Optimal fiscal policy when tastes are inherited and environmental quality matters *

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Abstract

In this paper, we analyze optimal fiscal policies in an overlapping generation framework with two types of externalities: aspirations in preferences and environmental quality that deteriorates with the use of both a dirty consumption good and a polluting input. We focus on second-best policies when the government finances an exogenous flow of public spending by using taxes. We find that environmental concerns and aspirations are crucial when solving the planner’s problem. Particularly, we find that the planner must always tax both the polluting input and the dirty consumption good, and that it must tax differently the dirty and the clean consumption good with a higher tax on the first one. Aspirations induce overconsumption of the young generation and an appropriate fiscal policy should always increase savings and investment. It is also possible to avoid labor taxes provided that both consumptions goods are taxed at positive rates.

Keywords: Overlapping generations, environment, aspirations, fiscal policy, second-best.

JEL classification: D62, E62, H21, H23

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1 Introduction

One of the main problems associated to environmental conservation is that short-lived individuals fail to internalize the long term effects of their decisions on environmental degradation. Present decisions affect future environment quality generating an intergenerational externality in which future unborn generations bear the costs imposed by current ones. The intergenerational aspect of the problem has lead several economists to use the overlapping generations framework (OLG) in order to study the issue of environmental conservation in a dynamic economy (see, for example, John and Pecchenino, 1994; Ono, 1996; Bovenberg and Heijdra, 1998). In the present paper, we follow this strand of the literature but we add a new kind of intergenerational externality trough the introduction of aspirations in preferences. These aspirations are inherited from the previous generation and are a frame of reference against which young age consumption is evaluated. This formulation implies that present choices are not only affected by the current capital stock and environmental quality levels but also directly by the consumption choices of the previous young generation. While aspirations have been introduced in the standard Diamond framework (see, for example, De la Croix, 1996; De la Croix and Michel, 1999; Artige et al., 2004; Alonso-Carrera et al., 2007), they have been largely disconnected from environmental concerns. A notable exception is the work of Aronsson and Johansson-Stenman (2014) who study the optimal provision of state variable public goods (global climate being the prime example) in a model where people care about relative consumption (including aspirations).

The importance of relative well-being has been highlighted in several empirical works such as Clark and Oswald (1996) or Ferrer-i Carbonell (2005) among others. The main conclusion being that utility does not only depend on present consumption but also on some reference point. Concerning more specifically aspirations, Becker (1992) has noted that individual behavior is affected by habits acquired as a child generating the intergenerational transfer of tastes. For example, Waldkirch et al. (2004) estimate that parental preferences explain between 5% and 10% of the preferences of their children after controlling for their respective incomes. Senik (2009) presents evidence showing that an individual’s well-being increase if he has done better in life than his parents. This large empirical evidence combined with the importance that endogenous preferences can play concerning environmental issues (see, for example, Brekke and Howarth, 2003; Welsch, 2009) lead us to include consumption aspirations in our framework.

The idea of aspirations has some similarities with the concepts of habit formation and status effects that have previously been introduced in models
dealing with environmental conservation. We can cite the works of Wend-ner (2003, 2005), Brekke and Howarth (2003) as well as Howarth (2006), which have focused on the environmental implications of different forms of consumption externalities. A related but different approach is the one of Schumacher and Zou (2008) who introduce habits in pollution and study the complex dynamic implications of such an assumption but focus only on the competitive equilibrium.

In the present paper we focus on a model where agents live for two periods, working only in the first and retiring in the second. Agents derive utility in both periods from environmental quality and the consumption of two goods, one of which is a polluting one. We furthermore assume that labor supply is endogenous and that aspirations affect both goods when young. On the production side, a representative firm produces output by using capital, labor and a polluting input. Environmental quality is a stock depleted by pollution and that regenerates at a given natural rate. Taxes which are arbitrary in the competitive case are used to finance public expenditures. Our objective is to study optimal fiscal policy in this model. In order to do so, taxes are chosen in order to maximize the discounted sum of utilities while taking the behavior of private agents as given.

Most of the OLG models studying at the same time environmental conservation and optimality focus on the planner’s problem and on the way to decentralize the first best solution trough an appropriate tax policy. Few studies have been conducted on second-best policies in this context. Notable exceptions are the works of Aronsson and Johansson-Stenman (2014) as well as the one of Nakabayashi (2010). The latter focuses on public sector efficiency in an OLG model with environmental quality.

An account of the results is as follows. The presence of environmental externalities induces the planner to set higher consumption taxes on the polluting good. The planner must also increase capital accumulation in order to bring the economy to the modified golden rule. Concerning labor income taxes, these can be discarded provided that the planner decides to tax both consumption goods. Finally, the polluting input must also be taxed.

