developed countries. Figure 1 illustrates the relationship between the public debt GDP ratio and the growth rate of per capita income from 1965 to 2010 in Japan. At a glance, the growth rate is negatively related to public debt outstanding. Figure 2 illustrates the relationship between the public debt GDP ratio and the fertility rate in the same period. We can observe that the fertility rate is also negatively related to public debt outstanding[1]. A proper public debt management is necessary not only for the sustainability of public finance but also for the sustainability of population.

[Figure 1 and 2 are here]

In this paper, we analyze how a public debt management rule affects the growth rate of per capita income and the fertility rate in an extended model of Lapan and Enders (1990). As for public debt management, we employ the Bohn's rule such as

$$\frac{T_t - G_t}{Y_t} = \phi + \beta \frac{D_{t+1} - D_t}{Y_t}$$

where T, G, Y, D stand for tax revenue, public expenditure, public debt, and GDP, respectively (Bohn (1998), Bohn and Inman (1996), Greiner and Semmler (2000), Greiner (2008)). It is well known that the sustainability of public finance needs $\beta > 0$, that is, the government's primary balance is an increasing function of the public debt newly issued.

Contributions of this paper are twofold. First, we derive conditions not only for the sustainability of public debt but also for the optimality of public debt. While many researchers analyze the sustainability of public debt (Bräuninger (2005), Yakita (2008), Futagami et al. (2008), Minea and Villieu (2013), and Kondo (2012) among others), the optimal debt management rule has been ignored.

Second, we derive a condition which clarifies the relationship between the Bohn's rule and the fertility rate. Lapan and Enders (1990) shows that an increase in public debt outstanding decreases fertility because, other things being equal, this change increases the price of children[2]. Although this mechanism is essential to our result, the fertility effect of policy restriction such as the Bohn's rule presented in our paper is a new finding.

The construction of the paper is as follows. In Section 2, we introduce a basic model. At first, we derive a temporary equilibrium in which public debt outstanding is given. Then, we analyze the dynamics of growth rate and fertility rate by tracing the path of public debt under a public debt management rule. In Section 3, we derive an optimal public debt policy. The final section concludes the paper.

[1] 1966 is the 43rd year of the sexagenary cycle, which is named the Fire Horse. Japanese people do not want to have children based on the superstition that women born in this year are destined to be unhappy.

[2] Wildasin (1990) also obtained similar results. Zhang (1997) analyzed a dymanic interaction among education, fertility, and growth rate when the public debt income ratio changes.

2 The model

2.1 Structure of the model

We extend the model of Lapan and Enders (1990) to include endogenous growth motivated by Arrow (1962) and formulated by Romer (1990) and Grossman and Yanagawa (1992).

It would be useful to summarize the model at first.

The population of the economy is given by

$$\frac{N_{t+1}}{N_t} = n_t$$

where N_t and n_t stand for a population size and a fertility rate at period t, respectively.

Individuals are born in each period and live for one period. The utility function of an individual born at period t is given by

$$W_t = u(c_t, n_t) + \delta W^*_{t+1}$$

where c_t stands for consumption. $0 < \delta < 1$ stands for a preference parameter attached to his representative child's welfare.

The household budget constraint is given by

$$(1 - \tau_t)(R_t a_t + w_t) = c_t + n_t(\phi_t + a_{t+1})$$

He supplies one unit of labor to earn wage income, w_t. Also, he receives a bequest, $R_t a_t$, which implies that his parent's bequest at period $t-1$, a_t, earns a gross rate of interest, R_t, in the capital market. The full income is taxed at a rate of $0 \leq \tau_t < 1$. The disposable income is allocated to consumption, c_t, a cost of rearing children, $n_t\phi_t$, and a bequest to his children, $n_t a_{t+1}$. That is, ϕ_t is a rearing cost per child, and a_{t+1} is a bequest per child.

The consumption good is produced by capital and labor. The technology is represented by a constant-returns-to-scale production function such as

$$Y_t = F(K_t, B_t L_t)$$

where Y_t, K_t, L_t stand for output, capital, and working time, respectively. The effective labor is given by $B_t L_t$ where $B_t > 0$ stands for a labor augmenting technology.

Assuming competitive factor markets, the wage rate and the interest rate are respectively given by

$$\begin{aligned} w_t &= \frac{\partial Y_t}{\partial L_t} \\ R_t &= \frac{\partial Y_t}{\partial K_t} \end{aligned}$$

The government budget constraint is given by

$$T_t + D_{t+1} = G_t + R_t D_t$$

where $T_t = N_t \tau_t (R_t a_t + w_t)$ is a total amount of tax revenue, G_t is a public expenditure, and D_t is a public debt outstanding at period t. The government redeems the debt for one period.

The market clearing conditions for labor, capital, and good are respectively given by

$$\begin{aligned} N_t &= L_t \\ K_t + D_t &= N_t a_t \\ Y_t &= N_t c_t + N_{t+1}\phi_t + G_t + K_{t+1} \end{aligned}$$

In this paper, we employ the following specifications to make the model tractable:

$$\begin{aligned} u_t &= (1-\beta)\ln c_t + \beta \ln n_t \\ F(K_t, B_t L_t) &= A K_t^{\alpha} (B_t L_t)^{1-\alpha} \\ B_t &= \frac{K_t}{L_t} \\ \phi_t &= \phi w_t \end{aligned}$$

The utility function is a log-linear type, in which the preferences for consumption and the number of children are weighted by $1-\beta$ and β, respectively. The production function is a Cobb-Douglas type, in which the income share of capital is $0 < \alpha < 1$. $A > 0$ stands for a time invariant total factor productivity. The labor augmenting technology at period t equals to the capital-labor ratio at period t. The cost per child is measured by a share of wage income, $\phi > 0$.

Finally, we assume

$$\beta > \delta$$

which implies that the preference for the number of children is greater than the preference for the welfare of a representative child. This assumption is necessary for the existence of the socially optimal solution (See Appendix).

2.2 Fiscal rules

We introduce into the model two fiscal rules such as

$$\begin{aligned} D_{t+1} - D_t &= \theta(T_t - G_t) \quad (\theta > -R^{-1}) && (1) \\ G_t &= g Y_t \quad (0 < g < 1) && (2) \end{aligned}$$

Equation (1) states that an additional public debt issued in period t is a linear function of the government's primary balance in period t. $\theta > -R^{-1}$ ensures that the low of motion of public debt is monotonic ($D_{t+1}/D_t > 0$). Equation (2) states that the share of government expenditure is constant over time.

The fiscal rule (θ, g) in equations (1) and (2) is of great advantage to interpret policy implications. First, equation (1) and the government budget constraint give

$$\frac{D_{t+1}}{D_t} = Rd(\theta) \tag{3}$$

where

$$d(\theta) = \frac{1+\theta R}{(1+\theta)R} \tag{4}$$

Equation (3) implies the growth rate of public debt is fully controlled by θ.[3] Assuming that $R = \alpha A > 1$, we know from equation (4) that $0 < d(\theta) < 1$and $d'(\theta) > 0$ for all $\theta > -R^{-1}$. The larger θ is, the higher is the debt growth.

