# Rollover risk and the social value of credibility*

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**Abstract**

This paper studies information disclosure when a financial supervisor cannot commit to reveal the situation of the banking sector truthfully. I present a bank-run model in which a regulator performs a stress test and chooses whether to disclose bank-specific information or only to release an aggregate report about the financial system. Information can be biased at a cost – the higher this cost, the more credible the regulator. If credibility is not too low, the disclosure policy is state-contingent and information manipulation is often effective, even though investors are aware of the regulator's incentives. If credibility is low enough, the regulator loses the ability to avoid systemic runs by manipulating aggregate information and must release bank-specific reports (truthful or not) in all states, in which case not all runs can be avoided. If misreporting bank-specific information is possible and supervisors lack credibility, some banks have to fail the stress test to generate confidence in those who do pass the test. The results have implications for institutional design. Ex ante, a social planner would choose an interior level of credibility.

**Keywords:** bank runs, stress tests, information disclosure, information manipulation, signaling games, endogenous information.

**JEL classification:** D82, D83, G01.

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# 1 Introduction

Stress testing has become an important regulatory tool since the 2008 financial crisis. However, the desirability of stress test results disclosure remains controversial. Not surprisingly, the many stress tests conducted in the last decade in different countries differed in the way supervisors handled results disclosure.[1] There is also evidence that, in some cases, regulators have tried to conceal bad news from the market, releasing information that turned out to be somewhat misleading.

This paper studies information disclosure when a financial regulator cannot commit to report stress test results truthfully. Regulators may have incentives to mask an undesirable scenario as an attempt to avoid runs on banks that would otherwise be solvent. Investors are aware of those incentives, and internalize them in their rollover decisions. I investigate how the possibility of biasing stress test results affects disclosure policies and financial stability. Understanding the links between information manipulation and financial stability helps answer the question of how credible we want our regulators to be.

I present a bank-run model where a regulator performs a stress test to learn banks' types. The regulator can choose whether to disclose bank-specific reports or to only release a report on the aggregate situation of the banking system. The regulator has the prerogative to bias those reports at a cost that is increasing in the amount of distortion. The higher the cost of producing biased information, the more credible the regulator in the eyes of investors. By manipulating information, the regulator can try to avoid inefficient coordination failures, but investors anticipate the regulator's incentives and lower their expectations of the true state of the economy.

Although in a benchmark case where the regulator has full credibility the equilibrium is unique, introducing the possibility of information manipulation generates multiple equilibria in the model. However, it is still possible to derive meaningful predictions. As long as the regulator has enough credibility, any equilibrium features a state-contingent disclosure policy. In bad states – i.e., when the proportion of good banks is low – the regulator discloses bank-specific information (truthfully); in intermediate states, the regulator opts for opacity and aggregate information manipulation, that is, it releases a biased aggregate signal about the banking sector as a whole; in good states, there is opacity and no bias. Information manipulation may be effective: the regulator causes a signal-jamming that boosts investors

[1] For instance, the U.S. regulatory authorities disclosed very detailed bank-specific results in 2009 (SCAP stress test), and in a subsequent stress test two years later (CCAR, 2011) only the macro-scenario was published, with no bank-level results.

beliefs in states where they would be otherwise too pessimistic, at the cost of deteriorating beliefs in states where there is more than enough optimism to avoid any runs.

When credibility is too low, though, the regulator loses the ability to use the disclosure of aggregate information as a means of generating confidence. The only option left is to release bank-level reports, which exposes some banks to undesirable runs. Two distinct cases are considered: (i) a case where only aggregate reports are subject to bias, and (ii) a case where both aggregate and idiosyncratic reports may be manipulated. In the former, the low-credibility equilibrium features full transparency in every state, and the most solid banks always survive, while the most fragile ones always suffer runs. In the latter, the low-credibility equilibrium also features bank-specific reports disclosure, but some banks' types are misreported. The same proportion of banks is labeled as good at all times. In low states of the world, the regulator is able to misrepresent some low-type banks' situation and avoid a run on more than just the most solid banks. On the other hand, in good states of the world, where most banks have high returns, some good banks must be sacrificed and fail the stress test in order to generate enough confidence that the banks that do pass the test are indeed solid. In fact, the idea that some banks must always fail stress tests for the exercise to be credible comes up often in policy circles.[2]

These results have implications for institutional design. Consider a social planner who can choose the amount of credibility regulators have in a previous stage. Increasing credibility too much reduces welfare since it precludes the financial authority from enhancing coordination in many states. However, when credibility is too low any aggregate report is meaningless to investors, so the regulator has no option but to disclose more disagreggate reports, exposing some banks to runs (sometimes, even the most solid ones). The paper suggests a central planner would choose an interior level of credibility, in which bank supervisors still have some room for maneuver during a financial crisis, but they are credible enough not to make stress test results uninformative.

The results of the paper seem consistent with evidence from the European experience in the early 2010s. The stress test conducted by the Committee of European Banking Supervisors (CEBS) in 2010 covering 91 European banks pointed to a total capital need of €3.5bn. A subsequent assessment of just the Irish banks conducted by independent consultants revealed a total capital need of €24bn.[3] European authorities came out of this episode with harmed

[2]For instance, when asked by the Financial Times in an interview, Daniele Nouy – head of the Single Supervisory Mechanism – agreed that that for the [stress-test] exercise to be credible there have to be failures (Financial Times, 2014).

[3]See Schuermann (2014).

credibility. In a new round of stress tests performed by the European Banking Authority (EBA), CEBS's successor, the degree of disclosure was much more extensive than in previous episodes, with bank-level results made available to the public. There is a widespread idea that European regulators lacked the credibility to be successful in generating confidence among investors by disclosing aggregate results, so they had to rely on more transparency. As argued by Schuermann (2014), "because credibility of European supervisors was rather low by that point, only with very detailed disclosure, bank by bank, [...] could the market do its own math and arrive at its own conclusions."

In 2014, the European Central Bank took charge of the supervision of all stress tests conducted in the Eurozone under the Single Supervisory Mechanism (SSM). Through the lens of the model presented here, the creation of the SSM could be interpreted as a step in the direction of raising credibility. In a survey conducted by Bloomberg in December of 2013, about 90% of the analysts interviewed believed the subsequent round of stress tests under the supervision of the ECB would be more credible than the previous one, conducted by the EBA (Bloomberg, 2013).

The results delivered by the paper are in line with the evidence in two dimensions: (i) one should expect there to be bias in stress test results in at least some states, and (ii) one should expect a lot more bank-specific information disclosure, regardless of the state of the economy, when regulators lack credibility.

The motivation for this paper is strengthened by the evidence presented by Bird et al. (2015). The authors empirically estimate the amount of bias in FED's stress test results. Their results point to the presence of both positive and negative bias in bank-specific disclosures in the CCAR's stress tests in the aftermath of the financial crisis.

This paper builds on Bouvard, Chaigneau and Motta (2015), who study information disclosure in a bank-run model in the tradition of Diamond and Dybvig (1983). Their model helps explain the well documented evidence that information disclosure is usually state-contingent, with more transparency in bad times and more opacity in good times. They also suggest regulators have a commitment problem in their disclosure policy that leads to excess opacity and a higher probability of systemic crises. In a very different environment, Goldstein and Leitner (2015) reach similar results. They suggest no disclosure is needed in good times, but some disclosure may be necessary in bad times to avoid a market breakdown. Revealing too much information in their model reduces welfare by destroying risk-sharing opportunities.[4]

[4] In a dynamic approach, He and Manela (2014) study private information acquisition and withdrawals decisions in a dynamic model with rumors about banks. They suggest that public provision of information

This paper contributes to this literature by studying how the possibility of biasing reports affects disclosure policies and the stability of the financial system.

Also related to this paper is the work by Edmond (2013), who studies a regime-change model in which a dictator engages in propaganda to try to avoid being overthrown. The dictator can add a bias to the mean of a private signal citizens receive concerning the strength of the regime. Here, I use a similar information manipulation technology, although the variable the regulator manipulates is common knowledge and the mechanism through which manipulation affects beliefs is different (there is not only a hidden action, but also an observable action that conveys information to the market).[5]

In Goldstein and Huang (2016), there is also a policymaker that attempts to affect agents' beliefs, but the effectiveness of the policy requires commitment. In their regime-change game, a policymaker can commit to abandon the regime if fundamentals fall below a threshold, so the survival of the regime conveys good news, a Bayesian persuasion effect. Part of the results presented here are also in the spirit of those in the Bayesian persuasion literature (see Benoît and Dubra (2011) and Kamenica and Gentzkow (2011)), but the persuasion effect arises from the introduction of bias by an informed sender, and there is no commitment.

The paper is also connected to the literature on self-fulfilling runs and global games. Games with strategic complementarities and common knowledge of fundamentals usually present multiple equilibria. As first shown by Carlsson and Van Damme (1993), the introduction of private information in these environments may lead to a unique equilibrium. This approach, known as global games, has been applied to a wide variety of settings, including currency attacks (Morris and Shin, 1998), debt pricing (Morris and Shin, 2004), and bank runs (Goldstein and Pauzner, 2005). Several papers in this tradition analyze the effects on financial stability of different policy interventions, such as government guarantees, liquidity injections, capital requirements and monetary policy. Examples include Rochet and Vives (2004), Bebchuk and Goldstein (2011), Morris and Shin (2014) and Allen et al. (2015).

Here, global games techniques are used in the rollover game played among investors. However, as in Angeletos, Hellwig and Pavan (2006) and Angeletos and Pavan (2013), uniqueness is not guaranteed. As shown by Angeletos, Hellwig and Pavan (2006), analyzing policies as comparative statics in global games models can be misleading since policies may have a signaling role. Multiplicity of equilibria may be restored when information is endogenous.

might promote stability by crowding out information acquisition by private depositors that can lead to runs.

[5]Other models where costly misreporting by an informed party is strategically adopted include Kartik, Ottaviani and Squintani (2007) and Kartik (2009).

Another branch of the literature related to this paper is concerned with the social value of information. Morris and Shin (2002) examine how the precision of public signals impacts welfare in a beauty contest model (where agents aim to take an action appropriate to the underlying fundamental, but also similar to the actions of others). They show that greater precision of public information may be detrimental to welfare, since agents may place too much weight on public signals relative to their private information due to the coordination motive. In Angeletos and Pavan (2004), though, increasing the precision of public information is always welfare improving. Angeletos and Pavan (2007) shed some light on this matter by providing conditions under which one or the other result holds.[6] This paper analyzes how the credibility of public information affects welfare (the social value of credibility), since the information agents observe can be biased by an informed party.

Here, the focus is on the the study of information disclosure in a setting where there is rollover risk and strategic complementarities among investors, but the paper abstracts from other frictions potentially important for the discussion. For example, Bond and Goldstein (2015) suggest that the disclosure of information by the government might reduce the informativeness of market prices, which could be useful to guide government policies. For a comprehensive analysis of the trade-offs involved in the disclosure of sensitive information from stress tests, see Goldstein and Sapra (2013). The authors debate pros and cons of stress test results disclosure in light of the existing literature on the subject and provide some policy recommendations.

The remaining of the paper is organized as follows. Section 2 presents the model. Section 3 describes the set of equilibria, presents comparative statics and analyzes welfare when only aggregate information is subject to manipulation. Section 4 characterizes the equilibria when both aggregate and bank-specific misreporting is possible and studies optimal credibility in this case. Section 5 concludes.

# 2 The model

This section presents the basic environment, which is based on Bouvard, Chaigneau and Motta (2015) and Morris and Shin (2000), describes the information manipulation and disclosure technologies, and characterizes investors' rollover decisions.

[6] For the role of information precision in regime-change games, see Iachan and Nenov (2015).

## 2.1 Environment

Consider a continuum $[0,1] \times [0,1]$ of investors endowed with one unit of the consumption good and a continuum of banks (or financial institutions in general) indexed by $i \in [0,1]$. All agents are risk neutral and there is no discounting among the three periods, $\tau = 0,1,2$. Investors can either store their unit of the consumption good or invest it in a bank at $\tau = 0$. Banks have access to a long-term investment opportunity that yields a gross return of $1 + r_i$ at $\tau = 2$ for each unit invested (the maximum scale is one). The net return $r_i$ is the sum of a common term and an idiosyncratic component. Specifically,

$$r_i = \tilde{\theta} + d_i,$$

where

$$d_i = \begin{cases} \eta & \text{with probability } p, \\ -\eta & \text{with probability } 1-p, \end{cases}$$

i.e., banks can be of a high or a low type. The proportion of high-quality banks $p$ is drawn from an uniform distribution on $[0,1]$, so at $t = 0$, $\mathbb{E}_0[d_i] = 0$, and the expected (net) return of the banking system is simply $\tilde{\theta}$. We can interpret the realization of $p$ at $\tau = 1$ as an aggregate shock to the return of the banking sector. As long as $\tilde{\theta} > 0$, the long-term investment technology is better than storage, so all agents choose to invest their unit of the consumption good in banks, and each bank has a unit-mass continuum of investors.

Banks face rollover risk. Investors can keep their unit invested in the bank until $\tau = 2$ or withdraw it at the interim period. Early liquidation of the long-term project is costly: if a proportion $l_i$ of bank $i$'s investors withdraws their unit at $\tau = 1$, the per-unit return at $\tau = 2$ becomes $r_i - \gamma l_i$, $\gamma > 0$. Hence, there are strategic complementarities in investors decisions, in the sense that their willingness to demand early withdrawal depends on their expectation about how other investors will behave.

The expected return of the banking system $\tilde{\theta}$ is drawn from a normal distribution with mean $\theta$ and precision $\beta_\theta$. At $\tau = 1$, investor $j$ receives a noisy signal about the fundamental, $x_j = \tilde{\theta} + \xi_j$, where $\xi_j$ is independently distributed (across investors in each bank) according to a normal distribution with mean 0 and precision $\beta_\xi$. Private information is important in this setting to obtain uniqueness of equilibrium in the rollover game. Since the introduction of the noisy signal is simply an equilibrium selection device, I will focus on the limiting case with both precisions approaching infinity, satisfying $\beta_\theta^2/\beta_\xi \to 0$.[7]

Moreover, the following parametric assumption is imposed.

**Assumption 1.**

$$0 < \theta - \eta < \theta < \frac{\gamma}{2} < \theta + \eta.$$

This assumption ensures (i) early liquidation is inefficient for both high- and low-quality banks; (ii) low-quality banks suffer run while high-quality banks do not when agents learn their bank's type[8]; and (iii) $\theta$ is low enough to trigger a run if agents act based solely on their prior information about $p$ (which captures situations in which stress tests are a desirable regulatory tool).[9]

## 2.2 Regulation

There is a financial regulator who has access to bank-specific information at the beginning of period $\tau = 1$ (obtained by performing a stress test, for example). The regulator learns $\{d_i\}_{i\in[0,1]}$ and, consequently, $p$. The informed regulator can then choose between two disclosure regimes: releasing bank-specific reports to investors about each bank's type, or only releasing an aggregate report about the proportion of good-type banks (p). Regarding information manipulation, two cases are considered: (i) only aggregate reports are subject to bias; and (ii) both aggregate and bank-level reports can be manipulated.

The regulator's objective is to maximize the expected return of the banking system minus the cost of manipulating information, which is proportional to the amount of distortion. If only aggregate information manipulation is possible, the cost of inflating the aggregate report by $b$ is $cb$, $c \geq 0$. In case bank-specific information manipulation is also allowed, the cost of misrepresenting the type of a proportion $b$ of banks is $cb$, regardless of whether bank-specific reports are disclosed or only the aggregate report is released.

The cost $c$ is meant to capture the expected punishment, or consequences in general, for the regulator in case its report is found to be false. One can interpret it as a pecuniary cost

[7]As common in the global-games literature, uniqueness of equilibrium in coordination games arises when there is enough private information relative to public information. This condition ensures that, even on the limiting case with both precisions approaching infinity, there is a unique equilibrium in the rollover game played among investors. For more details, see Morris and Shin (2000) and Morris and Shin (2003).

[8]This property will follow from the result presented in Lemma 1.

[9]If this third part of the assumption is eliminated, that is, if $0 < \theta - \eta < \frac{\gamma}{2} < \theta$, a regulator that can choose in a previous stage whether or not to perform a stress test would choose not to do so, since there would be no runs if investors had only their prior information about $p$. Hence, the interesting case is when Assumption 1 holds.

(being fired, paying a fine or losing a bonus, having the agency responsible for the stress test shut down) or a reputational cost.[10] Higher values of $c$ are interpreted as higher *credibility*: the higher the cost of fudging stress test results – or the more the regulator has at stake – the more credible the reports in the eyes of investors.

The regulator's action set is described in further details in Sections 3 and 4, since two alternative assumptions concerning the costs of information manipulation are made.

**Timeline** To summarize, the timeline of the model is as follows:

- At $\tau = 0$, investors deposit their unit of the consumption good in the banks, and banks invest in the long-term investment project;
- At $\tau = 1$, $\{d_i\}_{i\in[0,1]}$ is realized and observed by the regulator (through a stress test); the regulator makes its manipulation and disclosure decisions; after observing the regulator's disclosure choice and the available reports, investors in each bank decide whether to roll over or not.
- At $\tau = 2$, payoffs are realized.

## Investors' rollover decision

Before characterizing the optimal manipulation and disclosure policies for the regulator, we must determine the optimal rollover decision for investors. Standard global games techniques give us the following result.

**Lemma 1** (Morris and Shin, 2000)**.** *Investors in bank $i$ choose to roll over at $t = 1$ whenever $\theta + \mathbb{E}_1[d_i] > \gamma/2$, and to run whenever the inequality is reversed.*

*Proof.* See Appendix A.1. □

Lemma 1 implies that, if investors know their bank's type, investors in bank $i$ run if $d_i = -\eta$, and do not run if $d_i = \eta$, i.e., only the high-quality banks survive. Lemma 1 also implies that when there is uncertainty about banks' types, investors run whenever

$$P_1(d_i = \eta) < \bar{p} \equiv \frac{\gamma - 2(\theta - \eta)}{4\eta}, \tag{1}$$

[10]For instance, if a central banker has among its tasks the supervision of the banking sector and it has its reputation harmed by a controversial stress-test disclosure, the population could distrust its commitment to price stability, and even compromise its ability to remain in office.

and roll over if the inequality is reversed, where $P_1(d_i = \eta)$ is the probability investors assign for their bank being of high type conditional on the information they have at period $\tau = 1$.

With this result in hand, we can then solve the regulator's problem under each of the assumptions regarding the possibility of manipulating information: when only aggregate information manipulation is possible, and when both aggregate and idiosyncratic information are subject to bias.

# 3 Aggregate information manipulation

This section considers the case in which the regulator must report bank-specific information truthfully in case it opts to disclose it. However, the regulator cannot commit to tell the truth about the aggregate situation of the banking sector if it decides to withhold bank-specific information. It can transmit a delusive report about the aggregate state at a cost.

We can interpret full disclosure of bank-specific information as making spreadsheets containing data on the banks' financial situation available to the public. In such cases, fudging the information about the financial situation of a specific bank would be very difficult. The assumption here is that the cost of doing so is infinity. Still, manipulating aggregate information may be feasible since the public in general has no means of gathering information on all financial institutions to check if the aggregate report released by the regulator is accurate. Section 4 deals with the possibility of biasing bank-specific information as well.

After observing banks' types, the regulator makes two decisions: it chooses an observable action – disclosing or withholding bank-specific information – and a hidden action – how much bias (if any) to add to the aggregate report. Formally, after observing $p$ the regulator chooses $t \in \{0, 1\}$, where 1 represents transparency and 0 opacity, and a bias $b$ in order to form a report $z = p + b \leq 1$.[11] Investors always observe $z$, and in case of transparency, they also observe $\{d_i\}_{i \in [0,1]}$.

The regulator takes into account that if it chooses to disclose bank-specific information there is a run on low-quality banks while high-quality banks survive, and if it chooses opacity there is a massive run on banks if condition (1) holds, and no runs at all if the reversed inequality holds – as shown in Lemma 1.

