

Trading Arms With the Enemy : An Approach Based on Guns Versus Butter Models*

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Résumé

This paper explores the rationale of arms trade between two rival agents. We propose a “guns versus butter” framework in which guns produced are disentangled from guns used. By doing so, we are able to embrace arms trade. We design a three-stage game in which two agents first decide to trade arms (or not); then simultaneously choose their level of guns expenditure; last the prize at stake is shared with respect to a Contest Success Function. Our findings challenge the commonplace according which it would never be rational to trade arms with an enemy. Indeed, arms trade may be mutually beneficial even if agents are (potential) enemies. In particular, when agents have large initial resources we find that arms trade is neutral on both the intensity and on the balance of power. Consequently, it could be a subgame perfect equilibrium of the game if the seller has an advantage both in term of a military and non-military technology. In such a condition, the greatest share of the surplus generated by arms trade is always captured by the seller. Our framework allows to understand historical events like the arms trade between Germany and Britain on the eve of the World War I. We also discuss on the cases of Roman Empire’s arms trade policy, and the (aborted) sale of Mistral between France and Russia through our theoretical framework.

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“The Capitalists will sell us the rope with which we will hang them.”

Vladimir Ilyich Lenin (1870 – 1924)

1 Introduction

This famous quote attributed to Lenin well represents the existence of the temptation to trade arms with an enemy. On the one side, the ‘Capitalists’ described by Lenin sought short-term gains from the sale of arms. On the other side, ‘Soviets’ found an opportunity to fight harder against its sworn enemy. Consequently both the seller and the buyer – even if they are enemies – may benefit from arms trade. This topic requires a specific handling because arms are not usual goods. Indeed, by selling arms, the salesman, whether she is a person, a group or a country, exposes herself to conflict and robbery. In that sense it represents a very dangerous kind of trade. Surprisingly, in the economic literature, arms trade is often considered as the exchange of a good in an imperfectly competitive market (e.g. [Levine et al. \(1994\)](#), [García-Alonso \(1999\)](#), [García-Alonso and Levine \(2007\)](#) or [Gangopadhyay \(2014\)](#)). In these models, two sides are distinguished. The supply-side corresponds to few developed countries – mostly the United States, Russia, France, United Kingdom – able to harness the technology required to produce arms. They are mainly driven by economic motivations such as profits,¹ and by political ones such as peace promotion (see [Fauconnet et al. \(2016\)](#) for the case of France). On the other side of the market, the demand “depends on the security risk, or perceptions of internal and external threats” ([Gangopadhyay, 2014](#), p. 413) experienced by countries which are not able to produce enough arms. The demand is self-sustained through an arms race dynamics ([Richardson, 1960](#); [Intriligator, 1975](#)). Indeed, when a country buys arms, it may reinforce the perception of insecurity and therefore raise the demand of arms of neighboring countries. This mechanism is the major driver of the demand in traditional arms trade models ([Levine and Smith, 1995, 1997a,b](#)). These models assume that the only risk borne by the seller is an *indirect* one laying on potential instabilities resulting from the raise of regional tensions. By only considering risks related to regional instability and not direct ones, *i.e.* a (potential) *direct* conflict between the seller and the buyer,² explanations for arms supply are limited to the following argument “[i]f we do not sell the arms someone else will and [we] will get neither the money nor the security” [Levine et al. \(1994, p. 7, brackets are ours\)](#).³ This line of reasoning leads to the misleading intuition that countries would never sell to a direct enemy, and when they provide arms in an unstable region, it is without any consequences on their own

1. According to the Stockholm International Peace Research Institute (SIPRI), in 2013 global arms trade represented at least \$76 billion. Therefore, it embodies a huge commercial opportunity for arms producers.

2. To the best of our knowledge [Levine et al. \(1994\)](#)’s model is the only paper dealing with arms trade between rivals. Indeed in his framework, there is a “trade off [between] the economic benefits of the sale [...] against possibly negative security repercussions” (*ibid.* p.4, brackets are ours). Security repercussions are modeled by a one-argument function $SR = dW + eW^2$, where W is the aggregate stock of weapons. The authors consider that the seller and the buyer are enemies if $d < 0$ and $e \leq 0$.

3. A recent illustration was provided by the Slovak Prime Minister Robert Fico stating that “Arms are a normal business product. If we don’t sell them, somebody else will, but don’t come crying to me if a lack of arms deals causes the loss of jobs for our people” (quoted in [Fabok \(2016\)](#) from *News Now* : July 29, 2016).

security because other suppliers would have sold arms anyway.

The tacit reason for this “omission” is that selling arms to a direct enemy would strengthen her. It would necessarily reduce the probability of winning and therefore does not worth the money gained from trade. Numerous historical examples support this commonplace : “Documents from Charlemagne’s capitularies (768-814) prohibited the export of Frankish armour [...]. In 971, the Doge of Venice forbade the sale of arms to the Saracens against whom the ‘Orientals’ (Balkan peoples) were fighting. [...] Beginning in 1179, the Church also forbade the sale [of arms to Saracens...], on pain of excommunication or imprisonment, repeating this interdiction in 1215, 1245, 1304 and 1454” (Krause, 1992, p. 35). However, these numerous cases raise the following question : why enacting laws are necessary if there is no incentive to trade arms with an enemy in the first place? In a sense, prohibitions are one of the best justifications that such trade may be rational. The aim of this article is to bring to light conditions under which arms trade between enemies appear to be rational. Liberman (1996, pp. 159-166) provides a salient illustration of such a situation when he relates the Britain-Germany’s trade during the period 1870-1914. In a well-documented analysis he shows that Britain considered Germany as a *direct* threat, but still traded military materials with it. Indeed, “[w]ith new offensive naval forces and aggressiveness, and considerable overall power, Germany became a clear threat to Britain after the turn of the century” (*ibid.* pp. 159-160). However, “Britain was importing [...] militarily significant German manufactures, including steel bars, wire rods, plates and sheets, pig iron, khaki dye (for uniforms), optical equipment, high precision tools, and automobiles” (*ibid.* pp. 164-165). All this material is clearly intended to military use and have an impact on the balance of power between the two countries. Interestingly, even if “[t]he British felt that they were gaining less from Anglo-German trade than were Germans” (*ibid.* p. 163) they did not stop to trade. Although they were aware that “[t]he German army can conquer this country [*i.e.* Britain]” (*ibid.* p. 160, brackets added).⁴

In this article we explore the rationale of trading arms with a direct ennemy by modeling a dyadic relation between a seller and a buyer⁵. This strategy allows to abstract from the two traditional reasons used in the literature to explain arms trade. First, in a dyad, the demand of arms cannot be related to a dynamic of arms race with neighboring countries – because by definition of a dyadic relation there is none. Second, the supply of arms cannot be justified by arguments such as “they would obtain arms anyway” because there is only one supplier. Therefore, by studying a dyadic relation we circumvent traditional justifications for arms trade and we shed new light on the existence the incentive to trade arms with the enemy. For this purpose, we adopt the ‘guns’ versus ‘butter’ (hereafter ‘GVB’) framework in which two agents are involved

4. Another example was recently given by the magazine *Newsweek* (on 5/18/2015) which revealed that “entire groups of [afghan] soldiers manning various checkpoints have sold their weapons and ammunition to the insurgents” (Broder and Yousafzai, 2015, brackets are ours). According to this article, this phenomenon already happened during the Russian occupation of Afghanistan in the 1980’s : the *mujahedeen* were able to purchase military materiel directly from Afghan soldiers.

5. The dyadic approach is extensively used to study arms trade because it allows to have a better understanding on the relation between a seller and a buyer of arms (e.g. Bas and Coe (2012) and Hultquist (2013)).

in a military contest to grab a contested resource.⁶ This contest may be a conflict involving actual fights (*hot* conflicts), or can be limited to an exchange of threats (*cold* conflicts). These models depict agents engaged in a trade-off between ‘guns’ - *i.e.* activities directly related to the probability of winning the resource - and ‘butter’ - all others activities generating wealth. The conflict is seen as a *costly lottery* meaning that the relative amount of guns detained by each agent defines the share of the contested resource they grab.

The contribution of this article is two-fold. Firstly, from a methodological point of view, we enlarge the scope of GVB models by taking into account the option of arms trade. Indeed, GVB models usually do not differentiate between the guns produced and the guns used in the military contest. Without this distinction, they structurally cannot capture the logic of arms trade. In this article, we disentangle them. We develop a three-stage game with complete information in which the seller and the buyer first bargain over the price of guns to find a mutually advantageous agreement, if it exists. In the second stage, they simultaneously choose the amount guns they produce. In the third stage, guns are delivered if an agreement was reached in the first stage, then agents produce butter. Finally, the amount of guns detained by each agent determines the division of the contested resource. The second contribution is directly related to the results provided by our model. Indeed, we demonstrate that arms trade may be a Subgame Perfect Equilibrium of our game. It may take place if the guns offered by the seller are more *effective* - *i.e.* more lethal - than those produced by the buyer. Otherwise, the buyer would not agree to trade arms. We also find that the seller needs to be more productive in butter (*i.e.* non-military sector) than the buyer to avoid deviation at the time of the delivery (stage 3). This finding challenges the traditional view because it shows that selling arms to an enemy may be economically fruitful even when we abstract from traditional justifications proposed by the literature.

