Does an Optimal Voluntary Approach Flexibly Control Emissions from Heterogeneous Firms?*

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Abstract

We theoretically examine the potential of a voluntary policy (or quasi-regulation in other contexts than environmental issues) for emission pollution from a large number of heterogeneous firms under an asymmetric information case. Particularly, we focus on the performance and flexibility of the optimal voluntary policy as an alternative to inflexible mandatory regulations. Regardless of the type of heterogeneity, heterogeneous emission abatement technology level or emission size, the optimal voluntary policy performs better than the mandatory policy, and the optimal voluntary policy performs strictly better if there is non-zero probability that the regulator fails to introduce the mandatory policy. However, the perfectly flexible voluntary policy or different abatement (or emission) target for different types of firms is never optimal, and the inflexible voluntary policy, a uniform target for all types, is likely optimal if the majority of polluting firms has high abatement costs or a large emission size.

1 Introduction

Economists have advocated market-based approaches (MBAs) such as emission tax and tradable permits as an alternative to command and control approaches (CCAs) to address environmental problems. The CCAs were criticized because of their inflexibility in the sense that they cannot take into account the heterogeneity of abatement costs (and because of their inefficiency resulting from their inflexibility). In contrast with CCAs, MBAs can theoretically equalize marginal abatement costs between heterogeneous firms, or they can result in an efficient outcome. However, it is politically difficult to introduce efficient MBAs due to costs additional to abatement costs, such as tax payments or permit purchases. In fact, due to the political resistance of pollution-intensive

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industries that would have to bear a heavy tax burden, the introduction of carbon taxes has been time-consuming or postponed, or effective tax rates for such industries have been lower than those of other industries in many countries[1]. Moreover, under the administration of President Clinton, the carbon tax was withdrawn in the early 1990s, and no mandatory regulations (CCAs or MBAs) have been introduced in the United States at the federal level due to political resistance to climate legislation.

As alternatives to mandatory regulations for climate change mitigation, the US federal government has relied on a number of voluntary approaches (VAs), including the improvement of environmental performance beyond legal requirements such as quasi-legal policy instruments (soft law), co-regulation and self-regulation. The use of VAs has grown not only for climate change but also for other environmental problems in developed countries (Lyon and Maxwell, 2007)[2]. In addition to developed countries, some developing countries have experienced in adopting VAs (Segerson, 2013). The popularity of VAs stems mainly from their political acceptance. On the one hand, most polluting firms perceive that VAs are more flexible than CCAs and are less costly than MBAs. Therefore, these firms are likely to accept a voluntary approach when an environmental policy is introduced. On the other hand, governments are afraid to fail in the introduction of environmental policy, and as a result, they offer voluntary approaches to polluting firms due to their political acceptance.

Moreover, VAs can be implemented quickly relative to mandatory policies (in particular, MBAs) because no time-consuming legislation process is necessary to implement VAs. Such a property enables VAs to be implemented not only for environmental issues but also for other policy issues that need to be addressed as quickly and flexibly as possible. In fact, many Japanese companies voluntarily set their own targets for electrical load reduction with actual numerical values and have taken actions to achieve these targets during the summer of 2011, when load reduction in Tohoku, Kanto and Kansai areas was necessary due to the Tohoku Earthquake and the following nuclear accidents. Thanks to these voluntary reduction actions, the Japanese government did not have to rely on rolling blackouts to these areas in which the actions were implemented just after the Earthquake (from March 14-28, 2011). Thus, VAs have the potential to control demand in response to temporary supply shock as well as environmental issues.

In the above case of Japan's voluntary electrical load reduction, the experience of rolling blackouts just after the Tohoku Earthquake made a strong impression on many firms due to regulatory threats. As a result, flexibility in voluntary load reduction might have worked to mitigate the idea of rolling blackouts or mandatory policies. However, if the regulatory threat is weak, then firms targeted by VAs exploit the flexibility of VAs and reduce their demand by very little. Thus, the flexibility of VAs can be both advantageous and disadvantageous. In addition, the structure of costs

[1] For example, it took seven years to decide to introduce a carbon tax in Japan. The Ministry of Environment first proposed it in 2004. In many European countries, effective environmental tax rates for pollution-intensive industries were lower than in other industries. Please see Ekins and Speck (1999) for more detail on environmental taxes in European countries.

[2] There have been approximately 300 voluntary environmental agreements between firms and national governments in Europe and more than 87 voluntary environmental programs sponsored by the US Environmental Protection Agency.

due to reduction in demand might also affect whether the flexibility of VAs is their merit.

This paper examines conditions in which the flexibility of voluntary approach becomes a merit when a regulator has to implement a policy quickly to control demand by considering an emissions abatement policy. Taking into account that it is time consuming to gain the political acceptance of MBAs from polluting industries, the regulator is assumed to have two types of policy options, an inflexible (uniform) mandatory abatement target policy (CCA) and a voluntary abatement target policy (VA) that can be flexible (or different between firms). On the one hand, the mandatory policy might not be introduced due to political difficulties in the legislative process. On the other hand, the voluntary policy is "voluntary" in the sense that firms have a choice to reject the policy if they prefer the mandatory policy to the voluntary policy. Moreover, firms have the potential to abuse the flexibility of the voluntary policy in our model because abatement costs depend on each firm's characteristics, which are private information. To prevent firms from abusing the voluntary policy's flexibility and implement a socially desirable voluntary policy, the regulator allows the firms to verify their abatement cost. In this setting, we examine the flexibility of the socially desirable voluntary policy and its performance relative to the mandatory policy.

We find that regardless of the type of heterogeneity, the optimal voluntary policy is strictly better than mandatory policy if the regulator can fail to introduce the mandatory policy. We also find that the optimal voluntary policy is not perfectly flexible regardless of the type of heterogeneity. However, the optimal VA is likely flexible if many low abatement cost firms are majority and high abatement cost firms are not minority of minority (under heterogeneous technology case) or if medium or high abatement cost firms are minority (under heterogeneous emission size case). Thus, the optimal VP is more likely to be flexible under heterogeneous emission size case than it is under heterogeneous technology case.

Although many researchers have studied voluntary approaches to environmental protection due to an increase in the popularity of voluntary approaches, we are unaware of any theoretical study that addresses the potential and flexibility of voluntary policies targeted at a large number of firms under an asymmetric information case. Most researchers have focused on the case of a single polluter (e.g., Segerson and Miceli, 1998; Hansen, 1999; Segerson and Miceli, 1999; Glachant, 2007; and Fleckinger and Glachant, 2011) or multiple polluters without informational asymmetry (Lutz et al., 2000; Maxwell et al., 2000; Manzini and Mariotti, 2003; Dawson and Segerson, 2008; and Brau and Carraro, 2011). Only Lyon and Maxwell (2003) examined a voluntary program for multiple polluters with private information on their technology. However, the voluntary program in Lyon and Maxwell's model is assumed to provide all participating firms with a uniform subsidy. Thus, Lyon and Maxwell (2003) did not examine how voluntary approaches can flexibly address the emissions abatement of a large number of firms under an asymmetric information case.

The remainder of this paper is organized as follows. In the next section, we describe the model environment. Section 3 presents the main results. Section 4 examines the voluntary policy in different settings. Section 5 concludes this paper.

2 Setup

We consider a two-stage policy game played by a regulator and heterogeneous firms in an industry to examine voluntary policies' potential for emissions abatement of heterogeneous firms with private information. To achieve this aim, we consider a case where the benevolent regulator has strong negotiation power under the voluntary policy making process. Specifically, we focus on a voluntary agreement between the regulator and the industry (a group of the firms) in a case where the regulator makes a take-it-or-leave-it offer to the industry under the voluntary policy making process.

In the first stage, the regulator makes the take-it-or-leave-it offer of voluntary agreement to the industry and then, the industry decides whether to accept the offer. The industry accepts the offer when all firms prefer the voluntary policy to the mandatory policy. If the industry accepts the offer, the firms abate their emissions by following the agreement. If the industry rejects the offer, then, in the second stage, the regulator tries to introduce a mandatory policy for the industry with a probability of failure of $1-p$ $(0<p\leq 1)$. By introducing a possibility that the regulator fails to introduce the mandatory policy, we incorporate political difficulties in the mandatory policy-making process[3] as simply as possible to focus on firm heterogeneity. Thus, due to the risk of having no mandatory regulation, the regulator has an incentive to introduce the voluntary policy or to make the offer of the voluntary policy to the industry. Once the mandatory standard is introduced, all firms must follow it. The timing of the game is summarized as follows:

1. (Voluntary policy (VP)) The regulator makes the take-it-or-leave-it offer to the industry. If the industry accepts the offer, then firms abate their emissions by following the agreement.

2. (Mandatory policy (MP)) If the industry refuses the offer, then the regulator tries to introduce a mandatory policy for the industry. The mandatory policy is adopted with probability p and all firms comply. Otherwise, firms do not abate their emissions at all.

Firm i emits a pollutant at e_i before the introduction of environmental policy (e_i can be interpreted as a natural emission level or emission size) and it's emission abatement cost is given by $\frac{1}{2}c_i a_i^2$, where a_i and c_i are the actual emission abatement level and the slope of marginal abatement cost (MAC) of firm i. Although we examine the heterogeneity of both c_i and e_i, we first focus on the heterogeneity of abatement technology level (the slope of MAC), c, and let $e_i = e$ for all i (we consider the heterogeneity of emission size later). We assume that c is distributed over $[\underline{c}, \bar{c}]$ $(\bar{c} > \underline{c} \geq 0)$ with continuously differentiable probability density $f(c)$[4] and that the regulator knows $f(\cdot)$ but does not know the abatement technology level of any given firm. On the other hand, environmental damage due to firms' emissions is assumed to be $\delta \int_{\underline{c}}^{\bar{c}} (e - a(c)) f(c) dc = \delta e - \delta \int_{\underline{c}}^{\bar{c}} a(c) f(c) dc$.