I will look for Perfect Bayesian Equilibria in which investors play a cutoff strategy. When observing opacity, investors run if $z$ is smaller than a cutoff $\hat{z}$, and rollover their investments

[11] Naturally, the regulator cannot claim more than 100% of banks are of high quality, so the choice of $b$ must satisfy the restriction that $z \leq 1$.

if $z > \hat{z}$.[12]

## Regulator's problem

The financial regulator's objective is to maximize the return of the banking sector net of the costs of information manipulation. Suppose the regulator expects investors to run under opacity if $z < \hat{z}$ and to roll over otherwise. After observing the stress test results $\{d_i\}_{i\in[0,1]}$, and consequently $p$, the regulator chooses a bias $b$ (in order to form a report $z = p + b \leq 1$) and also chooses $t \in \{0, 1\}$, where 1 represents transparency (i.e., disclosure of bank-specific information) and 0 represents opacity (i.e. only an aggregate report $z$ is released). Given the agents' cutoff strategy $\hat{z}$, the regulator's problem can be written as

$$\max_{t\in\{0,1\},\, b\in[0,1-p]} U\left(p, t, b|\hat{z}\right) = t\left\{1 + p(\theta+\eta)\right\} + (1-t)\left\{1 + \left[\theta + (2p-1)\,\eta\right]\mathbb{1}_{\{p+b\geq\hat{z}\}}\right\} - cb \quad (2)$$

where $\mathbb{1}$ is an indicator function that assumes value 1 if the condition in braces is satisfied and 0 otherwise. The term multiplying $t$ is the aggregate return of the banking sector when the regulator chooses transparency – in which case bad banks suffer a run and good banks' long-term investments mature and pay the net return $(\theta + \eta)$. The term multiplying $(1 - t)$ is the aggregate return when the regulator chooses opacity – in which case either there is a massive run and all investors get 1 (which happens if $z < \hat{z}$) or there is no run and both good and bad banks' investments mature (which happens if $z \geq \hat{z}$).[13] The last term is the cost of manipulating information.

The solution to this problem yields two policy functions, $t(p)$ and $b(p)$, for a given $\hat{z}$. In equilibrium, $\hat{z}$ must be such that investors' rollover decisions are rational, taking into account the regulator's manipulation and disclosure policies. Beliefs are pinned down by Bayes rule whenever possible and off-the-equilibrium beliefs satisfy $\mu(p|z, 0) < \bar{p}$ for all $z < \hat{z}$, where $\mu(p|z, t)$ is investors' expectation of $p$ given the aggregate report and the disclosure decision observed. Furthermore, I impose any equilibrium must satisfy the intuitive criterion proposed by Cho and Kreps (1987), which requires off-the-equilibrium beliefs to be reasonable in the sense that investors cannot assign a positive probability to states in which the deviation is

[12] In Appendix A.3, it is shown that a strategy in which investors run when $z$ is larger than a cutoff and roll over otherwise cannot constitute an equilibrium.

[13] In any equilibrium, it must be that when $z = \hat{z}$ agents choose to roll over. Otherwise, the regulator's problem would not have a solution. When deciding the level of information manipulation, in some states the regulator would try to minimize $cb$ subject to $b > \hat{z} - p$.

dominated by the equilibrium strategy for the regulator.[14]

### 3.1 Benchmark case: full credibility

As a benchmark, consider the case where the regulator must report aggregate information truthfully.[15] In this case there is a unique equilibrium, in which the regulator chooses to disclose bank-specific information in bad times, and not to disclose it in good times. Proposition 1 states this result.[16]

**Proposition 1.** *When the regulator must report all information truthfully, in the unique equilibrium investors run under opacity if $z < \bar{p}$ and do not run otherwise. The regulator chooses transparency for $p < \bar{p}$ and opacity otherwise.*

*Proof.* See Appendix A.2. □

Since manipulating information is not possible, observing $z$ perfectly informs investors about the true state of the financial system. Whenever the aggregate state is high enough, revealing it is sufficient to avoid a run on all banks. Hence the regulator withholds banks-specific information, since disclosing it would cause an undesirable run on low-quality banks. Whenever the state is such that revealing aggregate information would not prevent a massive run on banks, the regulator chooses to disclose bank-specific information and avoid a run on high-quality banks.

### 3.2 General case: limited credibility

Now, consider the case with limited credibility (i.e., when the cost of manipulating information is finite). The regulator may have incentives to disclose an aggregate report that inflates the proportion of good banks in an attempt to help agents coordinate in rolling over. However, agents are aware of those incentives and may discount the report they receive when forming their expectations about the health of the financial system.

As before, there are equilibria in which the disclosure policy is state-contingent: the regulator reveals bank-specific information when the state is low and discloses only aggregate

[14]The intuitive criterion rules out some equilibria and allows us to derive sharp qualitative predictions. For instance, without the intuitive criterion there would always be an equilibrium with full transparency in all states in the general case presented in Section 3.2.

[15]This assumption is equivalent to making $c \to \infty$ in the general case in Section 3.2.

[16]This result is similar to the one in Bouvard, Chaigneau and Motta (2015) when the aggregate state of the banking system is common knowledge in their framework.

information in higher states. However, due to the possibility of information manipulation, now agents are not certain about the true value of $p$ when they observe opacity, so the disclosure policy will have a signaling role. By not disclosing bank-specific information the regulator conveys that the state of the financial system is not that bad (if $p$ was very low, the regulator would have disclosed more information). But under opacity investors may also expect $p$ to be smaller than the value reported, considering the regulator's incentives to inflate the aggregate state.

The signaling role of the disclosure policy in this setting generates multiplicity of equilibria. Proposition 2 below describes the set of equilibria in the general case where the regulator has limited credibility.

**Proposition 2.** *Let $p^b(z)$, $p^{b*}$, $z^*$, and $\bar{z}$ be given, respectively, by*

$$p^b(\hat{z}) = \frac{c\hat{z} - \theta + \eta}{c - \theta + \eta}, \tag{3}$$

$$z^* \equiv \frac{\gamma(c - \theta + \eta) - 2(\theta - \eta)(c - \theta)}{2\eta(2c - \theta + \eta)}, \tag{4}$$

$$p^{b*} \equiv \frac{c\gamma - 2(\theta - \eta)(c + \eta)}{2\eta(2c - \theta + \eta)}, \tag{5}$$

*and*

$$\bar{z} \equiv \frac{\gamma(c - \theta + \eta) - 2(\theta - \eta)(c - \theta - \eta)}{4c\eta}. \tag{6}$$

*The set of Perfect Bayesian Equilibria satisfying the intuitive criterion is as follows:*

1. *If $c > \theta - \eta$, for each $\left\{\hat{z}, p^b\right\}$ with $\hat{z} \in [z^*, \bar{z}]$ and $p^b = p^b(\hat{z})$ there is an equilibrium in which the regulator's strategy is given by*

$$t(p) = \begin{cases} 1 & \text{for } p \in [0, p^b), \\ 0 & \text{for } p \in \left[p^b, 1\right], \end{cases} \tag{7}$$

*and*

$$b(p) = \begin{cases} \hat{z} - p & \text{for } p \in \left[p^b, \hat{z}\right], \\ 0 & \text{for } p \in [0, p^b) \cup (\hat{z}, 1]. \end{cases} \tag{8}$$

2. *If $c < \theta - \eta$, the unique equilibrium features $\hat{z} > 1$ (investors run under opacity for all $z \in [0, 1]$) and $p^b > 1$ (regulator chooses transparency for all $p$).*

3. *If $c = \theta - \eta$, for each $\left\{\hat{z}, p^b\right\}$ with $\hat{z} = 1$ and $p^b \in \left[p^{b*}, 1\right]$ there is an equilibrium in which the regulator's strategy is given by (7) and (8). Full transparency, as in 2., is also an equilibrium.*

*Proof.* See Appendix A.3. □

For simplicity of the exposition, I hereafter refer to an equilibrium simply as a pair $\left\{\hat{z}, p^b\right\}$, given that these two thresholds are sufficient to characterize regulator's and investors' strategies in equilibrium: investors run under opacity whenever $z < \hat{z}$; under transparency, they run on low-quality banks and do not run on high-quality banks; the regulator chooses transparency for $p < p^b$, opacity and $b(p) = \hat{z} - p$ whenever $p \in \left[p^b, \hat{z}\right]$, and opacity with $b(p) = 0$ whenever $p > \hat{z}$. An equilibrium featuring transparency for all $p$ can be represented by $\left\{\hat{z}, p^b\right\}$ with $\hat{z}, p^b > 1$. Notice $p^{b^*} = p^b\left(z^*\right)$ and $\bar{p} = p^b\left(\bar{z}\right)$, so $\left\{z^*, p^{b*}\right\}$ and $\{\bar{z}, \bar{p}\}$ are the two extreme equilibria in item (i) (with the lowest and the highest thresholds $\hat{z}$, respectively). Also notice $z^*$ and $\bar{z}$ are smaller than one for any $c > \theta - \eta$.

Proposition 2 shows that, despite the existence of multiple equilibria, some meaningful predictions arise. If the credibility level is sufficiently large, all equilibria feature transparency in low states, opacity with information manipulation in intermediate states and opacity with truth-telling in high states. In both intermediate and high states, there are no runs. Under transparency, agents run on low-quality banks, but high-quality banks survive (as shown in Lemma 1). The intuition for this state-contingent policy is that if the proportion of good banks is too small, a truthful aggregate report would not convince investors to roll over their investments in all banks, and biasing the aggregate report in a sufficient amount to avoid a run would be too costly. Thus the best option for the regulator is to choose transparency and at least save good banks. For intermediate levels of $p$, lying about the aggregate state pays off: by inflating the report $z$, the regulator is able to avoid a coordination failure and save all banks. In high states, no manipulation is necessary, and there are no runs under opacity even under a truthful aggregate report.

If the credibility level is too low, though, the regulator loses the ability to help agents coordinate by manipulating aggregate information: the only equilibrium is the one with full transparency, in which case only the best banks survive. By withholding bank-specific information, a regulator with low credibility would trigger a run on all banks even if the proportion of good banks was equal to one. Investors discount too heavily the aggregate report released by a regulator that does not enjoy much credibility, so no report $z$ would ever convey that the state is good indeed. Anticipating that, the best strategy for such regulator is to opt for full transparency, minimizing losses and avoiding runs on the most solid banks.

**Sketch of the proof**

Suppose investors run under opacity whenever $z < \hat{z}$, and roll over otherwise. Also, given Lemma 1, under transparency an investor rolls over if her bank is of high type, and runs otherwise. Given these strategies, the regulator's choice boils down to choosing among (i) opacity and no bias; (ii) opacity and just the sufficient amount of bias to avoid any runs; and (iii) full transparency (i.e., bank-specific reports disclosure and no bias). Figure 1 depicts the regulator's payoffs under each of these options as a function of the aggregate state, $p$. $U^o$ represents the regulator's payoff under opacity and no bias, $U^t$ its payoff under transparency, and $U^b$ its payoff under opacity and manipulation. The optimal policy is the one that yields the upper contour of the three curves. Such policy is the one described in (7) and (8), and $p^b(\hat{z})$ is the function obtained by equating $U^t\left(p^b\right)$ and $U^b\left(p^b\right)$, for a given $\hat{z}$.

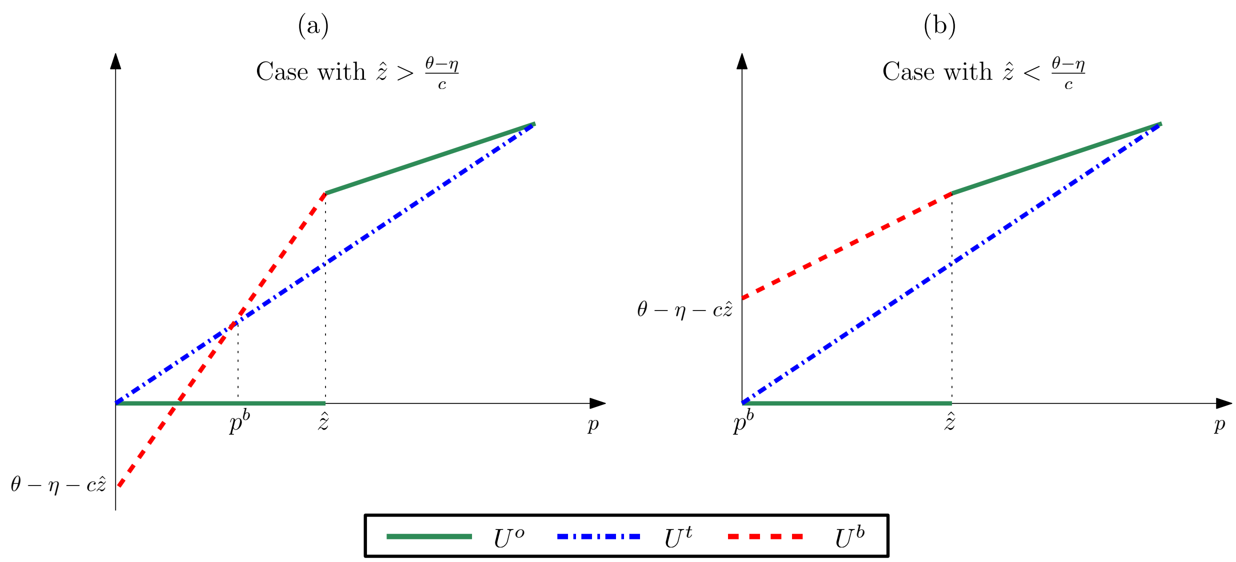



Figure 1: Regulator's payoffs for a given $\hat{z} \in [0, 1]$

Remember $\hat{z}$ is an equilibrium object. One must check if investors observing opacity and $z \geq \hat{z}$ are in fact willing to roll over. Considering the regulator's policies, conditional on the observation of opacity and $\hat{z}$ an investor expects $p$ to be uniformly distributed on the interval $\left[p^b, \hat{z}\right]$. If $c < \theta - \eta$, for any $\hat{z}$ the regulator's payoffs are as depicted in Figure 1b, and thus

$p^b = 0$. In this case, investors' expectation of $p$ conditional on opacity and $\hat{z}$ would be $\hat{z}/2$, which is always smaller than $\bar{p}$ (remember $\bar{p} > 0.5$). Hence, there cannot be any equilibrium featuring opacity in this case: the only equilibrium is the one with transparency in all states. If $c > \theta - \eta$, though, there exist values of $\hat{z}$ such that the regulator's payoffs are as depicted in Figure 1a. The set of equilibria in this case consists of all pairs $\left\{p^b, \hat{z}\right\}$ such that $p^b = p^b(\hat{z})$ and $\mathbb{E}\left[p|\hat{z}, t = 0\right] = \left(p^b + \hat{z}\right)/2 \geq \bar{p}$.

Finally, notice that under the state-contingent disclosure policy there are signals $z$ that are off-the-equilibrium, that is, they are never observed on the equilibrium path. Namely, all signals $z \in \left(p^b, \hat{z}\right)$ are received with probability zero. To sustain such an equilibrium with state-contingent disclosure, one must consider that off-the-equilibrium beliefs satisfy $\mu\left(p|z, 0\right) < \bar{p}$ for all $z < \hat{z}$. In many cases, those beliefs are completely unreasonable. As shown in Appendix A.3, only the equilibria in which $p^b \leq \bar{p}$ survive the intuitive criterion.

### Comparative Statics

The next proposition presents some comparative statics on the set of equilibria with respect to the cost of information manipulation when credibility is not too low, so all equilibria feature state-contingent disclosure policies. Conditional on $c$ being smaller than $\theta - \eta$, the equilibrium does not depend on $c$, since there is full transparency for all $p$. I check how the boundaries of the equilibrium set change as we change $c$.

**Proposition 3** (Comparative statics)**.** *If $c > \theta - \eta$, we have the following:*

$$\frac{\partial z^*}{\partial c} =< 0, \ \frac{\partial p^{b*}}{\partial c} > 0,$$

$$\frac{\partial \bar{z}}{dc} < 0, \ \frac{\partial \bar{p}}{\partial c} = 0.$$

*Proof.* See Appendix A.4. □

Analyzing the effect of an increase in $c$ on equilibrium strategies helps build some intuition about the model. If we assume investors always play according to the equilibrium with the smallest $\hat{z}$ (i.e., $\hat{z} = z^*$), an increase in $c$ causes a decrease in their threshold $\hat{z}$, that is, investors run less under opacity. The same comparative statics holds if we assume investors play according to the largest $\hat{z}$ (i.e., $\hat{z} = \bar{z}$), or any convex combination of the two (holding constant the weights as we change $c$).[17]

[17] If $\tilde{z} = \alpha z^* + (1-\alpha)\bar{z}$ and $\tilde{p}^b = p^b\left(\tilde{z}\right)$, $\alpha \in (0, 1)$, regardless of $c$, then the comparative statics is qualitatively identical to the case where the equilibrium is $\left\{z^*, p^{b*}\right\}$.

The effect on the equilibrium $p^b$'s is the opposite: for $c > \theta - \eta$, the range of $p$ for which transparency is optimal is (weakly) increasing in $c$ (it is strictly increasing in $c$ in the equilibrium with the smallest $\hat{z}$, $\left\{z^*, p^{b*}\right\}$, as well as in any convex combination of the two extremes). It means that, as long as $c > \theta - \eta$, the more credibility the regulator has, the larger the set of states for which transparency is the optimal policy.

In other words, a decrease in the credibility level makes (i) investors run more under opacity, and (ii) the regulator rely more on opacity and information manipulation. The reason is that as $c$ decreases, lying becomes a cheaper instrument to enhance coordination, so the regulator uses it more. However, lying also becomes less effective the smaller the cost of manipulating information. Investors discount the report more heavily, and thus require a higher $z$ not to run under opacity.

That is why the range of transparency only shrinks as we reduce $c$ down to some point. If credibility is too low, specifically if $c < \theta - \eta$, the aggregate report is completely disregarded by investors, so opacity would trigger a massive run on banks. Transparency is then the only option left in any state.[18]

In short, the relationship between the level of credibility and the range of states in which there is transparency in equilibrium is non-monotonic. Figure 2 illustrates the comparative statics with respect to $c$ when the equilibrium being played is the one with $\hat{z} = z^*$ (if in the equilibrium investors play some convex combination of $z^*$ and $\bar{z}$, the qualitative results are the same).

As $c$ decreases, the range of $p$ for which the regulator's optimal strategy is to introduce a bias on aggregate information to avoid runs increases, while the transparency region decreases, as well as the region where there is opacity but no bias. However, there is a discontinuity in the relationship between credibility and the amount of transparency. If $c$ is too low, the equilibrium features transparency in all states.

### Discussion

It is interesting to understand the mechanism through which information manipulation in this setting can be effective in boosting coordination. Consider the case with $c > \theta - \eta$ (as in Proposition 2, item 1). In a range of intermediate states, the manipulation policy is such that for all states in that range the report issued by the regulator is the same, as can be seen in the left panel of Figure 3. By pulling together all those states, the regulator causes

[18] When $c$ is exactly $\theta - \eta$, as shown in Proposition 2 there are equilibria with full transparency for all $p$ as well as equilibria with state-contingent disclosure policies.

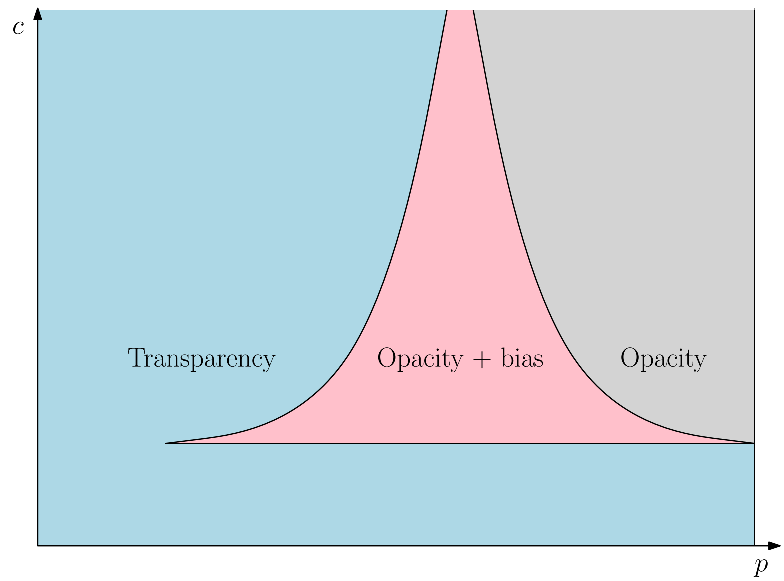



Figure 2: Comparative statics

a signal-jamming that boosts investors' beliefs in a region where beliefs would be otherwise too pessimistic, and deteriorates beliefs in a region where there would be optimism to spare. The blue area in the right panel of Figure 3 represents the gain in beliefs due to information manipulation, while the pink area represents the damage in beliefs.

