Simultaneous Innovation and Economic Growth

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Abstract

In practice, many innovators make the exact same innovation within a short time of each other. Motivated by this empirical regularity, the paper develops an expanding variety growth model which explicitly models firms' choice of a particular R&D project when an endogenously determined mass of many potential projects (ideas) is available. The paper emphasizes the lack of coordination inherent in this decision. These frictions lead to foregone innovation which generates a dynamic inefficiency. It decreases the growth rate and, at the same time, amplifies the fraction of wasteful simultaneous innovation. The economy grows because current innovation permanently reduces the severity of future coordination problems. The model features a "business-stealing" effect that induces over-investment in R&D as compared to the second-best. Implementing the constraint-efficient allocation requires a tax on R&D activities. Coordination frictions are of particular interest in the current context. The analysis suggests the inefficiencies associated with these frictions may be quite large in practice. The paper also analyzes firm-level data on patents which provides strong evidence in support of the ubiquity of simultaneous innovation.

Keywords: Growth, Simultaneous Innovation, Coordination Frictions, Search for Ideas.

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1 Introduction

On February 14, 1876 Alexander Bell filed a patent application for the telephone. Only two hours later Elisha Gray submitted a similar application for the exact same innovation. The same phenomenon of simultaneous innovation took place with the invention of the cotton gin, the steam engine, the computer, the laser, and many others. In particular, Lemley (2011) presents ample evidence that virtually every important innovation from history has been simultaneously innovated by more than one group of researchers. Moreover, simultaneous innovation is documented in more recent examples which are not limited to major innovations (see, for example, Cohen and Ishii (2005) and section five of the current paper).[1]

Motivated by this empirical regularity, the current paper investigates firms' decision to undertake a particular (out of many possible) research project in the context of economic growth. The analysis puts particular emphasis on the lack of coordination of firms' efforts inherent in this decision. These frictions are likely to be present in practice because of the size of the "market" for ideas.[2] Furthermore, such coordination requires each firm to know the portfolio of research projects of all of its rivals. This is particularly implausible in the current context given that firms actively employ secrecy as an intellectual property protection mechanism.[3] In addition, the paper analyzes firm-level data on patents, which provides evidence that strongly supports the ubiquity of simultaneous innovation.

This paper contributes an expanding variety growth model in which innovating firms direct their R&D efforts towards a particular research avenue (idea) out of an endogenously determined mass of potential projects.[4] Each idea is associated with a potential new variety

[1] Throughout the paper I use simultaneous innovation to include "quasi" simultaneous innovation: the event when more than one group of researchers make the same innovation within a relatively short period without being aware of each others' innovations.

[2] For example, this paper analyzes data on nearly $4,000$ Compustat firms covering nearly one million patents. Thus, it appears very implausible that this many firms can coordinate their research efforts (firm A directs effort towards project 1, firm B towards project 2, and so on) on this many projects.

[3] For a survey of the evidence see, for example, Hall *et al.* (2014).

[4] This paper makes a distinction between ideas (potential innovations) and innovations (ideas which have been brought to fruition as a consequence of costly R&D effort). This is in contrast to the previous literature on economic growth (Jones, 1995, 2002; Jones and Kim, 2014; Chiu *et al.*, 2015; Akcigit *et al.*, 2016; Bloom *et al.*, 2016) which has used ideas and innovations interchangeably.

that can be introduced to the market if the idea is innovated. At stage one of the innovation process firms enter the R&D sector, at stage two they direct their efforts, and at stage three they produce. The model emphasizes the coordination frictions inherent in firms' choice of research project.[5] To this end, I focus on the symmetric equilibrium in which firms use identical mixed strategies when directing their R&D efforts.

In equilibrium, each idea is innovated by a random number of firms that follows a Poisson distribution with parameter equal to the market tightness in the market for ideas (the ratio of firms to ideas). This tightness represents the average number of firms which simultaneously innovate the same idea and, thus, captures the level of the congestion in the market.[6] The equilibrium level of congestion is pinned down by the entry cost and the monopoly profits, whereas the absolute mass of entrants is ultimately determined by the pool of ideas. The frictions in the market for ideas lead to a concave varieties production function — higher aggregate R&D effort increases congestion and, hence, the possibility of (wasteful) simultaneous innovation. Knowledge is cumulative — each innovated idea allows firms to "stand on the shoulders of giants" and gain technological access to a number of new research projects. This intertemporal spillover effect is the ultimate source of growth in the economy — an expanding mass of ideas permanently alleviates future congestion problems, thus, permanently reducing the cost of discovering new varieties. Along the balanced growth path (BGP henceforth), the growth rate of the economy is determined by the growth rate of the mass of ideas, which in turn is endogenously determined by the market tightness and the coordination problems.

This paper argues that studying coordination frictions in the current context is particularly important. In equilibrium, a positive fraction of ideas are not innovated — a dynamic inefficiency that directly reduces the growth rate of the economy. Furthermore, innovation is very costly in terms of both time and effort and the possibility of simultaneous innovation

[5]For papers which focus on coordination frictions in contexts different from the current one see, for example, Julien *et al.* (2000), Burdett *et al.* (2001), and Shimer (2005).

[6]The possibility of simultaneous innovation in the current paper is a product of firms' decisions to direct their research efforts towards a particular R&D project and the coordination frictions inherent in this decision. In contrast, the simultaneous innovation present in some previous studies (Segerstrom *et al.*, 1990; Corriveau, 1994, 1998) is a by-product of discrete-time patent races.

implies that the resources devoted to wasteful duplication of effort can be quite high in practice. The coordination frictions in the economy amplify the fraction of wasteful simultaneous innovation and, hence, the associated inefficiency.[7] This is the case because of the dynamic inefficiency associated with coordination frictions. Coordination problems reduce the growth rate which rises the net present value of profits. This provides incentives for firms to tolerate higher congestion (lower chance of securing a monopoly position) which in turn increases the equilibrium fraction of wasteful duplication of effort. Together these inefficiencies imply a reduction in welfare which could be quite substantial. In particular, a calibrated version of the model implies that eliminating the frictions in the economy leads to a 13% gain in welfare, in consumption equivalent terms. This is the case because in the frictional economy about 34% of all potential projects are not innovated and the decentralized growth rate (of 1.75%) is only 66% of the growth rate (of 2.65%) in the frictionless economy. Furthermore, 39% of all innovations represent wasteful duplication of effort (a cost equal to 1.22% of GDP). This fraction is about 25% larger than that in the frictionless economy.

The decentralized equilibrium is also inefficient as compared to the second-best (the planer can choose the mass of R&D firms, but she cannot assign innovators to projects). A higher market tightness implies a higher fraction of ideas are innovated and, hence, a lower dynamic inefficiency. At the same time, it leads to higher congestion and, hence, higher static inefficiency The planner thus chooses the second-best market tightness which, on the margin, strikes a balance between these two inefficiencies. There are two externalities in the model which stem from the explicit modeling of firm's decision to direct their R&D efforts towards a particular idea and the inherent coordination frictions. First, due to the possibility of simultaneous innovation, the model features a "business-stealing" effect that leads to a congestion externality.[8] The marginal R&D entrant finds innovation profitable

[7] Even if firms can coordinate their efforts, several firms may choose to work on the same research project due to the usual "over-grazing" problem which is well-known in the patent-race literature (for a survey see, for example, Reinganum (1989)).

[8] This business-stealing effect is a direct consequence of the explicit modeling of firms' choice of research avenues and the inherent coordination frictions. It is thus different than the business-stealing effect examined in the previous literature (see, for example, Aghion and Howitt (1992), Corriveau (1994), and Corriveau (1998)).

even if a rival has already directed its research efforts towards the corresponding idea, as long as the entrant receives a patent for the innovation. In that event the rival is denied a patent and the entrant effectively steals the monopoly rents. The planner, on the other hand, finds the marginal entry beneficial only if no other firm has directed its research effort towards the corresponding idea. That is, on the margin, she values only the sole inventor. The congestion externality, thus, induces firms to over-invest in equilibrium. Second, there is a learning externality — the planner values innovation in part because it leads to an increase in the mass of ideas which permanently decreases future coordination problems.[9] Firms, on the other hand, do not have a mechanism through which to appropriate these ideas and hence do not price them. The size of the congestion externality is larger than that of the learning one, so implementing the second-best allocation requires the government to impose a tax on R&D spending. The calibration of the model implies the optimal tax rate is 118%, indicating the congestion externality is likely to be important in practice.[10]

This paper also examines firm-level data on patents granted between 1976 and 2006 which provides further evidence of the ubiquity of simultaneous innovation. Given the intuition from the theoretical model, if simultaneous innovation is present in practice, then an increase in the market tightness should be accompanied by a decrease in each firm's probability of securing a patent. To test this prediction, I use data on aggregate number of patent applications to proxy for the mass of innovations, data on the aggregate number of patents in force to proxy for the mass of research avenues, and data on firm-level controls proxy for the quantity and quality of firm-level patent applications. I find that the data provides strong support in favor of this prediction — the increase in congestion due to a one percent increase in the relevant mass of patent applications implies a 0.88% decrease in the probability of

[9]This positive externality is similar in spirit to the inter-temporal spillover effects present in previous models (see, for example, Romer (1990), Aghion and Howitt (1992), and Grossman and Helpman (1991)). In the present paper, the externality operates through the market for ideas — the planner values ideas because they alleviate the coordination problems in the economy.

[10]The model also features the usual appropriability externality — firms have less of an incentive to invest as compared to the planner because they do not fully appropriate the social value of the marginal variety. Because of this, it is unclear if the incentives to over-invest due to the congestion externality would dominate the incentives to under-invest due to the learning and appropriability externalities. Which of the two opposing effect dominates and subsequently whether the decentralized market tightness is lower or higher than the socially optimal one depends on parameter values.

receiving a patent.

1.1 Relationship to the Literature

The current analysis explicitly models firms' choice of direction of their R&D efforts and the coordination problems inherent in this decision. As such, the paper is related to a recent literature on economic growth which emphasizes matching, and other, frictions in the innovation process (see, for example, Perla and Tonetti (2014), Lucas and Moll (2014), Benhabib *et al.* (2014), Chiu *et al.* (2015), and Akcigit *et al.* (2016)). The work here complements that literature by examining a different source of friction. In particular, to the best of my knowledge, this is the first growth paper to emphasize search frictions in the market for ideas which take the form of coordination frictions. In contrast, previous growth models have focused on a search process which takes the form of arrival rate of innovations, a McCall-type search for innovations which features firms sampling from a distribution of heterogeneous technologies, or frictions in the market for innovations.[11]

The theoretical model in this paper differs from the existing literature on economic growth in a number of additional dimensions. First, the paper emphasizes firms' choice of research avenues by explicitly modeling the mass of available ideas. In particular, the analysis makes a distinction between potential innovations (ideas) and actual innovations. Second, the model features a scarce mass of potential research projects such as, for example, Grossman and Helpman (1991) and Klette and Kortum (2004).[12] Unlike these studies, the current paper explicitly models the decision of firms to direct their R&D activities and emphasizes the

[11]For papers which feature search as arrival rate of innovations see, for example, Aghion and Howitt (1992), Grossman and Helpman (1991), and Klette and Kortum (2004). For papers that feature a McCall-type search for heterogeneous technologies see, for example, Kortum (1997), Perla and Tonetti (2014), and Lucas and Moll (2014). For papers which focus on frictions in the market for innovations see, for example, Chiu *et al.* (2015) and Akcigit *et al.* (2016), which do not make a distinction between ideas and innovations. In particular, the market for ideas in the current paper (firm searching for a potential R&D project) is different from the "market for ideas" in Chiu *et al.* (2015) and Akcigit *et al.* (2016) where firms search for opportunities to trade the property rights over an innovation.

[12]In contrast, some previous studies (Romer, 1990; Corriveau, 1994, 1998; Kortum, 1997) have examined models which feature an abundance of research avenues, whereas some others (Aghion and Howitt, 1992; Segerstrom *et al.*, 1990) have examined models where a single avenue of research is available. For a recent review of the literature see, for example, Aghion *et al.* (2014).

coordination frictions inherent in this problem.[13] Third, in contrast to the previous literature, the current paper features an endogenously determined mass of ideas. Fourth, due to the scarcity of research avenues, in the present paper firms compete for ideas. This competition is different than the competition firms face at the product market or the innovation race which the previous literature has examined.[14].

Within the literature on industrial organization the two closest papers to this one are Kultti *et al.* (2007) and Kultti and Takalo (2008) which also feature search frictions in the market for ideas. In particular, in these papers there is the possibility of simultaneous innovation due to a matching technology which is the same as the equilibrium one in the current paper. Kultti *et al.* (2007) and Kultti and Takalo (2008) focus on intellectual property rights in a partial equilibrium framework with a fixed mass of ideas and without free entry into the innovation sector. In contrast, the present model focuses on a general equilibrium framework with growth, an endogenously determined mass of ideas, and an endogenously determined market tightness through free entry in the R&D sector.

The rest of the paper is organized as follows. Section two introduces the environment and characterizes the decentralized equilibrium. Section three examines the social planner's constrained-efficient allocation. Section four presents a numerical example. Section five details the empirical analysis. Section six concludes.

2 The Economy

Time is discrete and runs from 0 to infinity. There are three types of agents in the economy — a final good producer, a continuum of consumers with unit measure, and a continuum of R&D firms. The environment is an augmented version of the textbook model in Barro and Sala-i Martin (2003) Chapter 6 (BSM henceforth). To emphasize the novel features of

[13]In contrast, these papers do not focus on this decision and assume that firms can either perfectly coordinate their efforts (Grossman and Helpman, 1991) or cannot choose the direction of their research altogether (Klette and Kortum, 2004).

[14]See, for example, Segerstrom *et al.* (1990), Aghion and Howitt (1992), Corriveau (1994), Corriveau (1998), Aghion *et al.* (2005), and Acemoglu and Akcigit (2012)

the model, the only point of departure from BSM is in the R&D sector. In particular, the innovation process captures the possibility of simultaneous innovation through coordination frictions — potential R&D projects are scarce, R&D entrants can direct their research efforts towards a particular project, but cannot coordinate their innovative activities.

2.1 Final Good Sector

The final good is produced by a single firm, which behaves as a price taker, using the following technology

$$Y_t = AL^{1-\lambda} \int_0^{N_t} X_t^{\lambda}(n)dn, \quad 0 < \lambda < 1 \tag{1}$$

where Y_t is output, L is the fixed labor supply of households, N_t is the mass of intermediate varieties, and $X_t(n)$ is the amount of a particular variety n employed in production. The price of the final good is normalized to unity. The final good firm faces a competitive market for labor, which is hired at the wage w_t, and a monopolistically competitive market for varieties, where each variety n is bought at the price $P_t(n)$. As in BSM, the firm's maximizing behavior yields the wage $w_t = (1 - \lambda)Y_t/L$ and the inverse demand function for varieties $P_t(n) = \lambda AL^{1-\lambda}X_t^{\lambda-1}(n)$.

2.2 R&D Sector

The point of departure from the previous literature is at the R&D sector. There are three stages of innovation and the innovative process makes a distinction between potential innovations (ideas) and actual innovations (new varieties). At stage one, firms enter into the R&D sector at a cost η units of the final good. The mass of R&D entrants is denoted by μ_t and is to be determined in equilibrium. At stage two firms direct their innovative effort towards a particular R&D project from a finite mass ν_t of ideas.[15] The choice is private knowledge and firms cannot coordinate their efforts. To capture these coordination failures,

[15] In this model, as in Kortum (1997), firms cannot immediately undertake all possible R&D projects. At time t they can exert effort only towards innovating ideas in ν_t and all possible ideas that lay outside of the pool are technologically infeasible. This is different than, for example, Romer (1990) and Lucas and Moll (2014) in which all R&D projects are readily available to firms at all points in time.

I follow the previous literature on coordination frictions and focus on a symmetric equilibrium where firms use identical mixed strategies.[16] Due to the random realization of these strategies there is a chance that, in equilibrium, several firms simultaneously make the exact same innovation, i.e. they direct their R&D efforts towards the same idea. Ideas are identical and, if innovated, transform into exactly one new variety. Innovation takes one period — a firm which enters at time t is successful in innovating the chosen project at time $t+1$ for sure. Thus, the only source of uncertainty in the model is the random realization of firms' equilibrium mixed strategies. Each innovator applies for a patent which grants perpetual monopoly rights over the variety. Each innovation is protected by exactly one patent — if more than one firm simultaneously apply for a patent over the same innovation, then each has an equal chance of receiving the patent.

