Debt consolidation: Aggregate and distributional implications

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Abstract

This paper builds and solves numerically, by using Eurozone data, a closed-economy new Keynesian DSGE model in which the fiscal authorities are engaged in public debt reduction over time. The emphasis is on the aggregate and distributional implications of debt consolidation, where agent heterogeneity, and hence distribution, has to do with the distinction between "capitalists" and "workers". The paper studies how these implications depend on the specific fiscal policy instrument used for debt consolidation. There are two key results. First, if the criterion is total, or per capita, output (GDP), then the best policy mix found is to use the long term fiscal gain created by debt reduction so as to reduce the capital tax rate, and, during the early period of fiscal pain, to use spending cuts in order to bring public debt down. Second, if the criterion is equity in net incomes, the best recipe is to use the long term fiscal gain created by debt reduction so as to reduce the labor tax rate, and, during the early period of fiscal pain, to use capital taxes in order to bring public debt down.

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1 Introduction

The 2008 world crisis has, among other things, brought into the spotlight the need for debt consolidation in several European economies. Proponents claim that debt sustainability is necessary for the revival of these economies (see e.g. European Commission, 2015, and CESifo, 2016). Opponents, on the other hand, claim that debt consolidation worsens the recession and increases the public debt-to-GDP ratio at least in the short term; in addition, it is claimed that debt consolidation worsens inequality since fiscal austerity hurts the relatively poor. The latter can be a valid argument since spending cuts and/or tax rises can impact in different ways on different people/groups; even a uniform change in policy can have different effects simply because agents are heterogeneous.

This paper provides a quantitative study of the aggregate and distributional implications of debt consolidation in a new Keynesian DSGE model solved numerically using common parameter values and fiscal data from the Euro area. To study distributional implications, we obviously need a model with agent heterogeneity. There are many types of such heterogeneity. Here, we focus on a specific type which has always been popular in the related macro literature: the distinction between capitalists and workers. Capitalists are defined as those households who hold assets and own the firms. Workers are defined as those households with labor income only. This is related to the classic distinction between income going to capital and income going to labor. ¹ These two types are also called Ricardian and non-Ricardian or optimizing and liquidity constraint households.

The model is as follows. We use a rather standard New Keynesian DSGE model of a closed economy featuring imperfect competition and Rotemberg-type price fixities. The model is solved numerically employing commonly used parameter values and fiscal data from the Euro area. Then, we assume that the debt policy target in the feedback fiscal policy rules is below the data average (from 95% to 60%) and we study the aggregate and distributional implications of various policies aiming at such debt consolidation.

Results will be relative to the status quo (the status quo is defined as the case without debt consolidation). The main results are as follows. First, if the criterion is total, or per capita, output (GDP), then the best policy

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¹See Turnovsky (1995, p. 340) for the distinction between capital and labor income. See also the special issue of the Journal of Economic Dynamics and Control (2013) for heterogeneity and its types. See Lansing (2011) for a review of macro models on capitalists versus workers. Judd (1985) is probably the first paper on the implications of optimal tax policy on capitalists and workers.
mix is to use the long term fiscal gain (namely, the fiscal space created once debt has been reduced) so as to reduce the capital tax rate, and, during the early period of fiscal pain, to use spending cuts in order to bring public debt down.

Second, the above policy mix is Pareto efficient (i.e. both capitalists and workers get better off with debt consolidation). But, if we care about relative gains, there is a “social” cost: inequality (measured by the ratio of capitalist’s to worker’s net incomes) rises both in the new steady state and in the transition.

Third, if the criterion is equity in net incomes (although this comes a lower benefit relative to the above policy mix), the recipe is to use the long term fiscal gain so as to reduce the labor tax rate, and, during the early period of fiscal pain, to use capital taxes in order to bring public debt down.

Fourth, using labor taxes or consumption taxes during the early period of fiscal pain is a bad idea both in terms of total output and equity.

Although there are several papers on the aggregate implications of debt consolidation, there is relatively little on the joint study of aggregate and distributional implications.

Section 2 presents the model. Section 3 presents data, parameter values and the steady state solution. Section 4 explains how we model debt consolidation. The main results are in Section 5. Robustness checks in Section 6 close the paper. Details are in appendix sections.

2 Model

The model is a New Keynesian closed-economy model featuring imperfect competition and Rotemberg-type nominal rigidities, which is extended to include a relatively rich menu of fiscal policy instruments. The model comprises two social classes, called capitalists and workers.

2.1 Households

There are two types of households, a pool of identical capitalists and a pool of identical workers. The percentage of capitalists in the population is $v^k_t$, while that of workers is $v^w_t$. Hence, there are $\frac{v^w_t}{v^k_t}$ times more workers than capitalists, with total number of capitalists normalized to one (see also Lansing, 2015). Except for the fact that the population sizes of capitalists and workers at time $t$ are set exogenously, they are assumed to be constant over time ruling out occupational choice and mobility across groups.
Capitalists own the private firms, hold capital, money and government bonds and also receive labor income for their managerial services. Workers hold money and receive labor income for their labor services. Hence, only capitalists save in the form of capitals and bonds.

**Capitalists**

Each capitalist \( k \) acts competitively to maximize expected discounted lifetime utility taking prices and policy as given. Each \( k \) maximizes expected discounted lifetime utility:

\[
E_o \sum_{t=0}^{\infty} \beta^t U \left( c_t^k, n_t^k, m_t^k, g_t \right)
\]

where \( c_t^k \) is \( k \)'s consumption at \( t \), \( n_t^k \) is \( k \)'s hours of work at \( t \), \( m_t^k \) is \( k \)'s end-of-period real money balances, \( g_t \) is per capita government spending at \( t \), \( E_o \) is the rational expectations operator conditional on the current period information set and \( 0 < \beta < 1 \) is the time preference rate.

In our numerical solutions, we use a utility function of the form (see also Gali 2008):

\[
U \left( c_t^k, n_t^k, m_t^k, g_t \right) = \left[ \left( c_t^k \right)^{1-\sigma} - x_n \left( n_t^k \right)^{1+\eta} + x_m \left( m_t^k \right)^{1-\mu} + x_g \left( g_t \right)^{1-\zeta} \right]^{1-\sigma}
\]

where \( x_n, x_m, x_g, \sigma, \eta, \mu, \zeta \) are standard preference parameters.

The budget constraint of each \( k \) (written in real terms per capitalist) is:

\[
(1 + \tau_t^c) c_t^k + x_t^k + b_t^k + m_t^k = (1 - \tau_t^m) \left[ r_t^k k_{t-1}^k + d_t^k \right] + (1 - \tau_t^n) w_t^k n_t^k + R_{t-1} \left[ \frac{p_{t-1}}{p_t} b_{t-1}^k + \frac{p_{t-1}}{p_t} m_{t-1}^k - \tau_t^l k_{t-1}^k \right]
\]

where \( p_t \) is the price index and small letters denote real variables e.g. \( b_t^k = \frac{b_t^k}{p_t} \), \( d_t^k = \frac{d_t^k}{p_t} \). Here \( x_t^k \) is \( k \)'s real investment at \( t \), \( b_t^k \) is \( k \)'s end-of-period real government bonds at \( t \), \( d_t^k \) is \( k \)'s real dividends paid by firms at \( t \), \( w_t^k \) is workers’ real wage rate at \( t \), \( R_{t-1} \) is gross nominal return to government bonds between \( t - 1 \) and \( t \), \( r_t^k \) is gross real return to inherited capital between \( t - 1 \) and \( t \), \( \tau_t^l \) is real lump-sum.
taxes/transfers to each household $k$ from the government at $t$, $\tau^c_t$ is tax rate on consumption at $t$, $\tau^k_t$ is tax rate on capital income at $t$, $\tau^w_t$ is tax rate on labor income at $t$.

The motion of physical capital for each $k$ is:

$$k^k_t = (1 - \delta)k^k_{t-1} + x^k_t$$  \hspace{1cm} (4)

where $0 < \delta < 1$ is the depreciation rate of capital.

**Households-Workers**

Each worker $w$ has the same expected lifetime utility and instantaneous utility function as each capitalist $k$, that are given by (1) and (2) respectively, where now the index is $w$. Each $w$ acts competitively to maximize expected discounted lifetime utility taking prices and policy as given.

The budget constraint of each $w$ (written in real terms) is:

$$(1 + \tau^c_t^w) c^w_t + m^w_t = (1 - \tau^w_t) w^w_t n^w_t + \frac{p_{t-1}}{p_t} m^w_{t-1} - \tau^l_{t,w}$$  \hspace{1cm} (5)

where also small letters denote real variables per worker. $w^w_t = \frac{W^w_t}{p_t}$. Here $c^w_t$ is $w$’s consumption at $t$, $n^w_t$ is $w$’s hours of work at $t$ and $m^w_t$ is $w$’s end-of-period real money balances, $w^w_t$ is workers’ real wage rate at $t$ and $\tau^l_{t,w}$ is real lump-sum taxes/transfers to each household $w$ from the government at $t$.

**2.2 Firms**

The production sector consists of two sectors: the intermediate goods sector and the final goods sector. Following the literature on imperfect competition in product markets, we assume that the final goods sector is perfectly competitive, while each intermediate goods firm acts as a monopolist in its own market. The final good production “technology” is a constant elasticity (CES) bundler of intermediate goods. Profit maximization in the final goods sector (which is competitive) yields a downward sloping demand curve for intermediate goods producers. Intermediate goods firms choose factor inputs subject to this demand curve for their product facing Rotemberg-type nominal rigidities (the latter implies non-neutrality of money).

**Final goods firms**

Assume, for simplicity, that the single final good is produced by one firm. There is also a continuum (i.e. infinity) of intermediate goods firms that are
indexed along the unit interval. The production "technology" for the final good is a Dixit-Stiglitz type constant returns to scale technology:

\[ y_t = \left[ \int_0^1 [y_t(f)]^\frac{\phi-1}{\phi} df \right]^\frac{\phi}{\phi-1} \]  \tag{6} 

where \( y_t \) is the production of the final goods firm, \( y_t(f) \) is the production of the variety \( f \) produced monopolistically by the intermediate goods firm \( f \) and \( \phi > 0 \) is the elasticity of substitution across intermediate goods produced.