For any $p > \bar{p}$, if information manipulation was not a possibility, there would be no runs under opacity and a truthful aggregate report. When manipulation is possible, anticipating the regulator's incentive to inflate the report, investors only rollover if the report is at least $\hat{z} > \bar{p}$. That is because they know all intermediate states are pulled together: in any state $p \in \left[p^b, \hat{z}\right]$, the issued report is $\hat{z}$. This implies a twist in beliefs in an intermediate range of $p$ that makes it possible to avoid runs not only to the right of $\bar{p}$, but also in a range to the left of $\bar{p}$.

When $p < \bar{p}$, under a truthful report there would be a systemic run, but by manipulating information the regulator can improve confidence in the state of the economy in this region at the cost of worsening beliefs in states where there is more than enough optimism to avoid any runs. Hence, lowering investors' expectations of $p$ in higher states down to $\bar{p}$ may pay off. The size of the range where there is information manipulation in equilibrium depends on the cost $c$ for two reasons: the higher the cost, the less profitable it is for the regulator to use manipulation as a tool, but on the other hand, anticipating that the regulator is less prone to use it, the more effective manipulation is in increasing investors confidence.

If the cost of implementing information manipulation is too low (specifically, if $c < \theta - \eta$),

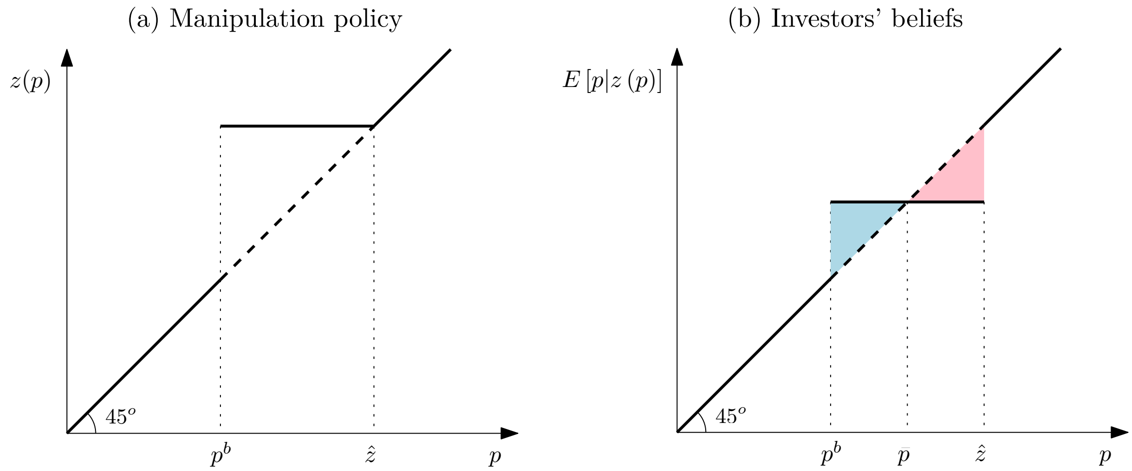



Figure 3: Aggregate report for a given $\hat{z} \in [0,1]$ and investors' beliefs

investors know the regulator would have incentives to use it even when $p$ is very low, so it becomes impossible to enhance coordination by pulling together low and high states. Even in very high states, the regulator loses the ability to convince investors the state is good indeed, because it would be too cheap to pretend this is the case if it was not. The only option left for the regulator is then to disclose bank-specific information and save the most solid banks at least.

The predictions of the model seem to be consistent with recent stress test disclosure episodes following the 2008 financial crisis. First, the model says there are always states in which there is information manipulation in equilibrium as long as credibility is not too low. Second, if the regulator's credibility is too low, we should expect a lot more transparency. European authorities responsible for regulating the financial system in the early 2010s released some controversial stress test results, as discussed in the introduction. The amount of capital the banking sector had to raise in order to be able to withstand possible downturns was found to be a lot larger than the regulators had previously announced. Regulators came out of these episodes with a harmed credibility. Stress tests performed later on adopted a much more transparent disclosure policy. For instance, the EBA's stress test in 2010/2011 exhibited a degree of disclosure much more extensive then previous episodes, making bank-level results available for download in spreadsheets, so that investors could check for themselves the banks' real situation. Analyzing the European experience through the lens of the model, we could say EBA had to opt for full transparency exactly because its credibility was too low at that point. The authorities were not able to generate confidence through the disclosure of aggregate results, and the only alternative was to expose weak financial institutions to save the strong ones.

## 3.3 Optimal credibility

The way credibility affects optimal information disclosure policies and investors' rollover decisions may have important implications for institutional design. When deciding which authority will be responsible for regulating the financial system and what will be its incentives, a social planner should consider how the credibility level affects the outcome of the coordination game among investors. Imagine the social planner can choose $c$, that is, the cost the financial regulator faces to manipulate aggregate reports, which can be thought of as the expected punishment or consequences imposed to a regulator for masking the situation of the banking system. A welfare analysis can tell us what would be the socially optimal level of $c$ from an ex ante perspective.

Assuming a given equilibrium $\left\{\hat{z}, p^b\right\}$ is being played, welfare in this economy can be written as

$$W = \int_0^{\min\left\{p^b,1\right\}} p(\theta+\eta)dp + \int_{\min\left\{p^b,1\right\}}^1 \left[\theta + (2p-1)\eta\right] dp, \tag{9}$$

which is the expected (net) aggregate return of the banking system ($W+1$ is total output or total consumption).

It is easy to see that the best equilibrium in terms of efficiency is the one with smaller $p^b$, that is, with $\hat{z} = z^*$ and $p^b = p^{b*}$ if $c \geq \theta - \eta$ (for smaller levels of $c$ the unique equilibrium features full transparency in all states). This equilibrium is the one where the regulator is able to avoid undesirable runs in more states.

Consider the case in which agents always play according to the best equilibrium. In the next subsection I discuss the issue of optimal credibility when this is not the case.[19] Proposition 4 determines the optimal credibility level from an ex ante perspective. It shows the planner would not opt for full credibility. An interior level of credibility allows the regulator to help agents coordinate by inflating the proportion of good banks, and it is good for society that the regulator has this prerogative. Hence, regulators' (expected) punishment for lying about the health of the banking sector should not be too harsh as to completely avoid information manipulation nor too low as to make opacity impractical. Figure 4 depicts the welfare as a function of the credibility level in this case.

[19] For instance, as long as the worst equilibrium is not played with probability one for all $c > \theta - \eta$, there is an interior credibility level that is optimal under the assumption that $c$ is picked from a countable set; neither full nor no credibility is optimal.

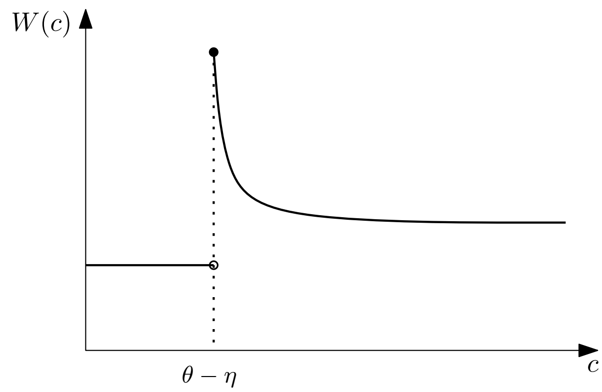



Figure 4: Welfare

**Proposition 4** (Optimal credibility)**.** *Suppose agents always play according to the best equilibrium. The socially optimal credibility level is* $c^* = \theta - \eta$*.*

*Proof.* See Appendix A.5. □

Proposition 4 can help explain why some financial regulators choose to delegate the responsibility for the performance of stress tests to independent consultants. One possibility is that the regulator lacks credibility and would have to disclose very detailed bank-specific information, while a more credible party – such as an international consultancy firm that probably would not jeopardize its credibility to avoid a run in some country's banks – may be able to only release aggregate results. Furthermore, one can also interpret the recent change in financial regulation in Europe – that put the European Central Bank in charge of all stress tests procedures from 2014 on – as an attempt to raise credibility in financial regulators in the Eurozone, since national regulators as well as some authorities such as the EBA did not enjoy sufficiently credibility to be able to have discretion over how much information to disclose.

### 3.3.1 Optimal credibility and multiple equilibria

Now, consider the case where agents can play according to any equilibrium $\left\{\hat{z}, p^b\right\}$. Figure 5 depicts the range of welfare that can be achieved for each value of $c$ depending on which equilibrium is selected.

Suppose there is a sunspot variable that determines which equilibrium will be played. Denote the implied distribution function over all possible equilibrium values of $p^b$ for each $c$ by $F(p^b; c)$. Also, assume $c$ must be chosen from a set $\mathcal{C}$ that is a discretization of $\mathbb{R}^+$.[20]

[20]Without requiring $\mathcal{C}$ to be a countable set, the planner's choice of $c$ could have no solution. For example, if the worst equilibrium happens with probability one when $c = \theta - \eta$ and the best one happens with probability

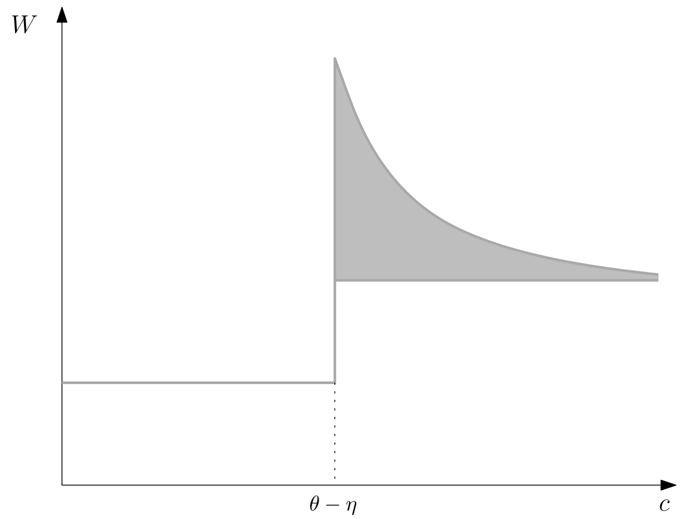



Figure 5: Welfare range

In this case, there is always an interior $c$ that is optimal, as illustrated in Figure 6. Squares represent the expected welfare for each level of $c \in \mathcal{C}$, that is, $\int W\left(p^{b'}\right) dF(p^{b'}; c)$, where $W(p^{b'})$ is the welfare when the the equilibrium value of $p^b$ is $p^{b'}$. The red square represents the maximum welfare.

Notice it can be the case that the maximum is not strict. For instance, if $F(.)$ assigns probability one to the worst equilibrium for all $c$, the planner would be indifferent between any $c \in \mathcal{C}$ satisfying $c > \theta - \eta$. Moreover, as long as the worst equilibrium happens with probability smaller than one for some $c > \theta - \eta$, the optimal $c$ lies in a bounded set, i.e., full credibility is not optimal.

The next section analyzes the case where both aggregate and bank-level reports are subject to bias. Although the equilibrium characterization is somewhat different for low levels of $c$, the main normative result of the paper remains the same: a social planner would not choose full credibility. However, the positive part of the results provides new insights when credibility is low.

# 4 Allowing bank-specific information manipulation

Now, suppose the regulator is also able to misreport bank-specific information. Specifically, assume the cost of misrepresenting the type of a proportion $b$ of banks is $cb$, regardless of whether bank-specific signals are sent out or only an aggregate report is released.

one when $c > \theta - \eta$, the planner would want to set $c$ as close as possible to $\theta - \eta$, satisfying $c > \theta - \eta$.

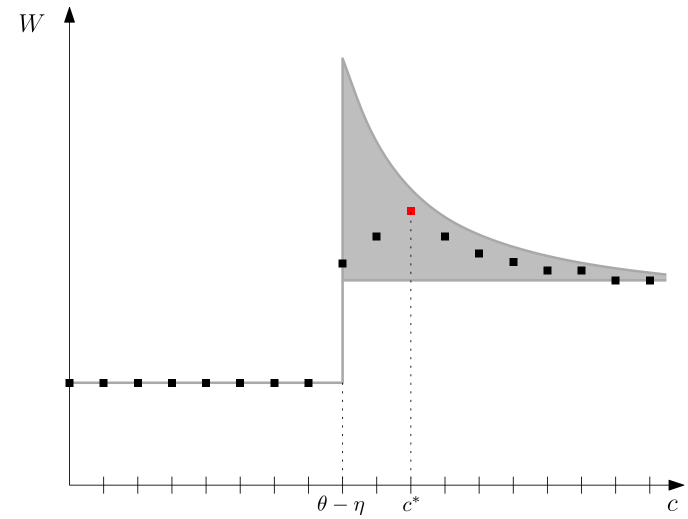



Figure 6: Optimal credibility

It is useful to restate the problem in the following way. Nature draws a type-profile $G \subseteq [0,1]$, such that $d_i = \eta$ for all $i \in G$ and $d_i = -\eta$ otherwise. In words, $G$ is the set of good banks. As before, $p$ denotes the proportion of banks of the good type, and therefore, it is the measure of $G$. After observing $G$, the regulator makes two choices: it chooses a $t \in \{0,1\}$ and a set $\hat{G} \subseteq [0,1]$ such that $n_i = \eta$ for $i \in \hat{G}$ and $n_i = -\eta$ otherwise. That is, $\hat{G}$ is the set of banks the regulator *claims* to be good. Choosing $t = 1$ means disclosing the bank-specific labels $n_i$ for all $i$ (thus investors observe the full $\hat{G}$). Choosing $t = 0$ represents the choice for opacity, in which case investors only observe the aggregate signal $z$, which is the measure of $\hat{G}$. The amount of bias the regulator introduces can now be denoted by $b = \int_0^1 \mathbb{1}_{\{n_i \neq d_i\}} di$, that is, it is the measure of banks whose types the regulator misrepresents (regardless of the disclosure policy $t$).

To summarize, the regulator observes $G$ and decides which indexes will be assigned to the group $\hat{G}$ and which will not; then, the regulator chooses whether to disclose the bank-specific reports (the full $\hat{G}$) or only the proportion of banks it claims to be good (the mass of the set $\hat{G}$).

## 4.1 Equilibria

Denote by $a(tn_i, z) \in \{0,1\}$ the strategy played by investors in bank $i$, where 0 represents running and 1 represents rolling over. It captures the fact that when $t = 0$, investors do not observe their bank-specific signal $n_i$, and when $t = 1$, they do.[21]

**Low credibility**

First, consider the case with $c \leq \theta - \eta$. The next proposition describes the equilibrium set in this case. An additional parametric assumption is needed to guarantee the existence of a pure strategy equilibrium. A sufficient condition for this to be the case is that Assumption 1 is replaced by the following:[22]

**Assumption 2.**

$$0 < \theta - \eta < \theta < \frac{\gamma}{2} < \theta + \ln(2)\eta.$$

The set of equilibria surviving the intuitive criterion in this case when manipulation costs are low is described in Proposition 5.

**Proposition 5.** *Consider $c < \theta - \eta$. Define*

$$\underline{z} \equiv \frac{c}{c + \theta + \eta} \tag{10}$$

*and $\tilde{z}$ as satisfying the following:*

$$\frac{\tilde{z}\left[1 - \ln\left(\tilde{z}\right)\right]}{2\tilde{z} - \tilde{z}\ln\left(\tilde{z}\right) + \left[1 - \tilde{z}\right]\ln\left(1 - \tilde{z}\right)} = \bar{p}. \tag{11}$$

*Also, let $B = [0,1] \setminus G$ be the set of banks of type $d_i = -\eta$.*

*The set of Perfect Bayesian Equilibria satisfying the intuitive criterion is as follows:*

*For any $z' \in [\underline{z}, \tilde{z}]$, there is an equilibrium in which investors decisions are given by*

$$a(0, z) = 0 \text{ for all } z, \tag{12}$$

$$a(-\eta, z) = 0 \text{ for all } z, \tag{13}$$

*and*

$$a(\eta, z) = \begin{cases} 1 & \text{if } z \leq z', \\ 0 & \text{if } z > z', \end{cases} \tag{14}$$

[21] Assuming investors observe the full $\hat{G}$ when $t = 1$ is analogous to saying they observe $n_i$ and $z$, since this is all they care about in order to make their rollover decision.

[22] Assumption 2 is sufficient, but not necessary. In the Appendix, more details are given on what would be a necessary and sufficient condition. Notice the only difference between Assumption 1 and Assumption 2 is the term $\ln(2)$, that is, now $\gamma/2$ is smaller than approximately $\theta + 0.7\eta$ instead of $\theta + \eta$.

*and regulator's decisions are given by*

$$t(p) = 1 \textit{ for all } p,$$

*and*

$$\hat{G}(p) = \begin{cases} G \cup F & p \leq z', \\ G' & p > z', \end{cases} \tag{15}$$

*where $F$ is any subset of $B$ of measure $z' - p$, and $G'$ is any subset of $G$ of measure $z'$. The regulator's policy implies*

$$z(p) = z' \textit{ for all } p.$$

*Proof.* See Appendix A.6. □

Proposition 5 states that, when $c < \theta - \eta$, in any equilibrium we have that: (i) investors run under opacity regardless of the aggregate report; (ii) investors observing a negative bank-specific report also run; and (iii) investors observing a positive bank-specific report roll over as long as the aggregate report is *low* enough. Notice the cutoff strategy in (iii) implies investors observing the good signal run if the regulator claims the proportion of good banks is too high. Also, any equilibrium has the property that the regulator chooses to send out bank-level reports in all states, as was the case in Section 3. However, now the regulator manipulates bank-specific information in a way that the proportion of banks that receive the high-type label is the same in all states – and it does so by misreporting the types of as few banks as possible.

Appendix A.6 presents the details on the construction of the equilibria and shows why no other set of strategies can constitute an equilibrium. Still, it is useful to discuss here why the set of strategies described in Proposition 5 are in fact consistent with equilibrium.

Fix investors' strategies as in (12), (13) and (14). Consider $p < z'$. If investors observing the high-type signal ($n_i = \eta$) roll over, then the marginal cost of saving a low-type bank is $c$ – that is, the cost of claiming a bad bank is good – and the marginal benefit of doing so is $\theta - \eta$. Since $c < \theta - \eta$, the regulator is always willing to save an additional (low-return) bank, but it can only do that up to some point: if it tries to save more than $z'$ banks, even investors observing $n_i = \eta$ would run.

Now consider $p > z'$. Given investor's strategies, the regulator must claim some of those $p$ good banks are in fact bad if it wants to achieve an aggregate proportion of good signals of $z'$, a necessary condition to avoid a massive run. If $z > z'$, even investors whose banks were

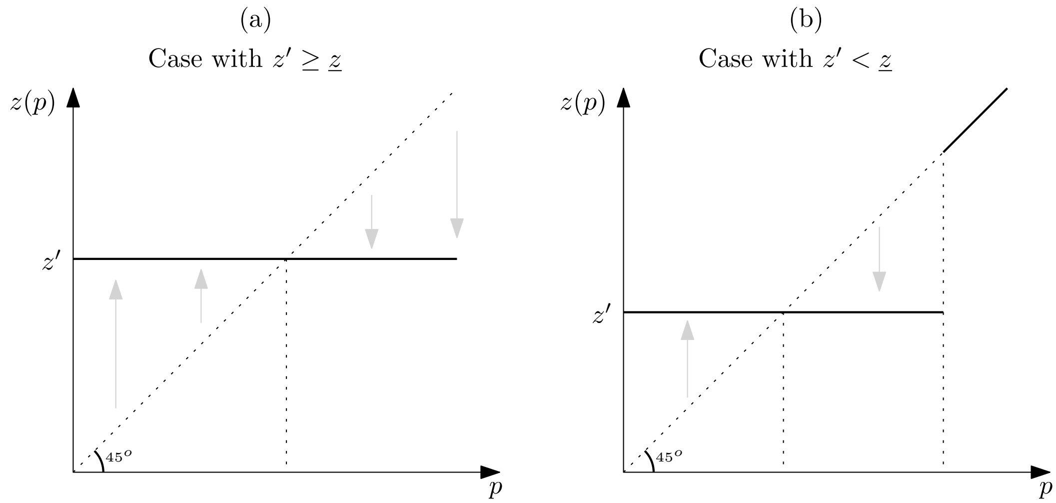



Figure 7: Regulator's strategy when $c < \theta - \eta$

labeled as good would run. But notice it may not always be worth bringing $z$ down to $z'$, that is, misreporting the types of $(p - z')$ good banks. The regulator only makes $z(p) = z'$ if the payoff of doing so is larger than zero, that is, if

$$z'(\theta + \eta) - c(p - z') \geq 0. \tag{16}$$

Figure 7 illustrates the $z(p)$ implied by the regulator's strategy.[23]

Suppose there is a region of states $[p', 1]$ in which labeling good banks as bad in a sufficient amount to achieve $z'$ is not worth it, as in Figure 7b. That cannot be an equilibrium, because investors observing $z > p'$ would know $\hat{G} = G$, that is, they would infer the regulator is sending out truthful signals about all banks, in which case investors observing $n_i = \eta$ would deviate from their equilibrium strategies and roll over. We must consider as candidates for equilibrium just the values of $z'$ such that (16) is satisfied for all $p$, which boils down to $z' \geq \underline{z}$.

