A test of rationality axioms with individual prices in a domestic production framework[1]

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Abstract

The purpose of this paper is to contribute to the current discussion on the compatibility of consumers' behavior in "real" life with the choice under consistency axioms by testing revealed preferences axioms using prices estimated at the individual level. The estimation of these individual prices is performed on a match of households' expenditures surveys panelized over four years with a Time Use survey, which allows the computation of a proxy for full prices at the individual level that operates in the test of rationality. A large number of preferences are revealed on this dataset compared to the literature, while the violations of rationality seem to be very limited. Finally, a demand system is estimated on the population of rational households and compared to the estimation on non-rational.

Key words : revealed preference, rationalization, domestic production, full price, strong separability.
JEL classification : C14, C33, D11, D12, D13, J22

Introduction

Rationality axiom have been mainly tested in the revealed preferences framework either using macro- data (recent tests have been for instance performed on money aggregates), or on semi-grouped data (Famulari, 1995). The tests which have been previously operated on micro-data usually conclude that the results of these tests are highly dependent on the dataset (aggregated vs microdata) and the number of periods. For instance, in the test performed on a four years Polish panel (Diaye et al., 2008) with prices changing with the socio-economic type of the households, violations of the integrability condition (the so called strong – SARP – or General – GARP – axioms of revealed preferences) are generally also violations of the anti-symmetry weak axiom (WARP) whenever the number of period is small. It appears also that results depend more on changing environments (unemployment, family size, location…) than on the agent or household's characteristics. Finally, a common trend on incomes reduces the violation potential (see a discussion by Chambers and Echenique, 2016, Chapter 5). It is thus crucial to operate the test either on a long panel, or in case of a short panel with prices characterized by large variations between periods and individuals.

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We propose in this paper to create proxies of full prices on a Polish panel matched with a Time Use survey, applying a generalization of Becker's model of the allocation of time model in order to model the substitution between time and monetary resources and estimate the opportunity cost of time for each household in the survey. Two types of non-parametric tests will be performed: first, the usual tests of the stability of preferences by the weak axiom (WARP) and the integrability of revealed preferences (i.e. their rationalization by a utility function) by the acyclicity axiom GARP; second, tests of the strong separability of the utility which may allow to calculate price elasticities by the Frisch's method based on income elasticities and the income flexibility (the inverse of the relative risk aversion indicator).

Having listed all households the behavior of which conforms to the assumptions of stable and integrable preferences, we will be able to estimate the demand function for this sub-population and to compare the income and price elasticities to those estimated for the whole population (containing non-rational households). In a second step, the test for strong separability define another more restricted sub-population for which the Frisch formulas are used to calculate price elasticities in order to compare them to the price elasticities estimated on full prices.

The domestic production model gives rise to full prices at the individual level which gives a solution to the problem of observing highly variable prices[3]. A previous paper on a similar Polish panel (Diaye et al., 2008) used local prices which depended both on the socio-economic characteristics of the household (4 social categories which are correlated with the household's location) and on 16 quarterly changes through four years of observation, giving rise to 64 price observations. Another possible method would be to use unit prices (derived from the observation of expenditures and of the corresponding quantities consumed), as proposed by Deaton to estimate price elasticities. The drawback lies here in the fact that these unit values only exist for food products (and are distorted by quality effects).

Section 1 presents the axioms which are tested. Section 2 gives the empirical definition of full prices while section 3 presents the application of rationality tests to this particular data. Section 4 introduces the dataset and the matching procedure between the family budget and the time use surveys. The last section contains the results of the non-parametric tests and the estimation of a demand system on those households which do not violate the rationality tests.

Section 1. Axioms of revealed preferences

The weak axiom (WARP) and the strong Ville-Houthakker axiom (SARP) are considered together with the Afriat-Varian Generalized axiom (GARP) which is close to SARP.

[3] Blundell et al. (2003) propose to solve the problem of the power of the tests (the fact that few situation allow to compare periods or individuals between which the price changes overrate the income changes) by using Engel curves to adjust the data. The calculation that we propose of individual full prices avoids this difficult operation.

Presentation of the WARP, SARP and GARP tests[4]

Let $D = \{(x_i, p_i)\}_{i=1}^{N}$ be a data set including bundles of goods $x_i \in R_+^n$ purchased at price $p_i \in R_+^n$. In order to define WARP (Weak Axiom of Revealed Preference), GARP (Generalized Axiom of Revealed Preference) and SARP (Strong Axiom of Revealed Preference), we need to recall the definition of the so-called revealed preference R :

$$\forall\, x_i, x_j, x_i \; R \; x_j \; if \; p_i x_j \leq \; p_i x_i \qquad (1)$$

The interpretation of R is the following: a bundle of goods x_i is *revealed preferred* to x_j , at the price vector p_i if x_j could have been chosen by the agent while x_i is chosen. Indeed at the price vector p_i, x_j is less expensive than x_i; but the latter is the one which is observed chosen by the agent.

Let us also state the definition of the so-called strict *revealed preference RS* :

$$\forall\, x_i, x_j, x_i \; RS \; x_j \; if \; p_i x_j < \; p_i x_i \qquad (2)$$

Let T(R) be the transitive closure of R, the SARP requires T(R) to be antisymmetric, the WARP (Samuelson 1948) requires the revealed preference relation R to be antisymmetric while the GARP requires the bilateral asymmetry of T(R) and RS[5] :

$$\forall\, x_i, x_j \in X, x_i \; T(R) \; x_j \; \rightarrow \; not(x_j \; RS \; x_i) \quad (3)$$

It is obvious that SARP implies GARP and WARP[6]. Also, GARP and SARP are empirically identical since the equality in relations (1) and (2) cannot be observed on empirical data (except if the strict revealed preference is defined by the Afriat index, i.e. if $p_i x_j < \; \theta p_i x_i$ with $\theta < 1$ in (2))[7]. However there is no relationship in general between GARP and WARP. When a data set $D = \{(xi, pi)\}_{i=1}^{N}$ fulfills SARP then there exists a stable local non-satiated order (a transitive, asymmetric, complete preference) which rationalizes the data set x (weak rationalization by GARP, strong rationalization by SARP, see Chambers and Echenique). Therefore, there exists a utility function that rationalizes the data set D. When the data set fulfills WARP then there exist a stable locally non-satiated complete and antisymmetric preference which rationalizes the data. This means that there is a function that rationalizes the data (but nothing can be said about the nature of this function).

