The Optimal Duration of Unemployment Benefits[*]

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Abstract: This paper studies the optimal duration of unemployment benefits in a basic job search model where a risk neutral unemployment insurance agency can not monitor the search effort of risk-averse workers. Unemployment assistance is taken as exogenous by the unemployment agency who chooses optimally the tax rate, the level of unemployment benefit and the maximum date of their exhaustion. We deal with non-stationarity complexities and show that the solution of the agency's problem always exists with, at some conditions, a finite optimal duration of unemployment benefits.

Keywords: Moral hazard, Search, Potential benefits duration, Unemployment insurance.

JEL Classification: D83, C61, C63, J65

1 Introduction

Unemployment insurance (UI) programs are generally limited in time but the fan of their potential duration is strikingly large across countries. For example, unemployment benefits last only 6 months in the United States, whereas in Europe they can vary from 6 (in United Kingdom) to 38 months (in the Netherlands). A special case is Belgium where the duration of unemployment benefits is not really determined but depends on numerous criteria such as the gender, the age or the local average duration of unemployment. In general, European countries offer a more generous level and duration of constant unemployment benefits and there is a widespread consensus that this institutional feature of European labor markets is associated with longer duration and higher aggregate level of unemployment (OECD, 2006). [1] The well-known argument (for these negative effects due to unemployment insurance) is that generous unemployment benefits discourage search

[1]More or less strict monitoring and sanctions or eligibility criteria are in the same time implemented to offset the disincentive effect of UI (Venn, 2012). Some empirical results suggest that these policies could quicken job finding (see Fredriksson and Holmlund (2006) for a review).

effort and exert a positive pressure on wages. So a limited period of eligibility intuitively appears as a suitable instrument for containing moral hazard problem created by weak monitoring of search efforts and the resulting financial costs.[2]

Beside the insurance system, there is always an assistance or solidarity system that is not subject to prior contributive participation and that gives some monetary support to those who are no more eligible for unemployment benefits. The maximum duration of unemployment benefits thus defines a time sequence of insurance and assistance policies that differs considerably among countries (as pointed out previously).

The purpose of this paper is to characterize the optimal duration of unemployment insurance in presence of social assistance.

This work is related to the literature on optimal unemployment insurance in search models with moral hazard which has been vastly enriched since the seminal papers of Baily (1978) and Shavell and Weiss (1979). Because of moral hazard, the optimal profile of unemployment benefits is endlessly decreasing and the optimal wage tax could increase with the length of previous unemployment (Hopenhayn and Nicolini, 1997, 2009). The reason is that a decreasing benefits profile and an increasing tax profile give stronger incentives to search by making more costly lasted job search. Whatever the policy instruments used, the definition of an optimal UI must finally balances its smooth consumption benefits to its discouraging effects on search effort. Smooth consumption can also be achieved by self-insurance devices if agents have access to financial markets and an optimal UI scheme should take into account this possibility if moral hazard problem exists. This assumption is consistent with evidences according to which search intensity is affected by the agent's wealth (Card, Chetty and Weber, 2007, Chetty 2008) but introducing free financial market access would make the problem intractable. So, for the sake of simplicity, savings or borrowing behaviors are not allowed.[3]

The issue we particularly address in our work is to know how to optimally choose the moment of decreasing unemployment payments.

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[2] There are indeed strong empirical results that extending eligibility duration have a negative effect on unemployment duration (see e.g. Lalive, (2007, 2008) ; Rothstein, 2011).

[3] A recent branch of the research on UI expectedly shed some light on how savings and borrowing behaviors can affect the optimal timing of unemployment benefits. Firstly, Kacherlakota (2004) considered a model where job search effort and savings are unobservable. He showed that if the disutility of effort is linear in the probability of getting a job, the optimal level of unemployment benefits should be constant. Shimer and Werning (2008) use a job search model where the assumption of hidden reservation wage is combined with constant absolute risk aversion (CARA) preferences. The optimal policy is characterized by a flat profile of benefits and taxes and a free access to savings by the means of a riskless asset. In other words, with CARA utility, the optimal insurance design becomes independent of the agent's wealth. However, constant relative risk preferences only alter marginally their results. With hidden savings, CARA utility function and binary unobservable search effort, Mitchell and Zhang (2010) demonstrate that the agent is not saving or borrowing constrained but his consumption decreases more than what suggested by Hopenhayn and Nicolini (1997). Rendahl (2012) finally points up the importance of the CARA hypothesis and, relaxing it, shows that the optimal profile of UI crucially depends on the wealth level of the agent. The optimal profile of consumption is decreasing but the optimal sequence of unemployment benefits is increasing, precisely to efficiently compensate the fall in consumption.

Davidson and Woodbury (1997) are the first who question the optimal termination of benefits in a search and matching framework[4] where wages are exogenous and number of jobs fixed. In their model, utility function is linear but risk aversion comes from the assumption that search costs are convex in search effort. With the help of numerical exercises, they conclude that the optimal duration of unemployment for risk-averse workers should always be infinite, which renders suboptimal almost all existing UI programs. As noticed by Fredriksson and Holmlund (2006), they do not examine the optimal time sequence of unemployment benefits since they only compare a situation where unemployment benefits are unlimited to a situation where benefits are limited then arbitrarily set to zero. Fredrikson and Holmlund (2001) also argue that the assumption of an exogenous social payment is the cause of the optimal infinite duration in the paper of Davidson and Woodburry (1997).

More recently, some papers using the recursive contract methodology investigate the effects of human capital depreciation and duration dependence on the design of an optimal unemployment insurance system. Pavoni (2009) confirms the optimal decreasing profile of unemployment benefits and finds that unemployment payments are bounded below by an endogenous assistance level if human capital depreciation is sufficiently fast. Pavoni and Violante (2007) provide the social planner with three unemployment policies: unemployment insurance, costly search effort monitoring and social assistance. Taxes and wage subsidies complete the labor markets policies. As in Pavoni (2009), job finding probabilities and wages are essentially driven in their model by human capital skills evolution, as optimal search effort is constant during unemployment. With human capital depreciation, the optimal time sequence of policies begins with decreasing unemployment benefits, switches in job-search monitoring and then turns into social assistance both with constant payments. In the latter state, the social planner only insures the unemployed with zero search effort required. This result comes directly from the binary search effort and the exogenous human capital depreciation assumptions. Indeed, when the level of human capital is too low, the exit rate of unemployment is mechanically also very weak and it is no more optimal to induce costly search effort. Numerical simulations of the model give some estimates of the duration of each policy to illustrate quantitatively their normative results. However, the role of exogenous human capital depreciation could be overestimated. Spinnewijn (2013) considers the introduction of a training program during unemployment to alleviate the negative effects of human capital depreciation and discusses how it affects the optimal design of unemployment insurance. With training, search incentives and the optimal profile of consumption will depends on the initial value

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[4]The paper of Fredriksson and Holmlund (2001) considers the optimal profile of unemployment benefits in a Pissarides (2000) framework where search efforts and wages are endogenous but stationary that renders the analysis mathematically tractable. Stationarity comes from the assumption that the necessary waiting duration to change state is a random variable that follows a Poisson process. UI is then on average duration limited. Using numerical simulations, they find that the optimal profile is decreasing. Coles (2008) also ends analytically at this result in a matching equilibrium model with endogenous constant wage but nonstationary search effort. Cahuc and Lehmann (2000) investigates the welfare consequences on long-term unemployed of a decreasing benefits scheme and conclude that a flat profile is optimal from a Rawlsian point of view.

of human capital. If human capital is below a certain level, the worker is encouraged to train with no incentives to search. When the human capital becomes above that level, it is optimal to give incentives to search. Contrary to Pavoni and Violante (2007) or Pavoni (2009), the social planner always gives incentives to search to the long term unemployed. The optimal profile of consumption is consequently decreasing for these workers while it could be constant for short-term unemployed.