Nominal profits of the final goods firm are defined as:

\[ p_t y_t - \int_0^1 p_t(f)y_t(f)df \]  \tag{7} 

where \( p_t(f) \) is the price of variety \( f \).

The final goods firm chooses the quantity of every variety, \( y_t(f) \), to maximize its profits (more generally it would want to maximize the present discounted value of profits, but there is nothing that makes the problem interesting in a dynamic sense as it just buys the intermediate goods period by period, so maximizing value is equivalent to maximizing profits period by period) subject to its production "technology". The objective in real terms is given by:

\[
\max \left[ y_t - \int_0^1 \frac{p_t(f)}{p_t} y_t(f)df \right] \tag{8}
\]

Details of the above problem and its solution are in Appendix C.

**Intermediate goods firms**

There are \( f \) intermediate goods firms whose total mass is 1. Each firm \( f \) produces a differentiated good of variety \( f \) under monopolistic competition facing Rotemberg-type nominal fixities (see Leeper et al, 2013). Nominal profits of firm \( f \) are defined as:

\[
D_t(f) = p_t(f)y_t(f) - p_t^r r_t^{k_l} k_{l-1}(f) - W_t^w w_t^r(f) - W_t^k n_t^k(f) - \frac{\phi p}{2} \left( \frac{p_t(f)}{p_{t-1}(f)} - 1 \right)^2 p_t y_t \]  \tag{9} 

where $\phi^p$ is a parameter which determines the degree of nominal price rigidity and $\pi$ stands for the steady state value of the inflation rate. The quadratic cost that the firm $f$ faces when it changes the price of its product is proportional to the aggregate output.

All firms use the same technology represented by the production function (see also Hornstein et al., 2005):

$$y_t(f) = A_t \left[ k_{t-1}^k(f) \right]^{\alpha} \left[ n_t^k(f) \right]^\theta \left[ n_t^w(f) \right]^{1-\theta} \right]^{1-\alpha}$$

where $A_t$ is an exogenous TFP which is determined below.

Profit maximization by firm $f$ is also subject to the demand for its product coming from the solution of the final goods firm’s problem (see Appendix C for details):

$$p_t(f) = \left( \frac{y_t(f)}{y_t} \right)^{-\frac{1}{\phi}} p_t$$

Each firm $f$ chooses its price, $p_t(f)$, and its inputs, $k_t^k, n_t^k, n_t^w$, to maximize the sum of discounted expected real dividends, $\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t} D_t(f) / p_t$, subject to the demand for its product and its production function. The objective in real terms is given by:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t} \left[ \frac{p_t(f)}{p_t} y_t(f) - r_t^k k_{t-1}(f) - w_t^w n_t^w(f) - w_t^k n_t^k(f) - \frac{\phi^p}{2} \left( \frac{p_t(f)}{p_{t-1}(f)} \pi \right)^2 y_t \right]$$

where $\Xi_{0,0+t}$ is a discount factor taken as given by the firm $f$. Details of the above problem and its solution are in Appendix D.
2.3 Government budget constraint

The budget constraint of the "consolidated" public sector expressed in real terms is:

\[
\left[1 + \frac{v^w}{v^k}\right] g_t + R_{t-1} \frac{p_{t-1}}{p_t} b_{t-1}^k + \frac{p_{t-1}}{p_t} \left[ m_t^k + \frac{v^w}{v^k} m_t^w \right] = b_t^k + \left[ m_t^k + \frac{v^w}{v^k} m_t^w \right] + \\
+ \tau_t^c \left[ c_t^k + \frac{v^w}{v^k} c_t^w \right] + \\
+ \tau_t^k \left[ r_t^k k_{t-1}^k + d_t^k \right] + \\
+ \tau_t^n \left[ w_t^k n_t^k + \frac{v^w}{v^k} w_t^w n_t^w \right] + \\
+ \left[ \tau_t^{l,k} + \frac{v^w}{v^k} \tau_t^{l,w} \right]
\]

(13)

all variables have been defined above. As above, small letters denote per capitalist and per worker real variables.

In each period, one of the fiscal policy instruments has to follow residually to satisfy the government budget constraint (see below).

2.4 Decentralized Equilibrium (given Policy)

We now combine all the above to solve for a symmetric Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) every type of household maximize utility; (ii) every firm maximize profit; (iii) all constraints, including government budget constraint, are satisfied; and (iv) all markets clear.

To proceed with the solution, we need to define the policy regime. Regarding monetary policy, we assume, as is usually the case, that the nominal

\[\int_0^1 c_t^k dk \equiv c_t^k, \int_0^1 c_t^w dw \equiv \frac{v^w}{v^k} c_t^w, \int_0^1 k_t^k dk \equiv k_t^k, \int_0^1 d_t^k dk \equiv d_t^k, \int_0^1 n_t^k dk \equiv n_t^k, \int_0^1 n_t^w dw \equiv \frac{v^w}{v^k} n_t^w, \int_0^1 m_t^k dk \equiv m_t^k, \int_0^1 m_t^w dw \equiv \frac{v^w}{v^k} m_t^w, \int_0^1 g_t dk + \int_0^1 g_t dw \equiv \left[1 + \frac{v^w}{v^k}\right] g_t, \int_0^1 b_t^k dk \equiv b_t^k, \int_0^1 \tau_t^{l,k} dk \equiv \tau_t^{l,k}, \int_0^1 \tau_t^{l,w} dw \equiv \frac{v^w}{v^k} \tau_t^{l,w}.\]
interest rate, \( R_t \), is used as a policy instrument, while money balances are endogenously determined. Regarding fiscal policy, we assume that, in the transition, tax rates and public spending \( \tau^c_t, \tau^k_t, \tau^n_t, \tau^l,k_t, \tau^l,w_t \) and \( g_t \), are set exogenously, while the end-of-period public debt, \( b_t \), follows residually from the government budget constraint (see Section 4 for a discussion of public financing cases).

Appendix E presents the dynamic DE system. It consists of 16 equations in 16 variables \([c^k_t, c^w_t, y_t, \pi_t, m^k_t, m^w_t, b^k_t, x^k_t, mc_t, w^k_t, n^k_t, n^w_t, r^k_t, r^l,k_t, r^l,w_t, d_t]_{t=0}^{\infty}\). This is given the independently set policy instruments, \([R_t, \tau^c_t, \tau^k_t, \tau^n_t, \tau^l,k_t, \tau^l,w_t, g_t]_{t=0}^{\infty}\), technology \([A_t]_{t=0}^{\infty}\), and initial conditions for the state variables. All these variables have been defined above except for \( \pi_t \) and \( mc_t \) where \( \pi_t \) is the inflation rate, defined as \( \pi_t \equiv \frac{p_t}{p_{t-1}} \), and \( mc_t \) is the firms’ real marginal cost as defined in Appendix D.2.

### 2.5 Policy rules

Following the related literature, we focus on simple rules for the exogenously set policy instruments, which means that the monetary and fiscal authorities react to a small number of macroeconomic indicators. In particular, we allow the nominal interest rate, \( R_t \), to follow a standard Taylor rule, meaning that it can react to inflation and output as deviations from a policy target, while we allow the distorting fiscal policy instruments, namely, government spending as share of output, \( s^g \), tax rate on consumption, \( \tau^c \), tax rate on capital income, \( \tau^k \), and tax rate on labor income, \( \tau^n \), to react to public debt, again as deviations from a policy target. The target values are defined below.

In particular we use policy rules of the functional form:

\[
\log \left( \frac{R_t}{R} \right) = \phi_{\pi} \log \left( \frac{\pi_t}{\pi} \right) + \phi_y \log \left( \frac{y_t}{y} \right) \tag{14}
\]

\[
s^g = s^g - \gamma_{l} (l_{t-1} - l) \tag{15}
\]

\[
\tau^c = \tau^c + \gamma_{l} (l_{t-1} - l) \tag{16}
\]

\[
\tau^k = \tau^k + \gamma_{l} (l_{t-1} - l) \tag{17}
\]

\[
\tau^n = \tau^n + \gamma_{l} (l_{t-1} - l) \tag{18}
\]
where $\phi_{\pi}, \phi_{y}, \gamma_{c}^{k}, \gamma_{l}^{k}$ and $\gamma_{n}^{k}$ are feedback policy coefficients of positive value, variables without time subscripts denote target values, and where

$$l_{t} \equiv \frac{R_{t}b_{t}}{y_{t}}$$

(19)

denotes the end-of-period public debt burden as share of GDP.

### 2.6 Final Equilibrium system
(given feedback policy coefficients)

The final equilibrium system consists of the 16 equations of the DE presented at the end of Appendix E, the 5 feedback policy rules, the definition of $l_{t}$ presented in Subsection 2.5. We thus end up with 22 equation in 22 variables

$$[c_{t}^{k}, c_{t}^{w}, y_{t}, \pi_{t}, m_{t}^{k}, m_{t}^{w}, b_{t}^{k}, x_{t}^{k}, mc_{t}, w_{t}^{k}, n_{t}^{k}, w_{t}^{w}, n_{t}^{w}, r_{t}^{k}, k_{t}^{k}, d_{t}, R_{t}, s_{t}^{g}, \tau_{c}^{k}, \tau_{l}^{k}, \tau_{n}^{k}, l_{t}]_{t=0}^{\infty}.$$ Among them, there are 16 non-predetermined or jump variables,

$$[c_{t}^{k}, c_{t}^{w}, y_{t}, \pi_{t}, x_{t}^{k}, mc_{t}, w_{t}^{k}, n_{t}^{k}, w_{t}^{w}, n_{t}^{w}, r_{t}^{k}, d_{t}, s_{t}^{g}, \tau_{c}^{k}, \tau_{l}^{k}, \tau_{n}^{k}]_{t=0}^{\infty}$$

and 6 predetermined or state variables $[m_{t}^{k}, m_{t}^{w}, b_{t}^{k}, k_{t}, R_{t}, l_{t}]_{t=0}^{\infty}$. This is given the TFP, initial conditions for the state variables and the values of coefficients in the feedback policy rules.