[4] The test of strong separability is discussed in Appendix D.

[5] See Varian 1982, Chambers and Echenique (2016, pp. 35-37) and Suzumura (2016) who name it HARP (Houthaker's axiom) and gives a slightly different definition of SARP.

[6] See Chambers and Echenique, Chapter 3, and Suzumura, p. 47.

[7] A much more interesting difference concerning the acyclity axioms is between Houthakker's axiom HARP, defined on discrete cycles (for a finite number of bundles in the cycle) and Ville's VARP, defined for continuous cycles. The latter is the true integrability condition, while Houthakker's SARP includes the quasi-concavity of the utility function (i.e. the negative semi-definitness of the Slutsky matrix). See on this point Gardes and Garrouste, 2006.

The test of the weak and strong axioms is generally made by filtering the population according to the corresponding inequalities between total expenditures for different systems of prices. This usual method necessitates a considerable amount of time, as the number of possible preferences increases rapidly with the number of couples of expenditures bundles which are compared. In our case, axioms are tested comparing four periods for each household, which sum to six possible violations of the weak axiom and 14 possible violations of the strong axiom for each household[8].

Section 2. Theory and empirical definition of the full prices

Becker (1965) considers a set of *final goods* which are the arguments of the consumer direct utility of the consumer. In order to simplify the analysis, Becker states that a separate activity i produces the final good i using a unique market good in quantity x_i and unit time t_i per unit of activity i (an hypothesis which can be generalized easily). Finally, the two factors of the domestic production functions can be supposed to be complementary, as in Becker's (1965) analysis, or substitutable, as in Gardes (2016). These two assumptions allow to derive full prices which can be measured on or dataset matching monetary expenditures and time use.

The opportunity cost of time is derived from a model of direct utility maximization in a domestic production framework (Gardes, 2014) which is presented in Appendix A. The opportunity cost of time ω is calculated in this model as the ratio of the marginal utilities for time over the marginal utility of money.

We propose two definitions of full prices, based on alternative assumptions about the substitutability between time and monetary expenditures. First, the derivation of the equation defining the full expenditures in terms of its monetary and time component allows calculating the full price for the model presented in section 2. In that case, full prices depend on the estimates of the parameters of the utility and domestic production functions α, β and the opportunity cost of time ω. The alternative definition relies on the complementarity of the two factors which characterizes the Becker's model. In that model, the full price is the sum of the monetary price and the cost of time to produce one unit of the final good. It can be approximated by a formula which depends only on the estimate of the opportunity cost of time and the household's monetary expenditure and time used for the domestic production of the final good. Finally, a relation is established between the two definitions.

2.1. Full prices for substitutable factors

Following Becker and Michael (1983, equation 10 p. 383)[9], the full expenditure can be written as the sum of its monetary and time components:

[8] Boelaert (2015) proposed a new algorithm using graph theory, which diminishes dramatically the computation time and is thus much more performant in case of abundant data.
[9] Suggested by Anil Alpman (see Alpman and Gardes, 2017).

$$\mathbf{p^{f1}_{iht} z_{iht} = p_{it} x_{iht} + \omega_{ht} t_{iht}} \qquad (4)$$

with p^f and p the full and the monetary prices corresponding to the quantities z and x of the commodity and of the corresponding market good, for commodity i, household h and time t (the time index will be omitted thereafter as we consider households surveyed during the same period).

The full price is the derivative of the full expenditure over z, which writes for the Cobb-Douglas specification of the domestic production functions:

$$\mathbf{p^{f1}_{iht} = p_{it} \frac{\partial x_{ih}}{\partial z_{ih}} + \omega_{ht} \frac{\partial t_{ih}}{\partial z_{ih}}} \qquad (5)$$

The optimization program (equation AI.1) gives rise to the first order condition:

$$\mathbf{\frac{t_{ih}}{x_{ih}} = \frac{p_{ih}}{\omega_{ih}} \frac{\beta_{ih}}{\alpha_{ih}}}$$

corresponding to equations (6) in section 1. Writing the quantity of the commodity $\mathbf{z_{ih}}$ in terms, either of t or x, gives:

$$\mathbf{t_{ih} = \frac{1}{a_i} z_{ih} \left(\frac{p_{ih}\beta_{ih}}{\omega_h \alpha_{ih}}\right)^{\alpha_{ih}} \text{ and } x_{ih} = \frac{1}{a_i} z_{ih} \left(\frac{\omega_{ih}\alpha_{ih}}{p_h \beta_{ih}}\right)^{\beta_{ih}}} \qquad (6)$$

so that the full price becomes:

$$\mathbf{p^{f1}_{iht} = \frac{1}{a_i} p_{ih}^{\alpha_{ih}} \omega_{ht}^{\beta_{ih}} \left\{ \left(\frac{\beta_{ih}}{\alpha_{ih}}\right)^{\alpha_{ih}} + \left(\frac{\alpha_{ih}}{\beta_{ih}}\right)^{\beta_{ih}} \right\}} \qquad (7)$$

This derivation of $\boldsymbol{\omega}, \boldsymbol{\alpha}$ **and** $\boldsymbol{\beta}$ at the individual level allows identifying the full price for each household ($\mathbf{a_i}$ **being supposed constant across the population**).

