Enlarging the collective model of household behaviour: A revealed preference analysis

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Abstract

We use a comprehensive model of strategic household behaviour in which the spouses' expenditure on each public good is decomposed into autonomous spending and coordinated spending *à la* Lindahl. We obtain a continuum of semi-cooperative regimes parameterized by the relative weights put on autonomous spending, by each spouse and for each public good, nesting full cooperative and non-cooperative regimes as limit cases. Testing is approached through revealed preference analysis, by looking for rationalisability of observed data sets, with the price of each public good lying between the maximum and the sum of the hypothesized marginal willingnesses to pay of the two spouses. Once rationalised, an observed data set always allows to identify the sharing rule, except when both spouses contribute in full autonomy to some public good (a situation of local income pooling).

Keywords: semi-cooperative household behaviour, revealed preference analysis, rationalisability, sharing rule identification.

JEL classification: D11, C72, H41.

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1 Introduction

The popular model of household behaviour, known as the 'collective model' and initiated by Chiappori (1988, 1991), has managed to integrate the fact that within-household decision-making is generally a multi-person process. Assuming full cooperation this model has generated testable restrictions in spite of being parcimonious in describing the decision process itself. A more explicit approach is Nash bargaining (with Nash cooperative solution) as applied to household behaviour by Lundberg and Pollak (1993), in which the threat (or

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disagreement) point is taken to be a Nash non-cooperative equilibrium in a game of voluntary contributions to public goods.[1] Lundberg and Pollack, though, focus on the 'separate spheres equilibrium', where each spouse is responsible for a distinct set of goods and services within the household. Recent work on this non-cooperative intra-household game[2] by Lechene and Preston (2005, 2011) and Browning, Chiappori and Lechene (2010) has shown that generically only two types of non-cooperative equilibrium can emerge in the game: "separate spheres" proper and "separate spheres up to one public good" (*i.e.* spouses do not contribute jointly to more than one public good within the household). Income pooling holds[3] at this second type of equilibrium, meaning that only the household total income matters not its distribution. Since most empirical studies reject the income pooling hypothesis[4] and since the separate spheres equilibrium remains a very particular case (say, justified by a traditional gender partition), we have argued in a previous paper[5] that the set of household agreements could not be reduced to the two extreme kinds, fully efficient or fully non-cooperative ones. There we introduced the possibility of testing semi-cooperative agreements by defining a concept of 'household θ-equilibrium' in a strategic model of household consumption, where the given vector parameter θ determines the degree of autonomy of each spouse for each public good within the household, the two extreme regimes remaining full cooperation (θ identically nil) and full non-cooperation (θ identically one). Hence, by varying this vector parameter, we get a continuum of household consumption models between the fully cooperative and the fully non-cooperative model. Our purpose in the present paper is to further explore methodologically the testability of this comprehensive equilibrium concept.

In testing behavioural models, two general approaches have been used in the literature. One is to take a flexible parameterization of the demand system and to derive testable local properties, such as properties of the (pseudo-) Slutsky matrix. In the collective case, Browning and Chiappori (1998) show that the pseudo-Slutsky matrix can be written as the sum of a symmetric negative semidefinite matrix and a rank 1 'deviation' matrix, and can be used to discriminate the collective model from the (less general) unitary model. In the fully non-cooperative case, the rank of the deviation matrix generally increases (Lechene and Preston, 2011), and increases even more in the semi-cooperative case (d'Aspremont and Dos Santos Ferreira, 2014). Unfortunately, each kind of effects increases the requirements for empirical testing. A more general argument against the parametric tests approach is that the functional structure

[1] This threat point can very well be the realized outcome (*e.g.* with transaction costs). Another possibility is to assume that the threat point coincides with the utility obtained after divorce (Manser and Brown 1980, McElroy and Horney, 1981).

[2] Noncooperative models have also been studied by Ulph (1988) and Chen and Woolley (2001). For a survey, see Donni (2007).

[3] Income pooling is already present in unitary models (Samuelson, 1956, and Becker, 1974a, b) as well as divorce threat bargaining models (Manser and Brown, 1980, and McElroy and Horney, 1981)

[4] For references, see Browning and Chiappori, (1998) and Vermeulen (2002).

[5] d'Aspremont and Dos Santos Ferreira (2014)

used is not verifiable *per se* and the tested model may be rejected due to mispecification. For that reason, Cherchye, De Rock and Vermeulen (2007) have adopted another approach, a non-parametric one, to test the collective model. No functional specification is assumed and observed quantity and price data are *rationalised* by means of revealed preference axioms. The same non-parametric approach is used to test the fully non-cooperative model by Cherchye, Demuynck and De Rock (2011) and to test non-cooperative models with caring[6] by Cherchye et al. (2015).

Here we shall follow the same approach (using the same revealed preference axiom) for our comprehensive model and derive necessary and sufficient conditions for the θ-rationalisability of an observed data set. These conditions coincide with the necessary and sufficient conditions derived by Cherchye, Demuynck and De Rock (2011) for full cooperative rationalisability (when θ is identically nil) and they coincide with the ones they derive for full non-cooperative rationalisabilty (when θ is identically one). As a practically useful corollary, we can show as well that these conditions are verifiable by simple Mixed Integer Programming methods, combining linear constraints with binary integer variables.

An important property that we get is that each spouse income can be empirically identified on the basis of the observed data set if this set is θ-rationalised, except when $\theta_k^A = \theta_k^B = 1$, for both spouses A and B and for some public good k. In other words, the *sharing rule* is identifiable except in cases of local income pooling.[7]

Another property is nonnestedness. Cherchye, Demuynck, De Rock (2011) construct two data sets to show that one type of rationalisability (cooperative or non-cooperative) does not imply the other. We shall exhibit another data set which is θ-rationalisable, but not for these two limit cases. This does not exclude, as we will show, that the same data set can be θ-rationalised for a large set of θs, including the extremes.

In the next section, we start by recalling the model and the concept of household θ-equilibrium introduced in d'Aspremont and Dos Santos Ferreira (2014). Then, using the generalised axiom of revealed preference (GARP), we derive the necessary and sufficient conditions for θ-rationalisability of an observed data set and interpret these conditions in terms of marginal willingness to pay for the public goods within the household. The property of sharing rule identifiability is finally shown to hold for all θs except when identically one for some public

[6] Each individual is assumed to have a (personal) Bergson-Samuelson Social Welfare Function (SWF) through which she/he "cares" about the utility of each member in her/his household. This is in contrast with Samuelson consensus model (1956) where all members care and have the same SWF and with Becker (1974a, b) where only one member (say the husband) cares. By varying the degree of intrahousehold caring for each individual member, Cherchye et al. (2015) also obtain a continuum of household consumption models between the fully cooperative model and the fully noncooperative model without caring.