The article proceeds as follows. The first section introduces the foundations of the basic model. In the second section, we resolve the three-stage game. In the third section, we notably focus on how the surplus generated by arms trade is divided between the two agents. We show that the seller always takes the lion’s share. In the fourth section, we propose an extension of the basic model in which guns production is exogenously fixed. It allows to account for situation in which the agent does not have a full ability to the quantity of arms she produces. In the fifth section we explore two cases : the Mistral’s (dis)agreement between France and Russia and the Roman empire - barbarians relation regarding arms trade. The last section concludes.

2 The model

In basic GVB models, two risk-neutral agents compete to grab a contested resource, $V > 0$. It may be, for example, an unoccupied territory or an unexploited natural resource. In this kind of game, agents initially possess a fixed amount of secured primary resources, $R_i > 0$ for $i = 1, 2$, that can be devoted to either guns

6. For an exhaustive overview of the scope of the contest theory, see [Konrad \(2009\)](#) or [Van Long \(2013\)](#). Seminal contributors to the use of contest theory for human warfare are [Hirshleifer \(1988, 1989\)](#) and [Skaperdas \(1992\)](#). This literature is well reviewed in [Garfinkel and Skaperdas \(2007\)](#).

spending (M_i) or productive activities (B_i) such as :

$$R_i \equiv B_i + M_i, \quad i = 1, 2. \quad (1)$$

This general specification is very helpful to capture the underlying motivations of conflict and the trade-off faced by agents. Indeed, investing in guns raises the probability to obtain the contested resources, but we see from (1) that it also decreases the production of butter. Note that previous literature uses M_i for both guns produced and guns used in conflict situations.

This article explores a situation in which arms can be produced by an agent and used by another one, through trade. In order to model this distinction we define G_i as the amount of guns produced by agent i . For ease of reading, we consider that one unit of resources is transformed at a one-to-one rate in guns. As a result, it may be said that agent i produces G_i units of guns.

Without loss of generality, let agent 1 be the seller of guns, and agent 2 the buyer. Arms trade is modeled as a transfer of $S > 0$ units of guns from agent 1 to agent 2 at a strictly positive price, τ . The seller cannot offer more arms she produced and the buyer cannot buy more than the resources she detains such that $S \leq \min(G_1, R_2/\tau)$. On the other hand, the *autarky* – defined as a situation without arms trade – is characterized by $S = 0$ and $\tau = 0$. Now we have introduced arms trade, one has to cautiously distinguish three levels of measurement for guns (see [Tableau 1](#)). Guns produced refer to guns that are manufactured by a given agent. Guns used represent the amount of guns available to grab the contested resources. In traditional GVB models, these two variables are undistinguished. Another important variable is the spending in guns, net of the earnings from the sale. It should be noted that if the price τ is sufficiently high (*i.e.* for $\tau > G_1/S$), guns exports of agent 1 entirely cover the costs of guns produced. In other terms, agent 1’s net guns spending may be negative.

TABLE 1 – Guns produced, used and resources devoted to guns

	Guns produced	Guns used	Net guns spending
Agent 1 (seller)	G_1	$G_1 - S$	$M_1 = G_1 - \tau S$
Agent 2 (buyer)	G_2	$G_2 + S$	$M_2 = G_2 + \tau S$

The share of the contested resource grabbed⁷ is given by a Tullock CSF ([Tullock, 1980](#)), meaning that only the ratio of guns used matters. Agent 1 uses the guns she produced minus the ones she sold (*i.e.* $G_1 - S$), and symmetrically agent 2 uses $G_2 + S$ guns in the conflict. Therefore, for $(G_1, G_2, S) \in \mathbb{R}_+^3$ the CSF assigns

7. There are mainly two ways to interpret the outcome of the CSF. It could be either the probability to win the entire contested resource (*probabilistic view*) or sharing of it. These two interpretations are perfectly equivalent if i) agents are risk-neutral and ii) the contested resource is fully divisible ([Garfinkel and Skaperdas, 2007](#), p. 663). These two conditions are fulfilled in our setting.

to agent i her probability to grab the contested resource. More formally, we have $p_i : \mathbb{R}_+^3 \rightarrow [0, 1]$ such that :

$$p_1(G_1, G_2, S) = \begin{cases} \frac{[\theta(G_1 - S)]^m}{[\theta(G_1 - S)]^m + [G_2 + \theta S]^m} & \text{if } G_1 + G_2 > 0, \\ \frac{1}{2} & \text{otherwise,} \end{cases} \quad (2)$$

and $p_2(G_1, G_2, S) = 1 - p_1(G_1, G_2, S)$, where θ is the relative effectiveness of one unit of guns produced by agent 1 with respect to one produced by agent 2 (Grossman and Kim, 1995). In particular, $\theta > 1$ refers to an advance in military technology of agent 1 over agent 2. Lastly, m is a ‘‘decisiveness parameter’’ (Hirshleifer, 2000) scaling the degree to which a difference in relative strength translates into the probability of winning the conflict. We assume that $m \in (0, \infty)$ which means that the technology of conflict is neither fully stochastic nor perfectly discriminant.⁸ It should be noticed that p_i is monotonically increasing in agent i ’s military capacity and monotonically decreasing in agent j ’s one, with $i, j = 1, 2$ and $j \neq i$.⁹

Last, payoffs are defined as follows, for any $S \geq 0$:

$$\pi_i(G_1, G_2, S) = p_i(G_1, G_2, S)V + \beta_i(R_i - M_i), \quad (3)$$

where β_i is the ‘butter’ productivity. To ease reading of our results, let $\beta_2 = 1$. As a result, β_1 captures the relative ‘butter’ productivity between agent 1 and agent 2. In particular, $\beta_1 > 1$ represents a situation in which agent 1 is more productive in ‘butter’ production than agent 2. The net guns spending, M_i , depends on the position of the agent i such as $M_1 = G_1 - \tau S$ and $M_2 = G_2 + \tau S$ (see Tableau 1).

We now address the design of the game. We develop a three-stage game with complete information in order to include arms trade in GVB models. In the first stage, agents consider the opportunity to trade a fixed amount $S > 0$ of guns. They bargain over the price, τ , in order to find a mutually advantageous agreement. If at least one agent finds no interest in the transaction, bargaining fails and arms trade does not occur. Otherwise, arms trade takes place. In the second stage, each agent simultaneously chooses the number of guns she produced, G_i for $i = 1, 2$, taking the decision of her contestant as given. Finally, in the third stage, agents produce butter, and the contested resource is distributed among agents according to (2). Since we have a three-stage game with complete information, the equilibrium concept employed is the sub-game perfect equilibrium (SPE) proposed by Selten (1975). Solutions for the model are determined iteratively, in a standard fashion, by backward induction.

8. We examine in Appendix A the $m = 0$ and $m \rightarrow \infty$ ’s cases.

9. For a complete analysis of axioms fulfilled by the ratio-form CSF, see Skaperdas (1996) or Corchón (2007).

3 Resolution of the game

We now address the resolution of our three-stage game. Note that in the rest of this article we indicate by a superscript ‘ T ’ the situation of arms trade, and by ‘ A ’ the *autarky*. We begin by analyzing the last stage of the game in which agents produce butter and the prize is distributed. Then, we focus on the production of guns and, last, we deal with the arms trade agreement.

3.1 Stage 3 : guns’ delivery, conflict and resulting payoffs

The third stage consists of three successive steps. Firstly, if an agreement was reached in the first period, $S > 0$ guns are delivered to agent 2, and agent 1 receives τS simultaneously. However, in our framework, there is no institution able to enforce arms transactions. Accordingly, agents may have interest to deviate from the original agreement signed in stage 1. In this case deal is off : the seller is not paid but she does not deliver any arms.

