Preliminary and incomplete

On the provision of impure public goods: A general equilibrium comparison of different social norms

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March 13, 2017

Abstract: This paper compares some of the most common or debated social systems of public good provision in a unified dynamic general equilibrium framework. We focus on impure public goods and services, namely, publicly provided private goods, like education and health. Our main aim is to answer questions like "who should pay for these goods?" and "who should produce these goods?". We study and rank a wide range of social systems ranging from state production without user charges, to state production with user charges or to full privatization; we also study mixed systems. In our quest for the best system, we address both efficiency and distribution issues.

Acknowledgements: We thank Kostas Angelopoulos, Harris Dellas, Saqib Jafarey, Christos Kotsogiannis, Jim Malley, Hyun Park, Peter Sørensen and Petros Varthalitis for discussions and comments. We thank seminar participants, in particular Arye Hillman and Assaf Razin, at the conference on "Public policy and public sector reforms" held at Delphi, Greece, on 5-6 June 2015. Any errors are entirely our own.

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1. Introduction

The 2008 world financial and economic crisis has brought into the spotlight the need to reform the public sector. In addition to the obvious task, which is public debt sustainability, there is the classic issue of how to improve the "provision" of public goods and services without increasing the social burden for this provision.[1] As is well recognized, the word "provision" needs clarification. Publicly provided goods and services are classified according to their degree of publicness, their form of production and their way of financing (see e.g. Atkinson and Stiglitz (1980, chapters 15 and 16) and Cullis and Jones (1998, chapters 3 and 5)).

Regarding their type, namely, the degree of publicness they possess, public goods are distinguished between pure and impure. In modern democracies, national defense, police and the court system are usually treated as pure public goods. As such, they are typically produced by state firms and are offered uniformly and free of charge (free of charge means they are paid by the general taxpayer). However, most publicly provided goods and services are actually impure or quasi-public, which means that they are excludable and/or congestable.[2] Education and health are the most commonly found examples, but the list also includes child care, elderly care, family services, and many others like motorways and public museums (see e.g. Cullis and Jones (1998, chapter 12), Hillman (2009, chapter 3), Blomquist et al. (2010) and Picot et al. (2015)).[3] These goods bear the characteristics of private goods, but, nevertheless, in many countries, they are publicly provided, usually on distribution grounds.

The production and finance of impure public goods have always been open, and politically debated, issues. To start with, production and finance are two distinct components of provision (see e.g. Atkinson and Stiglitz (1980, chapter 16)). Regarding

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[1] See e.g. Sørensen (2016) for a recent survey of the economics of the public sector with emphasis on the provision of public goods. See also the special issue of *The Economist*, January 8th 2011, on the challenges faced by public sectors after the 2008 shock.

[2] We will use the terms "impure public goods", "quasi-public goods" or "publicly provided private goods" interchangeably.

[3] Actually, it is widely believed that it is difficult to find examples of pure public goods (see e.g. Cullis and Jones, 1998, p. 50). According to the ECB, pure public goods include the COFOG categories "defense" and "public order safety" only, which amount to less than 5% of GDP in most countries (see ECB, Monthly Bulletin, 2009, April). The so-called merit goods, namely goods that are underestimated in value by individuals, and club goods, namely goods whose consumption is excludable, are also examples of impure public goods (see e.g. Cullis and Jones, 1998, chapter 3). According to Blomquist et al. (2010),

their way of production, impure public goods can be produced by state firms but their production can also be contracted out to private firms, the so-called private providers, with the government still financing their cost; or they can be fully privatized, meaning that now ownership also changes hands. Regarding their way of financing, impure public goods, either produced by state firms or by private firms, can be provided free of charge (meaning that their cost is covered indirectly by the general tax payer), or they can be provided with user charges, or with a mix of general taxes and user charges.[4] The standard classification of various ways of provision (meaning both production and finance) of impure public goods is summarized in Table 1 (for a more detailed classification, see Cullis and Jones, 1998, Table 5.2, and Picot et al., 2015, Figures 1.1 and 1.2).

Table 1: Provision of impure public goods

State producers			**Private producers**		
"free" of charge	mix	with user charges	"free" of charge	mix	with user charges

Interestingly, although the evaluation of the above systems has been a central topic in most textbooks on public economics and there has always been a debate about the advantages and disadvantages of each of those systems in policy circles, there has not been - as far as we know - a formal comparison of them in a unified micro-founded dynamic general equilibrium setup. This is our goal in this paper.[5] We construct a general equilibrium framework, which allows us to study the different features and implications of different social systems of impure public good provision. We focus on impure public goods and services because, as said already above, their provision is more

public provision of private goods is common and often is of the order of 15% of GDP in most developed countries.

[4] In the case of public production with user charges, ownership remains in the hands of the government but there are more instruments on the public revenue side. In the cases of private providers and privatization, there is a change in ownership. On the other hand, private providers differ from full privatization because, in the case of private providers, the government keeps financing the cost of public good provision (although it does not produce them itself).

[5] See e.g. the books edited by Picot et al. (2015) and Philippopoulos (2016) for papers on this theme. Here we differ because we attempt to compare the main social systems of (impure) public good provision in a micro-founded dynamic general equilibrium set up. This allows us to quantify their differences in a consistent way.

debated, especially politically, than those of pure public goods.[6] We study both production and finance, as well as address both aggregate and distributional implications. By aggregate, we mean, for instance, per capita output and welfare, whereas, by distribution, we mean differences in income and welfare between private sector agents and public employees. Distributional implications, and a potential conflict of interests, are at the heart of the debate on the reform of the public sector. Our main aim is to give quantitative answers to questions like “Who should pay for these goods? Their users or the general tax payer?” and “Who should produce these goods?”.

We find it natural to start with three polar systems of impure public good provision, which are studied in sections 2, 3 and 4 respectively (polar cases are obviously not observed in practice but their study helps to understand better the working of different social systems). In section 2, we will first consider the case in which the impure public good is produced by state firms and the amount produced is provided uniformly and free of user charges to all agents (i.e. paid by general taxes). This regime will serve as a benchmark and will be called the status quo so that comparisons will be with respect to this. In turn, in section 3, we will study the case in which, although there is still public production as in the status quo case, now individuals are free to choose the amount they wish by paying a price (typically called a user price). That is, now there is a market-based mechanism that allocates the publicly produced good according to private demands.[7] Specifically, private agents’ optimization problem gives, among other things, the individual or private demand for this good as a negative function of the user price and, then, this user price emerges as a consequence of the sum of the voluntary individual demands and the quantity produced optimally by state firms. Then, in section 4, we will consider the case in which the sector of impure public goods and services is fully privatized so that now, not only there is a market-based mechanism for its allocation as in the previous regime, but also production or supply decisions are made by private firms. A special case of privatization is the case of “private providers”, where the government outsources the production of impure public goods/services to private firms but it finances itself the cost of this production. It should be noted that, in all these cases,

[6] While there are several options regarding the provision of impure public goods, like those listed in Table 1, there are fewer options regarding the provision of pure public goods. In the case of pure public goods, the search is for schemes that can possibly lead to better work incentives and hence higher public sector efficiency (see e.g. Sørensen (2016), Gomez (2016) and Economides et al. (2016)).

we also allow for social externalities so that the publicly provided private good is not fully private.

Since production and finance are separate issues, it is important to clarify how production decisions are made in each of the above cases. In the status quo regime, we will start with the baseline case in which the inputs used by state firms (e.g. public spending on public wages, goods purchased from the private sector and public investment) are exogenously set as in the data; here we will use data from the euro zone since 2001 although our qualitative results are not sensitive to the data set used. But, even within the status quo regime, we will also allow for more sophisticated ways of decision-making by state firms; for instance, we study the case in which the government sets a shadow price for the public good and tells state firms to choose their inputs and the associated output so as to, say, maximize profits at those shadow prices (see e.g. Atkinson and Stiglitz, 1980, chapter …) or that state firms minimize their cost subject to an output target set by the government. Naturally, we also assume optimizing behavior (profit max or cost min) on the part of firms in the case in which state firms are allowed to charge user prices as well as in the case of full privatization. Thus, in all cases, except from the baseline version of the status quo regime where inputs are exogenously set as in the data, we assume that the firms producing the impure public good enjoy full autonomy and act optimally as their private counterparts do; this makes our results comparable across regimes.

It should be already clear from the above that, even if we leave aside political economy issues and even if we assume that state and private firms share the same objectives and the same technology, there can be different outcomes, both in terms of efficiency and distribution. For instance, one would expect that the introduction of user prices would make a difference, since they give individuals a way to express their willingness to pay for the provision of impure public goods; user prices also mean extra revenues for the government so they affect the tax-spending mix. Ownership can also make a difference, since any profits made by state firms go to the government budget while any profits made by privatized firms benefit specific income groups only.

In turn, after having studied the above three polar systems, we will study mixed ones which are perhaps closer to reality. This is in section 6. We will assume that,

[7] Obviously, this is not feasible in the case of pure public goods where private demand functions cannot be well-defined.

although we are in a market-based system with user prices charged either by state firms or by privatized firms, there is supplementary public finance. In particular, we will study three such cases. First, the case in which the government subsidizes a fraction of user charges, in other words, individuals are free to choose the amount they wish, but they have to pay a fraction of the market price only, with the rest paid by the general government budget; second, the case in which there is also a minimum uniform amount freely provided, so that households have to pay user prices only if they wish to top up; and, thirdly, a case with vouchers in which each individual is given a voucher in the form of tax exception with the obligation to use this voucher in order to pay the cost of the public good provided to him/her individually.

All the above described systems are listed for convenience in Table 2 below.[8]

Table 2: Systems of provision of impure public goods studied

A	Public production with uniform provision and free of charge
B	Public production with non-uniform provision and user charges
C	Privatization with non-uniform provision and user charges
D	As in B or C, plus subsidization paid by the government
E	As in B or C, plus a minimum uniform provision provided by the government
F	As in B or C, plus vouchers paid by the government

Finally, in section 7, we add political economy type distortions in the regimes with user charges. Trying to keep a balanced view, we add one distortion on the side of the public sector and one distortion on the side of the private sector. In particular, we assume that, when production takes place in the public sector, the wage rate of public employees does not reflect their marginal productivity, as assumed so far when state firms act optimally, but, instead, it is set in an ad hoc way. For instance, it is set proportionally to the wage rate in the private sector; this assumption is consistent with evidence in many countries (see e.g. Economides et al., 2016, and the references cited therein). On the other hand, when production takes place by privatized firms, we assume

[8] See also the introduction in the book edited by Picot et al. (2015) for a discussion of a similar menu of systems of public-good provisioning, ranging from state-based provision to market-based provisioning. Their focus is on the provision of infrastructure goods.

that these firms do not act competitively but, instead, enjoy monopolistic power and hence make extra profits. Again this is a commonly believed social fear in case of privatization (see e.g. Drazen, 2000). We wish to stress that there are obviously many other political economy type distortions that could be considered. We focus on these two only to capture the general idea of the tradeoff between policy and market distortions.

The vehicle of analysis is a neoclassical growth model augmented with three distinct types of households (capitalists, private workers and public employees), two types of firms (private firms and state firms) and a rather rich menu of fiscal policy instruments. The model is described in detail in section 3.1 below.

Our main results are as follows. First, irrespectively of whether the impure public good is produced optimally or not and irrespectively of whether the producers are state or privatized firms, the introduction of user prices leads, other things equal, not only to higher per capita income and welfare, but also to higher individual net income for all types of agents. In other words, the introduction of a market-based mechanism in the form of user prices turns out to be Pareto efficient, at least when the criterion is agents' net incomes. The mechanism is as follows. The introduction of user prices for the impure public good allows for the creation of a new market and the latter helps all types of agents to realize or internalize that, in order to enjoy this good, which is not provided "freely" anymore, they need to have a higher income. This realization improves private agents' incentives pushing them to work harder and to save more. As a result, all individual gross and net incomes rise, and this allows agents to increase their private consumption, despite the fact that they now have to pay by themselves for the impure public good.

Second, in all cases in which a regime switch results in aggregate or efficiency gains (like optimization on the part of state firms, introduction of user prices, or privatization), larger tax bases allow a cut in distorting tax rates, like the labor income tax rate. This development generates a second round of beneficial effects in the sense that the decrease of the labor income tax rate triggers a further improvement of individual incentives to work and save.

Third, comparison of the various systems with user charges reveals that the type of production (state, or privatized, firms) leads to quite similar results, not only qualitatively but also quantitatively. In other words, to the extent that there is a market mechanism for the impure public good, and state firms act optimally as their private counterparts would do in the case of full privatization, the matter of ownership is not of great importance and this is both in terms of aggregate outcomes and distribution. However, this equivalence breaks down when one adds political economy issues (see below).

Fourth, when we study mixed public financing systems, where the cost of the impure public good is co-financed by both the individual user and the government, a main result is that if the market mechanism is distorted (say, by heavy government subsidization) and, at the same time, individuals are free to choose the quantity they want, it is better not to have a market mechanism at all and rely instead on centralized policy mechanisms like in the status quo regime. Simply put, distorted prices push agents to overuse the good. To say the same thing differently, a centralized uniform provision without user prices works better than a market mechanism with distorted prices.

Fifth, inequality is lower when there is a mix of a minimum uniform provision financed by the general tax payer and voluntary market-based top ups than when there are only market-based top ups. In particular, while the net income of private agents (capitalists and workers) falls in case we also allow for a minimum uniform provision, the net income of public employees rises. On the other hand, in the case in which only capitalists (or "the rich") pay user prices when they decide to top up, while private workers and public employees make use of a minimum uniform amount only provided free of charge, the net income of public employees falls considerably, so this popular type of social policy is self-defeating. This happens because the loss of "freedom to choose" distorts incentives and makes these agents poorer ex post.