2.2. Proxies of full prices for complementary factors

Becker’s full price can be written:

$$\mathbf{p^{f2}_{iht} = p_{it} + \omega_h \tau_{iht}} \qquad (8)$$

with $\boldsymbol{\tau_{ih}}$ the time use necessary to produce one unit of the activity i. Suppose that a Leontief technology allows the quantities of the two factors to be proportional to the activity:

$\mathbf{x_{ih} = \xi_{ih} z_{ih}}$ **and** $\mathbf{t_{ih} = \theta_{ih} z_{ih}}$, so that: $\mathbf{t_{ih} = \tau_{ih}} \boldsymbol{x}_{\mathbf{ih}}$ with $\mathbf{\tau_{ih} = \frac{\theta_{ih}}{\xi_{ih}}}$

This case corresponds to an assumption of complementarity between the two factors in the domestic technology, which allows calculating a proxy for the full price of activity i by the ratio of full expenditure over its monetary component:

$$\pi_{iht} = \frac{(\mathbf{p_{it}}+\boldsymbol{\omega}_{\mathbf{ht}}\boldsymbol{\tau}_{\mathbf{ih}})\mathbf{x_{ih}}}{\mathbf{p_{it}x_{ih}}} = \frac{\mathbf{p_i}+\boldsymbol{\omega}_{\mathbf{h}}\boldsymbol{\tau}_{\mathbf{ih}}}{\mathbf{p_{it}}} = \mathbf{1} + \frac{\boldsymbol{\omega}_{\mathbf{h}}\boldsymbol{\tau}_{\mathbf{ih}}}{\mathbf{p_{it}}} = \frac{\mathbf{1}}{\mathbf{p_{it}}}\mathbf{p^{f2}_{iht}} \qquad (\mathbf{9})$$

Note that this second definition can be also derived from equation (2) with $\frac{\partial \mathbf{x_{ih}}}{\partial \mathbf{z_{ih}}} = \mathbf{1}$ and $\frac{\partial \mathbf{x_{ih}}}{\partial \mathbf{z_{ih}}} = \boldsymbol{\tau}_{\mathbf{ih}}$. Under the assumption of a common monetary price p_i for all households in a survey made during the same period, this ratio contains all the information on the differences of full prices between households deriving from their opportunity cost for time $\boldsymbol{\omega_h}$ and the coefficient of production $\boldsymbol{\tau}_{\mathbf{ih}}$. If the monetary price p changes between households or periods, the full price can be computed as the product of this proxy π_{ih} with p_{ih}: $\mathbf{p^{f2}_{ih}} = \mathbf{p_{iht}}\boldsymbol{\pi}_{\mathbf{ih}}$. With these definitions, it is possible to measure the full prices, observing only monetary and full expenditures by equation (6).

The first definition of prices corresponds to a complete substitution between the two factors in the model which is used in section 1 to estimate the opportunity cost of time, since the Cobb-Douglas domestic production functions are characterized by a unitary elasticity of substitution between the two factors[10]. It relies on the estimation of three parameters: $\boldsymbol{\alpha}, \boldsymbol{\beta}$ **and** $\boldsymbol{\omega}$. On the other hand, the second definition supposes no substitution between the two domestic production factors but it may give a more robust measure of the full prices since it depends only on the estimation of the households' opportunity cost of time $\boldsymbol{\omega}$. Both definitions of the full prices will be used in the estimation. However, there exists a simple relation between these two definitions of the full prices. Using equations (6) we obtain:

$$\mathbf{p^{f1}_{iht}} = \frac{\mathbf{1}}{\mathbf{a_i}}\mathbf{p_i^{\alpha_i}}\left(\frac{\mathbf{m_{ih}}}{\boldsymbol{\omega}_{\mathbf{ht}}\mathbf{t_{ih}}}\right)^{\boldsymbol{\beta}_{\mathbf{i}}}\left\{\mathbf{1} + \frac{\boldsymbol{\omega}_{\mathbf{h}}\mathbf{t_{ih}}}{\mathbf{p_{it}}}\right\} \qquad (\mathbf{10})$$

so that their logarithmic transforms differ only by $\boldsymbol{\beta_i}\,\mathbf{log}\,\frac{m_{ih}}{t_{ih}}$ on a cross-section:

$$\mathbf{log\,p^{f1}_{iht}} = \mathbf{constant} + \boldsymbol{\beta}_{\mathbf{i}}\,\mathbf{log}\,\frac{\mathbf{m_{ih}}}{\boldsymbol{\omega}_{\mathbf{h}}\mathbf{t_{ih}}} + \mathbf{log}\,\boldsymbol{\pi}_{\mathbf{ih}} \qquad (\mathbf{11})$$

with prices $\mathbf{p_{it}}$ set to one for all commodities observed in one survey.

[10] Note that the empirical evidence on the elasticity of substitution between time and money in the domestic production functions (see Hamermesh, 2008 and Canelas et al, 2015) gives rather estimates between 0.4 (for food) and 1 for other expenditure groups.

Two hypotheses were necessary to derive full prices from monetary and time expenditures: first, the domestic production functions are supposed to be either Leontief functions with constant coefficients of production (for the second definition) or Cobb-Douglas functions (for the first definition); second no joint production exists, which may be more easily verified for broad categories of activities such as housing and eating.

An important difficulty for such an application of the domestic production model lies in the valuation of time. A Cobb-Douglas specification of the utility and of the domestic production functions allows estimating locally (for each household) the opportunity cost of time by means of the first order conditions for the substitution between time and monetary resources used for the domestic production. Estimations for France (see Gardes, 2014) show that this estimated opportunity cost is close in average to the minimum wage rate and is positively indexed on the household's net wage and on income (conditional to net wage).

Two hypotheses are necessary to derive full prices from monetary and time expenditures: first, domestic production functions are supposed to be Leontief functions with constant production coefficients (note that the domestic production functions are specific for each household) or Cobb-Douglas production functions the parameters of which are estimated locally; second no joint production exists using a common monetary or time expenditure, which may be more easily verified for broad categories of activities such as housing and food (see Pollack-Wachter, 1976, for a discussion).