[7] Cherchye, Demuynck, De Rock (2015) already show that the sharing rule is identifiable for fully cooperative-rationalised data set and not for noncooperative-rationalised data set. Sharing rule identifiability also fails in models with caring, except when full cooperation is rationalised (Cherchye *et al.*, 2015).

good. Section 3 is devoted to empirical verifiability from a methodological point of view. First, we show that a simple mixed integer program can be used. Then, using this method, we construct examples to study non-nestedness. In section 4 we conclude.

2 Semi-cooperative household behaviour

In this section we define a comprehensive concept of household equilibrium which will encompass, as two limit cases, the concepts of cooperative and non-cooperative equilibrium already well-studied in the literature. This is achieved by introducing cooperation via a decentralised mechanism à la Lindahl. Our concept allows to cover a continuum of intermediate cases of semi-cooperative equilibria.

2.1 The model

Consider a two-adult household and denote by A the wife and by B the husband. Consumption within the household consists of goods that are either private or public, according to a contractual arrangement, supposed to be initially made by the spouses. Let $\left(q^A, q^B\right) \in \mathbb{R}_+^{2n}$ be the vector of consumption by the two spouses of the n private goods and $Q \in \mathbb{R}_+^m$ the consumption vector of the m public goods. The preferences of each spouse J ($J = A, B$) are represented by the utility function $U^J\left(q^J, Q\right)$, defined on $\mathbb{R}_+^{n+m}$, continuous, increasing and concave. The vector of private good prices $p \in \mathbb{R}_{++}^n$ and the vector of public good prices $P \in \mathbb{R}_{++}^m$ are given. The first private good, assumed to be always desired, is taken as numéraire ($p_1 = 1$). Although only the household income Y may be observable, each spouse J is supposed to know her/his income $Y^J \geq 0$, with $Y^A + Y^B = Y > 0$.

As in d'Aspremont and Dos Santos Ferreira (2014), we assume that the two spouses have agreed on some mechanism to share the financing of public consumption. For each public good k, each spouse, say the wife, makes two announcements: her desired household consumption Q_k^A and her voluntary contribution g_k^A. This means that she is ready to pay $P_k g_k^A$ for good k, for instance by buying it directly in the market place, leaving a fraction $\left(Q_k^A - g_k^A\right)/Q_k^A$ of the household expenditure of this good to her husband. Symmetrically, the husband will be ready to pay $P_k g_k^B$ for good k, leaving $\left(Q_k^B - g_k^B\right)/Q_k^B$ of the household expenditure of the same good to his wife. A first budgetary arrangement consists in allowing both spouses to contribute autonomously to public good k, by spending $P_k g_k^A$ and $P_k g_k^B$, respectively. If this stands for all public goods, we should end up with a *non-cooperative* equilibrium in the resulting *game with voluntary contributions to public goods*. An opposite budgetary arrangement consists in letting each spouse, say the husband, participate up to a non-manipulable fraction $\left(Q_k^A - g_k^A\right)/Q_k^A$ to a common fund allocated to public good k, so that he will have to pay a tax $\left(\left(Q_k^A - g_k^A\right)/Q_k^A\right) P_k Q_k^B$ to finance his desired household consumption Q_k^B of this good. If this stands for all public goods, we should end up with a *Lindahl equilibrium* resulting from this mechanism for financing public goods *cooperatively.*

Intermediate, semi-cooperative, schemes are however possible. We assume that the initial marriage agreement fixes, for each spouse J and each public good k, the proportions θ_k^J and $1-\theta_k^J$ (with $\theta_k^J \in [0,1]$) applying to each one of the two financing schemes, respectively. Given these agreed proportions, the announcements of the other spouse and the market prices, each spouse J is confronted with the following budget constraint:

$$pq^J + \sum_{k=1}^{m} \left(\theta_k^J P_k g_k^J + \left(1-\theta_k^J\right) \frac{Q_k^{-J} - g_k^{-J}}{Q_k^{-J}} P_k Q_k^J \right) \le Y^J. \tag{1}$$

On the left hand side of this inequality, the first term is the spouse's expenditure on private goods, and the second term is the sum of the spouse's expenditure on each public good k, decomposed into an autonomous spending $\theta_k^J P_k g_k^J$ and a coordinated spending *à la* Lindahl $\left(1-\theta_k^J\right)\left[\left(Q_k^{-J} - g_k^{-J}\right)/Q_k^{-J}\right] P_k Q_k^J$. This is the budget constraint that each spouse faces while maximizing her/his utility.

2.2 The household θ-equilibrium

A game is thus defined where the payoffs are the spouses' utility functions. The strategies of each spouse J are the quantities $\left(q^J, g^J, Q^J\right) \in \mathbb{R}_+^{n+2m}$, denoting respectively the quantities of private goods, the voluntary contributions and the desired household consumptions for the various public goods. For each spouse J, these strategies have to satisfy the budget constraint plus a feasibility constraint, whereby the desired quantities Q^J should be equal to the aggregate voluntary contributions $g^J + g^{-J}$. This leads to the following definition.