Also, for the set of strategies described in Proposition 5 to be an equilibrium, we need investor's strategies to be optimal, considering that they take the regulator's actions into account when forming their beliefs. On the equilibrium path, investors use Bayes' rule to form their expectations. Lemma 1 implies that an investor observing $z = z'$ and $n_i = \eta$ roll over whenever

$$P\left(d_i = \eta | n_i = \eta; z'\right) \geq \bar{p},$$

[23]Since in equilibrium the regulator never misrepresents the types of good and bad banks at the same time, one could think of its *equilibrium* strategy simply as functions $t(p)$ and $z(p)$, conditional on this restriction.

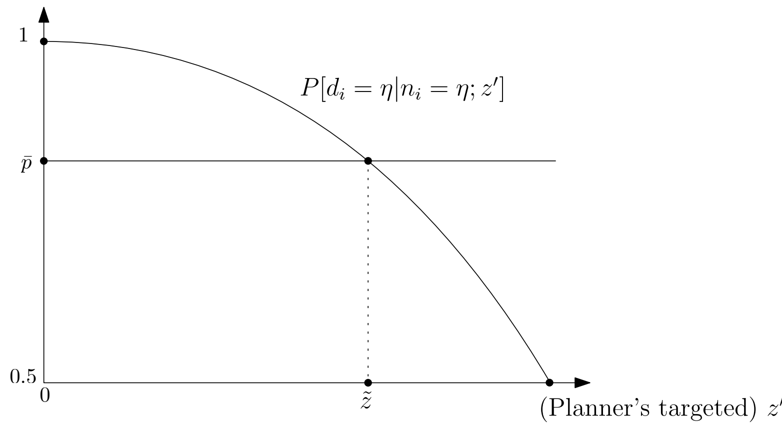



Figure 8: Investors' beliefs when $z(p) = z'\ \forall p$

which, using Bayes' rule, becomes

$$\frac{z'\left[1-\ln\left(z'\right)\right]}{2z'-z'\ln\left(z'\right)+\left[1-z'\right]\ln\left(1-z'\right)} \geq \bar{p}. \tag{17}$$

Figure 8 depicts the left- and the right-hand side of equation (17). $\tilde{z}$ is the value of $z'$ that satisfies (17) with equality, i.e., it is the maximum value of $z'$ that, when targeted by the regulator in all states, is capable of generating enough confidence in investors observing a high bank-specific signal for them to be willing to roll over.

Once again, there is multiple equilibria. Any $z' \in [\underline{z}, \tilde{z}]$ is consistent with equilibrium. Assumption 2 guarantees this set is not empty.

**Intuition** The effect in place here can be thought of as a "Bayesian persuasion" effect. Investors are aware the regulator always claims a given proportion of banks is of good type, so their prior about $p$ is unchanged: before looking at the "label" assigned to their banks, investors expect half the banks to have low return. However, by observing that quite a few banks were labeled as bad and her bank is not one of them, the probability an investor assigns of her bank being of good type increases. The pool from which her bank is drawn gets better the more banks are tagged as bad. If $z'$ were equal to one, that would not happen. The regulator would be happy to send $n_i = \eta$ for all $i$, no matter $p$. But then $P\left(d_i = \eta | n_i = \eta; z'\right)$ would be 0.5. As illustrated in Figure 8, the smaller the targeted $z'$, the more confident investors in the group of banks that receive the good label are that their banks are good indeed.

Notice when a low state happens, regulators are better off in the case manipulating bank-specific signals is a possibility: it is able to save more than just the $p$ good banks. If nature draws a high $p$, the regulator is in fact worst off. If bank-specific information manipulation was not a possibility, all $p$ banks would be saved. Now, a measure $p - z'$ of banks must be "thrown under the bus" in order to the remaining $z'$ good banks not to suffer any runs.

The idea that, to generate confidence that the banks a regulator claims to be good are in fact good, some banks must be labeled as bad seems to be present in the minds of investors and policy makers. In October of 2013, the head of the European Central Bank, Mario Draghi, told Bloomberg TV that some banks needed to fail to make the battery of tests credible. Months later, the head of the Single Supervisory Mechanism (SSM) demonstrated that she shares the same view.[24] The model presented in this section is consistent with that. If the authorities responsible for the conduction of stress tests lack credibility, the only way to generate confidence and avoid runs on banks that do pass the test is to make enough banks fail the test.

### High credibility

Now, consider the case with $c > \theta - \eta$.[25] For the sake of the exposition, it is useful to make one additional assumption in this case. Assume choosing $t = 0$ entails a cost $\varepsilon$ arbitrarily small, while choosing $t = 1$ does not. This assumption can be interpreted as a (very small) "preference for transparency" in the regulator's payoff function. As shown in the appendix, this assumption is not necessary to prove the normative results in the next subsection – namely that an interior credibility level is optimal –, but it helps reduce the set of equilibria and facilitates the exposition.[26] Under this additional assumption, the equilibrium set when the regulator has the prerogative to manipulate both aggregate and idiosyncratic information is identical to the case in which only aggregate information manipulation is possible, when $c > \theta - \eta$. Hence, the equilibrium set in this case is as presented in Proposition 2, item 1.

The basic intuition is that, if investors roll over when their bank is labeled as good, that is, when $n_i = \eta$, the marginal benefit of misrepresenting the type of a low-type bank is $\theta - \eta$,

[24]When asked by the Financial Times on February 2014: *"Do you agree with those who say that for the exercise to be credible there have to be failures?"*, Daniele Nouy asked: "*Well, the president of the ECB has said that that was one condition so I will certainly agree (...) and it seems precisely what markets expect from such an exercise; so, yes, probably that's the case"* (Financial Times, 2014).

[25]When $c = \theta - \eta$, the set of equilibria includes both the equilibria described in this subsection and the ones in the previous one.

[26]Signaling games often present a large set of equilibria, many of which depend upon insensible off-the-equilibrium beliefs. That is the case here. Appendix A.7 present the two additional equilibrium configurations that emerge in an intermediate range of values of $c$ if we make $\varepsilon = 0$. All the equilibria presented in the paper remain unchanged under this additional assumption.

and the marginal benefit of doing so is $c > \theta - \eta$, so the regulator would never want to incur in idiosyncratic information manipulation to save one additional (low-type) bank. However, depending on investors' strategies, in principle the regulator could manipulate idiosyncratic signals of a small group of banks to avoid a run in a larger group of banks, so ruling out other equilibria is not straightforward. Appendix A.6 exhausts all possibilities and shows the set of equilibria surviving the intuitive criterion is the same as in Proposition 2, item 1.

With these results in hand, we can now analyze welfare in this richer environment and compare the normative results under the possibility of bank-specific information manipulation to the case where only aggregate information is subject to bias.

## 4.2 Optimal credibility

Consider the low credibility case, where $c \leq \theta - \eta$. For a given equilibrium value of $z' \in [\underline{z}(c), \tilde{z}]$, with $\underline{z}$ and $\tilde{z}$ satisfying (6) and (11), respectively, welfare is given by

$$W = \int_0^{z'} [p(\theta+\eta) + (z'-p)(\theta-\eta)]\, dp + \int_{z'}^1 z'(\theta-\eta)\, dp = (\theta+\eta)z' - \eta z'^2.$$

Notice the welfare in this case is increasing in $z'$. The lower bound of $z'$, $\underline{z}(c)$, is decreasing in $c$, and $\underline{z}(0) = 0$. It means that when $c$ goes to zero, there is always an equilibrium in which a systemic run happens in all states – the worst possible scenario. On the other extreme, the upper bound of $z'$, $\tilde{z}$, does not depend on $c$, so the same holds for the upper bound of the welfare function when $c < \theta - \eta$.

If $c > \theta - \eta$, for a given equilibrium value of $p^b \in \left[p^{b^*}(c), \bar{p}\right]$, with $p^{b^*}$ and $\bar{p}$ satisfying (5) and (1), respectively, welfare is as in (9), which simplifies to

$$W = \frac{(\theta-\eta)}{2} p^{b^2} - p^b(\theta-\eta) + \theta.$$

Figure 9 illustrates the two possible forms the welfare correspondence can assume, depending on parameters. Appendix A.8 presents all welfare computations in more details. We can derive essentially the same welfare conclusions as in Section 3.3:

- As long as the worst equilibrium is not always played, full credibility is not optimal;
- If the worst equilibrium is always played, any $c > \theta - \eta$ is optimal;
- If any equilibrium is equally likely, the optimal $c$ lies in a neighborhood of $\theta - \eta$.

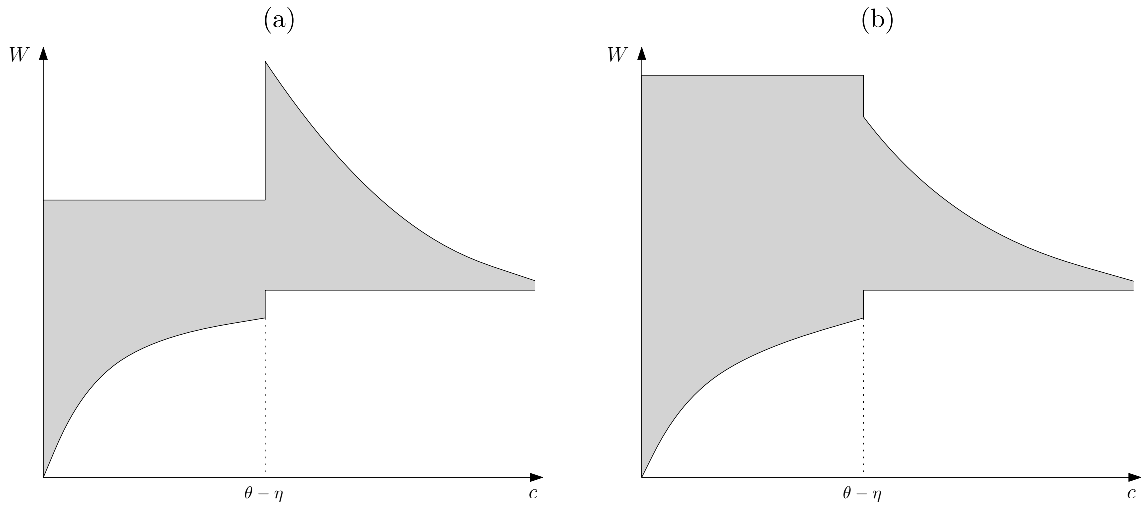



Figure 9: Welfare correspondence: two possibilities

Notice that when the regulator has the possibility of misreporting banks' types, depending on parameters and on the equilibrium selected, it is possible to achieve a higher level of welfare with values of $c$ smaller than $\theta - \eta$. When bank-specific information manipulation is possible, having to disclose bank-specific reports may not be as bad. When bad states are realized, runs can be avoided in more than just the good banks. On the other hand, when a high aggregate state is realized, some good banks must fail. Welfare is larger the smaller the range of states in which good banks must be sacrificed and the more banks can be saved by engaging in bank-specific misreporting in low states. The more severe the coordination problem is – which is captures by $\bar{p}$ – the larger the set of states in which good banks must be sacrificed and the less a regulator can explore manipulation in its favor. Figure 9a illustrates the welfare range when $\bar{p}$ is low, and Figure 9b, when $\bar{p}$ is high.

To summarize, the main message of the welfare analysis conducted in Section 3.3 remains the same. As long as the worst equilibrium is not played with probability one for all $c$, full credibility is not optimal. Moreover, if any equilibrium is equally likely to be played, there is an interior credibility level (in a neighborhood of $\theta - \eta$) that is strictly optimal.

# 5 Final remarks

This paper studies information manipulation and disclosure in financial crises. It shows that a regulator that has the possibility of misreporting information as an attempt to avoid runs will often do so. As long as the regulator has sufficient credibility, it pays off to incur in costly information manipulation of aggregate information, since it is very effective in boosting the stability of the financial sector. However, if the regulator's credibility is too low, it completely loses the ability to enhance confidence by releasing only aggregate information about the banking sector, so releasing bank-specific reports is the only option – and it causes part of the banks to fail.

Also, when misreporting banks' types is a possibility, bias goes in both directions: in low states, the proportion of good banks is inflated, and in high states, the proportion of good banks is underreported to generate confidence in the stress-test exercise. To convince investors that banks that passed the stress test are in good shape, some other banks have to fail – sometimes, even the strong ones.

These results have implications for institutional design. A social planner would choose an interior level of credibility, giving regulators room for maneuver in a crisis situation, but setting credibility in a high enough level so that opacity is still feasible, or at least, bank-specific signals are not completely disregarded. The results presented here are in line with evidence from recent stress test disclosure episodes.

# References


Allen, Franklin, Elena Carletti, Itay Goldstein, and Agnese Leonello, 2015, Government guarantees and financial stability, *CEPR Discussion Paper* DP10560.

Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan, 2006, Signaling in a global game: Coordination and policy traps, *Journal of Political Economy* 114, 452–484.

Angeletos, George-Marios, and Alessandro Pavan, 2004, Transparency of information and coordination in economies with investment complementarities, *The American Economic Review* 94, 91–98.

Angeletos, George-Marios, and Alessandro Pavan, 2007, Efficient use of information and social value of information, *Econometrica* 75, 1103–1142.

Angeletos, George-Marios, and Alessandro Pavan, 2013, Selection-free predictions in global games with endogenous information and multiple equilibria, *Theoretical Economics* 8, 883–938.

Bebchuk, Lucian, and Itay Goldstein, 2011, Self-fulfilling credit market freezes, *Review of Financial Studies* 24, 3519–3555.

Benoît, Jean-Pierre, and Juan Dubra, 2011, Apparent overconfidence, *Econometrica* 79, 1591–1625.

Bird, Andrew, Stephen Karolyi, Thomas Ruchti, and Austin Sudbury, 2015, Bank regulator bias and the efficacy of stress test disclosures, *SSRN* 2626058.

Bloomberg, 2013, Draghi builds stress-test credibility in ecb bank review http://www.bloomberg.com/news/articles/2013-12-12/draghi-builds-stress-test-credibility-as-ecb-readies-bank-review.

Bond, Philip, and Itay Goldstein, 2015, Government intervention and information aggregation by prices, *The Journal of Finance* 70, 2777–2812.

Bouvard, Matthieu, Pierre Chaigneau, and Adolfo Motta, 2015, Transparency in the financial system: Rollover risk and crises, *The Journal of Finance* 70, 1805–1837.

Carlsson, Hans, and Eric Van Damme, 1993, Global games and equilibrium selection, *Econometrica* 61, 989–1018.

Cho, In-Koo, and David Kreps, 1987, Signaling games and stable equilibria, *The Quarterly Journal of Economics* 179–221.

Diamond, Douglas, and Philip Dybvig, 1983, Bank runs, deposit insurance, and liquidity, *The Journal of Political Economy* 91, 401–419.

Edmond, Chris, 2013, Information manipulation, coordination, and regime change, *The Review of Economic Studies* 80, 1422–1458.

Financial Times, 2014, Transcript of interview with Danièle Nouy https://www.ft.com/content/3e0420e2-91a8-11e3-adde-00144feab7de.

Goldstein, Itay, and Chong Huang, 2016, Bayesian persuasion in coordination games, *The American Economic Review* 106, 592–596.

Goldstein, Itay, and Yaron Leitner, 2015, Stress tests and information disclosure, Manuscript.

Goldstein, Itay, and Ady Pauzner, 2005, Demand–deposit contracts and the probability of bank runs, *the Journal of Finance* 60, 1293–1327.

Goldstein, Itay, and Haresh Sapra, 2013, Should banks' stress test results be disclosed? an analysis of the costs and benefits, *Foundations and Trends in Finance* 8, 1–54.

He, Zhiguo, and Asaf Manela, 2014, Information acquisition in rumor-based bank runs, *The Journal of Finance* 71, 1113–1158.

Iachan, Felipe, and Plamen Nenov, 2015, Information quality and crises in regime-change games, *Journal of Economic Theory* 158, 739–768.

Kamenica, Emir, and Matthew Gentzkow, 2011, Bayesian persuasion, *The American Economic Review* 101, 2590–2615.

Kartik, Navin, 2009, Strategic communication with lying costs, *The Review of Economic Studies* 76, 1359–1395.

Kartik, Navin, Marco Ottaviani, and Francesco Squintani, 2007, Credulity, lies, and costly talk, *Journal of Economic Theory* 134, 93–116.

Morris, Stephen, and Hyun Song Shin, 1998, Unique equilibrium in a model of self-fulfilling currency attacks, *American Economic Review* 88, 587–597.

Morris, Stephen, and Hyun Song Shin, 2000, Rethinking multiple equilibria in macroeconomic modeling, *NBER Macroeconomics Annual* 15, 139–182.

Morris, Stephen, and Hyun Song Shin, 2002, Social value of public information, *The American Economic Review* 92, 1521–1534.

Morris, Stephen, and Hyun Song Shin, 2003, Global games: Theory and applications, in M. Dewatripont, L. Hansen, and S. Turnovsky, eds., *Advances in Economics and Econometrics*, Cambridge University Press.

Morris, Stephen, and Hyun Song Shin, 2004, Coordination risk and the price of debt, *European Economic Review* 48, 133–153.

Morris, Stephen, and Hyun Song Shin, 2014, Risk-taking channel of monetary policy: a global game approach, Manuscript.

Rochet, Jean-Charles, and Xavier Vives, 2004, Coordination failures and the lender of last resort: was bagehot right after all?, *Journal of the European Economic Association* 2, 1116–1147.

Schuermann, Til, 2014, Stress testing banks, *International Journal of Forecasting* 30, 717–728.

# A Proofs

## A.1 Proof of Lemma 1

Conjecture a threshold equilibrium in which investors run on bank $i$ at the end of period $t = 1$ if their expectation of $\tilde{\theta}$ is smaller than a threshold $\rho_i^*$, and do not run if it is larger. Investor $j$'s expectation of $\tilde{\theta}$ given her private information is given by

$$\rho_j = \mathbb{E}\left[\tilde{\theta}|x_j\right] = \frac{\beta_\theta \theta + \beta_\xi x_j}{\beta_\theta + \beta_\xi}.$$

The threshold $\rho_i^*$ is given by the indifference condition

$$\mathbb{E}\left[r_i|\rho_i^*\right] = \rho_i^* + \mathbb{E}\left[\eta_i - \gamma l_i|\rho_i^*\right] = 0,$$

which is equivalent to

$$\rho_i^* + \mathbb{E}\left[\eta_i\right] = \gamma\Phi\left(\sqrt{\alpha}\left(\rho_i^* - \theta\right)\right), \tag{18}$$

where $\Phi\left(.\right)$ is the standard normal distribution function and $\alpha = \frac{\beta_\theta^2\left(\beta_\theta+\beta_\xi\right)}{\beta_\varepsilon\left(\beta_\theta+2\beta_\xi\right)}$. Following Morris and Shin (2000), condition $\alpha\gamma^2 \leq 2\pi$ guarantees equation (18) to have a unique solution. Moreover, the equilibrium in threshold strategies is in fact the unique equilibrium, as can be shown by iterated elimination of strictly dominated strategies. Finally, making $\beta_\xi \to 0$ we have that $\rho_j \to \tilde{\theta}$ for all $j$, and since $\alpha \to 0$,

$$\rho_i^* \to \frac{\gamma}{2} - \mathbb{E}\left[\eta_i\right].$$

If we also assume $\beta_\theta \to \infty$, as long as we make $\alpha \to 0$ by assuming $\frac{\beta_\theta^2}{\beta_\xi} \to 0$ to guarantee uniqueness, we have $\rho_i^* \to \theta$, so in equilibrium investors run on bank $i$ whenever $\theta + \mathbb{E}\left[\eta_i\right] < \gamma/2$ and roll over whenever $\theta + \mathbb{E}\left[\eta_i\right] > \gamma/2$.

□

## A.2 Proof of Proposition 1

Consider $z(p) = p$ for all $p$. Given $\hat{z}$, the optimal policy for the regulator is to set

$$t(p) = \begin{cases} 1 & \text{if } p < \hat{z}, \\ 0 & \text{if } p \geq \hat{z}. \end{cases}$$

Since the report is always truthful, Lemma 1 implies investors run on all banks whenever $\mathbb{E}_1[p|z] = z > \bar{p}$, and do not run otherwise. Hence, $\hat{z} = \bar{p}$.