We start our analysis by integrating in a Mortensen (1977) framework the fact that insurance and social assistance roughly describe a decreasing profile of unemployment payments insofar as the latter program is less generous. This pattern of unemployment transfers indeed constitutes a common feature in most OECD countries. We then consider social assistance as a given policy for an unemployment agency which has to optimally chose the level of insurance benefits, their duration and the financing tax.

In our model, risk-averse workers receive an exogenous wage and pay tax to finance benefits. Search effort is unobservable so the unemployment agency never stop inducing unemployed to search, whatever the length of unemployment spell or unemployment status. Search cost is a continue function of search effort. As in Mortensen (1977), the search effort of an eligible unemployed monotonically converges to a stationary level at the exhaustion of the benefits. Another important feature of our model is that the budget constraint is supposed to be balanced in average but not at every time.

The main contribution of our paper is to provide some general conditions to obtain a finite optimal duration of unemployment benefits. The model under consideration that will be referred to as "the unemployment agency's problem" can be regarded as a dynamic optimization problem where the objective function, i.e. the expected utility of an eligible unemployed, as well as the inter-temporal budget constraint, are derived from solutions of differential equations. Even in our simple framework, non-stationarity brings significant complexities and we have to use some tricks to make the problem analytically more exploitable. Indeed, we extract some key properties of the model in order to rewrite a three dimensional optimization problem (in the tax rate, the level and the duration of unemployment benefits) to an implicit two dimensional one (in the tax rate and the level of unemployment benefits). We then explicitly define the duration of unemployment benefits as a function of the differences between the expected utility of an employee and unemployed workers ones. We succeed this way in circumventing some analytical difficulties caused by the non stationarity of search effort as with limited benefits, search effort increases as the worker nears the expiration of benefits (Mortensen, 1977; Van der Berg, 1990).

The paper unrolls as follows. In next section we present the agency's problem. Section **??** remodels the optimization problem in a three-dimensional model to make it tractable. In Section 4, we show that our problem can be rewritten in a more exploitable two-dimensional model and reformulate the agency's problem. In section 5, we calibrate the model to the US economy and solve it numerically, checking that the condition for a unique optimal non-zero finite duration of unemployment holds (Theorem 4.1). Section 7 concludes.

2 The model

Our framework is a standard continuous job search model *à la* Mortensen (1977) where mortal risk-averse workers have no access to financial markets and so cannot save or borrow. As in Hopenhayn and Nicolini (1997), we assume only search decisions, gross wage w consequently is set at an exogenous level. We also consider a two-tiered unemployment compensation system and so two types of job-seekers : those eligible to unemployment insurance benefits b and those who after a maximum period of eligibility T are still jobless. The latter benefit from the welfare or assistance system generally less generous. After T, the environment becomes stationary as assistance payments for noneligible unemployed, z, are unlimited. We assume, eventually, that insurance unemployment benefits and the employment tax rate denoted by τ are constant over time.

2.1 Worker's behavior

In the considered framework, each worker maximizes her expected discounted utility, which writes

$$E_0 \int_0^\infty e^{-rt} \left[u(x) - v(\varepsilon_t) \right] dt,$$

where $r \in (0, 1)$ is the discount rate, x the consumption and E_0 the expectation operator.

We assume that the instantaneous utility function of consumption $u(x)$ has standard properties, strictly increasing and concave whereas the effort ε_t is an additively source of disutility.

The search cost function is strictly positive and convex and, as for example in Chetty (2008), is defined by

$$v(\varepsilon_t) = c \frac{\varepsilon_t^{1+\alpha}}{1 + \alpha}, \text{ with } \alpha > 0.$$

In our partial model, an employee cannot search for another job and be laid off but, as an unemployed worker, can die according to a Poisson process with arrival rate μ. The expected utility of this employee is then

$$E = \frac{u(w - \tau)}{r + \mu}.$$

An unemployed worker chooses his search effort level ε and exit out of unemployment at rate $\lambda(\varepsilon)$. For the sake of simplicity and without loss of generality, we assume that $\lambda(\varepsilon) = \varepsilon$. The expected utility of an eligible unemployed, denoted by $U(t)$, satisfies the following Bellman equation

$$
\begin{aligned}
rU(t) - \dot{U}(t) &= \max_{\varepsilon_t} \{ u(b) - v(\varepsilon_t) + \lambda(\varepsilon_t) \left[E - U(t) \right] - \mu U(t) \} \qquad (2.1) \\
U(T) &= U_a,
\end{aligned}
$$

where U_a is the expected utility of a noneligible jobless that we will explicit later. As in Coles (2008), the flow value of being eligible unemployed at t equals the flow payoff

$u(b) - v(\varepsilon_t)$ augmented by the average gain of status change $\lambda(\varepsilon_t) [E - U(t)]$ and lowered by the loss caused by death, which occurs at rate μ. At the exhaustion of the insurance benefits period T, an eligible jobless becomes a noneligible one. Therefore his expected utility is U_a, what is formalized by the terminal condition $U(T) = U_a$. The first order condition for optimal search effort at t is given by

$$\varepsilon(t) = \left(\frac{E - U(t)}{c} \right)^{\frac{1}{\alpha}}. \tag{2.2}$$

It is straightforward that if $E = U(t)$ then $\varepsilon(t) = 0$ as there is no gain to search for a job. Moreover, the profile of $U(t)$ determine completely, in an opposite way, the evolution of $\varepsilon(t)$. Noneligible unemployed has a stationary environment since z is unlimited over time. The discounted expected utility U_a thus satisfies the following Bellman equation :

$$rU_a = u(z) - v(\varepsilon_a) + \lambda(\varepsilon_a)(E - U_a) - \mu U_a \tag{2.3}$$

and the first order condition for optimal search effort in this case is given by

$$\varepsilon_a = \left(\frac{E - U_a}{c} \right)^{\frac{1}{\alpha}} \tag{2.4}$$

Rearrange equation (**??**) taking into account the optimal search effort gives the following differential equation

$$\dot{U}(t) = (r + \mu)U(t) - \frac{\alpha}{1 + \alpha} c^{\frac{-1}{\alpha}} [E - U(t)]^{\frac{1+\alpha}{\alpha}} - u(b), \tag{2.5}$$

$$U(T) = U_a.$$

Observe that U_a is defined through equality (**??**) that can be rewritten as

$$(r + \mu)U_a - \frac{\alpha}{\alpha + 1} c^{\frac{-1}{\alpha}} (E - U_a)^{\frac{1+\alpha}{\alpha}} - u(z) = 0. \tag{2.6}$$

We will characterize in the next section the properties of the solution of this non linear differential equation.

2.2   Agency's behavior

Let $B(t)$, B_a, B_e be the value costs associated with insurance, assistance and work, respectively. The actuarially fair relation between benefits and taxes for an unemployed eligible worker is represented by the following Bellman equation

$$rB(t) = b + \varepsilon(t) [B_e - B(t)] - \mu B(t) + \dot{B}(t), \tag{2.7}$$

$$B(T) = B_a,$$

where B_e and B_a are given by

$$B_e = \frac{-\tau}{(r + \mu)}, \tag{2.8}$$

$$rB_a = \varepsilon_a [B_e - B_a] - \mu B_a. \tag{2.9}$$

The expected gain from an employee is given by (2.8) and the gain from an current noneligible unemployed is given by (2.9). Let us notice that the unemployment agency do not finance and provide the welfare payment z but captures the tax employment from a former noneligible unemployed. Then the inter-temporal budget constraint of the unemployment agency takes the form of the following first-order differential equation

$$\dot{B}(t) = (r + \mu + \varepsilon(t))B(t) + \frac{\tau}{(r + \mu)}\varepsilon(t) - b, \tag{2.10}$$

$$B(T) = B_a, \quad \text{with } B_a = -\frac{\tau\varepsilon_a}{(r+\mu)(r+\mu+\varepsilon_a)}.$$

This last relation is, from the point of view of the unemployment insurance agency, the financial counterpart of the worker's position on the labor market as the flow of taxes and benefits are contingent to the worker's job status. The unemployment agency collects employment taxes τ and distributes unemployment benefits b such as the net inter-temporal cost of an eligible unemployed at date 0 is nil. We do not impose a balanced budget at each date. As search is a private information, the agency then chooses b, τ and T so as to maximize worker's utility under the budget constraint. We then have to solve the following optimization problem

$$\max_{T,b,\tau} U(0) \tag{2.11}$$
$$\text{subject to : } B(0) = 0$$

where $U(.)$ and $B(.)$ are solutions of the differential equations (??) and (??), respectively. To solve analytically this problem we could be tempted to determine these explicit solutions but we shall see in the next section that if it is possible for the latter, it is not possible (in general) for the former. However, we circumvent this obstacle by fully using the intrinsic properties of equation (??).