2.3. Quality effects

Quality effects are likely to exist in full price and expenditure data. Indeed, an increase (in the cross-section dimension i.e. between two households) of the full price for commodity (activity) i may result either from the difference (between the two agents) of the opportunity cost ω or from the difference of their time allocated to activity i. Both causes may increase the quality of this activity, by means of an increased productivity (which can be supposed to be positively related to ω) or of the time devoted to i. This endogenous quality appears in the same form as in Deaton's technique used to estimate price-elasticities on local prices after removing the quality incorporated in unit values. In our matched dataset, local prices are replaced by the individual full prices for each household and the quality effects are removed later for a second version of the tests using the Deaton's procedure.

Section 3. Recovering revealed preferences

3.1. The case of substituable factors

The inequality which reveals preferences for an individual I between its choices in periods t and t' writes, with the full price defined by (4) and the quantity of the commodity z_i by (AI.1):

$$\mathrm{p}_{\mathrm{iht}}^{f_1} z_{iht} > \mathrm{p}_{\mathrm{iht}}^{f_1} z_{iht'} \leftrightarrow \sum_i \frac{1}{a_i} p_{it}^{\alpha_{ih}} \omega_{ht}^{\beta_{ih}} \left\{ \left(\frac{\beta_{ih}}{\alpha_{ih}}\right)^{\alpha_{ih}} + \left(\frac{\alpha_{ih}}{\beta_{ih}}\right)^{\beta_{ih}} \right\} m_i^{\alpha_i \gamma_i} t_i^{\beta_i \gamma_i}$$
$$> \sum_i \frac{1}{a_i} p_{it}^{\alpha_{ih}} \omega_{ht}^{\beta_{ih}} \left\{ \left(\frac{\beta_{ih}}{\alpha_{ih}}\right)^{\alpha_{ih}} + \left(\frac{\alpha_{ih}}{\beta_{ih}}\right)^{\beta_{ih}} \right\} m_{it'}^{\alpha_i \gamma_i} t_{it'}^{\beta_i \gamma_i}$$

which cannot be evaluated since the levels of the monetary prices p_{it} are unknown (except if all monetary prices are set to one for survey of period t).

Another way to test these inequalities relies on the definition of full expenditures given by (1):

$$\mathrm{p}_{\mathrm{iht}}^{f_1} z_{iht} = p_{it} x_{iht} + \omega_{ht} t_{iht} > \mathrm{p}_{\mathrm{iht}}^{f_1} z_{iht'} = p_{it} x_{iht'} + \omega_{ht} t_{iht'}$$

In that equation, only $p_{it} x_{iht'}$ is unobserved. We can suppose, as in the case of complementary factors, that the ratio of monetary prices $\frac{p_{it'}}{p_{it}}$ is measured by means of prices indices I_{it} so that $p_{it} x_{iht'} = \frac{I_{it}}{I_{it'}} p_{it'} x_{iht'}$ can be measured as soon as the monetary expenditure for market good i: $p_{it'} x_{iht'}$ is observed in the matched surveys.

3.2. The case of complementary factors: full prices $p_{iht}^{f_2}$ with their proxies π_{iht}

Suppose the actual (monetary) price for one unit of commodity i in period t is noted p_{it}^m but is not observed: only a price index $I_t p_i^m$ is observed for that commodity. The actual price thus writes $p_{it}^m = \mu_{it_0} \mathrm{I_t p_i^m}$ where the unknown multiplier μ_{it_0} equals the actual price in t_0 (unobserved variables are in italic). With π_{ih} being the proxy for the full price of the commodity for individual h, the full price can be written: $p_{iht}^{f_2} = \mu_{it_0} \mathrm{I_t (p_i^m) \pi_{iht}}$. The inequalities used to test the axioms of revealed preferences are based on the value of consumptions bundles in different period with prices corresponding either to the current period, or to another period when the consumer have chosen another bundle. These values can be computed as soon as the values of each commodity (measured with the current price $p_{ith}^{f_2}$ or with the price for another period $p_{it'h}^{f}$): $dep_{iht}^{f_2} = p_{iht}^{f_2} x_{iht}$ and $p_{iht'}^{f_2} x_{iht}$ are known for all couples of periods t and t' (note that μ_{it_0} and thus p_{iht}^{f} are not observed). These values can be written in terms of monetary and full expenditures as follows:

$$\mathrm{p}_{\mathrm{ith}}^{f_2} \mathrm{x_{iht}} = \mu_{\mathrm{it_0}} \mathrm{I_t (p_i^m) \pi_{iht} x_{it}} = \mu_{\mathrm{it_0}} \mathrm{I_t (p_i^m) C_{ith}^f}$$

with $\mathrm{C_{iht}^f = \pi_{iht} x_{it}} = \frac{dep_{iht}^m}{p_1^m}$ the full expenditure for commodity i by agent h in t computed with the proxies of full prices.

$$p^{f_2}_{iht}x_{iht'} = \mu_{it_0}I_t(p^m_i)\pi_{iht}x_{it'} = \mu_{it_0}I_t(p^m_i)\frac{\pi_{iht}}{\pi_{iht'}}C^f_{iht'} \quad (12)$$

Finally, this consumption writes: $p^{f_2}_{ith}x_{iht'} = \frac{dep^f_{iht}}{dep^m_{iht}}\frac{I_t(p^m_i)}{I_{t'}(p^m_i)}dep^m_{iht'}$ where all these ratios are observed.

The inequality between consumption in t' (valued by prices in t) and total consumption in t (valued by the same prices) is thus: $p^{f_2}_{ith}x_{iht'} < p^{f_2}_{ith}x_{iht}$ which is computable knowing the price index I(p) and monetary and full expenditures (valued by the proxies π of full prices).

Section 4. The dataset

The Polish panel of family expenditures contains 3052 households over four years (1997-2000). This panel is matched with one Time use survey conducted in 2003-2004 over approximately 10 000 households (20 000 individuals)[11]. Six activities have been defined: food, housing, clothing, transport, leisure and various expenditures (including health services). Their monetary, time and full expenditures as well as full prices differ both among households and across periods, as shown in Appendix. These two surveys and the matching procedure, based on a correspondence between households with the same demographic structures, are described in Appendix B.