□

## A.3 Proof of Proposition 2

### Regulator's decision

Suppose investors run under opacity whenever $z < \hat{z}$ and roll over otherwise. It is easy to see that $\hat{z} < 0$ is never an equilibrium. If agents never run under opacity, the optimal strategy for the regulator would be opacity (and no manipulation) for all $p$. But when observing any $z = p < \bar{p}$ , agents would want to deviate and run (given Lemma 1). If $\hat{z} > 1$, that is, if agents always run under opacity, the optimal policy for the regulator is to choose transparency for all $p$, that is, $p^b > 1$. It is still to be determined the regulator's solution for a given $\hat{z} \in [0, 1]$. The regulator's problem in (2) boils down to choosing, in each state $p$, between (i) transparency and $b(p) = 0$; (ii) opacity with $b(p) = 0$; or (iii) opacity with $b(p) = \hat{z} - p$ (for $p < \hat{z}$), that is, setting a bias just large enough to avoid a run.[27] Let $U^o$ denote the regulator's payoff under opacity and no manipulation, $U^b$ denote its utility under opacity and manipulation ($b = \hat{z} - p$) and $U^t$ denote its utility under transparency (for simplicity, I subtract the constant 1 from all expressions):

$$U^o(p) = [\theta + (2p - 1)\eta]\, \mathbb{I}_{\{p \geq \hat{z}\}},$$

$$U^b(p) = [\theta + (2p - 1)\eta - c(\hat{z} - p)]\, \mathbb{I}_{\{p < \hat{z}\}} + U^o(p)\mathbb{I}_{\{p \geq \hat{z}\}},$$

$$U^t(p) = p\,(\theta + \eta)\,.$$

[27]Notice no other value of $b$ is rationalizable. If $b \in (0, \hat{z} - p)$, the regulator pays a positive cost of lying $cb$ and derives no benefit, since $z = p + b < \hat{z}$ and there is a massive run on banks, so this value of $b$ is always dominated by $b = 0$. $b > \hat{z} - p$ is always dominated by $b = \hat{z} - p$, since at this level any run is avoided and further increasing $b$ only increases the cost of information manipulation.

The maximized utility of the regulator will be the upper contour of these three functions, which are depicted in Figure 1.

The solution to the regulator's problem in (2) is as follows: if $\hat{z} > 1$, $t(p) = 1$ and $b(p) = 0$ for all $p$; if $\hat{z} \in [0, 1]$,

$$t(p) = \begin{cases} 1 & \text{if } p < p^b, \\ 0 & \text{if } p \in \left[p^b, 1\right], \end{cases}$$

$$b(p) = \begin{cases} 0 & \text{if } p \in [0, p^b) \cup (\hat{z}, 1], \\ \hat{z} - p & \text{if } p \in \left[p^b, \hat{z}\right], \end{cases}$$

where

$$p^b(\hat{z}) = \begin{cases} \frac{c\hat{z} - \theta + \eta}{c - \theta + \eta} & \text{if } \hat{z} > \frac{\theta - \eta}{c}, \\ 0 & \text{if } \hat{z} \leq \frac{\theta - \eta}{c}. \end{cases} \tag{19}$$

Notice if $c < \theta - \eta$, $p^b(\hat{z}) = 0$ for all $\hat{z} \in [0, 1]$.

Now, suppose investors run under opacity whenever $z > \hat{z}$ and roll over otherwise. It is easy to see that for $z < \hat{z}$ the optimal choice for the regulator is $t(p) = 0$ and $b(p) = 0$, and for $p > \hat{z}$, it never pays off to set $z < \hat{z}$ (the regulator either biases $z$ down to $\hat{z}$ to avoid runs under opacity, or chooses transparency). Hence, investors observing any $z < \hat{z}$ know $p = z$. Whenever the observed $z$ is smaller than $\bar{p}$, investors then want to deviate from their equilibrium strategies and run. Hence, there can be no equilibrium in this form. We must look for an equilibrium where investors roll over under opacity when $z$ is larger than a cutoff.

**Investors' decision**

In equilibrium, investors must prefer to roll over whenever $z \geq \hat{z}$, and to run when $z < \hat{z}$. Thus, given Lemma 1, $\hat{z}$ is such that $\mathbb{E}[p|z, t = 0] \geq \bar{p}$ for $z \geq \hat{z}$ and $\mathbb{E}[p|z, t = 0] < \bar{p}$ for $z < \hat{z}$. Given the regulator's disclosure and manipulation policies, an agent receiving the threshold message $\hat{z}$ and no bank-specific information expects $p$ to be uniformly distributed on $\left[p^b, \hat{z}\right]$.

Consider the case with $c < \theta - \eta$. Since $p^b(\hat{z}) = 0$ for any $\hat{z} \in [0, 1]$, the posterior of an investor observing the threshold message $\hat{z}$ and opacity is that $p|\hat{z} \sim U(0, \hat{z})$. Her expectation of $p$ is then $\hat{z}/2$, which is smaller than $\bar{p}$ for any $\hat{z} \leq 1$. Hence, there is no equilibrium with $\hat{z} \in [0, 1]$, and the unique equilibrium features $\hat{z} > 1$, transparency for all $p$ and no information manipulation. This proves Proposition 2, item (ii).

Now, consider the case with $c \geq \theta - \eta$. One equilibrium can be constructed using the indifference condition:

$$\mathbb{E}\left[p|\hat{z}, t=0\right] = \frac{p^b + \hat{z}}{2} = \bar{p},$$

which yields

$$\hat{z} = z^* \equiv \frac{\gamma(c-\theta+\eta) - 2(\theta-\eta)(c-\theta)}{2\eta(2c-\theta+\eta)} \leq 1.$$

When investors observe the report $z^*$ and opacity, they are indifferent between running on banks or not. So for $z > z^*$, they roll over. However, receiving a signal below $z^*$ is off the equilibrium. To sustain this equilibrium, off-the-equilibrium beliefs must satisfy $\mu\left(p|z,0\right) < \bar{p}$ for $z < z^*$. Substituting $z^*$ in (19), we have:

$$p^b(z^*) = p^{b*} \equiv \frac{c\gamma - 2(\theta-\eta)(c+\eta)}{2\eta(2c-\theta+\eta)} > 0.$$

Hence, there is an equilibrium in which investors run under opacity whenever $z < z^*$, the regulator chooses transparency for $p < p^{b*}$, opacity and information manipulation (with $b(p) = z^* - p$) for $p \in \left[p^{b*}, z^*\right]$ and opacity with $b(p) = 0$ for $p > z^*$.

This is not the only equilibrium, though. Suppose investors play according to a cutoff $\hat{z} \in (z^*, 1]$. Given that $p^b(\hat{z}) > p^{b*}$ for all $\hat{z} > z^*$, investors receiving the threshold message $\hat{z}$ and observing opacity expect $p$ to be higher than $\bar{p}$, so this is also an equilibrium as long as off-the-equilibrium beliefs satisfy $\mu(p|z,0) < \bar{p}$ for $z < \hat{z}$. Hence, there is an equilibrium for each cutoff $\hat{z} \in (z^*, 1]$, where the regulator chooses transparency for $p < p^b(\hat{z})$, opacity with manipulation for $p \in \left[p^b(\hat{z}), \hat{z}\right]$ and opacity with no manipulation for $p > \hat{z}$. Moreover, $\hat{z} > 1$ and $p^b > 1$, that is, transparency for all $p$, is also an equilibrium.

Finally, notice when $c = \theta - \eta$, we have $z^* = 1$. In this case, the curves $U^b(p)$ and $U^t(p)$ coincide, so the regulator is indifferent between transparency and opacity for all $p$. As long as the regulator's strategy features a $p^b \geq p^{b*}$, investors observing $z = 1$ and opacity expects $p \geq \bar{p}$. Thus, any pair $\left\{\hat{z}, p^b\right\}$ with $p^b \in \left[p^{b*}, 1\right]$ constitutes an equilibrium.

### Ruling out some equilibria

In the case with $c > \theta - \eta$, there are equilibria that does not survive Cho and Kreps (1987)'s intuitive criterion.

Suppose an equilibrium in which investors play according to a cutoff $\hat{z} \in [0,1]$ under opacity and $p^b(\hat{z}) > \bar{p}$. Consider an agent observing a report $z' = \hat{z} - \xi$, for some $\xi > 0$, and opacity. The regulator's payoff from sending a report $z'$ under the most optimistic belief possible about investors response (i.e., investors roll over) is $U^b(p) = \theta + (2p-1)\,\eta - c(z' - p)$, so the maximum benefit of deviating from the equilibrium strategy is

$$B(p) = \begin{cases} c\xi & \text{if } p \in \left[p^b(\hat{z}), z'\right], \\ (1-p)\,(\theta-\eta) - c(\hat{z}-\xi-p) & \text{if } p < p^b\,(\hat{z})\,. \end{cases} \tag{20}$$

Since $B(p)$ is negative for all $p < p^b(z')$, the intuitive criterion imposes investors cannot assign positive probability for $p$ being smaller than $p^b(z')$. But if $p^b(\hat{z}) > \bar{p}$, for $\xi$ sufficiently small we have that $p^b(z') > \bar{p}$ as well. Hence, investors cannot expect $p$ to be smaller than $\bar{p}$ when observing an off-the-equilibrium $z$ slightly smaller than $\hat{z}$. We can then rule out any equilibria where $\hat{z}$ is such that $p^b(\hat{z}) > \bar{p}$ and restrict attention to equilibria satisfying the restriction that $p^b(\hat{z}) \leq \bar{p}$, which implies

$$\hat{z} \leq \bar{z} \equiv \frac{\gamma(c-\theta+\eta) - 2(\theta-\eta)(c-\theta-\eta)}{4c\eta} < 1.$$

Therefore, when $c > \theta - \eta$ the set of equilibria surviving the intuitive criterion reduces to all $\left\{\hat{z}, p^b\right\}$ with $\hat{z} \in [z^*, \bar{z}]$ and $p^b = p^b(\hat{z})$ as given in (19). When $c = \theta - \eta$, though, the maximum benefit of deviating from the equilibrium strategy given in equation (20) is positive for all $p$, so no equilibrium can be ruled out.

□

## A.4 Proof of Proposition 3

Consider $z^*$, $p^{b*}$, $\bar{z}$ and $\bar{p}$ as given by (4), (5), (6) and (1), respectively. Deriving each of these expressions with respect to $c$ when $c > \theta - \eta$ we have:

$$\frac{\partial z^*}{\partial c} = \frac{(\theta-\eta)(\gamma - 2(\theta+\eta))}{2\eta\,(\theta-\eta-2c)^2} < 0,$$

$$\frac{\partial p^{b*}}{\partial c} = -\frac{(\theta-\eta)(\gamma - 2(\theta+\eta))}{2\eta\,(\theta-\eta-2c)^2} > 0,$$

$$\frac{\partial \bar{z}}{dc} = \frac{(\theta-\eta)(\gamma - 2(\theta+\eta))}{4\eta^2} < 0,$$

$$\frac{\partial \bar{p}}{\partial c} = 0.$$

Assumption 1 guarantees the inequalities hold.

□

## A.5 Proof of Proposition 4

Assume agents always play according to the best equilibrium possible, that is, $\left\{z^*, p^{b*}\right\}$ and let $W(c)$ denote the welfare for a given $c$. The proof of Proposition 4 follows from the fact that the aggregate welfare in (9) is decreasing in $c$ for all $c \geq \theta - \eta$ and the fact that the welfare when $c < \theta - \eta$ is smaller than the welfare for any $c \geq \theta - \eta$.

For $c > \theta - \eta$, we have

$$\frac{dW(c)}{dc} = -\frac{c(\theta-\eta)^2\left(\gamma - 2\theta - 2\eta\right)^2}{4\eta^2\left(2c - \theta + \eta\right)^3} < 0.$$

Since for all $c < \theta - \eta$,

$$W(c) = \int_0^1 p\left(\theta + \eta\right) dp$$

and

$$W(\theta - \eta) = \int_0^{p^{b*}(c)} p(\theta+\eta)dp + \int_{p^{b*}(c)}^1 \left[\theta + (2p-1)\,\eta\right] dp > \int_0^1 p\left(\theta+\eta\right) dp,$$

the value of $c$ that maximizes welfare is $c^* = \theta - \eta$. Also, notice that

$$\lim_{c\to\infty} W(c) = \int_0^{\bar{p}} p(\theta+\eta)dp + \int_{\bar{p}}^1 \left[\theta + (2p-1)\eta\right] dp > \int_0^1 p\left(\theta+\eta\right) dp,$$

which finishes explaining Figure 4.

□

## A.6 Proof of Proposition 5

Consider $c < \theta - \eta$. To simplify notation, I denote the profile of bank-specific reports $\hat{G}$ chosen by the regulator as a correspondence that depends on the aggregate state $p$, instead of depending on the type profile $G$, since for the regulator the measure of $G$ is all that matters. The next two Lemmas are useful for the proof of Proposition 5.

**Lemma 2.** *Let $\mathbb{E}\left[d_i|z\right]$ be the updated prior about a bank $i$ investor's type after observing $z$ and $t = 1$, but not taking $n_i$ into account. It cannot be that $\mathbb{E}\left[d_i|n_i, z\right] \geq \mathbb{E}\left[d_i|z\right]$, for $n_i = \eta, -\eta$, with strict inequality holding for one of them. That is, the observation of the* *bank-specific signals $n_i$ cannot make investors receiving both types of signals more optimistic about their types than their prior.*

*Proof.* By the law of total expectations,

$$\mathbb{E}\left[d_i|z\right] = P\left(n_i = \eta\right)\mathbb{E}\left[d_i|n_i = \eta; z\right] + \left[1 - P\left(n_i = \eta\right)\right]\mathbb{E}\left[d_i|n_i = -\eta; z\right].$$

Since $\mathbb{E}\left[d_i|z\right]$ is a convex combination of $\mathbb{E}\left[d_i|n_i = \eta; z\right]$ and $\mathbb{E}\left[d_i|n_i = -\eta; z\right]$, if one of them is larger than $\mathbb{E}\left[d_i|z\right]$ , the other one must be smaller than it. □

**Lemma 3.** *Suppose when $t = 1$ investors observing $n_i = -\eta$ run regardless of $z$, that is, $a(-\eta, z) = 0\ \forall z$. Define $F$ as any subset of $B = [0,1]\setminus G$ of measure $z' - p$, for $z' > p$. Also, define $G'$ as any subset of $G$ of measure $z'$, for $z' < p$.*

1. *If $a(\eta, z) = \begin{cases} 1 & \text{if } z \geq z', \\ 0 & \text{otherwise,} \end{cases}$ then whenever $t(p) = 1$ is optimal we have that $\hat{G}(p) = [0,1]$.*

2. *If $a(\eta, z) = \begin{cases} 1 & \text{if } z \leq z', \\ 0 & \text{otherwise,} \end{cases}$ then whenever $t(p) = 1$ is optimal we have that $\hat{G}(p) = G \cup F$ for $p < z'$, and for $p > z'$,*

$$\hat{G}\left(p\right) = \begin{cases} G' & \text{if } p \leq z'\left(\frac{c+\theta+\eta}{c}\right), \\ G & \text{if } p > z'\left(\frac{c+\theta+\eta}{c}\right), \end{cases}$$

*which implies $z(p) = \begin{cases} z' & \text{if } p \leq z'\left(\frac{c+\theta+\eta}{c}\right), \\ p & \text{if } p > z'\left(\frac{c+\theta+\eta}{c}\right). \end{cases}$*

*Proof.* Let $a(\eta, z) = 1$ if $z \geq z'$ and $a(\eta, z) = 0$ otherwise. If investors observing a $z$ high enough $n_i = \eta$, since the cost of misreporting the type of an additional bank is $c$ and the benefit of avoiding a run on an additional bad bank is $\theta - \eta > c$, it is always profitable to save an additional bank. Hence, the regulator chooses $\hat{G} = [0,1]$ for all $p$.

Now let $a(\eta, z) = 1$ if $z \leq z'$ and $a(\eta, z) = 0$ otherwise. Again, the regulator is always willing to pay the marginal cost $c$ if it implies saving an additional bad bank, but now there is a systemic run whenever $z > z'$. If disclosing bank-specific signals is optimal, when $p < z'$ the regulator sets $\hat{G} = G \cup F$ – that is, it biases up the reports of a mass $z' - p$ of bad banks, the maximum amount of bias possible that does not trigger a systemic run. When $p > z'$, the regulator sets $\hat{G} = G'$ – that is– biases down the reports of a measure $p - z'$ of good banks to avoid a run – only if misreporting the types of $p - z'$ banks is worth it, that is, only if

$$z'(\theta + \eta) - c(p - z') \geq 0.$$

Otherwise, the best choice is to disclose the true type profile, that is, $\hat{G} = G$. □

The proof of Proposition 5 consists of checking all possible combinations of threshold strategies for investors and analyzing what would be the regulator's optimal strategy to see whether that combination of strategies constitutes an equilibrium or not.

Lets start by looking for an equilibrium in which investors roll over under opacity if $z$ is high enough.

**Case I: Suppose $a(0, z) = 1$ for $z \geq \hat{z}$ and 0 otherwise.**

**Subcase 1: Suppose $a(-\eta, z) = 0$ for all $z$ and $a(\eta, z) = \begin{cases} 1 & \textbf{if } z \geq z', \\ 0 & \textbf{otherwise.} \end{cases}$**

Notice the regulator has no incentives to misreport a good-type bank for bad. If any bias is desirable, it is a positive bias. We can think of the regulator's strategy in this case simply as the choice of $t \in \{0, 1\}$ and a $z \in [0, 1]$. Given investors' strategies, the regulator's payoff when choosing opacity is given by

$$U(0, z, p) = \mathbb{1}_{\{z \geq \hat{z}\}} \{p(\theta + \eta) + (1 - p)(\theta - \eta) - c|z - p|\} + \mathbb{1}_{\{z < \hat{z}\}} \{-c|z - p|\}. \tag{21}$$

Its payoff when sending out bank-specific reports is

$$U(1, z, p) = \begin{cases} [p(\theta + \eta) + (z - p)(\theta - \eta - c)]\, \mathbb{1}_{\{z \geq z'\}} - (z - p)c\mathbb{1}_{\{z < z'\}} & \text{if } z \geq p, \\ [z(\theta + \eta) - (p - z)c]\, \mathbb{1}_{\{z \geq z'\}} - (p - z)c\mathbb{1}_{\{z < z'\}} & z < p. \end{cases}$$

Notice for every $p$, opacity dominates transparency. So the planner chooses $t(p) = 0$ for all $p$, $z(p) = \hat{z}$ for $p \leq \hat{z}$ and $z(p) = p$ for $p > \hat{z}$. But, as we know from Section 3.2, there can be no equilibrium with $\hat{z} \in [0, 1]$ in this case, since $\mathbb{E}[d_i|\hat{z}] < \gamma/2 - \theta$ and thus agents observing $\hat{z}$ would be willing to deviate and run. Thus, if there is an equilibrium in this form, it must be such that investors run under opacity for all $z$ and the regulator always discloses bank-specific information, that is $t(p) = 1$ for all $p$. By Lemma 3, the solution to the regulator's problem is to set $z(p) = 1$ for all $p$. Notice this cannot be an equilibrium, though. Under this strategy for the regulator, investors observing $n_i = \eta$ will have the expectation that $\mathbb{E}[d_i|\eta, 1] = 0.5$ and run. Hence, there is no equilibrium in this form.