Second, equations (2), (3), and the government budget constraint give

$$\tau_t = \frac{g + [1 - d(\theta)]\frac{RD_t}{Y_t}}{1 + \frac{RD_t}{Y_t}} \tag{5}$$

where we have used $T_t = \tau_t(Y_t + R_t D_t)$. Equation (5) is a tax rate which balances the government budget in each period. Without public debt, a simple rule applies, that is, $\tau_t = g$. If $D_t > 0$, then a slack debt management (a large θ) decreases the tax rate. However, the effect of public debt outstanding on the tax rate depends on θ. From equation (5), we get

$$\tau_t - g = [1 - g - d(\theta)]\frac{\frac{RD_t}{Y_t}}{1 + \frac{RD_t}{Y_t}}$$

We know, given that $D_t > 0$, $\tau_t \gtreqless g$ and $\partial\tau_t/\partial D_t \gtreqless 0$ if $d(\theta) \lesseqgtr 1 - g$. The case-by-case effect of θ on the relationship between τ_t and D_t is attributed to the debt management rule (3). If $d(\theta) < 1 - g$, then equation (3) limits the upper bound of debt issue, which requires additional tax revenue to finance interest payments. In this case, the tax rate is positively related to the public debt outstanding. On the other hand, if $d(\theta) > 1 - g$, the tax rate is lower than the public expenditure-GDP ratio, which implies debt financing is dominant. In this case, the tax rate is negatively related to the public debt outstanding because a large amount of debt outstanding allows the government to issue new debt, by equation (3). Of course it is another question whether the slack debt management rule is sustainable or not. We will answer the question after deriving the equilibrium and dynamics of capital and public debt.

2.3 Fertility and bequests

To derive the equilibrium, we solve for the household maximization problem. This problem is formulated as

$$\max \sum_{i=0}^{\infty} \delta^i [(1-\beta)\ln c_{t+i} + \beta \ln n_{t+i}]$$

subject to the budget constraints,

$$(1 - \tau_{t+i})(R_{t+i}a_{t+i} + w_{t+i}) - c_{t+i} - n_{t+i}(\phi_{t+i} + a_{t+i+1}) = 0 \quad (i = 0, 1, 2, ...)$$

[3] The primary balance (PB) is also fully controlled by θ,

$$T_t - G_t = \frac{R-1}{1+\theta} D_t$$

If the government is a debtor (creditor), then PB is positive (negative). The absolute value of PB in period t is decreasing in θ, taking the interest payment in period t, $(R-1)D_t$ as given.

The first-order conditions require

$$\begin{aligned}
\frac{1-\beta}{c_t} - \lambda_t &= 0 \\
\frac{\beta}{n_t} - \lambda_t(\phi_t + a_{t+1}) &= 0 \\
-\lambda_t n_t + \delta\lambda_{t+1}(1-\tau_{t+1})R_{t+1} &= 0 \\
(1-\tau_t)(R_t a_t + w_t) - c_t - n_t(\phi_t + a_{t+1}) &= 0
\end{aligned}$$

where λ_t stands for a multiplier attached to the budget constraint in period t.

The first equation states that marginal benefit of consumption equates to the marginal cost. The second equation states that the benefit of having one more child equates to the cost, that is, the sum of rearing costs and bequest per child. The third equation stands for the optimality condition for bequests. Because one unit of increase in bequest needs n_t unit of resources, it costs by $\lambda_t n_t$ in utility terms. This bequest improves welfare of a representative child by $\lambda_{t+1}(1-\tau_{t+1})R_{t+1}$ in utility terms, which also benefits the parent by discounting it by δ. The last equation is the budget constraint in period t.

Solving them, the fertility rate and the bequest per child are respectively given by

$$n_t = \frac{(\beta-\delta)(1-\tau_t)(R_t a_t + w_t)}{\phi_t - \frac{w_{t+1}}{R_{t+1}}} \tag{6}$$

$$a_{t+1} = \frac{\delta}{\beta-\delta}\phi_t - \frac{\beta}{\beta-\delta}\frac{w_{t+1}}{R_{t+1}} \tag{7}$$

There exists a trade-off between fertility and per capita bequests in this model. Equation (7) implies that wage growth decreases bequest, which depresses capital accumulation. The reason is that a parent who wants to transfer income to his representative child decreases bequest if the child becomes rich. Accordingly, the fertility increases because the price of child falls, which is represented in the denominator in equation (6).

2.4 Temporary equilibrium

In this section, we derive a temporary equilibrium in which both the growth rate and the fertility rate are determined.

First, let us define the ratio of interest payment of public debt to GDP by

$$x_t = \frac{RD_t}{Y_t}$$

In our model, x_t is a unique state variable. Because $x_t = \alpha D_t/K_t$, the initial condition $x_0 = \alpha D_0/K_0$ is predetermined.

From equation (5) and $R_t a_t + w_t = (1 + x_t)Y_t/N_t$, the household income is given by

$$(1-\tau_t)(R_t a_t + w_t) = (1 - g + d(\theta)x_t)\frac{Y_t}{N_t}$$

The household income increases with x_t because a large x_t implies that the government issues a large amount of public debt, which decreases the tax rate.

The present value of the next period wage income is given by

$$\frac{w_{t+1}}{R_{t+1}} = \frac{(1-\alpha)Y_{t+1}}{RN_{t+1}} = \frac{(1-\alpha)Y_{t+1}}{Rn_tN_t}$$

Substituting them into equation (6), and using $\phi_t = \phi w_t = \phi(1-\alpha)Y_t/N_t$, we get the fertility rate such as

$$n_t = \frac{1}{\phi(1-\alpha)}\left[(\beta-\delta)(1-g+d(\theta)x_t) + \frac{1-\alpha}{R}\frac{Y_{t+1}}{Y_t}\right] \tag{8}$$

The fertility rate is positively related to both the public debt ratio, x_t, and the growth rate, Y_{t+1}/Y_t. The positive effect of x_t is the income effect because household income increases with x_t. The positive effect of Y_{t+1}/Y_t is accounted for by the trade-off between fertility and per capita bequests. Economic growth improves welfare of the next generation, which decreases intergenerational transfers motivated by altruism. Then, it reduces a perceived price of children, which makes fertility to increase.

Given x_t, equation (8) shows a linear relationship between the rates of growth and fertility. We call this line by H-line because this relationship arises from household decision making about fertility.

Next, we examine the capital market clearing condition. From equation (7), the capital supply is given by

$$N_{t+1}a_{t+1} = \frac{\delta}{\beta-\delta}\phi(1-\alpha)Y_tn_t - \frac{\beta}{\beta-\delta}\frac{1-\alpha}{R}Y_{t+1}$$

Using equation (3) and $RK_{t+1} = \alpha Y_{t+1}$, the capital demand is given by

$$K_{t+1} + D_{t+1} = \frac{\alpha}{R}Y_{t+1} + d(\theta)x_tY_t$$

Then, the capital market clearing condition gives

$$n_t = \frac{1}{\delta\phi(1-\alpha)}\left[(\beta-\delta)d(\theta)x_t + \frac{\beta-\delta\alpha}{R}\frac{Y_{t+1}}{Y_t}\right] \tag{9}$$

Given x_t, equation (9) shows a linear relationship between the rates of growth and fertility. A high growth rate is associated with a high fertility rate because increased capital supply encourages economic growth. We call this line by K-line because this relationship arises from the capital market clearing condition.

[Figure 3 is here]

Figure 3 illustrates a temporary equilibrium in period t. K-line is steeper than H-line because $\beta > \delta$. Therefore, the intercept of K-line should be smaller than that of H-line in order that the growth rate is positive. This requires

$$x_t < \frac{\delta(1-g)}{(1-\delta)d(\theta)} \tag{10}$$

If equation (10) is satisfied, then the growth rate and the fertility rate are respectively given by

$$\frac{Y_{t+1}}{Y_t} = R\left[\delta(1-g) - (1-\delta)d(\theta)x_t\right] \tag{11}$$

$$n_t = \frac{(\beta-\delta\alpha)(1-g) + [\beta-\delta-(1-\alpha)(1-\delta)]d(\theta)x_t}{\phi(1-\alpha)} \tag{12}$$

The growth rate in equation (11) is decreasing in x_t. Taking Y_t as given, a large x_t implies a large amount of public debt outstanding in period t. Then, the debt management rule (3) allows the government to issue a large amount of public debt in period t, which crowds out private capital in period $t+1$.