Stage three is as in BSM. Firms which hold a valid patent supply their variety in a monopolistically competitive market. Both the average and marginal costs of production are normalized to unity so profits are given by $\pi_t(n) = (P_t(n) - 1)X_t(n)$, where n denotes a particular variety. Furthermore, the value of holding a monopoly over a variety n at time t, V_t, is given by

$$V_t(n) = \sum_{i=t+1}^{\infty} d_{it}\pi_i(n)$$

where d_{it} is the stochastic discount factor.

2.3 Laws of Motion

A necessary condition for positive long term growth in the model is that the mass of ideas, ν_t, grows at a positive rate. I follow Kortum (1997) and Romer (1990), among others, and assume that knowledge is cumulative. In particular, patenting an idea at time t allows firms to "stand on the shoulders of giants" and gain access to $M > 1$ new research avenues at $t+1$. Thus, unlike previous growth models, in the current one the mass of ideas is endogenously

[16]See, for example, Julien *et al.* (2000), Burdett *et al.* (2001), and Shimer (2005).

determined in equilibrium. Once an idea is innovated, it is no longer a potential R&D project and so it is removed from the pool.[17] Thus the net increase in the pool of ideas from innovating one new variety is $M-1$. Because of the coordination frictions, there is a positive probability that an idea is not innovated (denotes by $1-\zeta_t$). Thus, the law of motion for ideas is given by

$$\nu_{t+1} = (1-\zeta_t)\nu_t + \zeta_t M \nu_t$$

This is similar in spirit to Romer (1990)'s assumption that firms become more productive in creating new varieties as N_t grows. There is a notable difference, however. The nonrivalry of knowledge in this model is translated into a larger pool of potential R&D projects, rather than a lower entry cost. This law of motion implies that more research today makes it easier to innovate in the future. This is because, in equilibrium, a higher mass of ideas, ν_t, reduces the severity of the coordination problems in the "market" for ideas. In particular, the model emphasizes the link between current innovation and future coordination problems. As ζ_t depends on firms' optimal behavior and the coordination frictions, so does the learning which takes place in the model.

There are N_t intermediate good producers in the economy and each of them has a monopoly over a particular variety. As each innovated idea is transformed into a new variety, it follows that

$$N_{t+1} = N_t + \zeta_t \nu_t$$

2.4 Households

Consumers are infinitely lived and supply their labor inelastically. The discount factor is β and the per-period utility function is $U(C_t) = \ln C_t$. Consumers can save by accumulating assets, which in this economy are claims on intermediate firms' profits. In particular, households have access to a mutual fund that covers all intermediate good firms. I denote by a_t the amount of shares held by the representative household at the beginning of

[17] In particular, each innovation is protected by a patent, so no firm has an incentive to imitate at a late date. Thus, the idea no longer represents a profitable R&D project and is, hence, no longer in ν_t.

period t. Shares represent claims on all future profits and each period all profits are redistributed as dividends. This implies that the total assets of the household entering period t are $a_t \int_0^{N_t} (\pi_t(n) + V_t(n))dn$. At time t households decide on the shares they would like to hold at $t+1$, a_{t+1}. The mutual fund at that time covers all firms which exist at time $t+1$, N_{t+1}. Thus, the household's problem is given by

$$\max_{\{C_t, a_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln C_t$$

$$\text{subject to } a_{t+1} \int_0^{N_{t+1}} V_t(n)dn = a_t \int_0^{N_t} (\pi_t(n) + V_t(n))dn + w_t L - C_t$$

The first order conditions imply the Euler equation below

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \Big(\int_0^{N_{t+1}} (\pi_{t+1}(n) + V_{t+1}(n))dn \Big) \Big(\int_0^{N_{t+1}} V_t(n)dn \Big)^{-1}$$

The intuition is standard — consumers equate the marginal benefit of consumption at time t with the discounted marginal utility at time $t+1$ times the gross rate of return on their shares a_{t+1}.

2.5 Equilibrium

I restrict the analysis to the following set of parameter values: $\eta \leq (1-\lambda)\beta(\lambda^2 A)^{1/(1-\lambda)} L / [\lambda(M-\beta)]$. This ensures that the entry cost is not too high, so that firms have an incentive to enter the R&D sector.[18] The usual profit maximization of intermediate good firms along with the demand function imply that $P_t(n) = 1/\lambda$, hence, $X := X_t(n) = (\lambda^2 A)^{1/(1-\lambda)} L$. Moreover, this implies that output has the following form $Y_t = (\lambda^{2\lambda} A)^{1/(1-\lambda)} L N_t$. Thus, every intermediate good firm yields the same per period profits of $\pi := \pi_t(n) = X(1-\lambda)/\lambda$. This implies that $V_t := V_t(n) = \sum_{i=t+1}^{\infty} d_{it}\pi$ — every firm is equally valuable. Since each variety

[18] The assumption is actually stronger than the one we require for at least some firms to have an incentive to enter the R&D sector, $\eta \leq (1-\lambda)\beta(\lambda^2 A)^{1/(1-\lambda)} L / [\lambda(1-\beta)]$. The additional restriction is imposed so that in the economy without coordination problems (analyzed in subsection 2.7) firms find it profitable to innovate all available ideas in equilibrium.

carries the same amount of profits, the stage two equilibrium strategy of firms is to direct their R&D effort towards each idea with equal probability.[19] This implies the following equilibrium outcome.

Proposition 1. *The number of firms which direct their R&D effort towards a particular idea follows a Poisson distribution with mean θ_t, where $\theta_t \equiv \mu_t/\nu_t$.*

The proof is fairly standard, nonetheless I include it in the appendix for convenience. Proposition 1 summarizes the effects of coordination frictions in the economy. In particular, the random realization of firms' equilibrium strategies gives rise to the standard urn-ball matching technology.[20] In equilibrium, the average number of firms which innovate simultaneously is θ_t. This ratio of firms to ideas represents the market tightness in the market for ideas and captures the level of congestion in the economy. A higher θ_t implies more firms, on average, innovate the same idea simultaneously. The matching technology and the market tightness also summarize the amount of wasteful duplication of effort in the economy. In particular, whenever m firms direct their research efforts towards a particular idea, $m-1$ of these firms make a wasteful duplicative innovation. Hence, given the matching technology, a fraction $\omega_t \equiv 1-(1-e^{-\theta_t})/\theta_t$ of all innovations are wasteful.[21]

In equilibrium, an R&D firm becomes a monopolist with probability $\sum_{m=0}^{\infty} Pr($ exactly m rival firms direct their research effort towards the particular idea$)/(m+1) = \sum_{m=0}^{\infty} e^{-\theta_t}\theta_t^m/(m+1)! = (1-e^{-\theta_t})/\theta_t$. This probability captures the business-stealing effect in the model. If a firm innovates a particular idea then a rival has a certain chance of stealing that firm's monopoly rents by simultaneously innovating and securing a patent over the innovation.[22] Thus, higher congestion increases the expected number of rivals, which

[19] I follow the literature on coordination frictions (see, for example, Julien *et al.* (2000)) and derive the optimal behavior for firms when there are finite number of ideas. The result is then obtained by taking the limit as $\nu_t \to \infty$, keeping the ratio μ_t/ν_t constant.

[20] See, for example, Wolinsky (1988), Lu and McAfee (1996), Julien *et al.* (2000), and Burdett *et al.* (2001).

[21] Each R&D firm makes an innovation, so the total number of innovations each period is μ_t. The total number of useful innovations equals the number of new varieties, $(1-e^{-\theta_t})\nu_t$. Thus, the fraction of all innovations which are not useful is simply $1-(1-e^{-\theta_t})/\theta_t$.

[22] Moreover, simultaneous innovation does not destroy the monopoly rents in the sense that firms expect a positive revenue stream even if they are not the sole inventor. In this sense,the simultaneous innovation which takes place in the model is different from what some previous papers have focused on. For example,

lowers each firm's chance of securing a patent. Given free entry, it follows that

$$\eta = \frac{1 - e^{-\theta_t}}{\theta_t} V_t \tag{2}$$

The free entry condition implies that the level of congestion firms are willing to tolerate is solely governed by the expected profits and the cost of entering the R&D sector. Higher profits (or lower costs) imply that firms can tolerate a lower chance of securing a monopoly position and, hence, a higher market tightness. Thus, the relative profitability of R&D innovations pins down the equilibrium market tightness, whereas the total number of new varieties is ultimately governed by the mass of available research avenues.

The frictions in the market for ideas also affect the varieties production function. If $R_t = \eta\mu_t$ represents the aggregate amount of R&D effort in the economy, then the varieties production function is given by

$$\text{New Varieties} = (1 - e^{-\frac{R_t}{\eta\nu_t}})\nu_t \tag{3}$$

The function exhibits decreasing marginal returns because higher aggregate R&d effort leads to higher congestion in the market for ideas. As R_t increases the market gets more congested (θ_t increases), which increases the probability of simultaneous innovation and, thus, the fraction of wasteful duplicative innovation, ω_t. The equation also shows the impact of learning on the severity of the coordination frictions. As the mass of ideas increases, the varieties production function shifts up and, at the same time, the congestion in the market for ideas is alleviated. Thus, the cost of creating new varieties in the future is permanently reduced. Furthermore, this allows the economy to exhibit higher aggregate research effort, R_t, while keeping the R&D intensity, $\eta\theta_t$, and the congestion constant.

Corriveau (1998) allows for firms to simultaneously make sufficiently different innovations so that each innovator secures a patent. In this event all innovators earn zero profits because of Bertrand competition. In the present paper, simultaneous innovation does not affect the market structure since firms make the exact same innovation (or a one which is sufficiently close to be covered under the same patent). Furthermore, the business-stealing effect and the underlying simultaneous innovation in this paper are a product of firms' decisions to direct their research effort towards a particular research avenue and the implied coordination frictions in that decision.

Given the matching process, we can also construct the laws of motion for ideas and varieties in equilibrium. In particular, in equilibrium $\zeta_t = 1 - e^{-\theta_t}$. Hence,

$$\nu_{t+1} = e^{-\theta_t}\nu_t + (1 - e^{-\theta_t})M\nu_t$$
$$N_{t+1} = N_t + (1 - e^{-\theta_t})\nu_t$$

The concavity of the varieties production function caries over to the laws of motion. In particular, the growth rate of the mass of ideas is $(1 - e^{-\theta_t})(M - 1)$ (which along the BGP equals the growth rate of the economy). Each innovated idea increases the mass of ideas by $M - 1$ but each period only a fraction $1 - e^{-\theta_t}$ of all ideas are innovated. This is because of the coordination frictions which imply that each idea is left uninnovated with probability $e^{-\theta_t}$ (the probability that no firm directs its research efforts towards that particular idea). Thus, the presence of coordination frictions scales down the growth rate of the economy, as compared to the growth rate in the frictionless economy (which is given by $M - 1$). Thus, the frictions in the model generate a dynamic inefficiency which curbs the growth rate.

Since all firms receive the same profits, it follows that

$$V_t = \beta \frac{C_t}{C_{t+1}} \Big(\pi + V_{t+1}\Big) \tag{4}$$

Hence, the stochastic discount factor is $d_{it} = \beta^i C_t / C_{t+1}$ in equilibrium.

Given the consumers' budget constraint, free entry, and the law of motion for varieties it is straightforward to derive the economy wide resource constraint which takes the usual form — output is distributed towards consumption, production of intermediaries, and investment in R&D.

$$Y_t = N_t X + \mu_t \eta + C_t \tag{5}$$

2.6 Balanced Growth Path

Since the focus of this paper is on long-run growth, I focus on the BGP of the economy, where output, consumption, varieties, ideas, and the mass of entrants all grow at constant (but possibly different) rates. I denote the growth rate of any variable x along the BGP by g_x. The following proposition characterizes the growth rates of the variables in the economy.

Proposition 2. *Output, varieties, consumption, entry into R&D, and the stock of ideas all grow at the same rate along the BGP. Namely,* $g \equiv g_Y = g_C = g_N = g_\mu = g_\nu = (1-e^{-\theta})(M-1)$*, where* θ *is the value of the market tightness along the balanced growth path.*

The proof is in the appendix. As in BSM Y_t, C_t, N_t, and μ_t all grow at the same rate. In the present model, the mass of ideas, ν_t, also grows at this same rate. In fact, the growth rate of the mass of ideas is the key driver behind the positive growth rate in the economy. The learning which takes place in the model emphasizes the link between current innovation and future coordination problems. Innovating today increases the mass of ideas available for innovation in the future. This intertemporal spillover alleviates future coordination frictions permanently. This reduces the chance a rival would steal the firm's patent and, hence, permanently decreases the cost of securing a monopoly position.[23] This, then induces higher entry into R&D up to the point where congestion reaches its BGP level. Thus, an expanding mass of research avenues allows the economy to exhibit more R&D effort without exacerbating the coordination problems, i.e. increasing ω_t.

It is convenient to solve the model by looking at the stable ratios θ, $\frac{\nu}{N}$ and $\frac{C}{N}$, where omitted time subscripts represent values along the BGP. From the law of motion of ideas and varieties, and from $g_N = g_\nu$, it follows that $\frac{\nu}{N} = M - 1$. Next, from the resource constraint it follows that

$$\frac{C}{N} = \frac{1+\lambda}{\lambda}\pi - \eta\theta(M-1)$$

[23] The average cost of securing a monopoly position is $\eta/Pr(\text{monopoly}) = \eta\theta_t/(1-e^{-\theta_t})$, which is decreasing in ν_t.

which solves for $\frac{C}{N}$ as a function of θ. Lastly, we can use the fact that $g_C = g_\nu$, the Euler equation, and the law of motion for ν_t to find an implicit solution for the market tightness.

$$e^{-\theta} + (1 - e^{-\theta})M = \beta\Big(1 + \frac{\pi}{\eta}\Big(\frac{1 - e^{-\theta}}{\theta}\Big)\Big) \tag{6}$$

Even though we cannot explicitly solve for θ, it is straightforward to characterize the properties of the decentralized solution. In particular, it is fairly straightforward to establish uniqueness of the solution since the right hand side of (6) is strictly decreasing in θ and the left hand side is strictly increasing. Furthermore, the following comparative statics hold

Proposition 3. *The equilibrium market tightness, θ is:*

- *increasing in π and β*
- *decreasing in η and M*

The proof is in the appendix. Intuitively, an increase in profits raises the value of being a monopolist, V_t, which through free entry implies higher mass of R&D firms and, hence, higher equilibrium market tightness. Similarly, a higher entry cost, η, discourages entry into R&D which decreases the market tightness. An increase in β or a decrease in M both lead to an increase in the stochastic discount factor, $\beta C_t/C_{t+1}$, along the BGP. Hence, firms place a higher value on future profits, which increases the value of a patent, V_t, and ultimately the market tightness.

2.7 Comparison with the Frictionless Economy

The coordination frictions in the model are associated with two inefficiencies: i)a dynamic one — a fraction $e^{-\theta_t}$ of all research avenues are not undertaken; ii) a static one — a fraction ω_t of all innovations are wasteful. These frictions are particularly important in the current context. As innovation is, in practice, quite costly in terms of both time and effort, the static inefficiency is likely to be associated with a large waste in terms of output. Furthermore, the dynamic inefficiency lowers the growth rate of the economy. This leads to foregone future consumption that is likely to have substantial welfare effects.

This section emphasizes these inefficiencies by comparing the BGP of the economy to the BGP of the frictionless economy. In particular, the only difference in that economy to the economy with coordination problems is that firms can coordinate their research efforts.[24] Let superscript c denote the value of any variable along the BGP in the frictionless economy. Evidently, when firms can coordinate their research efforts, all research avenues are undertaken and, hence, all ideas are innovated. However, the frictionless economy may feature a positive fraction of wasteful duplication of effort due to the usual "over-grazing" problem.[25] Nonetheless, this waste, ω^c, in the frictionless economy is smaller than the one in the frictional economy because of the dynamic inefficiency. The lower growth rate in the frictional economy decreases the rate with which firms discount future profits, hence, increasing the relative profitability of a monpoly position. This induces higher entry into the R&D sector and, ultimately, a higher fraction of wasteful innovation. The following proposition gives the results.