Proposition 1. (Profitable deviation) *At least one agent has an incentive to deviate from the initial agreement if $\beta_1 < 1$.*

Démonstration. Let p_1^T and p_1^D be respectively the share of the contested resource obtained by agent 1 if the agreement is respected, and the one there is a deviation. By definition, $p_1^T < p_1^D$ meaning that if no arms are delivered to the buyer, the outcome of the agent 1’s CSF would be higher that if arms are transferred. Agent 1 deviates if her gain from the sale is inferior to the loss of resources obtained due to arms trade : *i.e.* if $\beta_1 \tau S < (p_1^D - p_1^T) V$. In a similar fashion, agent 2 deviates from the original agreement as soon as $\tau S > (p_1^D - p_1^T) V$. It directly comes that a deviation from the original agreement always occurs for $\beta_1 < 1$. \square

[Proposition 1](#) involves that an agreement signed in the first stage of the game is not binding if the seller is less productive in butter than the buyer. The direct consequence is that no arms trade is feasible in such conditions. The intuition behind [Proposition 1](#) is the following : if the seller is more productive in butter than the buyer, she would be able to fruitfully invest the revenues from the sale in butter. If not, at least one agent would loose from the trade. As a result, contract are broken by one side in the third stage, and no arms are transferred. On the contrary, if there is no deviation, arms trade takes place.

Secondly, agent i produces butter with the available input $(R_i - M_i)$. Thirdly, the contested resource is distributed among agents according to (2). The repartition does not necessary hinges on an actual fight, but it may be seen as a exchange of threats leading to the repartition of the contested resource. As noted by [Garfinkel and Skaperdas \(2007, p. 652\)](#) : “[a]ctual, overt conflict does not necessarily have to occur but arming can be used, as often in reality, as a bargaining tool and as a deterrent within a larger economic context.” At the end of the game agent i receives (3).

3.2 Stage 2 : production of guns

In the second stage of the game agents observe the result of the bargaining concerning arms trade. If it fails, arms trade does not occur and we have $S = \tau = 0$. On the contrary, if the bargaining stage is a success, agents trade $S > 0$ arms at a price, $\tau > 0$. It also should be noticed that in this stage we only deal with the case of “sufficiently” rich agents. It means that, if one agent does not have any primary resources (R_i) to either buy or sell arms, the agreement cannot be reached in the first stage. Consequently, we only study here the case in which R_i is sufficiently high regarding τ and S .

We first deal with the case in which no agreement was reached in the first stage of the game – the *autarky*. In this situation, arms produced equal arms used. Treating her enemy’s choice of guns as given, agent $i = 1, 2$ chooses the level of guns maximizing her payoffs (3) with respect to the technology of conflict given by (2). First order conditions for an interior solution in *autarky* are given by $\partial\pi_i/\partial G_i = 0$, for $i = 1, 2$ and $\tau = S = 0$. We show in [Appendix B](#) that for all $m \in (0, \infty)$ the Nash equilibrium, labeled $(\tilde{G}_1^A, \tilde{G}_2^A)$, satisfies

$$\frac{\tilde{G}_2^A}{\tilde{G}_1^A} = \beta_1. \quad (4)$$

Two interpretations may be derived from (4). First, at the equilibrium, the relative guns production of agent 2 (captured by $\tilde{G}_2^A/\tilde{G}_1^A$) is inversely proportional to her relative productivity in butter ($1/\beta_1$). In accordance with ricardian models of international trade, agents tend to specialize according to their advantage. For example, if agent 1 is much more productive in butter than her enemy, she would devote more resources to butter and would decrease her efforts to grab the contested prize, all things being equal. Second, (4) exhibits that the value of $m \in (0, \infty)$ leaves unchanged the ratio of guns produced (and used) of the two agents. In other terms, the ratio is invariant to the level of decisiveness of the conflict. However, the value taken by $(\tilde{G}_1^A, \tilde{G}_2^A)$ are more sensitive. By dividing both sides of (4) by θ , we obtain $\tilde{G}_2^A/(\theta\tilde{G}_1^A) = \beta_1/\theta$. The CSF exhibited in (2) is such that $p_1 = 1/[1 + (\beta_1/\theta)^m]$. Thus, at the equilibrium, guns production in *autarky* is defined such as :

$$\tilde{G}_1^A = \frac{\left(\frac{\beta_1}{\theta}\right)^m mV}{\left[1 + \left(\frac{\beta_1}{\theta}\right)^m\right]^2 \beta_1}, \quad \text{and} \quad \tilde{G}_2^A = \frac{\left(\frac{\beta_1}{\theta}\right)^m mV}{\left[1 + \left(\frac{\beta_1}{\theta_1}\right)^m\right]^2}. \quad (5)$$

We assume the second order conditions fulfilled (see [Appendix B](#)) ensuring that values reported in (5) correspond to a maximum for each agent. In particular, the number of guns produced positively depends on the value of the contested prize, V and on the scale of the technology of conflict, m . On the other hand, it negatively hinges on the advantages detained by agent 1 both in production and in guns effectiveness. Last, equilibrium exhibited in (5) echoes to the traditional lottery literature : in the symmetric case $\theta = \beta_1 = 1$ and for $m = 1$, we obtain the standard result that agents devoted a quarter of the value of the prize to grab it ([Pérez-Castrillo and Verdier, 1992](#)).

Consider now the case in which agents struck a deal on the first stage of the game. Similar to the *autarky*, agent i maximizes (3) considering the decision of her contestant as given and a CSF as (2). However, we have $S > 0$ and $\tau > 0$. According to the first order conditions, the Nash equilibrium in case of arms trade, labeled $(\tilde{G}_1^T, \tilde{G}_2^T)$, is characterized by :

$$\frac{\tilde{G}_2^T + \theta S}{\tilde{G}_1^T - S} = \beta_1 \quad (6)$$

This equilibrium condition in the trade situation looks like (4), but differs in one point : agents also take into account the relative effectiveness of guns, θ . From the agent 2's perspective, θS corresponds to the number of guns she would have produced by herself to replicate the effectiveness of S . For example, if $\theta > 1$, agent 2 needs to produce $G_2 = \theta S > S$ to obtain the same share of V without arms trade. According to (6), the ratio between the number of guns used by agent 2 in “ G_2 equivalent” (*i.e.* $\tilde{G}_2^T + \theta S$) and the number of guns used by agent 1 ($\tilde{G}_1^T - S$) equals β_1 , and is stable for any $m \in (0, \infty)$. In other words, the relative strength is invariant, whatever the decisiveness of conflict.

The Nash equilibrium derived from (6) is given by :

$$\tilde{G}_1^T = \frac{\left(\frac{\beta_1}{\theta}\right)^m mV}{\left[1 + \left(\frac{\beta_1}{\theta}\right)^m\right]^2 \beta_1} + S, \quad \text{and} \quad \tilde{G}_2^T = \frac{\left(\frac{\beta_1}{\theta}\right)^m mV}{\left[1 + \left(\frac{\beta_1}{\theta}\right)^m\right]^2} - \theta S. \quad (7)$$

We assume the second order conditions fulfilled (see [Appendix B](#)). The number of guns produced positively depends on the value of the contested resource and on the agent 1's technological advance (both in terms of guns, θ , and butter, β_1). Logically, we also find that S is negatively (resp. positively) linked with the number of guns produced by agent 2 (resp. agent 1).

Proposition 2. (Arms trade's neutrality) *At the equilibrium, the distribution of the contested resource remains the same whether they trade arms, or not. Agent 1's share is given by $p_1^j = 1/[1 + (\beta_1/\theta)^m]$, for $j = T, A$.*

Démonstration. According to (4) and (6) we deduce that $\tilde{G}_2^A/\tilde{G}_1^A = (\tilde{G}_2^T + \theta S)/(\tilde{G}_1^T - S)$. By dividing both sides by θ , we obtain the $\tilde{G}_2^A/(\theta\tilde{G}_1^A) = (\tilde{G}_2^T + \theta S)/[\theta(\tilde{G}_1^T - S)]$. It means that the relative strength of agent 2 relative to agent 1 is the same whether arms trade occurs, or not. As a result, there exists a unique $\kappa > 0$ such as $\tilde{G}_2^A = \kappa(\tilde{G}_2^T + \theta S)$ and $\tilde{G}_1^A = \kappa[\theta(\tilde{G}_1^T - S)]$. Moreover, a Tullock CSF as described in (2) fulfilled the *scale invariance axiom* meaning that “winning probabilities must remain constant to equiproportional changes in all contenders' efforts” ([Cubel and Sanchez-Pages, 2015](#), p. 9). Consequently $p_i(\tilde{G}_1^A, \tilde{G}_2^A, 0) = p_i(\tilde{G}_1^T, \tilde{G}_2^T, S)$ holds by definition for $i = 1, 2$. Lastly, the proof for $p_1^j = 1/[1 + (\beta_1/\theta)^m]$, for $j = T, A$ is direct considering

(4) and (6).