Regarding the fertility rate in equation (12), we assume

$$\beta - \delta - (1-\alpha)(1-\delta) < 0 \tag{13}$$

Equation (13) implies the positive income effect of x_t on fertility is dominated by the negative growth effect. In equation (8), one unit of increase in x_t has an income effect on fertility by $(\beta-\delta)d(\theta)/[\phi(1-\alpha)]$. On the other hand, this change has a negative growth effect by $R(1-\delta)d(\theta)$ in equation (11). Then, the decreased growth rate decreases fertility by $(1-\delta)d(\theta)/\phi$ in equation (8). Equation (13) states that the latter negative effect on fertility is larger than the former positive effect.

This assumption is not so restrictive. Appendix shows that equation (13) implies that the share of child-rearing cost among resources allocated to consumption is smaller than the share of labor income at the social optimum[4].

We use a growth rate of per capita income as a welfare index. From equations (11) and (12), we get the growth rate of per capita income, $y_t = Y_t/N_t$, such as

$$\frac{y_{t+1}}{y_t} = \frac{R\phi(1-\alpha)\left[\delta(1-g) - (1-\delta)d(\theta)x_t\right]}{(\beta-\delta\alpha)(1-g) + [\beta-\delta-(1-\alpha)(1-\delta)]d(\theta)x_t} \tag{14}$$

Specifically, the following proposition summarizes the results.

Proposition 1 *Given x_t, the growth rate, the fertility rate, and the growth rate of per capita income are given by equations (11), (12), and (14), respectively. With assumption (13), all the rates are decreasing in x_t.*

Proof. From equation (11), we get $\partial(Y_{t+1}/Y_t)/\partial x_t = -R(1-\delta)d(\theta) < 0$. From equation (12), we get $\partial n_t/\partial x_t = [\beta-\delta-(1-\alpha)(1-\delta)]d(\theta)/[\phi(1-\alpha)] < 0$ with assumption (13). From equation (14), we get

$$\frac{\partial}{\partial x_t}\left(\frac{y_{t+1}}{y_t}\right) = \frac{-R\phi(1-\alpha)(\beta-\delta)(1-g)d(\theta)}{\{(\beta-\delta\alpha)(1-g) + [\beta-\delta-(1-\alpha)(1-\delta)]d(\theta)x_t\}^2} < 0$$

because $\beta > \delta$. ■

[4] Equation (13) is rewritten as

$$\frac{\beta-\delta}{(1-\beta)+(\beta-\delta)} < 1-\alpha$$

Appendix shows the left-hand side stands for an optimal share of child-rearing cost among resouces allocated to consumption, $N_{t+1}\phi_t/(N_t c_t + N_{t+1}\phi_t)$.

2.5 Dynamics and long-run equilibrium

In this section, we examine the dynamics of x_t. From equations (3) and (11), we get a first-order difference equation of x_t such as

$$x_{t+1} = \frac{d(\theta)x_t}{\delta(1-g) - (1-\delta)d(\theta)x_t} \equiv \Phi(x_t;\theta) \tag{15}$$

The denominator in equation (15) is positive from equation (10). We need $x_t = \alpha D_t/K_t > -\alpha$ for $\forall t \geq 1$ in order to ensure bequests are positive, $a_{t+1} > 0$. Therefore, the initial condition requires $\Phi(x_0;\theta) > -\alpha$, that is,

$$x_0 > -\frac{\alpha\delta(1-g)}{(1-\alpha+\alpha\delta)d(\theta)} \equiv x^- \tag{16}$$

Equation (15) has two steady states. One is $x_t = 0$ and another is

$$x^* = \frac{1}{1-\delta}\left[\frac{\delta(1-g)}{d(\theta)} - 1\right] \tag{17}$$

From equation (17), we know

$$x^* \gtreqless 0 \Leftrightarrow d(\theta) \lesseqgtr \delta(1-g) \Leftrightarrow \theta \lesseqgtr \hat{\theta}$$

where

$$\hat{\theta} = \frac{\delta(1-g) - R^{-1}}{1-\delta(1-g)} \tag{18}$$

Because the sign of x^* is critical for the dynamics of x_t, we examine two regimes separately.

[Figure 4 and 5 are here]

First, assume that the government adopts a slack debt management rule, $\theta > \hat{\theta}$. Figure 4 illustrates a dynamics of x_t. Under a slack debt management rule, the pubic debt ratio converges to $x^* < 0$ if and only if $x_0 \in (x^-, 0)$, that is, the government is a creditor initially. If the government is initially indebted, then the ratio continues to increase. In this case, both the fertility rate and the growth rate of per capita income continue to decrease from Proposition 1. Under a slack debt management rule, public debt is not sustainable if the government is initially indebted.

Second, assume that the government adopts a tight debt management rule, $\theta < \hat{\theta}$. Figure 5 shows that the public debt ratio converges to zero if and only if $x_0 \in (x^-, x^*)$. Even if the government is initially indebted, public debt could be sustainable. A tighter rule is needed if the government bears a large amount of public debt outstanding because equation (17) states the upper limit x^* is decreasing in θ. From Proposition 1, we conclude that both the fertility rate and the growth rate of per capita income continue to increase in transition under a tight debt management rule.

The following proposition summarizes the result.

Proposition 2 *Assume that the government is initially indebted. Under a slack debt management rule ($\theta > \hat{\theta}$), the public debt is not sustainable. Both rates of fertility and per capita income growth continue to decrease. Under a tight debt management rule ($\theta < \hat{\theta}$), the public debt ratio converges to zero. Both rates of fertility and per capita income growth continue to increase in transition.*

Proposition 2 suggests that a set of observations that both fertility and growth rates fall, and public debts continue to increase is attributed to a slack debt management. If it is the case, then the regime switch is needed not only to make public debt sustainable but also to raise both rates of fertility and per capita income growth.

At the end of this section, we briefly summarize the optimality of the long-run equilibrium. The growth rate of per capita income would be lower than the optimal one because individuals underestimate private capital because of the externality. In turn, the fertility rate would be higher than the optimal because a perceived cost of children is also underestimated. Specifically, we have the following proposition.

Proposition 3 *(Optimality) Under a tight debt management rule, the long-run growth rate is lower than the optimal growth rate, and the long-run fertility rate is higher than the optimal. The growth rate would be optimal if the government were a creditor,*

$$x_\infty = -\frac{(1-\alpha)\delta(1-g)}{\alpha(1-\delta)d(\theta)} < 0 \tag{19}$$

The fertility rate would be optimal if the government were a debtor,

$$x_\infty = \frac{(1-\alpha)\delta(1-g)}{[(1-\alpha)(1-\delta)-(\beta-\delta)]d(\theta)} > 0 \tag{20}$$

Unfortunately, each case cannot be achieved by the tight debt management rule.

Proof. Appendix shows that the optimal growth rate and the optimal fertility rate are respectively given by

$$\begin{aligned} \left(\frac{Y_{t+1}}{Y_t}\right)^* &= \delta(1-g)A \\ n^* &= \frac{(\beta-\delta)(1-g)}{\phi(1-\alpha)} \end{aligned}$$

It can be easily shown that the growth rate in equation (11) is optimal if equation (19) is satisfied, and that the fertility rate in equation (12) is optimal if equation (20) is satisfied. However, both cases are not satisfied because $x_\infty = 0$ under the tight debt management rule. ■

If the initial government is a creditor, the long-run growth rate could be optimal under a slack debt management rule, although this case is less realistic. The long-run fertility rate is higher than the optimal because x^* is negative. Specifically, the following remark summarizes the result.