To solve this non-linear difference equation system, we take a first order approximation around the steady state. We will work as follows. We first solve for the steady state of the model numerically employing common parameters values and data from the Euro area. The next section (Section 3) will present this steady state solution, or what we call status quo. In turn, we study the transition dynamics, under various policy scenarios when we depart from the status quo and travel to a new reformed steady state with lower public debt than in the status quo solution.

### 3 Data, parameterization and steady state solution

This section solves numerically the above model economy by using conventional parameters and data from the Euro area. As we shall see, the model’s steady state solution will resemble the main empirical characteristics of the Euro area.
3.1 Parameter values and economic policy

Table 1 reports the baseline parameter values for technology and preferences, and Table 2 reports the values of exogenous policy variables, used to solve the above model economy. The time unit is meant to be a quarter. Regarding parameters for technology and preferences, we use relatively standard values often employed by the business cycle literature. The public debt-to-GDP ratio is very close to the average value for the Euro area in 2015, as taken from the report on public finances in EMU, 2015. Public spending and tax rate values are calibrated so that they yield the above value for the public debt-to-GDP ratio and, at the same time, they are close to the data averages of the European economy over 2008-2011. These fiscal data are obtained from OECD, Economic Outlook No. 89.

Let us discuss, briefly, the values summarized in Table 1. Using the Euler equation for bonds, the value of time preference rate, $\beta$, follows so as to be consistent with the average value of the real interest rate in the data, 0.0075 quarterly (see Table 2) or 0.03 annually. The share of capital in income, $\alpha$, and the number of capitalists in the population, $v^k$, are set at 0.33 and 0.2 respectively. The parameter $\theta$ is calibrated so that we obtain a reasonable value for the ratio of capitalist’s wage to worker’s wage, $\frac{w^k}{w_w}$, which, in our model, equals 1.68. The elasticity of intertemporal substitution, $\sigma$, the inverse of Frisch labor elasticity, $\eta$, and the price elasticity of demand, $\phi$, are set as in Andrés and Doménech (2006) and Gali (2008) in related studies. The real money balances elasticity, $\mu$, is taken from Pappa and Neiss (2005), who estimate this value using UK data; this implies an interest-rate semi elasticity of money demand equal to -0.29 which is a common value in this literature. Regarding preference parameters in the utility function, $\chi_m$, is chosen so as to obtain a value of real money balances as share of output equal to 1.97 quarterly, or 0.49 annually, which is close to the data (when we use the M1 measure, the average value in the annual data is around 0.5), $\chi_{h}$, is chosen so as to obtain steady state labor hours equal to 0.28, while $\chi_g$ is arbitrarily set at 0.1 which is a common valuation of public goods in related utility functions. We set Rotemberg’s price adjustments cost parameter, $\phi^p$, at 30 which corresponds to approximately 33 percent of the firms re-optimizing each quarter in a Calvo pricing model, as in Keen and Wang (2007). Several related studies of the Euro area featuring Calvo price mechanism also set the probability of price readjustment at 1/3 (see e.g. Gali et al., 2001). Concerning the exogenous variable, the $A_t$ is constant over time and equal to 1.

The effective tax rates on consumption, capital and labor are respectively
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$v^w$</td>
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<td>---------------</td>
<td>--------</td>
<td>-------------------------------------------</td>
</tr>
<tr>
<td>( R )</td>
<td>1.0075</td>
<td>long-run nominal interest rate</td>
</tr>
<tr>
<td>( \tau_c )</td>
<td>0.20</td>
<td>consumption tax rate</td>
</tr>
<tr>
<td>( \tau_k )</td>
<td>0.29</td>
<td>capital tax rate</td>
</tr>
<tr>
<td>( \tau_n )</td>
<td>0.39</td>
<td>labor tax rate</td>
</tr>
<tr>
<td>( s^g )</td>
<td>0.24</td>
<td>government consumption spending as share of output</td>
</tr>
<tr>
<td>(-s^l)</td>
<td>0.2</td>
<td>government transfers as share of output</td>
</tr>
<tr>
<td>( \lambda^{l,k} )</td>
<td>0.2</td>
<td>percentage of total transfers to capitalists</td>
</tr>
</tbody>
</table>

\( \tau_c = 0.2, \tau_k = 0.29 \) and \( \tau_n = 0.39 \). These values are very close to the data averages for the Euro area over 2008-2011. The long-run nominal interest rate is 1.0075 quarterly for the Euro area in the same time period. Lump-sum taxes/transfers as share of output, \( s^l \), and total public spending as share of output, \( s^g \), are set -0.2 and 0.24 respectively so that their sum, \(-s^l + s^g\), to be close to the data for the same time period as well. The public debt-to-output ratio follows residually from the model and is equal to 3.8 quarterly (or 0.95 annually). This value is very close to the average value for the Euro area in 2015 (3.6 quarterly or 0.94 annually).

The government imposes/gives a percentage, \( \lambda^{l,k} \), of total lump-sum taxes/transfers to the class of capitalists, while a percentage, \( \lambda^{l,w} \equiv 1 - \lambda^{l,k} \), to the workers. Assuming that \( \lambda^{l,k} \) equals the percentage of capitalists in the population then the lump-sum taxes/transfers per capitalist equals those of per worker (See Appendix F for details). In other words, transfers are distributed according to the population.

Regarding fiscal (tax-spending) policy instruments along the transition, fiscal instruments can also react to the current state of public debt as deviation from their steady state values,\(^3\) where this reaction is quantified by the feedback policy coefficients in the policy rules (15)-(18). Here, we simply set the feedback coefficients of government spending at 0.1 (i.e. \( \gamma^g = 0.1 \)) which is necessary for dynamic stability in most experiments, while we switch off all other fiscal reactions to debt.\(^4\) These baseline values of feedback fiscal

---

\(^3\)Since policy instruments react to deviations of macroeconomic indicators from their steady state values, feedback policy coefficients do not play any role in steady state solutions. Also, recall that “money is neutral” in the long run, so that the monetary policy regime also do not matter to the real economy at the steady state.

\(^4\)These values are close to those found by optimized policy rules in related studies (see e.g. Schmitt-Grohé and Uribe (2007) and Philippopoulos et al. (2014)). They are also consistent
Table 3: Steady state solution or the “status quo”

<table>
<thead>
<tr>
<th>Variables</th>
<th>Solution</th>
<th>Variables</th>
<th>Solution</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2.3255</td>
<td>$l$</td>
<td>3.8273</td>
<td></td>
</tr>
<tr>
<td>$c^k$</td>
<td>0.5940</td>
<td>$x^k$</td>
<td>0.3379</td>
<td></td>
</tr>
<tr>
<td>$c^w$</td>
<td>0.2089</td>
<td>$d$</td>
<td>0.3876</td>
<td></td>
</tr>
<tr>
<td>$n^k$</td>
<td>0.1920</td>
<td>$\pi$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$n^w$</td>
<td>0.3237</td>
<td>$mc$</td>
<td>0.8333</td>
<td></td>
</tr>
<tr>
<td>$k^k$</td>
<td>16.0926</td>
<td>$y^k$</td>
<td>0.8657</td>
<td></td>
</tr>
<tr>
<td>$b^k$</td>
<td>8.8342</td>
<td>$y^{w^e}$</td>
<td>0.2089</td>
<td></td>
</tr>
<tr>
<td>$m^k$</td>
<td>1.5807</td>
<td>$c/y$</td>
<td>0.6147</td>
<td>0.57</td>
</tr>
<tr>
<td>$m^w$</td>
<td>1.1645</td>
<td>$b/y$</td>
<td>3.7988</td>
<td>3.76</td>
</tr>
<tr>
<td>$r^k$</td>
<td>0.0401</td>
<td>$x/y$</td>
<td>0.1453</td>
<td>0.18</td>
</tr>
<tr>
<td>$w^k$</td>
<td>1.3459</td>
<td>$m/y$</td>
<td>2.6830</td>
<td></td>
</tr>
<tr>
<td>$w^w$</td>
<td>0.7981</td>
<td>$k/y$</td>
<td>6.9201</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Parameters and policy variables as in Tables 1 and 2

policy coefficients are reported in Table 1. We report that our main results are robust to changes in these values.

3.2 Steady state solution or the “status quo”

Table 3 reports the steady state solution of the model economy when we use the parameter values in Table 1 and the policy instruments in Table 2. The solution makes sense and the resulting great ratios are close to their values in the actual data (recall that, since the time unit is meant to be a quarter, stock variables-like debt, capital and money balances- need to be divided by 4 to give the annual values). At this steady state, called the “status quo”, the residually determined public financing variable is public debt. In what follows, we will depart from this solution to study the aggregate and distributional implications of various policy experiments.

4 How we model debt consolidation

In this section, following Philippopoulos et al. (2016), we explain how we model debt consolidation.

with calibrated or estimated values by previous research(see e.g. Leeper et al.(2009), Forni et al.(2010), Coenen et al. (2013), Cogan et al.(2013), Erceg and Linde(2013)).
How we model debt consolidation

We assume that the government aims at reducing the share of public debt from approximately 95% \( \approx \frac{38}{4} \times 100\% \) of GDP, which is the steady state value and is also close to the data, to the target value of 60%. We choose the target value of 60% simply because it has been the reference rate of the Maastricht Treaty (we report however that our qualitative results are not sensitive to the value of the debt target assumed). Obviously, debt reductions have to be accommodated by adjustments in the tax-spending policy instruments, which, in our model, are the output share of public spending, and the tax rates on capital income, labor income and consumption. What described just above naturally implies the following trade-off. Debt consolidation implies an inter-temporal trade-off between fiscal pain in the short term (i.e. spending has to fall and/or taxes have to rise) and fiscal gain in the medium and long term once debt has been reduced (i.e. now spending can rise and/or taxes can fall). This is the standard logic in the literature on fiscal consolidation. Results will be relative to the status quo (the status quo is defined as the case without debt consolidation).

This inter-temporal trade-off also implies that the implications of consolidation depend heavily on the public financing policy instruments used, namely, which policy instrument adjusts endogenously to accommodate the exogenous change in fiscal policy (see also e.g. Leeper et al., 2009, and Davig and Leeper, 2011). Specifically, these implications depend both on which policy instrument bears the cost of adjustment in the early period of adjustment and on which policy instrument is anticipated to reap the benefit, once consolidation has been achieved.