Work in Progress: full descriptive analysis of the dataset and the variability of full pricest

Section 5. Results of the rationality tests

3.1. Rationality axioms (WARP, SARP)

These results concern the comparison between the four annual observations (1997-2000) for each household and full prices p^{f_2} for the case of complementary factors. First, a large number of preferences are revealed by full expenditures and full prices: 46 % of potential preference relation are realized over the whole population (31 % for the smaller of the two preferences between two periods), which is much larger than in the dataset where prices have a more limited variations between the period for which the household is observed (for instance 64 possible values for the prices defined by region and profession of the Polish panel studied by Diaye et al. (2015).

In spite of the large number of revealed preferences, the violations of the weak axiom concern only 18% of households in the whole population, which corresponds to an average violation equal to 4% for each of the six possible changes between two years (taking into account multiple violations for one household, which amount to 22% of all violating households). This means that comparing two periods the probability that revealed preferences

[11] The dataset and the match of the two surveys have been prepared by C. Starzec.

change is only 4%. This figure appears to be greater but rather comparable to those obtained in the analysis on individual data using prices across regions and professions (8.5% of violating households for the population studied in Diaye et al., 2008).

The violations of the strong axiom are much scarcer: for only 3.9% of the population are revealed preferences not integrable (representable by a utility function). All these cases correspond to households violating the stability of preferences condition (the weak axiom). Half of these violations of the strong axiom concern households characterized by multiple cycles of preferences.

These violations of the weak and the strong axiom seem to be significantly greater for aged households, as if possible changes of their living conditions impose new contraints or different conditions to their choices through time. The violation rate increases with the family size, bachelor being more rational than couples with children in these tests.

Table1
Violation rates of rationality axioms (%)

	(1) % Warp violation	(2) % Multiple Warp violation in (1)	(3) % Sarp violation	(4) % Multiple Sarp violation in (3)	(5) % Sarp violation without Warp violation	(6) % Sarp violations in (1)	Size of the population
All population	17.90 (0.69)	21.57 (0.74)	3.90 (0.35)	47.06 (0.90)	0 (0)	21.76 (0.75)	3052
Single	28.75 (2.56)	24.44 (2.43)	7.67 (1.50)	50.00 (2.83)	0 (0)	26.67 (2.50)	313
Couples without children	19.39 (1.24)	25.25 (1.36)	4.80 (0.67)	48.98 (1.56)	0 (0)	24.75 (1.35)	1021
Couples with children	15.14 (0.89)	17.96 (0.95)	2.72 (0.40)	40.91 (1.22)	0 (0)	17.96 (0.95)	1618
Young	13.88 (1.38)	22.99 (1.68)	3.03 (0.68)	36.84 (1.93)	0 (0)	21.84 (1.65)	627
Middle-aged	14.86 (0.90)	17.24 (0.96)	2.82 (0.42)	45.45 (1.26)	0 (0)	18.97 (0.99)	1561
Aged	26.39 (1.50)	25.44 (1.48)	6.48 (0.84)	51.79 (1.70)	0 (0)	24.56 (1.46)	864

Standard errors into parentheses.
[1] Afriat index equal to e=0.
Young households: Head of the household younger than 36 years; middle-aged: between 36 and 55; aged: older than 55.

The power of the test of the strong axiom can be measured considering how the total expenditure must be modified in order to suppress acyclic behavior: the inequalities corresponding to the strong axiom will never be verified supposing that the total actual expenditures is decrease by a sufficiently small deflating coefficient $e = (1 - \lambda)$ with λ a positive number. The Afriat's efficiency index is the supremum over the all numbers e such that the revealed preference is acyclic (see Chambers and Echenique, 2016, pp. 72-73). This index equals 0.9, with only 0.13 % SARP and 1.7 % WARP violations remaining as shown in Table 2, which indicates that violations of rationality disappear for an error of less than 10% on expenditures.

Table 2
Afriat Efficiency Index

Index	1	0.975	0.95	0.90	0.875
% WARP violations	0.179	0.154	0.075	0.017	0.002
% SARP violations	0.039	0.035	0.016	0.0013	0

Work in progress: computation of other power indices (Bronars index)..

Work in Progress: (i) Test for full prices p^{f_2} for the case of complementary factors.
(ii) Bronars's power index.
(iii) Test of the *strong separability of the utility (see Appendix D);*
(iv) Tests in the cross-section on sub-populations of similar households grouped by neuronal non parametric methods or cells defined by the Crawford-Pendakur method.

4. Estimation of demand functions for the sub-population of rational households

The estimation of the Almost Ideal demand system for non-violating households (the weak and the strong axioms) gives rise to significantly different income and price elasticities from those derived for violating households for at least one third of the expenditures (see tables in Appendix C). This important result shows that the estimation of a complete demand system for the whole population, including one fifth of non-rational households (according to the non-parametric tests), gives biased estimates which could be corrected using non-parametric tests of rationality before the estimation.

Work in Progress: Comparison to the estimation on samples of the same size as the sub-population of violating households.

Conclusion

Full prices defined by means of an opportunity cost of time estimated in a domestic production scheme prove to be much more volatile than price indexes differentiated by region or households types (which moreover are usually not available). This unusual definition of prices may result in a small number of revealed preferences, because there exists no intersection of budget lines between the four observations for each household. In this case, the test of rationality axioms would be impossible with this type of data. The number of preferences revealed by full cost is thus a first important result of this analysis. These prices allow operating efficiently rationality tests for a dataset with a limited number of periods. The estimation of a demand system considering only rational households in term of revealed preferences differs significantly of the estimation over the whole population. This indicates that this procedure should be adopted whenever a time use survey is available together with family budget data.

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Appendix A
The estimation of the opportunity cost of time: A model of the allocation of time with an endogenous value of time

The model is fully presented in Gardes (2014, 2016).