Remark 4 *Assume that (i) the initial government is a creditor ($x_0 \in (x^-, 0)$), and (ii) $\delta(1-g) < \alpha$. Then, under a slack debt management rule,*

$$d(\theta) = \frac{\delta}{\alpha}(1-g) \tag{21}$$

the long-run growth rate coincides with the optimal growth rate.

Proof. In the long-run, x_t converges to $x^* < 0$ because $\delta(1-g) < d(\theta) < 1$. Because the equilibrium stays at a balanced growth path, the growth rate is equal to $D_{t+1}/D_t = Rd(\theta)$. Using equation (21) and $R = \alpha A$, we can verify the growth rate coincides with the optimal growth rate. ■

3 Optimal policy

In this section, we consider not only the sustainability of public debt but also the optimality. Because the basic model contains capital externality, private capital is underestimated in a decentralized economy. In order to correct the inefficiency, an additional policy instrument is needed. We employ a Pigouvian subsidy for bequest received. Let us denote the subsidy rate by τ_b.[5] The household budget constraint in period t is modified as

$$(1+\tau_b)R_t a_t + w_t - \bar{\tau}_t = c_t + n_t(\phi_t + a_{t+1})$$

where $\bar{\tau}_t$ stands for a lump-sum tax, which is endogenously determined to balance the government budget.

Solving the household maximization problem, the fertility rate and the bequest per child are respectively given by

$$n_t = \frac{(\beta-\delta)[(1+\tau_b)R_t a_t + w_t - \bar{\tau}_t]}{\phi_t - \frac{w_{t+1}-\bar{\tau}_{t+1}}{(1+\tau_b)R_{t+1}}} \tag{22}$$

$$a_{t+1} = \frac{\delta}{\beta-\delta}\phi_t - \frac{\beta}{\beta-\delta}\frac{w_{t+1}-\bar{\tau}_{t+1}}{(1+\tau_b)R_{t+1}} \tag{23}$$

The government budget constraint is modified as

$$T_t + D_{t+1} = S_t + G_t + R_t D_t \tag{24}$$

where $T_t = N_t\bar{\tau}_t$ stands for a total amount of tax revenue, and $S_t = N_t\tau_b R_t a_t$ stands for a total amount of bequest subsidy.

Because the government's primary balance in period t is given by $T_t - S_t - G_t$, the debt management rule is modified as

$$D_{t+1} - D_t = \theta(T_t - S_t - G_t) \tag{25}$$

From equations (24) and (25), we get

$$\frac{D_{t+1}}{D_t} = Rd(\theta), \quad d(\theta) = \frac{1+\theta R}{(1+\theta)R} \tag{26}$$

[5] We assume the subsidy rate is constant over time because the optimal rate is constant as seen below.

which is the same as equation (3).

Without loss of generality, we assume the lump-sum tax is measured by a share of wage income,

$$\bar{\tau}_t = \tau_t w_t \tag{27}$$

Substituting equations (26) and (27) into equation (24), and using $S_t = \tau_b R(K_t + D_t) = \tau_b(\alpha Y_t + RD_t)$ and $G_t = gY_t$, the tax rate is given by

$$\tau_t = \frac{g + \alpha\tau_b + (1 + \tau_b - d(\theta))x_t}{1 - \alpha} \tag{28}$$

where we have used $x_t = RD_t/Y_t$. If $\tau_b = -\tau_t$, then equation (28) becomes equation (5). Following the same procedure as the basic model, we get an optimal policy rule.

Proposition 5 *Assume that $\delta(1-g)A > 1$ and*

$$x_0 > -\frac{1-g}{d(\theta)}$$

Under a policy rule (θ^*, τ_b^*) such that

$$\tau_b^* = \frac{1-g}{\alpha} - 1 \tag{29}$$

and

$$d(\theta^*) < \frac{\delta}{\alpha}(1-g) \tag{30}$$

the public debt-GDP ratio converges to zero. If the government is initially indebted, then the growth rate decreases and the fertility rate increases in transition. Both rates coincides with the social optimal in the long-run.

If $\delta(1-g)/\alpha < 1$, then equation (30) gives an upper limit of θ^*,

$$\theta^* < \frac{\delta(1-g) - A^{-1}}{\alpha - \delta(1-g)}$$

If $\delta(1-g)/\alpha \geq 1$, then equation (30) is satisfied for any $\theta^* \geq -R^{-1}$.

Proof. See Appendix. ■

4 Concluding remarks

For two decades, Japan has suffered from a low rate of economic growth, a low rate of fertility, and a high rate of public debt accumulation. The history shows the related public policies were not effective. In this paper, we tried to find a solution to resolve the triple difficulty. Our result implies that introducing a tight debt management rule serves for the purpose. A proper public debt management increases both the growth rate of per capita income and the fertility rate, and achieves an optimal resource allocation in the long run. We believe our finding is meaningful not only for Japan but also for developed countries that suffer from the same difficulties as Japan.

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Appendix

[Social optimum]

The social optimum is the solution of

$$\max \sum_{i=0}^{\infty} \delta^i [(1-\beta)\ln c_{t+i} + \beta \ln n_{t+i}]$$

subject to the resource constraints,

$$(1-g)Ak_{t+i} = c_{t+i} + \phi_{t+i} n_{t+i} + n_{t+i} k_{t+i+1} \quad (i = 0, 1, 2, ...)$$

taking k_0 as given. Here, $k_t = K_t/N_t$ stands for per capita capital. The public expenditure-GDP ratio, g, is constant over time.

The optimality conditions require

$$\begin{aligned} \frac{1-\beta}{c_t} - \lambda_t &= 0 \\ \frac{\beta}{n_t} - \lambda_t(\phi_t + k_{t+1}) &= 0 \\ -\lambda_t n_t + \delta \lambda_{t+1}(1-g)A &= 0 \\ (1-g)Ak_t - c_t - \phi_t n_t - n_t k_{t+1} &= 0 \end{aligned}$$

where λ_t is a multiplier attached to the resource constraint in period t.

Assumption

$$\phi_t = \phi(1-\alpha)Ak_t$$

Solving them, we get

$$\begin{aligned} c_t &= (1-\beta)(1-g)Ak_t \\ n_t &= \frac{(\beta-\delta)(1-g)}{\phi(1-\alpha)} \\ k_{t+1} &= \frac{\delta}{\beta-\delta}\phi(1-\alpha)Ak_t \end{aligned}$$

The optimal growth rate of per capita income is given by

$$\frac{k_{t+1}}{k_t} = \frac{\delta}{\beta-\delta}\phi(1-\alpha)A$$

The optimal resource allocation is given by

$$\begin{aligned} \frac{N_t c_t}{Y_t} &= (1-\beta)(1-g) \\ \frac{N_{t+1}\phi_t}{Y_t} &= (\beta-\delta)(1-g) \\ \frac{K_{t+1}}{Y_t} &= \delta(1-g) \end{aligned}$$

[Proof of Proposition 5]

Using equation (28), the household income and a present value of the net wage income in the next period are respectively given by

$$\begin{aligned}(1+\tau_b)R_t a_t + w_t - \bar{\tau}_t &= \frac{Y_t}{N_t}(1-g+d(\theta)x_t) \\ \frac{w_{t+1}-\bar{\tau}_{t+1}}{(1+\tau_b)R_{t+1}} &= \frac{(1-\alpha)(1-\tau_{t+1})Y_{t+1}}{(1+\tau_b)RN_{t+1}}\end{aligned}$$

Substituting them into equation (22), the fertility rate is given by

$$n_t = \frac{1}{\phi(1-\alpha)}\left[(\beta-\delta)(1-g+d(\theta)x_t) + \frac{(1-\alpha)(1-\tau_{t+1})}{(1+\tau_b)R}\frac{Y_{t+1}}{Y_t}\right]$$

If $\tau_b = -\tau_{t+1}$, this equation is the same as equation (8). In this model, however, x_t has an additional negative effect on fertility. Equation (26) implies that a larger amount of public debt outstanding in period t induces a larger amount of public debt outstanding in period $t+1$. Then, equation (28) implies that the tax rate in period $t+1$ increases. Because the representative child's income decreases, parents want to increase bequests, which, in turn, decreases fertility.