In the policy experiments we consider below, we experiment with fiscal policy mixes, which means that fiscal authorities are allowed to use different instruments in the transition and in the steady state. For instance, let us assume that, in the reformed steady state, it is the capital tax rate that takes advantage once the public debt has been reduced. In the transition to this reformed steady state, all fiscal instruments are available and, consequently, one of them is used, as in the policy rules in Subsection 2.5, to bring public debt down.

In particular, we work as follows. We first solve and compare the status quo steady state solution to the steady state of reformed solutions for every case of adjusting fiscal policy instrument. We will then study the transitional results. The latter means that we log-linearize the model around the new steady-state solution of each reformed economy and then check its saddle-path stability when we use as initial conditions for the state variables their
Table 4: Values of the adjusting fiscal policy instruments in steady state

<table>
<thead>
<tr>
<th>Adjusting Instrument</th>
<th>Status quo</th>
<th>New steady state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>0.29</td>
<td>0.2684</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>0.39</td>
<td>0.3711</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.20</td>
<td>0.1829</td>
</tr>
<tr>
<td>$s^g$</td>
<td>0.24</td>
<td>0.2487</td>
</tr>
</tbody>
</table>

values in the status quo steady state.

We report that we use Matlab for computing steady state solutions, checking saddle-path stability and computing dynamics (Matlab routines are available upon request).

In all cases, we will study both aggregate and distributional implications. Regarding aggregate outcomes, we look, for instance, at per capita output. Regarding distribution, we compute separately the income of the representative capitalist vis-à-vis that of the representative worker. The above values are then compared to their respective values had we remained in the status quo economy permanently.

5 Main results

5.1 Steady state results

We start with comparison of steady state solutions. Recall that in the SQ steady state, fiscal policy instruments were set as in the data and $b/y$ followed residually, while in the reformed steady state $b/y$ is cut to 60% so that one of the fiscal policy instruments follows residually meaning that $s^g$ is allowed to rise or $\tau^k, \tau^n, \tau^c$ are allowed to fall. Table 4 reports each case so that we examine each public financing case separately. That is, we investigate how the implications of debt consolidation depend on the public financing policy instrument used. Namely, which fiscal policy instrument should take advantage of the switch to a more efficient economy with lower debt and higher output?

Aggregate implications (efficiency)

Results for output in the SQ and the reformed economy under various public financing scenarios are shown in Table 5. As one would expect, in terms of the aggregate economy, our numerical results imply that it is better to
allow capital taxes to take advantage of the fiscal space. The superiority of capital tax rate is consistent with the well-known result that capital taxes are particularly distorting in the medium-run and long-run (see Chamley, 1986). Therefore, the most efficient way of using the fiscal space generated, once debt has been brought down, is to cut capital tax rate.

### Distributional implications (equity)

Results for net incomes are reported in Table 6. Since there are two different income groups in the society - capitalists and workers - the income gains from each particular structural reform may be distributed unequally. We first look at the net income of each agent, $y^k$ and $y^w$. Our results show that, relative to the status quo, both social groups gain with debt consolidation independently of what is the adjusting instrument in the steady state (see Table 6). But a key question is who gains more. Even if a policy reform produces a win-win outcome (Pareto efficient), here in the sense that both $y^k$

---

5The net income of the capitalist is defined as $y^k_i = -\tau^k_i c^k_i + (1 - \tau^k_i)\rho_i \eta^k_i + \delta^k_i + (1 - \tau^k_i)w^k_i n^k_i - r^k_i$, while that of the worker is defined as $y^w_i = -\tau^w_i c^w_i + (1 - \tau^w_i)w^w_i n^w_i - r^w_i$. 

---

Table 5: Total output (GDP) in steady state

<table>
<thead>
<tr>
<th>Adjusting Instrument</th>
<th>New steady state</th>
<th>% Change relative to the SQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>2.3648</td>
<td>+1.69%</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>2.3496</td>
<td>+1.04%</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>2.3336</td>
<td>+0.35%</td>
</tr>
<tr>
<td>$s^g$</td>
<td>2.3336</td>
<td>+0.35%</td>
</tr>
</tbody>
</table>

Note: Steady state value of the total output in the status quo (SQ) is 2.3255.

Table 6: Net income of capitalists and net income of workers in steady state

<table>
<thead>
<tr>
<th>Adjusting Instrument</th>
<th>Status quo</th>
<th>New steady state</th>
<th>% Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y^k$</td>
<td>$Y^w$</td>
<td>$Y^k/Y^w$</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>0.866</td>
<td>0.209</td>
<td>4.145</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>0.866</td>
<td>0.209</td>
<td>4.145</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.866</td>
<td>0.209</td>
<td>4.145</td>
</tr>
<tr>
<td>$s^g$</td>
<td>0.866</td>
<td>0.209</td>
<td>4.145</td>
</tr>
</tbody>
</table>

Note: $Y^k$ stands for the net income of capitalist and $Y^w$ stands for the net income of worker.
Table 7: Present value of total output (GDP) over different time horizons when the adjusting instrument in the steady state is the tax rate on capital ($\tau^k$)

<table>
<thead>
<tr>
<th>Adj.Instr.</th>
<th>$Y_5$</th>
<th>$Y_{10}$</th>
<th>$Y_{20}$</th>
<th>$Y_{40}$</th>
<th>$Y_{60}$</th>
<th>$Y_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>11.29</td>
<td>22.33</td>
<td>43.44</td>
<td>81.43</td>
<td>114.14</td>
<td>220.20</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>10.70</td>
<td>22.05</td>
<td>43.40</td>
<td>81.40</td>
<td>114.10</td>
<td>220.16</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>11.11</td>
<td>22.27</td>
<td>43.52</td>
<td>81.50</td>
<td>114.21</td>
<td>220.27</td>
</tr>
<tr>
<td>$s^g$</td>
<td>11.74</td>
<td>23.06</td>
<td>44.40</td>
<td>82.45</td>
<td>115.17</td>
<td>221.26</td>
</tr>
<tr>
<td><strong>Status quo</strong></td>
<td><strong>11.30</strong></td>
<td><strong>22.23</strong></td>
<td><strong>43.02</strong></td>
<td><strong>80.43</strong></td>
<td><strong>112.71</strong></td>
<td><strong>217.37</strong></td>
</tr>
</tbody>
</table>

Note: $Y_t$ stands for the discounted future values of total output (GDP) for the next $t$ periods after the fiscal consolidation takes place.

and $y^w$ rise, relative outcomes can also be important. Actually, the political economy literature has pointed out several reasons for this, including political ideology, envy, habits, etc. In our model, distributional implications can be measured by changes in the ratio of net incomes, $y^k/y^w$.

Departing from the status quo, the ratio $y^k/y^w$ rises, or equivalently inequality rises, when the instrument that takes advantage of the fiscal gains created in the steady state is the tax rate on capital. This policy is the most efficient, as well as Pareto efficient, but not equitable. For this reason perhaps we often observe workers opposing to such a reform. In terms of equity, the best outcome takes place when we use the fiscal space created in the medium- and long-run by debt consolidation in order to reduce the labor tax rate. Such a policy causes the ratio $y^k/y^w$ to fall, or equivalently inequality to fall.

Putting things together, in the reformed steady state, a policy that both increases all incomes and reduces income inequality is to cut the labor tax rate. On the other hand, if we focus on efficiency only, the best way of using the fiscal space is to cut the capital tax rate.

5.2 Transition results

We next study what happens in the transition as we depart from the status quo steady state and travel towards the new reformed steady state with lower public debt.
Table 8: Present value of total output (GDP) over different time horizons when the adjusting instrument in the steady state is the tax rate on labor ($\tau^n$)

<table>
<thead>
<tr>
<th>Adj. Instr.</th>
<th>$Y_5$</th>
<th>$Y_{10}$</th>
<th>$Y_{20}$</th>
<th>$Y_{40}$</th>
<th>$Y_{60}$</th>
<th>$Y_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>11.36</td>
<td>22.36</td>
<td>43.30</td>
<td>80.99</td>
<td>113.48</td>
<td>218.87</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>10.88</td>
<td>22.12</td>
<td>43.31</td>
<td>81.06</td>
<td>113.54</td>
<td>218.92</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>11.06</td>
<td>22.15</td>
<td>43.25</td>
<td>81.00</td>
<td>113.49</td>
<td>218.88</td>
</tr>
<tr>
<td>$s^g$</td>
<td>11.81</td>
<td>23.10</td>
<td>44.33</td>
<td>82.15</td>
<td>114.67</td>
<td>220.08</td>
</tr>
<tr>
<td><strong>Status quo</strong></td>
<td><strong>11.30</strong></td>
<td><strong>22.23</strong></td>
<td><strong>43.02</strong></td>
<td><strong>80.43</strong></td>
<td><strong>112.71</strong></td>
<td><strong>217.37</strong></td>
</tr>
</tbody>
</table>

Note: $Y_t$ stands for the discounted future values of total output (GDP) for the next $t$ periods after the fiscal consolidation takes place.

**Aggregate implications (efficiency)**

Results for the present discount value of total output over different time horizons along the transition to a new reformed steady state are shown in Tables 7 and 8. Every table corresponds to a different new reformed steady state depending on which fiscal policy instrument takes advantage of the fiscal space created by debt consolidation. Specifically, in Table 7, it is the tax rate on capital, $\tau^k$, that decreases in steady state, while in Table 8 it is the tax rate on labor, $\tau^n$, that falls. Every row of a table shows the present discount value of total output over different time horizons. The key message is that if the criterion is total, or per capita, output (GDP), then the best policy mix found in the transition is to use the long term fiscal gain (namely, the fiscal space created once debt has been reduced) to reduce the capital tax rate and, during the early period of fiscal pain, to use spending cuts to bring public debt down.