Proposition 4. *In the frictionless economy all ideas are innovated and the growth rate equals* $M - 1$*. Furthermore,* $\omega > \omega^c$ *and* $\theta < \theta^c$*.*

A proof is included in the appendix. Intuitively, when firms can coordinate their R&D activities all ideas are innovated because each of them represents an opportunity to gain a profitable monopoly position. Thus, the dynamic inefficiency is absent in the frictionless economy. The fact that all ideas are innovated directly translates to a higher growth rate.

Furthermore, the frictionless economy may still feature a positive fraction of wasteful duplication of effort, ω^c, but this is always smaller than ω. Hence the coordination frictions amplify the inefficiency associated with simultaneous innovation. To see this, first observe that when there are no frictions firms may exert some wasteful duplication of effort because they can steal each others rents. In particular, firms will enter until all expected profits are

[24] The proof of Proposition 4 explicitly defines the process of coordination.

[25] This is a well-known result in the patent-race literature (see, for example, Reinganum (1989)).

dissipated. This "over-grazing" phenomenon is well-known from the patent-race literature (see, for example, Reinganum (1989)). However, $\omega > \omega^c$ because of the dynamic inefficiency associated with coordination failures. As some of the research avenues are not undertaken, there is foregone innovation which decreases the growth rate of the economy. This, in turn, increases the stochastic discount factor, which raises the net present value of the stream of monopoly profits. Hence, firms have a higher incentive to enter the R&D sector which leads to higher congestion and, ultimately, to a higher fraction of innovations which represent a wasteful duplication of research.

Furthermore, the frictionless economy features a lower fraction of wasteful duplication of R&D effort, even though $\theta^c > \theta$. This is the case because, for a given market tightness the presence of coordination frictions reduces an entrant's chance of securing a monopoly position. In particular, the probability of securing a patent in the frictional economy for a given tightness $\tilde{\theta}$, $(1 - e^{-\tilde{\theta}})/\tilde{\theta}$, is only a fraction $1 - e^{-\tilde{\theta}}$ of the same probability in the frictionless economy, $1/\tilde{\theta}$. This is, again, due to the dynamic inefficiency associated with coordination frictions. As firms cannot coordinate their efforts only a fraction $1 - e^{-\tilde{\theta}}$ of ideas are patented. Thus, even though the number of patent applications per idea, $\tilde{\theta}$, is the same in both economies, in the frictional one there are relatively less patents to be distributed among innovators, which decreases each entrant's chance of securing a monopoly position. Hence, firms have a lower incentive to enter the R&D sector. This is true even though the dynamic inefficiency increases the value of holding a monopoly position. In other words, the decrease in the probability of securing a patent dominates the increase in the net present value of profits, ultimately reducing incentives to enter the R&D sector and decreasing the market tightness.

3 Constrained-Efficient Allocation

This section examines the planner's constrained-efficient (i.e. the second best) allocation — the planner chooses the optimal BGP allocations subject to the coordination frictions in the market for ideas. Because of this the planner chooses a market tightness which strikes a balance between the static and dynamic inefficiencies. The model features two externalities which are directly affected by the frictions in the economy. First, there is the learning externality associated with the accumulation of ideas. Firms do not have a mechanism through which they can appropriate the ideas which come about from their innovations, so they do not value these ideas. The planner, on the other hand, values them because they alleviate future coordination problems and, thus, reduce the cost of discovering new varieties in the future. Second, there is the congestion externality which comes from the difference between the fraction of privately and socially beneficial entry into R&D. The planner values the marginal entry only if the firm is the "sole inventor", i.e. the idea would not have been innovated had the firm not entered the market for ideas. Firms, on the other hand, value entry as long as they receive a patent, even though this may have caused a rival's patent application to be rejected. Thus, the congestion externality is a form of business-stealing effect — if a firm innovates a particular idea a rival can innovate as well and patent the innovation, effectively stealing the monopoly rents of the firm. The model also features the usual appropriability externality, as in BSM. The monopoly and learning externalities are both positive ones, while the congestion externality is a negative one. Hence, the planner's solution may feature either lower or higher market tightness, depending on which externality dominates. This is true, even though the magnitude of the congestion externality is larger than that of the learning one. That is, in the absence of the appropriability externality, the decentralized economy features a higher market tightness and subsequently over-investment in R&D.

I restrict the planner's problem by imposing symmetry in the intermediate varieties, i.e. $X_t(n) = X_t(n')$ for any varieties n and n'. By symmetry of the production technology, however, this is without loss of generality. Thus, the planner faces the problem of choosing production of varieties, consumption, a mass of varieties, a mass of ideas, and the market tightness in order to maximize welfare subject to the resource constraint, the law of motion for ideas, the law of motion for varieties, and the coordination frictions.

$$\max_{\{C_t, X_t, \theta_t, N_t, \nu_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln C_t$$

$$AL^{1-\lambda} N_t X_t^{\lambda} = N_t X_t + C_t + \eta \theta_t \nu_t \tag{7}$$

$$N_{t+1} = N_t + (1 - e^{-\theta_t}) \nu_t \tag{8}$$

$$\nu_{t+1} = e^{-\theta_t} \nu_t + (1 - e^{-\theta_t}) M \nu_t \tag{9}$$

Maximizing with respect to X_t yields the usual solution for varieties $X^* := X_t = (\lambda A)^{1/(1-\lambda)} L$. As in BSM the difference between the planner's solution and the decentralized outcome comes from the monopoly pricing of intermediate goods. Let $\pi^* = X^*(1-\lambda)/\lambda$ which is the implied per period monopoly profits at the socially optimal level of intermediate varieties. Then, the rest of the first order conditions are

$$[C_t]: \quad \beta \frac{C_t}{C_{t+1}} = \frac{\phi_{t+1}}{\phi_t} \tag{10}$$

$$[N_{t+1}]: \quad h_t = h_{t+1} + \phi_{t+1} \pi^* \tag{11}$$

$$[\nu_{t+1}]: \quad \lambda_t = \lambda_{t+1} \Big(e^{-\theta_{t+1}} + (1 - e^{-\theta_{t+1}}) M \Big) + h_{t+1}(1 - e^{-\theta_{t+1}}) - \phi_{t+1} \eta \theta_{t+1} \tag{12}$$

$$[\theta_t]: \quad \eta = e^{-\theta_t} \Big(\frac{h_t}{\phi_t} + \frac{\lambda_t}{\phi_t}(M-1) \Big) \tag{13}$$

where ϕ_t, h_t, λ_t are the multipliers associated with (7), (8), and (9), respectively. From (10) and (11), it follows that

$$\frac{h_t}{\phi_t} = \beta \frac{C_t}{C_{t+1}} \Big(\pi^* + \frac{h_{t+1}}{\phi_{t+1}} \Big) \tag{14}$$

The above equation characterizes the planner's valuation of varieties. From the optimization problem, the ratio h_t/ϕ_t is the shadow price of a variety in terms of the final good. This shadow price is determined analogously to the value of a variety in the decentralized equilibrium (given by (4)): the value of a variety equals the discounted sum of per period profits, π^*, and the continuation value h_{t+1}/ϕ_{t+1}. There are only two differences — the level of profits is different and the planner chooses a different growth rate which affects the stochastic discount factor.

From the first order conditions (10) and (12) we get

$$\frac{\lambda_t}{\phi_t} = \beta \frac{C_t}{C_{t+1}} \Big(-\eta\theta_{t+1} + (1 - e^{-\theta_{t+1}}) \Big(\frac{h_{t+1}}{\phi_{t+1}} + \frac{\lambda_{t+1}}{\phi_{t+1}}(M-1) \Big) + \frac{\lambda_{t+1}}{\phi_{t+1}} \Big) \tag{15}$$

The value of an idea is the discounted sum of several terms. First, we have the dividend, $-\eta\theta_{t+1}$, which represents the average cost of R&D per idea. It captures the intuition that unlike other assets which carry positive returns, an idea is only valuable if it is innovated. Hence, the planner finds it costly to keep a stock of ideas because these ideas divert resources away from consumption and production and into R&D. The second term represents the capital gain from innovation. The term $(1 - e^{-\theta_{t+1}})$ captures the probability an idea is innovated. The term in the brackets is the social benefit from innovating — the benefit from one extra variety, h_{t+1}/ϕ_{t+1}, plus the benefit of the extra ideas that would be added to the pool because of innovation, $\lambda_{t+1}/\phi_{t+1}(M-1)$. Lastly, the idea carries the continuation value λ_{t+1}/ϕ_{t+1}. Equation (15) highlights ideas as an important asset that the planner prices accordingly. They allow the economy to generate new varieties and alleviate future coordination problems. Moreover, their mass is endogenously determined through their law of motion which is governed by the matching technology and the market tightness.[26]

Equation (13) illustrates the two externalities in the model driven by coordination frictions. First, the congestion externality manifests through the difference in the fraction of socially and privately beneficial innovations. The planner finds the marginal entry beneficial only if the firm is the sole inventor, i.e. with probability $e^{-\theta_t}$. Firms, on the other hand, value entry even if they duplicate an innovation, as long as they receive a patent for it. In particular, firms can steal a competitor's monopoly rents which gives rise to the business-stealing effect in the model. Thus, the probability of a privately beneficial innovation is

[26] This is in contrast to previous models, such as Romer (1990), Corriveau (1994), and Corriveau (1998), where firms can take any research avenue they want and ideas are abundant, so they are not valued. In other models where firms can only take one specific avenue for research, such as in Aghion and Howitt (1992) and Segerstrom *et al.* (1990), ideas are not priced either because they are not a stock which the planner can expand. This is also true for models where ideas are many but scarce and are a fixed stock which the planner cannot affect, such as Grossman and Helpman (1991) and Klette and Kortum (2004).

$(1-e^{-\theta_t})/\theta_t > e^{-\theta_t}$. Hence, the congestion exterlaity induces firms to over-invest in R&D.[27]

Second, the equation captures the learning externality. Firms cannot appropriate the benefit of any ideas that come about from their innovations, so they do not price them. The planner, on the other hand, values these ideas because they permanently alleviate coordination problems. In particular, more innovation today increases the amount of future research avenues, which allows the economy to innovate more varieties without increasing the congestion problems, effectively permanently reducing the cost of discovering new varieties.[28]

Free entry in the decentralized economy implies that firms enter the R&D sector until the entry cost, η, equals the value of the monopoly position, V_t, times the probability of securing such a position, $(1 - e^{-\theta_t})/\theta_t$. Thus, firms ignore the contribution of their entry to the dynamic and static inefficiencies associated with the coordination frictions. The planner, on the other hand, chooses entry so as to strike a ballance between the two inefficiencies (equation (13)). She equates the expected cost of wasteful innovation, $\eta \times Pr(\text{duplication of effort}) = \eta(1 - e^{-\theta_t})$, to the expected benefit of increasing the mass of innovated ideas. This benefit equals the value of the extra variety, h_t/ϕ_t, (in terms of the final good) plus the value of the extra research avenues in the future $(M-1)\lambda_t/\phi_t$ (in terms of the final good) net of the entry cost, η, times the probability of not making a wasteful innovation, $e^{-\theta_t}$. Equating the two gives the entry condition in (13).[29]

[27]The business-stealing effect in the model is a consequences of firms' choice of R&D project and the coordination frictions inherent in this decision. It is thus different than the business-stealing effect examined in the previous literature (see, for example, Corriveau (1994) and Corriveau (1998)).

[28]The average cost of discovering one new variety is $\eta/Pr(\text{sole inventor}) = \eta e^{\theta_t}$, which is decreasing in the mass of ideas. Thus, the learning externality is different from the intertemporal spillover externality examined in some previous research (such as Romer (1990)) which serves to directly reduce the cost of future research. This externality manifests its self through the explicit modeling of the firm's decisions to direct their innovative activities towards a particular project and the implied coordination frictions. Thus, the reduction of the cost of future research due to learning is an endogenous object in the model.

[29]It is also worth noting that when evaluating the benefit of entry the planner takes into account that reducing the mass of uninnovated ideas directly impacts the growth rate which decreases the discount rate and consequently the value of varieties, h_t/ϕ_t, and ideas, λ_t/ϕ_t.

Proposition 5. *Along the BGP the second best satisfies*

$$\left(\frac{\nu}{N}\right)^* = M - 1$$

$$\left(\frac{C}{N}\right)^* = \pi^* - \eta\theta^*(M-1) \tag{16}$$

$$e^{-\theta^*} + (1 - e^{-\theta^*})M = \beta\Big(1 + \frac{\pi^*}{\eta}e^{-\theta^*} + (1 - e^{-\theta^*} - \theta^* e^{-\theta^*})(M-1)\Big) \tag{17}$$

Proof is in the appendix. The difference between the planner's solution for the market tightness, (17), and the decentralized one, (6), comes from the aforementioned externalities. To see clearly how they affect the solution, let us define the implied rate of return in the decentralized equilibrium (along the BGP) by

$$r := \frac{C_{t+1}}{\beta C_t} - 1 = \frac{\pi}{\eta}\Big(\frac{1 - e^{-\theta}}{\theta}\Big) \tag{18}$$

which is nothing but the rate of return on a unit investment in R&D — π is the flow of profits and $(1 - e^{-\theta})/\theta$ is the probability of securing a monopoly position. The implied rate of return in the planner's allocation is defined by

$$r^* := \frac{C^*_{t+1}}{\beta C^*_t} - 1 = e^{-\theta^*}\Big(\frac{\pi^*}{\eta} - \theta^*(M-1)\Big) + (1 - e^{-\theta^*})(M-1) \tag{19}$$

which is the social rate of return on a unit investment in R&D. First, the planner eliminates the monopoly distortion, so the flow of profits is π^*. Second, she values the marginal innovation only when the firm is the sole inventor, which occurs with probability $e^{-\theta^*}$. In that event, the net return is given by the normalized profits, π^*/η, less the normalized "storage cost" of the new research avenues, $\theta^*(M-1)$. Third, each innovation increases the mass of ideas, so the permanent decrease in future congestion yields the return of $(1 - e^{-\theta^*})(M-1)$.

In BSM, the decentralized economy allocates too few resources for innovation because of the appropriability externality. In the present paper the same effect is in play, but the planner may find the decentralized R&D effort to be too high, depending on the size of the other two externalities. First, the congestion externality tends to push the decentralized economy to exhibit too much innovation. This is the case because firms find a fraction $(1 - e^{-\theta})/\theta$ of all entries profitable. At the same time the planner values the marginal entry with probability $e^{-\theta}$. As the planner's fraction is lower, she would like to reduce R&D effort in the economy. However, the planner derives additional benefit from entry because she values ideas as well. In particular, if ideas are more valuable to the planner she is more likely to induce higher entry than the decentralized economy. The resulting net effect from these externalities may push the decentralized equilibrium to exhibit either too little or too much innovation, depending on parameter values.

Furthermore, the magnitude of the congestion externality is larger than that of the learning externality. To see this clearly, we can decompose the difference between the planner's valuation of the benefit of entry and the firm's valuation of this benefit. At the second-best this difference is given by

$$\mathcal{A} + \mathcal{L} + \mathcal{C} = \eta - \Big(\frac{1 - e^{-\theta^*}}{\theta^*}\Big)V^* \tag{20}$$

where $\mathcal{A}$, $\mathcal{L}$, and $\mathcal{C}$ denote the appropriability, learning, and congestion externalities; $V^* := \beta\pi/(e^{-\theta^*} + (1 - e^{-\theta^*})M - \beta)$ is the value of having a monopoly position at the second best level of the market tightness. The right hand side of (20) gives the difference between the planner's valuation of the benefit of entry, η, and the firm's, V^* times the probability of securing a patent. If the sum of the three externalities is positive, then the appropriability and learning externalities dominated and the decentralized economy exhibits too little innovation. If, on the other hand, the congestion externality dominates then there is over-investment in equilibrium.[30] Then, one can decompose the sum of the three externalities in the following

[30]Since $V^*(1 - e^{-\theta^*})/\theta^*$ is strictly decreasing in the tightness, it follows that when $V^*(1 - e^{-\theta^*})/\theta^*$ is smaller than the entry cost η, firms have an incentive to decrease entry into R&D in the decentralized equilibrium. Hence, $\theta < \theta^*$. If, on the other hand, the quantity is larger than the entry cost, then firms in the decentralized equilibrium have an incentive to increase entry in the R&D sector and hence $\theta > \theta^*$.

manner

$$\mathcal{A} := \left(\left(\frac{h}{\phi}\right)^* - V^*\right)\left(\frac{1-e^{-\theta^*}}{\theta^*}\right) \tag{21}$$

$$\mathcal{L} := \left(\frac{\lambda}{\phi}\right)^* \left(e^{-\theta^*}(M-1)\right) \tag{22}$$

$$\mathcal{C} := -\left(\frac{h}{\phi}\right)^* \left(\left(\frac{1-e^{-\theta^*}}{\theta^*}\right) - e^{-\theta^*}\right) \tag{23}$$

Thus, $\mathcal{A}$ is the measure of how much more would the planner value entry than the firm if the appropriability externality was the only one in the model. $\mathcal{L}$ and $\mathcal{C}$ measure the same difference if the only externality in the model was learning and congestion, respectively. Then, the following result holds

Proposition 6. *The size of the congestion externality is larger than that of the learning externality. That is, $|\mathcal{C}| > \mathcal{L}$.*

A proof is included in the appendix. The net effect of the two externalities associated with the frictions in the model is to push the economy towards over-investment. Whether or not this effect dominates the distortion due to the appropriability externality, however, depends on parameter values.