2.3 The optimal contract: an overview of the main results

As described in the previous subsection we are interested in solving problem (??) with respect to nonnegative values of b and τ and positive values of T verifying the set of constraints (s1)-(s3) given below:

(s1) The utility and cost functions U and B satisfy the ordinary differential equations (??) and (??), respectively, and are such that

$$U(T) = U_a, \quad U(0) > U_a, \tag{2.12a}$$
$$B(T) = B_a, \quad B(0) = 0. \tag{2.12b}$$

(s2) The following additional conditions on the parameters are needed:

$$\tau \geq 0, \quad w - \tau \geq b, \tag{2.13a}$$
$$b \geq z. \tag{2.13b}$$

(s3) The discounted utility U_a at time T is implicitly given by (**??**).

Let us discuss briefly some of the conditions above. In our principal-agent framework, we suppose that the unemployment agency wants to propose to each agent that enters unemployment a contract that define $\{b, \tau, T\}$ such as the insured unemployment situation is better than the social assistance, subject to an appropriate budget constraint. Constraints (**??**) formalize this assumption and overall assure that a positive date of exhaustion T is a relevant parameter of our problem. Indeed, if $U(t) = U_a$, it comes that $\varepsilon(t) = \varepsilon_a$ and the environment of the model become stationary. Recall that for the sake of tractability, we presume a two-tier unemployment insurance scheme where unemployment payments and tax rate are constant over time. We then start by proceeding with the assumption that $U(0) > U_a$ and let the analysis assert wether this strict inequality is optimal or not. The interpretation of the additional conditions (s2) is obvious insofar as there is no rational incentive that drives a worker to accept a contract that enforces her to pay a tax in order to receive less that what she would have with social assistance.

In Theorem **??**, we provide a first mathematical formulation of the agency's problem that involves the unknown b and τ together with an additional parameter. It is worth noticing that a natural approach to problem (**??**) (at first sight) would require to computing explicitly the formulation of $U(0)$, which is not a simple task. However, we overcome this drawback by exploiting some key elements in our analysis.

Next, in Theorem **??**, we prove that the agency's problem can be recast as a two-dimensional optimization problem over a nonempty and bounded set with a geometrical aspect that depends on the fixed parameters. Specifically, we establishes that for some bounded and nonempty set $\Gamma \subset \mathbb{R}^2$ and for some real-valued mapping J (continuous and lower bounded over Γ) that any optimal solution $(\bar{b}, \bar{\tau}, \bar{T})$ of problem (**??**) can be alternatively obtained by an element $(\bar{b}, \bar{\tau}) \in \text{argmin}_\Gamma J$ (namely $(\bar{b}, \bar{\tau}) \in \Gamma$ and solves $\min_\Gamma J$), together with a positive optimal duration $\bar{T} = \bar{T}(\bar{b}, \bar{\tau})$ (depending on $\bar{b}$ and $\bar{\tau}$). This latter formulation puts out to the following two possibilities: either there exists a minimizer of J over Γ, which ensures the existence of optimal finite positive values of b, τ and T; or there exists no minimizer of J over Γ. The second case could be related to any situation in which the optimal duration is zero or infinite. We then solve numerically the model to deal with the issue.

3   Remodeling the agency's problem

A natural way to handle the maximization problem (**??**) would consists in solving the two non-linear differential equations (**??**) and (**??**). This is far from being a simple task regarding the former. Such an approach would lead us to face many technical difficulties. So we use some tricks so as to avoid them. The objective of this section is to provide a more tractable formulation of our model. As announced in the previous section, we

(sometimes) work with the two differences $W(t) = E - U(t)$ and $W_a = E - U_a$ instead of the expected utilities themselves. Let us stress that the unemployment benefits and the tax rate are assumed to be constant (for allowing us to reach this accommodation). As already mentioned we deals with only finite positive values of T. This approach is not restrictive at all. Note indeed that any (possible) optimal value $T = +\infty$ should necessarily appear as a limiting case of our strategy. In the whole section, we work with the existence of a two-tiered insurance unemployment scheme as given, i.d. a positive finite duration of unemployment T with $U(0) > U_a$.

In order to obtain a more suitable formulation of the worker's problem, we introduce the set $\mathcal{C}$ of elements (b, τ, U_0, T) such that:

(c1) b and τ fulfill condition (??);
(c2)   there exists some mapping $U : [0, T] \to \mathbb{R}$ verifying (??) together with
$U(0) = U_0$, $U(T) = U_a$ and $U_a < U_0$.

The set $\mathcal{C}$ plays a crucial role in our methodology. Indeed it will be proved (see Theorem ??) that the value of the initial cost can be expressed as $B(0) = \mathcal{B}(b, \tau, W_a, W(0))$. So it is not difficult to see that the worker's problem can be rewritten as

$$\max_{b, \tau, T, U_0} U_0 \tag{3.1a}$$

$$\text{subject to}: \quad \mathcal{B}(b, \tau, W_a, E - U_0) = 0, \tag{3.1b}$$

$$(b, \tau, U_0, T) \in \mathcal{C}, \tag{3.1c}$$

where $E = (1/\eta)u(w - \tau)$. A relevant simplification of the set $\mathcal{C}$ is then provided through the analysis of the utility function (by Theorem) so as to deduce a more tractable formulation of problem (??).

3.1   Analysis of the utility function

To begin with, we put out some properties of the utility function U relative to its values $U(0)$ (at time $t = 0$) and U_a (at some positive time T). Let us recall from (??) that the utility function U satisfies the following ordinary differential equation

$$\dot{U}(t) = \eta U(t) - \beta \left[E - U(t) \right]^{\frac{1+\alpha}{\alpha}} - u(b), \tag{3.2}$$

where η and β are positive constants (independent of b, τ and T) such that

$$\eta = r + \mu, \qquad \beta = \left(\frac{\alpha}{\alpha+1} \right) c^{-\frac{1}{\alpha}}. \tag{3.3}$$

The solution of such a system cannot be obtained (in general) on closed form, but we discuss its feasibility and we exhibit some of its useful properties. For the sake of simplicity, we introduce the two mappings F and H defined for $s \in \mathbb{R}_+$ by

$$F(s) = \eta s + \beta s^{\frac{\alpha+1}{\alpha}}, \tag{3.4a}$$

$$H(s) = F(s) - \gamma, \quad \text{where } \gamma = u(w - \tau) - u(b). \tag{3.4b}$$

Then it is obviously checked from the new variables $W = E - U$ and $W_a = E - U_a$ that (**??**) and condition (**??**) can be equivalently written as

$$\dot{W}(t) = H(W(t)), \quad W(T) = W_a, \tag{3.5}$$

with the additional condition $W(0) < W_a$. It can be furthermore noticed that W_a is given by (**??**) through the following equality

$$F(W_a) = \kappa, \quad \text{where } \kappa = u(w - \tau) - u(z). \tag{3.6}$$

Let us observe that γ and κ (introduced in (**??**) and (**??**)) are nonnegative parameters (under conditions (**??**)), since u is assumed to be strictly increasing on $(0, +\infty)$. Consequently, as F is strictly increasing, we deduce that there exist unique nonnegative values W_* and W_a such that $F(W_*) = \gamma$ (namely $H(W_*) = 0$) and $F(W_a) = \kappa$. These two specific values W_a (only depending on τ) and W_* (depending on b and τ) are obviously defined by

$$W_a = F^{-1}(u(w - \tau) - u(z)), \quad W_* = F^{-1}(u(w - \tau) - u(b)). \tag{3.7}$$

The next lemma shows us that the profile of the solution W of (**??**) on $[0, T]$ depends on the relative values of W_a and W_* (see Fig. **??**). Note also that the function $H(.)$ has convenient properties that are exploited in Lemma **??** for investigating existence, uniqueness and regularity of W (by using the Cauchy-Lipschitz theorem).