[3]Fleckinger and Glachant (2011) pointed out that the "assumption that the adoption of legislation is subject to uncertainty is both realistic and common in papers dealing with voluntary abatement" (p.43). The simplest way to incorporate the uncertainty of adopting the legislation into a model is to assume that p is purely exogenous like Segerson and Miceli (1998). We follow Segerson and Miceli's method. See p.43 of Fleckinger and Glachant (2011) for other ways to incorporate the uncertainty of adopting the legislation in the literature.

[4]We normalize the number of firms to unity.

The regulator's objective is to minimize social cost, the sum of damage and aggregate emissions abatement cost given by

$$\delta e - \delta \int_{\underline{c}}^{\bar{c}} a(c) f(c) dc + \int_{\underline{c}}^{\bar{c}} \frac{1}{2} c [a(c)]^2 f(c) dc. \tag{1}$$

We assume that the mandatory policy forces any firm to abate the same amount, a_{MP}, irrespective of the abatement technology level. This assumption seems reasonable because the traditional "command-and-control" approach has been criticized as being inflexible and cost-ineffective whereas the voluntary approach is considered to be flexible. It is also possible to consider other strands of mandatory policies that rely on economic incentive such as emission tax or an emission trading scheme rather than the traditional "command-and-control" approach. Although the economic incentive approach is cost-effective in contrast with the "command-and-control" approach, the introduction of the "effective" economic incentive approach has been much more politically fraught and difficult than that of the "command-and-control" approach due to opposition from manufacturing sectors and industry associations. This is because the economic incentive approach imposes tax payments or permit purchases as well as emissions abatement cost on polluting industries or firms. As a result of the opposition to the economic incentive approach, the feasible mandatory policy instruments generating the highest social welfare might be a uniform emission standard. Or, like Japanese rolling blackouts that we explained in the introduction, the regulator cannot help adopting inflexible policy as the mandatory policy to quickly address an emission pollution problem because it's too hard and time consuming to enact a bill of complex policy. We consider such cases.

The mandatory inflexible standard (MP) is

$$\begin{aligned} a_{MP} &= \arg\min_{0 \le a_{MP} \le e} \; p\left\{ \delta(e - a_{MP}) + \int_{\underline{c}}^{\bar{c}} \frac{1}{2} c a_{MP}^2 f(c) dc \right\} + (1-p)\delta e \\ &= \arg\min_{0 \le a_{MP} \le e} \; \frac{1}{2} E[c] a_{MP}^2 - \delta a_{MP} \\ &= \frac{\delta}{E[c]} \end{aligned}$$

where $E[c] = \int_{\underline{c}}^{\bar{c}} c f(c) dc$. Thus, the mandatory policy maximizes (minimizes) social benefit from emissions abatement by firms with average technology level (social cost due to emissions by firms with average technology level).

Unlike the mandatory policy, abatement levels of individual firms can be different under voluntary policy. For example, under the voluntary policy, it is possible that firms with low abatement technology level abate less than those with high abatement technology level do, whereas both kinds of firms abate the same amount, a_{MP}, under the mandatory policy. To set abatement level depending on abatement technology level, the regulator has to differentiate firms' abatement technology level. The regulator could do so by using monetary transfer between the regulator (government) and firms that depends on abatement technology level reported by firms (type in a standard screening model). However, because such a monetary transfer is a kind of emission tax or subsidy for polluting firms, the introduction of the monetary transfer is also too hard in a case we consider here.

In reality, to verify or explain their activity related to environmental issues, firms adopt costly certificated environmental management systems (EMSs) such as ISO14001, make efforts to issue detailed environmental reports (with a third party certification) or do both, for instance. In other contexts than environmental issues, listed firms in many countries have to explain publicly why they do not comply a corporate governance code which is an example of quasi-regulation. Non-complying firms have to make enough effort that their public explanation is convincing to stakeholders (particularly, to their investors). Thus, firms make costly verification and/or make efforts to convince stakeholders in voluntary approaches or quasi-regulations. In this paper, the cost resulting from efforts to make convincing explanation as well as costs due to verification is considered as verification cost. Concretely, let verification cost of firm with technology level c be $v(c)(\geq 0)$. The verification *per se* is assumed to generate no social benefit or to just waste resources.

At the stage of voluntary policy negotiation, the regulator offers the industry a voluntary abatement rule a that depends on verification cost ($a(c) = a(v(c))$) and is semi-continuous. Then, the total cost of firm c that convinces the regulator that its abatement technology level is c' is $\frac{1}{2}c(a(c'))^2 + v(c')$ and social cost under the voluntary policy is

$$\int_{\underline{c}}^{\bar{c}} \left[\delta(e - a(c')) + \frac{1}{2}c(a(c'))^2 + v(c')\right] f(c)dc.$$

By the Revelation Principle, we can focus on a direct mechanism to analyze what happens in equilibrium. Therefore, we can write the regulator's problem under the voluntary policy as

$$\min_{a(c),\ s(c)} \int_{\underline{c}}^{\bar{c}} \left[\delta(e - a(c)) + \frac{1}{2}c[a(c)]^2 + v(c)\right] f(c)dc \tag{2}$$

subject to

$$\text{(IC)} \quad c = \arg\min_{c'}[\frac{1}{2}c(a(c'))^2 + v(c')] \quad \forall c \tag{3}$$

$$\text{(PC)} \quad \frac{1}{2}c(a(c))^2 + v(c) \leq p\frac{1}{2}c(a_{MP})^2 \quad \forall c \tag{4}$$

where IC is the incentive compatibility condition and PC is the participation condition. The participation condition is that firms' cost under the voluntary policy is smaller than their expected cost under the mandatory policy (taking into account the possibility that the regulator fails to introduce the mandatory policy). Otherwise, the industry rejects the voluntary policy rule proposed by the regulator.

Because $\frac{1}{2}c[a(c)]^2$ satisfies the single-crossing property, the IC condition can be rewritten as $a'(c) \leq 0$ (monotonicity) and $\frac{1}{2}c(a(c))^2 + v(c) = \frac{1}{2}[\underline{c}a(\underline{c})^2 + \int_{\underline{c}}^{c}[a(\tilde{c})]^2 d\tilde{c}]$ (LIC[5]) like the standard

[5]Local incentive compatibility.

screening model. In addition, from monotonicity and LIC, we have

$$\begin{aligned}\frac{1}{2}c(a(c))^2 + v(c) &= \frac{1}{2}[\underline{c}[a(\underline{c})]^2 + \int_{\underline{c}}^{c}[a(\tilde{c})]^2 d\tilde{c}] \\ &\leq \frac{1}{2}[\underline{c}[a(\underline{c})]^2 + [a(\underline{c})]^2\int_{\underline{c}}^{c} 1 d\tilde{c}] = \frac{1}{2}c[a(\underline{c})]^2.\end{aligned}$$

As a result, we hereinafter use a condition $a(\underline{c}) \leq \sqrt{p}a_{MP}$ (PC′) instead of using PC condition. Finally, using LIC, we can rewrite the regulator's objective function as

$$\delta e + \frac{1}{2}\underline{c}[a(\underline{c})]^2 + \int_{\underline{c}}^{\bar{c}} \left\{\frac{1}{2}[a(c)]^2[1-F(c)] - \delta a(c) f(c)\right\} dc.$$

Therefore, the regulator's problem under the voluntary policy rewrite as follows;

$$\min_{a(c)} \ \delta e + \frac{1}{2}[\underline{c}a(\underline{c})^2] + \int_{\underline{c}}^{\bar{c}} \left\{\frac{1}{2}[a(c)]^2[1-F(c)] - \delta a(c) f(c)\right\} dc \tag{5}$$

subject to

$$\text{(Monotonicity)} \quad a'(c) \leq 0 \quad \forall c \in [\underline{c}, \bar{c}] \tag{6}$$

$$\text{(PC}'\text{)} \quad a(\underline{c}) \leq \sqrt{p}a_{MP} \tag{7}$$

3 Results

Before we show and discuss our results, we make the following definitions related to properties of VP:

Definition 1. Let a VP that $a(c) = \sqrt{p}a_{MP}$ and $s(c) = 0$ for all c be inflexible VP.

Definition 2. VP is flexible if the inflexible VP is not optimal. Particularly, VP is perfectly flexible if $a(c) = \delta\frac{f(c)}{1-F(c)}$ for all c or if there exists $a(c)$ that is strictly monotonic decreasing.

From property of MP, a_{MP} is the best among uniform abatement targets. However, it is impossible to introduce a_{MP} as VP due to the PC condition (if $p < 1$). Our "inflexible VP" is the nearest uniform abatement target to a_{MP} and the best uniform abatement target among uniform abatement targets satisfying the PC condition. If the inflexible VP is the best among abatement policies satisfying PC and IC conditions, then flexibility of VP is not its advantage relative to the MP. However, if the inflexible VP is not optimal, then VP's flexibility can be its advantage. We define such a case as "VP is flexible".

From the F.O.C for (5), we can easily see that $a(c) = \delta\frac{f(c)}{1-F(c)}$ for all c is the optimal voluntary policy for the regulator's problem (5) without constraints (6) and (7). In the standard screening model, there exist distribution of abatement technology level, $f(c)$, such that $a(c) = \delta\frac{f(c)}{1-F(c)}$ for all c that is the optimal voluntary policy for the regulator's problem (5) with constraints (6) and (7) and the VP is perfectly flexible. However, the following proposition states that VP cannot be perfectly flexible.