Sixth, in the last section of the paper, we introduce political economy concerns. Given the wide range available (see e.g. Sørensen (2016)), we are selective. In particular, in the case with public production and user charges, we assume that the wage rate paid to public sector employees is not determined according to their marginal productivity but instead follows an ad-hoc rule, as is widely believed to be the case in most industrialized countries (see e.g. Economides et al. (2016)), whereas in the case with private production and user charges, we assume that the newly privatized firms have monopolistic power and hence make extra profits (see e.g. Drazen, 2000, chapter …). When we compare these two popular political economy scenarios, our results imply that, at least under commonly used parameterizations, private ownership is superior to state ownership both in terms of aggregate economy and individual incomes.

The rest of the paper is organized as follows. Section 2 presents the status quo economy. User prices under state production are in section 3. Section 4 studies the case of privatization. Section 5 considers the objective of cost minimization. Mixed financing systems are studied in section 6. Political economy extensions are in section 7. Section 8 closes this work. An Appendix provides technical details.

2. State production and free of charge uniform provision

In this section, we study the case in which the impure public good is produced by state firms and then the amount produced is provided uniformly to all agents without user charges. In other words, the production of the impure public good is in the hands of the state and the associated cost is paid by the general tax payer. We will start assuming that the inputs used for this production are exogenously set as observed in the data and then study the case in which these production decisions are chosen optimally. This regime will serve as a benchmark in what follows.

2.1 Informal description of the model and discussion of the key assumptions

We build on the baseline neoclassical growth model. There are three types of households, two types of firms and goods, as well as a government. The model is real so that we abstract from money and monetary policy.

Regarding households, we distinguish among capitalists, private workers and public workers (or public employees). Capitalists hold capital and government bonds and also receive labor income for their managerial services. They also own the private firms and so receive their profits. Private workers work in private firms and public employees work in the state firms. We assume (this is for simplicity) that only capitalists participate in financial and capital markets. The difference between public and private workers is that they earn different wages.[9]

Regarding firms, we distinguish between private and public or state firms. Private firms produce a single private good by choosing capital and labor inputs supplied by

[9] Public and private employees can differ in many other dimensions, like job security and non-monetary privileges (see e.g. Economides et al., 2016). Here, we focus on differences in wages only.

capitalists and private workers (in the robustness section, they can also make use of productivity-enhancing public goods/services). State firms produce a single public good/service by using public employees, public capital and a part of the private good. The public good produced provides utility-enhancing services to all households, but, as already said, in the robustness section, it can also provide productivity-enhancing public services to private firms.

Regarding fiscal policy, to finance total public spending, including the cost of the public good/service produced, the government levies distorting taxes and issues bonds. In the next sections, we will also allow for user charges for the use of the public good. As said above, we focus on impure public goods/services. However, we report that our main results are not altered when we add pure public goods/services that are provided freely.

The population size at time t is N_t. Among N_t, there is a pool of identical capitalists indexed by $k=1,2,...,N_t^k$, a pool of identical private workers indexed by $w=1,2,...,N_t^w$ and a pool of identical public employees indexed by $b=1,2,...,N_t^b$, where $N_t^k+N_t^w+N_t^b=N_t$ at each t. Equivalently, the population shares are denoted as $v_t^k \equiv N_t^k/N_t$, $v_t^b \equiv N_t^b/N_t$ and $v_t^w \equiv 1-v_t^k-v_t^b$, and where $v_t^k+v_t^w+v_t^b=1$. There are also $f=1,2,...,N_t^f$ identical private firms producing a single private good, where, for simplicity, the number of private firms equals the number of capitalists plus the number of private workers, namely, $N^f=N^k+N^w$. Similarly, we assume that there are $g=1,2,...,N^g$ identical state firms producing a single public good and that the number of state firms equals the number of public employees, namely, $N^g=N^b$. All this is about scaling and is not important to our qualitative results. Note that the fractions of the three agents in total population are exogenously set and are assumed to remain constant over time. We thus rule out occupational choice and mobility across groups.[10]

The above described status quo model is an enriched version of the framework used by most of the related literature (see e.g. Finn, 1998, Cavallo, 2005, Ardagna, 2007, Pappa, 2009, Linnemann, 2009, Forni et al., 2009, Fernández-de-Córdoba et al., 2010, and Economides et al., 2014, 2016, 2017).

[10] See e.g. Acemoglu (2009, chapter 23) for occupational choice although in a model with capitalists and workers only. See …. for occupational choice in a model including public employees.

We now model the above story.

2.2 Households

We start by modeling the behavior of the three distinct types of households.

Households as capitalists

There are $k = 1,2,..., N_t^k$ identical capitalists. Each capitalist derives utility from private consumption, c_t^k, leisure, $1 - he_t^k$, and per capita public goods provision, $\overline{y}_t^g$ (where, in equilibrium, $\overline{y}_t^g \equiv \frac{N^g y_t^g}{N} \equiv v^g y_t^g$). The discounted lifetime utility of each k is:

$$\sum_{t=0}^{\infty} \beta^t u(c_t^k, e_t^k, \overline{y}_t^g) \tag{1a}$$

where $0 < \beta < 1$ is the private discount rate.

For simplicity, we will use a log-linear form for the period utility function (we report that our main results do not depend on the type of the utility function used):

$$u(c_t^k, e_t^k, \overline{y}_t^g) = \mu_1 \log c_t^k + \mu_2 \log(1 - he_t^k) + \mu_3 \log \overline{y}_t^g \tag{1b}$$

where μ_1, μ_2 and μ_3 are positive preference parameters reflecting the importance of private consumption, leisure and public goods respectively.

The budget constraint of each k is:

$$(1+\tau_t^c)c_t^k + [k_{t+1} - (1-\delta)k_t^k] + b_{t+1}^k - b_t^k = (1-\tau_t^k)(r_t k_t^k + \pi_t^k) + (1-\tau_t^l)w_t^k e_t^k h + r_t^b b_t^k + g^{tr,k} \tag{2a}$$

where k_t^k and b_t^k are each capitalist's capital and public debt holdings respectively at the end of period $t-1$, δ is the depreciation rate of private capital, π_t^k is profit distributed by private firms to each capitalist, e_t^k is each capitalist's labor effort (whereas h is a fixed amount of time), r_t and r_t^b are the returns to private capital and public debt respectively, w_t^k is the wage rate earned by capitalists, τ_t^c, τ_t^k, τ_t^l are tax rates on consumption, capital income, and labor income respectively, and $g^{tr,k}$ is a transfer payment made by the government to each capitalist.

Each k chooses the paths $\{c_t^k, e_t^k, k_{t+1}^k, b_{t+1}^k\}_{t=0}^{\infty}$ to maximize (1a)-(1b) subject to (2a). The first-order conditions include (2a) and:

$$\frac{\mu_2}{(1-he_t^k)} = \frac{\mu_1(1-\tau_t^l)w_t^k}{(1+\tau_t^c)c_t^k} \tag{2b}$$

$$\frac{(1+\tau_{t+1}^c)c_{t+1}^k}{(1+\tau_t^c)c_t^k} = \beta[1-\delta+(1-\tau_{t+1}^k)r_{t+1}] \tag{2c}$$

$$\frac{(1+\tau_{t+1}^c)c_{t+1}^k}{(1+\tau_t^c)c_t^k} = \beta(1+r_{t+1}^b) \tag{2d}$$

Equation (2b) is the first-order condition for work effort, while equations (2c) and (2d) are the well-known Euler conditions for capital and bond holdings respectively. Therefore, the budget constraint in (2a), along with equations (2b)-(2d), summarize the optimal behavior of each k.

Households as workers in the private sector

There are $w = 1, 2, ..., N_t^w$ identical private workers. As said, workers do not save, and thereby their problem is static. Thus, each w maximizes:

$$u(c_t^w, e_t^w, \bar{y}_t^g) = \mu_1 \log c_t^w + \mu_2 \log(1-he_t^w) + \mu_3 \log \bar{y}_t^g \tag{3}$$

subject to the budget constraint:

$$(1+\tau_t^c)c_t^w = (1-\tau_t^l)w_t^w e_t^w h + g_t^{tr,w} \tag{4a}$$

where w_t^w is the wage rate earned by private workers, and $g^{tr,w}$ is a transfer payment made by the government to each private worker.

Each worker chooses c_t^w and e_t^w in each period. The first-order conditions are the constraint (4a) above and the optimality condition for work effort:

$$\frac{\mu_2}{(1-he_t^w)} = \frac{\mu_1(1-\tau_t^l)w_t^w}{(1+\tau_t^c)c_t^w} \tag{4b}$$

which is similar to (2b) above.

Households as public employees

There are $b = 1,2,...,N_t^b$ identical public sector employees. Public employees, like workers, do not to save. Therefore, each b faces a problem similar to that of private workers and so it maximizes:

$$u(c_t^b, e_t^b, \bar{y}_t^g) = \mu_1 \log c_t^b + \mu_2 \log(1-he_t^b) + \mu_3 \log \bar{y}_t^g \tag{5}$$

subject to the budget constraint:

$$(1+\tau_t^c)c_t^b = (1-\tau_t^l)w_t^g e_t^b h + g_t^{tr,b} \tag{6a}$$

where w_t^g is the wage rate earned by public employees, and $g^{tr,b}$ is a transfer payment made by the government to each public employee.

Each public employee chooses c_t^b and e_t^b in each period. The first-order conditions are the constraint (6a) above and the optimality condition for work effort:

$$\frac{\mu_2}{(1-he_t^b)} = \frac{\mu_1(1-\tau_t^l)w_t^g}{(1+\tau_t^c)c_t^b} \tag{6b}$$

which is similar to (2b) and (4b) above.

2.3 Production of private goods and private firms

We now model private firms and the production of the private good. There are $f = 1,2,...,N^f$ identical private firms. Each firm uses capital (supplied by capitalists) and labor services (supplied by both capitalists and private workers) to produce a single private good in a perfectly competitive market (see …. below for imperfect competition).

In each period, the profit of each firm f is:

$$\pi_t^f \equiv y_t^f - r_t k_t^f - w_t^w e_t^{f,w} - w_t^k e_t^{f,k} \tag{7}$$

where the production function is assumed to be:

$$y_t^f = A(k_t^f)^{\alpha_1}(A^k e_t^{f,k} + A^w e_t^{f,w})^{\alpha_2} \tag{8}$$

and where k_t^f is the firm's capital input, $e_t^{f,k}$ is capitalists' labour services used by the firm, $e_t^{f,w}$ is workers' labour services used by the firm, and $0 < \alpha_1, \alpha_2 < 1$, A, A^k, A^w are usual technology parameters (see also e.g. Hornstein et al., 2005, who distinguish between different types of labor services in a similar manner).

It should be said that, in a richer model specification, we have also considered the case in which the per capita publicly produced good, $\overline{y}_t^g$, provides, in addition to utility-enhancing services, productivity-enhancing services that benefit private firms, in the sense that this type of services increases private firms' productivity. Since the main results do not change, we relegate this extension to the Appendix.[11]

Each private firm acts competitively maximizing (7) subject to (8) in each period. The first-order conditions for the three inputs are simply:

$$w_t^w = \frac{\alpha_2 A^w y_t^f}{(A^k e_t^{f,k} + A^w e_t^{f,w})} \tag{9a}$$

$$w_t^k = \frac{\alpha_2 A^k y_t^f}{(A^k e_t^{f,k} + A^w e_t^{f,w})} \tag{9b}$$

$$r_t = \frac{\alpha_1 y_t^f}{k_t^f} \tag{9c}$$

which imply $\pi_t^f = 0$.

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[11] In particular, the private production function changes from (8) to $y_t^f = A(k_t^f)^{\alpha_1}(A^k e_t^{f,k} + A^w e_t^{f,w})^{\alpha_2}(\overline{y}_t^g)^{1-\alpha_1-\alpha_2}$, where the newly introduced term, $\overline{y}_t^g$, is taken as given by private firms in the status quo regime. In the

2.4 Public sector

We now model the public sector. We first model the way in which state enterprises produce the public good and then present the consolidated budget constraint of the public sector.

Production of public goods-services and state firms

Working as in the case of private firms, we assume that there are $g = 1, 2, ..., N^g$ identical state firms producing the single public good. The cost of producing the single public good for each state firm g is:

$$w_t^g l_t^g + g_t^g + g_t^i \tag{10}$$

where l_t^g is the labor input used by each state firm, g_t^g is goods purchased from the private sector and used for the production of the public good by each state firm and g_t^i is investment spending by each state firm. As said above, w_t^g is the wage rate paid in the public sector.

The production function of each firm is assumed to be (see also ….):

$$y_t^g = A^g (k_t^g)^{\theta_1} (l_t^g)^{\theta_2} (g_t^g)^{1-\theta_1-\theta_2} \tag{11}$$

where k_t^g denotes the stock of capital used by the government at the beginning of the current period and $0 < \theta_1, \theta_2 < 1$, A^g are usual technology parameters.

The stock of each state firm's capital evolves over time as:

$$k_{t+1}^g = (1-\delta^g) k_t^g + g_t^i \tag{12}$$

where $0 < \delta^g < 1$ is the depreciation rate.