Specifically, substituting equation (28) into the above equation, and using $x_{t+1}Y_{t+1}/Y_t = Rd(\theta)x_t$, we get

$$n_t = \frac{1}{\phi(1-\alpha)}\left[(\beta-\delta)(1-g+d(\theta)x_t) - \left(1-\frac{d(\theta)}{1+\tau_b}\right)d(\theta)x_t + \frac{1}{R}\left(\frac{1-g}{1+\tau_b}-\alpha\right)\frac{Y_{t+1}}{Y_t}\right] \tag{A1}$$

A second term in the square bracket in equation (A1) stands for the negative fertility effect as described above.

Next, let us turn to the capital market. From equation (23), aggregate capital supply is given by

$$N_{t+1}a_{t+1} = \frac{\delta}{\beta-\delta}\phi(1-\alpha)Y_t n_t - \frac{\beta}{\beta-\delta}\frac{(1-\alpha)(1-\tau_{t+1})}{(1+\tau_b)R}Y_{t+1}$$

Using aggregate capital demand such as

$$K_{t+1} + D_{t+1} = \frac{\alpha}{R}Y_{t+1} + d(\theta)RD_t$$

the market clearing condition gives

$$n_t = \frac{1}{\delta\phi(1-\alpha)}\left\{(\beta-\delta)d(\theta)x_t + \frac{1}{R}\left[(\beta-\delta)\alpha + \frac{\beta(1-\alpha)(1-\tau_{t+1})}{1+\tau_b}\right]\frac{Y_{t+1}}{Y_t}\right\}$$

Again, this equation is the same as equation (9) if $\tau_b = -\tau_{t+1}$. Using equation (28) and $x_{t+1}Y_{t+1}/Y_t = Rd(\theta)x_t$, this equation becomes

$$n_t = \frac{1}{\phi(1-\alpha)}\left[\left(\frac{\beta d(\theta)}{\delta(1+\tau_b)}-1\right)d(\theta)x_t + \frac{1}{R}\left(\frac{\beta(1-g)}{\delta(1+\tau_b)}-\alpha\right)\frac{Y_{t+1}}{Y_t}\right] \tag{A2}$$

Equations (A1) and (A2) determines a temporary equilibrium. Like the basic model, the coefficient of Y_{t+1}/Y_t in equation (A2) is greater than that in equation (A1) because $\beta > \delta$. Therefore, the intercept of equation (A1) should be greater than that of equation (A2) in order that the growth rate is positive. This requires

$$\delta(1+\tau_b)(1-g) + [\delta(1+\tau_b) - d(\theta)]\, d(\theta)x_t > 0$$

This condition gives a relationship between the debt management rule and a requirement for the range of the public debt ratio,

$$\begin{cases} x_t > -\frac{\delta(1+\tau_b)(1-g)}{[\delta(1+\tau_b)-d(\theta)]d(\theta)} & \text{if } d(\theta) < \delta(1+\tau_b) \\ \text{any } x_t & \text{if } d(\theta) = \delta(1+\tau_b) \\ x_t < \frac{\delta(1+\tau_b)(1-g)}{[d(\theta)-\delta(1+\tau_b)]d(\theta)} & \text{if } d(\theta) > \delta(1+\tau_b) \end{cases} \tag{A3}$$

From equations (A1) and (A2), we get

$$\frac{Y_{t+1}}{Y_t} = R\left[\delta(1+\tau_b) + \frac{\delta(1+\tau_b) - d(\theta)}{1-g} d(\theta)x_t\right] \tag{A4}$$

$$n_t = \frac{1}{\phi(1-\alpha)}\left\{\beta(1-g) - \delta\alpha(1+\tau_b) - \left[1 - \beta + \frac{\alpha(\delta(1+\tau_b) - d(\theta))}{1-g}\right] d(\theta)x_t\right\} \tag{A5}$$

If $d(\theta) < \delta(1+\tau_b)$, which is a relevant case as shown below, the growth rate is increasing in x_t, and the fertility rate is decreasing in x_t.

Finally, we examine a law of motion of x_t.

First, assume that the government adopts a rule $d(\theta) = \delta(1+\tau_b)$. In this case, $x_t = x_0$ for all t because the equilibrium stays on a balanced growth path in which $D_{t+1}/D_t = Y_{t+1}/Y_t = R\delta(1+\tau_b)$. The long-run growth rate is optimal if the government sets the subsidy rate at

$$\tau_b^* = \frac{1-g}{\alpha} - 1 \tag{A6}$$

The reason is that equation (A6) makes the rate of return of private capital coincide with the social rate of return, $(1+\tau_b^*)R = A(1-g)$.

However, the fertility rate in equation (A5) coincides with the optimal if and only if $x_0 = 0$. Therefore, the rule $d(\theta) = \delta(1+\tau_b)$ is not an optimal policy in general.

From equations (26) and (A4), we get a difference equation of x_t such as

$$x_{t+1} = \frac{(1-g)d(\theta)x_t}{\delta(1+\tau_b)(1-g) + [\delta(1+\tau_b) - d(\theta)]d(\theta)x_t} \tag{A7}$$

If $d(\theta) \neq \delta(1+\tau_b)$, equation (A7) has two steady states. One is $x_t = 0$, and another is

$$x^* = -\frac{1-g}{d(\theta)} < 0 \tag{A8}$$

The dynamics of x_t depends critically on whether $d(\theta)$ is larger or smaller than $\delta(1+\tau_b)$.

[Figure 6 and 7 are here]

Figure 6 illustrates a dynamics of x_t when $d(\theta) < \delta(1+\tau_b)$. Assume that the initial condition satisfies $x_0 > x^*$. Then, the public debt ratio converges to zero. We can verify that the condition (A3) is satisfied for all t. If the government is initially indebted, $x_0 > 0$, then the growth rate decreases and the fertility rate increases in transition.

The long-run equilibrium stays on a non-balanced growth path because

$$\lim_{t\to\infty} \frac{Y_{t+1}}{Y_t} = \delta(1+\tau_b)R > \frac{D_{t+1}}{D_t}$$

Suppose that the government sets the subsidy rate at τ_b^* in equation (A6). Then, we know

$$d(\theta) < \delta(1+\tau_b^*) \Leftrightarrow d(\theta) < \frac{\delta}{\alpha}(1-g) \tag{A9}$$

Under a policy rule (τ_b, θ) such that equations (A6) and (A9) are satisfied, the public debt-GDP ratio, $D_t/Y_t = x_t/R$, converges to zero. In the long-run, we get