Proposition 1. $\frac{f(c)}{1-F(c)}$ *must be strictly increasing for some c or the optimal VP cannot be perfectly flexible.*

Proof. Suppose $d[f(c)/(1-F(c)]/dc \leq 0$ for all c. Define $h(c) = -\log(1-F(c))$ and $g(c) = h(c) - (c - \underline{c})h'(\underline{c})$. Then, $h''(c) \leq 0$ because $h'(c) = f(c)/(1-F(c))$. In addition, $g(\underline{c}) = 0$, $g'(\underline{c}) = 0$ and $g''(c) \leq 0$. Therefore, $g(c) \leq 0$. However, $g(\bar{c}) = h(\bar{c}) - (\bar{c} - \underline{c})h'(\underline{c}) = +\infty$ because $F(\bar{c}) = 1$. This is contradiction. Therefore, $d[f(c)/(1-F(c)]/dc > 0$ for some c and the optimal VP cannot be perfectly flexible. □

Note that we do not take into account the PC condition to show that the optimal VP cannot be perfectly flexible. We just show monotonicity does not hold for some abatement technology level if $a(c) = \delta\frac{f(c)}{1-F(c)}$ for all c. Thus, the optimal abatement levels of some firms with different technology levels must be the same even though we ignore the PC condition. Even for some interval with $\frac{f(c)}{1-F(c)} \geq 0$ (let it be $[c_1, c_2]$), the optimal abatement levels might be the same due to the PC condition (for example, $a(c) = \sqrt{p}a_{MP}$ for $c \in [c_1, c_2]$ if $\delta\frac{f(c)}{1-F(c)} > \sqrt{p}a_{MP}$). Unlike the standard screening model, technology levels for which $a(c) = \delta\frac{f(c)}{1-F(c)}$ is the optimal abatement might be few. Therefore, we focus on whether the VP is flexible or not and social welfare under the VP relative to the MP. Before we analyze social welfare under VP, we define properties of VP related to social welfare.

Definition 3. VP is risk-eliminating if the inflexible VP generates higher social welfare than the MP (a_{MP}) regardless of firms' distribution $f(\cdot)$ whenever $p < 1$.

In our model, introduction of the MP might fail (with probability $1-p$) due to political resistance to the MP but any VPs satisfying the PC condition can be implemented for sure. Thus, VP eliminates risk of no emissions abatement in a case when the regulator would try to introduce the MP. As a result of eliminating such risk, the VP may generate higher social welfare than the MP even though the VP is inflexible as the MP is. We define such property of VP as "risk-eliminating". If the inflexible VP generares lower social welfare than the MP does in some cases (VP is not risk-eliminating) but the optimal VP always generates higher social welfare than the MP does, then flexibility is considered as an advantage of VP relative to the MP.

By using these definitions of properties, we characterize property of VP in the following proposition.

Proposition 2. *VP is risk-eliminating but is not always flexible. VP is flexible if and only if*

$$E[c|c \geq k] > c_{MP} + k \quad \exists k \in [\underline{c}, \bar{c}]. \tag{8}$$

where $E[c|c \geq k] = \int_k^{\bar{c}} cf(c)dc/[1 - F(k)]$ *and* $c_{MP} = E[c]/\sqrt{p} = \delta/(\sqrt{p}a_{MP})$.

Proof. See Appendix A.1. □

Corollary 1. *VP is not flexible if* $f(\cdot)$ *is weakly monotonically increasing.*

Proof. See Appendix A.2. □

From (8), Proposition 2 claims that VP is flexible if, for any technology inefficiency level (k) as above mentioned, the expected value of technology inefficiency level which is higher than k is lower than $E[c]/\sqrt{p} + k$. Inequality (8) evaluates the impact of a slight decrease in $a(k)$ on social cost for $k \in [\underline{c}, \bar{c}]$ when the inflexible VP is adopted. Reduction in emissions abatement target decreases abatement costs but increases environmental damage and verification costs. In addition, if the regulator reduces $a(k)$ by ϵ, due to IC condition or $a'(c) \leq 0$ for all c, it must also reduce the emission abatement of firms with a higher technology inefficiency level than k ($a(c)$ for all $c > k$) by ϵ. Therefore, if the regulator reduces a at k by ϵ, decrease in abatement costs of firms is equal to $\epsilon \int_k^{\bar{c}} c\sqrt{p}a_{MP} f(c)dc$, while increase in environmental damage is $\delta[1 - F(k)]\epsilon$ and increase in verification costs $k\sqrt{p}a_{MP}[1 - F(k)]\epsilon$ from $v'(c) = -ca(c)a'(c)$. By dividing the decrease in abatement costs and the increase in environmental damage and verification costs by $\sqrt{p}a_{MP}[1 - F(k)]\epsilon$, we have $E[c|c \geq k]$ (LHS of (8)), c_{MP}(the first term of RHS) and k (the second term of RHS). Thus, (8) is equivalent that the marginal social benefit from reducing emissions abatement at k from $\sqrt{p}a_{MP}$ to $\sqrt{p}a_{MP} - \epsilon$ (LHS) is greater than the marginal social costs from doing so (RHS). Proposition 2 implies that the regulator should decrease emissions abatement of firms whose abatement technology level is k (and greater than k due to IC the condition) if the marginal benefit from such a decrease (decrease in abatement costs) is smaller than the marginal cost (increase in environmental damage and verification costs).

If c_{MP} is large, (8) is not likely to hold. When is c_{MP} large? From the proof of Corollary 1, we can see that c_{MP} is large or (8) does not hold if $f(\cdot)$ is weakly monotonically increasing. Moreover, from this fact, we can guess that c_{MP} is large or (8) is not likely to hold if the distribution of firms is biased toward a high inefficiency level (high c). This is intuitive because the main target of the mandatory standard consists of firms with a high inefficiency level in such a case. The probability that the mandatory policy is adopted, p, also affects c_{MP}. c_{MP} is large or (8) is unlikely to hold due to the PC condition if p is small or the introduction of the mandatory standard is likely to fail. Finally, (8) does not hold if the difference in technology inefficiency level is small or in particular, $\underline{c}$ is greater than $\bar{c}/2$. This indicates that the efficiency loss of the inflexible VP is small relative to verification cost if the difference in technology inefficiency level is small.

If c_{MP} is small, (8) is likely to hold or VP is likely to flexible. c_{MP} is small if the distribution of firms is biased toward a low inefficiency level (low c), the mandatory policy is very likely to be adopted, and the difference in technology inefficiency level is large. Figure 1 gives a numerical example where VP is flexible. The firms' distribution (PDF) of figure 1 is biased toward a low inefficiency level, and therefore, c_{MP} is not large ($c_{MP} = 2.254$) if the mandatory policy is very likely to be adopted ($p = 0.9$). Because some firms have very inefficient technology (very high c), for lower-middle k, the difference in k and $E[c|c > k]$ is large. Therefore, there exists k such that $c_{MP} + k < E[c|c > k]$ or VP is flexible. Actually, around 1.6 where $E[c|c \geq k] = 4.1 > c_{MP}(= 2.254) + k(= 1.6)$, abatement level sharply decreases and after the sharp decrease in abatement level, abatement level changes little until around 5 that is the upper limit of c.

We give another example of firms' distribution where VP is flexible.

$$f(c) = \begin{cases} \frac{7}{4} & (1 \leq c < 1.5) \\ \frac{1}{28} & (1.5 \leq c \leq 5) \end{cases}$$

In this case, $E[c \mid c \geq 1.5] = 3.25$ and $E[c] = 1.5$. Therefore, $c_{MP} + 1.5 < E[c|c > 1.5]$ holds if p is very close to 1. From this example, we can guess that VP can be flexible if firms with efficient technology are majority and firms with inefficient technology are not a minority of minority. However, if firms with inefficient technology are not a minority of minority, $E[c|c \geq k]$ is very likely not to be large and as a result, VP is not flexible.

Finally, we have to mention that VP is risk-eliminating. PC condition determines whether VP is risk-eliminating. More specifically, it is crucial that the inflexible VP satisfies PC condition with equality for all firms or expected abatement costs of all firms under the inflexible VP are the same as those under the MP. In the next section, we characterize property of VP under different settings to examine what makes VP risk-eliminating or flexible.

4 Different settings

4.1 Progressive environmental damage case

Previously, environmental damage was considered to be linear: that is, environmental damage due to one firm's emission has no relation to that due to other firms' emission. This assumption is fairly plausible, but we can consider another case, i.e. environmental damage due to one firm's emission is positively related to that due to other firms' emission. Formally, such environmental damage can be written as $\frac{\delta}{2}(e - \int a(c)f(c)dc)^2$. In this case, the mandatory standard is given by $a'_{MP} = \frac{\delta e}{\delta + E[c]}$ and the inflexible VP is given by $\sqrt{p}a'_{MP}$ for all c (because both IC and PC do not change at all). Then, we have the following proposition.

Proposition 3. *VP is risk-eliminating but is not always flexible. VP is flexible iff*

$$E[c|c \geq k] > c'_{MP} + k \ \exists k \in [\underline{c}, \bar{c}]$$

where $c'_{MP} = [E[c] + \delta(1 - \sqrt{p})]/\sqrt{p} = \delta e/(\sqrt{p}a'_{MP}) - \delta$.

Proof. See Appendix A.3. □

Note that if $p = 1$, this necessary and sufficient condition coincides with that in linear environmental damage case because $c_{MP} = c'_{MP}$ when $p = 1$ (in the linear case, $E[c|c \geq k] \leq c_{MP} + k \ \forall k \in$

$[\underline{c}, \bar{c}]$ where $c_{MP} = E[c]/\sqrt{p})$. This coincidence is quite natural because due to property of the mandatory policy, marginal abatement cost of firm with average abatement technology $E[c]$ is equal to marginal damage and its value is given by $E[c]a_{MP}$ (in the linear case and $E[c]a'_{MP}$ in the progressive case) if all firms abate their emissions by a_{MP} (a'_{MP} in the progressive case). Of course, c_{MP} and c'_{MP} are different by $\delta(1 - \sqrt{p})/\sqrt{p}$ if $p < 1$. This difference is the impact of change in emission abatement by some firm on marginal damage of other firms' emissions under the progressive environmental damage. However, like the linear case, the LHS is decrease in abatement costs and the RHS is the sum of increase in environmental damage and verification cost if we multiply the both sides by the amount of abatement ($\sqrt{p}a'_{MP}$) and the number of firms whose technology level is lower than k ($\int_k^{\bar{c}} f(c)dc$).