The direct utility U depends on the consumption of final goods in quantities z_i which are produced by the household using the monetary expenditures used to buy the market goods and the time used for the corresponding activity (for instance transportation). Cobb Douglas specifications for the utility and the domestic production functions are chosen in order to allow the calculation of the opportunity cost of time as the ratio of the marginal utilities of monetary expenditures m_i and time use t_i for each activity i. Note that all the parameters of these two functions are estimated locally (i.e. for each household in the dataset). The optimization program is (all variables correspond to a household h which index is omitted in the equations):

$$\max_{m_i,t_i} u(Z) = \prod_i a_i z_i^{\gamma_i} \; with \; z_i = b_i m_i^{\alpha_i} t_i^{\beta_i} \quad \text{(A1)}$$

under the full income constraint:

$$\sum_i (m_i + \omega t_i) = w t_w + \omega(T - t_w) + V \qquad \text{(A2)}$$

with ω the valuation of time in the domestic production $T - t_w = \sum_i t_i = T_d$, w the wage rate, $wt_w$the household's wage and V other monetary incomes. Note that the opportunity cost of time ω may differ from the market wage w whenever there exist some imperfection on the labor market or if the disutility of labor is smaller for domestic production.

In order to estimate the opportunity cost of time, the utility function is re-written:

$$u(Z_i) = \prod_i a_i Z_i^{\gamma_i} = \prod_i a_i b_i \left[\prod_i m_i^{\frac{\alpha_i\gamma_i}{\sum \alpha_i\gamma_i}}\right]^{\sum \alpha_i\gamma_i} \left[\prod_i t_i^{\frac{\beta_i\gamma_i}{\sum \beta_i\gamma_i}}\right]^{\sum \beta_i\gamma_i} = a m'^{\sum \alpha_i\gamma_i} t'^{\sum \beta_i\gamma_i} \qquad \text{(A3)}$$

with m' and t' the geometric weighted means of the monetary and time inputs with weights $\frac{\alpha_i\gamma_i}{\sum \alpha_i\gamma_i}$ and $\frac{\beta_i\gamma_i}{\sum \beta_i\gamma_i}$. Deriving the utility over income Y and total leisure and domestic production time T_d gives the opportunity cost of time :

$$\omega = \frac{\frac{\partial u}{\partial T_d}}{\frac{\partial u}{\partial Y}} = \frac{\frac{\partial u}{\partial t'}\frac{\partial t'}{\partial T_d}}{\frac{\partial u}{\partial m'}\frac{\partial m'}{\partial Y}} = \frac{m' \sum \beta_i\gamma_i}{t' \sum \alpha_i\gamma_i} \frac{\frac{\partial t'}{\partial T_d}}{\frac{\partial m'}{\partial Y}} = \frac{\sum \beta_i\gamma_i}{\sum \alpha_i\gamma_i} \frac{T_d El_{t'/T_d}}{Y El_{m'/Y}} \qquad \text{(A4)}$$

The parameters of the utility (γ_i) and domestic production functions (α_i, β_i) are derived by the substitutions, first between time and money resources for the production of some activity, second between money expenditures (or equivalently time expenditures) concerning two different activities. These substitutions imply the system of equations:

$$m_i\gamma_j = m_j\gamma_i + \omega\gamma_i t_j - \omega\gamma_j t_i \qquad \text{(A5)}$$

which is estimated under the homogeneity constraint of the utility function: $\sum\gamma_i = 1$. In this system, the opportunity cost of time is over-identified, as well as all γ_j. The resulting estimates of the opportunity cost of time ω and the parameters γ_j of the utility function are then used to calculate α_i and β_i for each household, and finally the opportunity cost of time ω_h for each household in the population by equation (A4). These individual values of ω_h are finally used to value time and calculate the full expenditures and the proxies to full prices.

Appendix B
Description of the dataset and the matching procedure[12]

The Polish Family Budget Survey panel, 1997-2000

Household budget surveys have been conducted in Poland for many years. In the period analyzed, the annual total sample size was about 30 thousand households, which represent approximately 0.3% of all households in Poland. The data were collected by a rotation method on a quarterly basis. The master sample consists of households and persons living in randomly selected dwellings. This was generated by, a two-stage, and in the second stage, two-phase sampling procedure. The full description of the master sample generating procedure is given by Kordos and Kubiczek (1991).

Master samples for each year contain data from four different sub-samples. Two sub-samples started to be surveyed in 1996 and finished the four-year survey period in 2000. They were replaced by new sub-samples in 2000. Another two sub-samples of the same size were started in 1997 and followed through 2000. Over this four years period on every annual sub-sample it is possible to identify households participating in the surveys during all four years. The checked and tested number of households is 3052. The available information is as detailed as in the cross-section surveys: the usual socio-economic characteristics of households and individuals, as well as information on income and expenditures. A large part of this panel, containing demographic and income variables, is included in the comparable international data base of panels in the framework of the PACO project (Luxembourg) and is publicly available.

Prices and price indices are those reported by the Polish Statistical Office (GUS) for main expenditure items. They are observed quarterly and differentiated by 4 social categories: *workers, retired, farmers, and dual activity persons (farmers and workers).* This distinction implicitly covers the geographical distribution: workers and the retired live mostly in large and average size cities, farmers live in the countryside and dual activity persons live mostly in the countryside and in small towns. For food, price variations are taken into account at the individual observation level.

The period 1997-2000 covered by the Polish panel is the second panel covers years 1997 to 2000 corresponding to the beginning of post transition period in Poland.

The Time use survey 2003-2004

The Time Use Survey conducted in 2003-2004 is the fourth study on this subject matter in the history of the National Statistical Office (pol. GUS) and, at the same time, the first one executed in cooperation with Eurostat. The previous ones took place in the following years: 1968, 1976, 1984. They reflect the image of time management in the society of those years, however, due to methodological and organizational differences their results are difficult to compare.

[12] Prepared by Christophe Starzec.