To specify the level of output produced by each state firm, y_t^g, and in turn the total amount of the public good provided to the society, we obviously have to specify the

market-based regimes that follow below, the private firm pays user prices for this new factor. See amounts of the productive factors used in equation (11). We start with the simple case in which the three factor inputs are exogenously set at values implied by the actual data, meaning that the total number of public employees as share of population, as well as total spending on public investment, total public spending on the wage bill and total public spending on goods purchased from the private sector, all three as shares of GDP, will be set as in the data (see subsection …. below). Nevertheless, below, we will also study richer cases in which the amounts of factor inputs used, and hence the level of the public good produced and provided, are determined optimally (see subsection … below).

Government budget constraint

The within-period budget constraint of the consolidated public sector is (written in aggregate terms):

$$N^k g_t^{tr,k} + N^w g_t^{tr,w} + N^b g_t^{tr,b} + (1+r_t^b) N^k b_t^k + N^g (w_t^g l_t^g + g_t^g + g_t^i) =$$
$$= N^k b_{t+1}^k + \tau_t^c (N^k c_t^k + N^w c_t^w + N^b c_t^b) + \tau_t^k N^k (r_t k_t^k + \pi_t^k) + \tau_t^l (N^k w_t^k e_t^k h + N^w w_t^w e_t^w h + N^b w_t^g e_t^b h) \quad (13)$$

where one of the fiscal variables, $b_{t+1}^k, \tau_t^c, \tau_t^k, \tau_t^l$, needs to follow residually to close the budget (see below).

2.5 Decentralized equilibrium

As said, we start with the case in which the public inputs (or, strictly speaking, the public spending items on these inputs as shares of output) used for the production of the public good are set exogenously as in the data. The equilibrium system of the above economy is presented in Appendix A. It consists of 23 equations in 23 endogenous variables, which are: $\{c_t^k, c_t^w, c_t^b\}_{t=0}^\infty$, $\{e_t^k, e_t^w, e_t^b\}_{t=0}^\infty$, $\{k_{t+1}^k, k_{t+1}^g\}_{t=0}^\infty$, $\{r_t, r_t^b, w_t^k, w_t^w, w_t^g\}_{t=0}^\infty$, $\{y_t^f, \pi_t^f\}_{t=0}^\infty$, $\{y_t^g, \pi_t^g\}_{t=0}^\infty$, $\{g_t^g, g_t^i, g_t^{tr,k}, g_t^{tr,w}, g_t^{tr,b}\}_{t=0}^\infty$, and one of the fiscal variables $\{b_{t+1}^k, \tau_t^c, \tau_t^k, \tau_t^l\}_{t=0}^\infty$ which follows residually to satisfy the government budget constraint in each period. In all numerical simulations below, the residually determined fiscal variable, along the transition path, will be the end-of-period public debt, whereas, in the steady state, the residually determined fiscal instrument will be the labor income tax rate with the public

Appendix … for this model.

debt to GDP ratio set to its average data value. This equilibrium system is given the values of the exogenously set policy instruments (τ_t^k, τ_t^l, τ_t^c, s_t^w, s_t^g, s_t^i, s_t^{tr}, v^g, σ^k, σ^b), the fraction of capitalists or self-employed in the population (v^k), the fraction of public sector employees in the population (v^b) and initial values for the state variables.

3.6 Parameterization

The system is solved numerically using common values for technology and preference parameters as well as data averages for the policy variables (here we use data for the euro area over 2001 to 2012). These values are listed in Table 1 below. The time unit is meant to be a year. Before we discuss Table 1, we report from the outset that we have conducted a rather rich sensitivity analysis and our qualitative results are robust to changes in these values (details are available upon request).

Table 1 here

Consider first parameter values. In the private sector production function, following the related literature, the Cobb–Douglas exponents of labor and capital are set respectively at 0.67 and 0.33. The TFP parameters in the production functions of private and state firms, A^f and A^g respectively, are both normalized at 1. The time discount rate, β, is set at 0.9. The weight given to public goods/services in the private utility function is set at 0.05, which is within the range used in the related literature.[12] The other two preference parameters related to private consumption and leisure, μ_1 and μ_2, are set at 0.35 and 0.6 respectively; these values imply in turn hours of work within usual ranges. The private and public capital depreciation rates, δ and δ^g, are both set at 0.05. In the public sector production function, the Cobb–Douglas exponents of public employment and public capital are set respectively at 0.569 and 0.1078. These values correspond to payments to public wages and payments to public investment, expressed as shares of total public payments to all inputs used in the production of public goods, as they are in the data; in other words, $0.569 = s^w/(s^w + s^g + s^i)$ and $0.1078 = s^i/(s^w + s^g + s^i)$, where s^w is the output share of public spending on public wage payments in the data, s^g is the output share of public spending on goods and services purchased from the private sector in the data, and s^i is the output share of public spending on public investment in the data (for similar calibration practice, see the real business cycle literature and, in related models, see e.g. Linnemann 2009, and Economides et al., 2013). In turn, the Cobb-Douglas exponent of goods purchased from the private sector in the public sector production function follows residually so that it is 0.3232=1-0.569-0.1078. We also set $A^w = A^b = 1$ so that all employees (private and public sector) offer the same type of labor services, whereas we calibrate A^k so as to get a positive labor supply on the part of the capitalists.

Consider next the values of policy variables. The share of public employees in total population, v^b, is set at 0.215, which is the average value in the data. The share of capitalists, defined as those who are self-employed, is set at 0.148 as in the data. The data values of s^w, s^g and s^i (which were defined above), as well as the data value of s^{tr} (which denotes total transfers as share of GDP), are respectively 0.132, 0.075, 0.025 and 0.2170. The effective tax rates on consumption, capital income and labor income, (τ^c, τ^k and τ^l) are set at 0.1938, 0.2903 and 0.378 as in the euro area data. In the steady state, the public debt to GDP ratio is set at 0.9, which is the data average value, while, we will treat τ^l as the endogenously determined fiscal variable in all steady state solutions.

2.7 Solution and discussion of steady state results

We focus on steady state results (transition results can also be provided). The first column 1 in Table 2 reports the steady state solution of the equilibrium system in subsection 3.5, when we use the parameter values and the exogenous policy variables in Table 1. The solution is, in general, well defined.

Table 2 here

Regarding the aggregate economy, the solution does relatively well at mimicking the GDP ratios of the key macroeconomic aggregates, like consumption and capital, as

[12] Chari et al. (1995) use a zero value. In contrast, Guo and Lansing (1999) use a high value, around 0.36, in a similar utility function.

shares of GDP, in the data. Notice also that the value of the endogenously determined labor tax rate is very close to its actual value in the data. Regarding distribution, the solution implies that the capitalists (or “the rich”) enjoy the highest net income, the highest consumption level and the highest utility level. Private workers consume more, and work harder, than public employees; in terms of utility, the benefit from consumption more than outweighs the pain of work, so that private workers are better off than public employees. We believe all this is in accordance with common belief.

2.8 A caveat

The above regime will serve as a benchmark for the reformed economies studied in what follows. Before we move on to these reformed economies, we need to repeat that so far we have modeled the state firm in an exogenous way. In particular, we set the inputs as observed in some data and then, via a production function, we derived the level of public goods produced. Although this case can also be thought of as being close to reality, a criticism might be that, by doing so, we have not treated state firms fairly, in the sense that, in all other regimes studied in the next sections below, state and private firms will be assumed to act optimally. Optimizing behavior on the part of state firms, within the status quo regime, is postponed until subsection 3.5 below.

3. State production with user prices

In this section, we continue to assume that it is state firms that produce and provide the impure public good, as it was also the case in the previous section, but now its cost is covered by user charges paid by individual users. Hence, now individuals are free to choose the amount they wish by paying a price (typically called the user price). We continue to work with polar cases so that the production of the public good is now fully financed by user charges.

3.1 Informal description of the new system

There is now a new market and a new (user) price for the impure public good. Individuals face a uniform user price and, given this price, individual demand functions are derived, as in the case of private goods. In other words, private agents’ optimization problem gives, among other things, the individual demand for this good as a negative function of the user price. Then, this demand side, together with the supply side coming from profit-maximizing state firms, will determine the equilibrium (user) price for the publicly provided impure public good.

3.2 What changes in the model relative to the model in section 2

Here we model what changes relative to the previous section.

Households

Since now there is not a single or uniform level of provision but different households are free to choose the amount of the publicly provided private good they wish by paying the user price, p_t^g, the within-period utility function and the budget constraint of each capitalist change to:[13]

$$u(c_t^k, e_t^k, g_t^k) = \mu_1 \log c_t^k + \mu_2 \log(1 - he_t^k) + \mu_3 \log g_t^k \tag{17a}$$

$$(1+\tau_t^c)c_t^k + [k_{t+1} - (1-\delta)k_t^k] + b_{t+1}^k - b_t^k + p_t^g g_t^k = (1-\tau_t^k)(r_t k_t^k + \pi_t^k) + (1-\tau_t^l)w_t^k e_t^k h + r_t^b b_t^k + g^{tr,k} \tag{17b}$$

where g_t^k is the amount of the publicly provided good purchased by each k.

Similarly, for each private worker, and for each public employee, we have respectively:

$$u(c_t^w, e_t^w, g_t^w) = \mu_1 \log c_t^w + \mu_2 \log(1 - he_t^w) + \mu_3 \log g_t^w \tag{18a}$$

$$(1+\tau_t^c)c_t^w + p_t^g g_t^w = (1-\tau_t^l)w_t^w e_t^w h + g_t^{tr,w} \tag{18b}$$

$$u(c_t^b, e_t^b, g_t^b) = \mu_1 \log c_t^b + \mu_2 \log(1 - he_t^b) + \mu_3 \log g_t^b \tag{19a}$$

$$(1+\tau_t^c)c_t^b + p_t^g g_t^b = (1-\tau_t^l)w_t^g e_t^b h + g_t^{tr,b} \tag{19b}$$

[13] Notice that we start with the case in which the publicly provided good, sold at a user price, is fully private (see also Atkinson and Stiglitz, 1980, …). In subsection 4.5 below, we will generalize the model by allowing for social externalities generated by the individual use of this good. On the other hand, it should be pointed out that, if the publicly provided good is purely public, individual demand functions cannot be derived so that it is not possible to impose (market) user prices and hence to establish a relationship between prices and quantities; in this case, decisions on public goods have to be made through political, not market, mechanisms (see Buchanan, 1968, for an early study). Within the context of our model, a way of making the publicly provided good purely public is to assume away congestion (see section … below).

where g_t^w is the amount of the publicly provided good purchased by each w, and g_t^b is the amount of the publicly provided good purchased by each b.

We thus have three new first-order conditions (one for each type of household):

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^k} = \frac{\mu_3}{g_t^k} \tag{17c}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^w} = \frac{\mu_3}{g_t^w} \tag{18c}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^b} = \frac{\mu_3}{g_t^b} \tag{19c}$$

where (17c), (18c) and (19c) give the demand for the impure public good by the capitalist, the worker and the public employee respectively. Thus, now the optimal behavior of the three types of households is summarized by equations (17b)-(17c), (2b)-(2d), (4a)-(4b), (18b)-(18c), and (6a), (19b)-(19c).

State-owned firms charging user prices and acting optimally

On the supply side, there are $g = 1, 2, ..., N^g$ state firms, as above, but now these firms act optimally charging the user price. Each one of them is assumed to maximize its present value, which is the discounted sum of net cash flows, where the period net cash flow is revenue minus current expenditure on factors of production.[14]

Thus, each state firm g maximizes:

$$\sum_{t=0}^{\infty} (\beta^g)^t [p_t^g y_t^g - w_t^g l_t^g - g_t^g - g_t^i] \tag{20}$$

subject to the same technology constraints as in the previous regime, that is, equations (11) and (12).

The first-order conditions for the three inputs, $\{k_{t+1}^g, l_t^g, g_t^g\}_{t=0}^{\infty}$, are respectively:

[14] Notice that, under certain conditions specified in Appendix …, this dynamic problem is equivalent to a static profit maximization problem. See also Sargent (1987, chapter …).

$$1 = \beta^{g} \left[1 - \delta^{g} + \frac{\theta_1 y_{t+1}^{g}}{k_{t+1}^{g}} p_{t+1}^{g} \right] \tag{21a}$$

$$w_t^g = \frac{\theta_2 y_t^g}{l_t^g} p_t^g \tag{21b}$$

$$g_t^g = (1 - \theta_1 - \theta_2) y_t^g p_t^g \tag{21c}$$

which are similar to (16a)-(16c) above.

Government budget constraint

The within-period government budget constraint, presented in equation (13), changes to:

$$\begin{aligned} & N^k g^{tr,k} + N^w g^{tr,w} + N^b g^{tr,b} + (1 + r_t^b) N^k b_t^k + N^g (w_t^g l_t^g + g_t^g + g_t^i) = N^g p_t^g y_t^g + \\ & N^k b_{t+1}^k + \tau_t^c (N^k c_t^k + N^w c_t^w + N^b c_t^b) + \tau_t^k N^k (r_t k_t^k + \pi_t^k) + \tau_t^l (N^k w_t^k e_t^k h + N^w w_t^w e_t^w h + N^b w_t^g e_t^b h) \end{aligned} \tag{22}$$

where the new term $N^g p_t^g y_t^g$ is the total revenue obtained from the sale of the impure public good.

Market-clearing condition for the excludable public good

Since we have a new market, namely, the market for the impure public good, the market-clearing condition is:

$$N^g y_t^g = N^k g_t^k + N^w g_t^w + N^b g_t^b \tag{23}$$

The left hand-side of equation (23) depicts the supply of the impure public good coming from the state firms, whereas the right hand-side gives the total demand for it coming from the three types of agents. This equilibrium condition will determine the associated price, p_t^g.