The paper is organized as follows. In section 2 we present the model as well as the competitive equilibrium. Section 3 derives the main results of the paper concerning optimal fiscal policy. Finally, section 4 summarizes the main conclusions.
2 Model

Consider an overlapping generation economy of identical agents. Each agent lives for two periods, young and old age, and works only during the first period. The size of the population is constant and normalized to one. The lifetime utility function of a generation born in period $t$ is given by $U_t$ and takes the following form:

$$U_t(.) = \theta \ln(c_t - \rho_c c_{t-1}) + \gamma \ln(z_t - \rho_z z_{t-1}) + \epsilon \ln(1 - n_t) + \eta \ln E_t$$

$$+ \beta (\theta \ln d_{t+1} + \gamma \ln d^z_{t+1} + \eta \ln E_{t+1}),$$

(1)

where $c_t$ is the clean consumption level of a representative young generation, $z_t$ is the dirty consumption level (energy consumption can be considered as an example), $n_t$ represents the share of time spent working while $E_t$ is the level of environmental quality. When old, the representative generation derives utility from future clean consumption $d_{t+1}$, future dirty consumption $d^z_{t+1}$ as well as future environmental quality $E_{t+1}$. The discount factor of the agent is given by $\beta$. The utility function exhibits aspirations in young age where the representative generation compares consumption levels with the ones of the previous young generation (his parents) as in de la Croix (1996), de la Croix and Michel (1999) or Alonso-Carrera et al. (2007). The parameters $0 < \rho_c < 1$ and $0 < \rho_z < 1$ indicate the importance of both kind of aspirations in the utility function. As it is clear from expression (1), the model does not consider habits or aspirations for the old generation. This can be justified by empirical evidence showing that aspirations are less important for older persons. Clark and Oswald (1996) show for example that reported satisfaction levels increase with age. Older persons putting less weight on comparisons in their welfare evaluation.

When young, the representative agent splits his wage between both kind of consumption goods and savings $a_t$:

$$(1 + \tau^c_t)c_t + (1 + \tau^z_t)P^z_t z_t + a_t = (1 - \tau^w_t)w_t n_t,$$

(2)

where $w_t$ is the wage rate, $P^z_t$ is the exogenous relative price of dirty consumption, $\tau^c_t$, $\tau^z_t$ and $\tau^w_t$ are respectively clean consumption, dirty consumption and labor income taxes that the representative generation faces.

When old, the representative agent retires and consumes the income from his savings that he allocates once again between both kind of consumption goods:

$$(1 + \tau^c_{t+1})d_{t+1} + (1 + \tau^z_{t+1})P^z_{t+1} d^z_{t+1} = [1 + \tau_{t+1} r_{t+1} (1 - \tau^c_{t+1})]a_t,$$

(3)

where $r_{t+1}$, $\tau^c_{t+1}$ and $\tau^z_{t+1}$ are respectively taxes on future clean consumption, future dirty consumption and savings while $r_{t+1}$ is the net return of savings.
Output is produced by a representative firm with a production function \( F(k_t, n_t, x_t) \) using as inputs the current capital stock \( k_t \), labor \( n_t \) and a polluting input \( x_t \) (energy used in the production process is also a good example). The production function exhibits constant returns to scale and is increasing and concave in all three arguments. Since we are in a perfectly competitive setup, prices equal marginal productivities and we obtain:

\[
\begin{align*}
   w_t &= F_{n_t}(k_t, n_t, x_t), \\
   (1 + \tau_t^x)P_t^x &= F_{x_t}(k_t, n_t, x_t), \\
   r_t + \delta &= F_{k_t}(k_t, n_t, x_t).
\end{align*}
\]

\( P_t^x \) is the exogenous relative price of the dirty input, \( \tau_t^x \) is a tax on that same input and \( \delta \) is the capital depreciation rate. We suppose that one unit of the final good can be transformed into \( 1/P_t^z \) units of the dirty consumption good and into \( 1/P_t^x \) units of the polluting input.

The evolution of environmental quality is given by:

\[
E_{t+1} = E_t + b(E_t - E_t) - \kappa_z(z_t + d_t) - \kappa_x x_t,
\]

where \( E \) is the natural level of environmental quality, \( b \) is the natural regeneration rate of the environment, \( \kappa_z \) measures the polluting impact of dirty consumption while \( \kappa_x \) measures the one of the polluting input.