As with the decentralized solution, one cannot explicitly solve for the market tightness, due to the form of the matching technology. Nonetheless, we can characterize the properties of the solution. In particular, one can describe the comparative statics of this solution.

Proposition 7. *The second best market tightness, θ^*, is:*

- *increasing in π^* and β*
- *decreasing in η and M*

Proof is in the appendix. Intuitively, an increase in the implied profits, π^*, increases the planner's valuation of each variety and each idea. Hence, each entry is now more beneficial, so the planner increases the market tightness, which decreases the value of entry due to increased congestion, until the value of the marginal entry reaches the entry cost η. Similarly, an increase in the entry cost, η, requires the planner to extract more benefit from the marginal entry which reduces the market tightness and decreases congestion. At the same time, an increase in η decreases the value of an idea because it decreases the dividends.

An increase in β increases the stochastic discount factor, so the value of a variety and an idea increases because the stream of future profits is now more valuable. This increases the value of the marginal entry and hence, the planner increases the market tightness. An increase in M, on the other hand, increases the discount rate of profits. This is because higher M increases the growth rate which subsequently decreases the value of future consumption. This is the case even though an increase in M implies a higher benefit of entry due to an increase in the growth rate (each new variety carries more ideas in the future) which induces the planner to set a higher market tightness. At the optimum, the first effect dominates.

3.1 Implementing the Second Best

In BSM the planner's solution can be implemented using a subsidy on the purchases of intermediate goods. In the present model such a subsidy is still necessary to eliminate the dead-weight loss from monopoly and the appropriability externality, but it is not sufficient to achieve the planner's allocation. This is due to the congestion and learning externalities. To implement the second best, the planner needs to impose a tax on the entry into R&D. This is because the congestion externality is larger than the learning one, so the over-investment effect of the former dominates the under-investment effect of the latter.

In particular, suppose that the government imposes a subsidy on the purchases of intermediate varieties at a rate s and a tax on R&D activities at a rate τ. Furthermore, if the government keeps a balanced budget through the means of lump-sum transfers, then the optimal policy is summarized below.

Proposition 8. *The optimal subsidy on the purchase of intermediate varieties is given by*

$s^* = 1 - \lambda$. *The optimal tax rate on R&D entry is given by*

$$\tau^* = \frac{\beta\pi^*(1-e^{-\theta^*})}{\eta\theta^*(e^{-\theta^*} + (1-e^{-\theta^*})M - \beta)} - 1 > 0$$

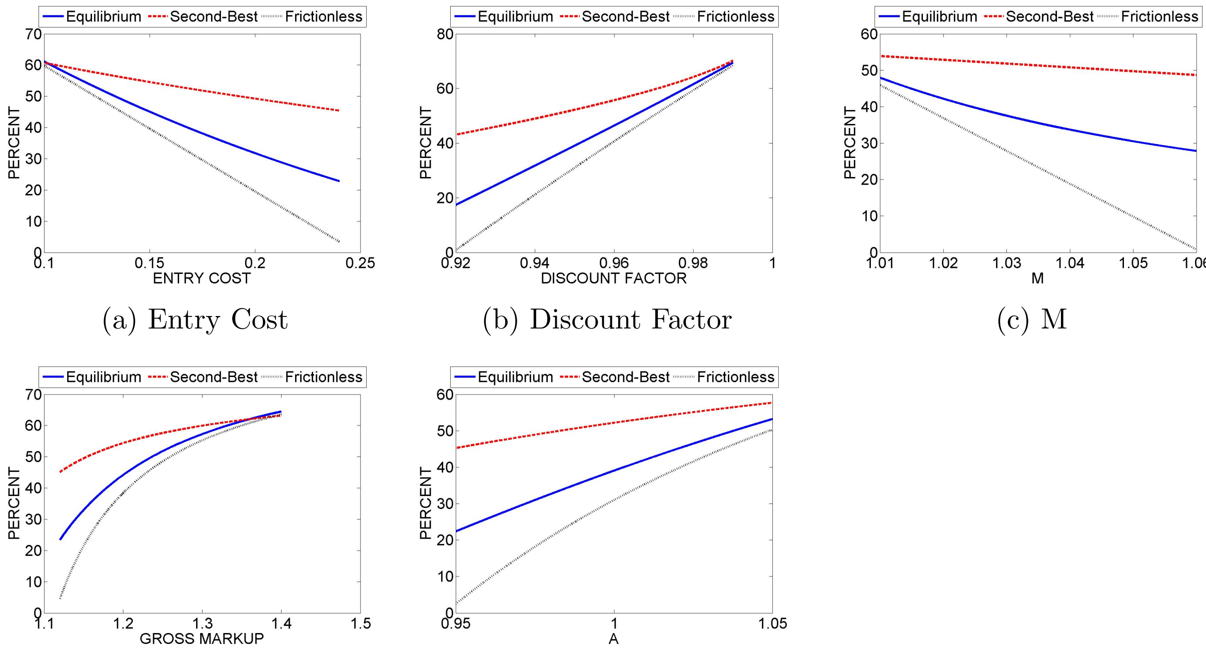
A proof is included in the appendix. This optimal s^* is the same rate as in BSM. Unlike in BSM, however, the tax rate is not zero, instead it is positive. The optimal tax rate is devised such that firms internalize the inefficiencies which stem from coordination frictions. That is, the tax rate is such that firms balance the dynamic and static inefficiencies in the same way the planner does.

4 Numerical Example

This section further investigates the features of the model through the means of a numerical example. In particular, it further emphasizes the practical important of the inefficiencies associated with the frictions in the economy. In the interest of practical relevance, I calibrate the economy to match key moments of the U.S. economy. The model is calibrated at annual frequency, so the discount factor, β, is set to 0.95. To calibrate the entry cost, η, I use data on patents and patent applications from the U.S. Patent and Trademark Office (USPTO henceforth). In the model, all firms apply for a patent that allows them to secure a monopoly position. Thus, the number of patent applications is μ_t. Of these firms only a fraction $(1-e^{-\theta})/\theta$ are successful in securing a patent. Hence, the probability of having a successful patent application is simply $(1-e^{-\theta})/\theta$. Using data on patent applications and patents granted for the period from 1966 to 2011, I set $\theta = 1.0876$ to match the average probability of 0.60957.[31] This yields $\eta = 0.1715$. Set $M = 1.0265$ to match the average growth rate of non-farm GDP for the same period, which turns out to be 1.7546%. Lastly, I set the markup to 17.431% ($\lambda = 1/1.17431$) to match the average R&D share of non-farm GDP for the period of 3.1194%.[32] The productivity parameter, A, and labor, L, are both normalized

[31]The data on patent grants is by year of application.

[32]The data on R&D expenditures and GDP is taken from the Bureau of Economic Analysis. The data on GDP is for non-farm GDP in 2009 chained dollars from NIPA table 1.3.6. The data on nominal R&D



(a) Entry Cost (b) Discount Factor (c) M

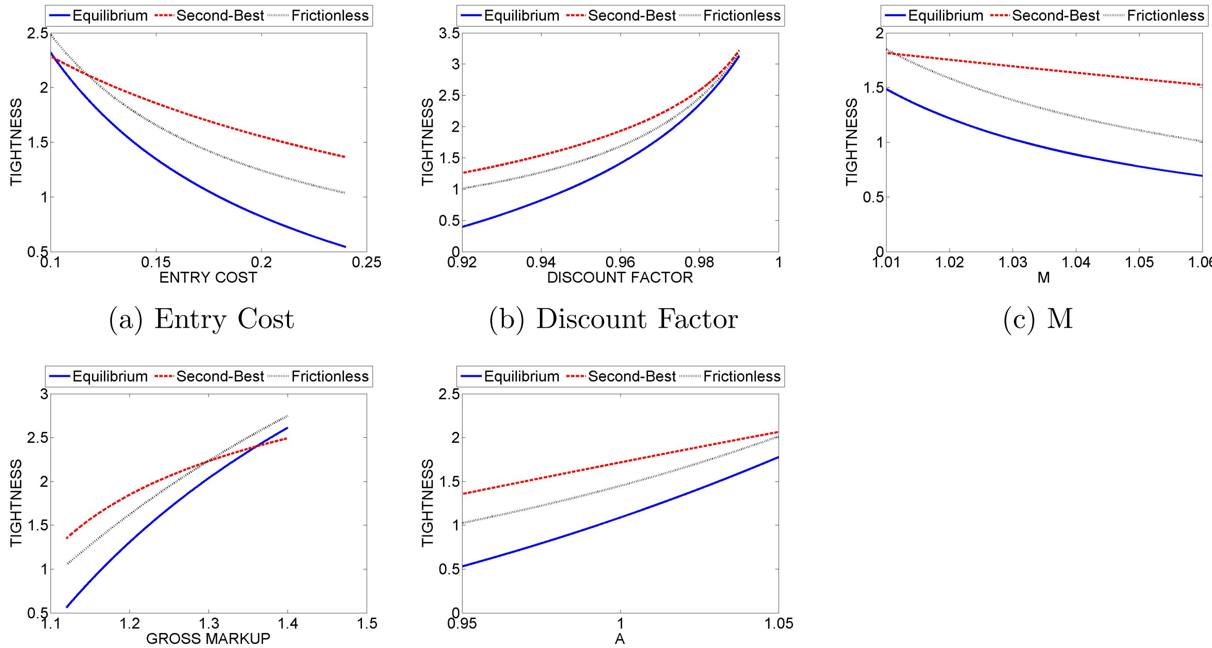
(d) Markup (e) A

Figure 1: Percentage of Wasteful Innovations

to unity.

The calibration implies the probability of simultaneous innovation in the decentralized economy is substantial — 66% of all innovators face at least one competitor who has simultaneously innovated the same idea. This implies that the fraction of wasteful innovation, ω, is 39%. This translates to a loss of 1.22% of GDP. Figure 1 shows the percentage of wasteful innovations for a range of the parameter values. It is substantial for most parameter values. Furthermore, ω is about 25% larger than that in the frictionless economy, $\omega^c = 31\%$, (Figure 1) even though the tightness, θ, is about 25% smaller than the tightness in the frictionless economy, $\theta^c = 1.4491$ (Figure 2). At the same time the decentralized economy features a large fraction of research avenues which are not undertaken — 33.7%. This inefficiency directly translates to a decrease in the growth rate due to the foregone innovations — the corresponding growth rate in the economy with perfect coordination is $M - 1 = 2.65\%$.

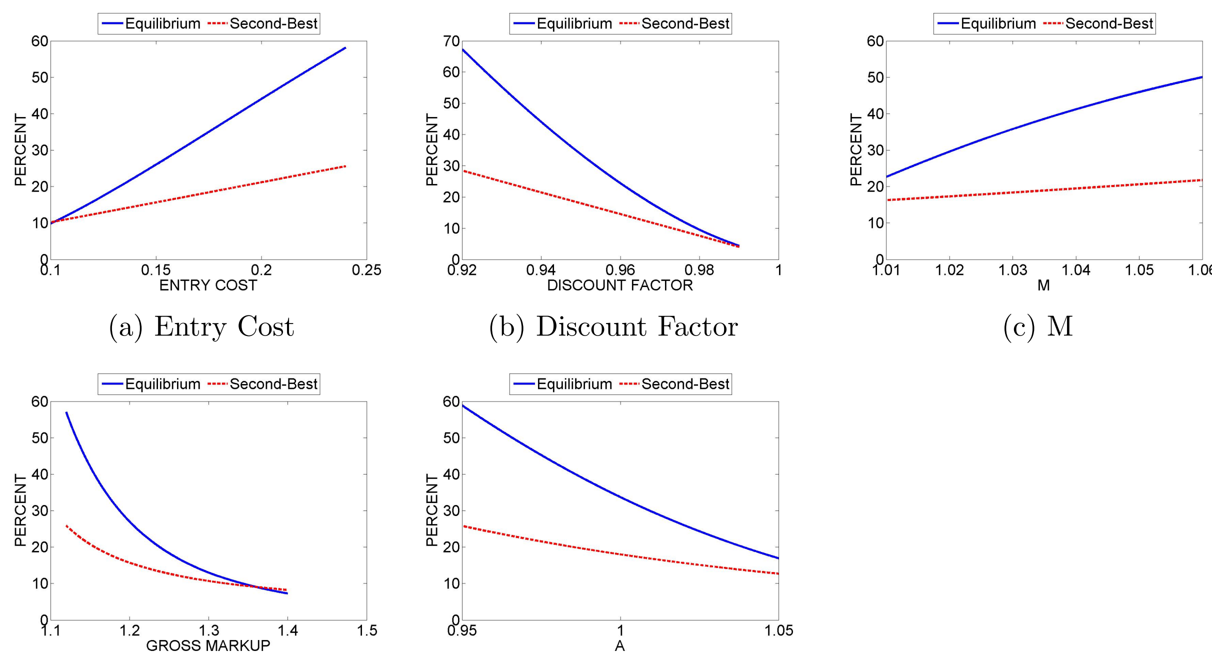
expenditures is from NIPA table 5.6.5 and includes private fixed investment in R&D (including software). To obtain the series on real R&D investment, I deflate the nominal series using the implicit GDP price deflator from NIPA table 1.1.9.