**Distributional implications (equity)**

Results for the ratio of the present discount value of net income of capitalists to that of workers over different time horizons along the transition to a new reformed steady state are shown in Tables 9 and 10. Every table corresponds to a different new reformed steady state depending on what is the fiscal policy instrument that takes advantage of the fiscal space created by debt consolidation. Specifically, in Table 9, it is the tax rate on capital, $\tau^k$, that decreases in steady state, while in Table 10 it is the tax rate on labor, $\tau^n$, that falls. Every row of a table shows the ratio of the present value of net income of capitalists to that of workers over different time horizons. Furthermore,
Table 9: Ratio of present value of net income of capitalist and that of worker over various time horizons when the adjusting instrument in the steady state is the tax rate on capital ($\tau^k$)

**Steady state value in the status quo is 4.1446**

<table>
<thead>
<tr>
<th>Adj. Instr.</th>
<th>$\frac{Y^k}{Y^w}$</th>
<th>$\frac{Y^k_{10}}{Y^w_{10}}$</th>
<th>$\frac{Y^k_{20}}{Y^w_{20}}$</th>
<th>$\frac{Y^k_{40}}{Y^w_{40}}$</th>
<th>$\frac{Y^k_{60}}{Y^w_{60}}$</th>
<th>$\frac{Y^k_{\infty}}{Y^w_{\infty}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>3.40</td>
<td>3.52</td>
<td>3.72</td>
<td>3.94</td>
<td>4.03</td>
<td>4.13</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>4.12</td>
<td>4.19</td>
<td>4.24</td>
<td>4.27</td>
<td>4.27</td>
<td>4.27</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>3.99</td>
<td>4.08</td>
<td>4.15</td>
<td>4.20</td>
<td>4.22</td>
<td>4.24</td>
</tr>
<tr>
<td>$s^g$</td>
<td>4.14</td>
<td>4.19</td>
<td>4.23</td>
<td>4.25</td>
<td>4.25</td>
<td>4.26</td>
</tr>
</tbody>
</table>

Note: $Y^k_T$ and $Y^w_T$ stand for the PV of net income of capitalist and worker respectively for the next $T$ periods after the fiscal consolidation.

---

Table 10: Ratio of present value of net income of capitalist and that of worker over various time horizons when the adjusting instrument in the steady state is the tax rate on labor ($\tau^n$)

**Steady state value in the status quo is 4.1446**

<table>
<thead>
<tr>
<th>Adj. Instr.</th>
<th>$\frac{Y^k}{Y^w}$</th>
<th>$\frac{Y^k_{10}}{Y^w_{10}}$</th>
<th>$\frac{Y^k_{20}}{Y^w_{20}}$</th>
<th>$\frac{Y^k_{40}}{Y^w_{40}}$</th>
<th>$\frac{Y^k_{60}}{Y^w_{60}}$</th>
<th>$\frac{Y^k_{\infty}}{Y^w_{\infty}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>3.41</td>
<td>3.48</td>
<td>3.61</td>
<td>3.78</td>
<td>3.87</td>
<td>3.98</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>4.08</td>
<td>4.11</td>
<td>4.14</td>
<td>4.15</td>
<td>4.14</td>
<td>4.13</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>3.81</td>
<td>3.90</td>
<td>3.98</td>
<td>4.03</td>
<td>4.05</td>
<td>4.08</td>
</tr>
<tr>
<td>$s^g$</td>
<td>4.14</td>
<td>4.19</td>
<td>4.23</td>
<td>4.25</td>
<td>4.25</td>
<td>4.26</td>
</tr>
</tbody>
</table>

Note: $Y^k_T$ and $Y^w_T$ stand for the PV of net income of capitalist and worker respectively for the next $T$ periods after the fiscal consolidation.
we also check whether these values are lower than the steady state value in the status quo, 4.1446. (If they are, then the reform improves equality relative to the status quo.) The main message is that during the early period of fiscal pain, it is better to increase the capital tax rate in order to bring debt down and this holds independently of what the adjusting instrument in the new reformed steady state is (see Tables 9 and 10). Actually, the best policy mix in terms of equity during the transition arises when we use capital taxes to bring public debt down and this is combined with labor tax cuts in the new reformed steady state.

6 Robustness

We finally check the sensitivity of our results. All results reported below are available upon request. Our results are robust to changes in all key parameter values. Among the latter, we have extensively experimented with changes in the values of the parameter in percentage of capitalists in the population, $v^k$, the Rotemberg adjustment pricing cost parameter in the firm’s problem, $\phi^p$, the coefficient of capital tax rate on public debt, $\gamma^k$, the coefficient of labor tax rate on public debt, $\gamma^n$, the coefficient of consumption tax rate on public debt, $\gamma^c$, the coefficient of government spending on public debt, $\gamma^g$, the coefficient of interest rate on inflation, $\phi^\pi$, whose values are relatively unknown empirically. We report that our main results do not change within $0.15 \leq v^k \leq 0.3, 5 \leq \phi^p \leq 105, 0.05 \leq \gamma^k \leq 0.30, 0.05 \leq \gamma^n \leq 0.30, 0.05 \leq \gamma^c \leq 0.30, 0.05 \leq \gamma^g \leq 0.30, 0.1 \leq \phi^\pi \leq 0.30$. It is worth to mention that there is no stability with or without debt consolidation when $\gamma^q$, with $q \in (k,n,c,g)$, is zero.

Appendix A  Households-Capitalists

This appendix provides details and the solution of capitalist’s problem. The mass of this type of household is 1. Each capitalist $k$ acts competitively to maximize expected discounted lifetime utility.

A.1 Capitalist’s problem

Each $k$’s expected discounted lifetime utility is:

$$E_o \sum_{t=0}^{\infty} \beta^t U(c^k_t, n^k_t, m^k_t, g_t)$$
where \( c_i^k \) is \( k \)'s consumption at \( t \), \( n_i^k \) is \( k \)'s hours of work at \( t \), \( m_i^k \) is \( k \)'s end-of-period real money balances, \( g_i \) is per capita government spending at \( t \), \( E_x \) is the rational expectations operator conditional on the current period information set and \( 0 < \beta < 1 \) is the time preference rate.

We will use a utility function of the form:

\[
U(c_i^k, n_i^k, m_i^k, g_i) = \left[ \frac{(c_i^k)^{1-\sigma}}{1-\sigma} - x_n (n_i^k)^{1+\eta} + x_m (m_i^k)^{1-\mu} + x_g (g_i)^{1-\zeta} \right]
\]  

(21)

where \( x_n, x_m, x_g, \sigma, \eta, \mu, \zeta \) are standard preference parameters.

The budget constraint of each \( k \) (written in nominal terms) is:

\[
(1 + \tau_i^c) p_t c_i^k + p_t x_i^k + B_i^k + M_i^k = (1 - \tau_i^k) [r_i^k p_t k_{t-1}^k + D_i^k] + \\
+ (1 - \tau_i^n) W_i^k n_i^k + R_{t-1} B_{t-1}^k + \\
+ M_{t-1}^k - T_{t}^{l,k}
\]

(22)

where \( p_t \) is the price index and small letters denote real variables.Here \( x_i^k \) is \( k \)'s real investment at \( t \), \( B_i^k \) is \( k \)'s end-of-period nominal government bonds at \( t \), \( M_i^k \) is \( k \)'s end-of-period nominal money holdings at \( t \), \( D_i^k \) is \( k \)'s nominal dividends paid by firms at \( t \), \( W_i^k \) is \( k \)'s nominal wage rate at \( t \), \( k_i^k \) is \( k \)'s end-of-period capital , \( R_{t-1} \geq 1 \) is gross nominal return to government bonds between \( t - 1 \) and \( t \), \( r_i^k \) is gross real return to inherited capital between \( t - 1 \) and \( t \), \( T_{t}^{l,k} \) is nominal lump-sum taxes/transfers to each household \( k \) from the government at \( t \), \( \tau_i^c \) is tax rate on consumption at \( t \), \( \tau_i^k \) is tax rate on capital income at \( t \) and \( \tau_i^n \) is tax rate on labor income at \( t \).

Dividing by \( p_t \), the budget constraint of each \( k \) in real terms is:

\[
(1 + \tau_i^c) c_i^k + x_i^k + b_i^k + m_i^k = (1 - \tau_i^k) [r_i^k k_{t-1}^k + d_i^k] + \\
+ (1 - \tau_i^n) w_i^k n_i^k + R_{t-1} \frac{P_{t-1}}{P_t} b_{t-1}^k + \\
+ \frac{P_{t-1}}{P_t} m_{t-1}^k - \tau_i^{l,k}
\]

(23)

where, as above, small letters denote real variables, i.e. \( b_i^k \equiv \frac{b_i^k}{P_t}, m_i^k \equiv \frac{M_i^k}{P_t}, d_i^k \equiv \frac{D_i^k}{P_t}, w_i^k \equiv \frac{W_i^k}{P_t}, \tau_i^{l,k} \equiv \frac{\tau_i^{l,k}}{P_t} \).

The motion of physical capital for each \( k \) is:

\[
k_{t}^k = (1 - \delta) k_{t-1}^k + x_{t}^k
\]

(24)

where \( 0 < \delta < 1 \) is the depreciation rate of capital.
A.2 Capitalist k’s optimality conditions

Each $k$ acts competitively taking prices and policy as given.

The first order conditions include the budget constraint, the law of motion of physical capital above and:

$$\frac{(c^k_t)^{-\sigma}}{1 + \tau^c_t} = \beta \frac{(c^k_{t+1})^{-\sigma}}{1 + \tau^c_{t+1}} \left[ (1 - \delta) + (1 - \tau^k_{t+1}) r^k_{t+1} \right]$$  \hspace{1cm} (25)

$$\frac{(c^k_t)^{-\sigma}}{1 + \tau^c_t} = \beta R_t \frac{p_t}{p_{t+1}} \frac{(c^k_{t+1})^{-\sigma}}{1 + \tau^c_{t+1}}$$  \hspace{1cm} (26)

$$x^m(m^k_t)^{-\mu} = \frac{(c^k_t)^{-\sigma}}{1 + \tau^c_t} \left[ (1 - \tau^w_t) R_t^w w^k_t \right]$$  \hspace{1cm} (27)

$$x^m(m^k_t)^{-\mu} = \frac{(c^k_t)^{-\sigma}}{1 + \tau^c_t} - \beta \frac{(c^k_{t+1})^{-\sigma}}{1 + \tau^c_{t+1}} \frac{p_t}{p_{t+1}}$$  \hspace{1cm} (28)

Eqs. (25) and (26) are respectively the Euler equations for capital and bonds, Eq. (27) is the optimality condition for work hours and Eq. (28) is the optimality condition for money balances.