We also easily show that if $f(\cdot)$ is weakly monotonically increasing, inflexible VP is optimal. This result is also the same as linear damage case. Thus, function form of environmental damage does not affect the condition for the optimality of inflexible VP very much. However, results are a bit different if information asymmetry of abatement costs results from emission size rather than abatement technology level. We show this in the next subsection.

4.2 Heterogeneous emission size case

We will see the case where e is heterogeneous between firms but c is homogeneous (and environmental damage is linear). Let e_i be distributed in $[\underline{e}, \bar{e}]$ with probability distribution $f(\cdot)$ that satisfies $f(\bar{e}) > 0$. In this case, we define MP and VP by emission level (e_{MP} and e_{VP}, respectively) rather than abatement level because the regulator cannot observe e_i and as a result, the regulator cannot specify abatement level of individual firms unless all firms are perfectly differentiated by verification. We define the inflexible VP in the same manner as heterogeneous technology case and let emission level of the inflexible VP be $e_{\bar{VP}}$ (for details of $e_{\bar{VP}}$ as well as IC and PC conditions, see Appendix A.4). Then, we have the following proposition that show when VP is flexible or when VP is inflexible.

Proposition 4. *The optimal VP generates higher social welfare than the MP but VP is neither perfectly flexible nor risk-eliminating. The condition for the flexibility of VP depend on whether the mandatory policy can be adopted for sure ($p = 1$).*

(i) Suppose that $p = 1$. Social cost under the optimal VP (SCVP) is strictly smaller than social cost under MP(SCMP) if the VP is flexible. The VP is flexible if and only if $c[1 - F(e)] - \delta f(e) > 0$ for some $e \in (\underline{e}, \bar{e})$.

(ii) Suppose that $p < 1$. SCVP is strictly smaller than SCMP. Inflexible VP is optimal if $cf(e) + \delta f'(e) \geq 0$ $\forall e$.

Proof. See Appendix A.5. □

This proposition shows sufficient conditions that VP is inflexible. VP is inflexible unless the distribution of firms' emission size, $f(e)$, is strongly biased toward small or medium size. If $f(e)$ is strongly biased toward small or medium size, $f(e)$ is very likely to sharply decrease in some range or $cf(e) + \delta f'(e) \geq 0 \ \forall e$ does not hold in the range. Therefore, if the distribution of firms' emission size is not strongly biased toward small or medium size, VP is very likely to be inflexible. In addition, if $cf(e) + \delta f'(e) \geq 0 \ \forall e$, $c[1 - F(e)] - \delta f(e)$ increases with e and $-\delta f(\bar{e}) < 0$ at $\bar{e}$. Thus, $cf(e) + \delta f'(e) \geq 0 \ \forall e$ is a sufficient condition for the optimality of inflexible VP or $c[1 - F(e)] - \delta f(e) \leq 0 \ \forall e$ regardless of whether the mandatory policy can be adopted for sure or not.

However, VP is likely (partially) flexible if $f(e)$ sharply decreases ($cf(e) + \delta f'(e) < 0$) for some range (but not short range). For example, the distribution of firms' emission size is strongly biased toward small or medium size like a normal distribution with low variance and small or medium mean. Actually, figure 2 gives an example when VP is flexible. In this example, the distribution of firms' emission size is strongly biased toward medium size. VP(e_{VP}) has inflexible part ($3.5 \leq e \leq 5.8$; $e_{VP} = 3.62$ and $e_{MP} = 3.5$) and flexible part ($5.8 < e \leq 10$). VP is also likely flexible if some medium or large size firms are few because if$f(e)$ is very small for some medium or large e, then $f(e)$ is very likely to sharply decrease around this e.

In contrast with heterogeneous technology level case, VP is not risk-eliminating under heterogeneous emission size case. The PC condition plays a key role in generating this. Firms' emission level under the VP that satisfies the PC condition with equality when verification cost is zero is increasing with their emission size if $p < 1$ or the regulator might fail to introduce the mandatory policy. This implies that the inflexible VP can be too lenient for small emission size firms and that they accept a more stringent emission target than the inflexible VP. Actually, the inflexible VP is too lenient from the viewpoint of social welfare if the distribution of firms' emission size is strongly biased toward small size or the number of firms sharply decreases as emission size increases[6]. In addition, the inflexible VP is more lenient if the regulator is more likely to fail to introduce the mandatory policy (p is small) and as a result, VP is more likely to be flexible (if $f(e)$ sharply decreases for some range).

Although VP is not risk-eliminating or the inflexible VP might generate lower social welfare than the MP does, a perfectly flexible VP, $a(e) = p(e - e_{MP})$[7], guarantees the same social welfare as the MP does and as a result, the optimal VP always generates higher social welfare than the MP does. In this sense, flexibility is an advantage of the VP which the MP does not have in heterogeneous emission size case, whereas certain policy implementation ("risk-eliminatingness") is an advantage of the VP which the MP does not have in heterogeneous technology level case. In addition, VP is flexible with many types of firms' distribution relative to heterogeneous technology level case. VP is flexible as long as firms with medium or high abatement cost are minority in heterogeneous emission size case (so, VP can be flexible even if firms with low abatement cost are minority). On the other hand, VP is flexible (only) if firms with low medium abatement cost are majority and firms with high abatement cost are not minority of minority in heterogeneous technology level case. These

[6]In the Appendix A.5, we give an numerical example where the inflexible VP is worse than the MP ($p = 0.9$ $\delta = c = 1$. $\underline{e} = 1$, $\bar{e} = 10$ and $f(x) = ke^{-x^2}$ where $k = 1/(\int_1^{10} e^{-x^2} dx)$.).

[7]This abatement is equal to the expected abatement (of type e) under the MP and the sum of abatement and verification costs under this abatement is weakly smaller than the expected abatement cost under the MP.

facts indicate that observability of individual firms' emission level before policy introduction affects advantage of the VP over the MP due to flexibility. Flexibility is beneficial in the unobservable case relative to the observable case.

5 Conclusion

This paper examines voluntary policies' potential for the emissions abatement of a large number of firms under an asymmetric information case and flexibility of the optimal voluntary policy by developing a game theoretic model of emissions abatement policy. In our model, all firms are subject to an inflexible emission standard under the mandatory regulation, whereas firms' emission level under voluntary policy can depend on their abatement cost if they submit credible evidence on their abatement cost to a regulator. We show that perfectly flexible voluntary policy is not the most socially desirable voluntary policy in the feasible set of voluntary policies that is characterized by firms' commitment to emissions abatement with submission of credible evidence on their cost information.. We also show when flexible voluntary policy is the most socially desirable voluntary policy and compare social welfare under the optimal voluntary policy and the mandatory standard.

We found that regardless of the type of heterogeneity, the optimal voluntary policy is strictly better than mandatory policy if the regulator can fail to introduce the mandatory policy and the optimal voluntary policy is not perfectly flexible. However, the more likely the regulator is to fail to introduce the MP, the more likely the optimal VP is to be flexible under heterogeneous emission size case and the less likely the optimal VP is to be flexible under heterogeneous technology case. In addition, the optimal VP is likely flexible if many low abatement cost firms are majority and high abatement cost firms are not minority of minority (under heterogeneous technology case) or if medium or high abatement cost firms are minority (under heterogeneous emission size case). Thus, the optimal VP is more likely to be flexible under heterogeneous emission size case than it is under heterogeneous technology case.

Judging from our results that VP is flexible only if relatively high abatement cost firms are minority, most firms face the same abatement (emission) target in most cases. However, when relatively high abatement cost firms are minority, the VP should enable them to receive exceptional treatment if they verify their abatement cost or explain why they receive the treatment. This approach to emission abatement is very similar to comply or explain approach to corporate governance. Therefore, our results imply that traditional inflexible regulations by hard law underperform quasi-regulations that require firms to comply with environmental standard by "soft law" or explain why they do not comply.

In our model, we implicitly assumed that the abilities to carry out verification of abatement costs are homogeneous among firms although emissions abatement cost are heterogeneous among firms. However, one way to extend our model is to introduce the heterogeneity of abatement technology among firms. Given that most voluntary approaches cannot enforce a firm's commitment, it would be interesting to consider the case in which voluntary policies are not enforceable. Exploring these types of extensions remains an endeavor for future research.

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A Appendix

A.1 Proof of Proposition 2

Proof. Let us assume $\bar{c} > \frac{\delta}{\sqrt{p}a_{MP}}$ hereinafter (otherwise, the Proposition is already solved). From (5), we just have to minimize $\int_{\underline{c}}^{\bar{c}} \left\{\frac{1}{2}[a(c)]^2[1-F(c)] - \delta a(c)f(c)\right\} dc$.