3.3 Decentralized equilibrium

The new equilibrium system is presented in Appendix B. It consists of 32 equations in 32 endogenous variables, which are $\{c_t^k, c_t^w, c_t^b\}_{t=0}^{\infty}$, $\{e_t^k, e_t^w, e_t^b\}_{t=0}^{\infty}$, $\{k_{t+1}^k, k_{t+1}^g\}_{t=0}^{\infty}$, $\{g_t^k, g_t^w, g_t^b\}_{t=0}^{\infty}$, $\{r_t, r_t^b, w_t^k, w_t^w, w_t^g\}_{t=0}^{\infty}$, $\{y_t^f, k_t^f, e_t^{f,k}, e_t^{f,w}, \pi_t^f\}_{t=0}^{\infty}$, $\{y_t^g, l_t^g, \pi_t^g, p_t^g, w_t^g\}_{t=0}^{\infty}$, $\{g_t^g, g_t^i, g_t^{tr,k}, g_t^{tr,w}, g_t^{tr,b}\}_{t=0}^{\infty}$, and one fiscal policy instrument that follows residually to satisfy the government budget constraint in each period. Notice that now, in contrast to the previous regime, the public spending shares (s_t^w, s_t^g and s_t^i) become endogenous variables being determined by the optimality conditions of the state firm (details are in Appendix B). On the other hand, (s_t^{tr}, v^g, σ^k, σ^b, τ^c, and τ^k), as well as, the fractions of capitalists (or self-employed) and public sector employees in population (v^k and v^b respectively) continue to be exogenous.

The new system will be solved numerically using the same parameter values and policy variables as in Table 1.

3.4 Solution and discussion of steady state results

The second column in Table 2 reports the steady state solution of the new economy. As above, at steady state, the residually determined fiscal policy instrument is the labor tax rate. The solution is again well defined.

Regarding aggregate outcomes or efficiency, comparison of the solution in column 1 (status quo) to the solution in column 2 (optimizing state firms charging user prices) reveals that a switch to the regime in column 2 would generate substantial efficiency gains. In particular, the levels of private and total output (y_t^f and y_t respectively) are much higher in column 2 than in column 1. The same applies to per capita utility (u_t). On the other hand, notice that the level of public output (y_t^g) falls relative to the status quo in column 1.

Notice that the switch to a more efficient macro-economy with larger tax bases allows for a big cut in the labor tax rate (τ_t^l), which serves as the residually determined fiscal policy instrument in our steady states solutions. The cut in a particularly distorting tax rate, like the labor tax rate, triggers a further improvement of individual incentives to work and save. This “public financing effect” resembles the “double dividend effect” in the environmental economics literature; namely, a regime switch that leads to a more efficient economy also allows for a cut in relatively distorting taxes which generates a second round of beneficial effects.

Regarding distribution, the net income of all agents is higher in column 2 than in the status quo case in column 1 of Table 2. That is, in terms of net income, the switch to a market-based mechanism in column 2 is Pareto efficient. In terms of utility, as we switch from column 1 to column 2, the utility of capitalists and workers gets higher in column 2, while public employees are worse off in column 2 than in the status quo in column 1, although this happens only because their work effort is higher and public good provision is lower. Focusing on relative changes in net income as we move from column 1 to column 2, the gains are distributed in a regressive way: this regime switch leads to lower net income inequality for private workers, but higher net income inequality for public employees, vis-à-vis capitalists. In other words, although the net income of public employees also rises as we move to a more efficient economy, this rise is smaller than the rise in the net income of capitalists, so that public employees gain by less and their relative net income inequality worsens.[15]

3.5 Interpretation of results and decomposition to various effects

The above results imply that there can be substantial gains when we switch from the status quo economy studied in section 2 to the reformed economy studied in this section. However, it should be stressed that two things are different when we compare these two regimes. First, state firms act optimally in the present section, while their decisions were ad hoc (as implied by the data on public spending) in the status quo economy in section 2. Second, in the current section, we have replaced taxes with user prices as a way of financing the impure public good. Which change is more important and can account for the substantial gains derived in the reformed economy? Namely, what fraction of the total effect is generated by optimizing behavior on the part of state firms and what fraction is generated by the extra market? Decomposing the two effects will also help us to understand the propagation mechanisms and the intuition behind our results.

We will work in steps. We start with the case in which, although we remain within the status quo regime without user prices, we allow the state firms to act optimally.

[15] The same happens in terms of utility: capitalists and private workers are better off relative to the status quo in column 1 (although they enjoy less leisure), while public employees are worse off.

The effect from optimizing state firms

As said, in the status quo economy in section 2, we modeled the state firm (strictly speaking, the determination of its inputs and then, via the production function used, the associated level of public good production) in an exogenous way. In particular, we set the inputs as observed in some data and then, via a production function, we derived the level of public goods produced. Although this can be thought of as being close to reality, a criticism might be that, by doing so, we have not treated state firms fairly. Therefore, we now study the case of optimum public good production/provision within the status quo regime.

As is well known, there is a wide range of models of optimum provision of public goods (pure or impure) ranging from first-best allocations in fully controlled economies *à la* Samuelson (1954) to political economy models of voting and rent seeking or to models with varying degrees of autonomy given to state enterprises (see e.g. Atkinson and Stiglitz (1980, chapters 15 and 16) for an early review and Sorensen (2016) for a recent one). Here, we will mainly follow the last strand of this literature. In addition, with an eye to what follows below, we will start with the case in which the government sets a shadow price for the public good and, in turn, given this price, state enterprises choose their inputs, and hence the level of public good production, to maximize their profits. It should be said that we start with profit maximizing behavior on the part of state enterprises simply because we want to make our results directly comparable to those in the rest of the paper (when we introduce privatization, we find it natural to assume that the objective of private firms is profit maximization). Nevertheless, in section 6, we will also study cost minimization; in that case, the government will set an output target and, given this target, state firms will choose their inputs to minimize costs.

Since, under a uniform and free of charge provision, there are no private demand functions for the impure public good, and so there are no market prices either, we need to assume a shadow price for the solution of the state firm's profit maximization problem. Here, we assume that public sector prices are set equal to their social marginal value. In particular, the shadow price of the public good relative to the private good is defined to be the weighted sum of the marginal rates of substitution across the three social groups, where the weights are their fractions in population (Atkinson and Stiglitz

(1980, chapter 16) call it the average marginal rate of substitution). This implies (see Appendix B for details):

$$p_t^g = \frac{\dfrac{\mu_3}{v^k g_t^k + v^w g_t^w + v^b g_t^b}}{\dfrac{\mu_1}{(1+\tau_t^c)}\left[\dfrac{v_t^k}{c_t^k} + \dfrac{v_t^w}{c_t^w} + \dfrac{v_t^b}{c_t^b}\right]} \tag{14}$$

Taking this shadow price, p_t^g, as given, each state firm g chooses its inputs to maximize profits:[16]

$$\sum_{t=0}^{\infty} (\beta^g)^t [p_t^g y_t^g - w_t^g l_t^g - g_t^g - g_t^i] \tag{15}$$

subject to the same technology constraints as above, namely, equations (11) and (12).

The first-order conditions for the three inputs, $\{k_{t+1}^g, l_t^g, g_t^g\}_{t=0}^{\infty}$, are respectively:

$$1 = \beta^g \left[1 - \delta^g + \frac{\theta_1 y_{t+1}^g}{k_{t+1}^g} p_{t+1}^g\right] \tag{16a}$$

$$w_t^g = \frac{\theta_2 y_t^g}{l_t^g} p_t^g \tag{16b}$$

$$g_t^g = (1 - \theta_1 - \theta_2) y_t^g p_t^g \tag{16c}$$

where (16a)-(16c) equate the marginal product of each factor of production to its cost.

The new equilibrium system is presented at the end of Appendix A. Numerical steady state solutions, by using the same parameter and policy variable values as above, are reported in the second column of Table 3. In this table, namely Table 3, for convenience and comparison, the first column repeats the status quo solution (this is as in Table 2) and the last column repeats the solution with user prices and optimizing state prices (this is as in Table 2 again). Observe that the output shares of public spending on public wage payments, s^w, and on goods and services purchased from the private sector,

s^g, which are now optimally chosen and linked to each factor's productivity, have much lower values than the respective ones in the first column, where these spending items were set as in the data. These changes in policy variables (especially, the fall in s^w) can in turn explain the aggregate and distributional implications.

Table 3 here

Regarding aggregate outcomes or efficiency, comparison of column 2 to column 1 in Table 3, reveals, as one would expect, that optimization on the part of state firms, other things equal, leads to a more efficient economy: the levels of private and total output are higher in column 2 than in column 1. On the other hand, the level of public output falls. Notice that again the switch to a more efficient macro-economy with larger tax bases allows for a cut in the labor tax rate (which serves as the residually determined fiscal policy instrument in our steady states solutions) which triggers a new round of efficiency gains. Regarding distribution, optimal decision-making on the part of state firms in column 2, makes capitalists and workers better off, but it makes public employees worse off. This happens both in terms of net incomes and utilities. Therefore, a switch to the regime in column 2 is not Pareto efficient.

Therefore, a switch from the status quo economy, where the inputs were exogenously set according to the actual data, to an economy where the same impure public good is produced optimally by state firms, leads, other things equal, to a more efficient economy but this comes at the cost of making public employees worse off both in terms of net income and utility. We label this the "state-firm optimization effect".[17] Recall that all this is when the cost of the impure public good is financed by the general tax payer.

The effect from adding user prices

Comparison of the solutions in columns 1 and 2 in Table 3 (status quo with ad hoc decisions by state firms and status quo with optimizing state firms, respectively) to the

16 Notice that, under certain conditions specified in Appendix … , this dynamic problem is equivalent to a static profit maximization problem. See also Sargent (1987, chapter …).

17 Although these implications are reasonable, we do not want to take a strong normative position. State firms may have to hit multiple objectives and this does not allow them to make the most efficient decisions (se e.g. Dewatripont et al., 1999).

solution in the last column of the same Table (profit maximizing state firms charging user prices) reveal that are substantial aggregate gains not only relatively to the status quo in column 1 but also relatively to column 2. In other words, even when we depart from a case in which state firms act optimally, the addition of user prices, other things equal, generates significant gains at aggregate level. In other words, a comparison of column 2 to the last column gives an idea of the importance of adding a new market (namely, a market for the impure public good).

The intuition behind the efficiency gains in the reformed economy with user prices in the last column of Table 3 vis-à-vis the status quo cases in columns 1 and 2 is as follows. In the reformed economy, the introduction of the new market for the impure public good helps all types of agents to realize/internalize that, in order to enjoy the impure public good, which is not provided freely anymore, they need to have a higher income. Algebraically, this internalization occurs through the new optimality conditions, equations (15c), (16c) and (17c). This realization/internalization improves private agents' incentives pushing them to work harder and to save more (notice the clear rise in work effort as we move from columns 1 and 2 to column 3). As a result, all individuals' gross and net income increase, and this allows agents to increase their private consumption, despite the fact that they now have to pay by themselves for the impure public good.

Therefore, the addition of a market-based mechanism for the impure public good generates gains in addition to those generated by optimal behavior on the part of state firms. We label this the "extra market effect".

3.6 Adding social externalities

In the model with user prices above, we assumed away externalities. In particular, in the model in this section, the publicly provided good was assumed to be fully private. Now, to make the results in this section directly comparable to those in the status quo regime in section 3, we allow for social externalities generated by the individual or private use of this publicly provided good (education and health are the classic examples of such goods, as argued in the Introduction above).

To account for social externalities, we will use the modeling of e.g. Alesina et al. (2005) by assuming that spending on the "public" good by one agent creates positive spillovers for all other agents and this is captured by a parameter $0 \leq \gamma \leq 1$. In particular, in order to make this case easily comparable to the cases studied so far, we assume that the utility functions of the three household types are:

$$u(c_t^k, e_t^k, \tilde{g}_t^k) = \mu_1 \log c_t^k + \mu_2 \log(1 - he_t^k) + \mu_3 \log \tilde{g}_t^k \tag{24a}$$

$$u(c_t^w, e_t^w, \tilde{g}_t^w) = \mu_1 \log c_t^w + \mu_2 \log(1 - he_t^w) + \mu_3 \log \tilde{g}_t^w \tag{24b}$$

$$u(c_t^b, e_t^b, \tilde{g}_t^b) = \mu_1 \log c_t^b + \mu_2 \log(1 - he_t^b) + \mu_3 \log \tilde{g}_t^b \tag{24c}$$

where we define $\tilde{g}_t^k = g_t^k + \gamma\left[\frac{N^k g_t^k + N^w g_t^w + N^b g_t^b}{N} - g_t^k\right]$, $\tilde{g}_t^w = g_t^w + \gamma\left[\frac{N^k g_t^k + N^w g_t^w + N^b g_t^b}{N} - g_t^w\right]$, $\tilde{g}_t^b = g_t^b + \gamma\left[\frac{N^k g_t^k + N^w g_t^w + N^b g_t^b}{N} - g_t^b\right]$ and where $\gamma \geq 0$ measures the degree of external effects. Notice that if we set $\gamma = 1$, then $\tilde{g}_t^k = \tilde{g}_t^w = \tilde{g}_t^b = v^k g_t^k + v^w g_t^w + v^b g_t^b = v^g y_t^g \equiv \bar{y}_t^g$, which is as in the status quo regime in section 2.