□

Proposition 2 states that arms trade does not impact on the outcome of the CSF. In other words, it depicts a “if we don’t sell, they will produce by themselves” principle. For example, if S is high, agent 1 will produce more guns in order to counterbalance the reduction of resources grabbed. Similarly, agent 2 will reduce her own production of guns due to the fact that she bought more effective arms. Regardless, the distribution of V remains the same whether agents trade arms in the first stage, or not. Therefore, only three parameters matter here : the relative productivity in butter, β_1 , the relative effectiveness of guns, θ and the scale of the technology of conflict m . In particular, $\beta_1 > \theta$ (agent 1 has a comparative advantage in butter) translates into a lower share of V because agent 1 devotes relatively more efforts in butter. Contrariwise, if $\beta_1 < \theta$, agent 1 would specialize into guns and would grab a higher share of the the contested resource.

Corollary 1. (Invariance of the intensity of conflict) *The intensity of conflict, measured in terms of total guns used, does not hinge on arms trade.*

Démonstration. We know from **Proposition 2** that the distribution of the contested resource is the same in case of arms trade or *autarky*. In accordance with (4-7), it is easy to show that :

$$\theta\tilde{G}_1^A = \theta(\tilde{G}_1^T - S), \quad \text{and} \quad \tilde{G}_2^A = \tilde{G}_2^T + \theta S. \quad (8)$$

To put it another way, the strength of each agent – in G_2 equivalent – is identical if arms trade is decided in the first stage, or not. Consequently, the intensity of the conflict remains the same.

□

Proposition 2 and **Corollary 1** exhibit that arms trade does not have influence on the conflict itself (both in terms of outcome and intensity). Interestingly, a parallel can be drawn with the traditional literature on arms trade. Indeed, there is kind of mechanism such as “if agent 1 does not sell arms to agent 2, she will produce by herself anyway”. This result finds its roots in two of our assumptions. First, counterbalancing is possible because there is no significant difference, apart from their effectiveness, between arms produced by agent 1 and agent 2 : one machine rifle may well be replaced by ten machetes.¹⁰ Second, we assume that agents are rich enough to always play their optimal strategies. Consequently, agents can always compensate arms trade by spending more, or less, effort on guns production. In such conditions, both the outcome and the intensity of the conflict are not impacted by arms trade.

10. Clearly, it is not the case when a country decides to sell a nuclear weapon to a country which does not detain the military technology to enrich uranium. Such a weapon cannot be counterbalanced by thousands of machetes. The literature dealing with this particular trade is mostly known as “arms proliferation” (e.g. [Jo and Gartzke \(2007\)](#) or [Anderton and Carter \(2009, chapter 10\)](#)).

3.3 Stage 1 : arms trade agreement

We now turn to the first stage of the game in which agents bargain over price of trading S guns. In order to stay tractable, let $\tilde{\pi}_i^A$ be the agent i 's payoffs when she produces \tilde{G}_i^A such that : $\tilde{\pi}_i^A = \pi_i(\tilde{G}_1^A, \tilde{G}_2^A, 0)$. Similarly, consider $\tilde{\pi}_i^T = \pi_i(\tilde{G}_1^T, \tilde{G}_2^T, S)$. The bargaining is successful if arms trade is payoffs-enhancing for both agents.¹¹ However, if at least one $\tilde{\pi}_i^A > \tilde{\pi}_i^D$ for $i = 1, 2$, then bargaining fails and agents only produce their own guns (*autarky*). In order to define conditions under which it is rational to trade arms with the enemy, we need to prove two intermediate results sketching the behavior of the seller and the buyer of guns.

Lemma 1. (Supply of arms) *For all $S > 0$, $\partial\tilde{\pi}_1^T/\partial\tau > 0$ and the lowest price that the buyer is willing to pay is $\underline{\tau} = 1$.*

Démonstration. First, the sign of $\partial\tilde{\pi}_1^T/\partial\tau$ can be easily found by introducing (7) into (3). Indeed, we know from Proposition 2 that Nash equilibrium verifies $p_1^j = 1/[1 + (\beta_1/\theta)^m]$, for $j = T, A$. Therefore, we have $\partial\tilde{\pi}_1^T/\partial\tau = \beta_1 S > 0$: revenues from the sale allow to produce more butter. Second, the seller's reservation price, $\underline{\tau}$, is the minimum price for which she accepts to sell arms. By definition, we have $\underline{\tau} = \max(0, \{\tau | \tilde{\pi}_1^T - \tilde{\pi}_1^A = 0\})$. Due to the arms trade neutrality regarding the conflict, we have $\tilde{\pi}_1^T - \tilde{\pi}_1^A = S\beta_1(\tau - 1)$, i.e. $\underline{\tau} = 1$. □

Lemma 1 expresses a quite logical intuition : for a given quantity of guns, S , a higher price leads to higher payoffs for the seller. Moreover, to be beneficial, the price needs to be at least equal to the production cost of one guns which is normalized to one in our model.

Lemma 2. (Demand of arms) *For all $S > 0$, $\partial\tilde{\pi}_2^T/\partial\tau < 0$ and the buyer's reservation price is $\bar{\tau} = \theta$.*

Démonstration. Similar to Lemma 1's proof, we deduce from (3), (7) and Proposition 2 that $\tilde{\pi}_2^T/\partial\tau = -S < 0$. In other words, the payoffs' decrease is linearly related to the amount of guns bought. Moreover, the reservation price of the buyer is price beyond which buying the S guns is not profitable. More formally, it corresponds to $\bar{\tau} = \max(0, \{\tau | \tilde{\pi}_2^T - \tilde{\pi}_2^A = 0\})$. We obtain that $\tilde{\pi}_2^T - \tilde{\pi}_2^A = S(\theta - \tau)$ and therefore $\bar{\tau} = \theta$. □

Lemma 2 highlights that, during the bargaining, the buyer prefers the lowest price possible. Moreover, agent 2 wants to buy arms if and only if $\tau \leq \theta$. At this condition it is always payoffs-enhancing to substitute guns production by guns importations, because the effectiveness of guns bought is sufficiently high regarding their price.

We are now able to formulate our third proposition.

11. Without loss of generality, we assume that when an agent is indifferent between trading arms or not, she would accept the trade.

Proposition 3. (Existence of a zone of agreement) *The zone of agreement, defined by $Z = [\underline{\tau}, \bar{\tau}]$, exists when $\underline{\tau} \leq \bar{\tau}$, that is for $\theta > 1$.*

Démonstration. Direct considering [Lemma 1](#) and [Lemma 2](#). □

[Proposition 3](#) is central because it states that agents disputing over a resource may have mutual benefits to trade arms with each other. As soon as $\theta > 1$, a zone of agreement exists. In other words, a military technology gap in favor of the seller is necessary. Otherwise, the buyer would prefer to produce guns by herself. This point leads helps to characterize the identity of the seller in our framework.

Corollary 2. (The identity of the seller) *If arms trade takes place, the seller of guns is the agent who is able to produce more effective guns ($\theta > 1$) and who is more productive in butter ($\beta_1 > 1$).*

Démonstration. Direct considering [Proposition 1](#) and [Proposition 3](#). □

According to [Corollary 2](#) agent 1 has to detain both a more advanced technology of conflict and production. First, the seller has to produce more effective arms otherwise the buyer would sell it. Second, the seller also needs to be more effective in butter in order to avoid a deviation in the contract in the third stage of the game. This double advantage that needs to detain the seller seems in line with the empirical evidence on arms trade. Indeed, according to the SIPRI, since the end of the Cold War the major guns exporters are : U.S.A., Russia, the United Kingdom, France and Germany. All these countries are clearly labeled as ‘developed’ ones and have high productivity both in guns and butter industries. It tends to go along with [Corollary 2](#).

We now address the determination of the price. Let A be the disagreement point defined by the vector $A = (\bar{\pi}_1^A, \bar{\pi}_2^A)$. It corresponds to an *autarky* situation in which the seller and the buyer cannot reach an agreement on the price. In our model, each agent faces a classical bargaining problem $\langle Z, A \rangle$, where Z is compact and convex. Both agents are fully informed about the bargaining structure, including each others’ payoffs. In order to deal with this situation, we use the Nash Bargaining Solution ([Nash, 1950, 1953](#)) – hereafter, ‘NBS’ – which is a static procedure allowing to abstract away some question related to the bargaining (e.g. who makes the first offer? How agents settle? etc.).