There is a government which collects taxes in order to finance a given level of public expenditures \( G_t \):

\[
G_t = \tau_t^c(c_t + d_t) + \tau_t^z P_t^z (z_t + d_t) + \tau_t^w w_t n_t + \tau_t r_t a_{t-1} + \tau_t^x P_t^x x_t.
\]

Market clearing implies that the future capital stock should equal to the savings of the young generation so that:

\[
k_{t+1} = a_t.
\]

In order to solve the problem of the representative generation, we will use the intertemporal budget constraint given by:

\[
\frac{(1 + \tau_t^c)c_t + (1 + \tau_t^z)P_t^z z_t - (1 - \tau_t^w)w_t n_t}{1 + r_{t+1}(1 - \tau_{t+1})} + \frac{(1 + \tau_{t+1}^c)d_{t+1} + (1 + \tau_{t+1}^z)P_{t+1}^z d_{t+1}}{1 + r_{t+1}(1 - \tau_{t+1}^z)} = 0.
\]

The problem of the representative generation is to maximize lifetime utility subject to the intertemporal budget constraint. We have the following
Lagrangian:

\[
L = U^t(.) - \lambda_t \left[ (1 + \tau^c_t) c_t + (1 + \tau^z_t) P^z_t z_t - (1 - \tau^w_t) w_t n_t \\
+ \frac{(1 + \tau^c_{t+1}) d_{t+1} + (1 + \tau^z_{t+1}) P^z_{t+1} d^z_{t+1}}{1 + r_{t+1}(1 - \tau^w_{t+1})} \right],
\]

(11)

where \( \lambda_t \) is the Lagrange multiplier associated to the optimization problem.

The first order conditions (FOC) are given by:

\[
\begin{align*}
\frac{\theta}{c_t - \rho_c c_{t-1}} &= \lambda_t (1 + \tau^c_t), \\
\frac{\gamma}{z_t - \rho_z z_{t-1}} &= \lambda_t P^z_t (1 + \tau^z_t), \\
\frac{\beta \theta}{d_{t+1}} &= \lambda_t (1 + \tau^c_{t+1}) + \frac{1}{1 + r_{t+1}(1 - \tau^w_{t+1})}, \\
\frac{\beta \gamma}{d^z_{t+1}} &= \lambda_t P^z_{t+1} (1 + \tau^w_{t+1}) + \frac{1}{1 + r_{t+1}(1 - \tau^w_{t+1})}, \\
\frac{\epsilon}{1 - n_t} &= \lambda_t (1 - \tau^w_t) w_t.
\end{align*}
\]

Using the previous FOC we obtain the following equilibrium conditions:

\[
\begin{align*}
\frac{1}{(1 + \tau^c_t)(c_t - \rho_c c_{t-1})} &= \frac{\beta [1 + r_{t+1}(1 - \tau^c_{t+1})]}{(1 + \tau^c_{t+1}) d_{t+1}}, \\
\frac{1}{(1 + \tau^z_t) P^z_t (z_t - \rho_z z_{t-1})} &= \frac{\beta [1 + r_{t+1}(1 - \tau^z_{t+1})]}{(1 + \tau^z_{t+1}) P^z_{t+1} d^z_{t+1}}, \\
\frac{\theta}{(1 + \tau^c_t)(c_t - \rho_c c_{t-1})} &= \frac{\epsilon}{(1 - \tau^w_t)(1 - n_t) w_t}, \\
\frac{\theta}{(1 + \tau^z_t)(c_t - \rho_c c_{t-1})} &= \frac{\gamma}{(1 + \tau^w_t) P^z_t (z_t - \rho_z z_{t-1})}, \\
\frac{\theta}{(1 + \tau^c_{t+1}) d_{t+1}} &= \frac{\gamma}{(1 + \tau^w_{t+1}) P^z_{t+1} d^z_{t+1}}.
\end{align*}
\]

Expression (17) represents the intertemporal allocation of the clean consumption good between young and old-age. Expression (18) does the same for the dirty consumption good. Expression (19) describes the trade-off between leisure and clean consumption when young while expression (20) describes the intratemporal allocation of consumption when young. Finally, expression (21) does the same for old-age consumption.
Before defining our competitive equilibrium, we need to derive the feasibility constraint which is given by:

\[ c_t + d_t + P^z_t(z_t + d^z_t) + k_{t+1} - (1 - \delta)k_t + G_t + P^x_t x_t = F(k_t, n_t, x_t). \tag{22} \]

**Definition 1:**

Given a sequence of policies \( \{\tau^c_t, \tau^z_t, \tau^r_t, \tau^w_t\} \) and a set of initial conditions \( \{k_0, d_0, d^z_0\} \), a competitive equilibrium of this economy is a sequence of allocations \( \{c_t, d_t, z_t, d^z_t, a_t\} \) with production plans \( \{k_t, n_t, x_t\} \) and prices \( \{r_t, w_t, P^z_t, P^x_t\} \) such that:

(i) the allocations \( \{c_t, d_{t+1}, z_t, d^z_{t+1}, a_t\} \) solve the consumer’s problem given prices \( \{r_{t+1}, w_t, P^z_t, P^x_t\} \) and policies \( \{\tau^c_t, \tau^c_{t+1}, \tau^z_t, \tau^z_{t+1}, \tau^r_t, \tau^r_{t+1}, \tau^w_t\} \);

(ii) the production plans \( \{k_t, n_t, x_t\} \) solve the firm’s problem given prices \( \{r_t, w_t, P^x_t\} \) and the policy \( \{\tau^r_t\} \);

(iii) the government budget constraint holds at each period,

(iv) the labor, capital and goods markets clear,

(v) feasibility is satisfied at each period.