**Subcase 2: Suppose** $a(-\eta, z) = 0$ **for all** $z$ **and** $a(\eta, z) = \begin{cases} 1 & \textbf{if } z \leq z', \\ 0 & \textbf{otherwise.} \end{cases}$

Investors payoff from opacity is as in (21), and investors payoff from bank-specific disclosure is

$$U(1, z, p) = \begin{cases} [p(\theta+\eta) + (z-p)(\theta-\eta-c)]\,\mathbb{1}_{\{z\leq z'\}} - (z-p)c\mathbb{1}_{\{z>z'\}} & \text{if } z \geq p, \\ [z\,(\theta+\eta) - (p-z)c]\,\mathbb{1}_{\{z\leq z'\}} - (p-z)c\mathbb{1}_{\{z>z'\}} & z < p. \end{cases}$$

Again, opacity dominates, but as is the previous case, there can be no equilibrium with $\hat{z} \leq 1$. Hence, $t(p) = 1$ for all $p$. From Lemma 3, regulator's manipulation policy is

$$z(p) = \begin{cases} z' & \text{if } z'(\theta+\eta+c) \geq cp, \\ p & \text{otherwise.} \end{cases}$$

Notice the solution becomes simply $z(p) = z'$ for all $p$ if

$$\begin{aligned} & z'(\theta+\eta+c) > cp \text{ for all } p \\ \iff \; & z' > \frac{c}{c+\theta+\eta} \equiv \underline{z} \end{aligned} \tag{22}$$

Suppose this condition is violated, so there is an interval $(\tilde{p}, 1]$ in which it is not worth misrepresenting banks types, i.e., the solution to the regulator's problem features $b(p) = 0$ for $p \geq \tilde{p}$, $\tilde{p} = z' \frac{(\theta+\eta+c)}{c}$. But in this case agents observing $n_i = \eta$ and $z > \tilde{p}$ know their bank is of high-type with probability one, so they want to deviate and roll over. Hence this cannot be an equilibrium. There can be no equilibrium in this form if condition (22) does not hold. We must check if there is a $z'$ consistent with equilibrium satisfying condition (22) holds.

By Lemma 1, we know investors' roll over whenever the probability they assign for their banks being of high type is larger than $\bar{p}$. Thus, an investor observing the high-type message and $z'$ roll over if $P\left(d_i = \eta | n_i = \eta; z'\right) \geq \bar{p}$. I hereafter omit the notation denoting "conditional on $z'$" for simplicity. Using Bayes' rule,

$$P\left(d_i = \eta | n_i = \eta\right) = \frac{P\left(n_i = \eta | d_i = \eta\right) P\left(d_i = \eta\right)}{P(n_i = \eta | d_i = \eta) P\left(d_i = \eta\right) + P(n_i = \eta | d_i = -\eta) P\left(d_i = -\eta\right)}.$$

We have that $P(d_i = \eta|z') = P(d_i = -\eta|z') = 0.5$, and

$$P(n_i = \eta|d_i = \eta) = \int_0^{z'} 1dp + \int_{z'}^1 \frac{z'}{p}dp = z'\left[1 - \ln(z')\right],$$

$$P(n_i = \eta|d_i = -\eta) = \int_0^{z'} \frac{z' - p}{1 - p}dp + \int_{z'}^1 0dp = z' + [1 - z']\ln(1 - z').$$

Hence,

$$P(d_i = \eta|n_i = \eta) = \frac{z'\left[1 - \ln(z')\right]}{2z' - z'\ln(z') + [1 - z']\ln(1 - z')}. \tag{23}$$

Notice

$$\frac{\partial P(.)}{\partial z'} < 0 \ \forall p, \ \lim_{z' \to 0} P(.) = 1, \text{ and } \lim_{z' \to 1} P(.) = 0.5.$$

An investor receiving the bank-specific signal $n_i = \eta$ rolls over if

$$\frac{z'\left[1 - \ln(z')\right]}{2z' - z'\ln(z') + [1 - z']\ln(1 - z')} \geq \bar{p}, \tag{24}$$

and runs otherwise. Denote by $\tilde{z}$ the value of $z'$ satisfying (24) with equality.

Now, consider an investor observing the bank-specific signal $n_i = -\eta$. By Lemma 2, since $P(d_i = \eta|z') = 0.5$ and $P(d_i = \eta|n_i = \eta; z') \geq 0.5$, we must have that $P(d_i = \eta|n_i = -\eta; z') \leq 0.5$, and thus investors observing $n_i = -\eta$ are in fact willing to run.

There is still to be checked if $\tilde{z}$ satisfies condition (22). Our parametric assumptions imply

$$\underline{z} \equiv \frac{c}{c + \theta + \eta} \leq \frac{\theta - \eta}{2\theta} < \frac{1}{2}.$$

If $\tilde{z}$ is always larger than this upper bound, we have an equilibrium. Equation (24) implies that a sufficient condition for $\tilde{z} > 1/2$ is that

$$\frac{\gamma}{2} < \theta + \ln(2)\eta \approx \theta + 0.69\eta.$$

Under this sufficient condition, there is always an equilibrium in this form, for every $z' \in [\underline{z}, \tilde{z}]$.

**Subcase 3: Suppose**$a(-\eta, z) = \begin{cases} 1 & \textbf{if } z \leq \tilde{z}_1, \\ 0 & \textbf{otherwise,} \end{cases}$ **and** $a(\eta, z) = \begin{cases} 1 & \textbf{if } z \leq \tilde{z}_2, \\ 0 & \textbf{otherwise.} \end{cases}$

Let $p' = \min\{\tilde{z}_1, \tilde{z}_2, \hat{z}\}$. Notice that for $p < p'$, the optimal strategy is always $t(p) = 1$ and $\hat{G} = G$ (which means $z(p) = p$). For $p \geq p'$, it is never optimal to choose a manipulation strategy that makes $z(p) < p'$ (it would involve a manipulation cost and no additional benefit). Thus observing $t(p) = 1$ and $z < \tilde{z}$ conveys that in fact $d_i = n_i$. Investors observing the low signal would then deviate and run. There can be no equilibrium in this form.

**Subcase 4: Suppose** $a(-\eta, z) = \begin{cases} 1 & \textbf{if } z \geq \tilde{z}_1, \\ 0 & \textbf{otherwise,} \end{cases}$ **and** $a(\eta, z) = \begin{cases} 1 & \textbf{if } z \geq \tilde{z}_2, \\ 0 & \textbf{otherwise.} \end{cases}$

Consider $\hat{z} < \max\{\tilde{z}_1, \tilde{z}_2\}$. Given investors' strategies, the regulator's optimal policy for $p \leq \hat{z}$ would be $t(p) = 0$ and $\hat{G} = G \cup H$, where $H$ is any subset of $B = [0, 1] \setminus G$ of measure $(\hat{z} - p)$ – implying $z(p) = \hat{z}$. For $p \in (\hat{z}, \max\{\tilde{z}_1, \tilde{z}_2\})$, the regulator would choose $t(p) = 0$ and $\hat{G} = G$ (thus $z(p) = p$), and for $p > \max\{\tilde{z}_1, \tilde{z}_2\}$, the regulator would set $\hat{G} = G$ and either $t(p) = 0$ or $t(p) = 1$. But then there is no $\hat{z} \in [0, 1]$ that would constitute an equilibrium, since $\mathbb{E}[p|\hat{z}] < \bar{p}$ for all possible $\hat{z} \in [0, 1]$.

If $\hat{z} > 1$, case in which the regulator's policy features $t(p) = 1\ \forall p$, the regulator would choose $\hat{G} = G$ for $p > \max\{\tilde{z}_1, \tilde{z}_2\}$, and for $p \leq \max\{\tilde{z}_1, \tilde{z}_2\}$, $z(p)$ would never be larger than $\max\{\tilde{z}_1, \tilde{z}_2\}$. Then, observing $z > \max\{\tilde{z}_1, \tilde{z}_2\}$ conveys that $n_i = d_i$, so investors observing the low signal would run. This cannot be an equilibrium.[28]

Now, consider $\hat{z} \in [0, 1]$ and $\hat{z} > \max\{\tilde{z}_1, \tilde{z}_2\}$.The optimal policy for the regulator would feature $t(p) = 1$ and $\hat{G} = G$ (and thus$z(p) = p$) for $p \in \max(\{\tilde{z}_1, \tilde{z}_2\}, \hat{z})$, $t(p) = 1$ and $z(p) \leq \max\{\tilde{z}_1, \tilde{z}_2\}$ for $p \leq \max\{\tilde{z}_1, \tilde{z}_2\}$($\hat{G}$ would imply at most the misrepresentation of $(\max\{\tilde{z}_1, \tilde{z}_2\} - p)$ bad banks for good in order for all runs to be avoided) and $t(p) = 0$ or $t(p) = 1$ and $\hat{G} = G$ for $p \geq \hat{z}$. Hence, observing $t(p) = 1$ and $p \in \max(\{\tilde{z}_1, \tilde{z}_2\}, \hat{z})$ conveys the bank-specific information is truthful, and thus investors observing $n_i = -\eta$ deviate and run.

There is still to be considered the case with $\hat{z} = \max\{\tilde{z}_1, \tilde{z}_2\}$. If $z_2 < z_1 = \hat{z}$, the regulator chooses $\hat{G} = G$ for $z \geq \hat{z}$ and the smaller amount of bias to achieve $z(p) = \hat{z}$ for $p < \hat{z}$, and it is indifferent between $t(p) = 1$ or $t(p) = 0$ for all $p$. But $t(p) = 1$ is never an equilibrium for $p > \hat{z}$, since $\mathbb{E}[d_i|n_i, z] = n_i$ for $z \in [\hat{z}, 1]$ and thus investors observing $n_i = -\eta$ would run.

[28]Notice $\hat{z}, \tilde{z}_1, \tilde{z}_2 > 1$ cannot be as equilibrium as well, since the regulator would choose $\hat{G} = G\ \forall p$ and either $t(p) = 0$ or $t(p) = 1$, so investors would eventually have incentives to deviate.

Then any equilibrium candidate must have $t(p) = 0$ for $p > \hat{z}$. Also, $t(p) = 0$ for all $p \leq \hat{z}$ cannot be an equilibrium, since $\mathbb{E}[p|\hat{z}] \leq 0.5 < \bar{p}$, and all investors would deviate and run. There is still to be checked if $t(p) = 1$ for $p \leq \hat{z}$ and $t(p) = 0$ for $p > \hat{z}$ can constitute an equilibrium. Investors observing $\hat{z}$ expect $p \sim U(0, \hat{z})$ and know there is only positive bias (bad banks misrepresented as good), for all $p < \hat{z}$. Thus $\mathbb{E}[d_i = \eta | n_i = -\eta, z = \hat{z}] = 0$, and all investors observing $n_i = -\eta$ deviate and run. Hence, there is no equilibrium in this form. Finally, if $z_1 < z_2 = \hat{z}$, the solution to the regulator's problem is analogous to the Case 5 presented below when $z_2 < z_1 < \hat{z}$, and this form of equilibrium can also be ruled out.

**Subcase 5: Suppose** $a(-\eta, z) = \begin{cases} 1 & \textbf{if } z \geq \tilde{z}_1, \\ 0 & \textbf{otherwise,} \end{cases}$ **and** $a(\eta, z) = \begin{cases} 1 & \textbf{if } z \leq \tilde{z}_2, \\ 0 & \textbf{otherwise.} \end{cases}$

First, suppose $\hat{z} < \min\{\tilde{z}_1, \tilde{z}_2\}$. The solution to the planner's problem involves $z(p) = \hat{z}$ for $p < \hat{z}$ (by choosing any$\hat{G}$ carrying the smallest amount of bias to achieve $z = \hat{z}$, that is, only bad banks are misrepresented for good) and $\hat{G} = G$ for $p \geq \hat{z}$. If $\tilde{z}_2 < \tilde{z}_1$, $t(p) = 0$ for all $p$. If $\tilde{z}_1 < \tilde{z}_2$, the regulator is indifferent between $t(p) = 1$ or $t(p) = 0$ for $p \in [\tilde{z}_1, \tilde{z}_2]$, and for $p \notin [\tilde{z}_1, \tilde{z}_2]$, $t(p) = 0$. Since $z(p) = \hat{z}$ and $t(p) = 0$ for all $p < \hat{z}$, there can be no equilibrium in this form, because all investors observing an aggregate signal $\hat{z}$ would deviate and run. If $\hat{z} = \tilde{z}_2 < \tilde{z}_1$, the same reasoning applies. If $\hat{z} = \tilde{z}_1 < \tilde{z}_2$, it would also be optimal for the regulator to set $t(p) = 1$ for any subset of $[0, \tilde{z}_2]$. But then, since for any $p < \tilde{z}_1$ there is only positive bias, observing $n_i = -\eta$ and $z = \tilde{z}_1$ conveys $d_i = -\eta$, so there can be no equilibrium with $t(p) = 1$ for some $p < \tilde{z}_1$. Also, there can be no equilibrium with $t(p) = 0$ for all $p < \tilde{z}_1$, because $\mathbb{E}[p|\tilde{z}_1] = \tilde{z}_1/2 < \bar{p}$.

Now, consider $\tilde{z}_1 < \tilde{z}_2 < \hat{z}$. The regulator's solution involves the smallest amount of bias to achieve $z(p) = \tilde{z}_1$ for $p < \tilde{z}_1$, $\hat{G} = G$ (and thus$z(p) = p$) for $p \in [\tilde{z}_1, \tilde{z}_2]$ and $z(p) \geq \tilde{z}_2$ for $p > \tilde{z}_2$, and choosing $t(p) = 1$ for $p \leq \tilde{z}_2$. Thus, $\mathbb{E}[d_i | n_i = -\eta; z] = -\eta$ for $z \in (\tilde{z}_1, \tilde{z}_2)$, and this cannot be an equilibrium. If $\tilde{z}_1 < \tilde{z}_2 = \hat{z}$, the planner chooses $z(p) = \tilde{z}_1$ for $p < \tilde{z}_1$, $z(p) = p$ for $p \geq \tilde{z}_1$, and $t(p) = 1$ for $p < \hat{z}$ and $t(p) = 0$ otherwise. Hence, once again $\mathbb{E}[d_i | n_i = -\eta; z] = -\eta$ for $z \in (\tilde{z}_1, \tilde{z}_2)$, and this cannot be an equilibrium. If $\tilde{z}_1 = \tilde{z}_2 < \hat{z}$, the planner chooses $z(p) = \tilde{z}_1$ and $t(p) = 1$ for all $p$ smaller than some $p' < \hat{z}$. But by Lemma 2, agents observing $n_i = -\eta$ and $n_i = \eta$ cannot both be willing to roll over in this case. Thus, there is no equilibrium in this form. Finally, consider $\tilde{z}_1 = \tilde{z}_2 = \hat{z}$. The regulator chooses $z(p) = \hat{z}$ for $p < \hat{z}$ and $z(p) = p$ for $p \geq \hat{z}$, and is indifferent between $t = 0$ or $t = 1$ for all $p$. Notice $t(p) = 1$ for $p > \hat{z}$ cannot be an equilibrium, since investors observing $n_i = -\eta$ would deviate and run. Also, $t(p) = 0$ for all $p < \hat{z}$ cannot be an equilibrium, since $\mathbb{E}[p|\hat{z}] < \bar{p}$. But $t(p) = 1$ for $p < \hat{z}$ is also not an equilibrium, due to Lemma 2.

Finally, consider $\tilde{z}_2 < \tilde{z}_1 < \hat{z}$. Notice if the regulator chooses $t = 1$ and $\hat{G} = G$, its payoff is giveb by

$$U(t = 1, z = p, p) = \begin{cases} (1-p)(\theta - \eta) & p \in [\tilde{z}_1, \hat{z}], \\ p(\theta + \eta) & p \in [0, z_2]. \end{cases} \tag{25}$$

The payoff from choosing opacity and the smaller amount of bias to achieve $z(p) = \hat{z}$ when $p < \hat{z}$ is

$$U(t = 0, z = \hat{z}, p) = (1-p)(\theta - \eta) + p(\theta + \eta) - c(\hat{z} - p). \tag{26}$$

If the regulator manipulates the smaller amount of banks possible to achieve $z = z_1$, its payoff is:

$$U(t = 1, z = \tilde{z}_1, p) = \begin{cases} (1-p)(\theta - \eta) + (p - \tilde{z}_1)(\theta + \eta - c) & p \in [\tilde{z}_1, \hat{z}], \\ (1 - \tilde{z}_1)(\theta - \eta) - c(\tilde{z}_1 - p) & p \in [0, \tilde{z}_1]. \end{cases} \tag{27}$$

Notice for $p \in [\tilde{z}_1, \hat{z}]$, (25) is dominated by (27), and for $p \in [0, \tilde{z}_1]$, (25) is dominated by (26). Depending on $\eta$ and $c$, it may be profitable to engage in double–misreporting, that is, to assign a bad label to good banks, and at the same time, to assign a good label to bad banks – and this way, it is possible to save some good banks instead of bad ones, that have smaller return. The next expression denotes the payoff the regulator gets when achieving $z = \tilde{z}_1$ by saving as much good-type banks as possible by using double-misreporting:

$$U(t = 1, z = \tilde{z}_1, p;\ \text{DM}) = \begin{cases} p(\theta + \eta) + (1 - \tilde{z}_1 - p)(\theta - \eta) - cp - c\tilde{z}_1 & p \leq 1 - \tilde{z}_1 \\ (1 - \tilde{z}_1)(\theta + \eta) - c(2 - \tilde{z}_1 - p) & p > 1 - \tilde{z}_1 \end{cases} \tag{28}$$

It turns out double-misreporting is profitable in comparison to misreporting the smallest measure of banks to achieve $\tilde{z}_1$ whenever $\eta > c$:

$$U(t = 1, z = \tilde{z}_1, p) - U(t = 1, z = \tilde{z}_1, p;\ \text{DM}) = \begin{cases} 2(c - \eta)\tilde{z}_1 & p \in [\tilde{z}_1, \hat{z}],\ p \leq 1 - \tilde{z}_1, \\ 2(c - \eta)(1 - p) & p \in [\tilde{z}_1, \hat{z}]\ \ p > 1 - \tilde{z}_1, \\ 2(c - \eta)p & p \in [0, \tilde{z}_1],\ p \leq 1 - \tilde{z}_1, \\ 2(c - \eta)(1 - \tilde{z}_1) & p \in [0, \tilde{z}_1],\ p > 1 - \tilde{z}_1. \end{cases}$$

First consider $c > \eta$, that is, the case in which double-misreporting is ruled out. The choice is then between transparency with $z = z_1$ and opacity with $z = \hat{z}$.

$$D_1 = U(t = 0, z = \hat{z}, p) - U(t = 1, z = \tilde{z}_1, p) = \begin{cases} 2cp + (\theta + \eta - c)z_1 - c\hat{z} & p \in [z_1, \hat{z}] \\ 2\eta p + (\theta - \eta + c)z_1 - c\hat{z} & p \in [0, \tilde{z}_1] \end{cases}$$

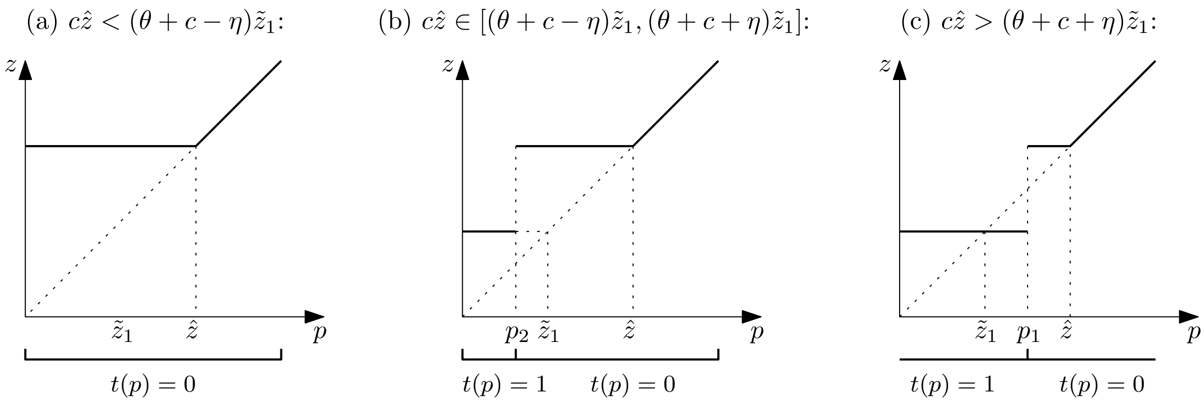
Notice both are increasing in $p$. If $c\hat{z} < (\theta + \eta + c)\,\tilde{z}_1$, $D_1 > 0$ for all $p \in [\tilde{z}_1, \hat{z}]$, thus $t(p) = 0$ and $z(p) = \hat{z}$ for $p \in [\tilde{z}_1, \hat{z}]$. If $c\hat{z} > (\theta + \eta + c)\,\tilde{z}_1$,

$$\begin{cases} t(p) = 0,\ z(p) = \hat{z} & p \in [p_1, \hat{z}]\,,\ p_1 \equiv \frac{c\hat{z} - (\theta+\eta-c)\tilde{z}_1}{2c} \\ t(p) = 1,\ z(p) = \tilde{z}_1 & p \in [\tilde{z}_1, p_1) \end{cases}$$

If $c\hat{z} < (\theta - \eta + c)\,\tilde{z}_1$, $D_1 > 0$ for all $p \in [0, \tilde{z}_1]$, and thus $t(p) = 0$ and $z(p) = \hat{z}$ for all $p \in [0, \tilde{z}_1]$. If $c\hat{z} > (\theta + \eta + c)\,\tilde{z}_1$, $D_1 < 0$ for at all $p \in [0, \tilde{z}_1]$. If $c\hat{z} \in [(\theta - \eta + c)\,\tilde{z}_1, (\theta + \eta + c)\,\tilde{z}_1]$,

$$\begin{cases} t(p) = 0,\ z(p) = \hat{z} & p \in [p_2, \hat{z}]\,,\ p_2 \equiv \frac{c\hat{z} - (\theta+c-\eta)\tilde{z}_1}{2\eta} \\ t(p) = 1,\ z(p) = \tilde{z}_1 & p \in [0, p_2) \end{cases}$$

The next figure sums up the regulator's policies depending on the position of $\tilde{z}_1$ and $\hat{z}$ when $c > \eta$.