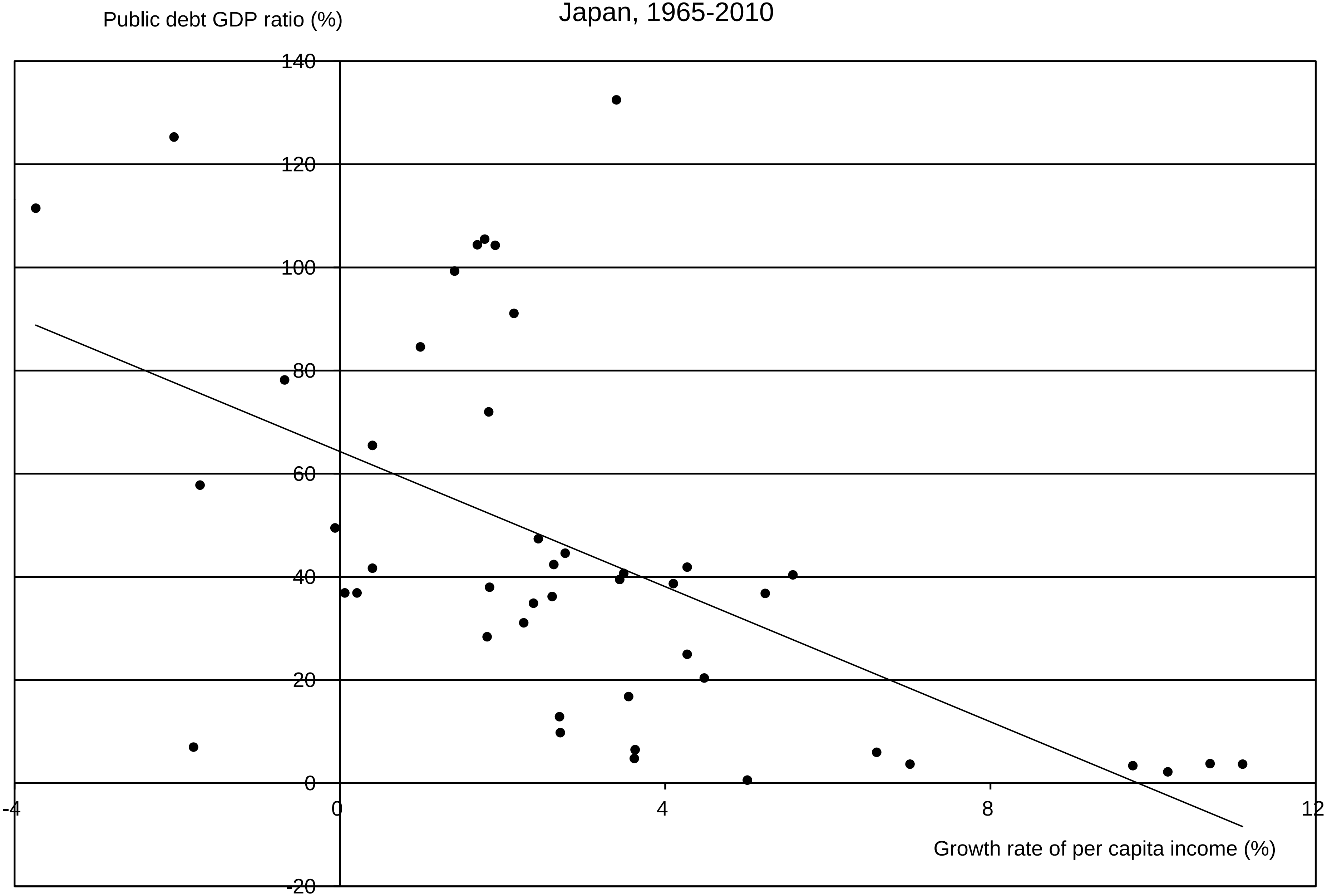
$$\begin{aligned} \frac{Y_{t+1}}{Y_t} &= \delta A(1-g) \\ n_t &= \frac{(\beta-\delta)(1-g)}{\phi(1-\alpha)} \end{aligned}$$

which implies that both rates are optimal.

Figure 7 shows a dynamics of x_t when $d(\theta) > \delta(1+\tau_b)$. In the long-run, the public debt ratio converges to $x^* < 0$ if and only if the initial government is a creditor, $x_0 \in (x^-, 0)$. Here, x^- stands for an initial value which ensures $x_1 > -\alpha$. The long-run equilibrium is not optimal because $x^* < 0$. If the initial government is indebted, then the public debt is not sustainable.

Finally, equation (A9) can be solved on a case-by-case basis. If $\delta(1-g)/\alpha < 1$, then there exists a $\hat{\theta} > -R$ such that equation (A9) is satisfied for any $\theta^* \in (-R^{-1}, \hat{\theta})$. If $\delta(1-g)/\alpha \geq 1$, then equation (A9) is satisfied for any $\theta^* > -R^{-1}$.

Figure 1. Public debt and economic growth
Japan, 1965-2010



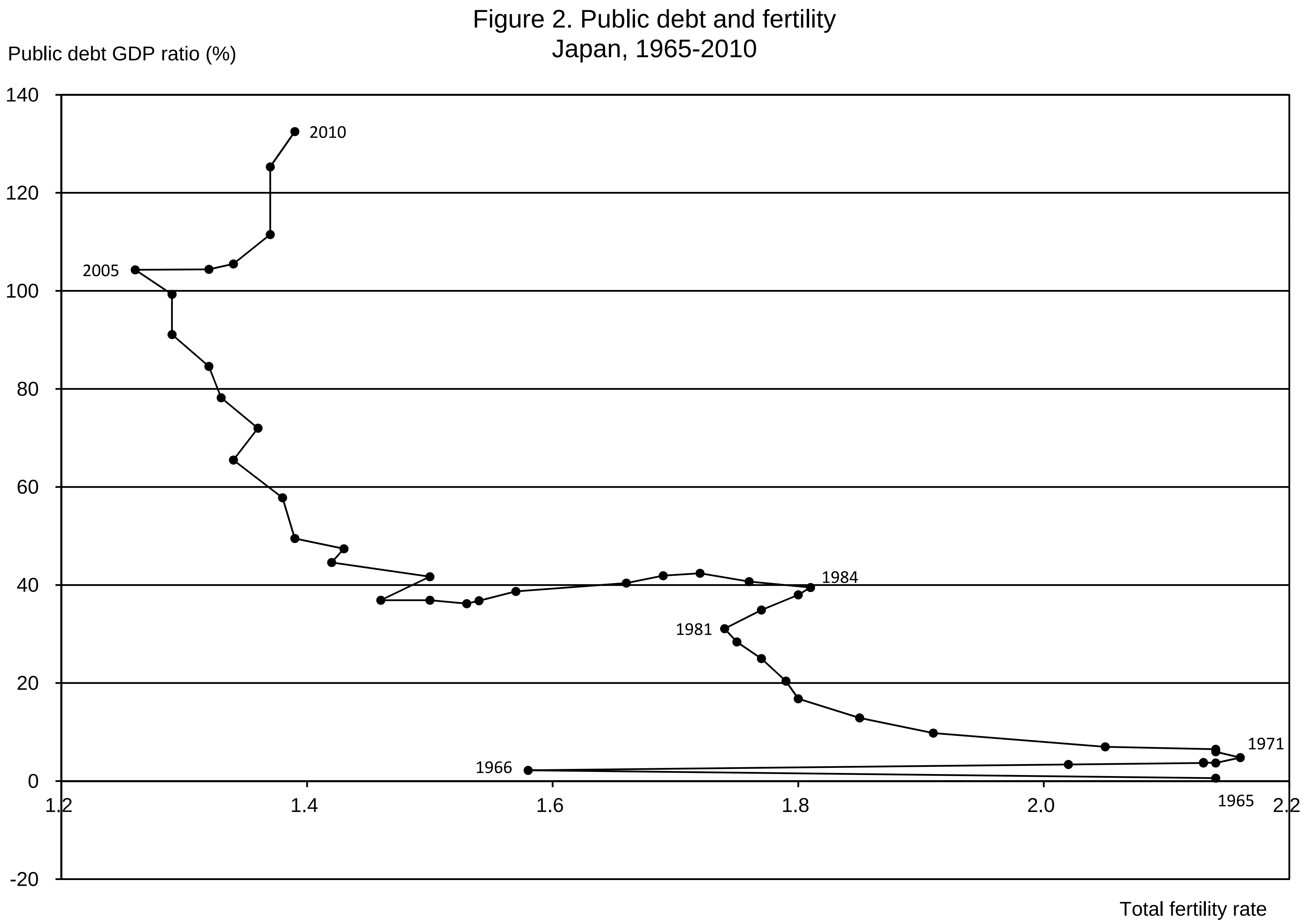


Figure 2. Public debt and fertility
Japan, 1965-2010
Public debt GDP ratio (%)
140
120
100
80
60
40
20
0
-20
2010
2005
1984
1981
1971
1966
1965
1.2
1.4
1.6
1.8
2.0
2.2
Total fertility rate