Proposition 4. (Subgame Perfect Equilibrium) *Arms trade between two enemies is a SPE of the game if and only if $\theta > 1$. The equilibrium price is unique and is such as $\tilde{\tau} = (\theta + 1)/2$.*

Démonstration. [Proposition 3](#) states that a zone of agreement exists if and only if $\theta > 1$. This condition ensures that at least one mutually beneficial agreement may be reached. Moreover, considering the bargaining problem $\langle Z, A \rangle$, the NBS exists and is unique. Indeed, Z is compact and the objective function is continuous and strictly quasi-concave. The equilibrium price is the solution of the following program $\tilde{\tau} = \arg \max_{\tau} (\tilde{\pi}_1^T - \tilde{\pi}_1^A)(\tilde{\pi}_2^T - \tilde{\pi}_2^A)$. We deduce from [Lemma 1](#) and [Lemma 2](#)'s proof that $(\tilde{\pi}_1^T - \tilde{\pi}_1^A)(\tilde{\pi}_2^T - \tilde{\pi}_2^A) = \beta_1 S^2 (\tau - 1)(\theta - \tau)$. It can easily be shown that the NBS is :¹²

$$\tilde{\tau} = \frac{\theta + 1}{2} \tag{9}$$

□

[Proposition 4](#) exposes in what conditions arms trade between two enemies may be a rational option. Few words are required about the Nash Bargaining procedure. In particular, it fulfills the symmetry axiom : as the strategies available for both agents are the same (*trade* or *autarky*), the only difference between agents is “included in the mathematical description of payoffs” ([Nash, 1953](#), p. 137). Consequently, the only threat available in the bargaining is a decrease in the *autarky* payoffs of the enemy. That is why it seems quite logical to observe in (9) that the agreement price is linearly related to the agent 1's guns relative effectiveness. Indeed, when θ is high, it lowers the *autarky* payoff of agent 2, therefore it reinforces the ability of agent 1 to obtain a high price ($d\tilde{\tau}/d\theta > 0$).

4 Surplus analysis and incentive to deviate

Arms trade, when it occurs, generated a positive surplus. We propose in this section to analyze its repartition between the two enemies. Let $\Delta\tilde{\pi}_i = \tilde{\pi}_i^T - \tilde{\pi}_i^A$ be the supplementary payoffs obtain by agent i from arms trade. [Figure 1](#) represents gains that each agent obtains. Arms trade is mutually advantageous if the two lines are above the horizontal axis (zone of agreement). We know from (9) that the solution of the game lies between 1 and θ . However, it does not provide any information about the surplus sharing between the two agents. We propose to compare the equilibrium price, $\tilde{\tau}$, the price which equally splits the surplus generated by arms trade (the so-called “split-the-surplus solution”, labeled τ^{SS}). Graphically, it corresponds to the intersection point of the two lines representing gains from arms trade. Following [Anbarci et al. \(2002\)](#), this price is such that $\tau^{SS} = \{\tau \mid \tilde{\pi}_1^T - \tilde{\pi}_1^A = \tilde{\pi}_2^T - \tilde{\pi}_2^A\} = (\theta + \beta_1)/(\beta_1 + 1)$. The comparison of the equilibrium and the split-the-surplus price leads to the following proposition.

Proposition 5. (Surplus sharing) *When arms trade between enemies occurs, the seller always obtains the highest share of the surplus generated by the sale.*

Démonstration. The buyer extracts the highest share of the surplus generated by the trade if she manages

12. Indeed, the first derivative of the Nash Product with respect to τ is $\beta_1 S^2 (\theta - 2\tau + 1)$. Moreover, it should be noted that the second derivative is negative which ensures that (9) corresponds to a maximum.

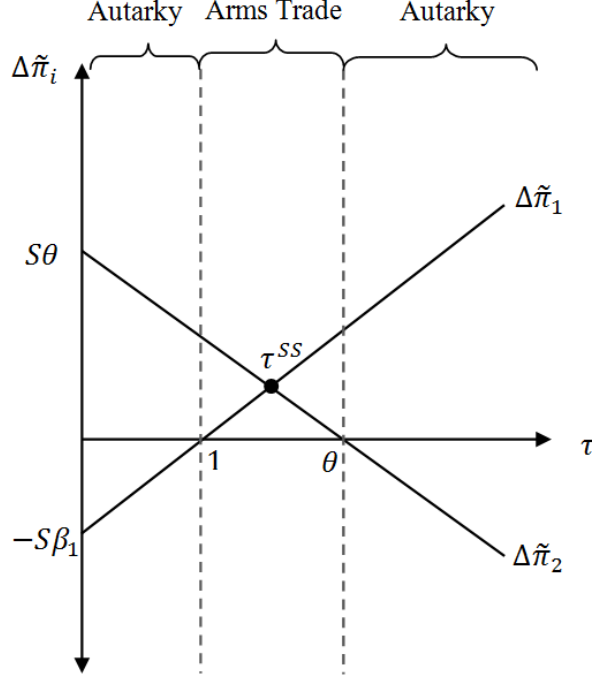


FIGURE 1 – Gains from arms trade

to obtain a price such as $\tilde{\tau} \geq \tau^{SS}$. By rearranging terms it requires that :

$$\left(\frac{\theta - 1}{2}\right) \left(\frac{\beta_1 - 1}{\beta_1 + 1}\right) \geq 0$$

We saw that arms trade requires $\beta_1 \geq 1$ and $\theta \geq 1$. Consequently, we directly see that agent 1 obtains the largest share of the surplus from the trade. □

Following [Proposition 5](#), arms trade seems to lead to an increase in economic wealth inequality between the two agents. Indeed, if the seller is initially more productive in general, it also captures the most important share of the surplus generated by the trade. In other words, no convergence may be initiated by the trade of arms. ¹³

Lastly, we can provide a finer understanding of the incentive to deviate from the arms trade agreement when $m = 1$. [Proposition 1](#) stated that there always is a deviation if the seller is less productive in butter than the buyer ($\beta_1 < 1$). Due to the values of (7) and (9) it can be shown that the buyer never deviates from the original agreement (all computational details are provided in [Appendix C](#)). Indeed, she would obtain higher payoff in case of deviation if $2\beta_1 - 1 + \theta < 0$. As we know that an arms trade agreement requires $\theta > 1$, it directly comes that the buyer would never want to deviate in stage 3. From her perspective, the

¹³. This assertion should be cautiously taken into consideration. Indeed, it should be reminded that our model is not a dynamic one. If we take into account the time dimension, arms trade may involve important technology transfers from the seller to the buyer. Buying arms may be a way to significantly improve her own guns effectiveness in the future.

seller deviates as soon as $\beta_1 < 2\theta/(\theta - 1)$. This condition means that the respect of the agreement in stage 3 depends both on the productivity in guns and in butter. Indeed, it appears that for $\theta \rightarrow \infty$, the seller would deviate for $\beta_1 < 1$. On the other hand, if θ is small enough, the seller would have always interest to deviate (e.g. for $\theta \rightarrow 1$, she does not respect the deal signed in period 1 as long as $\beta_1 < \infty$). To summarize, all things being equal, arms trade is more likely to occur when the seller is sufficiently more productive in butter than the buyer and if the guns she sold are not too much effective. Contrariwise, contracts are not binding and arms trade does not take place in the first stage of the game (due to backward reasoning).

5 The model with exogenous guns production

In [Section 3](#), any sale (resp. purchase) may be perfectly compensated by an increase (resp. decrease) in the guns produced. We now consider that such compensation is not possible by taking into account an exogenous guns production, labeled \bar{G}_i , $i = 1, 2$, in the second stage of the game. They decide to trade arms considering a fixed stock they possess. This assumption is in line with existing situations. For example, if agents are countries, military budgets are highly related to political factors and often disconnected from optimal decisions, as computed in (5) and (7). In particular, the Public Choice School deeply investigates the inefficiencies related to bureaucracy processes (e.g. [Mueller \(2003, chapter 16\)](#)). It fully applies to guns production. In that sense, [Rattinger \(1975, p. 575\)](#) notes for the military “regardless of the threat posed by international environment, cuts in the defense budget [...] have to be avoided by all means.” There is a high inertia in the determination of the budget allocated to guns activities, that is why it seems consistent to consider the case of exogenous guns production. Another justification may be found in the power detained by defense firms and lobbies. They are able to influence decisions leading to a gap between actual and optimal guns production. Quoting [Smith \(1995, p. 75\)](#) : “[t]he choices which arise from such bargaining and logrolling are unlikely to satisfy rationality requirements.”¹⁴ By considering that $G_i = \bar{G}_i$, we capture the lack of matching between the guns production and security needs.