In the competitive equilibrium, government policies are arbitrary and in the following we will study optimal fiscal policies in this economy taking as given the behavior of each generation.

### 3 Optimal fiscal policy

In this section we will study the optimal fiscal policy in the case where the government has access to some commitment technology preventing it from revising the optimal policy over time. In order to do so, we will follow the so-called primal approach developed by Lucas and Stokey (1983) as well as Chari and Kehoe (1999). The approach consists in letting the government choose an optimal allocation (instead of the different tax rates) but also to reduce the set from which it can do so. As explained in Erosa and Gervais (2002), using instead the dual approach makes it impossible to analytically characterize the optimal policy. An implementable allocation should satisfy the intertemporal budget constraint of the representative generation as well as the first order conditions of the competitive equilibrium. Combining these elements we can obtain the so-called implementability constraint which, since we are in an overlapping generations model, is different for each generation. In order to obtain this constraint, we multiply the intertemporal budget
constraint of the representative generation by its Lagrange multiplier ($\lambda_t$). In the present case, we have:

$$\lambda_t \left[ (1 + \tau_t^c) c_t + (1 + \tau_t^z) P_t^z z_t + \frac{(1 + \tau_{t+1}^c) d_{t+1} + (1 + \tau_{t+1}^z) P_{t+1}^z d_{t+1}}{1 + r_{t+1}(1 - \tau_{t+1}^r)} - (1 - \tau_t^w) w_t n_t \right] = 0.$$  

(23)

By using the first order conditions of the competitive equilibrium in order to substitute for taxes and prices, we obtain our implementability constraint for generation $t$:

$$\left[ \frac{\theta c_t}{c_t - \rho_c c_{t-1}} + \frac{\gamma z_t}{z_t - \rho_z z_{t-1}} + \beta (\theta + \gamma) - \frac{\epsilon n_t}{1 - n_t} \right] = 0.$$  

(24)

The objective of the planner is then to maximize the discounted sum of utilities subject to the implementability constraints, the feasibility constraint as well as the law of motion for environmental quality. Concerning the initial old generation, we proceed as in Chari et al. (1996) and adopt the convention that $\tau_0^r$ is fixed in order to avoid the possibility of lump sum taxation. The split of income between both consumption goods $d_0$ and $d_{0}^z$ is also given. We rewrite the problem in the following way: the objective function $W^t$ for generation $t$ includes lifetime utility as well as generation $t$’s implementability constraint:

$$W^t = U^t + \mu_t \left[ \frac{\theta c_t}{c_t - \rho_c c_{t-1}} + \frac{\gamma z_t}{z_t - \rho_z z_{t-1}} + \beta (\theta + \gamma) - \frac{\epsilon n_t}{1 - n_t} \right],$$  

(25)

where $\mu_t$ is the multiplier corresponding to the implementability constraint of the generation born in period $t$. The planner then maximizes the discounted sum ($\zeta > 0$ being the discount factor of the planner) of $W^t$ functions subject to the feasibility constraint and the law of motion for environmental quality:

$$\max \sum_{t=0}^{\infty} \zeta^t W^t,$$  

subject to:

$$c_t + d_t + P_t^z (z_t + d_t^z) + k_{t+1} + G_t + P_t^x x_t - F(k_t, n_t, x_t) - (1 - \delta) k_t = 0$$  

$$E_{t+1} - E_t - b(E - E_t) + \kappa z_t + d_t^z + \kappa x t = 0,$$

given initial conditions $\{k_0, d_0, d_0^z, E_0, c_{-1}, z_{-1}\}$.

The multipliers of both constraints are given respectively by $-\zeta^t \mu_t^1$ and $\zeta^t \mu_t^2$. 

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The FOC are given by:

\[ W_{c_t}^t + \zeta W_{c_t}^{t+1} = \mu_1^t, \quad (27) \]
\[ W_{z_t}^t + \zeta W_{z_t}^{t+1} - P^z_t \mu_1^t = -\kappa_z \mu_1^t, \quad (28) \]
\[ W_{d_{t+1}}^t = \zeta \mu_1^{t+1}, \quad (29) \]
\[ W_{n_t}^t = -F_n \mu_1^t, \quad (31) \]
\[ \zeta \mu_1^{t+1} (1 - \delta + F_{\kappa t+1}) = \mu_1^t, \quad (32) \]
\[ \mu_1^t (P^x_t - F^x_t) = \kappa_x \mu_1^t, \quad (33) \]
\[ W_{E_{t+1}}^t + \zeta W_{E_{t+1}}^{t+1} = -\mu_2^t + \zeta (1 - b) \mu_2^{t+1}, \quad (34) \]