($\Rightarrow$) Suppose that $a(c) = \sqrt{p}a_{MP}$ for all $c \in [\underline{c}, \bar{c}]$ is not optimal when $E[c|c \geq k] \leq c_{MP} + k \quad \forall k \in [\underline{c}, \bar{c}]$. Then, because $\dot{a}(c) \leq 0$ for all c must hold, optimal $a(c)$ must be strictly smaller than $\sqrt{p}a_{MP}$ for $c \geq \tilde{c}$. or for $c > \tilde{c}$. Let us define the following infinitedesimal function for sufficiently small positive value η and $0 < \lambda < 1$:

$$\ell(c) = \begin{cases} 0 & \text{when } c \leq \tilde{c} \\ \lambda(\sqrt{p}a_{MP} - a(c)) & \text{when } \tilde{c} \leq c \leq \tilde{c} + \eta \\ \lambda(\sqrt{p}a_{MP} - a(\tilde{c}+\eta)) & \text{when } \tilde{c} + \eta < c. \end{cases}$$

In addition, let us define $\epsilon = \lambda(\sqrt{p}a_{MP} - a(\tilde{c}+\eta))$ and $\hat{c} = \tilde{c} + \eta$. By adding $\ell(c)$ to $a(c)$, the increment of $\int_{\underline{c}}^{\bar{c}} \left\{\frac{1}{2}[a(c)]^2[1-F(c)] - \delta a(c)f(c)\right\} dc$ is

$$\begin{aligned} \int_{\underline{c}}^{\bar{c}} a(c)\ell(c)[1-F(c)]dc - \delta\epsilon \int_{\hat{c}}^{\bar{c}} f(c)dc &= \int_{\hat{c}}^{\bar{c}} a(c)\epsilon[1-F(c)]dc - \delta\epsilon \int_{\hat{c}}^{\bar{c}} f(c)dc \\ &= \epsilon \int_{\hat{c}}^{\bar{c}} a(c)[1-F(c)]dc - \delta\epsilon \int_{\hat{c}}^{\bar{c}} f(c)dc \\ &< \epsilon \int_{\hat{c}}^{\bar{c}} \sqrt{p}a_{MP}[1-F(c)]dc - \delta\epsilon \int_{\hat{c}}^{\bar{c}} f(c)dc \\ &= \epsilon\sqrt{p}a_{MP} \left\{ [c(1-F(c))]_{\hat{c}}^{\bar{c}} + \int_{\hat{c}}^{\bar{c}} cf(c)dc \right\} - \delta\epsilon[1-F(\hat{c})] \\ &= \epsilon\sqrt{p}a_{MP} \left\{ \int_{\hat{c}}^{\bar{c}} cf(c)dc - (\hat{c} + \delta/\sqrt{p}a_{MP})[1-F(\hat{c})] \right\} \leq 0. \end{aligned}$$

This contradicts with optimality of $a(c)$. Therefore, if $E[c|c \geq k] \leq c_{MP} + k \quad \forall k \in [\underline{c}, \bar{c}]$, the inflexible VP is optimal. Or if VP is flexible, $E[c|c \geq k] > c_{MP} + k \quad \exists k \in [\underline{c}, \bar{c}]$.

($\Leftarrow$) Suppose $a(c) = \sqrt{p}a_{MP}$ for all c. Let us define the following infinitedesimal function for sufficiently small positive value ϵ where $k \in [\underline{c}, \bar{c}]$:

$$\ell_k(c) = \begin{cases} 0 & \text{when } c \leq k \\ k - c & \text{when } k < c \leq k + \epsilon \\ -\epsilon & \text{when } k + \epsilon < c. \end{cases}$$

By adding $\ell_k(c)$ to $a(c)$, the increment of $\int_{\underline{c}}^{\bar{c}} \left\{ \frac{1}{2}[a(c)]^2[1-F(c)] - \delta a(c) f(c) \right\} dc$ is

$$\begin{aligned}
\int_{\underline{c}}^{\bar{c}} \sqrt{p} a_{MP} \ell_k(c)[1-F(c)]dc + \delta\epsilon \int_k^{\bar{c}} f(c)dc &= -\int_k^{\bar{c}} \sqrt{p} a_{MP} \epsilon [1-F(c)]dc + \delta\epsilon \int_k^{\bar{c}} f(c)dc \\
&= -\epsilon\sqrt{p} a_{MP} \left\{ [c(1-F(c))]_k^{\bar{c}} + \int_k^{\bar{c}} cf(c)dc \right\} + \delta\epsilon[1-F(k)] \\
&= \epsilon\sqrt{p} a_{MP}[1-F(k)] \left\{ \frac{\delta}{\sqrt{p} a_{MP}} + k - \int_k^{\bar{c}} cf(c)dc/[1-F(k)] \right\} \\
&= \epsilon\sqrt{p} a_{MP}[1-F(k)] \left\{ c_{MP} + k - E[c|c \geq k] \right\}
\end{aligned}$$

This increment must be larger than zero if $a(c)$ is optimal. Therefore, if the inflexible VP is optimal, $E[c|c \geq k] \leq c_{MP} + k \quad \forall k \in [\underline{c}, \bar{c}]$. Or if $E[c|c \geq k] > c_{MP} + k \quad \exists k \in [\underline{c}, \bar{c}]$, VP is flexible.

Social cost under the inflexible VP ($a(c) = \sqrt{p} a_{MP}$ for all c) is always weakly smaller than SCMP because expected emission abatement cost under both policies are the same where expected environmental damage under the MP is weakly greater than that under optimal VP because $\sqrt{p} \geq p$ holds. Since the inequality of $\sqrt{p} \geq p$ is the strict inequality if and only if $p < 1$, social cost under the inflexible VP is always strictly smaller than SCMP if and only if $p < 1$. If the inflexible VP is not optimal, SCVP is strictly greater than SCMP regardless of the value of p. □

A.2 Proof of Proposition 1

To prove $c_{MP} + k \geq E[c|c \geq k]$, it is sufficient to prove $E[c] + k \geq E[c|c \geq k]$. Since $E[c] + k \geq E[c|c \geq k]$ apparently holds when $k = \underline{c}$ because $\underline{c} \geq 0$, it is sufficient to prove $\frac{\partial E[c|c \geq k]}{\partial k} \leq 1 \; \forall k$ because $\frac{\partial E[c]+k}{\partial k} = 1 \; \forall k$ clearly holds.

Note that $\frac{\partial E[c|c \geq k]}{\partial k} = \frac{\partial}{\partial k} \frac{\int_k^{\bar{c}} cf(c)dc}{1-F(k)} = (E[c|c \geq k] - k)\frac{f(k)}{1-F(k)} = (E[c|c \geq k] - k)\frac{f(k)}{\int_k^{\bar{c}} f(c)dc}$ holds. Here, $0 \leq E[c|c \geq k] - k \leq \bar{c} - k$ holds. Also, $\frac{f(k)}{\int_k^{\bar{c}} f(c)dc} \leq \frac{1}{\bar{c}-k}$ holds because $f(k) \leq f(c)$ is satisfied for all $c \in [k, \bar{c}]$. Thus, $\int_k^{\bar{c}} f(c)dc \geq f(k)(\bar{c} - k)$. Therefore, $(E[c|c \geq k] - k)\frac{f(k)}{\int_k^{\bar{c}} f(c)dc} \leq 1$ is satisfied and consequently $\frac{\partial E[c|c \geq k]}{\partial k} \leq 1$ is satisfied.

A.3 Proof of Proposition 3

Let us assume $\bar{c} \geq \frac{\delta(e - \sqrt{p} a'_{MP})}{\sqrt{p} a'_{MP}}$ here (otherwise, the Proposition is already solved.). In addition, $\dot{a}(c) \leq 0$ for all c must hold because firm's cost function is the same as linear damage case. Verification cost is calculated as$v(c) = \frac{1}{2}[\int_{\underline{c}}^{c} a(c')^2 dc' + p\underline{c} a'^2_{MP} - ca(c)^2]$. Firm's cost is $\frac{1}{2}[\int_{\underline{c}}^{c} a(c')^2 dc' + p\underline{c} a'^2_{MP}]$ Taking also into account the environmental damage, total social cost is $\frac{1}{2}\int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{c} a(c')^2 dc' f(c)dc + \frac{1}{2} p\underline{c} a'^2_{MP} \int_{\underline{c}}^{\bar{c}} f(c)dc + \frac{\delta}{2}[e - \int_{\underline{c}}^{\bar{c}} a(c)f(c)dc]^2$. Our interest is whether $a(c) = \sqrt{p} a'_{MP} \forall c$ minimizes $\frac{1}{2}\int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{c} a(c')^2 dc' f(c)dc + \frac{\delta}{2}[e - \int_{\underline{c}}^{\bar{c}} a(c)f(c)dc]^2$.

Proof. ($\Rightarrow$) Suppose that $a(c) = \sqrt{p} a'_{MP}$ for all $c \in [\underline{c}, \bar{c}]$ is not optimal when $E[c|c \geq k] \leq c'_{MP} + k \quad \forall k \in [\underline{c}, \bar{c}]$. Then, because $\dot{a}(c) \leq 0$ for all c must hold, optimal $a(c)$ must be strictly smaller than $\sqrt{p}a'_{MP}$ for $c \geq \tilde{c}$ or for $c > \tilde{c}$. Let us define the following infinitedesimal function for sufficiently small positive value η and $0 < \lambda < 1$:

$$\ell(c) = \begin{cases} 0 & \text{when } c \leq \tilde{c} \\ \lambda(\sqrt{p}a'_{MP} - a(c)) & \text{when } \tilde{c} \leq c \leq \tilde{c} + \eta \\ \lambda(\sqrt{p}a'_{MP} - a(\tilde{c} + \eta)) & \text{when } \tilde{c} + \eta < c. \end{cases}$$