Appendix B  Households-Workers

This appendix provides details and the solution of worker’s problem. The mass of this type of household is $v^w v^k$. Each worker $w$ acts competitively to maximize expected discounted lifetime utility.

B.1 Worker’s problem

Each worker $w$ has the same expected lifetime utility and instantaneous utility function as each capitalist $k$, that are given by (20) and (21) respectively, where now the index is $w$.

The budget constraint of each $w$ is in nominal terms:

$$(1 + \tau^c_t) p_t c^w_t + M^w_t = (1 - \tau^w_t) W^w_t n^w_t + M^w_{t-1} - T^l,w_t$$  \hspace{1cm} (29)

where $c^w_t$ is $w$’s consumption at $t$, $n^w_t$ is $w$’s hours of work at $t$, $M^w_t$ is $w$’s end-of-period nominal money holdings at $t$, $W^w_t$ is $w$’s nominal wage rate at $t$ and $T^l,w_t$ is nominal lump-sum taxes/transfers to each household $w$ from the government at $t$.  

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Dividing by $p_t$, the budget constraint of each $w$ in real terms is:

$$(1 + \tau_i^c)c_i^w + m_i^w = (1 - \tau_i^n)w_i^w n_i^w + \frac{p_t-1}{p_t} m_{t-1}^w - \tau_i^{l,w}$$ (30)

where small letters denote real variables e.g. $w_i^w \equiv \frac{W_i^w}{p_t}$, $\tau_i^{l,w} \equiv \frac{T_i^{l,w}}{p_t}$.

**B.2 Worker $w$’s optimality conditions**

Each $w$ acts competitively taking prices and policy as given.

The first order conditions include the budget constraint above and:

$$\frac{(c_i^w)^{-\alpha}}{x_i^n(n_i^w)^{\eta}} = \frac{1 + \tau_i^c}{(1 - \tau_i^n)w_i^w}$$ (31)

$$\frac{(c_i^w)^{-\alpha}}{1 + \tau_i^c} = \beta \frac{p_t}{p_{t+1}} \left[ \frac{(c_{i+1}^w)^{-\alpha}}{1 + \tau_{i+1}^c} \right] + x_m (m_i^w)^{1-\mu}$$ (32)

Eq.(31) is the optimality condition for work hours and Eq.(32) is the optimality condition for money balances.

**Appendix C  Final goods firms**

This appendix provides details and the solution of the final goods firm’s problem. There is a final goods firm that produces a single good and operates in a perfectly competitive environment.

**C.1 Final goods firm’s problem**

Nominal profits of the final goods producer are:

$$p_t y_t - \int_0^1 p_t(f) y_t(f) df$$ (33)

where $p_t(f)$ is the price of variety $f$, $y_t(f)$ is the production of the variety $f$ produced monopolistically by the intermediate goods firm $f$ and $y_t$ is the production of the final goods firm.
There is a final goods firm and a continuum (i.e. infinity) of intermediate goods firms. These firms are indexed along the unit interval. The production "technology" for the final good is:

\[
y_t = \left[ \int_0^1 \left[ y_t(f) \right]^{\phi-1} \phi^{-\phi} df \right]^{\frac{\phi}{\phi-1}}
\]

where \(\phi > 0\) is the elasticity of substitution across intermediate goods produced.

### C.2 Final goods firm’s optimality conditions

Under perfect competition, the final goods firm chooses the quantity of every variety, \(y_t(f)\), to maximize its profits (more generally it would want to maximize the present discounted value of profits, but there is nothing that makes the problem interesting in a dynamic sense as it just buys the intermediate goods period by period, so maximizing value is equivalent to maximizing profits period by period) subject to its production "technology" taking prices as given. The solution to the profit maximization problem gives:

\[
y_t(f) = \left( \frac{p_t(f)}{p_t} \right)^{-\phi} y_t
\]

or equivalently:

\[
p_t(f) = \left( \frac{y_t(f)}{y_t} \right)^{-\frac{1}{\phi}} p_t
\]

Notice that, the zero profit condition \(p_t y_t = \int_0^1 p_t(f) y_t(f) df\), along with Eq. (35) imply for the price index:

\[
p_t = \left\{ \int_0^1 [p_t(f)]^{1-\phi} df \right\}^{\frac{1}{1-\phi}}
\]
Appendix D  Intermediate goods firms

This appendix provides details and the solution of intermediate goods firm’s problem. The mass of these firms is normalized to 1. Each firm \( f \) produces a differentiated good of variety \( f \) under monopolistic competition facing a Rotemberg-type nominal fixities.

D.1 Intermediate goods firm \( f \)’s problem

Due to Rotemberg pricing, to the extent that the increase of a firm \( f \)’s price differs from the long run inflation rate this firm faces a quadratic adjustment cost,

\[
-\phi_p \left( \frac{p_t(f)}{p_{t-1}(f)} \right)^2 y_t.
\]

As stressed in Rotemberg (1982), this adjustment cost accounts for the negative effects of price changes on the customer-firm relationship and, consequently, creates an inefficiency wedge between output and demand, which is reflected by the term \( \left\{ 1 - \frac{\phi_p}{2} \left[ \frac{p_t(f)}{p_{t-1}(f)} - 1 \right] \right\}^{-1} \).

Nominal profits of intermediate goods firm \( f \) are:

\[
D_t(f) = p_t(f) y_t(f) - p_t r^k_{t-1}(f) - W^w_t n^w_t(f) - W^k_t n^k_t(f) - \frac{\phi_p}{2} \left( \frac{p_t(f)}{p_{t-1}(f)} - 1 \right)^2 p_t y_t,
\]

where \( \phi_p \) is a standard parameter which determines the degree of nominal price rigidity and \( \pi \) stands for the steady state value of inflation rate.

All firms use the same technology represented by the production function:

\[
y_t(f) = A_t \left[ k_{t-1}(f) \right]^\alpha \left[ \left( n^k_t(f) \right)^\theta n^w_t(f)^{1-\theta} \right]^{1-\alpha}
\]

where \( A_t \) is an exogenous TFP.

Under imperfect competition, profit maximization by \( f \) is subject to the demand function coming from the solution to the final goods firm’s problem, namely:

\[
p_t(f) = \left( \frac{y_t(f)}{y_t} \right)^{-\frac{1}{\phi}} p_t
\]

D.2 Intermediate goods firm \( f \)’s optimality conditions

Following the related literature, we follow a two step procedure. We first solve a cost minimization problem, where each \( f \) minimizes cost by choosing production factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of production factor.
prices and output produced by the firm. In turn, given this cost function, each \( f \) solves a maximization problem by choosing its price.

Each \( f \) chooses its input factors \( k_{t-1}(f), n_t^k(f), n_t^w(f) \), to minimize its real cost. The above cost minimization is subject to the production function of \( f \),
\[
y_t(f) = A_t \left[ (k_{t-1}(f))^\alpha (n_t^k(f))^\theta (n_t^w(f))^{1-\theta} \right]^{1-\alpha}.
\]

The solution to the cost minimization problem gives the input demand functions:
\[
r_k^t = mc_t \alpha \frac{y_t(f)}{k_{t-1}(f)} \quad \text{(41)}
\]
\[
w_k^t = mc_t \theta (1-\alpha) \frac{y_t(f)}{n_t^k(f)} \quad \text{(42)}
\]
\[
w_w^t = mc_t (1-\theta) (1-\alpha) \frac{y_t(f)}{n_t^w(f)} \quad \text{(43)}
\]

From the three above equations it arises that the associated minimum real cost function of \( f \) equals \( mc_t y_t(f) \). Where \( mc_t \) is the real marginal cost which it can be shown that equals:
\[
mc_t = \frac{1}{A_t} \left[ \frac{r_k^t}{\alpha} \left\{ \frac{w_k^t}{\theta(1-\alpha)} \right\}^\theta \left\{ \frac{w_w^t}{(1-\theta)(1-\alpha)} \right\}^{1-\theta} \right]^{1-\alpha} \quad \text{(44)}
\]
implying that \( mc_t \) is common for all firms since it only depends on production factor prices, parameters and technology which are common for all firms.

Then \( f \) chooses its price, \( p_t(f) \), to maximize the lifetime expected discounted real profits:
\[
\max E_0 \sum_{t=0}^\infty \Xi_{0,0+t} \left[ \frac{p_t(f)}{p_t} y_t(f) - r_k^t k_{t-1}(f) - w_w^t n_t^w(f) - w_k^t n_t^k(f) - \frac{\phi p_t(f)}{2} \left( \frac{p_t(f)}{p_{t-1}(f)} - 1 \right)^2 \right] \quad \text{(45)}
\]
where \( \Xi_{0,0+t} \) is a stochastic discount factor which arises from Euler for bonds and is defined as
\[
\Xi_{0,0+t} = \prod_{i=0}^{t-1} \left( \frac{1}{\mathcal{R}_i} \right) = \beta^t \prod_{i=0}^{t-1} \left( \frac{1 + r_f}{p_{t+1}} \right) \left( \frac{1 + r_{i+1}}{c_{t+1}} \right)^{-\sigma}.
\]

The above profit maximization is subject to the demand equation that the monopolistically competitive firm \( f \) faces, \( y_t(f) = \left( \frac{p_t(f)}{p_t} \right)^{-\phi} y_t \).
The first order condition gives:

\[
(1 - \phi) \frac{p_t(f)}{p_t} y_t(f) + \phi mc_t y_t(f) - \phi^p \left[ \frac{p_t(f)}{p_{t-1}(f)} \pi - 1 \right] y_t p_t(f) = \beta \phi^p \left( \frac{p_t}{p_{t+1}} \right) \left( \frac{1 + \tau^c_t}{1 + \tau^c_{t+1}} \right)^{-\sigma} \left[ 1 - \frac{p_{t+1}(f)}{p_t(f)} \pi \right] y_{t+1} (46)
\]

Thus, the behavior of \( f \) is summarized by Eqs. (41),(42),(43) and (46).

All firms solve the identical problem and they will set the same price, \( p_t(f) \), which implies that \( p_t(f) = p_t \).