Figure 3. Temporary equilibrium

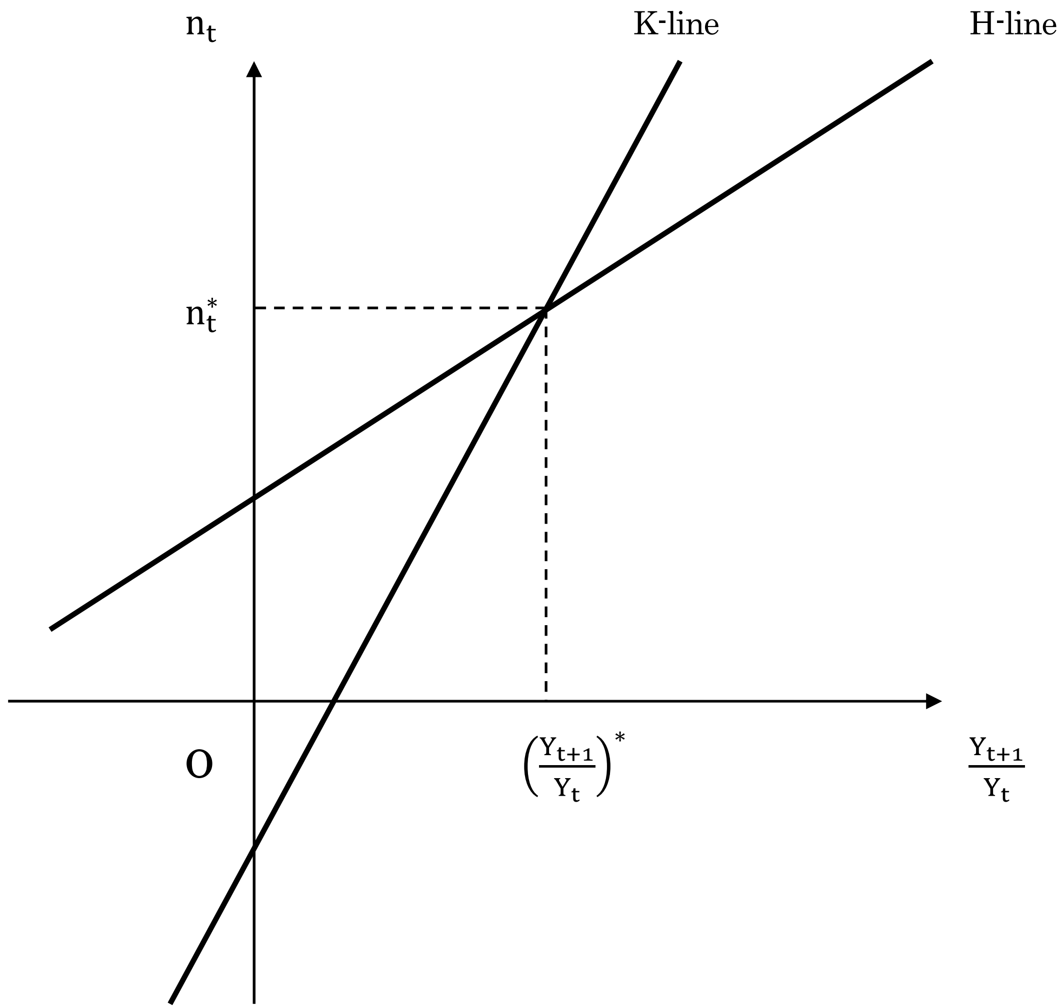


Figure 4. Slack debt management regime

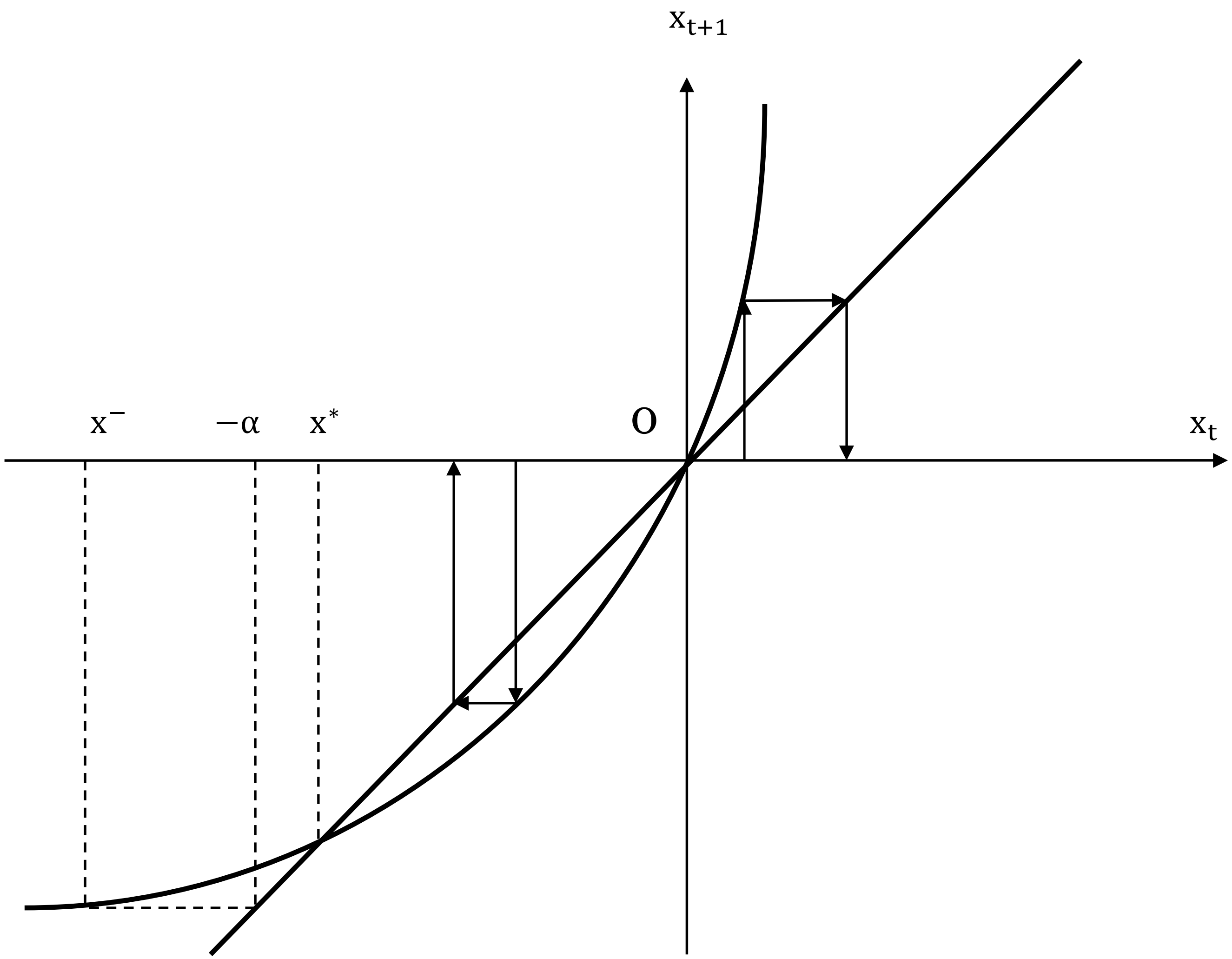


Figure 5. Tight debt management regime