Definition 1 *A vector* $\left(q^A, g^A, Q^A, q^B, g^B, Q^B\right) \in \mathbb{R}_+^{2(n+2m)}$ *is a* household θ-equilibrium *with degrees of autonomy* $\left(\theta^A, \theta^B\right) \in [0,1]^{2m}$ *if, for* $J = A, B$, *the strategy* $\left(q^J, g^J, Q^J\right)$ *solves the program:*

$$\max_{\left(\underline{q}^J, \underline{g}^J, \underline{Q}^J\right) \in \mathbb{R}_+^{n+2m}} U^J\left(\underline{q}^J, \underline{Q}^J\right) \tag{2}$$

$$\begin{aligned} s.t. \quad & p\underline{q}^J + \sum_{k=1}^{m} \left(\theta_k^J P_k \underline{g}_k^J + \left(1-\theta_k^J\right) \frac{Q_k^{-J} - g_k^{-J}}{Q_k^{-J}} P_k \underline{Q}_k^J \right) \le Y^J, \\ and \quad & \underline{Q}^J = \underline{g}^J + g^{-J}. \end{aligned}$$

Existence of a household θ-equilibrium for every $\left(\theta^A, \theta^B\right) \in [0,1]^{2m}$ has been proved in d'Aspremont and Dos Santos Ferreira (2014, Proposition 1), when the utility functions of the two spouses are strongly quasi-concave. The corresponding equilibrium outcome coincides, if $\left(\theta^A, \theta^B\right) \equiv (0,0)$, with the

Lindahl equilibrium outcome and, if $\left(\theta^A, \theta^B\right) \equiv (1,1)$, with the outcome of a non-cooperative equilibrium of the game with voluntary contributions to public goods.[8]

3 Revealed preference analysis

Two approaches have been used to test for household behaviour. One is to assume sufficient differentiability of the demand system (a parameterized system for empirical applications) and to derive testable local properties, such as properties of the Slutsky matrix. This is the approach of Browning and Chiappori (1998) to discriminate the collective model from the (less general) unitary model, an approach extended by d'Aspremont and Dos Santos Ferreira (2014) for the present comprehensive model. Another approach, that we shall here adopt, is the revealed preference approach which consists in rationalising given data sets in terms of a particular model. Such rationalisation is based on global conditions and is non-parametric (see Cherchye, De Rock and Vermeulen, 2007, for the collective model, and Cherchye, Demuynck and De Rock, 2011, for the non-cooperative model).

3.1 θ-Rationalisability: A necessary and sufficient condition

For the present model, we use the following more general definition of rationalisability.

Definition 2 *A data set $(p_t, P_t, q_t, Q_t)_{t \in T}$ is* θ-rationalisable *(for given degrees of autonomy $\left(\theta^A, \theta^B\right) \in [0,1]^{2m}$) if there exist pairs of continuous, increasing and concave utility functions $\left(U^A, U^B\right)$ defined on $\mathbb{R}_+^{n+m}$, of individual incomes $\left(Y_t^A, Y_t^B\right)_{t \in T} \in \mathbb{R}_+^{2|T|}$, of individual private consumptions $\left(q_t^A, q_t^B\right)_{t \in T} \in \mathbb{R}_+^{2n|T|}$ and of voluntary contributions to public goods $\left(g_t^A, g_t^B\right)_{t \in T} \in \mathbb{R}_+^{2m|T|}$, such that, for any $t \in T$,*

$$Y_t^A + Y_t^B = p_t q_t + P_t Q_t,\ q_t^A + q_t^B = q_t,\ g_t^A + g_t^B = Q_t$$

and such that $\left(q_t^A, g_t^A, Q_t, q_t^B, g_t^B, Q_t\right)$ is a household θ-equilibrium.

It should be noticed that this approach, as applied to the household, involves an identification problem as long as the observed data set contains only aggregate information on the household. In addition any rationalisation is determined

[8] As stated in d'Aspremont and Dos Santos Ferreira (2014, Proposition 2), the outcome of any θ-equilibrium with *separate spheres* (*i.e.* with $g_k^A g_k^B = 0$ for any public good k) coincides, even for $\left(\theta^A, \theta^B\right) \neq (1,1)$, with the outcome of an equilibrium of the game with voluntary contributions to public goods.

only up to the permutation of the spouses' decisions $\left(q_t^A, g_t^A\right)$ and $\left(q_t^B, g_t^B\right)$.[9]

Varian (1982) has established the connection between rationalisability for the individual consumer model and a property called the Generalized Axiom of Revealed Preference (GARP). In the present model this can be defined as follows.

Definition 3 *A data set* $(p_t, \tau_t, q_t, Q_t)_{t\in T}$ *satisfies the* Generalized Axiom of Revealed Preferences (GARP) *if there exists a* transitive *binary relation* $\mathcal{R}$ *such that, for any* $s, t \in T$, $p_t q_t + \tau_t Q_t \geq p_t q_s + \tau_t Q_s$ *implies* $(q_t, Q_t)\,\mathcal{R}\,(q_s, Q_s)$ *and* $(q_t, Q_t)\,\mathcal{R}\,(q_s, Q_s)$ *implies* $p_s q_s + \tau_s Q_s \leq p_s q_t + \tau_s Q_t$.

We can now establish a necessary and sufficient condition for an observed data set to be θ-rationalisable.

Theorem 1 *Consider the observed data set* $\mathcal{D} = (p_t, P_t, q_t, Q_t)_{t\in T} \in \mathbb{R}_{++}^{2(n+m)|T|}$ *and take as given the vector pair* $\left(\theta_t^A, \theta_t^B\right) \in [0,1]^{2m}$ *of degrees of autonomy. The following conditions are equivalent:*

(i) The data set $\mathcal{D}$ *is* θ*-rationalisable.*

(ii) For any $t \in T$*, there exist vector pairs* $\left(q_t^A, q_t^B\right) \in \mathbb{R}_+^{2n}$*,* $\left(g_t^A, g_t^B\right) \in \mathbb{R}_+^{2m}$ *and* $\left(\tau_t^A, \tau_t^B\right) \in \mathbb{R}_+^{2m}$ *such that:*

$$q_t^A + q_t^B = q_t,\ g_t^A + g_t^B = Q_t; \tag{T1.1}$$

for $J = A, B$ *and* $k = 1, ..., m$*,*

$$\tau_{tk}^J \leq \theta_k^J P_{tk} + \left(1 - \theta_k^J\right)\left(g_{tk}^J / Q_{tk}\right) P_{tk},\ \textit{with equality if}\ g_{tk}^J > 0; \tag{T1.2}$$

and, for $J = A, B$*,*

$$\textit{the hypothesized data set}\ \mathcal{D}^J = \left(p_t, \tau_t^J, q_t^J, Q_t\right)_{t\in T}\ \textit{satisfies GARP.} \tag{T1.3}$$