where \( W_{c_t}^t > 0 \) and \( W_{z_t}^t > 0 \) since the marginal utilities of both goods should be positive while \( W_{c_t}^{t+1} < 0 \) and \( W_{z_t}^{t+1} < 0 \) since aspirations have a negative impact on the utility of the following young generation. Furthermore, since the marginal cost of a unit of consumption \( \mu_1^t \) must be positive while the marginal cost of a unit of pollution \( -\mu_2^t \) is also positive, we know that \( W_{c_t}^t + \zeta W_{c_t}^{t+1} > 0 \) and \( W_{z_t}^t + \zeta W_{z_t}^{t+1} > 0 \). The derivatives of \( W \) with respect to our endogenous variables are given by:

\[ W_{c_t}^t = \frac{\theta}{c_t - \rho_c c_{t-1}} - \frac{\theta \mu_1 \rho_c c_{t-1}}{(c_t - \rho_c c_{t-1})^2}, \quad (35) \]
\[ W_{c_t}^{t+1} = -\frac{\theta \rho_c [c_{t+1} (1 - \mu_{t+1}) - \rho_c c_t]}{(c_{t+1} - \rho_c c_t)^2}, \quad (36) \]
\[ W_{z_t}^t = \frac{\gamma}{z_t - \rho_z z_{t-1}} - \frac{\gamma \mu_1 \rho_z z_{t-1}}{(z_t - \rho_z z_{t-1})^2}, \quad (37) \]
\[ W_{z_t}^{t+1} = -\frac{\gamma \rho_z [z_{t+1} (1 - \mu_{t+1}) - \rho_z z_t]}{(z_{t+1} - \rho_z z_t)^2}, \quad (38) \]
\[ W_{d_{t+1}}^t = \frac{\beta \theta}{d_{t+1}}, \quad (39) \]
\[ W_{d_{t+1}}^{t+1} = \frac{\beta \gamma}{d_{t+1}^{t+1}}, \quad (40) \]
\[ W_{n_t}^t = -\frac{\epsilon}{1 - n_t} - \frac{\epsilon \mu_t}{(1 - n_t)^2}, \quad (41) \]
\[ W_{E_{t+1}}^t + \zeta W_{E_{t+1}}^{t+1} = \frac{\eta (\beta + \zeta)}{E_{t+1}}. \quad (42) \]

With our FOC in hand we can determine conditions concerning the optimal tax rates. In the present case, we will consider that the planner cannot implement age-dependant taxes. This implies that he has to set the same
consumption taxes to the young and old generations living at the same time.

**Proposition 1**: At the optimum, the tax on dirty consumption $\tau^z_t$ is always higher than the one on the clean consumption good $\tau^c_t$ so that $\tau^z_t > \tau^c_t \geq 0$. Furthermore, the following condition should be satisfied:

$$-\frac{\zeta \kappa_z \mu^2_t d^z_t}{\beta \gamma} = W^t_{zt} + \zeta W^{t+1}_{zt} - P^z_t (W^t_{ct} + \zeta W^{t+1}_{ct}) > 0.$$ (43)

However, uniform taxation ($\tau^z_t = \tau^c_t$) prevails if $\mu^2_t = 0$ and marginal utilities net of aspirations are equal.

At the steady-state, we obtain:

$$\frac{\tau^z - \tau^c}{1 + \tau^z} = \frac{\zeta \kappa_z \eta (\beta + \zeta) d^z}{\beta \gamma [1 - \zeta (1 - b)] E} > 0,$$ (44)

so that $\tau^z > \tau^c \geq 0$ as well.

**Proof.** Using expressions (29), (30), the equality between marginal utilities in old age from the competitive equilibrium (expression (21)) and setting these expressions at time $t$ we obtain:

$$\frac{\tau^z_t - \tau^c_t}{1 + \tau^z_t} = -\frac{\zeta \kappa_z \mu^2_t d^z_t}{\beta \gamma}.$$ (45)

We know that $-\mu^2_t$ which represents the marginal cost of pollution is positive implying that $\tau^z_t > \tau^c_t$. However, since we apply the same consumption taxes to young and old generations, the result should also be valid for the young generation. Combining expression (27) and (28) we obtain:

$$-\kappa_z \mu^2_t = W^t_{zt} + \zeta W^{t+1}_{zt} - P^z_t (W^t_{ct} + \zeta W^{t+1}_{ct}) > 0.$$ (46)

Using expression (45) to substitute for the left hand side, we obtain:

$$\left(\frac{\tau^z_t - \tau^c_t}{1 + \tau^z_t}\right) \frac{\beta \gamma}{\zeta d^z_t} = W^t_{zt} + \zeta W^{t+1}_{zt} - P^z_t (W^t_{ct} + \zeta W^{t+1}_{ct}) > 0.$$ (47)