(a) Entry Cost (b) Discount Factor (c) M

(d) Markup (e) A

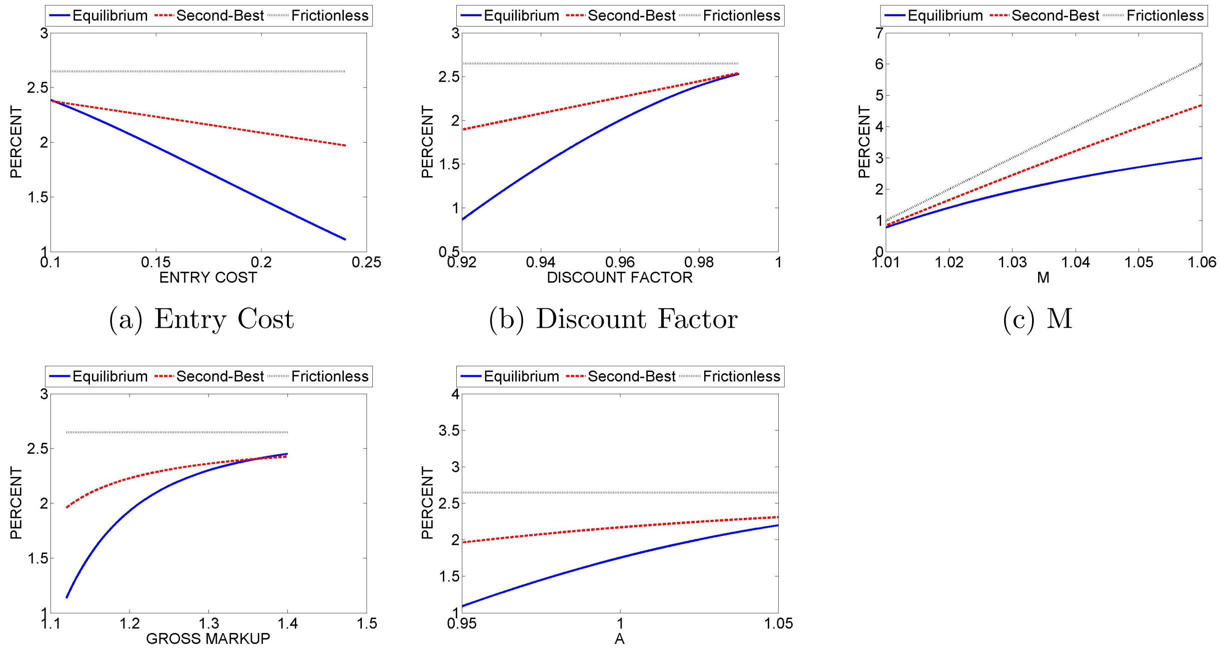
Figure 2: Market Tightness



(a) Entry Cost (b) Discount Factor (c) M

(d) Markup (e) A

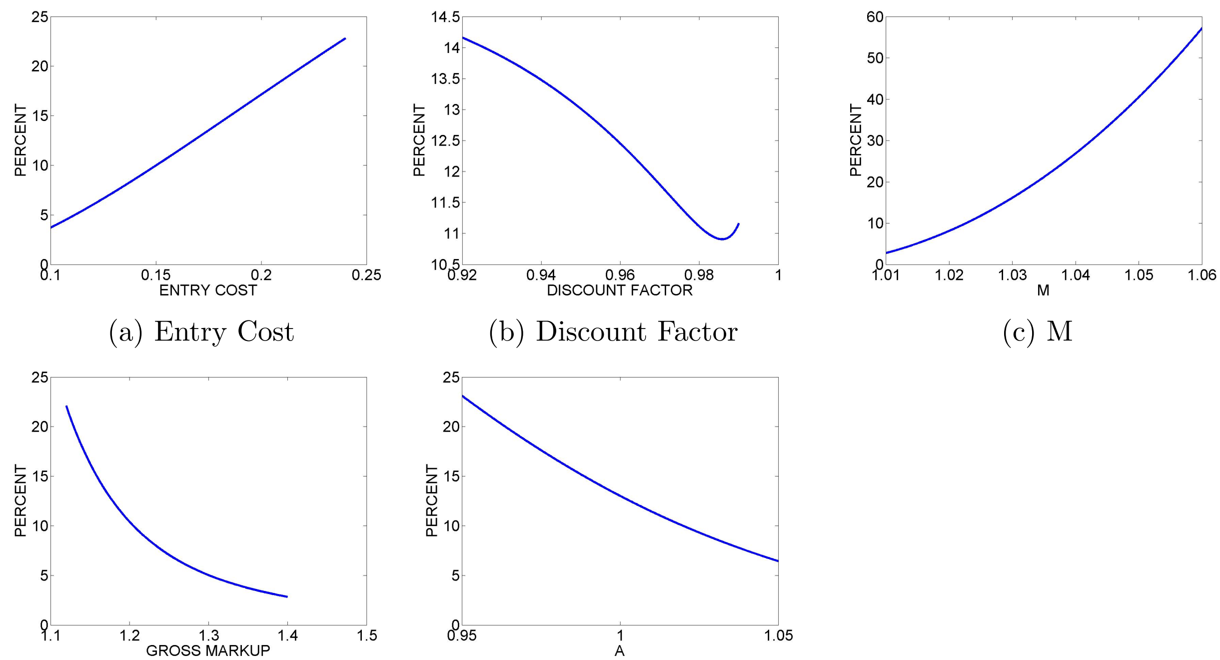
Figure 3: Percentage of Uninnovated R&D Projects



(a) Entry Cost (b) Discount Factor (c) M

(d) Markup (e) A

Figure 4: Growth Rate



(a) Entry Cost (b) Discount Factor (c) M

(d) Markup (e) A

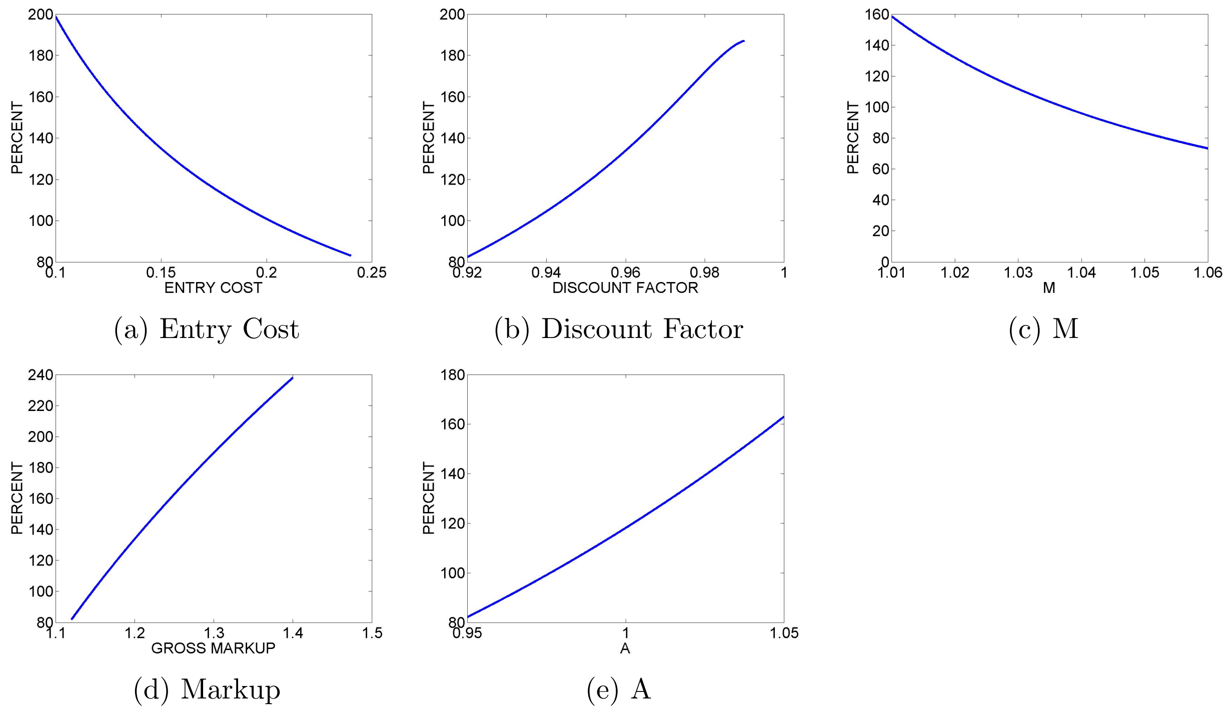
Figure 5: Welfare Gain from Eliminating the Coordination Frictions

The mass of foregone innovations is substantial for most parameter values (Figure 3) which translates to a sizable decrease in the growth rate (Figure 4). The reduction in the growth rate due to the frictions in the economy lead to large welfare losses. Eliminating the frictions generates a welfare gain of 13% in consumption equivalent terms.[33] This gain is quite substantial for most parameter values considered (Figure 5). Thus, the calibration implies that studying the coordination frictions in the current context is of particular importance.

The decentralized economy exhibits too little innovation — the second-best market tightness, θ^*, is 1.7154. Thus, in the planner's allocation 82% of all innovators face at least one rival who has innovated simultaneously and the percentage of innovations which represent a wasteful duplication of effort is 52%. Moreover, for all parameter values considered the percentage of wasteful innovation is quite large, i.e. it is at least 30% (Figure 1). In fact, it is larger than the corresponding fraction in the decentralized equilibrium for most parameter values. This is the case because the planner balances the effects of the dynamic and static inefficiencies. The second-best features a fraction of uninnovated research avenues of 18%. While this is still quite sizable, it is about half of that in the decentralized equilibrium. As Figure 3 shows, even though that probability is stubstantial for most parameter values it is also generally much lower than that in the decentralized equilibrium. This translates to the growth rate — although the planner's growth rate (of 2.17%) is still smaller than that under perfect coordination, it is considerably larger than the growth rate in the decentralized economy (Figure 4). Thus, although the duplication of effort can be quite sizable and lead to a large loss of resources (in terms of GDP), the planner generally prefers a higher market tightness. This second best tightness provides for a higher waste due to the duplication of effort but considerably decreases the other inefficiency associated with coordination frictions — the loss in welfare due to foregone innovation.

Even though the decentralized economy suffers from too little innovation ($\theta < \theta^*$), implementing the second-best requires a tax on R&D activities at the rate of $\tau^* = 118\%$. This is the case because the congestion externality is an order of magnitude larger than the learning one. In particular, once the optimal subsidy to intermediate good purchases is implemented

[33] A detailed explanation of the welfare calculations is included in the appendix.



(a) Entry Cost (b) Discount Factor (c) M

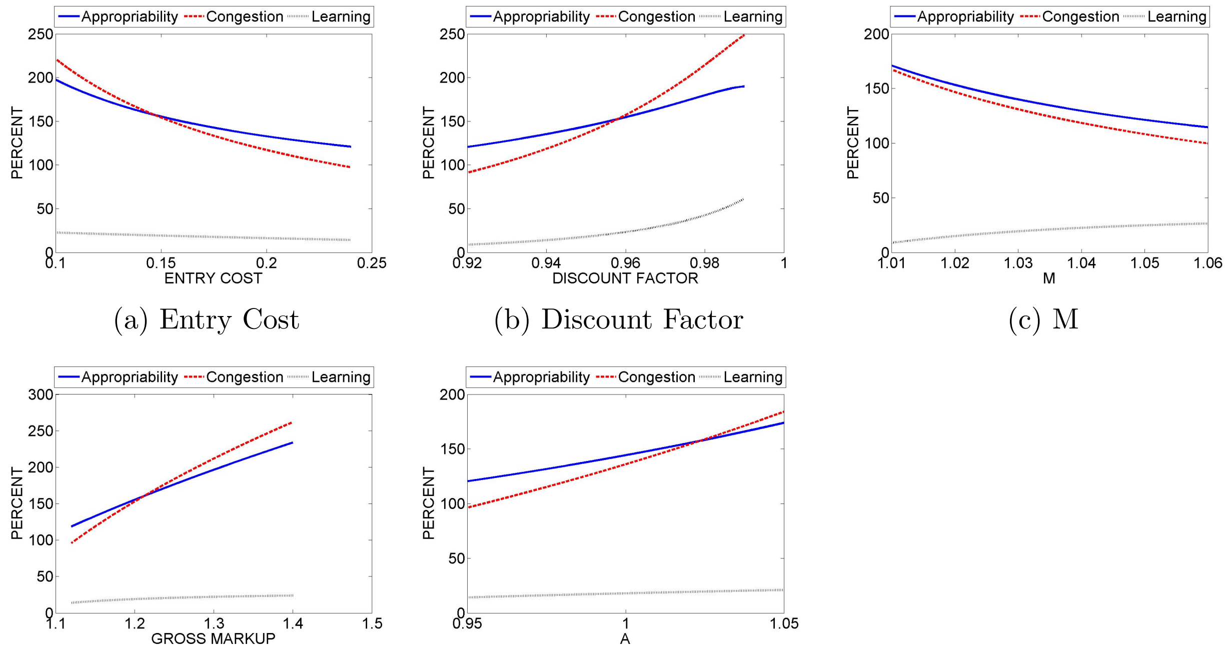
(d) Markup (e) A

Figure 6: Optimal Tax Rate

and the appropriability externality internalized, the decentralized market tightness is 4.2373, which is more than twice as large as the second-best. Moreover, the tax rate is considerably large for all parameter values considered — at least 50% (Figure 6). This suggests that the business-stealing effect due to coordination frictions is likely to be large in practice and important when considering policies.

Furthermore, the externalities as a percentage of the entry cost turn out to be $\mathcal{A} = 144\%$, $\mathcal{L} = 17.94\%$, and $\mathcal{C} = -136\%$. The relative size of these externalities remains roughly the same for the range of parameter values considered (Figure 7). Furthermore, the congestion externality is always comparable in magnitude to the appropriability externality. This provides further support for the intuition that the business-stealing effect is likely to be large in practice and relevant for policy makers.

Although the calibrated version of the model implies the decentralized economy exhibits too little innovation, this is not always the case. Figure 2 reports the decentralized tightness as a percentage of the second best when we vary each of the parameters. In particular,



(a) Entry Cost

(b) Discount Factor

(c) M

(d) Markup

(e) A

Figure 7: Externalities

for small values of the entry cost, η, and large markup (small λ) the second best market tightness is smaller than the decentralized one. This is the case because for these parameter values the congestion externality is large enough to dominate both the appropriability and learning externalities, so as to push the decentralized economy towards over-investment in equilibrium. Moreover, the ratio is increasing in the discount factor, the markup, the probability of success, and the productivity parameter A, whereas, it is decreasing in the entry cost and M.

5 Empirical Analysis

This section uses panel firm-level data on patents granted between 1976 and 2006 to provide further support for the ubiquity of simultaneous innovation. Some of the previous literature has focused on major historical innovations (see, for example, Lemley (2011)) while others, such as Cohen and Ishii (2005), have looked at more recent examples in the context of patents. Cohen and Ishii (2005) find that for the period between 1988 and 1996 0.6% of all granted patents in the U.S. were declared in interference (two innovators have filed for the same innovation within three months of each other).[34] Furthermore, this phenomenon is still observed today. For example, Siemens applied for a patent for a positron emission tomography scanner on April 23, 2013 (application number 13/868,256). Most claims are rejected because Philips (application number 14/009,666 filed on March 29, 2012 and application number 14/378,203 filed on February 25, 2013) has simultaneously made a similar innovation.[35]

Moreover, the findings in Cohen and Ishii (2005) are likely to significantly underestimate the amount of simultaneous innovation because of the tight time interval. In particular, when a firm innovates and applies for a patent the application is generally not disclosed by the USPTO until a patent is issued.[36] Given the considerable time lag associated with most patent grants, it is possible that a rival files for the same innovation one or even two years after the initial innovator without being aware of that initial application.[37] Although this case would not be counted as interference, the two firms did make the same innovation at almost the same time. In particular, it is not hard to find recent examples of patent applications which were rejected because a firm has previously filed a patent application more than 3 months in the past and the patent application was still kept secret at the time the rival files for a patent. For example, on November 1, 2012 Google Inc. filed a patent application (number 13/666,391) for methods, systems, and apparatus that provide content to multiple linked devices. All twelve claims are rejected because of simultaneous innovations made by Yahoo! Inc. (application number 13/282,180 with filing date October 26, 2011), Microsoft Corporation (application number 13/164,681 with filing date June 20, 2011), and

[34] For major innovations the period of interference is six months.

[35] The information on the patent applications is taken from the U.S. Patent and Trademark Office Patent Application Information Retrieval. Philips's applications were made public on January 23, 2014 and January 15, 2015. Siemens' patent application was rejected on September 10, 2015. The examiner rejected most claims under 35 U.S.C. 103 citing the two patent applications in the text, as well as a patent held by the National Institute of Radiological Sciences in Japan (patent application number 12/450,803).

[36] Applications filed on or after November 29, 2000 are generally made public 18 months after their effective filing date.

[37] For example, 98.87% of all patents granted in the U.S. between 1976 and 2006 had a patent grant lag of at least 1 year and 71.74% of at least 2 years.

Comscore Inc. (application number 13/481,474 with filing date May 25, 2012). In particular, all three patent applications which form the basis of rejection were not public at the time Google Inc. submitted its patent application.[38]

Motivated by the above considerations, I use firm-level patent data to test for the prediction in the model borne out by simultaneous innovation and the associated coordination frictions — higher aggregate number of patent applications implies that all else equal, each firm has a lower chance to secure a patent and hence, will have a lower number of patents granted. I find that the data does provide evidence in support of this hypothesis — controlling for the mass of ideas and firm-level quantity and quality of patent applications, a one percentage point increase in the relevant number of patent applications leads to 0.88% decrease in the average number of patents a firm receives.

5.1 Empirical Methodology

I adapt the empirical model of the patent production function (see, for example, Hall *et al.* (1986) and Fabrizio and Tsolmon (2014)) to account for the aggregate number of patent applications and patents in force. Those are included due to the intuition from the model in the preceding sections. If the phenomenon of simultaneous innovation is present, then a higher aggregate number of patent applications (keeping the stock of ideas fixed) implies that more firms will, on average, apply for the same innovation (since there is higher congestion) which means that each firm will have a lower chance of securing a patent and, controlling for firm-level quality and quantity of patent applications, a lower number of patents granted.