Proof. *Sufficiency ((ii)$\Longrightarrow$(i)).* Take any $J \in \{A, B\}$. Since $\mathcal{D}^J$ satisfies GARP (T1.3), we know by Afriat's theorem (*cf.* Varian 1982) that there exist numbers $\left(U_t^J, \lambda_t^J\right)_{t\in T} \in (\mathbb{R} \times \mathbb{R}_{++})^{|T|}$, such that

$$U_s^J \leq U_t^J + \lambda_t^J \left[\left(p_t, \tau_t^J\right)\left(q_s^J - q_t^J, Q_s - Q_t\right)\right] \tag{3}$$

for any $s, t \in T$. Define J's utility function as

$$U^J\left(q^J, Q\right) \equiv \min_{t\in T} \left\{U_t^J + \lambda_t^J \left[\left(p_t, \tau_t^J\right)\left(q^J - q_t^J, Q - Q_t\right)\right]\right\}, \tag{4}$$

so that U^J is continuous, increasing, concave, and such that for any $t \in T$, $U_t^J = U^J\left(q_t^J, Q_t\right)$. We prove that, for any t, $\left(q_t^J, g_t^J\right)$ maximises $U^J\left(q_t^J, g_t^J + g_t^{-J}\right)$

[9] One possible way of tackling this problem would be to use the exclusivity assumption suggested by Chiappori and Ekeland (2009), "whereby each member is the exclusive consumer of at least one good."

under the budget constraint (1) with income $Y_t^J \equiv p_t q_t^J + \sum_{k=1}^m P_{tk} g_{tk}^J$. More precisely, $U^J\left(q_t^J, g_t^J + g_t^{-J}\right) = U_t^J \geq U^J\left(q^J, g^J + g_t^{-J}\right)$ for any $\left(q^J, g^J\right)$ satisfying:

$$p_t q^J + \sum_{k=1}^m \left(\theta_k^J P_{tk} g_k^J + \left(1-\theta_k^J\right) \frac{Q_{tk} - g_{tk}^{-J}}{Q_{tk}} P_{tk}\left(g_k^J + g_{tk}^{-J}\right)\right) \leq Y_t^J. \tag{5}$$

Adding $\sum_{\left\{k:\ g_{tk}^J > 0\right\}} \theta_k^J P_{tk} g_{tk}^{-J}$ to both sides of this inequality, and using (T1.1) and (T1.2), we obtain:

$$\begin{aligned} & p_t q^J + \sum_{\left\{k:\ g_{tk}^J>0\right\}} \left(\underbrace{\theta_k^J P_{tk} \overbrace{\left(g_k^J + g_{tk}^{-J}\right)}^{Q_k^J} + \left(1-\theta_k^J\right) \frac{Q_{tk} - g_{tk}^{-J}}{Q_{tk}} P_{tk} \overbrace{\left(g_k^J + g_{tk}^{-J}\right)}^{Q_k^J}}_{\tau_{tk}^J Q_k^J}\right) \\ & + \sum_{\left\{k:\ g_{tk}^J=0\right\}} \underbrace{\theta_k^J P_{tk} g_k^J}_{\tau_{tk}^J g_k^J} \\ \leq \quad & p_t q_t^J + \sum_{\left\{k:\ g_{tk}^J>0\right\}} \left(\underbrace{\theta_k^J P_{tk} g_{tk}^{-J} + \overbrace{\theta_k^J P_{tk} g_{tk}^J + \left(1-\theta_k^J\right) P_{tk} \frac{g_{tk}^J}{Q_{tk}} Q_{tk}}^{P_{tk} g_{tk}^J}}_{\tau_{tk}^J Q_{tk}}\right). \end{aligned} \tag{6}$$

By adding $\sum_{\left\{k:\ g_{tk}^J=0\right\}} \tau_{tk}^J g_{tk}^{-J}$ to both sides of this inequality and using (T1.1), we see that it implies:

$$\begin{aligned} & p_t q^J + \sum_{\left\{k:\ g_{tk}^J>0\right\}} \tau_{tk}^J Q_k^J + \sum_{\left\{k:\ g_{tk}^J=0\right\}} \tau_{tk}^J \overbrace{\left(g_k^J + g_{tk}^{-J}\right)}^{Q_k^J} \\ \leq \quad & p_t q_t^J + \sum_{\left\{k:\ g_{tk}^J>0\right\}} \tau_{tk}^J Q_{tk} + \sum_{\left\{k:\ g_{tk}^J=0\right\}} \tau_{tk}^J \overbrace{g_{tk}^{-J}}^{Q_{tk}}, \end{aligned} \tag{7}$$

an inequality which can be written as $\left(p_t, \tau_t^J\right)\left(q^J - q_t^J, Q^J - Q_t\right) \leq 0$, so that

$$\begin{aligned} U^J\left(q^J, Q^J\right) & \equiv \min_{s \in T}\left\{U_s^J + \lambda_s^J\left[\left(p_s, \tau_s^J\right)\left(q^J - q_s^J, Q^J - Q_s\right)\right]\right\} \\ & \leq U_t^J + \lambda_t^J\left[\left(p_t, \tau_t^J\right)\left(q^J - q_t^J, Q^J - Q_t\right)\right] \leq U_t^J. \end{aligned}$$

Deviating from $\left(q_t^J, g_t^J\right)$ can only decrease $U^J\left(q^J, Q^J\right)$. As this is true for $J = A, B$, we may conclude that $\left(q_t^A, g_t^A, Q_t, q_t^B, g_t^B, Q_t\right)$ is a household θ-equilibrium, and hence that the data set $\mathcal{D}$ is θ-rationalisable.

Necessity ((i)$\Longrightarrow$(ii)). Suppose $\mathcal{D}$ is θ-rationalisable. Clearly, condition (T1.1) is then fulfilled for the corresponding quantities q_t^J and g_t^J, $J \in \{A, B\}$

and $t \in T$. Take any $J \in \{A, B\}$. By the FOC of spouse J's program at a household equilibrium (Lemma 1 of d'Aspremont and Dos Santos Ferreira, 2014, extended to the case of non-differentiability), we have

$$\begin{aligned} \text{for } h &= 1, ..., n,\ U^J_{q^J_{th}}\left(q^J_t, Q_t\right) \leq \lambda^J_t p_{th}, \text{ with equality if } q^J_{th} > 0, \\ \text{for } k &= 1, ..., m,\ U^J_{Q_{tk}} \leq \lambda^J_t \left(\theta^J_k P_{tk} + \left(1 - \theta^J_k\right) \frac{g^J_{tk}}{Q_{tk}} P_{tk}\right), \text{ with equality if } g^J_{tk} > 0, \end{aligned}$$