In addition, let us define $\epsilon = \lambda(\sqrt{p}a'_{MP} - a(\tilde{c} + \eta))$ and $\hat{c} = \tilde{c} + \eta$. By adding $\ell(c)$ to $a(c)$, the increment of $\frac{1}{2}\int_{\underline{c}}^{\bar{c}}[\int_{\underline{c}}^{c} a(c')^2 dc']f(c)dc + \frac{\delta}{2}[e - \int_{\underline{c}}^{\bar{c}} a(c)f(c)dc]^2$ is

$$\begin{aligned}
&- \delta e \int_{\underline{c}}^{\bar{c}} \ell(c)f(c)dc + \delta \int_{\underline{c}}^{\bar{c}} a(c)f(c)dc \int_{\underline{c}}^{\bar{c}} \ell(c)f(c)dc + \int_{\underline{c}}^{\bar{c}}[\int_{\underline{c}}^{c} a(c')\ell(c')dc']f(c)dc \\
=& - \delta e \epsilon \int_{\hat{c}}^{\bar{c}} f(c)dc + \delta \int_{\underline{c}}^{\bar{c}} a(c)f(c)dc \int_{\hat{c}}^{\bar{c}} \epsilon f(c)dc + \int_{\hat{c}}^{\bar{c}} \epsilon \int_{\hat{c}}^{c} a(c')dc' f(c)dc \\
<& - \delta e \epsilon \int_{\hat{c}}^{\bar{c}} f(c)dc + \delta \int_{\underline{c}}^{\bar{c}} \sqrt{p}a'_{MP} f(c)dc \int_{\hat{c}}^{\bar{c}} \epsilon f(c)dc + \int_{\hat{c}}^{\bar{c}} \epsilon \int_{\hat{c}}^{c} \sqrt{p}a'_{MP} dc' f(c)dc \\
=& - \epsilon\delta[e - \sqrt{p}a'_{MP}] \int_{\hat{c}}^{\bar{c}} f(c)dc + \epsilon\sqrt{p}a'_{MP} \int_{\hat{c}}^{\bar{c}} \int_{\hat{c}}^{c} dc' f(c)dc \\
=& \epsilon\sqrt{p}a'_{MP} \int_{\hat{c}}^{\bar{c}} (c - \hat{c})f(c)dc - \epsilon\delta[e - \sqrt{p}a'_{MP}] \int_{\hat{c}}^{\bar{c}} f(c)dc \\
=& \epsilon\sqrt{p}a'_{MP}[\int_{\hat{c}}^{\bar{c}} cf(c)dc - (\hat{c} + \delta e/(\sqrt{p}a'_{MP}) - \delta) \int_{\hat{c}}^{\bar{c}} f(c)dc] \leq 0
\end{aligned}$$

where inequality holds because $a(c) < \sqrt{p}a'_{MP}$ for $c \geq \tilde{c}$ or for $c > \tilde{c}$. This contradicts with optimality of $a(c)$. Therefore, if $E[c|c \geq k] \leq c'_{MP} + k \quad \forall k \in [\underline{c}, \bar{c}]$, the inflexible VP is optimal. Or if VP is flexible, $E[c|c \geq k] > c'_{MP} + k \quad \exists k \in [\underline{c}, \bar{c}]$.

($\Leftarrow$) Suppose $a(c) = \sqrt{p}a'_{MP}$ for all c. Let us define the following infinitedesimal function for sufficiently small positive value ϵ where $k \in [\underline{c}, \bar{c}]$:

$$\ell_k(c) = \begin{cases} 0 & \text{when } c \leq k \\ k - c & \text{when } k < c \leq k + \epsilon \\ -\epsilon & \text{when } k + \epsilon < c. \end{cases}$$

For any abatement function $\bar{a}(\cdot)$ which is infinitesimally different from $a(c) = \sqrt{p}a'_{MP}\forall c$, the set of the above infinitesimal functions constitutes the basis for $\bar{a} - a$, i.e. $\bar{a} - a$ can be expressed as the superposition of the above infinitesimal functions. Therefore, the necessary and sufficient condition that $a(c) = \sqrt{p}a'_{MP}\forall c$ is the (locally) best VP is that $\frac{1}{2}\int_{\underline{c}}^{\bar{c}}[\int_{\underline{c}}^{c} a(c')^2 dc']f(c)dc + \frac{\delta}{2}[e - \int_{\underline{c}}^{\bar{c}} a(c)f(c)dc]^2$ under abatement function $a + \ell_k$ is always (weakly) greater than that under abatement function a

for all $k \in [\underline{c}, \bar{c}]$. Taking the difference between them, this condition can be calculated as

$$
\begin{aligned}
&\delta[-e\int_{\underline{c}}^{\bar{c}} \ell_k(c)f(c)dc + \int_{\underline{c}}^{\bar{c}} \ell_k(c)f(c)dc \int_{\underline{c}}^{\bar{c}} \sqrt{p}a'_{MP}f(c)dc] + \int_{\underline{c}}^{\bar{c}}\{\int_{\underline{c}}^{c} \sqrt{p}a'_{MP}\ell(c')dc'\}f(c)dc \\
=&\delta[e - \sqrt{p}a'_{MP}]\epsilon \int_k^{\bar{c}} f(c)dc - \sqrt{p}a'_{MP}\epsilon \int_k^{\bar{c}} (c-k)f(c)dc \\
=&\delta\epsilon[e - \sqrt{p}a'_{MP}](1-F(k)) - \sqrt{p}a'_{MP}\epsilon \int_k^{\bar{c}} (c-k)f(c)dc \\
=&\sqrt{p}a'_{MP}\epsilon[(k + \delta e/(\sqrt{p}a'_{MP}) - \delta)(1-F(k)) - \int_k^{\bar{c}} cf(c)dc].
\end{aligned}
$$

This increment must be larger than zero if $a(c)$ is optimal. Therefore, if the inflexible VP is optimal, $E[c|c \geq k] \leq c'_{MP} + k \quad \forall k \in [\underline{c}, \bar{c}]$. Or if $E[c|c \geq k] > c'_{MP} + k \quad \exists k \in [\underline{c}, \bar{c}]$, VP is flexible.

Social cost under the inflexible VP ($a(c) = \sqrt{p}a'_{MP}$ for all c) is always weakly greater than SCMP because expected emission abatement cost under both policies are the same where expected environmental damage under the MP is weakly greater than that under optimal VP because $\sqrt{p} \geq p$ holds. Since the inequality of $\sqrt{p} \geq p$ is the strict inequality if and only if $p < 1$, social welfare under the inflexible VP is always strictly greater than SCMP if and only if $p < 1$. If the inflexible VP is not optimal, SCVP is strictly greater than SCMP regardless of the value of p. $\square$

A.4 MP, inflexible VP and IC and PC conditions under heterogeneous emission size case

Under heterogeneous emission size case, the MP is a (local) optimum of the following problem;

$$
\min_{e_P} \int_{e_P}^{\bar{e}} (\frac{1}{2}c(e-e_P)^2 + \delta e_P)f(e)de + \delta \int_{\underline{e}}^{e_P} ef(e)de.
$$

From the F.O.C for this problem. its local optimum, e_{MP}, is given by $e_{MP} = \int_{\underline{e}}^{\bar{e}} ef(e)de - \delta/c$ if $\underline{e} \geq \int_{\underline{e}}^{\bar{e}} ef(e)de - \delta/c$ or $\int_{\underline{e}}^{\bar{e}}\{c(1-F(e)) - \delta f(e)\}de \leq 0$ and characterized by

$$
\int_{e_{MP}}^{\bar{e}} [c(e - e_{MP}) - \delta]f(e)de = \int_{e_{MP}}^{\bar{e}} \{c(1-F(e)) - \delta f(e)\}de = 0 \tag{9}
$$

if $\underline{e} < \int_{\underline{e}}^{\bar{e}} ef(e)de - \delta/c$. Thus, if $e_{MP} \geq \underline{e}$, then $e_{MP} = \int_{\underline{e}}^{\bar{e}} ef(e)de - \delta/c$ and if $e_{MP} < \underline{e}$, e_{MP} satisfies $\int_{e_{MP}}^{\bar{e}}\{c(1-F(e)) - \delta f(e)\}de = 0$.

Let $e_{VP}(e)$ be emission target under VP for firms whose emission size is e. Then, like heterogeneous technology case, we can write the regulator's problem under the VP as

$$\min_{a(e),\ s(e)} \int_{\underline{e}}^{\bar{e}} \left[\delta(e - a(e)) + \frac{1}{2}c[a(e)]^2 + v(e)\right] f(e)de \tag{10}$$

subject to

$$\text{(IC)} \quad e = \arg\min_{e'}[\frac{1}{2}c(e - e' + a(e'))^2 I_{[e>e'-a(e')]} + v(e')] \quad \forall e \tag{11}$$

$$\text{(PC)} \quad \frac{1}{2}c(a(e))^2 + v(e) \leq p\frac{1}{2}c(e - e_{MP})^2 I_{[e>e_{MP}]} \quad \forall e. \tag{12}$$

where $a(e) = e - e_{VP}(e)$. We derive the inflexible VP, $e_{\bar{VP}}$, by using (12). It is given by $e_{\bar{VP}} = \sqrt{p}e_{MP} + (1 - \sqrt{p})\bar{e}$.

We can rewrite the IC and PC conditions. By the following lemma (Lemma 1), the IC condition can be written as $a'(e) \leq 1$ $\forall e$ and

$$v(e) = c\{A(e) - \frac{(a(e))^2}{2} + \frac{(a(\underline{e}))^2}{2}\}\ \forall e \tag{13}$$

where $A'(\cdot) = a(\cdot)$ and $A(\underline{e}) = 0$. (13) can be written in differentiable form as $c(1-a'(e))a(e) = v'(e)$ if $a(\cdot)$ and $s(\cdot)$ are differentiable. Note that the IC condition implies that $v(\cdot)$ is weakly monotonically increasing derived from $c(1-a'(e))a(e) = v'(e)$ and $a'(e) \leq 1$. Substituting (13) for the PC condition, the PC condition is simplified to:

$$A(e) \leq \frac{p(e - e_{MP})^2}{2} - \frac{(a(\underline{e}))^2}{2}\ \forall e. \tag{14}$$

Lemma 1. *IC condition* $\Leftrightarrow$ *(13) and* $a'(e) \leq 1$ *for all* e.