I include the number of patents in force for two reasons. First, an increase in the aggregate number of patent applications does not necessarily imply a higher market tightness, instead it might simply be a response to a higher mass of ideas. Thus, motivated by the assumption that knowledge is cumulative (discussed in the preceding sections), I use past innovation, captured

[38]The information on the patent applications is taken from the U.S. Patent and Trademark Office Patent Application Information Retrieval. Yahoo! Inc.'s application was made public on May 2, 2013, Microsoft's application was made public on December 20, 2012, and Comscore Inc.'s application was made public on December 20, 2012. Google's patent application was rejected on November 20, 2014. The examiner rejected the application under pre-AIA 35 U.S.C. 103(a) citing the three patent applications in the text.

by the number of patents in force, as a proxy for the current stock of research avenues. Thus, keeping patents in force constant, an increase in the aggregate number of patent applications corresponds to an increase in the market tightness. Second, patents might have a strategic aspect that allows their owner to block rivals from innovating, and subsequently patenting related innovations.[39] Thus, since a higher number of patent applications in the aggregate translates to a higher number of patents, I include the number of patents in force to ensure that the estimated effect of a higher aggregate number of patent applications captures only the congestion we are interested in and not the reduction in the number of patents because of the strategic effect of a higher number of rivals' patents.

Furthermore, as firm-level data on the quantity and quality of patent applications is not available, I proxy for these by using a set of firm-level controls. As the number of patents granted to each firm is a count variable, I estimate the following equation using the Poisson quasi-maximum likelihood estimator (Wooldridge, 1999):

$$\begin{aligned} E[P_{i,t}|I_t] = \exp\big[\sum_{i=0}^{L_{\text{Apps}}} \beta_i \Delta\ln(\text{Apps}_{t-i}) + \sum_{i=0}^{L_{\text{PatsInForce}}} \gamma_i \Delta\ln(\text{PatsInForce}_{t-i}) \\ + \alpha_i + t + \sum_{j=1}^{k} \sum_{i=0}^{L_j} \delta_{j,i} X_{j,t-i}\big] \end{aligned} \tag{24}$$

where $P_{i,t}$ is the number of successful patent applications for which firm i applied at time t, $\Delta\ln(\text{Apps}_t)$ is the growth rate of aggregate patent applications at time t, $\Delta\ln(\text{PatsInForce}_t)$ is the growth rate of the number of patents in force at time t, $X_{j,t}$ represents firm-level controls, and α_i represents firm-level fixed effects. The main coefficient of interest is β_1 — a negative estimate would provide a support for the model's prediction borne out of the possibility of simultaneous innovation that a higher number of patent applications increases congestion.

I include the growth rate of Apps_t, rather than its natural log because of unit root considerations.[40] The equation includes lags of $\Delta\ln(\text{Apps}_t)$ to account for the patent grant

[39] See, for example, Bessen and Meurer (2006), Hall *et al.* (2014), and Choi and Gerlach (2017).

[40] An augmented Dickey-Fuller test on $\ln(Apps_t)$ over the period $1964 - 2014$ yields a test statistic of 1.813

lag observed in the data. In particular, we would like to capture the congestion — how many firms work on the same innovation simultaneously. If a firm applies for a patent after its rival, but before the rival has received the patent we would like to count this a simultaneous innovation. This is because prior to November 2000 the USPTO did not have a policy of making most patent applications public. Thus, it is reasonable to assume that firms which apply for a patent at time t do not observe successful patent application filed by time t before the patent is granted. In particular, 28.26% of all patents granted in the U.S. between 1976 and 2006 have a grant lag of less than two years. Thus, the appropriate measure of the mass of innovations should include Apps_t and Apps_{t-1}, as most of the successful patent applications in that pool were not granted at time t. In contrast, most of the successful patent applications filed at time $t-2$ or earlier were granted by time t.[41] Hence, the coefficient on these estimates would likely not capture any of the congestion we are interested in, but will instead capture the effect of strategic blocking and learning. In particular, I do include $\Delta\ln(\text{Apps}_{t-2})$ in the regression equation for robustness and to capture any possible learning not absorbed by the number of patents in force.

Thus, the regression equation features

$$\begin{aligned}\sum_{i=0}^{L_{\text{Apps}}} \beta_i \Delta\ln(\text{Apps}_{t-i}) &= \beta_0 \Delta\ln(\text{Apps}_t) + \beta_1 \Delta\ln(\text{Apps}_{t-1}) + \beta_2 \Delta\ln(\text{Apps}_{t-2}) \\ &= \beta_0 \ln(\text{Apps}_t) + (\beta_1 - \beta_0)\ln(\text{Apps}_{t-1}) + (\beta_2 - \beta_1)\ln(\text{Apps}_{t-2})\end{aligned}$$

Hence, the coefficient β_1 captures the relevant congestion in the market — the percentage response of $E[P_{i,t}|I_t]$ to a one percent increase in the relevant mass of innovations, $(\text{Apps}_t+\text{Apps}_{t-1})$. Through the lens of the model, we can interpret β_1 as the elasticity of the probability of securing a monopoly position, $(1 - e^{-\theta})/\theta$, with respect to the mass of R&D firms, μ_t. To see this clearly, observe that we can decompose the average number of

with a 10% critical value of -2.6, whereas the same test on $\Delta\ln(\text{Apps}_t)$ over the period $1965 - 2014$ yields a test statistic of -5.991 with a 1% critical value of -3.587.

[41] 72.64% of all successful patent applications in the period between 1976 and 2006 in the U.S. have a grant lag of no more than two years.

successful patent applications as $E[P_{i,t}|I_t] =$Pr(the application is granted)$\times$Applications$_{i,t}$ $=$ $((1-e^{-\theta})/\theta)\times$Applications$_{i,t}$, where Applications$_{i,t}$ is the number of firm's innovations, i.e. patent applications which are of high enough quality to warrant a patent. As the firm-level quantity and quality of applications is independent of the aggregate number of patent applications, it follows that $\beta_1 = -(1-e^{-\theta}-\theta e^{-\theta})/(1-e^{-\theta})$.

I include lags of PatsInForce$_t$ to better proxy for the mass of ideas. Lastly, for consistency with the growth rate of patent applications, I include the growth rate of PatsInForce$_t$ rather than their natural log.[42]

5.2 Data and Variables

To construct the sample I started with the NBER Patent Data (Hall *et al.* (2001), HJT henceforth) which consists of $3,279,509$ unique patent-assignee observations and covers all utility patents granted by the USPTO between 1976 and 2006. To mitigate truncation problems, I drop all applications filed prior to 1975 and post 2002. Additionally, after dropping all observations for which information on assignees is not available, the sample size reduced to $2,550,892$ observations. This sample was then used to calculate the total number of patents per year of patent application filling date per assignee. The data was then matched with Compustat using the unique company identifier, gvkey. This resulted in a panel of $11,957$ firms covering $333,193$ observations and $1,061,995$ patents. After dropping observations which have missing firm-level control variables there remained $49,913$ observations on $5,901$ firms and $967,820$ patents. Lastly, I dropped 609 observations with only one firm-year observation and $1,335$ firms ($7,783$ observations) which have zero total patents in the sample years. This resulted in a data set of $41,566$ observations covering $966,688$ patents by $3,957$ firms.

Table 1 describes variables used and their sources. The dependent variable in the regressions, NumPats$_{i,t}$, is the count of patented inventions by firm and year of patent application.

[42]Also, there is strong evidence that ln(PatsInForce$_t$) contains a unit root. Its first difference appears to be $I(0)$, however. As the series appears to have a prominent break in its level, I apply a Zivot-Andres unit root test (Zivot and Andrews, 1992) to Δln(PatsInForce$_t$) for the period of $1965-2014$. The minimum $t-$statistic is -5.111 while the 5% critical value is -4.80.

Table 1: Variable Description and Sources

Variable	Description	Level	Data Source
Endogenous Variables			
$\text{NumPats}_{i,t}$	Count of patented inventions by application year	Firm	NBER Patent Data
Exogenous Variables			
Deflator_t	Implicit GDP deflator	Aggregate	U.S. Bureau of Economic Analysis (NIPA Table 1.1.9)
Apps_t	Total utility patent applications	Aggregate	USPTO (U.S. Patent Statistics Chart Calendar Years 1963 − 2015)
PatsInForce_t	Number of patents in force	Aggregate	USPTO (Historical Patent Data Files)
$\text{NomR\&D}_{i,t}$	Nominal private R&D expenditures	Firm	Compustat
$\text{NomSales}_{i,t}$	Nominal net sales	Firm	Compustat
$\text{Emp}_{i,t}$	Number of employees	Firm	Compustat
$\text{NomPPE}_{i,t}$	Nominal gross value of property, plant, and equipment	Firm	Compustat
$\text{R\&D}_{i,t}$	(Real) Private R&D expenditures	Firm	$100\times\text{NomR\&D}_{i,t}/\text{Deflator}_t$
$\text{Sales}_{i,t}$	(Real) Net sales	Firm	$100\times\text{NomSales}_{i,t}/\text{Deflator}_t$
$\text{PPE}_{i,t}$	(Real) Gross value of property, plant, and equipment	Firm	$100\times\text{NomPPE}_{i,t}/\text{Deflator}_t$

Table 2: Descriptive Statistics

Variable	N	N(Firms)	Mean	SD	Min	Median	Max
			Aggregate Variables				
$\Delta\ln(\text{Apps}_t)$	28	N/A	0.042	0.052	−0.084	0.045	0.121
$\Delta\ln(\text{PatsInForce}_t)$	28	N/A	0.013	0.024	−0.017	0.008	0.072
			Firm-Level Variables				
$\text{NumPats}_{i,t}$	41,566	3,957	23.257	120.708	0	1	4,344
$\ln(\text{R\&D}_{i,t})$	41,566	3,957	2.112	2.224	−6.468	2.041	9.359
$\ln(\text{Sales}_{i,t})$	41,566	3,957	5.226	2.546	−6.283	5.160	12.356
$\ln(\text{Emp}_{i,t})$	41,566	3,957	0.103	2.257	−6.908	−0.023	6.809
$\ln(\text{PPE}_{i,t})$	41,566	3,957	4.394	2.595	−3.468	4.131	12.981

Sample period: 1975 − 2002

The firm-level controls include current and one-period-lagged natural log of (i) firm's real expenditures in R&D, $\ln(\text{R\&D})_{i,t}$; (ii) company size, measured by the total number of employees, $\ln(\text{Emp})_{i,t}$; (iii) firm's real value of property, plant, and equipment, $\ln(\text{PPE})_{i,t}$; (iv) firm's real net sales, $\ln(\text{Sales})_{i,t}$. All real variables were deflated using the implicit GDP deflator (from NIPA table 1.1.9). The aggregate number of patent applications, Apps_t represents all utility patent applications submitted to the USPTO. The data on the numbers of patents in force, PatsInForce_t, is taken from the USPTO Historical Patent Data Files (Marco *et al.*, 2015). Table 2 provides summary statistics for the key variables, whereas Table 3 contains the correlations of firm-level variables.

Table 3: Correlations

Variables	$\text{NumPats}_{i,t}$	$\ln(\text{R\&D}_{i,t})$	$\ln(\text{R\&D}_{i,t-1})$	$\ln(\text{Sales}_{i,t})$	$\ln(\text{Sales}_{i,t-1})$	$\ln(\text{Emp}_{i,t})$	$\ln(\text{Emp}_{i,t-1})$	$\ln(\text{PPE}_{i,t})$
$\text{NumPats}_{i,t}$	1.000							
$\ln(\text{R\&D}_{i,t})$	0.378	1.000						
$\ln(\text{R\&D}_{i,t-1})$	0.382	0.977	1.000					
$\ln(\text{Sales}_{i,t})$	0.321	0.743	0.749	1.000				
$\ln(\text{Sales}_{i,t-1})$	0.317	0.726	0.741	0.984	1.000			
$\ln(\text{Emp}_{i,t})$	0.323	0.748	0.753	0.960	0.951	1.000		
$\ln(\text{Emp}_{i,t-1})$	0.322	0.738	0.754	0.955	0.958	0.991	1.000	
$\ln(\text{PPE}_{i,t})$	0.337	0.774	0.786	0.941	0.937	0.947	0.946	1.000
$\ln(\text{PPE}_{i,t-1})$	0.334	0.757	0.778	0.935	0.940	0.940	0.949	0.992

Sample period: $1975-2002$. $N = 41,566$. All values are significant at the 0.1% level.

5.3 Empirical Results

The results from the estimation, controlling for different number of lags of $\Delta\ln(\text{PatsInForce}_t)$, are given in Table 4. The estimates provide strong support in favor of the model's prediction that higher congestion reduces the probability a patent application is successful. The coefficient β_1 is negative and strongly significant in all specifications. As the model predicts, controlling for the mass of ideas, proxied by PatsInForce, impacts the estimates. In column 1, where we do not control for the patents in force, the magnitude of β_1 is about half that in the other columns. The seventh, eight, and ninth lags of $\Delta\ln(\text{PatsInForce}_t)$ are all insignificant and their inclusion does not affect the estimate of the coefficient of interest, β_1, thus in the robustness checks that follow I use the equation in column 8 as the benchmark.

The results are robust to including further lags of $\Delta\ln(\text{Apps}_t)$ (Table 5). Including the third and fourth lag does not affect the significance of β_1, although it increases slightly in magnitude. The results are also robust to changes in the firm-level controls. Table 6 presents estimates for the benchmark model when we vary the lag structure of firm-level controls. The significance of the coefficient of interest, β_1, does not change and its point estimate varies only slightly. This is true even in column 1 where we do not include any firm level controls. This suggests that firm's number of patent applications does not vary substantially from year to year and the quantity and quality of patent applications made by a firm may be reasonably captured by the firm-level fixed effects alone. The results are also robust to reasonable changes in the sample period (Table 7). In all cases considered β_1 remains negative and significant. The magnitude changes by only a little.