Appendix E  Decentralized equilibrium
(given policy)

We now combine all the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) every type of household maximize utility; (ii) every firm maximize profit; (iii) all constraints, including government budget constraint, are satisfied; and (iv) all markets clear.

The DE is summarized by the following conditions (quantities are in per capitalist and per worker terms):

\[
x_n(n_t^k)^{\eta}(c_t^k)^{\sigma} = \frac{(1 - \tau^c_t)}{(1 + \tau^c_t)} w_t (D1)
\]

\[
\frac{(c_t^k)^{-\sigma}}{(1 + \tau^c_t)} = \beta \left( \frac{(c_{t+1}^k)^{-\sigma}}{(1 + \tau^c_{t+1})} \right) [(1 - \delta) + (1 - \tau^k_{t+1})] (D2)
\]

\[
x_m(m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1 + \tau^c_t)} - \beta \frac{p_t}{p_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1 + \tau^c_{t+1})} (D3)
\]

\[
\frac{(c_t^k)^{-\sigma}}{(1 + \tau^c_t)} = \beta R_t \frac{p_t}{p_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1 + \tau^c_{t+1})} (D4)
\]

\[
k_t^k = (1 - \delta) k_{t-1}^k + x_t^k (D5)
\]

\[
c_t^k + \frac{v^w}{v^k} c_t^w + x_t^k + \left(1 + \frac{v^w}{v^k} \right) g_t = y_t \left[ 1 - \frac{\phi^p}{2} \left( \frac{p_t}{p_{t-1} \pi} - 1 \right)^2 \right] (D6)
\]
\[ x_n(n^w_t)^\eta(c^w_t)^\sigma = \frac{(1 - \tau_n^w)}{(1 + \tau_n^w)} w^w_t \]  
(D7)

\[ x_m(m^w_t)^-\mu = \frac{(c^w_t)^-\alpha - \beta}{1 + \tau_t^c} \frac{p_t}{p_{t+1}} \left[ \frac{(c^w_{t+1})^{-\alpha}}{1 + \tau_{t+1}^c} \right] \]  
(D8)

\[ (1 + \tau_t^c)c^w_t + m^w_t = \frac{p_{t-1}}{p_t} m^w_{t-1} + (1 - \tau_t^w) w^w_t n^w_t - \tau_t^{l,w} \]  
(D9)

\[ r^k_t = mc_t \frac{v_t}{k^k_{t-1}} \]  
(D10)

\[ w^k_t = mc_t \theta (1 - \alpha) \frac{v_t}{n^k_t} \]  
(D11)

\[ w^w_t = mc_t (1 - \theta) (1 - \alpha) \frac{y^k_t}{v^w_t n^w_t} \]  
(D12)

\[ y_t = A_t (k^k_{t-1})^\alpha \left[ \left\{ n^k_t \right\}^\theta \times \left\{ \frac{v^w_t}{v^k_t} n^w_t \right\} \right]^{1-\theta} \]  
(D13)

\[ d_t = y_t - mc_t y_t - \frac{\phi p}{2} \left( \frac{p_t}{p_{t+1} \pi} - 1 \right)^2 y_t \]  
(D14)

\[ (1 - \phi) + \phi mc_t - \phi \left[ \frac{p_t}{p_{t+1} \pi} - 1 \right] \left[ \frac{p_t}{p_{t+1} \pi} \right] y_{t+1} = \phi \beta \left[ \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right] \left( \frac{c^k_{t+1}}{c^k_t} \right)^-\alpha \left[ 1 - \frac{p_{t+1}}{p_t \pi} \right] y_{t+1} \]  
(D15)

\[
\left[ 1 + \frac{v^w}{v^k} \right] g_t + R_{t-1} \frac{p_{t-1}}{p_t} b^k_{t-1} + \frac{p_{t-1}}{p_t} m^{k}_{t-1} + \frac{v^w}{v^k} m^w_{t-1} = b^k_t + m^k_t + \frac{v^w}{v^k} m^w_t + \frac{\tau_t^c}{c^k_t + \frac{v^w}{v^k} c^w_t} + \frac{\tau_t^k}{r^k_t k_{t-1} + d_t} + \frac{\tau_t^n}{w^k_t n^k_t + \frac{v^w}{v^k} w^w_t n^w_t} - \tau_t^{l,k} \]  
(D16)
where \( n_t^k = n_t^k(f) \), \( n_t^w(f) = \frac{v_t^w}{v_t} n_t^w \), \( k_t^k = k_t^k(f) \), \( b_t = b_t^k \), \( d_t^k = d_t(f) \equiv d_t \), \( p_t(f) = p_t \), and \( y_t(f) = y_t \).

Thus, we have a system of 16 equations \([\text{D1)}-(\text{D16})\] in the 16 following endogeneous variables

\[
\begin{array}{c}
[k_t^k, c_t^k, y_t, p_t, m_t^k, m_t^w, b_t^k, x_t^k, m^c_t, w_t^k, n_t^k, w_t^w, n_t^w, t_t^k, k_t^k, d_t]_{t=0}
\end{array}
\]

Conclusively, the Decentralized Equilibrium is a sequence of

\[
\begin{array}{c}
[c_t^k, c_t^w, y_t, p_t, m_t^k, m_t^w, b_t^k, x_t^k, m^c_t, w_t^k, n_t^k, w_t^w, n_t^w, t_t^k, k_t^k, d_t]_{t=0}
\end{array}
\]

satisfying the equations \([\text{D1)}-(\text{D16})\) given:

a) technology \([A_t]_{t=0}^\infty\),

b) initial conditions for state variables \(k_{t-1}^k, b_{t-1}^k, A_{t-1}, m_{t-1}^k, m_{t-1}^w, R_{t-1}\)

c) policy

### Appendix F  Decentralized equilibrium

(given feedback policy coefficients)

We now rewrite the above equilibrium conditions, first, by using the inflation rate rather than price level and, second, by writing total public spending and total lump-sum taxes/transfers as shares of GDP, which is a more convenient form.

### F.1 Variables expressed in ratios

We define the gross inflation rate \(\pi_t \equiv \frac{p_t}{p_{t-1}}\). Defining above the exogenous total public spending as \(\left[1 + \frac{v^w}{v_t^k}\right] s_t^g \), we also find it convenient to express it as ratio of GDP, \(\left[1 + \frac{v^w}{v_t^k}\right] s_t^g = s_t^g y_t\). From this equation, we can express the per capita public spending, \(g_t\), as \(v_t^k s_t^g y_t\). Additionally, the total lump-sum taxes/transfers, \(\left[\tau_t^l + v_t^w s_t^l y_t\right]\), equal \(s_t^l y_t\), where as \(s_t^l\) are defined the lump-sum taxes/transfers as share of output. The government imposes/gives a percentage, \(\lambda_t^l\), of the total lump-sum taxes/transfers to the class of capitalists, while a percentage, \(\lambda_t^l\), to the workers. From the above, it arises that \(\tau_t^l = \lambda_t^l s_t^l y_t\) and \(\tau_t^l = \frac{v_t^k}{v_t^k} \lambda_t^l s_t^l y_t\) implying that, to the extent \(\lambda_t^l\) equals the percentage of capitalists in the population then it holds.
\( \tau_t^{l,k} = \tau_t^{l,w} = v^k s_t^l y_t \). Otherwise, if \( \lambda_t^{l,k} \) is larger than the percentage of capitalists in the population then the lump-sum taxes/transfers per capitalist are larger than those of per worker and vice versa.

### F.2 Final equations

Using the above, the final non-linear stochastic system is:

\[
x_n(n_t^k)^{(1 - \tau_t^w)(1 + \tau_t^c)} w_t^k \quad \text{(D1')}
\]

\[
\frac{(c_t^k)^{-\sigma}}{1 + \tau_t^c} = \beta \frac{(c_{t+1}^k)^{-\sigma}}{1 + \tau_t^c} [(1 - \delta) + (1 - \tau_t^c)r_{t+1}^k] \quad \text{(D2')}
\]

\[
x_m(m_t^w)^{-\mu} = \frac{(c_t^w)^{-\sigma}}{1 + \tau_t^c} - \beta \frac{1}{\pi_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{1 + \tau_t^c} \quad \text{(D3')}
\]

\[
\frac{(c_t^k)^{-\sigma}}{1 + \tau_t^c} = \beta R_t \frac{1}{\pi_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{1 + \tau_t^c} \quad \text{(D4')}
\]

\[
k_t^k = (1 - \delta)k_{t-1}^k + x_t^k \quad \text{(D5')}
\]

\[
c_t^k + \frac{v^w c_t^w + x_t^k + s_t^g y_t = y_t \left\{ 1 - \frac{\phi \pi}{2 \pi - 1} \right\} \quad \text{(D6')}
\]

\[
x_n(n_t^w)^{(1 - \tau_t^w)(1 + \tau_t^c)} w_t^w \quad \text{(D7')}
\]

\[
x_m(m_t^w)^{-\mu} = \frac{(c_t^w)^{-\sigma}}{1 + \tau_t^c} - \beta \frac{1}{\pi_{t+1}} \frac{(c_{t+1}^w)^{-\sigma}}{1 + \tau_t^c} \quad \text{(D8')}
\]

\[
(1 + \tau_t^c)c_t^w + m_t^w = \frac{1}{\pi_t} m_{t-1}^w + (1 - \tau_t^w)w_t^w n_t^w - v^k s_t^l y_t \quad \text{(D9')}\]

\[
r_t^k = mc_t \alpha \frac{y_t}{k_{t-1}^k} \quad \text{(D10')}
\]

\[
w_t^k = mc_t \theta (1 - \alpha) \frac{y_t}{n_t^k} \quad \text{(D11')}
\]
\[ w_t^w = mc_t(1 - \theta)(1 - \alpha) \frac{v^k}{v^w n_t^w} \]  

(D12')

\[ y_t = A_t[k_{t-1}^k]^\alpha \left[ (n_t^k)^{1-\theta} \times \left\{ \frac{v^w}{v^k n_t^w} \right\}^{1-\theta} \right]^{1-\alpha} \]  

(D13')

\[ d_t = y_t - mc_t y_t - \frac{\phi p}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 y_t \]  