Then the first-order conditions with respect to the quantity of the impure public good demanded by each individual change to:

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^k} = \frac{\mu_3}{g_t^k + \gamma[(v^k - 1)g_t^k + v^w g_t^w + v^b g_t^b]} \tag{25a}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^w} = \frac{\mu_3}{g_t^w + \gamma[v^k g_t^k + (v^w - 1)g_t^w + v^b g_t^b]} \tag{25b}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^b} = \frac{\mu_3}{g_t^b + \gamma[v^k g_t^k + v^w g_t^w + (v^b - 1)g_t^b]} \tag{25c}$$

The new equilibrium system is presented at the end of Appendix B. It consists of the same equations and the same endogenous variables as in the subsection 4.3, except from the first-order conditions with respect to the public good which now are as above.

Table 4 here

The new system is solved numerically by using the same parameter values and policy variables as in Table 1. Results with externalities are reported in Table 4, column

3. In this solution, we have set, for instance, $\gamma = 0.15$.[18] Inspection of these results reveals that the main message remains as before. Namely, even under social externalities, the switch (from the status quo economy to an economy with user prices) improves individual incentives and implies significant efficiency gains. Moreover, the distributional implications of user prices, at least qualitatively, remain unaffected by the presence of social externalities.

4. Privatization or change in ownership

In this section, we consider the case in which the sector of impure public goods and services is fully privatized so that now, not only there is a market-based mechanism for its allocation as in the previous section, but also production decisions are made by private firms. Hence, the main aim of this section is to study how important is the matter of ownership.[19]

4.1 Informal description of the new policy regime

In this section, both the production and the ownership of the excludable public good are privatized. Relative to section 3, the main difference is that now the impure public good is not produced by state-owned firms but it is instead produced by private firms, exactly like the private good. Moreover, it is capitalists who own these “new” private firms. This implies that if the production of the impure public good entails any profits, these are distributed to capitalists and not to the government as it was the case in section 3. Finally, the privatized firms, contrary to the state owned firms producing the impure public good in section 3, have access to the private capital market from which they hire the quantity of capital needed for the production process.

4.2 What changes in the model relative to the model in section 3

Here we model what changes relative to the previous section.

[18] We report that we get well-defined solutions in the range $0 \le \gamma \le 0.55$. Above 0.55, we $g_t^b < 0$, meaning that, when social externalities get relatively large, the low-income groups find it optimal to free ride on the other income groups.

[19] See e.g. Drazen (2000, chapter …) for a review of the early literature on privatization.

Households

The budget constraint of the capitalist changes to:

$$(1+\tau_t^c)c_t^k + [k_{t+1} - (1-\delta)k_t^k] + b_{t+1}^k - b_t^k + p_t^g g_t^k = (1-\tau_t^k)(r_t k_t^k + \pi_t^k + (v^g / v^k)\pi_t^g) + (1-\tau_t^l)w_t^k e_t^k h + r_t^b b_t^k + g^{tr,k} \tag{26}$$

That is, now the capitalists receive the profits of the privatized firms.

Privatized firms charging user prices and acting optimally

Each privatized firm, $g = 1, 2, ..., N^g$, maximizes profits given by:

$$\pi_t^g \equiv p_t^g y_t^g - r_t k_t^g - w_t^g l_t^g - g_t^g \tag{27}$$

subject to the same technology constraint as above, namely:

$$y_t^g = A^g (k_t^g)^{\theta_1} (l_t^g)^{\theta_2} (g_t^g)^{1-\theta_1-\theta_2}$$

The first-order conditions for the three inputs, $\{k_t^g, l_t^g, g_t^g\}_{t=0}^{\infty}$, are respectively:

$$r_t = \frac{\theta_1 y_t^g}{k_t^g} p_t^g \tag{28a}$$

$$w_t^g = \frac{\theta_2 y_t^g}{l_t^g} p_t^g \tag{28b}$$

$$g_t^g = (1-\theta_1-\theta_2) y_t^g p_t^g \tag{28c}$$

where equations (28a)-(28c) equate the marginal product of each factor of production to its return.

Government budget constraint

The within-period government budget constraint now changes to:

$$
\begin{aligned}
&N^k g^{tr,k} + N^w g^{tr,w} + N^b g^{tr,b} + (1+r_t^b) N^k b_t^k = N^k b_{t+1}^k + \tau_t^c (N^k c_t^k + N^w c_t^w + N^b c_t^b) + \\
&+ \tau_t^k N^k (r_t k_t^k + \pi_t^k + (N^g / N^k)\pi_t^g) + \tau_t^l (N^k w_t^k e_t^k h + N^w w_t^w e_t^w h + N^b w_t^g e_t^b h)
\end{aligned} \tag{29}
$$

Notice that the profits of the newly privatized firms do not enhance the government budge constraint as it was the case with the profits of the state firms in section 3. As already said, the profits of privatized firms, if any, are distributed to capitalists, and thereby are incorporated into their income. However, these profits, as any other source of income, are taxed, and, as a result, the tax proceeds are part of the government budget constraint.

4.3 Decentralized equilibrium

The new equilibrium system is presented in Appendix C. It consists of the same number of equations and the same endogenous variables as in the previous section with state firms and user prices, but now some equations (and, of course, definitions of some variables) differ.

The new system is solved numerically using the same parameter values and policy variables as in Table 1.

4.4 Solution and discussion of steady state results

The last column in Table 2 reports the steady state solution of the privatized economy. Comparison of the solutions in columns 2 and 3 (both with user prices and optimizing behavior on the part of the relevant firms) reveals that the two regimes are quite similar, not only qualitatively but also quantitatively (any differences are at the third decimal point only and we report that this is robust to changes in parameter values.[20] In other words, to the extent that there is a market mechanism for the impure public good and state firms in column 2 act optimally like their private counterparts do in column 3, the issue of ownership is not of great importance and this is both in terms of aggregate outcomes and distribution. It should be stressed, however, that so far we have worked in

[20] The slight superiority of the solution with state production is possibly due to two reasons. First, when the firms that produce the impure public good remain under state control, the revenues associated with the sale of this good, enhance the government budget, thus allowing for lower tax rates, relative to the case in which the same firms are privatized and where it is only the tax proceeds of profits that enter the government budget constraint. Second, while the capital used by state-owned firms to produce the impure public good, is accumulated through time via public investment spending (which again burdens the

an environment without differences in productivity between state-owned and privatized firms and that we have assumed away any political economy distortions (these assumptions are relaxed in section 8 below).

4. 5 Adding social externalities

As we did in the previous section, and in particular in subsection 3.6, we now add social externalities. We work exactly as explained in section 3.6 above, except that now the good is produced by privatized, rather than state, firms. The new equilibrium system is presented at the end of Appendix C.

The new system is solved numerically by using the same parameter values and policy variables as in Table 1. We also again set $\gamma = 0.15$, as we did in subsection 4.5 before. Results with externalities are reported in Table 4, the last column. Inspection of these results reveals that the main messages - regarding both aggregate and distributional implications of user prices - remain as in Table 2.

5. Cost minimization

We now assume that the objective of (state and privatized) firms producing the impure public good is cost minimization rather than profit maximization as assumed so far. Subsection 5.1 models the case in which state firms minimize costs without charging user prices; thus, this is within the status quo framework of section 2. Subsection 5.2 models the case in which state firms minimize costs and they also charge user prices; thus, this is like in section 3. Subsection 5.3 models the widely-met case in most industrial countries nowadays, in which privatized firms minimize costs with the government subsidizing production in case this entails losses; this is known as "private providers" and belongs to section 4 since the ownership is private as it was in section 4. Subsection 5.4 will sum up results under cost minimization.

In all cases with cost minimization, we obviously need a target value for output, denoted as $\tilde{y}_t^g$. For reasons of comparison to what we have done so far, we choose to consider two cases. First, we assume that the public output target, $\tilde{y}_t^g$, is set at the value

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government budget), in the case in which the same firms are privatized, they hire capital directly from the private capital market, thus crowding out the capital going to the firms producing the private good.

implied by the status quo regime where policy variables were exogenously set as in the data. Numerical results for this case are in Table 5a. Second, we assume that $\tilde{y}_t^g$ takes the value derived optimally by the status quo regime when state firms acted as profit maximizers. Numerical results for this case are in Table 5b. Then, in each of these two cases, given the associated output target, we compare the various systems of producing that level of output by assuming that, across all these systems, the producers (either public or private) act as cost minimizers. Before we start, we report that our qualitative results are robust to the value of $\tilde{y}_t^g$ assumed.

Tables 5a and 5b here

5.1 Cost minimizing state firms without user prices (as in section 3)

Given an output target, $\tilde{y}_t^g$, each state firm g chooses its inputs to minimize costs. Thus,

$$\sum_{t=0}^{\infty}(\beta^g)^t\left[w_t^g l_t^g + g_t^g + g_t^i + \lambda_t^g[\tilde{y}_t^g - A^g(k_t^g)^{\theta_1}(l_t^g)^{\theta_2}(g_t^g)^{1-\theta_1-\theta_2}]\right] \tag{30}$$

where λ_t^g is the multiplier associated with the output target (see also …).

The first-order conditions for the three inputs, $\{k_{t+1}^g, l_t^g, g_t^g\}_{t=0}^{\infty}$, are respectively:

$$1=\beta^g\left[1-\delta^g+\frac{\theta_1 y_{t+1}^g}{k_{t+1}^g}\lambda_{t+1}^g\right] \tag{31a}$$

$$w_t^g=\frac{\theta_2 y_t^g}{l_t^g}\lambda_t^g \tag{31b}$$

$$g_t^g=(1-\theta_1-\theta_2)y_t^g\lambda_t^g \tag{31c}$$

In this regime, the cost of production is financed by the general government budget. The new equilibrium system is presented in Appendix D and its numerical solution is in column 2, Tables 5a and 5b. Notice that in Table 5b, as expected, cost minimization is equivalent to profit maximization since the output target in the cost minimization problem is set at the value implied by profit maximizing state firms within the status quo regime.

5.2 Cost minimizing state firms with user prices (as in section 4)

The problem of the state firm is as in the previous subsection but now the cost of producing the good is financed by user charges which equalize supply to demand as well as (in case cost minimization of a given output level entails losses) by the general tax payer. The new equilibrium system is presented in Appendix D and its numerical solution is in column 3, Tables 5a and 5b.

5.3 Cost minimizing privatized firms without user prices (as in section 4) or the so-called private providers

A commonly used regime is the one in which the government outsources the production and provision of impure public goods/services to private firms, the so-called private providers. Garbage collection, maintenance of roads, public cleaning vehicles, etc, are examples of this case. In this subsection, we model this case. In particular, we assume that private firms, acting as cost-minimizers, produce a certain amount of the impure public good, while the government subsidizes production (in case the given output level entails losses) via its general budget.[21]

The new equilibrium system is presented in Appendix D and its numerical solution is in column 4, Tables 5a and 5b.

5.4 Discussion of results in Tables 5a-5b

Inspection of the results presented in Tables 5a and 5b reveals that the main message – regarding both aggregate and distributional implications of user prices - remains the same as in Table 2. Thus, firms' objective (profit maximization or cost minimization) is not important to our results for the ranking of various systems.

6. Mixed public financing cases

In this section, although we remain in a market-based system with user prices as in sections 3 and 4 above, there is also supplementary public finance. In particular, we will study three such cases. In subsection 6.1, we study the case in which the government subsidizes a fraction of user charges, in other words, individuals are free to choose the amount they wish, but they have to pay a fraction of the market user price only, with the rest paid through the general government budget. In subsection 6.2, we study the case in which there is also a minimum uniform amount freely provided, so that households have to pay user prices only if they wish to top up. Finally, in subsection 6.3, we study a case with vouchers in which each individual is given a voucher in the form of tax exception with the obligation to use this in order to pay the cost of the public good provided to him/her individually.

6.1 Mix of user prices and public finance

In this subsection, we revisit sections 3 and 4 but now we also allow for partial public finance of the impure public good. In particular, each agent chooses her demand for the impure public good knowing that she will pay only a fraction $0 \le \mu \le 1$ of user charges, while the rest $0 \le (1-\mu) \le 1$ will be covered by the government.

6.1.1 What changes in the model relative to the model in sections 3 and 4

Here we model what changes relative to sections 3 and 4.

Households

Since now households are aware of the fact that they will get back a fraction of their total spending on the impure public good, their budget constraints change.

For the capitalist, the budget constraint is:

$$(1+\tau_t^c)c_t^k + [k_{t+1} - (1-\delta)k_t^k] + b_{t+1}^k - b_t^k + \mu p_t^g g_t^k = (1-\tau_t^k)(r_t k_t^k + \pi_t^k) + (1-\tau_t^l) w_t^k e_t^k h + r_t^b b_t^k + g^{tr,k} \quad (32a)$$

$$(1+\tau_t^c)c_t^k + [k_{t+1} - (1-\delta)k_t^k] + b_{t+1}^k - b_t^k + \mu p_t^g g_t^k = (1-\tau_t^k)(r_t k_t^k + \pi_t^k + (v^g / v^k)\pi_t^g) + (1-\tau_t^l) w_t^k e_t^k h + r_t^b b_t^k + g^{tr,k} \quad (32b)$$

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[21] In practice, potential private providers make a bid for a contract that gives the winner the sole right to provide the good/service. See Sorensen (2016) for details and a principal-agent model of this regime,

where (32a) refers to the case in which state-owned firms produce the impure public good, whereas (32b) refers to the case in which these firms are privatized (in the latter case, the capitalist also gets any profits made by the newly privatized firms).