In order to stay tractable, we only present results for $m = 1$. We reason backward. The third stage remains identical. The second stage is modified such as it does not involve any decisions by the agents anymore : the quantities \bar{G}_1 and \bar{G}_2 are produced, independently of the occurrence of arms trade. The technology of

14. One illustration is associated with the case of France. Expectations in terms of means and objectives are defined in the “loi de programmation militaire” (*Military Program Law - MPL*). It covers several years and follows the official doctrine fixed in “Livres blanc sur la défense et la sécurité nationale” (*White paper on defense and security*). The current MPL provides quantitative goals in terms of procurement for each major defense platform. Given France’s current stock and operational needs, it was argued that 26 new Rafales (fighter aircraft built by Dassault Aviation) are necessary for the whole period. In our model, it corresponds to \bar{G}_1 . However, Dassault Aviation indicates that the production of at least 11 Rafales per year (*i.e.* a total of 66 Rafales) is required not to lose core competences and technological skills. This exigence of the defense firm corresponds to the \bar{G}_i in our model.

conflict described in (2) becomes :

$$p_1(S, \bar{G}_1, \bar{G}_2) = \begin{cases} \frac{\theta(\bar{G}_1 - S)}{\theta\bar{G}_1 + \bar{G}_2} & \text{if } \bar{G}_1 + \bar{G}_2 > 0, \\ \frac{1}{2} & \text{otherwise,} \end{cases} \quad (10)$$

and $p_2 = 1 - p_1$. It appears clearly from (10) that arms trade (*i.e.* $S > 0$) decreases the share of the contested resources grabbed by agent 1. As the guns production is fixed, no compensation can be made. Therefore, the neutrality of arms trade on the outcome of the CSF described in Proposition 2 is systematically transgressed here. In other words, once we consider exogenous guns production, the seller necessarily faces a tradeoff between not selling and securing the contested resources, and producing more butter with the revenue obtained from arms trade. This result seems logical but corresponds to a quite new element in the arms trade literature. Indeed, selling arms is usually considered as neutral because of the existence of multiple suppliers. Traditional models capture the existence of an aggregated threat which is stable according to the “if I am not selling arms, others will” principle. However, in our case if arms trade occurs, agent 1 would deliberately decrease her relative military strength, which lowers the share of the contested resource obtained in the third stage of the game.

Last, we turn to the first stage of the game. Using the same methodology as in Section 3, one can easily compute the reservation price of each agent : $\bar{\tau} = V\theta/(\theta\bar{G}_1 + \bar{G}_2)$ and $\underline{\tau} = V\theta/[(\theta\bar{G}_1 + \bar{G}_2)\beta_1]$. It leads to the final proposition of this article.

Proposition 6. (Subgame Perfect Equilibrium) *If the production of guns is exogenous, arms trade can occur if and only if $\beta_1 \geq 1$. It is a SPE of the game when $R_2 \geq \bar{G}_2 + \theta(\beta_1 + 1)SV/[\beta_1(\theta\bar{G}_1 + \bar{G}_2)]$.*

Démonstration. First, we notice that $\bar{\tau} = \underline{\tau}/\beta_1$ meaning that the agreement zone Z exists only if $\beta_1 \geq 1$. In other words, arms trade cannot be a SPE if agent 1 is strictly less productive in butter than agent 2. On the contrary, for $\beta_1 \geq 1$, a NBS exists and is unique. The level of price maximizing the Nash Product is defined by $\tilde{\tau} = \theta(\beta_1 + 1)V/[2\beta_1(\theta\bar{G}_1 + \bar{G}_2)]$. Second, as there is no compensation, agent 2 cannot diminish her own production of guns to buy the ones produced by agent 1. As a consequence, agent 2 accepts the agreement only if $R_2 \geq \bar{G}_2 + \tau^*S$, that is if :

$$R_2 \geq \bar{G}_2 + \frac{\theta(\beta_1 + 1)SV}{2\beta_1(\theta\bar{G}_1 + \bar{G}_2)} \quad (11)$$

□

Condition (11) corresponds to the participation constraint (PC) of agent 2 in a mutually advantageous arms trade.¹⁵ First of all, it is more difficult to fulfill the PC when S is high. Secondly V and θ exert a positive influence on the equilibrium price which in turn harden the agent 2’s budgetary constraint. The

15. See Appendix D for comparative statistics details

higher the value of contested resource's at stakes is, the more the seller requires a high price to compensate the decrease of her share of the contested resources grabbed. The line of reasoning is similar for the relative effectiveness of guns : a high θ allows agent 1 to get a high price. On the contrary, a high β_1 relaxes the budgetary constraint of agent 2 because agent 1 is more prone to accept a deal with a low price. The PC highlights the incentive

Corollary 3. (Defense lobbying and arms trade) *Powerful defense lobbies favor the sale of arms to an enemy.*

Démonstration. Powerful defense lobbies are able to set a high level of \bar{G}_1 , that can mismatch with operational needs. We directly see from (11) that a high \bar{G}_1 translates into a greater chance to see the emergence of arms trade. Indeed, due to a basic price-quantity relation, \bar{G}_1 is negatively linked with $\tilde{\tau}$.

□

Corollary 3 finds its roots in the technology of conflict described in (10). Indeed, $\partial^2 p_1 / \partial \bar{G}_1^2 = -2\theta^2(\bar{G}_2 + \theta S) / (\theta \bar{G}_1 + \bar{G}_2)^3 \leq 0$. In other terms, if defense lobbies impose a large \bar{G}_1 , the consequences of selling a little portion of it would be feeble. In parallel, production function exhibits constant return to scale : revenues from arms trade are reinvested in butter at a β_1 rate. If a country possesses a large stock of arms (superior to its operational needs), then it would be rational to increase a little the strength of its opponent if the economic consequences are sufficiently positive. If guns are not usual business, they still are business.

6 Discussion

In this article, we show that trading arms with an enemy may be rational from a theoretical perspective. However, real situations involve other variables that can impact arms trade decision. First, the duration of each stage of the game does matter. Producing arms takes time and the delay between the original agreement (stage 1) and the arms delivery (stage 3) could be years. However, the situation between two hostile countries is much more unstable and a geopolitical shock can jeopardize the agreement. In other words, an unexpected change may induce an unpredictable *deviation* in the third stage of the game. One illustration may be found in the recent case of the trade of Mistral class ships – a polyvalent helicopter carrier – between France (the seller *i.e.* agent 1) and Russia (the buyer, *i.e.* agent 2). In March 2010, an agreement concerning a sale of four Mistral was reached. At this time, France and Russia consider each other as a potential threat. For General Gomart, in charge of French military intelligence, Russia is a major military power “potentially dangerous” and was closely monitored.¹⁶ Moreover, the 2008 Russo-Georgian crisis revealed the Russia’s territorial ambition. From the Russian perspective, NATO countries were considered as the first “external

16. Quoted from the French National Assembly available here (in French) : <http://www.assemblee-nationale.fr/14/cr-cdef/14-15/c1415049.asp>.

military danger”¹⁷ in its 2010 Military Doctrine. It appears that one country considers the other as a *potential* threat, thus this case belongs to the scope of this article. Moreover, we can consider that $\beta_1 > 1$ and $\theta > 1$.¹⁸ Under these conditions, mutually advantageous agreement is reachable as in [Proposition 3](#). Moreover, as France is the seller, [Corollary 2](#) is also verified. Arms trade agreement was signed in 2010 (stage 1) and the French defense firm DCNS started Mistral production in 2011 (stage 2).

However, the 2013 Ukrainian crisis changed the whole picture. The climax was reached in 2014, when Crimea, a region of Ukraine with a majority of Russian speakers, voted for joining Russia in a referendum. At the same time, Russia supported the rebellion against the legitimate power in the East of Ukraine.¹⁹ In September 2014, French President Hollande declared the suspension of the delivery because international security was threatened by the aggressive behavior from Russia in Ukraine. Finally, the contract was definitely suspended on November 26, 2014. What happened in our framework? France deviation in the third stage is clearly related to the evolution of the 2013 Ukrainian crisis. Indeed, situation in Crimea was very alarming because it revealed the Russia’s wish to ‘annex’ Russian-speaking areas. In particular, it worried France regarding the Baltic region. Indeed, Estonia and Latvia have also a large share of Russians in their population – respectively 25.8% and 25.1%. However, the stakes are different here. Indeed, those two countries are NATO nations and if Russia shows aggression towards them, it would automatically involved each NATO member, France included, through the famous article 5 of the North Atlantic Treaty²⁰. In other words, Crimea crisis drastically changed the geopolitical context and increased the size of the stakes of the conflict, $V' > V$. As a result, if selling arms to Russia was perfectly rational in the first stage, France was induced to deviate in the third one. A justification may be found in the deviation condition highlighted in the proof of the [Proposition 1](#). One could easily see that the shift from V to V' make $\beta_1 \tau S < (p_1^D - p_1^T) V'$ is more easily fulfilled : considering the new situation it appeared rational for France to deviate from the original agreement.

Throughout this article, we claim that arms trade between enemies does not seem as irrational as considered by the literature. However, as we mentioned above, History is replete with examples of prohibitions regarding this kind of transfer. In this discussion we propose an explanation staying in the general framework adopted in this article. We choose more ancient case : the Roman Empire’s (‘agent 1’) prohibition of arms transfer outside the empire, and more specifically to Barbarians (‘agent 2’). In this early period, the military gap is kept relatively low (θ is close to one) due to a very slow military innovation ([Krause, 1992](#)). Moreover, regarding the means of arms production at this time, neither Roman Empire nor barbarian tribes were able

17. It should be noted that “a ‘danger’ is a situation with the potential ‘under certain conditions’ to develop into an immediate military threat” ([Keir, 2010](#), p. 1). In other words, OTAN represents a *potential* threat for Russia.