If the planner does not take into account environmental quality ($\mu^2_t = 0$), uniform taxation ($\tau^z_t = \tau^c_t$) prevails both for the old and the young generation. Concerning the last part of the proposition, in order to obtain expression (44) we combine condition (45) with the steady-state value for $\mu^2_t$ derived from expression (34).
The fact that the tax on dirty consumption $\tau^z_t$ is higher than the one on clean consumption $\tau^c_t$ is linked to the negative environmental externality produced by pollution. In this case, for the young, the marginal utility of dirty consumption net of aspirations is higher than the one for the clean consumption good. Another implication of these results is that while clean consumption taxes could be set to zero, the ones on dirty consumption should always be positive unless the planner does not take into account environmental quality in which case uniform taxation should prevail (Atkinson and Stiglitz, 1976). In that case, marginal utilities net of aspirations should be equal since there is no reason for the planner to induce agents to consume more of a particular good. At the steady-state, the result is maintained and we can observe that the difference between both consumption taxes is increasing in the planner’s discount factor. Intuitively, if the planner puts a relatively important weight to the welfare of future generations, he will increase the difference between dirty and clean consumption taxes in order to preserve environmental quality. The tax difference is also increasing in old age dirty consumption since a higher level of the latter deteriorates environmental quality. Finally, we can observe that the tax difference is decreasing in the level of environmental quality since with a relatively high level of the latter, differential taxation between both goods is less needed.

**Proposition 2:** At the optimum, capital taxes are bounded and must satisfy the following two conditions:

\[
\begin{align*}
\frac{1 + F_{k_{t+1}} - \delta}{F_{k_{t+1}} - \delta} \left( \frac{\tau^c_t - \tau^c_{t+1}}{1 + \tau^c_t} \right) &> \tau^r_{t+1}, \\
\frac{1 + F_{k_{t+1}} - \delta}{F_{k_{t+1}} - \delta} \left( \frac{\tau^z_t - \tau^c_{t+1}}{1 + \tau^z_t} \right) &> \tau^r_{t+1}.
\end{align*}
\]

(48)

(49)

In the case where $\rho_c = \rho_z = \mu^z_t = 0$, the previous conditions should be satisfied with equality.

At the steady-state where $\tau^c_t = \tau^c_{t+1} = \tau^c$ and $\tau^z_t = \tau^z_{t+1} = \tau^z$, the planner needs to subsidize savings ($\tau^r_{t+1} = \tau^r < 0$) unless $\rho_c = 0$ in which case $\tau^r = 0$ since condition (48) should then be satisfied with equality.

**Proof.** Using expressions (27), (29), (32) and the intertemporal allocation from the competitive equilibrium (expression (17)), we obtain the following expression:

\[
\begin{align*}
\beta \theta \left[ (1 + F_{k_{t+1}} - \delta)(\tau^c_t - \tau^c_{t+1}) - \tau^r_{t+1}(F_{k_{t+1}} - \delta)(1 + \tau^c_t) \right] \\
= -\zeta W_{c_{t+1}} - \frac{\mu_k \theta \rho_c c_{t-1}}{(c_t - \rho_c c_{t-1})^2} > 0,
\end{align*}
\]

(50)
since $W_{t+1} < 0$ implying that $\tau^{r}_{t+1}$ must satisfy condition (48). However we must also derive a condition concerning the dirty consumption good. In this case we obtain:

$$\beta \gamma \left[ (1 + F_{k_{t+1}} - \delta)(\tau^z_t - \tau^z_{t+1}) - \tau^r_{t+1}(F_{k_{t+1}} - \delta)(1 + \tau^z_t) \right]$$

$$= \frac{P^z_{t+1}}{P^z_t} \left[ -\zeta W^{t+1}_{z_t} + \frac{\mu^2 \gamma \rho z_{t-1} - \kappa \mu^2_t}{(z_t - \rho z_{t-1})^2} \right] > 0,$$  \hspace{1cm} (51)

since $W^{t+1}_{z_t} < 0$ implying that $\tau^{r}_{t+1}$ must also satisfy condition (49).

It is straightforward to see that if $\rho_c = \rho_z = \mu^2_t = 0$, both conditions should be satisfied with equality.

At the steady-state, $\tau^c_t = \tau^c_{t+1} = \tau^c$, $\tau^z_t = \tau^z_{t+1} = \tau^z$ and $\tau^r_{t+1} = \tau^r$ implying that condition (48) becomes $\tau^r < 0$ while condition (49) is now given by:

$$\frac{1 + F_k - \delta}{F_k - \delta} \left( \frac{\tau^z - \tau^c}{1 + \tau^z} \right) > \tau^r.$$  \hspace{1cm} (52)

From proposition 1, we know that $\tau^z > \tau^c$ implying that in order to satisfy both conditions we have $\tau^r < 0$. However, if $\rho_c = 0$, condition (48) should be satisfied with equality and thus $\tau^r = 0$.