for some subgradient $\left(U^J_{q^J_t}, U^J_{Q_t}\right) \in \partial U^J\left(q^J_t, Q_t\right)$ and some positive Lagrange multiplier λ^J_t. Take $\tau^J_{tk} \equiv U^J_{Q_{tk}}/\lambda^J_t \leq \theta^J_k P_{tk} + \left(1 - \theta^J_k\right)\left(g^J_{tk}/Q_{tk}\right) P_{tk}$, with equality if $g^J_{tk} > 0$, so that condition (T1.2) is satisfied. By concavity of the utility function U^J, for any $s, t \in T$,

$$U^J\left(q^J_s, Q_s\right) - U^J\left(q^J_t, Q_t\right) \leq \widetilde{U}^J_{q^J_t} \cdot \left(q^J_s - q^J_t\right) + \widetilde{U}^J_{Q_t} \cdot \left(Q_s - Q_t\right),$$

where $\left(\widetilde{U}^J_{q^J_t}, \widetilde{U}^J_{Q_t}\right) \in \partial U^J\left(q^J_t, Q_t\right)$ is *any* subgradient of U^J at $\left(q^J_t, Q_t\right)$. We thus obtain

$$\underbrace{U^J\left(q^J_s, Q_s\right)}_{U_s} \leq \underbrace{U^J\left(q^J_t, Q_t\right)}_{U_t} + \lambda^J_t \left[\left(p_t, \tau^J_t\right)\left(q^J_s - q^J_t, Q_s - Q_t\right)\right],$$

for any $s, t \in T$. By Afriat's theorem, we conclude that GARP applies to the hypothesized data set $\mathcal{D}^J = \left(p_t, \tau^J_t, q^J_t, Q_t\right)_{t \in T}$, so that condition (T1.3) is also satisfied, completing the proof. ■

3.2 Marginal willingness to pay for public goods

In the case of differentiability of the utility functions, the first order conditions for utility maximisation at a household θ-equilibrium for each spouse J and any public good k is given by:

$$\frac{\partial U^J\left(q^J, Q\right)/\partial Q_k}{\partial U^J\left(q^J, Q\right)/\partial q_1} \leq \theta^J_k P_k + \left(1 - \theta^J_k\right) \frac{g^J_k}{Q_k} P_k, \tag{8}$$

with equality if $g^J_k > 0$. Thus, the numbers τ^J_{tk} in condition (T1.2) of Proposition 1 can be interpreted as instances of J's marginal willingness to pay (MWTP) for public good k. By adding the MWTPs for the two spouses, we obtain:

$$\tau^A_{tk} + \tau^B_{tk} = \left(1 + \theta^A_k \frac{g^B_{tk}}{Q_{tk}} + \theta^B_k \frac{g^A_{tk}}{Q_{tk}}\right) P_{tk} \geq P_{tk}, \tag{9}$$

with equality if and only if $\theta^A_k g^B_{tk} = \theta^B_k g^A_{tk} = 0$. In particular, the equality is always satisfied in the cooperative case, where $\theta^A_k = \theta^B_k = 0$ for every k. The equality $\tau^A_{tk} + \tau^B_{tk} = P_{tk}$ is then an expression of the Bowen-Lindahl-Samuelson condition. Hence, θ-rationalisability coincides in this case with cooperative rationalisability, as defined by Cherchye *et al.* (2011), our Proposition 1 implying their Theorem 2.

If we now consider the non-cooperative case, where $\theta_k^A = \theta_k^B = 1$ for every k, we see that

$$\max\left\{\tau_{tk}^A, \tau_{tk}^B\right\} \leq P_{tk}, \tag{10}$$

the equality $\tau_{tk}^J = P_{tk}$ being satisfied for any k such that $g_{tk}^J > 0$. Thus, θ-rationalisability coincides in this case with non-cooperative rationalisability, as defined by Cherchye *et al.* (2011), our Proposition 1 implying their Theorem 3.

Putting together inequalities (9) and (10), we obtain, for any public good k:

$$\max\left\{\tau_{tk}^A, \tau_{tk}^B\right\} \leq P_{tk} \leq \tau_{tk}^A + \tau_{tk}^B, \tag{11}$$

a generalization of conditions (C.2) and (NC.2), for the cooperative and non-cooperative cases, respectively, in Cherchye *et al.* (2011).

3.3 Sharing rule identifiability

An important property, verified by the collective model (see Cherchye *et al.,* 2007), is that the *sharing rule* be identifiable, *i.e.* that each spouse income, Y_t^A and Y_t^B, be empirically identified on the basis of the observed data set, once rationalised (hence on the basis of the hypothesized data sets $\mathcal{D}^A$ and $\mathcal{D}^B$). This rationalisation is of course conditional on the supposed degrees of autonomy $\left(\theta^A, \theta^B\right)$, as it is usual when assuming full cooperation ($\theta^A = \theta^B \equiv 0$) or full non-cooperation ($\theta^A = \theta^B \equiv 1$).

Suppose first that, for some public good k, $\theta_k^J < 1$ for some J. Then condition (T1.2) for θ-rationalisability stated in Proposition 1, namely $\tau_{tk}^J = \theta_k^J P_{tk} + \left(1 - \theta_k^J\right)\left(g_{tk}^J / Q_{tk}\right) P_{tk}$ if $g_{tk}^J > 0$, allows to identify the voluntary contribution by each spouse to every public good k as either zero or

$$g_{tk}^J = \frac{\tau_{tk}^J / P_{tk} - \theta_k^J}{1 - \theta_k^J} Q_{tk} \tag{12}$$

for spouse J and $g_{tk}^{-J} = Q_{tk} - g_{tk}^J$ for the other spouse. Suppose now that, for some public good k, $\theta_k^A = \theta_k^B = 1$. If, say $g_{tk}^J = 0$ and $g_{tk}^{-J} = Q_{tk}$, the voluntary contributions of the two spouses are again identified.

If one of these two cases holds for every public good, the sharing rule can be identified. Spouse J's income ($J = A, B$) is simply $Y_t^J = p_t q_t^J + \sum_k P_{tk} g_{tk}^J$. It is only when, for some public good k, $\theta_k^A = \theta_k^B = 1$ and $g_{tk}^A g_{tk}^B > 0$ that the sharing rule is not identifiable, since the two equations (T1.2) can be trivially satisfied by any positive pair $\left(g_{tk}^A, g_{tk}^B\right)$ such that $g_{tk}^A + g_{tk}^B = Q_{tk}$. We then observe a situation of *local income pooling*. Such failure of the sharing rule identifiability, outside the case of *separate spheres* (when $g_{tk}^J = 0$ and $g_{tk}^{-J} = Q_{tk}$ for any k), was already emphasized by Cherchye *et al.* (2011) in the non-cooperative case. More generally, such failure still appears in non-cooperative regimes with caring, as introduced by Cherchye *et al.* (2015). By contrast, in our approach, the failure of identifiability of the sharing rule is confined to the case where both spouses contribute *fully autonomously* to some public good.