18. First, France is said to be more productive by Russia according to the World Economic Forum productivity ranking. Second, From an operational point of view, the Russian Admiral Vysotskiy argued that “a ship like Mistral would allowed the Black Sea fleet to accomplish its mission in 40 minutes, not 26 hours, which was how long it took us” ([Baker, 2011](#), p. 11). It means that France detained an advantage in military technology.

19. Russia suffered from international economic sanctions regarding this event. These sanctions also concern arms trade but they do not cover the period before August 2014. Mistral ships are then not included but this restriction.

20. It states that “[t]he Parties agree that an armed attack against one or more of them in Europe or North America shall be considered an attack against them all and consequently they agree that [...] each of them [...] will assist the Party or Parties so attacked by taking forthwith, individually and in concert with the other Parties, such action as it deems necessary, including the use of armed force, to restore and maintain the security of the North Atlantic area.” - [NATO \(1949\)](#)

to freely adapt the amount of guns produced in accordance to an eventual arms trade deal. As a result, this situation is well-depicted by the model with exogenous production of guns (see Section 5). In such a case, “with uniform military technology, [...] arms exports diminished one’s potential arsenal and augmented that of likely (or actual) enemies.” (Krause, 1992, p. 35). In other words, selling arms to Barbarians would have lead to a decrease in the security of the Roman Empire. Moreover, regarding the impressive military power detained by Roman Empire during the first four centuries AD, it appears quite reasonable to assume that $\bar{G}_1 > \bar{G}_2$. Lastly, it is largely admitted that agriculture and production system conferred to Roman Empire a clear advantage in terms of butter productivity ($\beta_1 > 1$). Following Proposition 6, everything was in place to allow arms trade. However, that is not what happened : the imperial laws clearly stated that arms trade without the frontiers of the Empire was prohibited. The main reason stems from the fact that the scope of our model is limited to conflicts over an exogenous resource. However, in the dispute opposing barbarians tribes and Roman Empire, there was much more at stake. The prize was not just an unoccupied territory, or a natural resources, but it was the Empire itself. Consequently, in case of a barbarian victory the whole empire would have collapsed.²¹ In our model, it would involve that property rights on the agent 1’s butter are not fully enforced. As a result, when agent 1 sells arms, she needs to take into account that the supplementary revenue from arms trade may also be appropriated by agent 2. The prize would thus take the following form : $V = v + (1 - \mu)\beta_1(R_1 - \bar{G}_1 + \tau S)$, with v is an exogenous component of the prize and $\mu \in [0, 1]$ represents the enforcement of property rights. When $\mu = 1$, the butter produced by Roman Empire is fully secured. On stark contrast, $\mu = 0$ means that the whole Empire may be potentially grabbed or invaded.²²

TABLE 2 – Numerical simulations on arms trade between Romain Empire and Barbarian tribes

μ	$\bar{\tau} \geq \underline{\tau}$	$\bar{\tau}$	Participation constraint (agent 2)
0	Yes	12.44	Transgressed
0.25	Yes	9.38	Transgressed
0.5	Yes	6.29	Transgressed
0.75	Yes	3.19	Fulfilled
1	Yes	0.07	Fulfilled

With : $\theta = 1.1$, $\beta_1 = 2$, $v = 1$, $R_1 = 100$, $\bar{G}_1 = 10$, $R_2 = 5$, $\bar{G}_2 = 1$ and $S = 1$

Tableau 2 presents the results of simulations of our model for different levels of property rights enforcement. Parameters’ value has been arbitrarily chosen in order to depict the Roman Empire’s situation. We assume that there is no clear advance in military technology ($\theta = 1.1$), but Roman Empire has an advantage in butter ($\beta_1 = 2$). It is also richer than barbarian tribes ($R_1 = 100$ and $R_2 = 5$), and their guns stocks are fixed ($\bar{G}_1 = 10$ and $\bar{G}_2 = 1$). Lastly, we consider the sale of one unit of guns ($S = 1$). We first notice that there

21. From the 3rd to the 5th Century, Gaul was frequently invaded by barbarian tribes ending the *pax romana* and slowly initiating the decline of the Roman Empire.

22. Due to the insecure property rights payoffs are modified as follows : $\pi_1 = p_1 [v + (1 - \mu)\beta_1(R_1 - \bar{G}_1 + \tau S)] + \mu\beta_1(R_1 - \bar{G}_1 + \tau S)$, and $\pi_2 = (1 - p_1) [v + (1 - \mu)\beta_1(R_1 - \bar{G}_1 + \tau S)] + R_2 - \bar{G}_2 - \tau S$.

always exists a zone of mutually advantageous arms trade between Roman Empire and Barbarian Tribes. This result is in line with [Proposition 6](#), because $\beta_1 > 1$, meaning that Roman Empire would be able to take advantage of revenues from the arms sale. However, we clearly see in [Tableau 2](#) that when μ is low, the price demanded by Roman Empire for arms is very high. Indeed, it means that an important share of the wealth of the Empire is subject to barbarian appropriation. Roman Empire is therefore less prone to accept arms trade, except if the price, $\tilde{\tau}$, is sufficiently high. If we also consider other elements increasing the stakes of the conflict (e.g. emperor's prestige, existence of a Roman's hegemony, potential conquests), the reservation price of the Roman Empire could be so high that it is equivalent to a legal ban. In other words, a slightly modified version of the model is able to explain these behaviors : arms trade prohibition could seem reasonable according to what the seller estimates she may lose.

7 Conclusion

In this article, we claim that there is no reason why arms trade between two enemies would always be considered as irrational. We develop a three-stage GVB model in which two enemies have the possibility to trade arms in the first period. In the second stage they produce arms. In the third stage, the transfer – if an agreement was reached in stage 1 – takes place and the contested resources is shared according to a contest success function. Our original approach allows both to enlarge the scope of GVB models and to have a more comprehensive understanding of arms trade throughout History. In particular, we argue against the commonplace stating that it is never rational to trade arms with an enemy. Indeed, under few (reasonable) conditions we find that a mutually arms trade agreement may be reached if the seller is more productive both in guns and in butter production (see [Proposition 3](#), [Corollary 2](#) and [Proposition 4](#)). With our [Proposition 5](#), we demonstrate that the seller always capture the largest share of the surplus generated by such a transfer. We extend our framework by considering exogenous guns production as when powerful lobbies can interfere with guns decisions. In this situation, we also find that arms trade may be a SPE of the game (see [Proposition 6](#)).

In the last section of this article, we explore two cases : the (aborted) Mistral's sale between France and Russia and the Roman empire's prohibition to trade arms. We hope to demonstrate that the theoretical framework we build may provide interesting insights on the arms trade phenomenon.

Finally, our model belongs to the contest theory's framework, which fits with a broad number of situations. As a matter of fact, we think our article may be useful for other kind of form of contests. Indeed, from a lottery perspective, our model allows to understand why an individual may sell tickets to her contestant – despite having a negative impact of her probability to win the prize. It may help to provide answer to questions such as : why sell a license to a competing firms? Why do rival soccer clubs trade soccer players?

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Appendices

Appendix A : Section 2’s computational details

THE MODEL WITH $m = 0$

In this specific case, the technology of conflict is fully noisy in the sense that the guns involved in the battlefield does not influence the probability of grabbing the prize which is randomly fixed. Without loss of generality, consider $p_1 = p_2 = 1/2$, for all values taken by G_1, G_2, S, τ . It is trivial to show that the SPE of the game is such that no agent spend any resources in military expenditures²³ – either to produce nor to buy guns. As a result, arms trade does not take place and agents devote their whole resources in guns activities, *i.e.* $\tilde{B}_i = R_i$. To conclude, arms trade is not feasible if the probability of success does not hinge on levels of guns used in the conflict.

THE MODEL WITH $m \rightarrow \infty$

In this case, the CSF is perfectly discriminant (the slightest difference in agents’ relative strength translates

23. This result directly echoes the work of Skaperdas (1992, p. 726) highlighting that “a sufficiently ineffective conflict technology is required for the existence of a fully cooperative equilibrium” (an absence of guns expenditures). Although the author excludes the fully ineffective technology of conflict ($m = 0$) of his scope, the intuition is the same : why investing resources in arms if it yields to nothing ?

into a guaranteed victory. This kind of technology of conflict is called all-pay auction (Hillman and Riley, 1989). For $i, j = 1, 2, i \neq j$ we have :

$$p_i = \begin{cases} 1 & \text{if } G_i > G_j, \\ 1/2 & \text{if } G_i = G_j, \\ 0 & \text{otherwise .} \end{cases}$$

In this case, there is no pure strategy Nash equilibrium (see Baye et al. (1996) for a characterization of the Nash equilibrium in mixed strategies).

