TAX COMPETITION, QUALITY AND QUANTITY OF PROVISION OF PUBLIC GOODS

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14th February 2017

Abstract

In this article, we study the behavior that local governments face to the choice of financing a certain quality of public goods provision. More especially, we inspect whether the local governments must tax the mobile capital or not. For this, we examine if the Samuelson rule for the optimal resources allocation between private-goods and public-goods is satisfied or not. We show that if local governments finance the public-goods in taxing the households without varying their tax rate on capital, then the optimality, as defined in Samuelson, is constrained by the funding of the quality of public-goods. Otherwise, taxing the capital modify the Samuelson rule. Thus, there is a supplementary cost supported by the households linked to a distorting tax.

JEL Classification: D00, H20, H41, H70, H71.
Keywords: Tax competition; Quality; Optimal provision of public-goods; Samuelson rule. Marginal cost of public funds; Public Economic.

1 INTRODUCTION

Over the past 15 years, many researchers have addressed the problem of tax competition in focusing on the inefficiency that may occur in the fiscal policies, that the local governments want to follow in order to attract a large number of private investors. Therefore, tax competition can lead to a local insufficiency of public-goods. To maintain a lower tax rate to attract capital, Oates (1972) proved that the local governments must provide an amount of a local public spending below the level at which the marginal benefits equalize the marginal cost, especially for the expenses that not directly benefit from private investors. Wildasin (1989) studied the fiscal externalities problem of tax policies that the local governments establish. Thus, since the work of Tiebout (1956), many extensions (like those of Zodrow and Mieszkowski (1986), Wilson (1986)) examined the tax competition problem in atomizing the number of jurisdictions and considering the mobile capital and immobile labor. Some works analyzed the impact of the mobility of labor and capital on the tax competition for the supply of public-goods. Bucovetsky and Wilson (1991) concluded, from the previous models of tax competition, that the local public-goods are underoptimised. In contrast, the use available tax by governments is efficient when both "source-taxation and residence-based taxation" taxes are available, even in the absence of wage taxation. Other work is focused on the impact of the types of taxes to see if the non-lump sum taxes have the same

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influences as the lump sum rates, where local governments use more than two fiscal instruments to finance public-goods provision.[1] Gordon (1986), and Bucovetsky and Wilson (1991), confirmed, that there would not exist a tax competition problem if the local governments would utilize head taxes or other forms of lump-sum taxation.

Nevertheless, in the current context of the World economic crisis, it is necessary for the affected countries to attract capital, increase tax revenues, and use these revenues effectively for financing public-goods. Thus, the introduction of the quality factor of public-goods can be a good element of attractiveness of investmets. This element is often "overlooked" in the literature of the tax competition theory and so we have the following question : Should local governments introduce the quality as a second characterizing factor of public-goods?

The local governments supply public-goods consisting of combination of a quantity and a "minimum quality" for economic agents that reside within their locality. Under this perspective it should be possible to upfront determine the quality standard to define the quality notion.[2] The determinants of the quality of public-goods can be related to either, the increasing demand of taxpayers in a context of economic crisis less willing to be treated as administered and intending to be treated more customerlike, or administration that suffers from numerous organizational and operational weaknesses. Under this perspective, and according to Samuelson (1954) public-goods are not only destinated to a final consumption, but also to support firm activities (knowledge, infrastructure, etc.), and sometimes necessary for market transactions (law of agreement, etc.). In this area, but in different puroses, several authors have analyzed the mechanism of tax competition and its effects in the presence of quality of public services on the localization of capital and households' welfare. Hoyt and Jensen (2001) showed how the differentiation of the quality of education can improve the differential impacts of tax competition and households' welfare. Gabe and Bell (2004) suggested that a local fiscal policy of reduced government spending with decreased public services may attract fewer firms.[3]

In this work, we study the importance of the quality of public-goods in a context of tax competition. We explain the existence of some consumer (households') reactions towards quality based on "the conventional rule of Samuelson 1954". We use the same assumptions as those of the models of Zodrow and Mieszkowski (1986). These authors used distorting taxes, which affect the optimal allocation of the public goods. This distortionary of taxes generates the notion of the marginal cost of public funds (MCPF). Indeed, there is an additional cost borne by households relating to the use of the distorting taxes. The consequence is that the Samuelson rule of optimality for public-goods is changed.