Case (a) cannot be an equilibrium because $\mathbb{E}\,[p|\hat{z}, t = 0] = \hat{z}/2 < \bar{p}$. Case (b) cannot be an equilibrium as well because $\mathbb{E}\,[d_i|n_i = -\eta, z = \tilde{z}_1, t = 1] = -\eta$ (since the probability of $d_i = \eta$ when $n_i = -\eta$ is zero, there are only high signals being misrepresented when $p \in [0, p_2]$). Now consider case (c), in which $p_1$ is always larger than (or equal to) $\tilde{z}_1$. Notice

$$\partial p_1/\partial \tilde{z}_1 < 0 \text{ and}$$

$$\lim_{\tilde{z}_1 \to 0} p_1 = \frac{\hat{z}}{2},$$

$$\lim_{\tilde{z}_1 \to \frac{c\hat{z}}{\theta+\eta+c}} p_1 = \frac{c\hat{z}}{\theta+\eta+c}.$$

The probability investors assign for their bank being of high type after observing $z = \tilde{z}_1$, $t = 1$ and $n_i = -\eta$ is given by

$$P(d_i = \eta | n_i = -\eta) = \frac{P(n_i = -\eta | d_i = \eta) P(d_i = \eta)}{P(n_i = -\eta | d_i = \eta) P(d_i = \eta) + P(n_i = -\eta | d_i = -\eta) P(d_i = -\eta)},$$

where

$$P(n_i = -\eta | d_i = \eta) = \int_{z_1}^{p_1} \frac{p - \tilde{z}_1}{p} \frac{1}{p_1} dp = 1 - \frac{\tilde{z}_1}{p_1} \left[1 + \ln(p_1) - \ln(\tilde{z}_1)\right],$$

$$P(n_i = -\eta | d_i = -\eta) = \int_0^{z_1} \frac{1 - \tilde{z}_1}{1 - p} \frac{1}{p_1} dp + \int_{z_1}^{p_1} \frac{1}{\bar{z}} dp = 1 + \frac{1}{p_1} \left[\tilde{z}_1 + (1 - \tilde{z}_1) \log(1 - \tilde{z}_1)\right],$$

and

$$P(d_i = \eta) = \frac{p_1}{2},\; P(d_i = -\eta) = 1 - \frac{p_1}{2}.$$

Since

$$\frac{\partial P(n_i = -\eta | d_i = \eta)}{\partial \tilde{z}_1} = -\frac{(\ln(p1) - \ln(\tilde{z}_1))}{p_1} < 0$$

and

$$\frac{\partial P(n_i = -\eta | d_i = -\eta)}{\partial \tilde{z}_1} = -\frac{\ln(1 - \tilde{z}_1)}{p_1} > 0,$$

we have that

$$\frac{\partial P(d_i = \eta | n_i = -\eta)}{\partial \tilde{z}_1} < 0.$$

Also, notice $\lim_{\tilde{z}_1 \to 0} P(d_i = \eta | n_i = -\eta) = \frac{p_1}{2}$ and $\lim_{\tilde{z}_1 \to p_1} P(d_i = \eta | n_i = -\eta) = 0$, that is, investors observing $n_i = -\eta$ are at best as optimistic as their priors updated by the observation of $\tilde{z}_1$, which is not enough for them to be willing to rollover. Hence, there is no equilibrium in this form when $c > \eta$.

Now, consider $c < \eta$ (so double-misreporting dominates biasing the least to achieve $z = \tilde{z}_1$). The choice is then between transparency with $z = \tilde{z}_1$ and double-misreporting and opacity with $z = \hat{z}$.

$$D_2 = U(t = 0, z = \hat{z}, p) - U(t = 1, z = z_1, p;\ \text{DM}) = \begin{cases} 2cp + (\theta - \eta + c)\tilde{z}_1 - c\hat{z} & p \leq 1 - \tilde{z}_1 \\ 2\eta p + (\theta + \eta - c)\tilde{z}_1 - c\hat{z} - 2(\eta - c) & p > 1 - \tilde{z}_1 \end{cases}$$

If $c\hat{z} < (\theta - \eta + c)\,\tilde{z}_1$, $D_2 > 0$ for all $p \in [0, \hat{z}]$, and thus $t(p) = 0$ and $z(p) = \hat{z}$ for all $p \in [0, \hat{z}]$. Now consider $c\hat{z} > (\theta - \eta + c)\,\tilde{z}_1$. Opacity is better whenever

$$p > \tilde{p} \equiv \frac{c\hat{z} - (\theta - \eta + c)\tilde{z}_1}{2c}$$

Notice since $c < \theta - \eta$, the upper bound for $\tilde{p}$ is

$$\tilde{p} \leq \frac{\hat{z}}{2} - \tilde{z}_1 \leq \frac{1}{2} - \tilde{z}_1. \tag{29}$$

We can compute the probability an agent observing $t = 1$, $z = \tilde{z}_1$ and $n_i = -\eta$ assigns to her bank being of high type:

$$P\left(d_i = \eta | n_i = -\eta; \tilde{z}_1\right) = \frac{P\left(n_i = -\eta | d_i = \eta\right) P\left(d_i = \eta\right)}{P\left(n_i = -\eta | d_i = \eta\right) P\left(d_i = \eta\right) + P\left(n_i = -\eta | d_i = -\eta\right) P\left(d_i = -\eta\right)}$$

Since $p \leq 1 - \tilde{z}_1$ for all $p \in [0, \tilde{p}]$, $P\left(n_i = -\eta | d_i = \eta\right) = 1$ – that is, investors in all high-type banks receive low signals. Also, $P\left(d_i = \eta\right) = \tilde{p}/2$ and $P\left(n_i = -\eta | d_i = -\eta\right) = \frac{1}{\tilde{p}} \int_0^{\tilde{p}} \frac{1 - \tilde{z}_1 - p}{1 - p} dp$, hence

$$P\left(d_i = \eta | n_i = -\eta; \tilde{z}_1\right) = \frac{\tilde{p}}{\tilde{p} + (2 - \tilde{p})\left[1 + \frac{\tilde{z}_1 \ln(1 - \tilde{p})}{\tilde{p}}\right]}.$$

Using the facts that this expression is increasing in $\tilde{z}_1$ and, by (29), $\tilde{z}_1 \leq 1/2 - p_1$, one can check that $P\left(d_i = \eta | n_i = -\eta; \tilde{z}_1\right)$ is always smaller than 0.5, and thus there can be no equilibrium in this form.

**Subcase 6: Suppose** $a(-\eta, z) = \begin{cases} 1 & \textbf{if } z \le \tilde{z}_1, \\ 0 & \textbf{otherwise,} \end{cases}$ **and** $a(\eta, z) = \begin{cases} 1 & \textbf{if } z \ge \tilde{z}_2, \\ 0 & \textbf{otherwise.} \end{cases}$

First, consider $\tilde{z}_1 \le \tilde{z}_2 \le \hat{z}$: For $p \in [0, z_2)$, choosing the policies with the smallest amount of bias to achieve each $z$, regulator's payoffs are

$$U(t = 1, z = p, p) = \begin{cases} (1-p)(\theta - \eta) & p \le z_1 \\ 0 & p > z_1 \end{cases} \tag{30}$$

$$U(t = 1, z = 0, p) = (1-p)(\theta - \eta) + p(\theta + \eta - c) \tag{31}$$

$$U(t = 1, z = 0, p) = (1-p)(\theta - \eta) + (p - z_1)(\theta + \eta - c), \; z > z_1 \tag{32}$$

$$U(t = 1, z = z_2, p) = p(\theta + \eta) + (z_2 - p)(\theta - \eta - c) \tag{33}$$

$$U(t = 0, z = \hat{z}, p) = (1-p)(\theta - \eta) + p(\theta + \eta) - (\hat{z} - p)c \tag{34}$$

Notice (30) and (32) are always dominated by (31) and (33) is always dominated by (34). Comparing (31) and (34),

$$U(t = 1, z = 0, p) \ge U(t = 0, z = \hat{z}, p) \iff p \le \frac{\hat{z}}{2}.$$

For $p \in [z_2, \hat{z})$, the choice is also between $t = 1,\ z = 0$, and $t = 0,\ z = \hat{z}$, and the latter is more profitable whenever $p \ge \frac{\hat{z}}{2}$. For $p \in [\hat{z}, 1]$, the regulator chooses $t(p) = 0$ and $z(p) = p$. Hence, the regulator's policy implies[29]

$$\begin{cases} t(p) = 1,\ z(p) = 0 & \text{for } p \le \frac{\hat{z}}{2}, \\ t(p) = 0,\ z(p) = \hat{z} & \text{for } p \in \left[\frac{\hat{z}}{2}, \hat{z}\right], \\ t(p) = 0,\ z(p) = p & \text{for } p \in [\hat{z}, 1]. \end{cases} \tag{35}$$

Since the regulator sends $n_i = -\eta$ for everyone when $p \in [0, \hat{z}/2]$, this signal is completely uninformative. Thus, agents expect their banks to be of high type with probability $\hat{z}/4$ (since

[29]Since there is no double-misreporting here, $z(p) = p$ means that $\hat{G} = G$.

their prior about $p$ is update to a uniform distribution on $[0, \hat{z}/2]$, and thus they must deviate and run.

If $\tilde{z}_1 \leq \hat{z} < \tilde{z}_2$, the regulator's strategy optimal strategy can also be described by (35), which cannot be an equilibrium for the same reason.

If $\tilde{z}_2 \leq \tilde{z}_1 < \hat{z}$, the optimal strategy for the regulator is to misreport the least to achieve the following:

$$\begin{cases} t(p) = 1,\, z(p) = 0 & \text{for } p \leq \frac{\tilde{z}_2}{2}, \\ t(p) = 1,\, z(p) = \tilde{z}_2 & p \in \left(\frac{\tilde{z}_2}{2}, \tilde{z}_2\right), \\ t(p) = 1,\, z(p) = p & p \in [\tilde{z}_2, \tilde{z}_1], \\ t(p) = 1,\, z(p) = \tilde{z}_1 & p \in \left(\tilde{z}_1, \frac{\hat{z}-\tilde{z}_1}{2}\right), \\ t(p) = 0,\, z(p) = \hat{z} & \text{for } p \in \left[\frac{\hat{z}-\tilde{z}_1}{2}, \hat{z}\right], \\ t(p) = 0,\, z(p) = p & \text{for } p \in [\hat{z}, 1]. \end{cases} \tag{36}$$

Notice this cannot be an equilibrium, since observing $z \in (\tilde{z}_2, \tilde{z}_1)$ reveals the regulator is disclosing bank-specific information truthfully, and then investors observing $n_i = -\eta$ deviate and run.

Finally, if $\hat{z} < \min[\tilde{z}_2, \tilde{z}_1]$, the regulator follows the strategy in (35) – or that strategy substituting $t(p) = 0$ by $t(p) = 1$ for $p \in [\tilde{z}_2, \tilde{z}_1]$ if $\tilde{z}_2 < \tilde{z}_1$, which yields the same payoff. Again, this is not an equilibrium since investors observing $n_i = -\eta$ deviate and run when $z < \hat{z}/2$.

**Case II: suppose $a(0, z) = 1$ for $z \leq \hat{z}$ and $0$ otherwise**

First, suppose $a(-\eta, z) = \begin{cases} 1 & \text{if } z \leq z_1, \\ 0 & \text{otherwise.} \end{cases}$ for all $z$ and $a(\eta, z) = \begin{cases} 1 & \text{if } z \leq z_2, \\ 0 & \text{otherwise.} \end{cases}$

In this case, whenever $p \in [0, \min\{z_1, z_2, \hat{z}\}]$, the regulator sets $\hat{G} = G$ (and thus$z(p) = p$) and it is indifferent between between $t(p) = 0$ or $t(p) = 1$. For $p > \min\{z_1, z_2, \hat{z}\}$, it is easy to see that the possible choices for $z(p)$ are $\{p, z_1, z_2, \hat{z}\}$. If $t(p) = 1$ for $p \in \mathcal{T} \subseteq [0, \min\{z_1, z_2, \hat{z}\}]$, observing $p \in \mathcal{T}$ conveys there is no information manipulation, and thus investors observing $n_i = -\eta$ deviate and run. If $t(p) = 0$ for all $p < \min\{z_1, z_2, \hat{z}\}$, observing $z < \min\{z_1, z_2, \hat{z}\}$ conveys that $p = z$. Hence, whenever $z < \min\{z_1, z_2, \hat{z}\}$ and $z < \bar{p}$, investors observing $z$ and opacity deviate and run. There is no equilibrium in this form, then.

Now, consider any other strategy for investors when observing bank-specific information, that is, an strategy in which either investors observing a high signal or the ones observing a low signal (or both) rollover only if $z$ is above a threshold. In this case, choosing $\hat{G} = G$ and $t(p) = 0$ is always the optimal strategy for $p \in [0, p']$ for some $p'$, and $z(p) \geq p'$ for $p > p'$. Hence, there are always values of $p < 1/2$ for which the optimal strategy is opacity and for which $\mathbb{E}[p|z(p)] = p < 1/2$. This as well cannot be an equilibrium.

This concludes the proof of Proposition 5. □

## A.7 Equilibria allowing bank-specific misreporting – Case $c > \theta - \eta$

When $c > \theta - \eta$, the only equilibria satisfying the intuitive criterion are the ones described in Proposition 2. This section shows how other possibilities are ruled out.

The proof that there is no equilibrium in which investors roll over under opacity if $z$ is smaller than a threshold is the same as in the case with $c < \theta - \eta$. All cases that follow assume investors roll over under opacity if $z \geq \hat{z}$ for some $\hat{z}$.

**Case 1: Suppose** $a(-\eta, z) = \begin{cases} 1 & \textbf{if } z \leq z_1, \\ 0 & \textbf{otherwise,} \end{cases}$ **and** $a(\eta, z) = \begin{cases} 1 & \textbf{if } z \leq z_2, \\ 0 & \textbf{otherwise.} \end{cases}$

The same reasoning as in the case with $c < \theta - \eta$ implies there is no equilibrium in this form. Let $p' = \min\{z_1, z_2, \hat{z}\}$. For $p < p'$, the optimal strategy is always $t(p) = 1$ and $\hat{G} = G$. For $p \geq p'$, it is never optimal to set a $\hat{G}$ such that $z(p) < p'$ (it would involve a manipulation cost and no additional benefit). Thus observing $t(p) = 1$ and $z < p'$ conveys that in fact $d_i = n_i$, so investors observing the low signal deviate and run.

**Case 2: Suppose** $a(-\eta, z) = \begin{cases} 1 & \textbf{if } z \leq z_1, \\ 0 & \textbf{otherwise,} \end{cases}$ **and** $a(\eta, z) = \begin{cases} 1 & \textbf{if } z \geq z_2, \\ 0 & \textbf{otherwise.} \end{cases}$

First, consider $\hat{z} \leq \min\{z_1, z_2\}$, $z_1, z_2 \in (0,1)$.For $p < \hat{z}$, the candidates for optimal policy for the regulator are the following (where $z$ is achieved by the smallest amount of bias possible, that is, with no double misreporting):

$$U(t = 1, z = p, p) = (1 - p)(\theta - \eta);$$

$$U(t = 1, z = p - \alpha, p) = (1 - p)(\theta - \eta) - \alpha(\theta + \eta - c);$$

$$U(t = 0, z = \hat{z}, p) = (1 - p)(\theta - \eta) + p(\theta + \eta) - c(\hat{z} - p).$$

Notice when $c > \theta + \eta$, it is never worth biasing the proportion of good banks down, but it may be worth doing so otherwise. If $c > \theta + \eta$, there exists $p' < \hat{z}$ such that $t(p) = 1$ and $n_i = d_i \; \forall i$ (thus $z(p) = p$) whenever $p < p'$, and $t(p) = 0$ and $z(p) = \hat{z}$ for $p \in [p', \hat{z}]$. Also, for $p > \hat{z}$, it is never optimal to set $z(p) < \hat{z}$. Thus, it cannot be an equilibrium, since investors observing the low signal and $z(p) = p < p'$ would deviate and run. Now consider $c < \theta + \eta$. There exists $p'' < \hat{z}$ such that the regulator sets $t(p) = 1$ and $n_i = -\eta$ (and thus $z(p) = 0$) for all $p < p''$, and $z(p) = \hat{z}$ and $t(p) = 0$ for $p \in [p'', \hat{z}]$. Again, it cannot be an equilibrium. Investors observing $z = 0$ and $n_i = -\eta$ know their low signal is completely uninformative, so they stick to their updated priors, namely that $p \sim U(0, p'')$, and thus they expect $p$ to be smaller than $0.5 < \bar{p}$ and run.

Now, consider $z_1 < \hat{z} \leq z_2$ or $z_1 < z_2 < \hat{z}$. The equilibrium can be ruled out by an analogous reasoning. If $c > \theta + \eta$, there is always an interval $[0, p']$ in which the regulator chooses $t(p) = 1$ and sends signals $n_i = d_i$ for all $i$, but then investors observing the low signal run (considering $z(p)$ is never smaller than $p'$ for $p > p'$). If $c < \theta + \eta$, there is always a region $[0, p'']$ in which $n_i = -\eta \; \forall i$, and since $\mathbb{E}[d_i | n_i = -\eta, z = 0] = p''/2 < \bar{p}$, investors deviate and run.

If $z_2 \leq \hat{z} \leq z_1$, the regulator's strategy is such that, if $c > \theta + \eta$, $t(p) = 1$ for $p \in [0, \hat{z}]$, with $\hat{G} = G$ for $p < p' = cz_2/(\theta + \eta + c)$ and $p \in [z_2, \hat{z}]$ and $z(p) = z_2$ for $p \in [p', z_2]$ (again, with no double misreporting). This cannot be an equilibrium since investors observing $z < p'$ and $n_i = -\eta$ would deviate. If $c < \theta + \eta$, $n_i = -\eta \; \forall i$ (and thus $z(p) = 0$) for $p < p'' = z_2/2$. In this region, investors observing the low signal deviate and run since the private signal is uninformative and the expectation of $p$ is smaller than $\bar{p}$.

Finally, consider $z_2 < z_1 < \hat{z}$. For $p \in [z_2, z_1]$, the regulator chooses $t(p) = 1$ and $n_i = d_i \; \forall i$ (and thus $z(p) = p$), while $z(p) \leq z_2$ for $p < z_2$ and $z(p) \geq z_1$ for $p > z_1$. Thus, observing $z \in (z_2, z_1)$ conveys that $n_i = d_i$, hence investors observing the low signal deviate and run.

There is no equilibrium in this form with $z_1, z_2 \in (0, 1)$, then. Notice, though, if $z_1 < 0$ and $z_2 > 1$, there is an equilibrium. It is the same as in the case where no bank-specific information manipulation is possible.

**Case 3: Suppose** $a(-\eta, z) = \begin{cases} 1 & \textbf{if } z \geq z_1, \\ 0 & \textbf{otherwise,} \end{cases}$ **and** $a(\eta, z) = \begin{cases} 1 & \textbf{if } z \leq z_2, \\ 0 & \textbf{otherwise.} \end{cases}$

Consider $\hat{z} < \min\{z_1, z_2\}$. For $p \geq \hat{z}$, the regulator chooses $\hat{G} = G$; if $z_2 < z_1$, $t(p) = 0$ for all $p \geq \hat{z}$ , and if $z_1 \leq z_2$, the regulator is indifferent between $t(p) = 1$ or $t(p) = 0$ for $p \in [z_1, z_2]$.