For the worker, the budget constraint is:

$$(1+\tau_t^c)c_t^w + \mu p_t^g g_t^w = (1-\tau_t^l)w_t^w e_t^w h + g_t^{tr,w} \tag{33}$$

For the public employees, the budget constraint is:

$$(1+\tau_t^c)c_t^b + \mu p_t^g g_t^b = (1-\tau_t^l)w_t^g e_t^b h + g_t^{tr,b} \tag{34}$$

Then, the first-order conditions with respect to the quantity of the impure public good demanded (one for each type of household) change to:

$$\frac{\mu_1 \mu p_t^g}{(1+\tau_t^c)c_t^k} = \frac{\mu_3}{g_t^k} \tag{35a}$$

$$\frac{\mu_1 \mu p_t^g}{(1+\tau_t^c)c_t^w} = \frac{\mu_3}{g_t^w} \tag{35b}$$

$$\frac{\mu_1 \mu p_t^g}{(1+\tau_t^c)c_t^b} = \frac{\mu_3}{g_t^b} \tag{35c}$$

Government budget constraint

Since now the government finances a fraction, $0 \leq (1-\mu) \leq 1$, of agents' spending on the impure public good, the within-period government budget constraints, presented in equations (22) and (29), change respectively to:

$$\begin{aligned} &N^k g^{tr,k} + N^w g^{tr,w} + N^b g^{tr,b} + (1+r_t^b)N^k b_t^k + N^g(w_t^g l_t^g + g_t^g + g_t^i) + (1-\mu)p_t^g(N^k g_t^k + N^w g_t^w + N^b g_t^b) = \\ &= N^g p_t^g y_t^g + N^k b_{t+1}^k + \tau_t^c(N^k c_t^k + N^w c_t^w + N^b c_t^b) + \tau_t^k N^k(r_t k_t^k + \pi_t^k) + \tau_t^l(N^k w_t^k e_t^k h + N^w w_t^w e_t^w h + N^b w_t^g e_t^b h) \end{aligned} \tag{36a}$$

although in a partial equilibrium setup. Here, we work as in Economides et al. (2014).

$$N^k g^{tr,k} + N^w g^{tr,w} + N^b g^{tr,b} + (1+r_t^b) N^k b_t^k + (1-\mu) p_t^g (N^k g_t^k + N^w g_t^w + N^b g_t^b) = N^g p_t^g y_t^g + N^k b_{t+1}^k +$$
$$+ \tau_t^c (N^k c_t^k + N^w c_t^w + N^b c_t^b) + \tau_t^k N^k (r_t k_t^k + \pi_t^k + (N^g / N^k) \pi_t^g) + \tau_t^l (N^k w_t^k e_t^k h + N^w w_t^w e_t^w h + N^b w_t^g e_t^b h) \tag{36b}$$

where (36a) refers to the case in which it is state-owned firms that produce the impure public good, whereas (36b) refers to the case in which these firms are privatized.

6.1.2 Decentralized equilibrium

The two new equilibrium systems, i.e. with state-owned firms and with privatized firms, are presented in Appendix E. They consist of the same number of equations and the same endogenous variables as in the sections 3 and 4 respectively with the changes shown above. The new systems are solved numerically using the same parameter values and policy variables as in Table 1.

6.1.3 Solution and discussion of steady state results

Results for various degrees of $0 \le \mu \le 1$, namely various degrees of user charges *vis-à-vis* public finance, are presented in Table 6. Recall that the polar case in which $\mu = 1$, namely the case in which the public good is financed by user charges paid by individual users only, is the case already studied in sections 3 and 4.

Table 6 here

Several interesting results emerge from Table 6. We first report that for very low values of μ, with our parameterization for $\mu < 0.25$, we cannot obtain well-defined solutions. This is because a very low value of μ means a heavy subsidization which pushes individual agents to demand a very high quantity of the impure public good. For relatively low values of μ, say 0.25, we do get solutions, but these solutions are inferior - both in terms of welfare and private output - not only relative to the polar cases with $\mu = 1$ but also relative to the status quo. This is for the same reason: a relatively heavy subsidization gives the wrong incentives; work effort of capitalists and workers falls relative to the status quo and the same applies to their net incomes. The production of the private good also falls relative to the status quo when $\mu = 0.25$. Only when μ gets high enough, say 0.5, so that private incentives are restored, the mixed system gets better than the status quo. Combining results, the main message is clear: if the market mechanism is distorted, it is better not to have a market mechanism at all and rely instead on centralized policy mechanisms.

Our computations also show that, if the criterion is the production of the private good, the value of μ that maximizes this type of output is 1, while, if the criterion is total output, the maximizing value of μ is around 0.87. This again makes sense: if the criterion is the production of the private good, we should choose a public finance scheme that delivers the best incentives for work and saving and this is $\mu = 1$. If, on the other hand, the criterion is total output, which includes the production of the impure public good, we should choose a public financing mix that is relatively biased towards the public good; hence $\mu < 1$.

6.2 Minimum uniform provision and voluntary top ups paid by user charges

In this subsection, and irrespectively of who is the owner of the firms producing the impure public good, we study the case in which there is also a minimum uniform amount freely provided, so that households have to pay user prices only if they wish to top up.[22]

6.2.1 What changes in the model relative to sections 3 and 4

Here we model what changes relative to the polar cases presented in sections 3 and 4.

Households

Since there is a minimum uniform level of impure public good provision freely available, $\bar{g}_t$, so that households pay user prices only if they wish to top up, the within-period utility functions of the three household types change to:

$$u(c_t^k, e_t^k, g_t^k) = \mu_1 \log c_t^k + \mu_2 \log(1 - he_t^k) + \mu_3 \log(\bar{g}_t + g_t^k) \tag{37a}$$

$$u(c_t^w, e_t^w, g_t^w) = \mu_1 \log c_t^w + \mu_2 \log(1 - he_t^w) + \mu_3 \log(\bar{g}_t + g_t^w) \tag{37b}$$

$$u(c_t^b, e_t^b, g_t^b) = \mu_1 \log c_t^b + \mu_2 \log(1 - he_t^b) + \mu_3 \log(\bar{g}_t + g_t^b) \tag{37c}$$

In turn the first-order conditions for the quantity demanded of the impure public good (one for each type of household) change to:

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^k} = \frac{\mu_3}{\overline{g}_t + g_t^k} \tag{38a}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^w} = \frac{\mu_3}{\overline{g}_t + g_t^w} \tag{38b}$$

$$\frac{\mu_1 p_t^g}{(1+\tau_t^c)c_t^b} = \frac{\mu_3}{\overline{g}_t + g_t^b} \tag{38c}$$

Government budget constraint

Since now the government also covers the cost of the minimum uniform amount of the impure public good, the within-period government budget constraint, presented in equations (22) and (29), now change respectively to:

$$p_t^g \overline{g}_t + N^k g^{tr,k} + N^w g^{tr,w} + N^b g^{tr,b} + (1+r_t^b)N^k b_t^k + N^g(w_t^g l_t^g + g_t^g + g_t^i) = N^g p_t^g y_t^g + N^k b_{t+1}^k +$$
$$+\tau_t^c(N^k c_t^k + N^w c_t^w + N^b c_t^b) + \tau_t^k N^k(r_t k_t^k + \pi_t^k) + \tau_t^l(N^k w_t^k e_t^k h + N^w w_t^w e_t^w h + N^b w_t^g e_t^b h) \tag{39a}$$

$$p_t^g \overline{g}_t + N^k g^{tr,k} + N^w g^{tr,w} + N^b g^{tr,b} + (1+r_t^b)N^k b_t^k = N^g p_t^g y_t^g + N^k b_{t+1}^k + \tau_t^c(N^k c_t^k + N^w c_t^w + N^b c_t^b) +$$
$$+\tau_t^k N^k(r_t k_t^k + \pi_t^k + (N^g / N^k)\pi_t^g) + \tau_t^l(N^k w_t^k e_t^k h + N^w w_t^w e_t^w h + N^b w_t^g e_t^b h) \tag{39b}$$

where (39a) refers to the case in which it is state-owned firms that produce the impure public good, whereas (39b) refers to the case in which the same firms are privatized.

Market-clearing condition for the excludable public good

The market-clearing condition is now:

$$N^g y_t^g = N^k g_t^k + N^w g_t^w + N^b g_t^b + N\overline{g}_t \tag{40}$$

[22] This subsection draws on Economides et al. (2017) who focus on tuition fees for publicly provided education services.

where, on the right hand-side of (40), the uniform – freely available – quantity of impure public good has been added.

6.2.2 Decentralized equilibrium

The new equilibrium system, for each case, i.e. with state-owned firms and privatized firms, is presented in Appendix E. It consists of the same number of equations as in the sections 3 and 4 respectively with the changes in the definition of the variables required by each case. However, since now the government provides freely to all households a minimum uniform quantity of the impure public good, the number of endogenous variables has increased by one relative to sections 3 and 4. To deal with this, we assume that the exogenous uniform provision is s fraction of total output, namely;

$$N\overline{g}_t = \overline{s}_t(N^k y_t^f + N^g p_t^g y_t^g) \tag{41}$$

where $\overline{s}_t$ is a new policy instrument measuring public spending on the minimum uniform provision of the impure public good expressed as share of total output.

The new system is solved numerically using the same parameter values and policy variables as in Table 1. We also set $\overline{s}_t$ at 0.05 which is close to the value of …

6.2.3 Solution and discussion of steady state results

Results are reported in Table 7. The main results remain as in Table 2 although, as one would expect, the aggregate or efficiency gains were stronger in Table 2. Namely, private incentives to work and save are stronger when there is a market-based mechanism only and this proves to be good for all types of output (private, total and even public). On the distribution side, on the other hand, inequality gets smaller when there is a mix of a minimum uniform provision and voluntary market-based top us than when there are only market-based top ups. In particular, while the net income and consumption of private agents (capitalists and workers) falls in case we also allow for a minimum uniform provision, the net income and consumption of public employees rises (see levels of income as well as relative incomes). We also study the extreme case in which only capitalists pay user prices if they want to top up, while private workers and public employees make use of the minimum uniform amount only which is provided free of charge (see columns 5 and 8 in Table 7). In this case, it is interesting to notice that the net income of public employees falls considerably so this type of social policy is self-defeating. This happens because the loss of freedom to choose distorts incentives and makes them poorer. Finally notice that, as above, the type of ownership is not important: the main results are the same with privatization.

Table 7 here

These results are practically similar to those in the literature on the public provision of private goods (see e.g. the review paper by Blomquist et al., 2010). Actually, Blomquist et al. (2010) show, in a model where the need for the publicly provided good is an increasing function of work hours (e.g. child care), that the optimally chosen tax rates play the same role as market prices (in our case user prices) in the sense that they “serve to induce agents to internalize the real resource cost of the publicly provided work-complement” and also “to deter mimicking behavior” in an incomplete information setup. They also show that a Pareto improvement can be achieved if we supplement a freely publicly provided private good with a properly designed optimal tax-public provision scheme. We believe that our results are complementary. In our work, we do not claim that the introduction of user prices delivers the best possible outcome. What we do claim is that the introduction of user prices Pareto dominates the outcome that would be achieved by “free” provision without user prices, other things equal. Blomquist et al. (2010), on the other hand, show that further gains can be achieved by a tax-transfer scheme that is optimally designed and replaces the market mechanism. However, we believe that what they suggest is more complex socially than the simple a-personal market mechanism studied here.

7. Some political economy issues

In this section, we introduce political economy issues. First, in the case with state firms and user charges studied in section 3, we assume that the wage rate paid to public sector employees is not determined according to their marginal productivity as assumed so far. Instead, we assume that the public sector wage rate follows an ad-hoc rule, as is widely believed to be the case in most industrialized countries. This could be justified, among others, on the grounds of public sector unions, which are powerful enough to exploit their position and earn extra benefits on behalf of their members. Second, in the case with privatized firms and user charges in section 4, we assume that the privatized firms enjoy monopolistic power and hence make extra profits. This is a common social fear behind privatization schemes. We realize, of course, that there is a plethora of other political economy stories associated with the production and finance of impure public goods. Unavoidably we are selective focusing on two of them, one on the labor side and one on the firm side, that seem to be at the center of the political debate.

7.1 Public wages and imperfect competition in product markets

Here we discuss what changes in terms of equations and endogenous variables relative to sections 3 and 4.

7.1.1 Public wages: What changes in the model relative to section 3

We start by modeling the case in which public sector employees earn a wage rate above their marginal productivity. In particular, we arbitrarily assume that the wage rate in the public sector equals the wage rate of private workers. Thus,

$$w_t^g \equiv w_t^w \tag{47}$$

That is, relative to the model in section 4, we simply replace the optimality condition for employment in the public sector with the ad hoc condition above.

Decentralized equilibrium

The new equilibrium system is as the one presented in Appendix B with the changes described above. The new system is solved numerically using the same parameter values and policy variables as in Table 1.

Solution and discussion of steady state results

Column 4 in Table 8 reports the steady state solution of the above economy. The solution is well defined. By comparing the solution in this column to the solution in the previous column (column 3), where the latter is the case analyzed in section 4, we conclude that, private incentives (at least regarding capitalists and private workers) to work and save are affected negatively from the presence of this sort of power on the part of public sector employees. On the distribution side, as expected, inequality gets worse for private workers and better for public employees, relative to the solution in column 3. Finally, notice that a switch from the status quo economy to an economy with user charges, as implied by the comparison of columns 1-2 with column 4, is still efficiency-enhancing, even in the presence of such distortion, although the efficiency gains are smaller than in section 3.