Lemma 3.1 *For any $T > 0$ and $W_a \geq 0$, problem (**??**) admits a unique classical solution W on $[0, T]$ that belongs to $C^\infty([0, T])$ and the following statements are reached:*
  (i1) $W_a > W_* \Rightarrow W$ *is increasing and, for $t \in [0, T[$, we have* $\quad W_* < W(t) < W_a$.
  (i2) $W_a = W_* \Rightarrow W \equiv W_*$ *on $[0, T]$.*
  (i3) $W_a < W_* \Rightarrow W$ *is decreasing and, for $t \in [0, T[$, we have* $\quad W_a < W(t) < W_*$.

Proof. See Appendix. ●

According to Lemma **??**, the expected utility of an insured unemployed $U(t)$ is decreasing, constant or increasing if the difference $W_a - W_*$ is respectively positive, null or negative. A decreasing expected utility $U(t)$ (obtained for $W_a > W_*$) means that unemployment benefits b is larger than the assistance payments z. When $W_a = W_*$ (namely $b = z$), we have $U(t) = U_a$ and so the expected utility of an insured jobless is independent of T. It can be verify from the cost function (**??**) that the budget constraint as well does not depend on T anymore and simply gives the level of tax rate τ with respect to the search effort ε. These results are very intuitive insofar as the only difference between the expected incomes of an insured unemployed and that of a non-insured jobless comes precisely from the gap between the level of unemployment benefits b and the assistance payments z. Note that the cases (i2) and (i3) in Lemma **??** are not compatible with our assumption that $U(0) > U_a$.
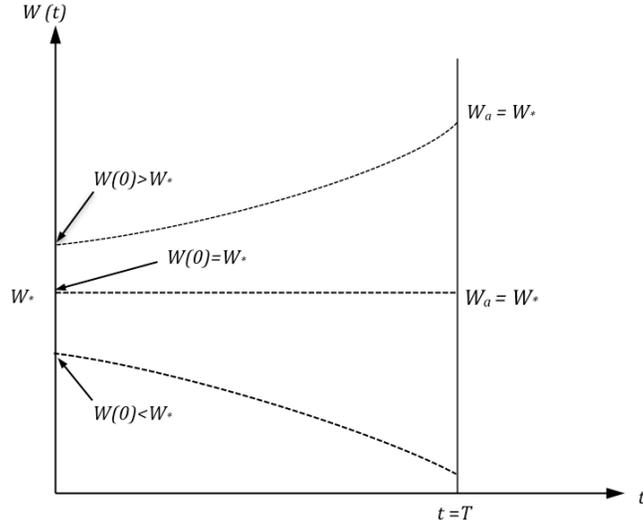
Figure 1: The evolution of $W(t)$ and the duration of unemployment

Figure **??** illustrates (for a positive and finite T) the evolution of the expected utility $U(t)$ and the search effort which is equal to the escape rate in our model. As in Mortensen (1977), the expected utility $U(t)$ declines and the search effort rises during insured unemployment, until the maximum benefit duration T. Once the unemployment benefits are exhausted, the search behavior as the gain from unemployment are stationary, respectively equal to ε_a and U_a. It could be easily checked that the entitlement effect pointed out by Mortensen (1977) also plays in our set up.

Observe from Lemma **??** that, for W satisfying (**??**), conditions (**??**) and (**??**) occurring in the set of constraints of the agency's problem can be replaced by

$$W_* \quad < \quad W(0) < W_a, \tag{3.8a}$$
$$b \quad > \quad z \quad \text{(which is equivalent to } W_* < W_a). \tag{3.8b}$$

Remark 3.1 Similarly, the set of constraints $\mathcal{C}$ arising in problem (**??**) can be also rewritten as the set of elements (b, τ, U_0, T) such that:

(c1)' $\tau \geq 0$, $w - \tau \geq b$ and $b > z$;

(c2)' $\exists\, W : [0, T] \to \mathbb{R}$ verifying (**??**) with $W(0) = E - U_0$ and $W_* < W(0) < W_a$.

The next lemma will be helpful for a key transformation of $\mathcal{C}$.

Lemma 3.2 *For any pair (W_0, W_a) of positive values verifying $W_* < W_0 < W_a$, there exist a unique positive time $T = \int_{W_0}^{W_a} \frac{1}{H(u)} du$ and some function $W \in C^\infty([0, T])$ that satisfies (**??**) with $W(0) = W_0$.*

Proof. See Appendix. •

The specific value of T appearing in Lemma **??** can be regarded as an integral formulation of the potential duration of benefits that depends on the differences of expected utility W_a and $W(0)$ and that will come in very handy for our analysis. Lemma **??** also shows us that, regarding condition (j2) related to the previous definition of $\mathcal{C}$, the existence of a mapping W verifying (**??**) can be omitted, so that (j2) can be reduced to $W_* < E - U_0 < W_a$, together with the specific value of T given in this lemma.

Now, we give a fruitful simplification of $\mathcal{C}$.

Lemma 3.3 *The constraints $\mathcal{C}$ arising in problem (**??**) can be rewritten as set of elements (b, τ, U_0, T) such that:*
   (j1) $\tau \geq 0$, $w - \tau \geq b$ *and* $b > z$;
   (j2) $W_* < E - U_0 < W_a$ *and* $T = \int_{E-U_0}^{W_a} \frac{1}{H(u)} du$.

Proof. This result is obtained as a straightforward consequence of Remark **??** and the Lemma **??**.

3.2 Exact formulation of the cost function

Contrary to the utility differential equation (**??**), the solution of cost differential equation (**??**) can be computed explicitly. The real issue of this subsection is to estimate the value at time $t = 0$ of the cost function $B(t)$ that derives from its knowledge at some positive time T.

The following lemma shows us that $B(0)$ can be formulated as a first step with respect to the quantity ψ defined by $\psi(t) = e^{-\eta t - \int_0^t \varepsilon(s)ds}$.

Lemma 3.4 *The cost function B at time $t = 0$ is given by*

$$B(0) = \left(B(T) + \frac{\tau}{\eta} \right) \psi(T) + (b + \tau) \int_0^T \psi(s)ds - \frac{\tau}{\eta}. \tag{3.9}$$

Proof. See Appendix. ●

The first term of equation (**??**) can be interpreted as the net cost of an non-insured unemployed. The second term is the net unemployment subsidy to an insured jobless as he does not pay tax, whereas the last term is the potential gain from an entirely life of employment, which lasts in average $1/\eta$ unit of time.

Computing the quantity ψ, its integral and the first expression in brackets that occur in the above formulation of $B(0)$ and substituting these last results into (**??**) amounts to the following lemma.

Lemma 3.5 *The differential cost equation (**??**) with $B(T) = B_a$ admits a unique classical solution on $[0, T]$ such that*

$$B(0) = \tau \frac{H(W(0))}{\rho(b)F'(W_a)} + (b + \tau)H(W(0)) \int_{W(0)}^{W_a} \frac{ds}{H^2(s)} ds - \frac{\tau}{\eta}, \tag{3.10}$$

*where $H(s) = F(s) - \gamma$, with $F(s) = \eta s + \beta s^{1+\frac{1}{\alpha}}$ and $\gamma = u(w - \tau) - u(b)$, while $\rho(b) = H(W_a(\tau))$, or equivalently (from (**??**) and (**??**))*

$$\rho(b) = u(b) - u(z). \tag{3.11}$$

Proof. See Appendix. ●

Lemma **??** provides an explicit expression of the cost function with respect to the differences $W(0)$ and W_a. It especially establishes that $B(0)$ is depending on T only through the value of $U(T)$.