The first condition implies that it is possible for the government to impose positive capital taxes provided that it compensates this outcome with a decrease in clean consumption taxes between two successive periods ($\tau^c_t > \tau^c_{t+1}$). Intuitively, higher consumption taxes today will still induce the representative generation to save more for the future. The second condition gives us a similar result except that here the planner takes into account the environmental impact of dirty consumption. Indeed the term between brackets in condition (49) can be written as $\tau^z_t - \tau^z_{t+1} = \tau^z_t - \tau^z_{t+1} + \tau^z_{t+1} - \tau^z_{t+1}$ which is composed of two elements: the potential increase in dirty consumption taxes across time and the internalization of the environmental externality in $t+1$.

The condition implies that it is more difficult to generate capital subsidies in this case. However, the reader should remember that both conditions should be satisfied at the same time implying that condition (48) might be enough to generate investment subsidies (for example in the case where $\tau^c_t = \tau^c_{t+1}$). Another implication of the condition is that a higher future marginal productivity of capital induces a decrease in a possible investment subsidy since the planner’s intervention is less needed. As explained in Atkinson and Sandmo (1980), an intervention on the capital stock should only be used in order to increase efficiency and ensure convergence to the modified golden rule. In the present framework, the presence of aspirations pushes the young generation.
to consume too much of both goods in the first period which leads the economy to excessive under-accumulation of capital justifying public intervention. Garriga (2001) and Erosa and Gervais (2002) show that it is optimal to tax (or subsidize) savings if agents have labor-leisure choices over their entire life. The result does not apply here since we suppose that the agent is retired in the second period of life and our results concerning capital taxation are only driven by aspirations and environmental externalities. It is interesting to compare the results to the case where aspirations affect only one of the two goods. Suppose that \( \rho_c > 0 \) and \( \rho_z = 0 \): in this case, due to the presence of the environmental externality, the results are qualitatively the same and once again conditions (48) and (49) should be satisfied. Suppose now that \( \rho_c = 0 \) and \( \rho_z > 0 \): in this case, the only change is that condition (48) should be satisfied with equality. It can be shown that both conditions are satisfied provided that \( \tau_t^z > \tau_t^c \) (which is always true since we study the case where \( -\mu_t^2 > 0 \)). These two cases show that our conditions are also valid when only one of the goods exhibit aspirations. Concerning the steady-state outcome, the planner needs to subsidize investment to compensate for aspirations since consumption taxes should be constant across time. This steady-state result is related to the fact that the planner cannot impose age-dependant taxes in the present framework. In the case where this is allowed, a capital subsidy would not be needed since different tax rates for the young and the old generations would be sufficient to guarantee the optimality of the intertemporal allocation. It is worth noticing that in the case where \( \rho_c = 0 \), the subsidy is equal to zero even if \( \rho_z > 0 \). This is due to the presence of the environmental externality which implies that the left hand side of expression (49) is positive at the steady-state. Finally, it can be observed that the only way for the capital stock to converge to the modified golden rule without any intervention is that the preferences of the young generations do not exhibit aspirations and that there are no environmental externalities.

**Proposition 3**: At the optimum, a zero tax on income \( (\tau_t^w = 0) \) is possible only if both consumption taxes are positive \( (\tau_t^c, \tau_t^z > 0) \).

**Proof.** Using expressions (27), (31) and the condition reflecting the allocation between consumption and leisure from the competitive equilibrium (expression (19)), we obtain:

\[
\frac{\gamma}{P_t(z_t - \rho_z z_{t-1})} \left( \frac{\tau_t^c + \tau_t^w}{1 + \tau_t^z} \right) = \frac{\mu_t \theta \rho_c c_{t-1}}{(c_t - \rho_c c_{t-1})^2} - \zeta W_{ct}^{t+1} + \frac{\mu_t \epsilon}{(1 - n_t)^2 F_{nt}} > 0. \tag{53}
\]
Due to the presence of the last term on the right hand side, $\tau_t^c + \tau_t^w > 0$ even in the case where $\rho_c = 0$. We can obtain the condition for the dirty consumption good by combining expressions (28), (31) and (19) and obtaining:

$$
\frac{P^z_t \theta}{c_t - \rho_c c_{t-1}} \left( \tau_t^z + \tau_t^w \right) = \frac{\mu_{t} \gamma \rho_z z_{t-1}}{(z_t - \rho_z z_{t-1})^2} - \zeta W_t^{z_{t+1}} - \kappa_z \mu_t^2 \\
+ \frac{P^z_t \mu_t \epsilon}{(1 - \eta_t)^2 F_n_t} > 0. \tag{54}
$$

Due to the presence of the last term on the right hand side, $\tau_t^z + \tau_t^w > 0$ even in the case where $\rho_z = \mu_t^2 = 0$.