Moreover, these estimates are consistent with the calibration of $\theta = 1.0876$ from the

Table 4: $\Delta\ln(\text{PatsInForce}_t)$ Lags table

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
$\Delta\ln(\text{Apps}_{t-1})$	−0.436**	−0.991***	−1.026***	−1.080***	−1.081***	−1.015***	−1.007***	−0.875***	−0.894***	−0.868***	−0.855***
	(0.168)	(0.200)	(0.213)	(0.223)	(0.228)	(0.233)	(0.232)	(0.216)	(0.230)	(0.217)	(0.221)
$\Delta\ln(\text{PatsInForce}_t)$		2.618**	3.167***	3.400***	3.396***	3.363***	4.121***	4.158***	4.193***	4.262***	4.515***
		(0.867)	(0.843)	(0.876)	(0.859)	(0.862)	(0.918)	(0.923)	(0.917)	(0.953)	(0.922)
$\Delta\ln(\text{PatsInForce}_{t-1})$			−0.684	2.007***	2.017***	1.960***	0.265	0.222	0.236	0.106	−0.105
			(0.718)	(0.528)	(0.554)	(0.569)	(0.640)	(0.651)	(0.680)	(0.621)	(0.723)
$\Delta\ln(\text{PatsInForce}_{t-2})$				−3.364***	−3.357***	−3.989***	−3.830***	−4.069***	−4.091***	−4.045***	−4.083***
				(0.822)	(0.788)	(0.800)	(0.785)	(0.840)	(0.802)	(0.783)	(0.773)
$\Delta\ln(\text{PatsInForce}_{t-3})$					−0.0229	−0.182	−1.040**	−1.146**	−1.149**	−1.154**	−1.101**
					(0.380)	(0.354)	(0.379)	(0.390)	(0.387)	(0.389)	(0.410)
$\Delta\ln(\text{PatsInForce}_{t-4})$						1.828**	2.789***	2.427***	2.391***	2.379**	2.340**
						(0.607)	(0.684)	(0.654)	(0.726)	(0.733)	(0.729)
$\Delta\ln(\text{PatsInForce}_{t-5})$							5.885***	6.134***	6.140***	6.189***	6.250***
							(1.041)	(1.114)	(1.099)	(1.103)	(1.089)
$\Delta\ln(\text{PatsInForce}_{t-6})$								3.096***	3.079***	2.940***	2.911**
								(0.881)	(0.931)	(0.887)	(0.893)
$\Delta\ln(\text{PatsInForce}_{t-7})$									−0.171	−0.260	−0.338
									(0.742)	(0.730)	(0.751)
$\Delta\ln(\text{PatsInForce}_{t-8})$										0.835	1.041
										(0.863)	(0.714)
$\Delta\ln(\text{PatsInForce}_{t-9})$											−0.642
											(0.717)
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Trend	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Applications	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N(Firms)	3,957	3,957	3,957	3,957	3,957	3,957	3,957	3,957	3,957	3,957	3,957
N	41,566	41,566	41,566	41,566	41,566	41,566	41,566	41,566	41,566	41,566	41,566
χ^2	495.5	532.6	567.7	603.1	607.4	655.8	682.9	704.3	707.8	857.4	860.9

Controls indicates the inclusion of $\ln(\text{R\&D}_{i,t})$, $\ln(\text{Sales}_{i,t})$, $\ln(\text{Emp}_{i,t})$, $\ln(\text{PPE}_{i,t})$, and their first lags. Applications indicates the inclusion of $\Delta\ln(\text{Apps}_t)$ and $\Delta\ln(\text{Apps}_{t-2})$. Robust standard errors, clustered by firm are in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 5: $\Delta\ln(\text{Apps}_t)$ Lags table

	(1)	(2)	(3)
$\Delta\ln(\text{Apps}_{t-1})$	-0.875^{***}	-0.971^{***}	-1.001^{***}
	(0.216)	(0.271)	(0.278)
$\Delta\ln(\text{Apps}_{t-3})$	No	Yes	Yes
$\Delta\ln(\text{Apps}_{t-4})$	No	No	Yes
Fixed Effects	Yes	Yes	Yes
Trend	Yes	Yes	Yes
Applications	Yes	Yes	Yes
Controls	Yes	Yes	Yes
PatsInForce	Yes	Yes	Yes
N(Firms)	3,957	3,957	3,957
N	41,566	41,566	41,566
χ^2	704.3	732.1	772.2

Controls indicates the inclusion of $\ln(\text{R\&D}_{i,t})$, $\ln(\text{Sales}_{i,t})$, $\ln(\text{Emp}_{i,t})$, $\ln(\text{PPE}_{i,t})$, and their first lags. PatsInForce indicates the inclusion of the growth rate of patents in force and its first six lags. Applications indicates the inclusion of $\Delta\ln(\text{Apps}_t)$ and $\Delta\ln(\text{Apps}_{t-2})$. Robust standard errors, clustered by firm are in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 6: Firm-Level Controls Table

	(1)	(2)	(3)	(4)	(5)
$\Delta\ln(\text{Apps}_{t-1})$	-0.863^{***}	-0.873^{***}	-0.875^{***}	-0.752^{***}	-0.716^{***}
	(0.228)	(0.220)	(0.216)	(0.203)	(0.204)
$\ln(\text{R\&D}_{i,t})$, $\ln(\text{Sales}_{i,t})$, $\ln(\text{Emp}_{i,t})$, $\ln(\text{PPE}_{i,t})$	No	Yes	Yes	Yes	Yes
$\ln(\text{R\&D}_{i,t-1})$, $\ln(\text{Sales}_{i,t-1})$, $\ln(\text{Emp}_{i,t-1})$, $\ln(\text{PPE}_{i,t-1})$	No	No	Yes	Yes	Yes
$\ln(\text{R\&D}_{i,t-2})$, $\ln(\text{Sales}_{i,t-2})$, $\ln(\text{Emp}_{i,t-2})$, $\ln(\text{PPE}_{i,t-2})$	No	No	No	Yes	Yes
$\ln(\text{R\&D}_{i,t-3})$, $\ln(\text{Sales}_{i,t-3})$, $\ln(\text{Emp}_{i,t-3})$, $\ln(\text{PPE}_{i,t-3})$	No	No	No	No	Yes
Fixed Effects	Yes	Yes	Yes	Yes	Yes
Trend	Yes	Yes	Yes	Yes	Yes
Applications	Yes	Yes	Yes	Yes	Yes
PatsInForce	Yes	Yes	Yes	Yes	Yes
N(Firms)	6,247	4,333	3,957	3,508	3,089
N	173,778	46,234	41,566	37,232	33,585
χ^2	354.5	577.0	704.3	706.5	788.7

PatsInForce indicates the inclusion of the growth rate of patents in force and its first six lags. Applications indicates the inclusion of $\Delta\ln(\text{Apps}_t)$ and $\Delta\ln(\text{Apps}_{t-2})$. Robust standard errors, clustered by firm are in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 7: Sample Period Table

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\Delta\ln(\text{Apps}_{t-1})$	-0.875^{***}	-0.776^{***}	-0.884^{***}	-0.839^{***}	-0.747^{***}	-0.777^{***}	-0.735^{***}
	(0.216)	(0.221)	(0.229)	(0.214)	(0.214)	(0.220)	(0.220)
Sample	$1975-2002$	$1975-2001$	$1975-2000$	$1976-2002$	$1977-2002$	$1976-2001$	$1977-2000$
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Trend	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Applications	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
PatsInForce	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N(Firms)	3,957	3,776	3,541	3,879	3,790	3,698	3,374
N	41,566	39,278	37,038	40,187	38,804	37,901	34,286
χ^2	704.3	664.8	682.3	680.3	660.9	662.2	673.8

Controls indicates the inclusion of $\ln(\text{R\&D}_{i,t})$, $\ln(\text{Sales}_{i,t})$, $\ln(\text{Emp}_{i,t})$, $\ln(\text{PPE}_{i,t})$, and their first lags. PatsInForce indicates the inclusion of the growth rate of patents in force and its first six lags. Applications indicates the inclusion of $\Delta\ln(\text{Apps}_t)$ and $\Delta\ln(\text{Apps}_{t-2})$. Robust standard errors, clustered by firm are in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 8: Patents By Technological Category

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\Delta\ln(\text{Apps}_{t-1})$	-0.875^{***}	0.0711	-1.946^{***}	-0.936^{*}	-0.578^{*}	-0.245	-0.586^{*}
	(0.216)	(0.221)	(0.353)	(0.447)	(0.287)	(0.230)	(0.278)
HJT Category	All	Chemical	Computers & Communications	Drugs & Medical	Electrical & Electronic	Mechanical	Others
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Trend	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Applications	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
PatsInForce	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N(Firms)	3,957	1,734	1,932	1,215	1,898	1,813	1,869
N	41,566	22,397	22,463	14,632	24,726	24,270	24,586
χ^2	704.3	226.4	1036.9	285.5	502.7	315.0	235.9

Controls indicates the inclusion of $\ln(\text{R\&D}_{i,t})$, $\ln(\text{Sales}_{i,t})$, $\ln(\text{Emp}_{i,t})$, $\ln(\text{PPE}_{i,t})$, and their first lags. HJT designates the technological category as defined in Hall *et al.* (2001). PatsInForce indicates the inclusion of the growth rate of patents in force in the corresponding technological category and its first six lags. Applications indicates the inclusion of $\Delta\ln(\text{Apps}_t)$ and $\Delta\ln(\text{Apps}_{t-2})$. $\Delta\ln(\text{Apps}_{t-3})$ and $\Delta\ln(\text{Apps}_{t-4})$ are also included in the estimation of column (4). Robust standard errors, clustered by firm are in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

preceding section. In particular, the calibrated value of θ implies that the elasticity is -0.44712 which is inside the 95% confidence interval of all estimates in Tables 5, 6, and 7, except for the benchmark estimation and column (3) in Table 5 for which the 95% confidence interval is slightly above that value.

As a last robustness check, I consider the implied congestion by patent category. To this end, I construct the variables $\text{NumPatsCat}j_{i,t}$, for $j = 1, \ldots, 6$, which represent the total number of successful patent applications in technological category j filed in year t by firm i. I use the classification in HJT, where the six technological categories are "Chemical", "Computers & Communications", "Drugs & Medical", "Electrical & Electronic", "Mechanical", and "Others". Table 8 presents the results. Column 1 is the benchmark specification and each of the other 6 columns uses $\text{NumPatsCat}j_{i,t}$ as the dependent variable. The results provide further support for the model's prediction. The coefficient β_1 is negative and significant at the 5% level for all categories except "Chemical" and "Mechanical".

It should be noted that, to the best of my knowledge, there is no data on the total number of patent applications filed within a year in a given technological category. Thus, in each specification I had to use the total number of patent applications. Hence, it is plausible that there is no evidence in favor of the congestion hypothesis in these two categories because of data limitation. Another plausible explanation is that most firms in these industries do not rely on patents to secure a monopoly position, but instead resort to other mechanisms such as secrecy or complexity. If this is the case, then patents do not capture most of the innovation that goes on in these categories, and hence, most of the congestion. In particular, survey data from Cohen *et al.* (2000) favors this explanation. When asked for what percentage of product innovations are patents considered an effective property rights protection mechanism the average response across all manufacturing firms is 34.83%. The response of firms in the "Food" and "Textiles" industries (both in HJT subcategory 11 of the "Chemical" technological category) was 18.26% and 20%, respectively. The response of "Mineral Products", "Metal", and "Steel" (subcategories 51 and 52 of HJT "Mechanical") was 21.11%, 20%, and 22%, respectively. Thus, if these industries do not find patents as effective it stands to reason that they do not rely heavily on patents.

6 Conclusion

This paper develops an endogenous growth model where firms' decisions to direct their R&D effort towards a particular research project out of an endogenously determined mass of potential research avenues and the coordination frictions implied in this choice play a central role. In equilibrium, the number of firms which innovate the exact same idea is a random variable which follows a Poisson distribution with a parameter given by the market tightness in the market for ideas. This matching technology gives rise to a concave aggregate varieties production function — the productivity of aggregate R&D investment is diminishing because higher levels of R&D effort imply higher congestion in the market for ideas, as captured by the market tightness. The tightness is, in turn, determined by the relative profitability of R&D projects because higher congestion implies innovating firms have a lower chance of securing a monopoly position. The ultimate source of growth in the economy is the expansion of the mass of ideas due to learning. Innovation today permanently reduces future coordination problems, effectively reducing the cost of discovering new varieties. The market tightness and the matching technology endogenously determine the growth rate of the economy.

The paper argues that studying coordination frictions is of particular interest in the current context. In particular, these frictions generate a dynamic inefficiency — each period a fraction of all ideas are uninnovated. This foregone innovation translates into a lower growth rate. Furthermore, coordination frictions amplify the fraction of wasteful simultaneous innovation. Furthermore, the simultaneous innovation that takes place in the model generates a business-stealing effect — whenever a firm innovates, there is a chance that a rival would simultaneously innovate the same idea and receive a patent for the innovation, effectively stealing the firm's monopoly rents. This business-stealing effect leads to a congestion externality which induces firms to over-invest in equilibrium. Because of this, implementing the second-best allocation requires the government to impose a tax on R&D activities.

The analysis suggests that these inefficiencies and the business-stealing effect are of particular practical interest. A calibration of the model implies that 34% of all ideas are uninnovated, which leads to a reduction in the growth rate by about one third. The welfare loss from the resulting foregone consumption is quite sizable — eliminating the coordination frictions leads to 13% gain in welfare. At the same time 39% of innovations are wasteful duplication of effort. A fraction that is 25% larger than the corresponding one in the frictionless economy. Furthermore, implementing the second-best requires the government to impose a tax on R&D spending at the rate of 118%. This is because the congestion externality due to the business-stealing effect is an order of magnitude larger than the learning one.

Lastly, this paper also analyzes firm-level data on patents granted between 1976 and 2006 to test for the prevalence of simultaneous innovation. I find that the data provides evidence in strong support for the presence of congestion in the market for ideas — all else equal, a one percent increase in the relevant mass of patent applications leads to a 0.88% decrease in the expected number of patents granted per firm.

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7 Appendix

7.1 Appendix A: Welfare Comparison

I follow Akcigit *et al.* (2016) and compare the welfare difference between the frictional and frictionless economies in consumption equivalent terms. In particular, consider the welfare of the frictional economy, W, and the frictionless economy, W^c, along their BGPs. Suppose at time $t = 0$, both economies start at the same initial position with $N_0 = N_0^c$. Now, welfare in the frictional economy is given by

$$W = \sum_{t=0}^{\infty} \beta^t \ln C_t = \ln\left((1+g)^{\frac{\beta}{(1-\beta)^2}} C_0^{\frac{1}{1-\beta}} \right)$$

Similarly, the welfare in the frictionless economy is given by

$$W^c = \ln\left((1+g^c)^{\frac{\beta}{(1-\beta)^2}} (C_0^c)^{\frac{1}{1-\beta}} \right)$$

Then, let α measure the fraction with which initial consumption in the frictional economy, C_0, must be increased for consumers to have the same welfare as people in the frictionless economy. Thus, α solves

$$\ln\left((1+g)^{\frac{\beta}{(1-\beta)^2}} (C_0(1+\alpha))^{\frac{1}{1-\beta}} \right) = W^c$$

Hence,

$$\alpha = e^{(1-\beta)(W^c - W)} - 1$$

This measure of welfare is used throughout the text.

7.2 Appendix B: Proofs Omitted from the Text

Proof of Proposition 1:

Proof. I follow previous literature (see, for example, Julien *et al.* (2000)) and threat the mass of entrants, μ_t and ideas, ν_t, as finite. Then the resulting equilibrium outcome is evaluated at the limit as $\mu_t, \nu_t \to \infty$ (keeping θ_t constant), so as to characterize the behavior in a market with continuum of firms and ideas.

First, look at the firm's probability of securing a monopoly position given that there are exactly n competitors that chose to innovate the same project, $Pr(\text{monopoly}|n)$. By assumption, each firm has an equal chance of securing a patent, so the probability is given by $1/(n+1)$. In a symmetric equilibrium all firms place the same probability s_i of directing their effort towards a particular idea i. Then, the chance that a firm would face exactly n

competitors for the idea is

$$Pr(n) = \binom{\mu_t - 1}{n} s_i^n (1 - s_i)^{\mu_t - 1 - n}$$

Hence, the probability of securing a monopoly position is given by

$$\begin{aligned} Pr(\text{monopoly}) &= \sum_{n=0}^{\mu_t - 1} Pr(\text{monopoly}|n) P(n) = \sum_{n=0}^{\mu_t - 1} \binom{\mu_t - 1}{n} s_i^n (1 - s_i)^{\mu_t - 1 - n} \frac{1}{n+1} = \\ &= \sum_{n=0}^{\mu_t - 1} \frac{(\mu_t - 1)!}{(n+1)!(\mu_t - 1 - n)!} s_i^n (1 - s_i)^{\mu_t - 1 - n} = \frac{1}{\mu_t} \sum_{n=0}^{\mu_t - 1} \binom{\mu_t}{n+1} s_i^n (1 - s_i)^{\mu_t - 1 - n} = \\ &= \frac{1}{\mu_t s_i} \left(\sum_{n=0}^{\mu_t} \binom{\mu_t}{n} s_i^n (1 - s_i)^{\mu_t - n} - (1 - s_i)^{\mu_t} \right) = \frac{(s_i + (1 - s_i))^{\mu_t} - (1 - s_i)^{\mu_t}}{\mu_t s_i} = \\ &= \frac{1 - (1 - s_i)^{\mu_t}}{\mu_t s_i} \end{aligned}$$

Next, I will show that $s_k = s_j$ for all $k, j \in \nu_t$. Suppose not. Then, there exists some k, j such that $s_k > s_j$. But for any $i \in \nu_t$, we have that

$$\frac{\partial Pr(\text{monopoly})}{\partial s_i} = \frac{\mu_t^2 s_i (1 - s_i)^{\mu_t - 1} - \mu_t [1 - (1 - s_i)^{\mu_t}]}{(\mu_t s_i)^2}$$

For any $s_i \in (0, 1)$, it follows that $Pr(\text{monopoly})$ is decreasing in s_i if and only if

$$(1 - s_i)^{\mu_t - 1} < Pr(\text{monopoly})$$

which clearly holds since $\mu_t \geq 2$. Now, for $s_i = 1$, we have that $\partial Pr(\text{monopoly})/\partial s_i = -1/\mu_t < 0$. Furthermore, it is easy to see that $\lim_{s_i \to 0} \partial Pr(\text{monopoly})/\partial s_i = -(\mu_t - 1)/2 < 0$. Hence, $Pr(\text{monopoly})$ is decreasing in s_i everywhere in its domain. Then, $s_k > s_j$ implies that $Pr_k(\text{monopoly}) < Pr(\text{monopoly})_j$, which then implies that $Pr_k(\text{monopoly}) V_{k,t} < Pr_j(\text{monopoly}) V_{j,t}$ since all varieties are equally profitable. Thus, $s_k > s_j$ cannot be an equilibrium. Hence, we must have $s_i = s_j$ for all $i, j \in \nu_t$. Thus, $s_i = 1/\nu_t$.