3.3 A three-dimensional model

From now on, the values W_a and W_* (given in (**??**)) are sometimes denoted by $W_a(\tau)$ and $W_*(b, \tau)$, respectively, so as to be more precise. We also consider sometimes the more precise notations $H_{(b,\tau)}(s)$ and $\gamma(b, \tau)$ (instead of $H(s)$ and γ) given for some values (b, τ, s) by $H_{(b,\tau)}(s) = F(s) - \gamma(b, \tau)$, where $\gamma(b, \tau) = u(w - \tau) - u(b)$, with $F(s) = \eta s + \beta s^{1+\frac{1}{\alpha}}$ (introduced in (**??**)).

Clearly, from Lemma **??**, we recall that the constraint $(b, \tau, U_0, T) \in \mathcal{C}$ arising in problem (**??**) can be expressed as the two conditions given by (j2) together with condition (j1) that can be also expressed as $(b, \tau) \in \Omega$, where Ω is the bounded set defined by

$$\Omega = \{(b, \tau) \in \mathbb{R}^2; \quad b > z, \quad 0 \le \tau \le w - b\}. \tag{3.12}$$

Note also by Theorem **??** that we have
$B(0) = \mathcal{B}(b, \tau, W_a, W(0)),$
where

$$\mathcal{B}(b, \tau, W_a, W(0)) = \frac{\tau H(W(0))}{\rho(b)F'(W_a)} + (b + \tau)H(W(0)) \int_{W(0)}^{W_a} \frac{ds}{H^2(s)} - \frac{\tau}{\eta}. \tag{3.13a}$$

Then the following theorem is reached as an immediate consequence of (**??**).

Theorem 3.1 *the maximal value of $U(0)$ we are seeking is nothing but the value $\bar{U}_0$ obtained from any solution $(\bar{b}, \bar{\tau}, \bar{U}_0)$ of the following optimization problem*

$$\max_{b, \tau, U_0} U_0 \tag{3.14a}$$

$$\textit{subject to}: \quad \mathcal{B}(b, \tau, W_a(\tau), E(\tau) - U_0) = 0, \tag{3.14b}$$

$$W_*(b, \tau) < E(\tau) - U_0 < W_a(\tau), \tag{3.14c}$$

$$(b, \tau) \in \Omega. \tag{3.14d}$$

Moreover, the corresponding optimal duration $\bar{T}$ associated with $(\bar{b}, \bar{\tau}, \bar{U}_0)$ is given by

$$\bar{T} = \int_{E(\bar{\tau}) - \bar{U}_0}^{W_a(\bar{\tau})} \frac{1}{H_{(\bar{b}, \bar{\tau})}(s)} ds. \tag{3.15}$$

It can be noticed that $\Omega = \emptyset$ if $w \leq z$, and that Ω has a nonempty interior otherwise (that is if $w > z$). Throughout this paper, we thus assume that $z \in]0, w[$.

Let us underline that the use of standard Lagrangian techniques does not seem appropriate for investigating the constrained optimization problem (**??**). Indeed, we have to keep in mind that our purpose is to find sufficient conditions for the existence of a solution of (**??**) (which ensures the existence of a fixed optimal duration of unemployment taking into account the set of constraints (**??**), (**??**) and (**??**)). To this aim, we simplify in the next section the above formulation of the model.

4 An implicit two-dimensional model

In this section we show that (**??**) can be rewritten as a more exploitable two-dimensional minimization problem. This strategy essentially brings the numbers of relevant variables down to only two, b and τ. Such a transformation will be obtained by introducing the new variables $y = E - U_0$ and $\nu = E - U_a$ (namely, $\nu = W_a$). Clearly, we have $U_0 = E - y = (1/\eta)u(w - \tau) - y$. So model (**??**) can be alternatively considered through the following formulation

$$\min_{b, \tau, y} \quad y - (1/\eta)u(w - \tau) \tag{4.1a}$$

$$\text{subject to:} \quad \Phi(b, \tau, y, \nu(\tau)) = 0, \tag{4.1b}$$

$$W_*(b, \tau) < y < \nu(\tau), \tag{4.1c}$$

$$(b, \tau) \in \Omega, \tag{4.1d}$$

where $W_*(b, \tau) := F^{-1}(u(w - \tau) - u(b))$, $\nu(\tau) := F^{-1}(u(w - \tau) - u(z))$, Ω is defined in (**??**) (also see Fig. **??**), and where the mapping Φ is defined by

$$\Phi(b, \tau, y, \nu) \;\; = \;\; \tau H(y) + \rho(b) H'(\nu) \left((b + \tau) H(y) \int_y^\nu \frac{ds}{H^2(s)} - \frac{\tau}{\eta} \right). \tag{4.2a}$$

Remark 4.1 It can be checked that (**??**) is equivalent to (**??**). Moreover, the value of U_0 (arising in problem (**??**)) at some optimal point $(\bar{b}, \bar{\tau}, \bar{y})$ of (**??**) is given by $U_0 = E(\bar{\tau}) - \bar{y}$, which is associated with an optimal duration $\bar{T} = \int_{\bar{y}}^{\nu(\bar{\tau})} \frac{1}{H_{(\bar{b}, \bar{\tau})}(s)} ds$ (according to (**??**)).

Now, we focus on simplifying (**??**). To that end, for $(b, \tau) \in \Omega$, we discuss the existence and uniqueness of some real $y(b, \tau) \in [W_*(b, \tau), \nu(\tau)]$ such that $\Phi(b, \tau, y, \nu(\tau)) = 0$. For the sake of legibility we sometimes omit the parameters b and τ in the formulations of $\nu(\tau)$ and $W_*(b, \tau)$.

Lemma 4.1 *Let $(b, \tau) \in \Omega$ (hence $b > z$, so that $\nu(\tau) > W_*(b, \tau)$) and consider the mapping $\Phi_{(b,\tau)}$ defined for $s \in]W_*(b, \tau), \nu(\tau)]$ by $\Phi_{(b,\tau)}(s) = \Phi(b, \tau, s, \nu(\tau))$. Then $\Phi_{(b,\tau)}$ is continuous and (strictly) decreasing on $[W_*, \nu]$. Moreover, $\Phi_{(b,\tau)}$ has a zero $y(b, \tau) \in [W_*(b, \tau), \nu(\tau)]$ (which is unique) iff the pair (b, τ) also satisfies*

$$\zeta(b, \tau) \geq 0, \quad \text{where } \zeta(b, \tau) := \frac{b+\tau}{F'(W_*(b,\tau))} - \frac{\tau}{\eta}. \tag{4.3}$$

In particular, under condition (??) with $(b, \tau) \in \Omega$, $\Phi_{(b,\tau)}$ admits a unique zero $y(b, \tau)$ on $[W_*(b, \tau), \nu(\tau)]$ such that :
- *if $\tau > 0$ and $\zeta(b, \tau) > 0$ then $y(b, \tau) \in]W_*(b, \tau), \nu(\tau)[$;*
- *if $\tau > 0$ and $\zeta(b, \tau) = 0$ then $y(b, \tau) = W_*(b, \tau)$;*
- *otherwise, that is if $\tau = 0$, then $y(b, \tau) = \nu(\tau)$.*

Proof. See Appendix. •

Remark 4.2 It is established in the proof of Lemma **??** that $\Phi_{(b,\tau)}(\nu(\tau))$ is negative and that the two values of $\Phi_{(b,\tau)}$ on the boundary of $[W_*, \nu]$ are given by

$$\Phi_{(b,\tau)}(\nu(\tau)) = \tau H(\nu(\tau)) \left(1 - \frac{1}{\eta} F'(\nu(\tau))\right), \tag{4.4a}$$

$$\Phi_{(b,\tau)}(W_*(b, \tau)) = H(\nu(\tau)) F'(\nu(\tau)) \left(\frac{b+\tau}{F'(W_*)} - \frac{\tau}{\eta}\right). \tag{4.4b}$$

These two values above are then used so as to reach the results of Lemma **??**, by means of the intermediate value theorem.