The paper is organized as follows; in Section 2, we expose the different assumptions of the model of Zodrow and Mieszkowski (1986). We extend the existing tax model by including the quality of public goods in the used utility function. In Section 3, we describe subsequently the social objectives of local governments in terms of supply of public goods. We present a resolution of the model. We give different configurations of the provision of public goods. We conclude in Section 4 with remarks.

[1] See Hoyt (1991), Krelove (1993), Burbidge and Myers (1994), Wilson (1995), Wildasin and Wilson (1996).

[2] According to Palmer et al. (1991), the quality notion can be viewed as a production of better services to satisfy a population, taking into account the technological and resourceful constraints. For Roemer and Montoya-Aguilar (1989), the quality of a public good is measured by the level at which it meets predefined standards.

[3] See also the works of Jud and Watts (1981), Henderson and Thisse (1997), Bénassy-Quéré et al. (2005), Fatica (2010), Ould abdessalam et al. (2014).

2 THE MODEL

Many models of tax competition consider a certain number of assumptions based on the model of Zodrow and Mieszkowsky (1986) that we proceed an extension. We consider an economy composed of M identical jurisdictions (local governments), with $M \geq 2$. Each local government i is inhabited by a set of homogeneous and sedentary representative households (normalized to unity). The representative resident possesses the whole of the local lands and has a fraction of an available capital stock K_i. Capital is perfectly mobile between local governments without travel costs. The total stock of capital is assumed to be fixed in economy $\sum_{i=1}^{M} K_i = \bar{K}$.

In each jurisdiction, the firms use capital to produce its output, this capital being perfectly mobile between the jurisdictions, and some locally fixed factor, such as land, which is held entirely by households in each local government. The production technology of a firm i, denoted $F_i(K_i)$ occurs through the inputs of capital K_i land, and a fixed factor. The production function is a decreasing scale return, twice continuously-differentiable, *i.e.*, $\partial F_i/\partial K_i > 0$ and $\partial^2 F_i/\partial K_i^2 < 0$. The capital is mobile and attracted by the local governments that offer a best return after taxation. The arbitrage condition equals the net return of capital in each local government $F_{K_i}(K_i) - t_i = \rho$, with ρ is the net return of the capital and t_i is the tax on mobile capital.[4] Assume that the households of each local government consumes a private good C_i and a public goods G_i, with a quality Q_i. The households preferences are represented by a utility function $U_i(C_i, G_i, Q_i)$ where $\partial U_i/\partial C_i > 0$, $\partial U_i/\partial G_i > 0$, and $\partial U_i/\partial Q_i > 0$, which respectively denote: the variation of the total utility resulting from the addition of one unit for the two types of goods C_i, G_i and quality Q_i. The marginal utility is positive, and the total utility increases with the consumed amount of goods $\partial^2 U_i/\partial C_i^2 < 0$, $\partial^2 U_i/\partial G_i^2 < 0$ and $\partial^2 U_i/\partial Q_i^2 < 0$.

2.1 LOCAL GOVERNMENTS

After explaining a number of basic assumptions the Zodrow and Mieszkowsky (1986) model of tax competition, we make an extension of the latter by including the quality of public-goods in the budget constraint of government i's. We ask here, the determining the scale of quantity public-goods and the quality being as follows the planning process by local governments. If the quality of a public-good is fixed by the local government, then quantity is determined by the local elects. The local governments provide a quantity of public-goods with a quality for the economic agents (the households), denoted $G_i + Q_i$, it is financed by a tax on mobile capital $t_i K_i$ and lump-sum tax on households H_i at its maximum level $H_i \leq \bar{H}$. The governments establishes some standards which define the quality of a public-good Q_i: the characteristics that should be respected by the transport infrastructures or the specific steps to improve the safety of the transport network. For instance, the organizations in the public education system are insured by the government and subject to the powers of the governments to the development of this public-service.[5] The governments requires certain expenditures called pedagogical qualities, which concern, for example, the equipment for computer sciences and electronic, audiovisual or other technologic equipment for teaching and having all high-quality media. Quality is a real factor of development and an attractiveness of investment (the transparency of public institutions, stability, the predictability of policy, rule of law and the regulatory environment).