4 Empirical verifiability

We shall not analyze here a particular sample of two-person households as done, for instance, in Cherchye *et al.* (2011) for the non-cooperative regime and in Cherchye *et al.* (2015) for the non-cooperative model with caring, both using the Russia Longitudinal Monitoring Survey (RLMS). Still, from a methodological point of view, it is important to investigate the empirical applicability of Theorem 1. This we will do first in a general perspective, second by constructing illustrative examples.

4.1 A Mixed Integer Programming formulation

Two important problems have to be solved. The first is the computational verifiability of the conditions of Proposition 1, taking as given the degrees of autonomy θ. The second is the identification of these degrees of autonomy. To solve the first problem, we can refer to Cherchye *et al.* (2011), where the non-cooperative rationalisability of an observed data set can be verified by solving a Mixed Integer Programming (MIP) problem. Following this approach, we shall handle the GARP conditions by defining binary variables $x_{ts}^J \in \{0,1\}$ for any pair $(t,s) \in T^2$, where $x_{ts}^J = 1$ means that $\left(q_t^J, Q_t\right) \mathcal{R}^J \left(q_s^J, Q_s\right)$ for the hypothesized set $\mathcal{D}^J = \left(p_t, \tau_t^J, q_t^J, Q_t\right)_{t\in T}$ and the revealed preference relation $\mathcal{R}^J$, for $J = A, B$. By applying Definition 3, we have:

$$p_t q_t^J + \tau_t^J Q_t - p_t q_s^J - \tau_t^J Q_s \leq C x_{ts}^J - \varepsilon \tag{P1}$$

(where C and ε are positive constants, arbitrarily large and small, respectively), ensuring that $x_{ts}^J = 1$ whenever the LHS is non-negative. Also,

$$p_s q_s^J + \tau_s^J Q_s - p_s q_t^J - \tau_s^J Q_t \leq C\left(1 - x_{ts}^J\right), \tag{P2}$$

ensuring that the LHS is non-positive whenever $x_{ts}^J = 1$. Thirdly, in order to ensure transitivity of $\mathcal{R}^J$, we have in addition:

$$x_{tv}^J + x_{vs}^J \leq 1 + x_{ts}^J. \tag{P3}$$

It should be noticed that constraints (P1)-(P3) are equivalent to condition (T1.3) of Theorem 1, covering all possible regimes.

In this general case, we have to introduce as variables not only the non-negative quantities of private goods q_t^A and q_t^B (as in Cherchye *et al.*, 2011), but also the spouses' non-negative contributions to public goods g_t^A and g_t^B, both constrained by the equalities (T1.1). As to the variables τ_{tk}^J representing the marginal willingnesses to pay for the public goods, each one is constrained as follows:

$$0 \leq \tau_{tk}^J \leq P_{tk}. \tag{P4}$$

These variables are in addition linked by condition (T1.2) to the contributions g_{tk}^J. In order to integrate this condition in the MIP problem, we define binary variables $z_{tk}^J \in \{0,1\}$, such that $z_{tk}^J = 1$ if and only if $g_{tk}^J = 0$, and write:

$$-z_{tk}^J + \varepsilon \quad \leq \quad g_{tk}^J \leq C\left(1 - z_{tk}^J\right) \tag{P5a}$$

$$-Cz_{tk}^J \quad \leq \quad \tau_{tk}^J - \left(\theta_k^J + \left(1 - \theta_k^J\right)\frac{g_{tk}^J}{Q_{tk}}\right) P_{tk} \leq 0. \tag{P5b}$$

We recall that C and ε are positive constants, arbitrarily large and small, respectively.

To summarize, we thus obtain a MIP problem in unknowns q_t^J, g_t^J and τ_t^J, plus the binary variables x_{ts}^J and z_t^J, related to the observed data set $(p_t, P_t, q_t, Q_t)_{t\in T}$ through the inequalities (P1) to (P5), the equalities (T1.1), plus the non-negativity constraints on the quantity variables.

The second problem concerning the empirical application of Theorem 1 is the identification of the degrees of autonomy θ. The standard approach to this problem considers only the two extreme cases where the θ_k^J's are either all equal to 0 (the fully cooperative regime) or all equal to 1 (the non-cooperative regime), and then checks if rationalisability is ensured or not using the MIP problem. Presently, the most convenient way to take other values of the degrees of autonomy into account is probably to follow the suggestion of Cherchye *et al.* (2015), namely "to conduct a grid search that checks the above problem (through MIP methods) for a whole range of possible values" (p. 21).

5 Examples

Cherchye *et al.* (2011) emphasized the importance of the non-nestedness property exhibited by the cooperative and non-cooperative regimes in the revealed preference approach in contrast with the parametric approach: "a data set that satisfies the cooperative condition does not necessarily satisfy the non-cooperative condition, and *vice versa*" (p. 1084). The non-nestedness property "is not a theoretical curiosity but also has empirical relevance" (*ibid.*). Indeed, given a set of observations to be rationalised, we may then hope to be able to falsify the rationalisability in terms of all regimes outside the relevant one (or, in our context, outside some neighborhood of the relevant one).

The non-nestedness property is shown in Cherchye *et al.* (2011) by using two examples, the first exhibiting a data set that is cooperatively but not non-cooperatively rationalisable, the second the other way round. By combining these two examples, we can easily construct (example 1 below) a data set which is semi-cooperatively rationalisable, but neither cooperatively nor non-cooperatively rationalisable, thus extending the non-nestedness property to semi-cooperation.

A general problem with the empirical application of the revealed preference approach to household behaviour remains however. This is the multiplicity of potential ways of rationalising the same set of observed data. As already mentioned, since the data are restrained to household aggregates, this multiplicity results from the difficulty in identifying who is who in the couple, also from the occurrence of income pooling and more generally from other sources of non-uniqueness for given degrees of autonomy. In addition, varying the degrees of autonomy can only increase this indeterminacy.[10]

The objective of our second example is to illustrate this indeterminacy. Some observed data set can be θ-rationalised for a large set of degrees of autonomy, that may even include the two limit cases.