Then, it follows that

$$Pr(i \text{ is matched with exactly } n \text{ firms}) = \binom{\mu_t}{n}\Big(\frac{1}{\nu_t}\Big)^n\Big(1-\frac{1}{\nu_t}\Big)^{\mu_t-n}$$

Taking the limit as $\mu_t, \nu_t \to \infty$ while keeping the ratio θ_t constant, we get that

$$Pr(i \text{ is matched with exactly } n \text{ firms}) \to \frac{\theta_t^n e^{-\theta_t}}{n!}$$

This concludes the proof.

■

Proof of Proposition 2:

Proof. First, it is obvious that $g_X = g_L = g_\pi = 0$. Next, from the production function, it follows that $g_Y = g_N$. Along the BGP g_C is constant, then from the Euler equation it follows that $g_V + \pi/V_t$ is constant. As g_V must be constant as well, it follows that $g_V = g_\pi = 0$. Next, from free entry, it follows that $g_\theta = 0$, which implies that $g_\mu = g_\nu$. From the laws of motion of ideas and varieties, it follows that $g_N = (1 - e^{-\theta_t})\nu_t/N_t$, hence, $g_N = g_\nu$. Then, from the resource constraint it is straightforward to establish that $g_C = g_N$. This concludes the proof.

■

Proof of Proposition 3

Proof. Totally differentiating both sides of (6) with respect to π yields

$$\frac{\mathrm{d}\theta}{\mathrm{d}\pi} = \frac{\beta}{\eta}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} > 0$$

which is positive since $1 - e^{-\theta} - \theta e^{-\theta} > 0$. As profits are increasing in A and decreasing in λ, the claims in the proposition follow. Similarly, totally differentiating (6) with respect to

β, η, M, and p yields

$$\frac{\mathrm{d}\theta}{\mathrm{d}\beta} = \Big[1 + \frac{\pi}{\eta}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big]\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} > 0$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}\eta} = -\frac{\beta\pi}{\eta^2}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big]\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} < 0$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}M} = -(1-e^{-\theta})\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} < 0$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}p} = e^{-\theta}\Big(\frac{\beta\pi}{\eta} - \theta(M-1)\Big)\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1}$$

From (6), it follows that $\beta\pi/\eta = (e^{-\theta} + (1-e^{-\theta})M - \beta)\theta/(1-e^{-\theta})$. Hence, θ is increasing in p if and only if $e^{-\theta} + (1-e^{-\theta})M - \beta > (1-e^{-\theta})(M-1)$, which always holds.

■

Proof of Proposition 4:

Proof. First, let us explicitly characterize the environment in the economy without frictions. The only difference to the economy with frictions is at the second stage in the innovation process. Coordination is achieved through the means of a centralized allocation of firms to ideas. In particular, upon entry, a Walrasian auctioneer directs firms' research efforts and assigns patents in the following way. First, if $\mu_t \leq \upsilon_t$, then each firm is directed towards a distinct project and each firm receives a patent. Second, if $\mu_t > \nu_t$, the auctioneer chooses ν_t firms at random, assigns each a distinct project, and grants each a patent over the corresponding variety. The rest $\mu_t - \nu_t$ firms are randomly assigned a project, but none of them receives a patent.

The assumption we have placed on the parameter vales, namely $\eta \leq (1-\lambda)\beta(\lambda^2 A)^{1/(1-\lambda)}L/[\lambda(M-\beta)]$, ensures that firms find all research avenues profitable. Hence, in equilibrium, all ideas are innovated, i.e. $\mu_t \geq \nu_t$, and each firm secures a patent with probability $Pr(\text{monopoly}) = 1/\theta_t$.

Hence, the laws of motion for ideas and varieties are given by

$$\nu_{t+1} = M\nu_t$$
$$N_{t+1} = N_t + \nu_t$$

Since the final good sector and the intermediate varieties production technology are as described in the text, it follows that in equilibrium it is still the case that $P_t(n) = 1/\lambda$, $X = (\lambda^2 A)^{1/(1-\lambda)}L$, $Y_t = (\lambda^{2\lambda} A)^{1/(1-\lambda)}LN_t$, $\pi = X(1-\lambda)/\lambda$, $V_t^c = \sum_{i=t+1}^{\infty} d_{it}\pi$, where the subscript c indicates the value of holding a monopoly position in the economy without coordination frictions. As all ideas are equally productive, the free entry condition is now given by

$$\eta = \frac{1}{\theta_t} V_t^c$$

Moreover, consumers face the same problem as in the text, so the Euler equation is analogous to (4):

$$V_t^c = \beta \frac{C_t}{C_{t+1}} \left(\pi + V_{t+1}^c \right)$$

Furthermore, the resource constraint is still given by (5).

One can establish in a manner analogous to that in the proof of Proposition 2 that we still have $g_Y = g_C = g_N = g_\mu = g_\nu$. However, now from the law of motion for ideas, it follows that $g_\nu = M - 1$. In particular, this is the case because firms find innovation profitable, so a mass $\mu_t \geq \nu_t$ enters the R&D sector and because the centralized allocation of firms to ideas ensures that there is no foregone innovation.

Next, using the laws of motion for ideas and varieties, it follows that along the BGP we still have, $\nu/N = M - 1$. Furthermore, from the resource constraint, it follows that we have

$$\frac{C}{N} = \frac{1+\lambda}{\lambda}\pi - \eta\theta^c(M-1)$$

where the subscript c indicates that the market tightness is evaluated at its value along the BGP in the economy without frictions. Lastly, using the free entry condition and the Euler equation, it follows that the market tightness is given by

$$\theta^c = \frac{\beta\pi}{\eta(M-\beta)} \tag{25}$$

Next, we can compare the percent of wasteful innovations in the two economies. In the economy without coordination problems there are μ_t innovations and ν_t of those are beneficial. Hence, $\omega^c = 1 - 1/\theta^c$. Then, observe that $V_t > V_t^c$ because the growth rate in the economy with coordination frictions, $(1-e^{-\theta})(M-1)$, is always smaller than the growth rate in the economy without frictions, $M-1$. Then, using the two free entry conditions, it follows that $\omega = 1 - \eta/V_t > 1 - \eta/V_t^c = \omega^c$.

Next, from (25) it follows that

$$\frac{\theta^c}{1-e^{-\theta}} = \frac{\beta\pi}{\eta(M-\beta)(1-e^{-\theta})} > \frac{\beta\pi}{\eta(1+(1-e^{-\theta})(M-1)-\beta)} = \frac{\theta}{1-e^{-\theta}}$$

where the inequality follows because $\beta < 1 \Rightarrow 1 + (1-e^{-\theta})(M-1) - \beta > (M-\beta)(1-e^{-\theta})$. Hence, $\theta^c > \theta$.

■

Proof of Proposition 5:

Proof.

Lemma 1. *Along the BGP, the second best features $g_\nu = g_N$.*

Proof. Proceed by contradiction. If $g_\nu > g_N$, then the resource constraint, (7), is eventually violated or $\theta_t \to 0$. But, if θ_t tends to 0, then $g_\nu \to (M-1)\mu_t/\nu_t \to 0$, since $\theta_t \to 0$ implies that $g_\mu < g_\nu$. Since, $g_N \geq 0$, this leads to a contradiction. Next, suppose that $g_\nu < g_N$. If $\theta_t \to$ constant > 0, then $g_N = (1-e^{-\theta_t})\nu_t/N_t \to 0$. If, on the other hand $\theta_t \to 0$, then $g_N = (1-e^{-\theta_t})\nu_t/N_t \to \mu_t/N_t \to 0$, since $\theta_t \to 0$ implies that $g_\mu < g_\nu$. But, $g_\nu \geq 0$, hence, we have a contradiction. Thus, $g_\nu = g_N$. ■

Since $g_\nu = g_N$ along the BGP, it follows that θ_t is also constant, for if not, then g_ν would not be constant. Then, from the resource constraint, (7), and the law of motion for ideas it follows that $g_C = g_N = g_\nu = (1 - e^{-\theta^*})(M-1)$, where θ^* is the second best market tightness. From (10), it follows that $1 + g_\phi = \beta(1+g_C)^{-1}$. From (11), it follows that $1 = g_h + \pi^*\phi_{t+1}/h_t$. Hence, $g_\phi = g_h$, since g_h is constant along the BGP. Then, (13) implies that $g_\lambda = g_\phi$ or $\lambda_t = 0$ along the BGP. But, if $\lambda_t = 0$, then (13) implies that $\eta = e^{-\theta^*}h_t/\phi_t$ and (12) implies that $\eta = (1 - e^{-\theta^*})h_t/(\theta^*\phi_t)$. Hence, it must be the case that $\theta^* = 0$ and $g_C = 0$. But then, (14) implies that $h_t/\phi_t = \beta\pi^*/(1-\beta)$. But, from (13) we have that $h_t/\phi_t = \eta/p$. Hence, it must be the case that $\eta = \beta\pi^* p/(1-\beta)$. But this is a contradiction, since by assumption $\eta < \beta\pi p/(1-\beta)$ and $\pi^* > \pi$. Hence, $\lambda_t > 0$ along the BGP and $g_\lambda = g_\phi$.

Then, using $g_\lambda = g_\phi = g_h$ together with $g_C = g_\nu = g_\mu = g_N = (1 - e^{-\theta^*})(M-1)$ and equations (13), (14, and (15) implies that θ^* solves

$$e^{-\theta^*} + (1 - e^{-\theta^*})M = \beta\Big(1 + \frac{\pi^*}{\eta}e^{-\theta^*} + (1 - e^{-\theta^*} - \theta^* e^{-\theta^*})(M-1)\Big)$$

Also, from the laws of motion for varieties and ideas and $g_\nu = g_N$, it follows that $\left(\frac{\nu}{N}\right)^* = M - 1$. Then, from the resource constraint, (7), it follows that $\left(\frac{C}{N}\right)^* = \pi^* - \eta\theta^*(M-1)$.

■

Proof of Proposition 6:

Proof. From equations (22) and (23), it follows that the magnitude of the congestion externality is larger than that of the learning externality if and only if

$$\left(\frac{h}{\phi}\right)^*\left(\frac{1 - e^{-\theta^*}}{\theta^*}\right) > e^{-\theta^*}\left(\left(\frac{h}{\phi}\right)^* + \left(\frac{\lambda}{\phi}\right)^*(M-1)\right) \tag{26}$$

From equations (13) and (14), it then follows that (26) holds if and only if

$$\frac{(1 - e^{-\theta^*})\beta\pi^*}{\theta^*\eta} > e^{-\theta^*} + (1 - e^{-\theta^*})M - \beta \tag{27}$$

Next, from the planner's solution, (17), it follows that

$$e^{-\theta^*} + (1 - e^{-\theta^*})M - \beta = \beta\Big(\frac{\pi^*}{\eta}e^{-\theta^*} + (1 - e^{-\theta^*} - \theta^* e^{-\theta^*})(M-1)\Big)$$

Then, $|\mathcal{C}| > \mathcal{L}$ if and only if $\pi^* - \eta\theta^*(M-1) > 0$. But this has to hold, from equation (16), as the second best allocation must feature $C_t > 0$.

■

Proof of Proposition 7

Proof. Totally differentiating (17) with respect to π^*, β, η, M, and p respectively and applying some algebra yields

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\pi^*} = \frac{\beta}{\eta}e^{-\theta^*}\left[e^{-\theta^*}(M-1) + p(1-\beta)\Big(e^{-\theta^*} + (1-e^{-\theta^*})M\Big)\right]^{-1} > 0$$

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\beta} = \Big(1 + \frac{\pi^*}{\eta}e^{-\theta^*} + (1 - e^{-\theta^*} - \theta^* e^{-\theta^*})(M-1)\Big)\left[e^{-\theta^*}(M-1) + p(1-\beta)\Big(e^{-\theta^*} + (1-e^{-\theta^*})M\Big)\right]^{-1} > 0$$

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\eta} = -\frac{\beta\pi^*}{\eta^2}e^{-\theta^*}\left[e^{-\theta^*}(M-1) + p(1-\beta)\Big(e^{-\theta^*} + (1-e^{-\theta^*})M\Big)\right]^{-1} < 0$$

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}M} = -\Big((1-\beta)(1-e^{-\theta^*}) + \beta\theta^* e^{-\theta^*}\Big)\left[e^{-\theta^*}(M-1) + p(1-\beta)\Big(e^{-\theta^*} + (1-e^{-\theta^*})M\Big)\right]^{-1} < 0$$

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}p} = \frac{1-\beta}{p}\Big(e^{-\theta^*} + (1 - e^{-\theta^*} - \theta^*)M\Big)\left[e^{-\theta^*}(M-1) + p(1-\beta)\Big(e^{-\theta^*} + (1-e^{-\theta^*})M\Big)\right]^{-1}$$

■

Proof of Proposition 8:

Proof. The government imposes a tax on R&D activities at a rate τ and subsidizes the purchase of intermediate varieties at a rate s. Furthermore, it keeps a balanced budget through the means of lump-sum transfers to households in the amount T_t. Thus, the government's budget constraint is given by

$$T_t = \int_0^{N_t} sP_t(n)X_t(n)dn - \tau\eta\mu_t$$

The final good firm chooses labor and intermediate inputs to maximize profits, now given by $Y_t - w_t L - \int_0^{N_t} (1-s) P_t(n) X_t(n) dn$. The first order conditions yield the same labor demand equation as in the text, $w_t = (1-\lambda) Y_t / L$, and an inverse demand function for intermediaries given by $P_t(n) = \lambda A L^{1-\lambda} X_t^{\lambda-1}(n)/(1-s)$.

At stage three of the innovation process, the monopolist faces an analogous problem as in the text. The only difference now is in the inverse demand function. Hence, in equilibrium, $P = 1/\lambda$, $X = [A\lambda^2/(1-s)]^{1/(1-\lambda)} L$, $\pi = (1-\lambda)X/\lambda$, $Y_t = [A(\lambda^2/(1-s))^\lambda]^{1/(1-\lambda)} L N_t$.

As in the economy without government intervention, all ideas are equally profitable, so the matching technology is as in the text. The free entry condition is now given by

$$\eta(1+\tau) = \frac{1-e^{-\theta_t}}{\theta_t} V_t$$

where the value of the monopoly position, V_t, is defined as in the text.

The laws of motion for ideas and varieties, and the Euler equation are as in the text. Hence, the value of the monopoly position is still given by (4). Furthermore, the resource constraint is still given by (5).

Along the BGP, we still have that $\nu_t / N_t = M - 1$, as the laws of motion for ideas and varieties are as in the text. Thus, from the resource constraint, (5) it follows that

$$\frac{C}{N} = \frac{1 - s - \lambda^2}{(1-\lambda)\lambda} \pi - \eta\theta(M-1)$$

Next, (4), the law of motion for ideas, and the free entry condition imply that

$$e^{-\theta} + (1-e^{-\theta})M = \beta\Big(1 + \frac{\pi}{\eta(1+\tau)}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big)$$

Then, setting $s = s^*$ implies that $\pi = \pi^*$ and setting $\tau = \tau^*$ implies that $\theta = \theta^*$. Thus, $C/N = (C/N)^*$.

Next, τ^* is positive if and only if

$$\frac{(1-e^{-\theta^*})\beta\pi^*}{\theta^*\eta} > e^{-\theta^*} + (1-e^{-\theta^*})M - \beta$$

The above is the exact same condition as (27) in the proof of Proposition 6, which has been shown to hold. Hence, $\tau^* > 0$.

$\blacksquare$