[4] Local government provides a public good that it finances by taxing the mobile capital at a tax rate $t_i \in [0, 1]$.

[5] The public education service, whose organization and operation are provided by the State, and subject to the responsibilities within the jurisdiction contribute to the development of this public service. In this domain, the State requires governments a spending educational quality. See the example Hoyt and Jensen (2001) on quality of provision of public good.

The local government determines the quantity of public-goods produced by taking into consideration the quality financed with a "part" of the global tax revenues, noted ε_i; with $\varepsilon_i \in [0,1]$.[6] Without a loss of generality, we shall explain the two extreme cases to finance the quality of public-good: (i) if $d\varepsilon_i = 0$, the funding the quality is close to its optimum level (ii) if $d\varepsilon_i$ tending toward unity $d\varepsilon_i \rightarrow 1$, the variation of funding the quality by government is at his maximum level. The budget constraint for the local government in terms of quantity and quality can be written as follows

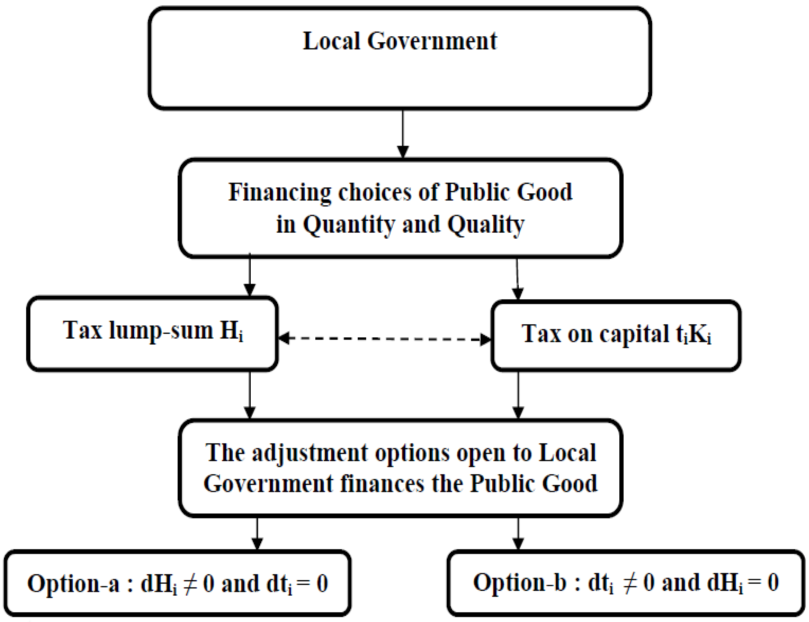
$$G_i + Q_i = t_i K_i + H_i \Rightarrow G_i = [t_i K_i + H_i] - Q_i \tag{1}$$

But, the quality of the public-goods is financed by a part ε_i of the total tax revenues, which gives us the function $Q_i(\varepsilon_i)$. Thus, the general form is: $Q_i(\varepsilon_i) = \varepsilon_i [t_i K_i + H_i]$. Replacing Q_i by its value $\varepsilon_i [t_i K_i + H_i]$ in the budget constraint $G_i = [t_i K_i + H_i] - Q_i$, the form of budget constraint local government, the quantity is the fonction the quality, where

$$G_i = [t_i K_i + H_i] - \varepsilon_i [t_i K_i + H_i] \Rightarrow G_i = [t_i K_i + H_i](1 - \varepsilon_i) \tag{1a}$$