(D14')

\[ (1 - \phi) + \phi mc_t - \phi^p \left[ \frac{\pi_t}{\pi} - 1 \right] \frac{\pi_t}{\pi} = \phi^p \beta \left[ \left( 1 + \tau_i^c \right) \left( \frac{c_{i+1}^k}{c_i^k} \right)^{-\sigma} \right] \left[ 1 - \frac{\pi_{t+1}}{\pi} \right] y_{t+1} \]  

(D15')

\[ s_t^g y_t + R_{t-1} \frac{1}{\pi_t} b_{t-1}^k + \frac{1}{\pi_t} \left[ \frac{m_{t-1}^k + v^w}{v^k m_{t-1}^w} \right] = b^k + \left[ m_{t-1}^k + \frac{v^w}{v_k m_{t-1}^w} \right] + \tau_i^c \left[ c_{i+1}^k + \frac{v^w}{v_k c_{i+1}^w} \right] + \]  

\[ + \tau_i^k \left[ r_{t-1}^k + d_t \right] + \]  

\[ + \tau_i^n \left[ w_t^k n_t^k + \frac{v^w}{v_k w_t^w n_t^w} \right] + \]  

\[ + s_t^l y_t \]  

(D16')

\[ \log \left( \frac{R_t}{R} \right) = \phi_n \log \left( \frac{\pi_t}{\pi} \right) + \phi_y \log \left( \frac{y_t}{y} \right) \]  

(D17')

\[ s_t^g = s^g - \gamma_t^g (l_{t-1} - l) \]  

(D18')

\[ \tau_i^c = \tau^c + \gamma_t^c (l_{t-1} - l) \]  

(D19')

\[ \tau_i^k = \tau^k + \gamma_t^k (l_{t-1} - l) \]  

(D20')

\[ \tau_i^n = \tau^n + \gamma_t^n (l_{t-1} - l) \]  

(D21')
The final equilibrium system consists of the 17 equations in 17 endogenous variables \( [c_k^t, c_w^t, y_t^t, \pi_t^t, m_k^t, m_w^t, b_k^t, x_k^t, m_c^t, w_k^t, n_k^t, n_w^t, n_t^t, r_t^t, k_t^t, d_t^t, l_t^t]_{t=0}^\infty \). This is given the 5 independently set monetary and fiscal instruments, \( [R_t^t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n]_{t=0}^\infty \), technology, \( [A_t^t]_{t=0}^\infty \), and initial conditions for the state variables, \( k_{-1}^k, b_{-1}^k, A_{-1}, m_{-1}^k, m_{-1}^w, R_{-1}, l_{-1} \). Recall that \( [R_t^t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n]_{t=0}^\infty \) follow the feedback rules specified above, while \( [s_t^f]_{t=0}^\infty \) remains constant and close to its average value in data.

Conclusively, we have a system of 22 equations \([(D1')-(D22')]\) in the 22 following endogeneous variables

\[
[c_k^t, c_w^t, y_t^t, \pi_t^t, m_k^t, m_w^t, b_k^t, x_k^t, m_c^t, w_k^t, n_k^t, n_w^t, n_t^t, r_t^t, k_t^k, d_t^t, R_t^t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n, l_t^t]_{t=0}^\infty
\]

Conclusively, the Decentralized Equilibrium is a sequence of

\[
[c_k^t, c_w^t, y_t^t, \pi_t^t, m_k^t, m_w^t, b_k^t, x_k^t, m_c^t, w_k^t, n_k^t, n_w^t, n_t^t, r_t^t, k_t^k, d_t^t, R_t^t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n, l_t^t]_{t=0}^\infty
\]

satisfying the equations \([(D1')-(D22')]\), given:

a) technology \( [A_t^t]_{t=0}^\infty \),

b) initial conditions for state variables \( k_{-1}^k, b_{-1}^k, A_{-1}, m_{-1}^k, m_{-1}^w, R_{-1}, l_{-1} \)

### Appendix G  Results

In this appendix we present, in form of tables, the outcomes of our experiments. These tables have been used for comparison reasons and led to the conclusions of our study, as they are presented in the main text. In particular Tables 11 and 12 show what are the aggregate implications in the transition when the adjusting instrument in the new reformed steady state is the tax rate on consumption and government spending respectively. Tables 13 and 14 show what are the distributional implications in the transition when the adjusting instrument in the new reformed steady state is the tax rate on consumption and government spending respectively.
Table 11: Present value of total output (GDP) over different time horizons when the adjusting instrument in the steady state is the tax rate on consumption ($\tau^c$)

<table>
<thead>
<tr>
<th>Adj. Instr.</th>
<th>$Y_5$</th>
<th>$Y_{10}$</th>
<th>$Y_{20}$</th>
<th>$Y_{40}$</th>
<th>$Y_{60}$</th>
<th>$Y_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>11.30</td>
<td>22.22</td>
<td>43.02</td>
<td>80.43</td>
<td>112.70</td>
<td>217.37</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>10.79</td>
<td>21.97</td>
<td>43.01</td>
<td>80.49</td>
<td>112.76</td>
<td>217.42</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>11.00</td>
<td>22.01</td>
<td>42.96</td>
<td>80.45</td>
<td>112.73</td>
<td>217.39</td>
</tr>
<tr>
<td>$s^g$</td>
<td>11.74</td>
<td>22.97</td>
<td>44.06</td>
<td>81.62</td>
<td>113.92</td>
<td>218.62</td>
</tr>
<tr>
<td><strong>Status quo</strong></td>
<td><strong>11.30</strong></td>
<td><strong>22.23</strong></td>
<td><strong>43.02</strong></td>
<td><strong>80.43</strong></td>
<td><strong>112.71</strong></td>
<td><strong>217.37</strong></td>
</tr>
</tbody>
</table>

Note: $Y_t$ stands for the discounted future values of total output (GDP) for the next $t$ periods after the fiscal consolidation takes place.

Table 12: Present value of total output (GDP) over different time horizons when the adjusting instrument in the steady state is public spending ($s^g$)

<table>
<thead>
<tr>
<th>Adj. Instr.</th>
<th>$Y_5$</th>
<th>$Y_{10}$</th>
<th>$Y_{20}$</th>
<th>$Y_{40}$</th>
<th>$Y_{60}$</th>
<th>$Y_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>11.30</td>
<td>22.23</td>
<td>43.02</td>
<td>80.43</td>
<td>112.71</td>
<td>217.37</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>10.79</td>
<td>21.96</td>
<td>43.00</td>
<td>80.49</td>
<td>112.75</td>
<td>217.42</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>10.97</td>
<td>21.98</td>
<td>42.94</td>
<td>80.42</td>
<td>112.70</td>
<td>217.36</td>
</tr>
<tr>
<td>$s^g$</td>
<td>11.74</td>
<td>22.97</td>
<td>44.06</td>
<td>81.62</td>
<td>113.92</td>
<td>218.62</td>
</tr>
<tr>
<td><strong>Status quo</strong></td>
<td><strong>11.30</strong></td>
<td><strong>22.23</strong></td>
<td><strong>43.02</strong></td>
<td><strong>80.43</strong></td>
<td><strong>112.71</strong></td>
<td><strong>217.37</strong></td>
</tr>
</tbody>
</table>

Note: $Y_t$ stands for the discounted future values of total output (GDP) for the next $t$ periods after the fiscal consolidation takes place.

Table 13: Ratio of present value of net income of capitalist and that of worker over various time horizons when the adjusting instrument in the steady state is the tax rate on consumption ($\tau^c$)

**Steady state value in the status quo is 4.1446**

<table>
<thead>
<tr>
<th>Adj. Instr.</th>
<th>$Y_k^5$</th>
<th>$Y_k^{10}$</th>
<th>$Y_k^{20}$</th>
<th>$Y_k^{40}$</th>
<th>$Y_k^{60}$</th>
<th>$Y_k^\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>3.46</td>
<td>3.52</td>
<td>3.64</td>
<td>3.81</td>
<td>3.90</td>
<td>4.02</td>
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<tr>
<td>$\tau^n$</td>
<td>4.13</td>
<td>4.16</td>
<td>4.19</td>
<td>4.20</td>
<td>4.19</td>
<td>4.17</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>3.89</td>
<td>3.96</td>
<td>4.03</td>
<td>4.08</td>
<td>4.10</td>
<td>4.12</td>
</tr>
<tr>
<td>$s^g$</td>
<td>4.10</td>
<td>4.12</td>
<td>4.14</td>
<td>4.15</td>
<td>4.15</td>
<td>4.15</td>
</tr>
</tbody>
</table>

Note: $Y_k^T$ and $Y_w^T$ stand for the PV of net income of capitalist and worker respectively for the next $T$ periods after the fiscal consolidation.
Table 14: Ratio of present value of net income of capitalist and that of worker over various time horizons when the adjusting instrument in the steady state is government spending ($s^g$)

Steady state value in the status quo is 4.1446

<table>
<thead>
<tr>
<th>Adj. Instr.</th>
<th>$\frac{Y^k}{Y^k_T}$</th>
<th>$\frac{Y^k}{Y^k_{10}}$</th>
<th>$\frac{Y^k}{Y^k_{20}}$</th>
<th>$\frac{Y^k}{Y^k_{40}}$</th>
<th>$\frac{Y^k}{Y^k_{60}}$</th>
<th>$\frac{Y^k}{Y^k_{\infty}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>3.47</td>
<td>3.53</td>
<td>3.65</td>
<td>3.83</td>
<td>3.92</td>
<td>4.03</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>4.14</td>
<td>4.18</td>
<td>4.21</td>
<td>4.22</td>
<td>4.21</td>
<td>4.19</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>3.85</td>
<td>3.94</td>
<td>4.03</td>
<td>4.09</td>
<td>4.11</td>
<td>4.13</td>
</tr>
<tr>
<td>$s^g$</td>
<td>4.11</td>
<td>4.13</td>
<td>4.15</td>
<td>4.17</td>
<td>4.17</td>
<td>4.17</td>
</tr>
</tbody>
</table>

Note: $Y^k_T$ and $Y^w_T$ stand for the PV of net income of capitalist and worker respectively for the next $T$ periods after the fiscal consolidation.

References