Lemma **??** gives us general conditions that guarantee that the budget constraint (**??**) is binding. More specifically, the first case contemplates in the Lemma **??** ensures a positive value of the potential duration of unemployment T with positive values of b and τ that satisfy the budget constraint (**??**). The second point stipulates that the budget constraint can be satisfied with a positive value of τ such that $U(0) = U_a$. From (**??**) it comes that $b = z$, and the insurance and assistance unemployment systems also coexist. In our model where assistance payments is not subject to any eligibility criteria, it seems obvious that if $b = z$ the optimal duration of T should be zero. Indeed, the existence of an unemployment agency has in this case only a negative effect on the gain from employment, and consequently on the exit rate from unemployment, through a positive tax rate. The last point of Lemma (**??**) simply stipulates that if $\tau = 0$ then $b = 0$ and only exists the social part of the unemployment treatment. The value of unemployment is then only U_a and there is no rationale behind problem (**??**).

Now, we show that (**??**) can be transformed into a two-dimensional minimization problem over the set Γ defined as follows:

$$\Gamma = \{(b, \tau) \in \Omega; \quad \zeta(b, \tau) > 0\}, \quad \text{where } \zeta \text{ is given in (??)}. \tag{4.5}$$

Remark 4.3 Clearly, Lemma **??** tells us that there exists a real-valued mapping y that is implicitly defined for $(b, \tau) \in \Gamma$ by

$$\Phi(b, \tau, y(b, \tau), \nu(\tau)) = 0. \tag{4.6}$$

Remark 4.4 Let us set $S(b, \tau) = \{y \in \mathbb{R} | \; W_*(b, \tau) < y < \nu(\tau), \quad \Phi(b, \tau, y, \nu(\tau)) = 0\}$ for $(b, \tau) \in \Omega$. From Lemma **??** we observe that $S(b, \tau) = \{y(b, \tau)\}$ if $(b, \tau) \in \Gamma$, and $S(b, \tau) = \emptyset$ otherwise.

The following theorem provides a more exploitable formulation of (**??**) that will be useful with regard to theoretical and numerical viewpoints.

Theorem 4.1 *The agency's problem (**??**) can be reformulated as*

$$\min_{b, \tau} \quad J(b, \tau), \quad subject \; to : (b, \tau) \in \Gamma, \tag{4.7}$$

with the objective function J defined for $(b, \tau) \in \Gamma$ by

$$J(b, \tau) := y(b, \tau) - (1/\eta)u(w - \tau), \tag{4.8}$$

*where $y(b, \tau) \in]W_*(b, \tau), \nu(\tau)[$ is the unique value on $[W_*(b, \tau), \nu(\tau)]$ verifying (**??**). Moreover, the value of U_0 at some point $(\bar{b}, \bar{\tau}) \in \operatorname{argmin}_\Gamma J$ is given by $U_0 = -J(\bar{b}, \bar{\tau})$, with a non-zero and finite optimal duration*

$$\bar{T} = \int_{y(\bar{b}, \bar{\tau})}^{\nu(\bar{\tau})} \frac{1}{H_{(\bar{b}, \bar{\tau})}(s)} ds. \tag{4.9}$$

Proof. Theorem **??** is deduced from Remarks **??** and **??** (also see Lemma **??**). ●

Theorem **??** establishes that the optimal duration linked to our problem is non-null and finite whenever problem (**??**) admits a solution that belong to Γ. However, this latter condition is not guaranteed at all. Let us underly that Γ, the set of constraints of the minimization problem (**??**), is bounded but not closed. So any existing minimizer of the objective function J over Γ does not necessarily belong to Γ but to its closure (denoted by $\bar{\Gamma}$). Note also that J is bounded below over Γ, as the mapping $y(b, \tau)$ is positive on Γ and as the utility function u is assumed to be continuous on $]0, +\infty[$, hence the mapping $u(w - \tau)$ is bounded on $\bar{\Gamma}$ and Γ. This latter fact clearly guarantees the existence of a minimizer of J over $\bar{\Gamma}$, provided that Γ is nonempty. Consequently, a minimizer of J over Γ may be located outside of Γ, which does not guarantee (in general) the existence of a non-zero and finite optimal duration.

5 Simulations

In this section we give an illustrative numerical example of our model based on the american economy. We first calibrate some unknown parameters to fit some keys characteristics of the american labor market. We then simulate the model to determine the optimal contract.

5.1   Calibration of the model

Let us consider the very usual CRRA function u defined by

$$u(x) = \tfrac{1}{1-\lambda}x^{1-\lambda} \text{ with } \lambda \neq 1. \tag{5.10}$$

We then have a separable functional form for the current utility as used by Chetty (2008), among others. λ is calibrated to be 2 but we discuss later of its impact on the results. The time unit is one month. The interest rate is therefore set to $r = 0.00327$ which corresponds to an annual rate of 4%. In the United-Sates, life expectancy at birth is somehow above 78 years whereas the minimum legal age at which people can work is 15 years (OECD, 2014). The life expectancy in our model is set at 63 years since we consider only the working age population, which makes the death monthly rate $\mu = 0.0013$. The duration of unemployment benefits T is set at 6 months which is the duration of regular UI benefits. To calibrate the unemployment insurance program, we follow Nakajima (2012), who takes into account both monetary and non-monetary benefits of unemployment. Eligible UI benefits b and non-eligible unemployment benefits z are then respectively set to 1393 and 961 dollars with a mean wage equals to 3202.

Parameters		Value
α	Curvative parameter of disutility from search	0.827
c	Level parameter of disutility from search	0.019
μ	Death rate	0.0013
r	Monthly interest rate	0.00327
λ	Coefficient of relative risk aversion	2
T	Maximum period of UI eligibility in months	6
w	Gross wage	3202
b	Unemployment insurance benefits	1393
z	Unemployment assistance benefits	961
$1/\varepsilon_0$	Mean duration of unemployment for eligible unemployed at 0	5
$1/\varepsilon_a$	Mean duration of unemployment for non eligible unemployed	4

Table 1: Calibration

Finally, we need to assign a value to the level parameter of disutility of search c and a value to the curvature parameter of disutility from search α, for which there are no available informations. We then calibrate it such that the mean duration of unemployment for an eligible and an non-eligible unemployed are respectively 4 and 5 months. We then uncover the value of $\{c, \alpha, W_a, W_0, \tau\}$ that solve the system composed of the following constraints: the first order conditions for optimal search effort (**??**) and (**??**), the equality that gives U_a (**??**), the budget constraint (**??**) and the expression of the potential duration of unemployment benefits (**??**) from theorem (**??**).