Proof of $\Rightarrow$

It is obvious that (13) holds for all $e \in [e_{MP}, \bar{e}]$ if the IC condition holds. Therefore, what we have to show is that $a'(e) \leq 1$ for all $e \in [e_{MP}, \bar{e}]$ if the IC condition holds. From the IC condition, $\forall e,\ e' \in [e_{MP}, \bar{e}]$,

$$\frac{1}{2}c(a(e))^2 + v(e) \leq \frac{1}{2}c(e - (e' - a(e')))^2 + v(e') \tag{15}$$

$$\frac{1}{2}c(a(e'))^2 + v(e') \leq \frac{1}{2}c(e' - (e - a(e)))^2 + v(e). \tag{16}$$

Without loss of generality, we can assume $e > e'$. Therefore, combining 15 and 16,

$$\frac{1}{2}c(e-(e'-a(e')))^2 - \frac{1}{2}c(a(e'))^2 \geq \frac{1}{2}c(a(e))^2 - \frac{1}{2}c(e'-(e-a(e)))^2 \tag{17}$$
$$(e-e')(e-e'+2a(e')) \geq (e-e')(e'-e+2a(e))$$
$$e-e' \geq a(e)-a(e')$$
$$\int_{e'}^{e} 1dt \geq \int_{e'}^{e} a'(t)dt$$

Therefore, $a'(e) \leq 1$ for all $e \in [\underline{e}, \bar{e}]$.

Proof of $\Leftarrow$

Suppose that the IC condition does not hold. Then, there exist e' and e'' (we can assume $e' > e''$without loss of generality) such that

$$\frac{1}{2}c(e'-(e''-a(e'')))^2 + v(e'') < \frac{1}{2}c(a(e'))^2 + v(e').$$

Therefore,

$$\frac{1}{2}c(e'-(e''-a(e'')))^2 + v(e'') - [\frac{1}{2}c(a(e''))^2 + v(e'')] < \frac{1}{2}c(a(e'))^2 + v(e') - [\frac{1}{2}c(a(e''))^2 + v(e'')].$$
$$\frac{1}{2}(e'-e''+a(e''))^2 - \frac{1}{2}(e''-e''+a(e''))^2 < A(e') - A(e'')$$
$$\int_{e''}^{e'} [e-e''+a(e'')]de < \int_{e''}^{e'} a(e)de.$$
$$\int_{e''}^{e'} [e-e'']de < \int_{e''}^{e'} [a(e)-a(e'')]de.$$

However, $\int_{e''}^{e'}[e-e'']de \geq \int_{e''}^{e'}[a(e)-a(e'')]de$ must hold because $a'(e) \leq 1$ implies that $e - e'' \geq a(e) - a(e'')$ for all $e \in [e'', e']$.

A.5 Proof of Proposition 4

Social cost under the MP, written as SCMP, is written as follows:

$$SCMP = p\int_{\underline{e}}^{\bar{e}} \{\frac{c(e-e_{MP})^2}{2} - \delta(e-e_{MP})\}I_{[e \geq e_{MP}]}f(e)de + \int_{\underline{e}}^{\bar{e}} \delta e f(e)de.$$

Then, let us define $\xi(e)$ as follows:

$$\xi(e) = \begin{cases} \frac{1}{2}p(e-e_{MP})^2 - A(e) - \frac{1}{2}(a(\underline{e}))^2 & \text{when } e \geq e_{MP} \\ 0 & \text{otherwise.} \end{cases}$$

Because of PC, $\xi(e) \geq 0$ always holds.

Social cost under a VP, written as SCVP, is calculated as follows:

$$
\begin{aligned}
SCVP &= \int_{\underline{e}}^{\bar{e}} [\{\frac{1}{2}ca(e)^2 + s(e) - \delta a(e)\} I_{[e \geq e_{MP}]} + \delta e] f(e) de \\
&= \int_{\underline{e}}^{\bar{e}} \{cA(e) + \frac{1}{2}c(a(\underline{e}))^2 - \delta a(e)\} I_{[e \geq e_{MP}]} f(e) de + \int_{\underline{e}}^{\bar{e}} \delta e f(e) de \\
&= \int_{\underline{e}}^{\bar{e}} \{c\frac{1}{2}p(e - e_{MP})^2 - c\xi(e) - p\delta(e - e_{MP}) + \delta\xi'(e)\} I_{[e \geq e_{MP}]} f(e) de + \int_{\underline{e}}^{\bar{e}} \delta e f(e) de \\
&= SCMP - \int_{\underline{e}}^{\bar{e}} \{c\xi(e) - \delta\xi'(e)\} f(e) de \\
&= SCMP - \int_{\underline{e}}^{\bar{e}} c\xi(e) f(e) de + [\delta\xi(e) f(e)]_{\underline{e}}^{\bar{e}} - \int_{\underline{e}}^{\bar{e}} \delta\xi(e) f'(e) de \\
&= SCMP - \int_{\underline{e}}^{\bar{e}} \{cf(e) + \delta f'(e)\}\xi(e) de + \delta\xi(\bar{e}) f(\bar{e}) - \delta\xi(\underline{e}) f(\underline{e})
\end{aligned}
$$

where the second equality holds due to IC condition (3). Because the regulator can always choose $\xi(e) = 0$ for all e and $SCVP = SCMP$ if $\xi(e) = 0$ for all e, $SCVP \leq SCMP$ and in particular, $SCVP < SCMP$ if $\xi(e) = 0$ for all e is not optimal. However, inflexible VP, $e_{\bar{VP}} = \sqrt{p}e_{MP} + (1 - \sqrt{p})\bar{e}$, might underperform relative to the MP. For example, $-\int_{\underline{e}}^{\bar{e}} \{cf(e) + \delta f'(e)\}\tilde{\xi}(e) de + \delta\tilde{\xi}(\bar{e}) f(\bar{e}) - \delta\tilde{\xi}(\underline{e}) f(\underline{e}) > 0$ where $\xi(e) = \frac{1}{2}p(e - e_{MP})^2 - \int_{\underline{e}}^{e} \max(\hat{e} - e_{\bar{VP}}, 0) d\hat{e} = \frac{1}{2}p(e - e_{MP})^2 - \frac{1}{2}(e - e_{\bar{VP}})^2 I_{e \geq e_{\bar{VP}}}$ when $p = 0.9$ $\delta = c = 1$. $\underline{e} = 1$, $\bar{e} = 10$ and $f(x) = ke^{-x^2}$ where $k = 1/(\int_1^{10} e^{-x^2} dx)$. Thus, in this case, $SCVP > SCMP$ if the VP is inflexible and VP is not risk-eliminating.

Next, we show that $\xi(e) = 0$ for all e is optimal if VP is perfectly flexible. Assume $\forall e$; $a'(e) < 1$ and $\exists e$ s.t. $\xi(e) > 0$. Due to the continuity of $\xi(\cdot)$, there exists open interval (e_1, e_2) s.t. $\forall e \in (e_1, e_2)$; $\xi(e) > 0$. In (e_1, e_2), PC is not binding. Then, consider the infinitesimal function as follows.

$$
\lambda(e) = \begin{cases} -\epsilon(e - e_1) & \text{when } e_1 \leq e \leq e_1 + \eta \\ -\epsilon\eta + \epsilon(e - e_1 - \eta) & \text{when } e_1 + \eta < e \leq e_1 + 3\eta \\ \epsilon\eta - \epsilon(e - e_1 - 3\eta) & \text{when } e_1 + 3\eta < e \leq e_1 + 4\eta. \\ 0 & \text{otherwise} \end{cases}
$$

where $e_1 + 4\eta \leq e_2$. If positive variables ϵ and η are sufficiently small, neither $a(e) + \lambda(e)$ nor $a(e) - \lambda(e)$ violates IC and PC. Such addition or subtraction of $\lambda(\cdot)$ only changes the value of $A(e)$ in $[e_1, e_1 + 4\eta]$. If addition or subtraction of $\lambda(\cdot)$ changes SCVP, then the optimality of $a(e)$ is violated. Therefore, $cf(e) + \delta f'(e) = 0$ holds for all $e \in [e_1, e_2]$. By adding and subtracting countably many $\lambda(\cdot)$, $\xi(e) = 0$ can be achieved for $e \in [e_1, e_2]$. Thus, $\xi(e) = 0$ for all e is optimal if VP is perfectly flexible. In addition, this implies that VP is perfectly flexible only if $\xi(e) = 0$ for all e is optimal.

When $p = 1$, the inflexible VP ($a(e) = e - e_{MP}$) satisfies $\xi(e) = 0$ for all e. This implies that VP cannot be perfectly flexible because by definition, VP is NOT flexible (nor perfectly flexible) if the inflexible VP is optimal. However, the VP can be flexible and if so, $SCVP < SCMP$. Suppose that the inflexible VP ($\xi(e) = 0$ for all e) is optimal and consider the following deviation from the inflexible VP; $\xi(\tilde{e}) = \epsilon$ holds for some $\tilde{e} \in (\underline{e}, \bar{e})$. Because $\dot{a}(e) \leq 1$, $\xi(e) = \epsilon$ for all $e \geq \tilde{e}$ must also hold. The difference in SCVP between the deviated VP and the inflexible VP is

$$-\epsilon \int_{\tilde{e}}^{\bar{e}} \{cf(e) + \delta f'(e)\} de + \delta \epsilon f(\bar{e}) = -\epsilon[c(1 - F(\tilde{e})) - \delta f(\tilde{e})].$$

This must be positive for all $\tilde{e} \in (\underline{e}, \bar{e})$ if the inflexible VP ($\xi(e) = 0$ for all e) is optimal. Therefore, VP is flexible and $SCVP < SCMP$ if $p = 1$ and $\frac{c}{\delta} > \frac{f(e)}{1-F(e)}$ for some $e \in (\underline{e}, \bar{e})$.