Table 8 here

7.1.2 Imperfect competition in the product market: What changes in the model relative to section 5

Here we model the case in which the newly privatized firms, producing the impure public good, enjoy monopolistic power and hence achieve extra profits over those justified in a perfectly competitive environment. To model this, we follow most of the related literature on imperfect competition in product markers by distinguishing between final goods producers and intermediate goods producers, where the former act competitively and the latter non-competitively. The technology that transforms intermediate goods to final goods is a la Dixit-Stiglitz. This practically means that, in equilibrium, imperfect competition is simply captured by a parameter which measures the degree of substitutability in intermediate inputs or equivalently the degree of competition in product markets. We define this parameter as $0 \leq \xi \leq 1$, where $\xi = 1$ can take us back to perfect competition in section 4. Modelling details are in Appendix F.

Decentralized equilibrium

The new equilibrium system is as the one presented in Appendix D with the changes described above. The new system is in Appendix G. It is solved numerically using the same parameter values and policy variables as in Table 1. Regarding the parameter $0 \leq \xi \leq 1$, which measures the degree of market power enjoyed by the newly privatized firms, we calibrate it so as to imply a profit-to-revenue ratio in the range of 5-15%. However, as is reported below, this is not crucial to the main conclusions.

Solution and discussion of steady state results

Columns 6, 7 and 8 in Table 8 report the steady state solution of the above modeled economy for alternative, abnormal, profit-to-total revenues ratios. These solutions are well defined. By comparing the solutions in columns 6-8 to the solution in column 5, the latter being the case analyzed in section 4, we conclude that private incentives (at least regarding capitalists and ex public sector employees) to work and save are affected negatively from the presence of monopolistic power in the market of the impure public good. Moreover, on the distribution side, inequality gets worse. However, a switch from the status quo economy to an economy in which there are user charges and privatized firms enjoying monopolistic power, as implied by the comparison of columns 1-2 with columns 5, 6 and 7, is still efficiency-enhancing, although the efficiency gains are smaller than in the case without monopolistic power.

7.2 Does ownership matter under political economy issues?

Comparison of column 4 to columns 6-8 in Table 8 reveals that now the regime of private production with user charges is slightly superior to the regime with public production and user charges, at least when the criterion is per capita private output or per capita total output. Individual incomes for capitalists and workers are higher in the former regime, whereas public sector employee’s individual income is higher in the latter. Regarding distribution, inequality seems to be lower, when it is measured as the net income of worker vis-à-vis the net income of capitalist, but higher, when it is measured as the net income of public sector employee vis-à-vis the net income of capitalist, in the privatization regime.

Therefore, once we assume the existence of political economy distortions, namely, extra benefits for public sector employees in the regime of public production with user charges, and monopolistic power for privatized firms in the regime of private production with user charges, the ranking of various social systems of public good provision, in terms of per capita private and/or total output, seem to change in favor of privatization. To put it differently, with common parameterizations, the distortion associated with wage determination in the public sector is more important quantitatively than the distortion associated with monopolistic power enjoyed by privatized firms. This is in terms of aggregate outcomes or efficiency. On the other hand, on the distribution side, the comparison is ambiguous.

8. Concluding remarks

In this paper, we have tried to quantify the aggregate and distributional implications of various social systems of impure public good provision. Although the main results have already been listed in the Introduction, here we wish to emphasize that the introduction of user prices (to a system with general taxes) can crowd in private incentives to work and save and hence improve aggregate efficiency. We also showed that – at least in some cases – the introduction of user prices can also reduce income inequality. All this is irrespectively of ownership. Thus, our results question the validity of some widely perceived views in public policy regarding the dilemma between efficiency and equity.

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Table 1: Baseline parameterization

Parameters and policy instruments	**Description**	**Value**
α_1	Share of capital in private production	0.33
α_2	Share of labor in private production	0.64
θ_1	Share of capital in public production	0.1078
θ_2	Share of employment in public production	0.5690
δ	Private capital depreciation rate	0.05
δ^g	Public capital depreciation rate	0.05
β	Time discount rate	0.9
μ_1	Preference parameter on private consumption in utility	0.35
μ_2	Preference parameter on leisure in utility	0.6
μ_3	Preference parameter on publicly provided goods in utility	**1-** $\mu_1 - \mu_2$
h	Fixed shift length of hours at work	1
s^w	Public wage payments as share of GDP (data)	0.132
s^g	Public purchases of goods-services as share of GDP (data)	0.075
s^i	Public investment as share of GDP (data)	0.025
s^{tr}	Government transfers as share of GDP (data)	0.2170
$s^{tr,k}$	Public transfers as share of GDP to capitalists (set)	0.0321
$s^{tr,w}$	Public transfers as share of GDP to private workers (set)	0.1382
$s^{tr,b}$	Public transfers as share of GDP to public sector employees (set)	0.0467
τ^c	Tax rate on consumption (data)	0.1938
τ^k	Tax rate on capital income (data)	0.2903
τ^l	Tax rate on labor income (data)	0.3780
B/Y	Public debt as a share of GDP (data)	0.9
v^k	Capitalists as share of population (data)	0.1480
v^w	**Workers as share of population (data)???**	**0.6370**
v^b	Public employees as share of population (data)	0.2150
v^f	Private firms as a share of population (set)	$v^k + v^w$
v^g	Public firms as a share of population (set)	v^b
ϕ	Degree of competition in private product market	$0 \le \phi \le 1$
μ	Fraction of subsidization of public goods	$0 \le \mu \le 1$
γ	Degree of social externalities	0.15
$\bar{s}_{y_g}$	Minimum provision of public output as share of GDP	0.03
A^f	Long-run TFP in private firms' production function	1
A^g	Long-run TFP in public firms' production function	1
A^k	Long-run labor productivity of capitalists	3
A^w	Long-run labor productivity of private workers	1
A^b	Long-run labor productivity of public sector employees	1
ξ	Degree of market power of privatized ex state-owned firms	0.85-1.0

Table 2: Steady state solutions in the three polar cases

Variable	Status quo	Reformed economies	
		Profit max state firms with user prices	**Profit max privatized firms with user prices**
c^k	0.6521	0.8442	0.8420
e^k	**0.1301**	**0.1908**	**0.1853**
k^k	2.8656	3.4534	3.5912
g^k	0.0470	0.1035	0.0998
y^k	0.7954	1.0169	1.0215
c^w	0.1774	0.2344	0.2322
e^w	**0.2902**	**0.3261**	**0.3260**
g^w	0.0470	0.0287	0.0275
y^w	0.1774	0.2344	0.2322
c^b	0.1483	0.1541	0.1521
e^b	**0.2726**	**0.2799**	**0.2792**
g^b	0.0470	0.0189	0.0180
y^b	0.1483	0.1541	0.1521
y^w / y^k	**0.2230**	**0.2305**	**0.2273**
y^b / y^k	**0.1864**	**0.1515**	**0.1489**
y	**0.2917**	**0.4040**	**0.4007**
y^f	**0.3717**	**0.4479**	**0.4441**
y^g	**0.2187**	**0.1753**	**0.1682**
p^g	-	1.3908	1.4394
c / y	0.8273	0.7607	0.7618
τ^l	**0.3652**	**0.1165**	**0.1248**
u^k	-0.3861	-0.2997	-0.2984
u^w	-0.9639	-0.9221	-0.9274
u^b	-1.0118	-1.0500	-1.0564
u	-0.8887	-0.8575	-0.8621
s^w	0.132	0.0738	0.0739
s^g	0.075	0.0419	0.0420
s^i	0.025	0.0043	-

Table 3: Decomposition of steady state effects as we switch to state production with user prices

Variable	Status quo	Status quo with profit max state firms	Non-optimizing state firms with user prices	Profit max state firms with user prices
c^k	0.6521	0.7863	0.7097	0.8442
e^k	**0.1301**	**0.1618**	**0.1606**	**0.1908**
k^k	2.8656	3.1096	3.2174	3.4534
g^k	0.0470	0.0283	0.1433	0.1035
y^k	0.7954	0.9418	0.8706	1.0169
c^w	0.1774	0.2187	0.1928	0.2344
e^w	**0.2902**	**0.3006**	**0.3158**	**0.3261**
g^w	0.0470	0.0283	0.0389	0.0287
y^w	0.1774	0.2187	0.1928	0.2344
c^b	0.1483	0.1187	0.1795	0.1541
e^b	**0.2726**	**0.2311**	**0.3086**	**0.2799**
g^b	0.0470	0.0283	0.0362	0.0189
y^b	0.1483	0.1187	0.1795	0.1541
y^w / y^k	**0.2230**	**0.2322**	**0.2215**	**0.2305**
y^b / y^k	**0.1864**	**0.1260**	**0.2062**	**0.1515**
y	**0.2917**	**0.3166**	**0.3730**	**0.4040**
y^f	**0.3717**	**0.4033**	**0.4173**	**0.4479**
y^g	**0.2187**	**0.1316**	**0.2502**	**0.1753**
p^g	-	1.2269	0.8447	1.3908
c / y	0.8273	0.8882	0.7143	0.7607
τ^l	**0.3652**	**0.2056**	**0.2840**	**0.1165**
u^k	-0.3861	-0.3683	-0.3222	-0.2997
u^w	-0.9639	-0.9248	-0.9661	-0.9221
u^b	-1.0118	-1.0818	-0.9884	-1.0500
u	-0.8887	-0.8762	-0.8756	-0.8575
s^w	0.132	0.0624	0.132	0.0738
s^g	0.075	0.0354	0.075	0.0419
s^i	0.025	0.037	0.025	0.0043

Table 4: Steady state solutions in the three polar cases with externalities in the reformed economies

Variable	Status quo	Status quo with profit max state firms	Profit max state firms with user prices	Profit max privatized firms with user prices
c^k	0.6521	0.7863	0.8399	0.8377
e^k	**0.1301**	**0.1618**	**0.1949**	**0.1895**
k^k	2.8656	3.1096	3.4600	3.5981
g^k	0.0470	0.0283	0.1134	0.1093
y^k	0.7954	0.9418	1.0129	1.0176
c^w	0.1774	0.2187	0.2350	2.2329
e^w	**0.2902**	**0.3006**	**0.3241**	**0.3240**
g^w	0.0470	0.0283	0.0270	0.0258
y^w	0.1774	0.2187	0.2350	0.2329
c^b	0.1483	0.1187	0.1577	0.1557
e^b	**0.2726**	**0.2311**	**0.2756**	**0.2749**
g^b	0.0470	0.0283	0.0159	0.0152
y^b	0.1483	0.1187	0.1577	0.1557
y^w / y^k	**0.2230**	**0.2322**	**0.2321**	**0.2289**
y^b / y^k	**0.1864**	**0.1260**	**0.1557**	**0.1530**
y	**0.2917**	**0.3166**	**0.4048**	**0.4014**
y^f	**0.3717**	**0.4033**	**0.4487**	**0.4450**
y^g	**0.2187**	**0.1316**	**0.1739**	**0.1669**
p^g	-	1.2269	1.4045	1.4538
c / y	0.8273	0.8882	0.7607	0.7618
τ^l	**0.3652**	**0.2056**	**0.1165**	**0.1248**
u^k	-0.3861	-0.3683	-0.3053	-0.3040
u^w	-0.9639	-0.9248	-0.9196	-0.9250
u^b	-1.0118	-1.0818	-1.0377	-1.0439
u	-0.8887	-0.8762	-0.8541	-0.8586
s^w	0.132	0.0624	0.0738	0.0739
s^g	0.075	0.0354	0.0419	0.0420
s^i	0.025	0.037	0.0043	-

Table 5a: Steady state solutions in the three polar cases when firms producing the impure public good act as cost minimizers

Variable	Status quo	Status quo with cost-min state firms	Cost-min state firms with user prices	Cost-min privatized firms with user prices
c^k	0.6521	0.7863	0.9062	0.8730
e^k	**0.1301**	**0.1618**	**0.2026**	**0.1711**
k^k	2.8656	3.1096	3.5454	3.4792
g^k	0.0470	0.0283	0.0794	0.0808
y^k	0.7954	0.9418	1.0835	1.0470
c^w	0.1774	0.2187	0.2538	0.2358
e^w	**0.2902**	**0.3006**	**0.3301**	**0.3284**
g^w	0.0470	0.0283	0.0222	0.0218
y^w	0.1774	0.2187	0.2538	0.2358
c^b	0.1483	0.1187	0.1271	0.1231
e^b	**0.2726**	**0.2311**	**0.2421**	**0.2460**
g^b	0.0470	0.0283	0.0111	0.0114
y^b	0.1483	0.1187	0.1271	0.1231
y^w / y^k	**0.2230**	**0.2322**	**0.2342**	**0.2252**
y^b / y^k	**0.1864**	**0.1260**	**0.1173**	**0.1175**
y	**0.2917**	**0.3166**	**0.4161**	**0.3950**
y^f	**0.3717**	**0.4033**	**0.4598**	**0.4368**
y^g	**0.2187**	**0.1316**	**0.1316**	**0.1316**
p^g	-	1.2269	1.9474	1.8435
c / y	0.8273	0.8882	0.7765	0.7743
τ^l	**0.3652**	**0.2056**	**0.0376**	**0.1081**
u^k	-0.3861	-0.3683	- 0.2969	- 0.2859
u^w	-0.9639	-0.9248	- 0.9107	- 0.9358
u^b	-1.0118	-1.0818	- 1.1133	- 1.1265
u	-0.8887	-0.8762	- 0.8634	- 0.8806
s^w	0.132	0.0624	0.0446	0.0501
s^g	0.075	0.0354	0.0253	0.0285
s^i	0.025	0.037	0.0026	-