5.2 Simulations results

Using the parameter values above, we illustrate first the evolutions of the search effort and the gains from employment. As expected from Lemma **??**, search effort increases monotonously over time (Fig. **??**) while the expected utility of an an eligible jobless, U_0, decreases (Fig. **??**).
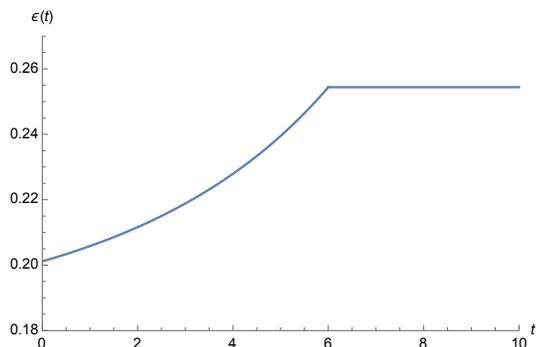

Figure 2: The evolution of $\varepsilon(t)$ and the duration of unemployment
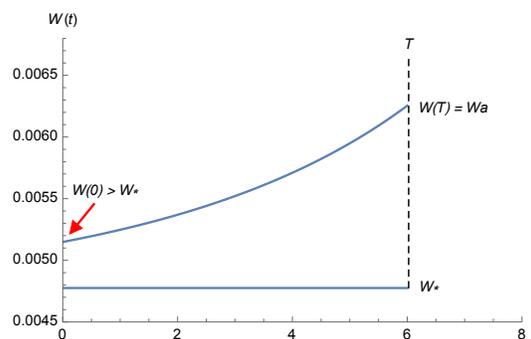

Figure 3: The gains from employment

The computation of the optimal contract is graphically illustrated in Figure **??** and Figure **??** and described in the first row of Table **??**. The optimal duration of unemployment benefits is 12.68 months, double the potential duration of the baseline model with more generous unemployment benefits but also a higher tax rate. The latter was equal to 21.9 in the baseline model *versus* 78.63 for the optimal contract. The greater generosity of the optimal contract reduces incentives to find a job since search efforts decrease significantly for the eligible unemployed to $\varepsilon_0 = 0.1$ (at $t = 0$) and, to a lesser extent, for the non-insured unemployed to $\varepsilon_a = 0.248$.
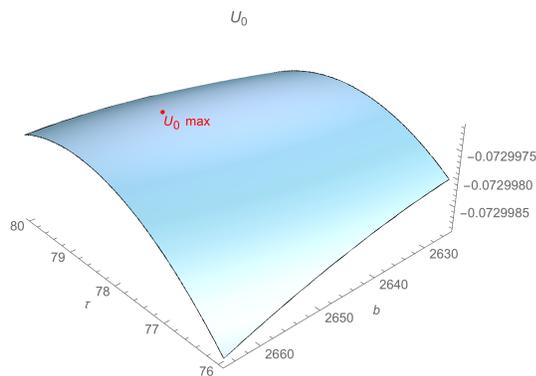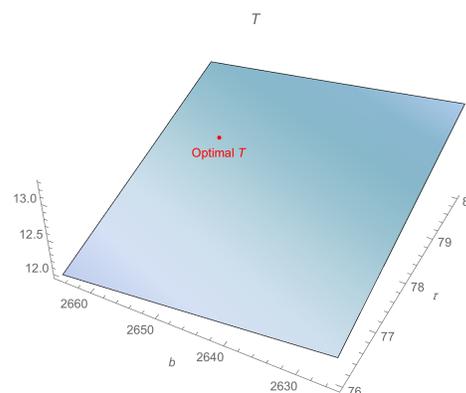

Figure 4: Expected utility at date 0

Figure 5: The optimal duration of unemployment benefits $\bar{T}$

Finally, we consider how the results can be affected when the risk aversion parameter λ is modified. In our all simulations, Lemma **??** holds and, by theorem **??**, the agency's problem **??** has a unique solution with a non-zero and finite optimal duration $\bar{T}$. Intuitively, the risk aversion parameter should play an important role in the definition of the optimal contract since a higher value of risk aversion makes both consumption smoothing and insurance more important for the worker. Rows 3-7 of Table **??** reports the results obtained when the risk aversion is gradually raised to 2.5. We can notice the striking increase of the optimal expected utility U_0 with higher values of λ and the trade-off between the level of unemployment benefits and the duration of their paiement. However, it is obvious that with such degrees of risk aversion, the optimal contract gives much lower incentives for employment than the baseline program. In our framework and for the parameters chosen, a later exhaustion date of unemployment benefits and longer duration of unemployment, as λ grows, do not compensate the financial gains from the fall in benefits, that leads to an increasing tax rate.

λ	b	$\bar{\tau}$	$\bar{T}$	U_0
2	2653.26	78.63	12.68	-0.073
2.1	2611.11	121.23	19.72	-0.031
2.2	2561.14	184.962	29.88	-0.013
2.3	2486.27	276.693	43.88	-0.0062
2.4	2382.94	402.69	61.60	-0.0034
2.5	2253.0	564.82	81.11	-0.0015

Table 2: Optimal unemployment insurance

6 Conclusion

This paper determines some sufficient conditions that ensure the existence a fixed optimal duration of unemployment benefits in a simple job search model. If a significant work have been done to determine the optimal profile of unemployment benefits, it was generally with the implicit assumption that the potential duration of unemployment benefits was infinite. It is true that a fixed and predictable date of exhaustion of benefits often brings inextricable or for the less hardly extricable technical difficulties since one has to deal with time dependency of search effort (see Mortensen, 1977). To stride over these drawbacks, some models assumes a given Poisson process for a random duration of benefits and run numerical exercises to determine the optimal profile. In so doing, they can find an infinite (e.g. Davidson and Woodbury, 1997) or a finite (e.g. Fredrikson and Holmlund, 2001) expected optimal duration.

Our paper is however a very first step and the numerical results provided are more illustrative than prescriptive. The model used here is very basic and many extensions can be contemplated by lifting some restrictions. In particular, we assume constant benefits over the spell of unemployment, a constant tax rate and an exogenous assistance

payment. Maybe a decreasing sequence of benefits with an assistance payment optimally chosen could be Pareto improving. Another important feature of our model is the absence of financial markets. In a sequential job search model, Shimer and Werning (2008) show how saving and borrowing behaviors can play a significant role in the definition of an optimal insurance unemployment system. It is probably the case in our model. These questions will be the subject of future investigations.

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7 Appendix

7.1 Proof of Lemma ??

Given W_a and T, we set $\phi(t) = W(T - t)$, hence
$\dot{\phi}(t) = -\dot{W}(T - t) = -H(W(T - t)) = -H(\phi(t))$.
It is not difficult to see that W satisfies (??) on $[0, T]$ with condition $W(T) = W_a$ if and only if ϕ satisfies on $[0, T]$ the following system

$$\dot{\phi} = -H(\phi), \text{ with } \phi(0) = W_a. \tag{7.1}$$

Clearly, from the Cauchy-Lipschitz theorem, we deduce that there exists a maximal positive time T_m (finite or infinite) that ensures existence and uniqueness of a classical solution to (??) on $[0, T_m)$. So, recalling that the mapping $z \to H(z)$ is increasing, we immediately obtain the following result:

(j1) $\phi(0) > W_* \Rightarrow \phi$ is decreasing and $W_* \leq \phi \leq W_a$ on $[0, T_m)$.
(j2) $\phi(0) = W_* \Rightarrow \phi \equiv W_*$ on $[0, T_m)$ (because W_* is the unique solution).
(j3) $0 \leq \phi(0) < W_* \Rightarrow \phi$ is increasing and $W_a \leq \phi \leq W_*$ on $[0, T_m)$.

As a consequence, by continuity of ϕ on $[0, T_m)$, we deduce that $\phi(T_m^-)$ is well-defined, which entails that $T_m = +\infty$ (otherwise, T_m won't be the maximal existence time of ϕ), so that $\phi \in C^1(\mathbb{R}_+)$. It can be also observe that ϕ is positive on $[0, +\infty)$, which readily amounts to $\phi \in C^\infty(\mathbb{R}_+)$. Therefore, assuming that $W_a > 0$ yields the existence and uniqueness of a positive solution W to (??) on $[0, T]$ such that $W \in C^\infty([0, T])$. The rest of the proof follows from the above results (j1)-(j3) together with $W(t) = \phi(T - t)$. ●

7.2 Proof of Lemma ??

Note that if (??) is not satisfied then by Lemma ?? we deduce that any solution W of (??) satisfies $W(0) \geq W_a$ (which contradicts condition $W(0) < W_a$). It follows that (??) is a necessary condition for the existence of some positive T and some function W verifying (??). Conversely, we assume now that (??) holds and then from Lemma ?? we know that there exists a unique function $W \in C^\infty([0, T])$ verifying (??) together with

$$\begin{aligned}
\dot{W}(t) &= H(W(t)), &(7.2a)\\
W(T) &= W_a, &(7.2b)\\
W_* &< W(0) < W_a. &(7.2c)
\end{aligned}$$