When $p < 1$, only perfect flexible VP ($\dot{a}(e) < 1$ for all e) satisfies $\xi(e) = 0$ for all e. We show that $\xi(e) = 0$ for all e is not optimal. First, consider a case where $e_{MP} \leq \underline{e}$. Suppose that $\xi(e) = 0$ for all e is optimal. Then, consider $\xi(e) = \epsilon$ for all e instead where ϵ is sufficiently small positive value. It increases SCVP by the amount of

$$\begin{aligned}
-\epsilon \int_{\underline{e}}^{\bar{e}} \{cf(e) + \delta f'(e)\} de + \delta \epsilon f(\bar{e}) - \delta \epsilon f(\underline{e}) =& -\epsilon \int_{\underline{e}}^{\bar{e}} cf(e) de - \epsilon \int_{\underline{e}}^{\bar{e}} \delta f'(e) de + \delta \epsilon f(\bar{e}) - \delta \epsilon f(\underline{e}) \\
=& -c\epsilon - \delta\epsilon[f(\bar{e}) - f(\underline{e})] + \delta \epsilon f(\bar{e}) - \delta \epsilon f(\underline{e}) \\
=& -c\epsilon < 0.
\end{aligned}$$

This contradicts with the optimality of $\xi(e) = 0$ for all e. Therefore, $\xi(e) = 0$ for all e is not optimal if $e_{MP} \leq \underline{e}$ and $p < 1$.

Next, consider a case where $e_{MP} > \underline{e}$. In this case, SCVP is equal to $SCMP - \int_{e_{MP}}^{\bar{e}} \{cf(e) + \delta f'(e)\}\xi(e) de + \delta \xi(\bar{e}) f(\bar{e})$. Suppose that $\xi(e) = 0$ for all e is optimal. Then, consider $\xi(e) = \epsilon$ for all $e \geq \tilde{e} \geq e_{MP}$ instead where ϵ is sufficiently small positive value. It increases SCVP by the amount of $\epsilon\{-\int_{e_{MP}}^{\bar{e}} \{cf(e) + \delta f'(e)\} de + \delta f(\bar{e})\}$. If $\xi(e) = 0$ for all e is optimal, the amount must be positive. Therefore, $\int_{\tilde{e}}^{\bar{e}} \{cf(e) + \delta f'(e)\}(e) de - \delta f(\bar{e}) < 0 \ \forall \tilde{e} \in [e_{MP}, \bar{e})$ holds. The LHS of this inequality can be rewritten as

$$c(1 - F(\tilde{e})) + \delta(f(\bar{e}) - f(\tilde{e})) - \delta f(\bar{e}) = c(1 - F(\tilde{e})) - \delta f(\tilde{e}).$$

Therefore, $c(1 - F(\tilde{e})) - \delta f(\tilde{e}) < 0 \ \forall \tilde{e} \in [e_{MP}, \bar{e})$ holds if $\xi(e) = 0$ for all e is optimal, and this implies that $\int_{e_{MP}}^{\bar{e}} [c(1 - F(e)) - \delta f(e)] < 0$. This contradicts that e_{MP}must satisfy $\int_{e_{MP}}^{\bar{e}} \{c(1 - F(e)) - \delta f(e)\} de = 0$ if $e_{MP} > \underline{e}$. Therefore, $\xi(e) = 0$ for all e is not optimal if $e_{MP} > \underline{e}$ and $p < 1$ and$\xi(e) = 0$ for all e is not optimal or VP is not perfectly flexible in any cases.

Finally, we show that a sufficient condition for the optimality of the inflexible VP is $cf(e) + \delta f'(e) \geq 0 \ \forall e$ (therefore, a necessary condition that VP is flexible is $cf(e) + \delta f'(e) < 0 \ \exists e$) when $p < 1$. Suppose that $cf(e) + \delta f'(e) \geq 0 \ \forall e$. Then, $A(\bar{e}) > 0$ must hold because our minimand is rewrited as follows;

$$\int_{\underline{e}}^{\bar{e}} \{\delta(e - a(e)) + cA(e) + \frac{1}{2}c(a(\underline{e}))^2\} f(e)de = \delta \int_{\underline{e}}^{\bar{e}} ef(e)de + \frac{1}{2}c(a(\underline{e}))^2$$
$$- \delta f(\bar{e})A(\bar{e}) + \int_{\underline{e}}^{\bar{e}} \{cf(e) + \delta f'(e)\}A(e)de.$$

Otherwise, by letting $a(e) = \min\{0, e - \bar{e} + \epsilon\}$, the minimand is

$$-\delta f(\bar{e})\frac{\epsilon^2}{2} + \{cf(\bar{e}) + \delta f'(\bar{e})\}\frac{\epsilon^3}{6} < 0$$

if ϵ is sufficiently small. When $A(\bar{e}) = 0$, the minimand is 0. This contradicts with optimality of $a(e)$. Therefore, $A(\bar{e}) > 0$.

$\int_{\underline{e}}^{\bar{e}} \{cf(e) + \delta f'(e)\}A(e)de$ is minimized by minimizing $A(e)$ for all e. However, $A(\bar{e})$ must be some positive value (letting it be $k < \frac{p}{2}(\bar{e} - e_{MP})^2$). Therefore, when $A(\bar{e})$ is fixed at k, $a(e) = 0$ should hold for as long an interval as possible and $a(e)$ increases around $\bar{e}$ as sharply as possible. Due to the IC condition, $\dot{a}(e) \leq 1$ must hold. Therefore, $a(e) = \min\{0, e - \bar{e} + a(\bar{e})\}$. In addition, $A(\bar{e}) = k > 0$ must hold. This $a(e)$ is the same as setting uniform emission standard e_U for all firms. Because $e_{\bar{V}P}$ for all firms is the best uniform emission standards that satisfies the PC condition, $a(e) = \min\{0, e - e_{\bar{V}P}\}$ is optimal.

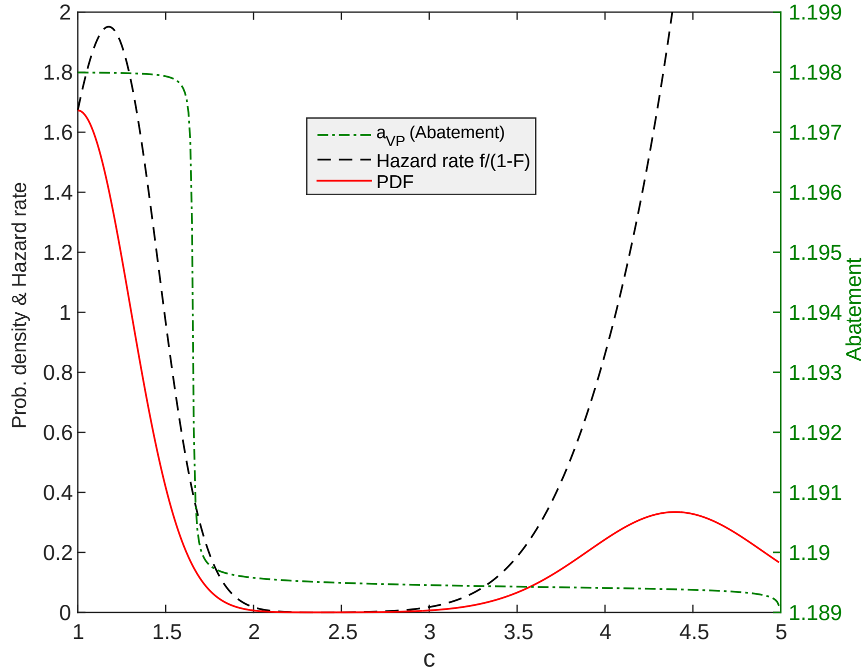

Figure 1: An example where VP is flexible ($p = 0.9$, $\delta = 3$, $c \in [1, 5]$, $f(c) = \phi(c; 1, 0.3) + \phi(c; 4.4, 0.5)/3$ and $c_{MP} = 2.2538$)

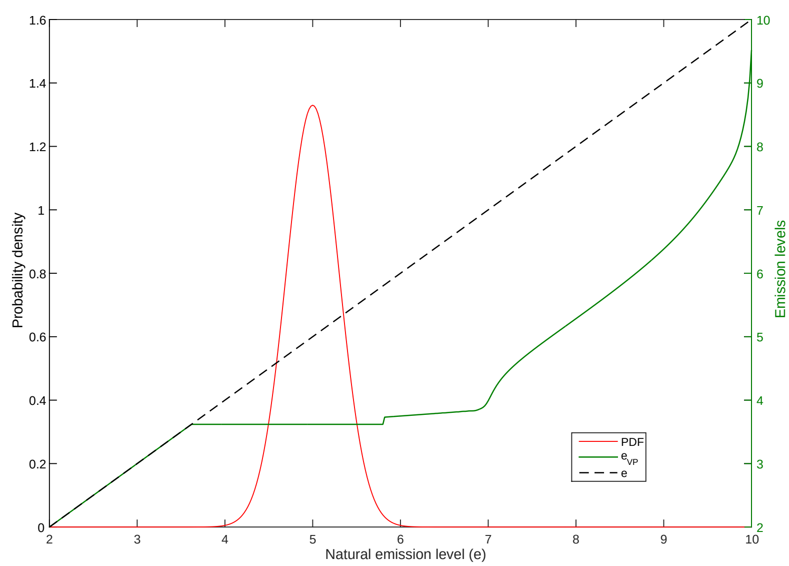

Figure 2: An example where VP is flexible (but not perfectly flexible) ($p = 0.9$, $\delta = 1.5$, $c = 1$, $e \in [2, 10]$ and $f(e) = \phi(e; 5, 0.3)$)