Example 1

We assume 7 observations, 7 public goods and no private goods. Observed prices and quantities are as follows (different lines corresponding to different observations, and different columns to different goods):

$$P = \begin{bmatrix} 7 & 4 & 4 & 1 & 1 & 1 & 1 \\ 4 & 7 & 4 & 1 & 1 & 1 & 1 \\ 4 & 4 & 7 & 1 & 1 & 1 & 1 \\ 7 & 7 & 7 & 1 & \varepsilon & \varepsilon & \varepsilon \\ 7 & 7 & 7 & \varepsilon & \varepsilon & \varepsilon & 1 \\ 7 & 7 & 7 & \varepsilon & 1 & 1 & \varepsilon \\ 7 & 7 & 7 & 1 & \varepsilon & 1 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & 100 & 11 & \varepsilon & 20 \\ \varepsilon & \varepsilon & \varepsilon & 20 & \varepsilon & 11 & 100 \\ \varepsilon & \varepsilon & \varepsilon & 5 & 10 & 10 & 5 \\ \varepsilon & \varepsilon & \varepsilon & 10 & 4 & 4 & 10 \end{bmatrix},$$

with ε positive but arbitrarily small. In each one of these two matrices the north-west 3x3 block corresponds to example 1 and the south-east 4x4 block to example 2 in Cherchye *et al.* (2011). In the complementary blocks, we have introduced the maximal prices and the minimal quantities in the given data of those two examples. The three first observations suffice to recover the authors' conclusion (concerning example 1) that non-cooperative rationalisation is impossible. Similarly, the last four observations suffice to recover their conclusion (concerning example 2) that cooperative rationalisation is impossible. In this example it seems natural to conjecture that the present data set is θ-rationalisable for the degrees of autonomy

$$\theta^A = \theta^B = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix},$$

that is for the two spouses behaving cooperatively with respect to the first three public goods, and non-cooperatively with respect to the last four. Using a MIP algorithm,[11] it is easy to verify this conjecture.

Example 2

[10] Of course, rationalisability of any kind may also fail, as it can be easily shown using example 1 in Cherchye *et al.* (2007).

[11] We have used Gusek software, which combines the SCIntilla based Text Editor (SciTE) plus the linear/integer programming solver GNU Linear Programming Kit (GLPK).

Take the case of 3 observations, 3 public goods and no private goods. We know that, for any observation $t \in \{s, v, w\}$ and for any public good $k \in \{1, 2, 3\}$, $\tau_{tk}^{J} Q_{tk} = \theta_k^J P_{tk} Q_{tk} + \left(1 - \theta_k^J\right) P_{tk} g_{tk}^J$. For simplicity, we assume that $\theta_k^J = \theta_k^{-J} = \theta_k$, for all k. Then, since $g_{tk}^J + g_{tk}^{-J} = Q_{tk}$, we get: $\tau_{tk}^J Q_{tk} + \tau_{tk}^{-J} Q_{tk} = (1 + \theta_k) P_{tk} Q_{tk}$, hence

$$\tau_{tk}^A + \tau_{tk}^B = (1 + \theta_k) P_{tk} \geq P_{tk}.$$

Let $\tau_{tk}^A = \gamma_{tk} P_{tk}$ and $\tau_{tk}^B = (1 + \theta_k - \gamma_{tk}) P_{tk}$, with $\theta_k \leq \gamma_{tk} \leq 1$. As $\gamma_{tk} \leq 1$ and $\theta_k \leq 1 + \theta_k - \gamma_{tk} \leq 1$, we have

$$\tau_{tk}^A = \gamma_{tk} P_{tk} \leq P_{tk} \text{ and } \tau_{tk}^B = (1 + \theta_k - \gamma_{tk}) P_{tk} \leq P_{tk}.$$

Thus, we satisfy the condition $\max\left\{\tau_{tk}^A, \tau_{tk}^B\right\} \leq P_{tk} \leq \tau_{tk}^A + \tau_{tk}^B$.

Also, using (12), we get:

$$g_{tk}^A = \frac{\gamma_{tk} - \theta_k}{1 - \theta_k} Q_{tk} \text{ and } g_{tk}^B = \frac{1 - \gamma_{tk}}{1 - \theta_k} Q_{tk}.$$

Now, assume an observed data set with the same cyclical structure as in Cherchye *et al.* (2011, Example 1):

$P_s = \begin{bmatrix} \alpha & \eta & \eta \end{bmatrix}$, $P_v = \begin{bmatrix} \eta & \delta & \eta \end{bmatrix}$, $P_w = \begin{bmatrix} \eta & \eta & \beta \end{bmatrix}$,

$Q_s = \begin{bmatrix} 1 \\ \varepsilon \\ \varepsilon \end{bmatrix}$, $Q_v = \begin{bmatrix} \varepsilon \\ 1 \\ \varepsilon \end{bmatrix}$, $Q_w = \begin{bmatrix} \varepsilon \\ \varepsilon \\ 1 \end{bmatrix}$,

where α, β, δ, η are all positive parameters and $\varepsilon \in (0, 1)$, not necessarily small.

In order to rationalise the observed data set, let us suppose WLOG that $Q_s \mathcal{R}\, Q_v \mathcal{R} Q_w$, implying by GARP, in the unitary model:

$$\begin{aligned} P_s (Q_s - Q_v) &\geq 0,\ P_v (Q_s - Q_v) \geq 0, \\ P_s (Q_s - Q_w) &\geq 0,\ P_w (Q_s - Q_w) \geq 0, \\ P_v (Q_v - Q_w) &\geq 0,\ P_w (Q_v - Q_w) \geq 0, \end{aligned}$$

that is,

$$\begin{aligned} (\alpha - \eta)(1 - \varepsilon) &\geq 0,\ (\eta - \delta)(1 - \varepsilon) \geq 0, \\ (\eta - \beta)(1 - \varepsilon) &\geq 0,\ (\delta - \eta)(1 - \varepsilon) \geq 0, \end{aligned}$$

and finally the conditions reduce to:

$$\beta \leq \eta = \delta \leq \alpha.$$

Hence, the unitary model is rejected as soon as these conditions are violated, as in the example of Cherchye *et al.* (2011), where $\eta = 4$ and $\beta = \delta = 7$.