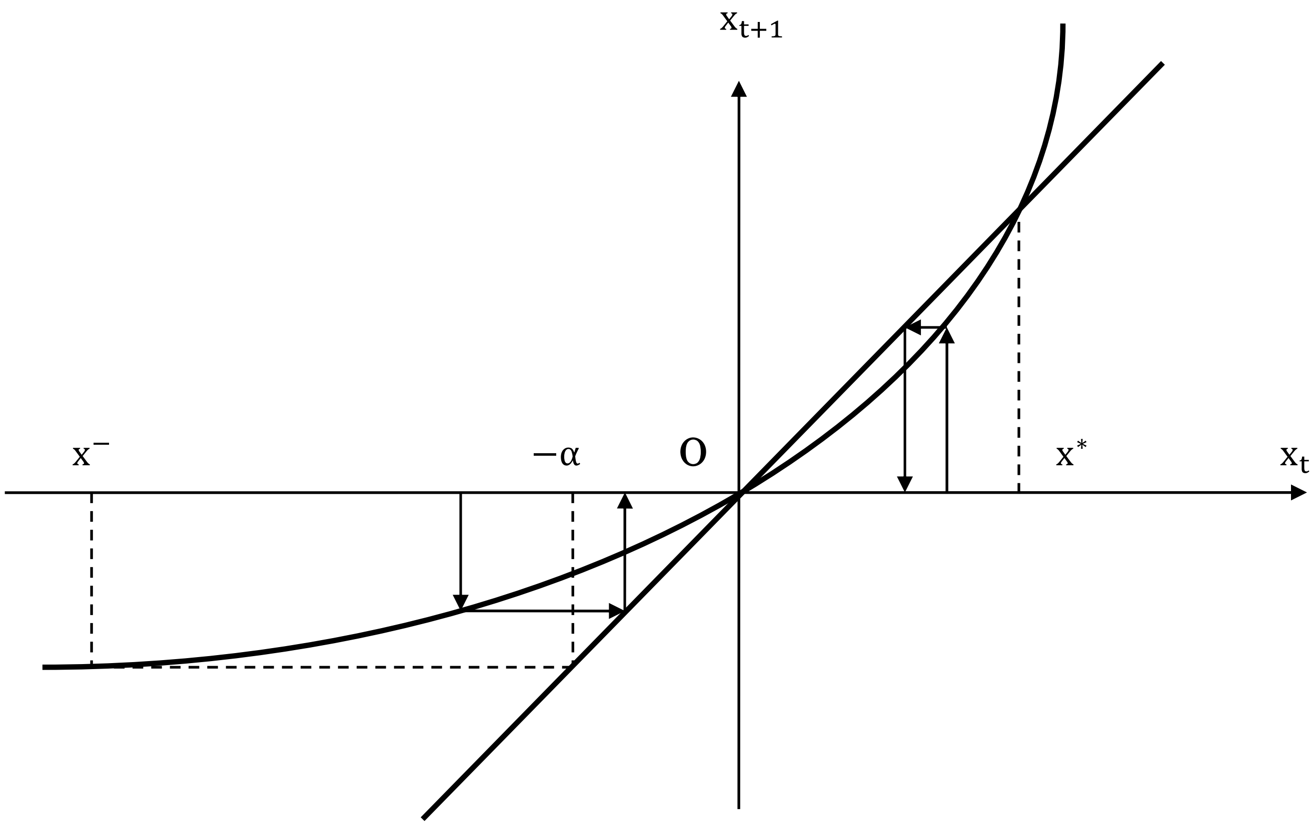


Figure 6. Dynamics when $d(\theta) < \delta(1 + \tau_b)$

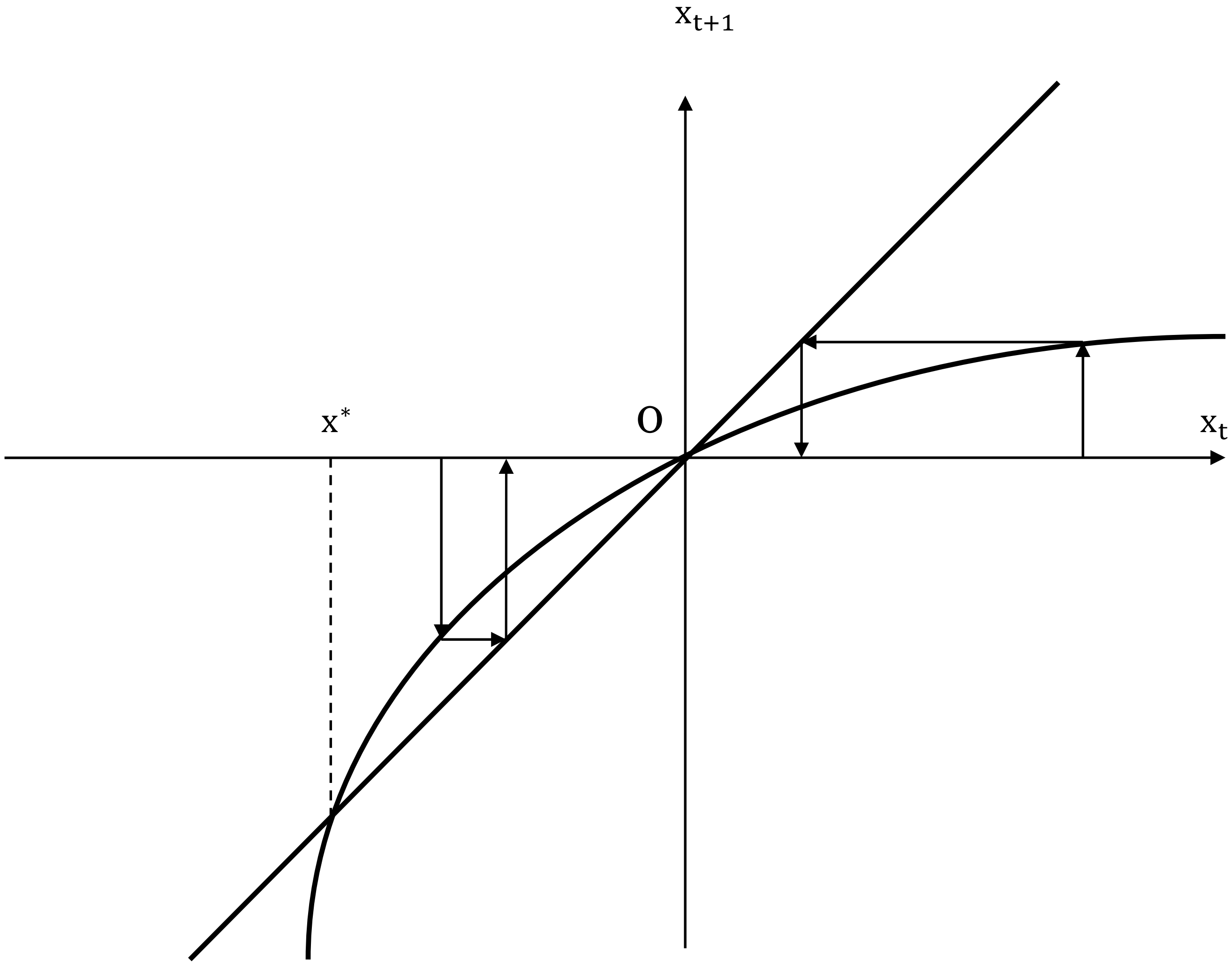


Figure 7. Dynamics when $d(\theta) > \delta(1 + \tau_b)$