Furthermore, we know by Lemma ?? that W is strictly increasing on $[0, T]$, while it is obvious that $T = \int_0^T ds$. Consequently, by the change of variable $u = W(s)$ (hence $du = \dot{W}(s)ds$), and recalling that $\dot{W}(s) = H(W(s))$, we obtain

$$T = \int_{W(0)}^{W(T)} \frac{1}{H(W(s))} du = \int_{W(0)}^{W_a} \frac{1}{H(u)} du. \tag{7.3}$$

As a result, by (??), we obtain $\int_{W_0}^{W(0)} \frac{1}{H(u)} du = 0$ (since $1/H$ is continuous and positive on the set $(W_*, +\infty)$ that also contains the values $W(0)$ and W_a. Then it is deduced that W fulfils (??), which ends the proof. ●

7.3 Proof of Lemma ??

Proof. Let us recall that the cost function B is given by the following differential equation $\dot{B}(t) = a(t)B(t) + f(t)$, where $a(t) = \eta + \varepsilon(t)$ (with some constant $\eta = r + \mu$) and $f(t) = \frac{\tau}{\eta}\varepsilon(t) - b$. From the considered equation, and setting $G_T(t) = \int_t^T a(s)ds$, we equivalently have $e^{-G_T(t)}\frac{d}{dt}\left(B(t)e^{G_T(t)}\right) = f(t)$, namely $\frac{d}{dt}\left(B(t)e^{G_T(t)}\right) = f(t)e^{G_T(t)}$. Now, integrating on $[t, T]$ amounts to

$$B(t)e^{G_T(t)} - B(T) = -\int_t^T f(\rho)e^{G_T(\rho)}d\rho. \tag{7.4}$$

From the definitions of f and a and observing that $G'_T(t) = -a(t)$ we also have

$$f(t) = \frac{\tau}{\eta}a(t) - (b + \tau) = -\frac{\tau}{\eta}G'_T(t) - (b + \tau), \tag{7.5}$$

so that we get

$$B(t)e^{G_T(t)} - B(T) = \frac{\tau}{\eta}[e^{G_T(\rho)}]_{\rho=t}^{\rho=T} + (b + \tau)\int_t^T e^{G_T(\rho)}d\rho,$$

where $[e^{G_T(\rho)}]_{\rho=t}^{\rho=T} = 1 - e^{G_T(t)}$ (since $G_T(T) = 0$). As a consequence of this observation we obtain

$$B(t)e^{G_T(t)} - B(T) = \frac{\tau}{\eta}(1 - e^{G_T(t)}) + (b + \tau)\int_t^T e^{G_T(\rho)}d\rho,$$

which, by $G_T(\rho) - G_T(t) = -G_\rho(t)$, yields

$$B(t) = (B(T) + \frac{\tau}{\eta})e^{-G_T(t)} + (b + \tau)\int_t^T e^{-G_\rho(t)}d\rho - \frac{\tau}{\eta}.$$

It follows the corresponding value of $B(0)$ (with respect to T and $B(T)$). ●

7.4 Proof of Lemma ??

Lemma ?? is an immediate consequence of the following result.

Lemma 7.1 *The quantity ψ, its integral and the first expression in brackets that occur in the above formulation of $B(0)$ are given by :*

$$\psi(T) = \frac{H(W(0))}{H(W(T))}, \tag{7.6a}$$

$$\int_0^T \psi(t)dt = H(W(0))\int_{W(0)}^{W(T)} \frac{ds}{H^2(s)}, \tag{7.6b}$$

$$B_a + \frac{\tau}{\eta} = \frac{\tau}{F'(W_a)}. \tag{7.6c}$$

Proof. We recall that the utility equation is written as

$$\dot{W} = \eta W + \beta W^{1+1/\alpha} - \gamma,$$

where $\beta = c^{1/\alpha}\frac{\alpha}{\alpha+1}$. Then, by deriving the above expression, we obtain

$$\ddot{W} = \eta\dot{W} + c^{1/\alpha}W^{1/\alpha}\dot{W}.$$

As W is (strictly) increasing, and recalling that $\epsilon = c^{1/\alpha}W^{1/\alpha}$, we deduce that

$$\epsilon = \frac{\ddot{W}}{\dot{W}} - \eta.$$

Next, by integration on $[0, t]$, we get

$$\eta t + \int_0^t \epsilon(s)ds = -\ln \frac{\dot{W}(0)}{\dot{W}(t)},$$

which by the definition of ψ amounts to (??). Now, from (??) we have

$$\int_0^T \psi(s)ds = H(W(0)) \int_0^T \frac{dt}{\dot{W}(t)},$$

hence, by the change of variable $s = W(t)$ (hence $ds = \dot{W}dt$), we obtain

$$
\begin{aligned}
\int_0^T \psi(t)dt &= H(W(0)) \int_{W(0)}^{W(T)} \frac{ds}{(\dot{W}(t))^2} \\
&= H(W(0)) \int_{W(0)}^{W(T)} \frac{ds}{(H(W(t)))^2} \\
&= H(W(0)) \int_{W(0)}^{W(T)} \frac{ds}{H^2(s)},
\end{aligned}
$$

that is (??). Let us recall that $B_a = -\dfrac{\tau \epsilon^a}{\eta(\eta + \epsilon^a)}$, where $\epsilon^a = \nu W_a^{1/\alpha}$ together with $\nu = c^{-1/\alpha}$. Then it can be checked that $B_a + \dfrac{\tau}{\eta} = \dfrac{\tau}{F'(W_a)}$, that is (??). $\bullet$

7.5   Proof of Lemma ??

For $(b, \tau) \in \Omega$, by setting $D = \Phi_{(b,\tau)}$ and by definition of $\Phi_{(b,\tau)}$ on $[W_*(b, \tau), \nu(\tau)]$ we obtain

$$D(\nu(\tau)) = \tau H(\nu(\tau))\left(1 - (1/\eta)H'(\nu(\tau))\right), \tag{7.7}$$

that is (??). So it is easily checked from the definition of H that $D(\nu(\tau))$ is some (negative) value. Furthermore, for $y \in (W_*(b, \tau), \nu(\tau)]$ we obtain the following derivatives:

$$D^{(1)}(y) = \tau H'(y) - H(\nu(\tau))H'(\nu(\tau))(b + \tau)\left(H'(y)\int_{\nu(\tau)}^y \frac{ds}{H^2(s)} + \frac{1}{H(y)}\right), \tag{7.8a}$$

$$D^{(2)}(y) = \tau H^{(2)}(y) + H(\nu(\tau))H'(\nu(\tau))(b + \tau)H^{(2)}(y)\int_y^{\nu(\tau)} \frac{ds}{H^2(s)}. \tag{7.8b}$$

Clearly, $D^{(2)}$ is nonnegative, so that $D^{(1)}$ is nondecreasing on $(W_*(b, \tau), \nu(\tau)]$. Furthermore, we readily have

$$D^{(1)}(\nu(\tau)) = -bH'(\nu(\tau)), \tag{7.9}$$

hence $D^{(1)}(\nu(\tau)) < 0$. It is then immediate that $D^{(1)}$ is negative on $(W_*(b, \tau), \nu(\tau)]$. So (by invoking continuity arguments) we deduce that D is decreasing on $[W_*(b, \tau), \nu(\tau)]$.

Moreover an easy computation gives us

$$\lim_{y \to W_*(b,\tau)} H(y) \int_y^{\nu(\tau)} \frac{ds}{H^2(s)} = \frac{1}{F'(W_*(b, \tau))}, \tag{7.10}$$

which (from the definition of D) amounts to

$$D(W_*(b,\tau)) = H(\nu(\tau))F'(\nu(\tau))\left((b+\tau)\frac{1}{F'(W_*(b,\tau))} - \frac{\tau}{\eta}\right), \qquad (7.11)$$

that is (**??**). The rest of the proof follows immediately from the previous arguments. $\bullet$