Table 5b: Steady state solutions in the three polar cases when firms producing the impure public good act as cost minimizers

Variable	Status quo	Status quo with cost-min state firms	Cost-min state firms with user prices	Cost-min privatized firms with user prices
c^k	0.6521	0.6187	0.7651	0.7399
e^k	**0.1301**	**0.1209**	**0.1739**	**0.1587**
k^k	2.8656	2.7944	3.3210	3.4968
g^k	0.0470	0.0470	0.1259	0.1262
y^k	0.7954	0.7584	0.9312	0.9147
c^w	0.1774	0.1672	0.2098	0.1998
e^w	**0.2902**	**0.2872**	**0.3203**	**0.3185**
g^w	0.0470	0.0470	0.0345	0.0341
y^w	0.1774	0.1672	0.2098	0.1998
c^b	0.1483	0.1644	0.1804	0.1809
e^b	**0.2726**	**0.2856**	**0.3052**	**0.3087**
g^b	0.0470	0.0470	0.0297	0.0309
y^b	0.1483	0.1644	0.1804	0.1809
y^w / y^k	**0.2230**	**0.2205**	**0.2254**	**0.2184**
y^b / y^k	**0.1864**	**0.2167**	**0.1937**	**0.1977**
y	**0.2917**	0.2845	**0.3868**	**0.3756**
y^f	**0.3717**	**0.3624**	**0.4307**	**0.4186**
y^g	**0.2187**	**0.2187**	**0.2187**	**0.2187**
p^g	-	-	1.0362	0.9998
c / y	0.8273	0.8204	0.7385	0.7338
τ^l	**0.3652**	0.4040	**0.2157**	**0.2553**
u^k	-0.3861	-0.3982	-0.3119	-0.3126
u^w	-0.9639	-0.9820	-0.9464	-0.9627
u^b	-1.0118	-0.9867	-0.9938	-0.9939
u	-0.8887	-0.8966	-0.8627	-0.8732
s^w	0.132	0.1705	0.1149	0.1270
s^g	0.075	0.0969	0.0654	0.0721
s^i	0.025	0.0100	0.0068	-

Table 6: Steady state solutions with a mix of user prices and public finance

Variable	Status quo	Status quo with profit max state firms	Profit max state firms with user prices			Profit max privatized firms with user prices		
			$\mu = 0.25$	$\mu = 0.87$	$\mu = 1$	$\mu = 0.25$	$\mu = 0.87$	$\mu = 1$
c^k	0.6521	0.7863	0.4409	0.8172	0.8442	0.4254	0.8145	0.8420
e^k	**0.1301**	**0.1618**	**0.0517**	**0.1839**	**0.1908**	**0.0257**	**0.1776**	**0.1853**
k^k	2.8656	3.1096	2.2249	3.3938	3.4534	2.4363	3.5466	3.5912
g^k	0.0470	0.0283	0.1415	0.1093	0.1035	0.1347	0.1053	0.0998
y^k	0.7954	0.9418	0.5521	0.9869	1.0169	0.5473	0.9919	1.0215
c^w	0.1774	0.2187	0.1147	0.2260	0.2344	0.1078	0.2235	0.2322
e^w	**0.2902**	**0.3006**	**0.2597**	**0.3230**	**0.3261**	**0.2596**	**0.3229**	**0.3260**
g^w	0.0470	0.0283	0.0368	0.0302	0.0287	0.0341	0.0289	0.0275
y^w	0.1774	0.2187	0.1147	0.2260	0.2344	0.1078	0.2235	0.2322
c^b	0.1483	0.1187	0.1424	0.1609	0.1541	0.1291	0.1584	0.1521
e^b	**0.2726**	**0.2311**	**0.2919**	**0.2858**	**0.2799**	**0.2871**	**0.2850**	**0.2792**
g^b	0.0470	0.0283	0.0457	0.0215	0.0189	0.0409	0.0205	0.0180
y^b	0.1483	0.1187	0.1424	0.1609	0.1541	0.1291	0.1584	0.1521
y^w / y^k	**0.2230**	**0.2322**	**0.2078**	**0.2290**	**0.2305**	**0.1969**	**0.2254**	**0.2273**
y^b / y^k	**0.1864**	**0.1260**	**0.2579**	**0.1630**	**0.1515**	**0.2359**	**0.1597**	**0.1489**
y	**0.2917**	**0.3166**	**0.3418**	**0.4042**	**0.4040**	**0.3213**	**0.4003**	**0.4007**
y^f	**0.3717**	**0.4033**	**0.2886**	**0.4402**	**0.4479**	**0.2707**	**0.4358**	**0.4441**
y^g	**0.2187**	**0.1316**	**0.2521**	**0.1863**	**0.1753**	**0.2347**	**0.1786**	**0.1682**
p^g	-	1.2269	2.1261	1.4657	1.3908	2.1541	1.5161	1.4394
c / y	0.8273	0.8882	0.4943	0.7409	0.7607	0.4961	0.7420	0.7618
τ^l	**0.3652**	**0.2056**	**0.6063**	**0.1520**	**0.1165**	**0.6302**	**0.1613**	**0.1248**
u^k	-0.3861	-0.3683	-0.4163	-0.3033	-0.2997	-0.4150	-0.3016	-0.2984
u^w	-0.9639	-0.9248	-1.1033	-0.9295	-0.9221	-1.1289	-0.9355	-0.9274
u^b	-1.0118	-1.0818	-1.0436	-1.0335	-1.0500	-1.0795	-1.0407	-1.0564
u	-0.8887	-0.8762	-0.9888	-0.8592	-0.8575	-1.0126	-0.8643	-0.8621
s^w	0.132	0.0624	0.1919	0.0826	0.0738	0.1926	0.0828	0.0739
s^g	0.075	0.0354	0.1090	0.0469	0.0419	0.1094	0.0470	0.0420
s^i	0.025	0.037	0.0113	0.0049	0.0043	-	-	-

Table 7: Steady state solutions with minimum provision of public goods and services

Variable	Status quo	Status quo with profit-max state firms	Profit max state firms with user prices			Profit max privatized firms with user prices		
			Without minimum provision	Minimum provision	Minimum provision (only capitalists pay)	Without minimum provision	Minimum provision	Minimum provision (only capitalists pay)
c^k	0.6521	0.7863	0.8442	0.7856	0.7990	0.8420	0.7875	0.8009
e^k	**0.1301**	**0.1618**	**0.1908**	**0.1839**	**0.1976**	**0.1853**	**0.1749**	**0.1914**
k^k	2.8656	3.1096	3.4534	3.2820	3.2538	3.5912	3.3983	3.3273
g^k	0.0470	0.0283	0.1035	0.0927	0.1102	0.0998	0.0802	0.0980
y^k	0.7954	0.9418	1.0169	0.9497	0.9617	1.0215	0.9574	0.9672
c^w	0.1774	0.2187	0.2344	0.2220	0.2341	0.2322	0.2199	0.2326
e^w	**0.2902**	**0.3006**	**0.3261**	**0.3081**	**0.2948**	**0.3260**	**0.3088**	**0.2954**
g^w	0.0470	0.0283	0.0287	0.0146	0	0.0275	0.0142	0
y^w	0.1774	0.2187	0.2344	0.2220	0.2341	0.2322	0.2199	0.2326
c^b	0.1483	0.1187	0.1541	0.1562	0.1294	0.1521	0.1498	0.1245
e^b	**0.2726**	**0.2311**	**0.2799**	**0.2604**	**0.2213**	**0.2792**	**0.2558**	**0.2167**
g^b	0.0470	0.0283	0.0189	0.0068	0	0.0180	0.0060	0
y^b	0.1483	0.1187	0.1541	0.1562	0.1294	0.1521	0.1498	0.1245
y^w / y^k	**0.2230**	**0.2322**	**0.2305**	**0.2338**	**0.2494**	**0.2273**	**0.2297**	**0.2405**
y^b / y^k	**0.1864**	**0.1260**	**0.1515**	**0.1645**	**0.1345**	**0.1489**	**0.1564**	**0.1288**
y	**0.2917**	**0.3166**	**0.4040**	**0.3863**	**0.3650**	**0.4007**	**0.3792**	**0.3880**
y^f	**0.3717**	**0.4033**	**0.4479**	**0.4257**	**0.4220**	**0.4441**	**0.4203**	**0.4183**
y^g	**0.2187**	**0.1316**	**0.1753**	**0.1679**	**0.1268**	**0.1682**	**0.1563**	**0.1177**
p^g	-	1.2269	1.3908	1.4450	1.2365	1.4394	1.4660	1.2553
c / y	0.8273	0.8882	0.7607	0.7540	0.8087	0.7618	0.7618	0.8149
$\bar{g}$	-	-	-	0.0116	**0.0109**	-	0.0114	0.0108
τ^l	**0.3652**	**0.2056**	**0.1165**	**0.1848**	**0.1567**	**0.1248**	**0.1917**	**0.1613**
u^k	-0.3861	-0.3683	-0.2997	-0.3194	-0.3162	-0.2984	-0.3185	-0.3161
u^w	-0.9639	-0.9248	-0.9221	-0.9299	-0.9435	-0.9274	-0.9350	-0.9468
u^b	-1.0118	-1.0818	-1.0500	-1.0305	-1.0916	-1.0564	-1.0444	-1.1021
u	-0.8887	-0.8762	-0.8575	-0.8612	-0.8825	-0.8621	-0.8673	-0.8869
s^w	0.132	0.0624	0.0738	0.0768		0.0739	0.0739	0.0502
s^g	0.075	0.0354	0.0419	0.0436		0.0420	0.0420	0.0285
s^i	0.025	0.037	0.0043	0.0045		-	-	-

Table 8: Steady state solutions in the three polar cases with political economy frictions

Variable	Status quo	Status quo with profit max state firms	Profit max state firms with user prices		Profit max privatized firms with user prices			
			$w^g = MPL^g$	$w^g = w^w$	$\xi = 1$	$\xi = 0.95$ (5%)	$\xi = 0.90$ (10%)	$\xi = 0.85$ (15%)
c^k	0.6521	0.7863	0.8442	0.7780	0.8420	0.8465	0.8511	0.8558
e^k	**0.1301**	**0.1618**	**0.1908**	**0.1766**	**0.1853**	**0.1832**	**0.1810**	**0.1788**
k^k	2.8656	3.1096	3.4534	3.3408	3.5912	3.5742	3.5572	3.5403
g^k	0.0470	0.0283	0.1035	0.1051	0.0998	0.0973	0.0947	0.0920
y^k	0.7954	0.9418	1.0169	0.9450	1.0215	1.0253	1.0290	1.0328
c^w	0.1774	0.2187	0.2344	0.2138	0.2322	0.2327	0.2332	0.2338
e^w	**0.2902**	**0.3006**	**0.3261**	**0.3210**	**0.3260**	**0.3264**	**0.3267**	**0.3271**
g^w	0.0470	0.0283	0.0287	0.0289	0.0275	0.0267	0.0260	0.0251
y^w	0.1774	0.2187	0.2344	0.2138	0.2322	0.2327	0.2332	0.2338
c^b	0.1483	0.1187	0.1541	0.2138	0.1521	0.1478	0.1435	0.1391
e^b	**0.2726**	**0.2311**	**0.2799**	**0.3210**	**0.2792**	**0.2753**	**0.2710**	**0.2665**
g^b	0.0470	0.0283	0.0189	0.0289	0.0180	0.0170	0.0160	0.0150
y^b	0.1483	0.1187	0.1541	0.2138	0.1521	0.1478	0.1435	0.1391
y^w / y^k	**0.2230**	**0.2322**	**0.2305**	**0.2263**	**0.2273**	**0.2270**	**0.2267**	**0.2263**
y^b / y^k	**0.1864**	**0.1260**	**0.1515**	**0.2263**	**0.1489**	**0.1442**	**0.1394**	**0.1347**
y	**0.2917**	**0.3166**	**0.4040**	**0.3908**	**0.4007**	**0.3998**	**0.3989**	**0.3981**
y^f	**0.3717**	**0.4033**	**0.4479**	**0.4333**	**0.4441**	**0.4430**	**0.4419**	**0.4407**
y^g	**0.2187**	**0.1316**	**0.1753**	**0.1868**	**0.1682**	**0.1632**	**0.1581**	**0.1528**
p^g	-	1.2269	1.3908	1.2622	1.4394	1.4837	1.5324	1.5861
c / y	0.8273	0.8882	0.7607	0.7607	0.7618	0.7636	0.7655	0.7674
τ^l	**0.3652**	**0.2056**	**0.1165**	**0.1999**	**0.1248**	**0.1224**	**0.1199**	**0.1175**
u^k	-0.3861	-0.3683	-0.2997	-0.3170	-0.2984	-0.2962	-0.2940	-0.2920
u^w	-0.9639	-0.9248	-0.9221	-0.9494	-0.9274	-0.9284	-0.9294	-0.9306
u^b	-1.0118	-1.0818	-1.0500	-0.9494	-1.0564	-1.0661	-1.0761	-1.0864
u	-0.8887	-0.8762	-0.8575	-0.8558	-0.8621	-0.8644	-0.8669	-0.8696
s^w	0.132	0.0624	0.0738	0.1423	0.0739	0.0704	0.0669	0.0633
s^g	0.075	0.0354	0.0419	0.0419	0.0420	0.0400	0.0380	0.0360
s^i	0.025	0.037	0.0043	0.0043	-	-	-	-

APPENDICES

to be inserted …