Proposition 3 tells us that we always need to use at least two instruments among the two consumption taxes and the income tax. This implies that the planner can avoid income taxes provided that he implements consumption taxes on both goods. Conversely, the planner can avoid clean consumption taxes if he implements positive income and dirty consumption taxes. In the case of uniform taxation, both kind of consumption taxes can be set to zero and a positive tax on income is sufficient as in Atkinson and Stiglitz (1976). All the results derived in Proposition 3 are independent of the presence of aspirations. Our results are a potential justification for green tax reforms that have been implemented in several European countries (see, for example, Speck et al., 2011) and in which the introduction of environmental consumption taxes were compensated by reductions in income taxes (Sweden in 1991, Norway and the Netherlands in 1992, Estonia in 2006) or in social contributions (Germany in 1999, Czech Republic in 2008).

**Proposition 4**: At the optimum, the government imposes a tax on the polluting input so that $\tau_t^x > 0$ unless $\mu_t^2 = 0$ in which case $\tau_t^x = 0$.

At the steady-state, we obtain:

$$
\tau_t^x = \frac{\zeta \kappa x \eta (\beta + \zeta) d}{P^x \beta \theta (1 - \zeta(1 - b)) E}. \tag{55}
$$

**Proof.** By combining expression (33) with the condition for the marginal productivity of energy from the competitive equilibrium (expression (5)), we obtain:

$$
\tau_t^x = -\frac{\kappa x \mu_t^2}{P^x \mu_t^1}. \tag{56}
$$

We know that $\mu_t^1 > 0$ since the marginal utility of present consumption is positive and that $\mu_t^2 < 0$. We can thus conclude that $\tau_t^x > 0$ unless
the planner does not take into account environmental quality in which case \( \mu_1^2 = \tau_i^e = 0 \).

Concerning the last part of the proposition, in order to obtain condition (55), we combine expression (56) with the steady-state values for \( \mu_1 \) (expression (29)) and \( \mu_2 \) (expression (34)).

If environmental quality matters to the planner, the latter will impose a pigouvian tax in order to correct the negative externality due to pollution. The standard Diamond and Mirrlees (1971) result implying that taxes on intermediate inputs should be zero is not valid any more in the present case. A similar result is found by De Miguel and Manzano (2006) in a RBC model where oil is used as a production input which also reduces welfare. In fact, in the present case, optimal taxes involve a compromise between the positive effect of the input on the production function and the negative effect on environmental quality and thus on utility. At the margin, both effects should be equal (see Baumol and Oates, 1988). At the steady-state, the result is preserved and the dirty input tax is increasing in the social discount factor. As in the case of the tax on dirty consumption, the higher the weight associated to the welfare of future generations, the higher is the tax on the polluting input in order to preserve environmental quality. Moreover, the tax is increasing in old-age clean consumption and decreasing in the level of environmental quality. In the first case, this is due to the fact that a higher level of old-age consumption implies larger levels of investment and thus a substitution of the dirty input by additional capital. In the second case, this is again due to the fact that a larger level of environmental quality decreases the need for a tax on the polluting input. The tax will induce a reduction in the use of the input and consequently a decrease in production and a lower marginal productivity of capital. The reader should however remember that in Proposition 2 we showed that the planner will increase savings and investment thus compensating the negative impact of the input tax and allowing the economy to converge to the modified golden rule. Since the capital stock is also an intermediate input, the result of Diamond and Mirrlees (1971) would suggest that capital should not be subsidized. However, the presence of externalities (here consumption externalities under the form of aspirations), induces the planner to modify the intertemporal allocation in order to converge to the modified golden rule.

4 Conclusion

This paper analyses optimal fiscal policies in an OLG model where preferences include aspirations and environmental quality. We focus on second-best
policies in a setup where the government needs to finance public expenditures. The main results of the paper highlight the need to impose different tax rates on both consumption goods with a higher tax on the dirty one. It is also possible to avoid labor taxes provided that both consumptions goods are taxed at positive rates. Concerning the polluting input, it should always be taxed in a pigouvian way. Moreover, aspirations induce overconsumption of the young generation and an appropriate fiscal policy should always increase savings and investment.

In terms of policy implications, our paper suggests that public spending should partially be financed by environmental taxes on dirty consumption goods and on polluting inputs. Our work also justifies recent green tax reforms that have introduced environmental consumption taxes while reducing income taxes or social contributions. Finally, the possible influence of aspirations on consumption decisions is a warning towards policy makers tempted to impose large capital taxes.

In the present paper, we have focused on the case of a representative generation ruling out the possibility of intragenerational heterogeneity. It would be interesting to focus next on the introduction of different types of agents allowing us to study redistribution policies. This topic is on our research agenda.

References