Considering the usefulness of the Time Use Survey, it should be noted that time is often "a product" more in deficit than money, and its use affects many spheres of family, personal and professional life, and is also reflected, in the macro scale, in economic processes and social issues. Time management applies not only to professional work, ways of spending free time, activity in the family, non-profit work, but also such issues as the efficiency of public transport and transport network systems, participation in culture, education, sport or leisure activities. It is extremely important to obtain the knowledge about time management in territorial sections, both regional and class of locality, since many of the observed behaviours and practices concerning time management may be strongly conditioned territorially. It is also vital to obtain the knowledge about time management for various age groups or levels of education. Hence, the results of the Time Use Survey are utilized in different ways. They are the source of basic measures of level and of life quality of the population, information helpful when taking up activities regarding social policy and labour market. They are used to evaluate the value of working time in a household, they are helpful in estimating for national accounts and in preparing a national working time balance, there are of practical use, for instance, in judicial decisions or when designing laws relating to pension plans and insurance for people dealing with homework, they are also used in order to analyze territorial differences in Poland, as well as international comparisons.

The Time Use Survey in 2003-2004 was conducted on a representative sample of about 10000 households and 20000 persons. It covered persons aged 10 and more being members of selected households. Duration of the survey amounted to 365 days. Selected households conducted the survey for two days, i.e. one weekday (Monday-Friday) and one the week-end day (Saturday or Sunday). The respondents recorded performed activities, main and secondary, describing them freely in fixed 10-minute time intervals, entering persons accompanying them when they were performing those activities and the place of performing of them or the means of transport. Main and secondary activities, recorded by the respondent were then coded according to list of codes developed on the basis of *"Harmonized European Time Use Survey Guidelines"* by adequately trained persons.
The activities recorded by the respondents found their place in the following basic groups:

1. Personal care
2. Employment
3. Study
4. Household and family care
5. Voluntary work in organizations and outside them; helping others, involvement in the activities of an organization and religious practices
6. Social life and entertainment
7. Sport and outdoor activities
8. Hobbies and games – hobbies and using the computer
9. Mass media
10. Travel
11. Unspecified time use.

The study covered the main and secondary activities, a subjective assessment of the time as pleasant or unpleasant, the place of activities and people accompanying the respondent when performing the activities. Moreover, data making it possible to characterise the respondents and their households were collected.

Matching methodology

We use a dataset which combines at the individual level the monetary and time expenditures into a common, unique good and services consumption structure by a statistical match of the information contained in two surveys: Family Budget Survey Panel (FBSP, 1997-2000, (GUS) and Time Use Survey (TUS,GUS 2003). This rather difficult exercise needs some arbitrary assumptions about the substitution between time use and monetary expenditures (see Gronau and Hammermesh, 2008 for a discussion and Hamermesh, 2008 and Canelas et al., 2008, for estimations of the elasticity of substitution between these inputs of the domestic production).

We define 8 types of activities or time use types compatible with the available data both from FBSP and TUS for instance for eating wich combine *Eating and cooking time* (FTB) and *food consumption* (FES). Other activities correspond to housing, clothing, transport, education, health, leisure and various consumptions. As the needed information is present only into two separate data sources (FBSP and TUS) we need to combine them. The statistical matching between the surveys was done. In this article the statistical match is done regressing the times for each activity over a common set of socio-economic characteristics of households which are present in both surveys. The estimated coefficients are used to predict these times for each household in the all four waves of the Family Budget Survey Panel.

The main methodological issue of the aggregation of time use and monetary expenditures is the estimation of the monetary value of the time spent on different domestic activities. The alternative costs of work at home can be obtained in two ways: (i) by multiplying the time spent on a given activity by the regional labour market price of work for the similar market activities or (ii) by multiplying domestic work time by the potential earnings (*opportunity costs*) which can be obtained by the given person on the labour market. The first (i) solution is certainly more precise than the second (ii), but needs detailed information on local market hourly wages for all equivalents of domestic production activities, which are rarely available (see for example Havrylyshyn,1976, Goldschmidt-Clermont, 1993).

Two methods have been used in this article to value the time spent on domestic activities. First, this value is simply the official minimum wage rate for this period in Poland, which is supposed to indicate the market wage for a low productive job. In the second method, when the time use is supposed to be perfectly exchangeable between market and non-market activities, the *opportunity cost* of non-market work is computed as the average actual net wage rates for all working individuals in the family, or by their expected hourly wage rate on the labor market for not working individuals (estimated separately for man and woman using the two-steps Heckman7 method). In the individually matched sample the total value of non-monetary activities can be obtained directly by adding man's and woman's contributions. Both evaluation methods are adjusted for income taxes and the estimated numbers of working days and hours.

Descriptive analysis

Table B1. Full prices p^{f_2} for 1997

	Mean	Standard error	Minimum	Maximum
Food	1.2630	0.1932	1.0486	4.1289
Housing	1.2160	0.3908	1	11.5871
Clothing	1.3454	1.2629	1	40.1501
Transport-com.	3.0749	4.0208	1.0030	39.8457
Leisure	10.2019	7.2555	1.0447	39.9823
Other	1.5720	0.6167	1.0131	19.1814

Filter: suppression of 3.9% households (price for transport and communication and leisure greater than 40 due to a small value of monetary expenditure, see equation 8).

Appendix C
Estimation of a demand system on rational consumers

1. Estimation for all households

PREDICTED SHARES, BUDGET AND (UN)COMPENSATED OWN-PRICE ELASTICITIES

	shares b/se	budget b/se	u_price b/se	c_price b/se
wfoodom97	0.402***	0.674***	-2.605***	-2.335***
	(0.026)	(0.029)	(0.097)	(0.124)
whousin~97	0.153***	1.752***	-2.763***	-2.495***
	(0.023)	(0.135)	(0.249)	(0.269)
wtransp~97	0.101***	1.454***	-1.021***	-0.875***
	(0.009)	(0.097)	(0.034)	(0.036)
wclothi~97	0.066***	1.288***	-1.065***	-0.980***
	(0.004)	(0.151)	(0.025)	(0.031)
wleisur~97	0.155***	0.840***	-1.232***	-1.102***
	(0.006)	(0.136)	(0.025)	(0.010)
wotherom97	0.124***	0.803***	-2.130***	-2.031***
	(0.004)	(0.205)	(0.047)	(0.065)