Appendix B : Section 3's computational details

STAGE 2 : FIRST ORDER CONDITIONS IN AUTARKY

In *Autarky* there is no arms transfer, *i.e.* $\tau = S = 0$. Considering (2) and (3) first order conditions (hereafter, 'FOCs') are defined as follows :

$$\begin{cases} \frac{\partial \pi_1}{\partial G_1} = 0 \\ \frac{\partial \pi_2}{\partial G_2} = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{(\theta G_1)^m m V (G_2)^m}{[(\theta G_1)^m + (G_2)^m]^2 G_1} = \beta_1 \\ \frac{(\theta G_1)^m m V (G_2)^m}{[(\theta G_1)^m + (G_2)^m]^2 G_2} = 1 \end{cases}$$

Therefore the equilibrium in *autarky* always verifies : $\tilde{G}_2^A / \tilde{G}_1^A = \beta_1$ as mentioned in (4).

STAGE 2 : SECOND ORDER CONDITIONS IN AUTARKY

We compute the second order conditions (hereafter 'SOCs') in a standard fashion :

$$\begin{cases} \frac{\partial^2 \pi_1}{(\partial G_1)^2} \leq 0 \\ \frac{\partial^2 \pi_2}{(\partial G_2)^2} \leq 0 \end{cases} \Leftrightarrow \begin{cases} (\theta G_1)^m (m+1) + (G_2)^m (1-m) \geq 0 \\ (\theta G_1)^m (m-1) - (G_2)^m (m+1) \leq 0 \end{cases}$$

Consider the equilibrium point, we know from (5) that $\tilde{G}_i^A > 0$ and $m > 0$ has been assumed. Consequently, by dividing the SOCs by $(\tilde{G}_1^A)^m$, one could use (4) to show that, at the equilibrium, SOCs are fulfilled for

$$\frac{\beta_1}{\theta} \geq \max \left\{ \left(\frac{m+1}{m-1} \right)^{\frac{1}{m}}, \left(\frac{m-1}{m+1} \right)^{\frac{1}{m}} \right\}$$

STAGE 2 : STATIC COMPARATIVE ANALYSIS AT $(\tilde{G}_1^A, \tilde{G}_2^A)$

Based on (5), it directly comes that $\partial \tilde{G}_i^A / \partial V \geq 0$ and $\partial \tilde{G}_i^A / \partial m \geq 0$, for $i = 1, 2$. Moreover, $\partial \tilde{G}_i^A / \partial \theta \geq 0$ if

and only if $\beta_1 > \theta$. Last, with respect with the SOCs $\partial \tilde{G}_1^A / \partial \theta \leq 0$ ²⁴ and $\partial \tilde{G}_2^A / \partial \theta \geq 0$ when $\beta_1 \leq \theta$.

STAGE 2 : FIRST ORDER CONDITIONS IN THE ARMS TRADE REGIME

Considering (2), (3) and that τ, S are strictly positive numbers, FOCs are given by :

$$\begin{cases} \frac{\partial \pi_1}{\partial G_1} = 0 \\ \frac{\partial \pi_2}{\partial G_2} = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{[\theta(G_1 - S)]^m mV [G_2 + \theta S]^m}{[(\theta(G_1 - S))^m + (G_2 + \theta S)^m]^2 (G_1 - S)} = \beta_1 \\ \frac{[\theta(G_1 - S)]^m mV [G_2 + \theta S]^m}{[(\theta(G_1 - S))^m + (G_2 + \theta S)^m]^2 (G_2 + \theta S)} = 1 \end{cases}$$

In particular, the equilibrium admits $(\tilde{G}_2^T + \theta S) / (\tilde{G}_1^T - S) = \beta_1$, which corresponds to (6).

STAGE 2 : SECOND ORDER CONDITIONS IN THE ARMS TRADE REGIME

$$\begin{cases} \frac{\partial^2 \pi_1}{(\partial G_1)^2} \leq 0 \\ \frac{\partial^2 \pi_2}{(\partial G_2)^2} \leq 0 \end{cases} \Leftrightarrow \begin{cases} (\theta(G_1 - S))^m (m + 1) + (G_2 + \theta S)^m (1 - m) \geq 0 \\ (\theta(G_1 - S))^m (m - 1) - (G_2 + \theta S)^m (m + 1) \leq 0 \end{cases}$$

We proceed as for the *autarky* situation except for the fact that we divide by $(\tilde{G}_1^T - S)^m$. According to (6) we find the exact same SOCs :

$$\frac{\beta_1}{\theta} \geq \max \left\{ \left(\frac{m+1}{m-1} \right)^{\frac{1}{m}}, \left(\frac{m-1}{m+1} \right)^{\frac{1}{m}} \right\}$$

STAGE 2 : STATIC COMPARATIVE ANALYSIS AT $(\tilde{G}_1^T, \tilde{G}_2^T)$

According to (7), signs of the first derivatives of \tilde{G}_1^T are the same as those found for \tilde{G}_1^A . And we one could see that $\partial \tilde{G}_1^T / \partial S > 0$. For \tilde{G}_2^T we also have the same first derivatives as \tilde{G}_d^A for V and β_1 . We also have $\partial \tilde{G}_2^T / \partial S < 0$. Finally, $\partial \tilde{G}_2^T / \partial \theta < 0$ is not tractable and depends on the value taken by all parameters of the model.

Appendix C : Section 4's computational details

In order to provide tractable results, we need to assume that $m = 1$. Based on that assumption, we can deduce from (3) and (7) that agent 1 would get $\theta \tilde{G}_1^T V / (\theta \tilde{G}_1^T + \tilde{G}_2^T) + \beta_1 (R_1 + \tilde{G}_1^T)$. She does not respect the

24. Indeed, we have that $\partial \tilde{G}_1^A / \partial \theta \leq 0 \Rightarrow \beta_1 / \theta \leq [(m-1)/(m+1)]^{(1/m)}$, which never holds if SOCs are fulfilled.

original deal in the third stage if she obtains more than $\tilde{\pi}_1^T$:

$$\tilde{\pi}_1^T - \frac{\theta \tilde{G}_1^T}{\theta \tilde{G}_1^T + \tilde{G}_2^T} V + \beta_1 (R_1 - \tilde{G}_1^T) < 0 \Leftrightarrow \frac{S(2\beta_1 - 1 + \theta)}{2} < 0$$

As $S > 0$ (because a contract was signed in the first stage of the game), we obtain that agent 2 will deviate as soon as : $2\beta_1 - 1 + \theta < 0$.

Similarly, agent 2 would deviate if :

$$\tilde{\pi}_2^T - \frac{\tilde{G}_2^T}{\theta \tilde{G}_1^T + \tilde{G}_2^T} V + R_1 - \tilde{G}_2^T < 0 \Leftrightarrow \frac{S(-\beta_1 + \theta\beta_1 - 2\theta)}{2} < 0$$

As $S > 0$, deviation occurs if $\beta_1 < 2\theta/(\theta - 1)$.

Appendix D : Section 5's computational details

SOME COMPARATIVE STATICS RELATED TO [PROPOSITION 6](#)

Let X be the LHS of (11) such that $X = \bar{G}_2 + \theta(\beta_1 + 1)VS / [2\beta_1(\theta\bar{G}_1 + \bar{G}_2)]$. X represents the amount of resources that must be spent by agent 2 to buy the S guns (her PC). We easily find the following first derivatives :

$$\frac{\partial X}{\partial \bar{G}_2} = 1 - \frac{\theta(1 + \beta_1)V}{2\beta_1(\theta\bar{G}_1 + \bar{G}_2)^2} \geq 0 \text{ if and only if } \beta_1 \geq \frac{1}{1 - 2\frac{(\theta\bar{G}_1 + \bar{G}_2)^2}{\theta VS}}$$

$$\frac{\partial X}{\partial \bar{G}_1} = -\frac{\theta(1 + \beta_1)V}{2\beta_1(\theta\bar{G}_1 + \bar{G}_2)^2} < 0,$$

$$\frac{\partial X}{\partial V} = \frac{\theta(1 + \beta_1)}{2\beta_1(\theta\bar{G}_1 + \bar{G}_2)} > 0,$$

$$\frac{\partial X}{\partial \theta} = \frac{(1 + \beta_1)VS\bar{G}_2}{2\beta_1(\theta\bar{G}_1 + \bar{G}_2)} > 0,$$

$$\frac{\partial X}{\partial \beta_1} = -\frac{\theta VS}{2\beta_1^2(\theta\bar{G}_1 + \bar{G}_2)} < 0.$$