Notice there is no equilibrium in this form with $\hat{z} < z^*$, as defined in (4) (the expectation of $p$ under opacity when $z = \hat{z}$ would be smaller than $\bar{p}$). If $\hat{z} \geq z^*$, there are equilibria in which for $p \in \left[0, \frac{c\hat{z}-\theta+\eta}{c-\theta+\eta}\right]$, $t(p) = 1$ and $n_i = d_i$ $\forall i$. For $p \in \left(\frac{c\hat{z}-\theta+\eta}{c-\theta+\eta}, \hat{z}\right)$, $t(p) = 0$ and $z(p) = \hat{z}$ (with a $\hat{G}$ that imposes the smallest amount of bias as possible), and for $p > \hat{z}$, $t(p) = 0$ and $\hat{G} = G$. This equilibrium is ruled out by the Intuitive Criterion, though. For instance, consider the observation of $t = 1$ and $z = 1$. The only state in which such strategy is not dominated by the equilibrium strategy is if $p = 1$. But this off-the-equilibrium belief is not compatible with the investors' strategy, which would prescribe that investors run when observing such deviation and a signal $n_i = \eta$. We can then rule out this equilibrium.

Now consider $z_1 < z_2 \leq \hat{z}$. For $p \in (z_1, z_2)$, the regulator chooses $t(p) = 1$ and $n_i = d_i$ for all $i$. For $p \notin (z_1, z_2)$, either $z(p) \leq z_1$ or $z(p) \geq z_2$. Thus, observing $z \in (z_1, z_2)$ conveys that $d_i = n_i$, and thus investors run when observing $n_i = -\eta$. This is not an equilibrium, then.

Consider now the case with $z_2 < \hat{z} \leq z_1$. For $p < \hat{z}$, the regulator chooses $t(p) = 0$ and $\hat{G} = G$. For $p \in [0, \hat{z}]$, the regulator chooses between the strategies (that uses the smallest amount of bias to achieve each $z$) yielding the following payoffs:

$$U(t = 1, z = p, p) = \begin{cases} p(\theta + \eta) & p \leq z_2, \\ 0 & p \in (z_2, \hat{z})\,; \end{cases}$$

$$U\left(t = 0, z = \hat{z}, p\right) = p\left(\theta + \eta\right) + \left(1 - p\right)\left(\theta - \eta\right) - c\left(\hat{z} - p\right);$$

$$U(t = 1, z = z_2, p) = \begin{cases} p(\theta + \eta) + (z_2 - p)(\theta - \eta - c) & p < z_2, \\ z_2\left(\theta + \eta\right) - c(p - z_2) & p \in [z_2, \hat{z}). \end{cases}$$

Since $c > \theta - \eta$, choosing $z(p) = z_2$ when $p < z_2$ is never optimal. Notice $U\left(t = 0, z = \hat{z}, p\right) \geq U(t = 1, z = p, p)$ if and only if $p \geq \frac{c\hat{z}-\theta+\eta}{c-\theta+\eta}$, and $U(t = 1, z = p, p) = 0$ when $p = 0$. Also, $U(t = 0, z = \hat{z}, p) \geq 0$ whenever $p \geq \frac{c\hat{z}-\theta+\eta}{2\eta+c}$, and thus $U(t = 0, z = \hat{z}, p)$ is larger than zero for all $p$ when $\hat{z} \leq \frac{\theta-\eta}{c}$. We also have that $U\left(t = 0, z = \hat{z}, p\right) \geq U(t = 1, z = z_2, p)$ for $p \in (z_2, \hat{z})$ if and only if $p \geq \frac{c\hat{z}-\theta+\eta+(\theta+\eta+c)z_2}{2(\eta+c)}$. Hence, if $\hat{z} \leq \frac{\theta-\eta+z_2(c-\theta+\eta)}{c}$, $U\left(t = 0, z = \hat{z}, p\right) \geq U(t = 1, z = z_2, p)$ for all $p \in (z_2, \hat{z})$. Thus, the regulator's policy given investors' strategies is the following:

(i) if $\hat{z} \leq \frac{\theta-\eta}{c}$, $t(p) = 0$ for all $p$, $z(p) = \hat{z}$ for $p \in [0, \hat{z}]$ and $z(p) = p$ otherwise. This cannot be an equilibrium, though, since $\mathbb{E}\left[p|\hat{z}\right] < \bar{p}$.

(ii) if $\hat{z} \in \left[\frac{\theta-\eta}{c}, \frac{\theta-\eta+(c-\theta+\eta)z_2}{c}\right]$, the regulator chooses $t(p) = 1$ and $z(p) = p$ (with $n_i = d_i$

for all $i$) for $p \in [0, p']$, $t(p) = 1$ for $p > \hat{z}$, with $z(p) = \hat{z}$ for $p \in [p', \hat{z}]$ and $z(p) = p$ for $p > \hat{z}$, where $p' = \frac{c\hat{z}-\theta+\eta}{c-\theta+\eta}$. This is not always an equilibrium. For this to be an equilibrium, we need that $\mathbb{E}[p|\hat{z}] = \frac{p'+\hat{z}}{2} \geq \bar{p}$. This is only true when

$$\hat{z} \geq \frac{2\bar{p}(c-\theta+\eta)+\theta-\eta}{2c-\theta+\eta} \equiv z^*. \tag{37}$$

To sum up, there is only an equilibrium in this format when $\hat{z} \in \left[z^*, \frac{\theta-\eta+(c-\theta+\eta)z_2}{c}\right]$. However, we can eliminate this equilibrium using the Intuitive Criterion. Consider a deviation in which the regulator chooses $z = 1$ and $t = 1$. Even under the most optimistic belief possible concerning investors responses, the only regulator type for which this strategy is not equilibrium-dominated is the one with $G = [0, 1]$ (thus $p = 1$). But than the off-the-equilibrium belief that the investor observing $n_i = \eta$ is of good type with probability smaller than $\bar{p}$ needed to sustain this equilibrium is not reasonable in the sense of the IC. We can thus eliminate this equilibrium.

(iii) if $\hat{z} \in \left[\frac{\theta-\eta+(c-\theta+\eta)z_2}{c}, \frac{\theta-\eta}{c} + \frac{z_2(2\eta+c)(\theta+\eta+c)}{c^2}\right]$, there can be an equilibrium in which $t(p) = 1$ for $p < p'' = \frac{c\hat{z}-\theta+\eta+(\theta+\eta+c)z_2}{2(\eta+c)}$ and $t(p) = 0$ otherwise, with $n_i = d_i$ for all $i$ (and thus $z(p) = p$) for $p \leq z_2$, $z(p) = z_2$ for $p \in (z_2, p'')$, $z(p) = \hat{z}$ for $p \in [p'', \hat{z}]$ and $z(p) = p$ for $p > \hat{z}$. This is an equilibrium as long as $(p'' + \hat{z})/2 \geq \bar{p}$. However, this equilibrium do not survive the IC, for the same reason as the previous case.

(iv) if $\hat{z} > \frac{\theta-\eta}{c} + \frac{z_2(2\eta+c)(\theta+\eta+c)}{c^2}$, whenever $p \in \left(\frac{(\theta+\eta+c)z_2}{c}, \frac{c\hat{z}-\theta+\eta}{2\eta+c}\right)$ the regulator chooses $t(p) = 1$ and $n_i = d_i$ for all $i$, and consequently $z(p) = p$. Since $z(p) \notin \left(\frac{(\theta+\eta+c)z_2}{c}, \frac{c\hat{z}-\theta+\eta}{2\eta+c}\right)$

for values of $p$ out of this interval, investors observing a $z$ in this interval and $n_i = -\eta$ deviate and run.

Finally, consider $z_2 \leq z_1 \leq \hat{z}$. We can easily apply the intuitive criterion to eliminate any possible equilibria in this case. Notice the regulator always sets $t(p) = 0$ and $\hat{G} = G$ for $p > \hat{z}$, and for $p \leq \hat{z}$ it is never profitable to increase $z$ above $\hat{z}$ (in which case all investors roll over under opacity). To sustain an equilibrium in this form, we need the out-of-equilibrium belief $P(d_i = \eta | n_i = \eta, z)$ to be smaller than $\bar{p}$ whenever $z > z_2$. But the only state in which one could expect $z$ to be equal 1 is when $p = 1$ (sending $z(p) = 1$ is dominated for any $p < 1$). Hence, an investor observing transparency, $z(p) = 1$ and $n_i = \eta$ would know her type is $d_i = \eta$ with probability 1, and then she would deviate and roll over.

**Case 4: Suppose** $a(-\eta,z)=\begin{cases}1 & \textbf{if } z\geq z_1,\\ 0 & \textbf{otherwise,}\end{cases}$ **and** $a(\eta,z)=\begin{cases}1 & \textbf{if } z\geq z_2,\\ 0 & \textbf{otherwise.}\end{cases}$

Consider $\hat{z}<\min\{z_1,z_2\}$. For $p<\hat{z}$, the regulator chooses either $t=1$ and no bias or $t(p)=0$ and just the smallest amount of bias so that $z(p)=\hat{z}$. For $p\geq\hat{z}$, $\hat{G}=G$. If $t(p)=0$ and $z(p)=\hat{z}$ for all $p<\hat{z}$, this is not an equilibrium, since $\mathbb{E}[p|\hat{z}]<\bar{p}$. If $t(p)=1$ for some $p<\hat{z}$, this is also not an equilibrium, since investors observing $n_i=\eta$ would deviate and roll over (since they know $d_i=n_i\ \forall i$ when $p<\hat{z}$ and $t=1$).

Now consider $\hat{z}>\max\{z_1,z_2\}$. For $p\in(\max\{z_1,z_2\},\hat{z})$, $t(p)=1$ and $n_i=d_i\ \forall i$ (hence $z(p)=p$). Since for $p\leq\max\{z_1,z_2\}$, the regulator certainly sets $z(p)\leq\max\{z_1,z_2\}$, and for $p\geq\hat{z}$, $z(p)=p$, observing $z\in(\max\{z_1,z_2\},\hat{z})$ conveys that there is no bias, and thus investors observing $n_i=-\eta$ deviate and run. This is not an equilibrium, then.

Consider now the case with $\hat{z}\in[\min\{z_1,z_2\},\max\{z_1,z_2\}]$. First, let $z_2<\hat{z}\leq z_1$. For $p\geq\hat{z}$, there is no bias ($z(p)=p$). For $p<\hat{z}$, we have the following options (all with the smallest bias to achieve $z$):

$$U(t=1,z=p,p)=\begin{cases}0 & p<z_2,\\ p(\theta+\eta) & p\in[z_2,\hat{z});\end{cases}$$

$$U(t=0,z=\hat{z},p)=p(\theta+\eta)+(1-p)(\theta-\eta)-c(\hat{z}-p);$$

$$U(t=1,z=z_2,p)=p(\theta+\eta)+(z_2-p)(\theta-\eta-c),\ p<z_2.$$

Defining $p_1=\frac{(c-\theta+\eta)z_2}{2\eta+c}$ and $p_2=\frac{c\hat{z}-\theta+\eta}{c-\theta+\eta}$, we have the following:

(i) If $\hat{z}\in[z^*,\bar{z}]$ and $z_2\leq p_2$, the regulator's strategy imply

$$z(p)=\begin{cases}p & p<p_1,\\ z_2 & p\in[p_1,z_2],\\ p & p\in(z_2,p_2),\\ \hat{z} & p\in[p_2,\hat{z}],\\ p & p>\hat{z}.\end{cases}$$

Since $c>\theta-\eta$, there is a non-empty interval of states $[0,p_1]$ in which the regulator chooses no bias and would indifferent between transparency and opacity if $\varepsilon=0$. Given the assumption that choosing $t=0$ entails a cost $\varepsilon>0$, the regulator chooses $t=1$ in this region. Hence, whenever $z < p_1$, investors observing $n_i = \eta$ deviate and roll over. If we assume $\varepsilon = 0$ and consider a strategy in which the regulator chooses $t(p) = 0$ for $p$ in that interval, this is compatible with an equilibrium. Thus, an equilibrium candidate has the following disclosure choices:

$$t(p) = \begin{cases} 0 & p \in [0, p_1] \cup [p_2, 1], \\ 1 & p \in (p_1, p_2). \end{cases}$$

The fact that $\hat{z} \geq z^*$ ensures investors observing opacity and $\hat{z}$ form beliefs compatible with their strategy of rolling over, and $\hat{z} \leq \bar{z}$ eliminates equilibria that do not satisfy the intuitive criterion. Notice that we can also eliminate any equilibria with $z_1 \leq 1$ applying the IC. Suppose $z_1 \leq 1$. Consider a deviation from the equilibrium strategy in which the regulator chooses $t(p) = 1$ for some $p > z_1$, and thus it discloses signals $n_i = \eta$ for $i$ in some $B'$ and $n_i = -\eta$ for $i \in G'$. Under the most optimistic belief possible (that all investors would roll over), the only type of regulator for which this strategy is not equilibrium-dominated is the one with $B = B'$ and $G = G'$. Hence, an investor observing $n_i = -\eta$ would deviate and run.

There is still to be checked if $P(d_i = \eta | n_i = \eta; z_2) \geq \bar{p}$, which must hold for the strategies described above to be an equilibrium (remember this equilibrium is ruled out under the assumption that $\varepsilon > 0$). Using Bayes' rule, we have

$$P(d_i = \eta | n_i = \eta; z_2) = \frac{P(n_i = \eta | d_i = \eta; z_2) P(d_i = \eta; z_2)}{P(n_i = \eta | d_i = \eta; z_2) P(d_i = \eta; z_2) + P(n_i = \eta | d_i = -\eta; z_2)[1 - P(d_i = \eta; z_2)]}$$

Since $P(d_i = \eta; z_2) = \frac{p_1 + z_2}{2}$ , $P(n_i = \eta | d_i = \eta; z_2) = 1$ and

$$P(n_i = \eta | d_i = -\eta; z_2) = \frac{1}{z_2 - p_1} \int_{z_2}^{p_1} \left[ \frac{z_2 - p}{1 - p} \right] dp = 1 - \frac{(1 - z_2)[\ln(1 - p_1) - \ln(1 - z_2)]}{z_2 - p_1},$$

then

$$P(d_i = \eta | n_i = \eta; z_2) = \frac{\frac{p_1 + z_2}{2}}{\frac{p_1 + z_2}{2} + \left\{ 1 - \frac{(1 - z_2)[\ln(1 - p_1) - \ln(1 - z_2)]}{z_2 - p_1} \right\} \left[ 1 - \frac{p_1 + z_2}{2} \right]},$$

where $p_1 = \frac{(c - \theta + \eta) z_2}{2\eta + c}$. If $P(d_i = \eta | n_i = \eta; z_2) \geq \bar{p}$, we have an equilibrium. Moreover, for this equilibrium to survive the IC, we need $\frac{\tilde{p}}{z_2} \leq \bar{p}$, where $\tilde{p} = \max\left\{0, \frac{c z_2 - \theta + \eta}{2\eta + c}\right\}$. $\tilde{p}/z_2$ is the smaller probability of the bank being of high type an investor observing $n_i = \eta$ and a $z$ slightly smaller than $z_2$ can expect.

Now, define $p' = \frac{c\hat{z} - \theta + \eta}{2\eta + c}$.

(ii) If $\hat{z} \in \left[\frac{c}{\eta+c}\bar{p} + \frac{\gamma-(\theta-\eta)}{2(\eta+c)}, \frac{(2\eta+c)\bar{p}+\theta-\eta}{c}\right]$ and $z_2 \in \left[\frac{c\hat{z}-\theta+\eta}{c-\theta+\eta}, \hat{z}\right]$, the regulator's strategy imply

$$z(p) = \begin{cases} p & p < p', \\ \hat{z} & p \in [p', \hat{z}], \\ p & p > \hat{z}. \end{cases}$$

We can once again rule out any equilibrium with $z_1 \leq 1$ due to the IC. Lets thus consider that investors observing the low signal always run. For $p > p'$, the regulator sets $t(p) = 0$, and for $p < p'$, the regulator is indifferent between $t = 0$ or $t = 1$ when $\varepsilon = 0$. However, if $\varepsilon > 0$, than $t(p) = 1$ for $p < p'$, and this is not an equilibrium because investors infer $d_i = n_i$ for all $i$ and those observing the low signal deviate and run.

If we assume $\varepsilon = 0$ and consider the strategy with $t(p) = 0$ for all $p$, this is in fact an equilibrium, since $(p' + \hat{z})/2 \geq \bar{p}$. For the equilibrium to satisfy the IC, we need $p' < \bar{p}$, which is guaranteed by the inequality $\hat{z} \leq \frac{(2\eta+c)\bar{p}+\theta-\eta}{c}$. Notice if $\gamma \geq \frac{2c(\theta+\eta)}{2\eta+c}$, the IC does not rule out any equilibrium since the upper bound of $\hat{z}$ in this case is 1, i.e., even for $\hat{z} = 1$, $p' \leq \bar{p}$. Moreover, notice this equilibrium only exists when $c \geq \frac{2\eta(\gamma-\theta-\eta)}{2(\theta+\eta)-\gamma} > \theta - \eta$. Otherwise, the lower bound for $\hat{z}$ is larger than one.

Finally, consider $z_1 < \hat{z} \leq z_2$. For $p \geq \hat{z}$, the best option for the regulator is $n_i = d_i$ for all $i$ (which implies $z(p) = p$). Notice it is never optimal for $p < \hat{z}$ to set $z(p) > \hat{z}$. Although the regulator is indifferent between transparency and opacity when $p > z_2$ (if $\varepsilon = 0$), any equilibrium candidate must have $t(p) = 1$ for all $p > \hat{z}$, otherwise investors observing the low signal would run. Notice, however, that even if there exists any equilibrium in this form, such equilibrium does not survive the intuitive criterion for the same reason presented earlier. After observing a deviation for transparency and $z > \hat{z}$, investors know the only type for whom such deviation is not equilibrium-dominated is the one with $B = \hat{B}$ and $G = \hat{G}$ (where $\hat{B}$ and $\hat{G}$ are the disclosed sets of alleged good and bad banks), and then an investor observing $n_i = -\eta$ would deviate and run. There is no equilibrium in this form surviving the intuitive criterion, then.

## A.8 Welfare under bank-specific information manipulation

For $c \leq \theta - \eta$, welfare is given by

$$W = \int_0^{z'} \left[p\left(\theta+\eta\right) + (z'-p)(\theta-\eta)\right] dp + \int_{z'}^1 z'\left(\theta-\eta\right) dp = (\theta+\eta)z' - \eta z'^2,$$

where $z' \in [\underline{z}(c), \tilde{z}]$, $\tilde{z}$ satisfying (11) and $\underline{z}(c) = c/(c+\theta+\eta)$. Notice $W$ is decreasing in $z'$. If the worst equilibrium is played, that is, the one with $z' = \underline{z}(c)$, we have that

$$\frac{\partial W\left(\underline{z}(c)\right)}{\partial c} = \frac{(\theta+\eta)\left(\theta^2+2\theta\eta+\eta^2+c\left(\theta-\eta\right)\right)}{\left(\theta+\eta+c\right)^3} > 0$$

$$\frac{\partial^2 W\left(\underline{z}(c)\right)}{\partial c^2} = -\frac{2\left(\theta+\eta\right)\left(\theta^2+3\theta\eta+\eta^2+c\left(\theta-\eta\right)\right)}{\left(\theta+\eta+c\right)^4} < 0,$$

and $W(\underline{z}(0)) = 0$. If the best equilibrium is always played, that is, the one with $z' = \tilde{z}$, welfare is independent of $c$ for all $c < \theta - \eta$ (since $\tilde{z}$ does not depend on $c$). One can verify that welfare in this case can be smaller or larger than the welfare when $c = \theta - \eta$ and the best equilibrium is played (described below), depending on parameters, as discussed in the main text.

For $c \geq \theta - \eta$, welfare is given by

$$W = \int_0^{p^b} p\left(\theta+\eta\right)dp + \int_{p^b}^1 \left[p\left(\theta+\eta\right) + (1-p)(\theta-\eta)\right]dp = \frac{(\theta-\eta)}{2}p^{b^2} - p^b\left(\theta-\eta\right) + \theta,$$

where $p^b \in \left[p^{b^*}(c), \bar{p}\right]$. Notice welfare is decreasing in $p^b$, and thus for every $c$ the best equilibrium is the one with $p^b = p^{b^*}(c)$, and the worst equilibrium is the one with $p^b = \bar{p}$, which does not depend on $c$. Substituting $p^{b^*}$ in the welfare expression and differentiating with respect to $c$, we have that for $c > \theta - \eta$, welfare is decreasing and convex in $c$. As $c \to \infty$, the equilibrium set converges to a unique equilibrium with $p^b = \bar{p}$.