So, let us assume that $Q_s \mathcal{R}^A Q_v \mathcal{R}^A Q_w$ and $Q_w \mathcal{R}^B Q_v \mathcal{R}^B Q_s$, implying by GARP:

$$
\begin{aligned}
\tau_s^A (Q_s - Q_v) &\geq 0,\ \tau_v^A (Q_s - Q_v) \geq 0,\ \tau_s^B (Q_s - Q_v) \leq 0,\ \tau_v^B (Q_s - Q_v) \leq 0,\\
\tau_s^A (Q_s - Q_w) &\geq 0,\ \tau_w^A (Q_s - Q_w) \geq 0,\ \tau_s^B (Q_s - Q_w) \leq 0,\ \tau_w^B (Q_s - Q_w) \leq 0,\\
\tau_v^A (Q_v - Q_w) &\geq 0,\ \tau_w^A (Q_v - Q_w) \geq 0,\ \tau_v^B (Q_v - Q_w) \leq 0,\ \tau_w^B (Q_v - Q_w) \leq 0.
\end{aligned}
$$

We thus get the following system of inequalities:

$$
\begin{aligned}
&\eta \max \{\gamma_{s2}, \gamma_{s3}\} \leq \alpha\gamma_{s1},\ \eta\gamma_{v3} \leq \delta\gamma_{v2} \leq \eta\gamma_{v1},\ \beta\gamma_{w3} \leq \eta \min \{\gamma_{w1}, \gamma_{w2}\},\\
&\alpha (1 + \theta_1 - \gamma_{s1}) \leq \eta \min \{1 + \theta_2 - \gamma_{s2}, 1 + \theta_3 - \gamma_{s3}\},\\
&\eta (1 + \theta_1 - \gamma_{v1}) \leq \delta (1 + \theta_2 - \gamma_{v2}) \leq \eta (1 + \theta_3 - \gamma_{v3}),\\
&\eta \max \{1 + \theta_1 - \gamma_{w1}, 1 + \theta_2 - \gamma_{w2}\} \leq \beta (1 + \theta_3 - \gamma_{w3}).
\end{aligned}
$$

Now, we take the values of the γ_{tk}'s which are the most favourable to verifying these inequalities, namely $\gamma_{s1} = 1$, $\gamma_{s2} = \theta_2$, $\gamma_{s3} = \theta_3$, $\gamma_{v1} = 1$, $\gamma_{v3} = \theta_3$, $\gamma_{w1} = \gamma_{w2} = 1$, $\gamma_{w3} = \theta_3$. With these values, the system of inequalities becomes:

$$
\begin{aligned}
\max \{\theta_2, \theta_3\} &\leq \alpha/\eta \leq 1/\theta_1,\\
\max \{\theta_1, \theta_2\} &\leq \beta/\eta \leq 1/\theta_3,\\
\frac{\theta_1 + \theta_3}{1 + \theta_2} &\leq \delta/\eta \leq \frac{2}{1 + \theta_2},\\
\max \{(1 + \theta_2) \delta/\eta - 1, \theta_3\} &\leq \gamma_{v2} (\delta/\eta) \leq \min \{(1 + \theta_2) \delta/\eta - \theta_1, 1\}.
\end{aligned}
$$

Let us observe that:

1. For θ-rationalisability, all degrees of autonomy are upper-bounded, possibly below one:

$$\theta_1 \leq \min \{\eta/\alpha, \beta/\eta, 2\delta/\eta\},\ \theta_2 \leq \min \{\alpha/\eta, \beta/\eta, 2\eta/\delta - 1\},\ \theta_3 \leq \min \{\alpha/\eta, \eta/\beta, 2\delta/\eta\}.$$

It is possible only if $\delta \leq 2\eta$.

2. The collective model ($\theta_1 = \theta_2 = \theta_3 = 0$) is clearly rationalisable whatever the positive parameters α, β, δ and η (under the constraint $\delta \leq 2\eta$).

3. Non-cooperative rationalisability ($\theta_1 = \theta_2 = \theta_3 = 1$) requires the very special condition that all prices are the same ($\alpha = \beta = \delta = \eta$),[12] a condition which is still stronger in this example than the condition allowing unitary rationalisability ($\beta \leq \eta = \delta \leq \alpha$).

4. Separate spheres for some observation t requires $\gamma_{tk} (1 + \theta_k - \gamma_{tk}) = 0$ for all k, which implies $\theta_k = 0$ for all k (since $\theta_k \leq \gamma_{tk} \leq 1$). It is only possible in the collective model.

For instance if we suppose $\eta < \min \{\alpha, \beta, \delta\}$, as in Cherchye *et al.* (2011), where $\eta = 4$ and $\alpha = \beta = \delta = 7$, θ-rationalisability prevails only for degrees of autonomy, uniform across the spouses, such that

$$(0, 0, 0) \leq (\theta_1, \theta_2, \theta_3) \leq (\eta/\alpha, 2\eta/\delta - 1, \min \{\eta/\beta, 2\delta/\eta\}),$$

hence $(\theta_1, \theta_2, \theta_3) \leq (4/7, 1/7, 4/7)$ in the example of Cherchye *et al.*

[12] In addition, non-cooperative rationalisability requires $\gamma_{v2} = 1$, hence (considering the values already imposed on the γ_{tk}'s) $\tau_t^A = \tau_t^B = P_t$ for any t.

6 Conclusion

Introducing a comprehensive concept of household equilibrium leads to a fundamental change of perspective in the empirical analysis of household behaviour. Instead of opposing two different kinds of household decision regimes, cooperative or non-cooperative, we have now a continuum of semi-cooperative regimes with the two former as limit cases. In terms of marginal willingness to pay for each public good, instead of focusing on the equality of the observed prices either to the maximum or to the sum of the MWTPs of the spouses, we should rather consider an interval between these two extremes. For empirical application, this new perspective is more demanding. As we have seen in example 2, given a set of observations rationalised for some regime (*i.e.* for some degree of autonomy θ), it should generally be rationalisable for other close regimes (in some neighbourhood of θ), and excluding other regimes (outside this neighbourhood) might not be easy.

In this new perspective, though, previous results are still valid. They can be embedded in more general statements. In particular this is verified here, from a theoretical point of view, for the theorem characterizing θ-rationalisability, and from an empirical point of view, for the applicability of the MIP algorithm.

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