* p<0.05, ** p<0.01, *** p<0.001

2. Estimation for households not violating the weak and strong axioms

PREDICTED SHARES, BUDGET AND (UN)COMPENSATED OWN-PRICE ELASTICITIES

	shares b/se	budget b/se	u_price b/se	c_price b/se
wfoodom97	0.402***	0.661***	-2.907***	-2.642***
	(0.030)	(0.034)	(0.137)	(0.169)
whousin~97	0.154***	1.712***	-3.068***	-2.805***
	(0.024)	(0.138)	(0.315)	(0.337)
wtransp~97	0.100***	1.461***	-1.014***	-0.868***
	(0.011)	(0.107)	(0.049)	(0.051)
wclothi~97	0.066***	1.406***	-1.080***	-0.987***
	(0.006)	(0.133)	(0.039)	(0.047)
wleisur~97	0.155***	0.882***	-1.243***	-1.106***
	(0.004)	(0.157)	(0.019)	(0.008)
wotherom97	0.124***	0.776***	-2.330***	-2.234***
	(0.006)	(0.215)	(0.079)	(0.103)

* p<0.05, ** p<0.01, *** p<0.001

3. Estimation for households violating the weak and strong axioms

PREDICTED SHARES, BUDGET AND (UN)COMPENSATED OWN-PRICE ELASTICITIES

	shares b/se	budget b/se	u_price b/se	c_price b/se
wfoodom97	0.410***	0.698***	-2.086***	-1.799***
	(0.066)	(0.074)	(0.159)	(0.227)
whousin~97	0.163*	1.895***	-2.258***	-1.950**
	(0.079)	(0.501)	(0.573)	(0.645)
wtransp~97	0.090***	1.488***	-0.989***	-0.856***
	(0.025)	(0.332)	(0.078)	(0.093)
wclothi~97	0.059***	0.937	-0.989***	-0.934***
	(0.006)	(0.666)	(0.046)	(0.031)
wleisur~97	0.157***	0.742*	-1.186***	-1.070***
	(0.028)	(0.334)	(0.117)	(0.086)
wotherom97	0.122***	0.823	-1.815***	-1.715***
	(0.008)	(0.691)	(0.104)	(0.095)

* p<0.05, ** p<0.01, *** p<0.001

Appendix D
A non-parametric test for strong separability

Definition: a direct utility function over two groups of commodities 1, 2, is *strongly separable* if it writes: $U(x)=u\{u_1(x_1), x_2\}$ with u increasing with u_1. Without this condition of strict monotonicity, the utility is said to be *separable*[13]. Gorman (1971) shows that strong separability is equivalent to the assumption that the set of bundles of the first group of commodities that are not worse than a given bundle x_1,is independent of the second group.

Afriat (1970)shows that the representative utility is strictly separable if and only if the following inequalities conditions are satisfied (see Russell, 1992, p. 126): given $(p^1,\ldots,p^T)$ and $(x^1,\ldots,x^T)>0$, there exist $(u^1,\ldots,u^T)$, $(u_1^1, u_1^2, \ldots, u_1^T)$, $(\beta^1,\ldots,\beta^T)$ and $(\lambda^1,\ldots,\lambda^T)>0$ such that: (a) $u_1^s \leq u_1^t + \beta^t p_1^t.(x_1^s - x_1^t), \forall\, s,t = 1,\ldots,T$

(b) $u^s \leq u^t + \lambda^t(\beta^t)^{-1}(u_1^s - u_1^t) + \lambda^t p_2^t.(x_2^s - x_2^t), \forall\, s,t$

These conditions are easily generalized to the case of n sub-groups instead of two: condition (a) must be verified for the (n-1) first, then (b) is written with (n-1) terms $\lambda^t(\beta^t)^{-1}(u_k^s - u_k^t)$ and the last term for x_n. The first n (a) conditions state that there exist, for each k sub-group, an utility function u_k^s rationalizing the choice for this sub-group. The last condition says that the overall utility writes under an additive separable scheme on these sub-utilities u_k^s and the last expenditure on sub-group n.

It can be easily verified that (a) implies that the second term in the left side of condition (b) is greater that the similar expression on expenditures:

$$(a) \rightarrow (b')\, u^t + \lambda^t(\beta^t)^{-1}(u_1^s - u_1^t) + \lambda^t p_2^t.(x_2^s - x_2^t) \leq u^t + \lambda^t p_1^t(x_1^s - x_1^t) + \lambda^t p_2^t.(x_2^s - x_2^t)$$

Which corresponds to the Garp condition for the whole expenditure over sub-groups 1 and 2. Thus, whenever this Garp condition (b') is not verified, (b) is also not verified which implies that the utility is not separable. Testing condition (b') thus allows to select the sub-population $\mathfrak{P}^{ns}$ in the reference population $\mathfrak{P}$ which does not fulfill this separabilty condition. Thus, it is possible to eliminate some among households in $\mathfrak{P}$ which do not verify the separability condition. Perhaps there remain in $\mathfrak{P}\backslash\mathfrak{P}^{ns}$ some households characterized by a non-separable utility. Rewriting (b') with an "Afriat index" μ<1 after the inequality may allow to recover part of these non-separable households:

$$(b'')\, u^t + \lambda^t(\beta^t)^{-1}(u_1^s - u_1^t) + \lambda^t p_2^t.(x_2^s - x_2^t) \leq \mu\{u^t + \lambda^t p_1^t(x_1^s - x_1^t) + \lambda^t p_2^t.(x_2^s - x_2^t)\}$$

We propose the following econometric strategy to recover the population of separable households:

(i) Test Garp condition (a) for all sub-groups of commodities;
(ii) Test (n times) condition (b') or (b") for the corresponding last sub-group;
(iii) Estimate income elasticities and the Frisch income flexibility on the sub-population $\mathfrak{P}\backslash\mathfrak{P}^{ns}$ of all households which have not been selected as non-separable.

[1313] See Russell (1992) and Blackorby et al. (1978) for a discussion of these definitions and their relationship.