The local government can finance the provision of public-goods in quantity and quality in two ways. First, the local government finances the public-good provision in taxing the households (*lump-sum tax*) $dH_i \neq 0$, without varying their tax rate on capital, $dt_i = 0$. Or secondly, the local government finances the public-goods provision through taxation of capital $dt_i \neq 0$ without varying the tax lump-sum $dH_i = 0$. The following diagram ulistrates this analysis

Diagram 1. Establishment of Public Goods Financed



[6]The variation of funding the quality $d\varepsilon_i$ is comprised between 0 and 1. When $d\varepsilon_i = 0$, this means that the government no longer has the possibility of increasing the fund the quality public-goods. Conversely, when $d\varepsilon_i \rightarrow 1$ this means that the government in question finance much quality and quantity.

3 OBJECTIVE OF LOCAL GOVERNMENTS

The purpose of a local government is to maximize the social welfare of residents (within its budget constraint. The following program shows this

$$\max_{t_i,H_i} U_i(C_i, G_i, Q_i) \qquad \textbf{(1b)}$$

$$\text{B/c.} \left| \begin{array}{ll} C_i = F_i(K_i) - (\rho + t_i)K_i + \rho(\bar{K}/M) - H_i & 1-(a) \\ G_i = (1-\varepsilon_i)\,[t_iK_i + H_i] & 1-(b) \end{array} \right.$$

The value $F_i(K_i) - (\rho + t_i)K_i$ corresponds to the land revenue paid by firms to lands owners. The amount $\rho(\bar{K}/M)$ is the return of capital invested by a resident, regardless of his place of residence. Thus, the representative households in a local government only deduces the inhabitant tax from his revenues and devotes all the rest to the consumption of the private good C_i. The condition of the first order give the following equation

$$1 + \text{MRS}_{G_i.C_i}\frac{dG_i}{dC_i} + \text{MRS}_{Q_i.C_i}\frac{dQ_i}{dC_i} = 0 \qquad \textbf{(2)}$$

To study this relation we begin by introducing the following definition of "the conventional rule of Samuelson (1954)".[7] The Samuelson rule is that the sum of the “disposible” revenu to pay for a marginal increase of one unit of public good between private good MRS^i must be equal to the cost of the marginal unit of public good in terms of a private good. *(i)* If the sum of MRS^i is greater than the cost of an additional unit, then households would be better off with one more unit of public good. *(ii)* If the sum of MRS^i is below the cost of an additional unit they would be better off with one unit less of public good. So optimal provison of the allocation of public good can exists only if the Samuelson rule is satisfied. When the implented tax is entirely "lump-sum tax", we must have the conventional rule of Samuelson (1954), which characterizes the optimality of a pure public-goods, *i.e.,* when $\text{MRS}^i = \text{MRT}^i = 1$.[8]

3.1 FUNDAMENTAL ASSUMPTIONS

Any government must take two fundamental decisions: the first one concerns the level of "provision of public-goods " offered to the residents, and the second one applies to the level of "taxes" and to the mode of distribution for taxes between households and capital. We describe subsequently the social objectives of local governments in terms of supply of public-goods and we present a resolution of the model in giving different configurations of the public-goods provision.

According the economic definition proposed by Samuelson (1954), the public goods are the necessary means in economic transactions on the markets. What is the impact of improvement in quality of the public-goods in economic transactions? A low quality of public-goods, causes deficiencies in supply of public-goods resulting in inefficiencies on the markets in terms of productivity or transaction costs. Nonetheless, a high quality of public-goods is an important factor in the economic development and attractiveness of the capital. It, is under this perspective that the question of the quality of the public-goods it is so important. The high quality of public-goods also serves to enhance the legitimacy of governments and as a consequence, is an important factor of revelation of the preferences of people. Based on the equation (2) that we shall study three assumptions on the behavior of households.

[7] See Annex 1 for the result of the equation (2).

[8] The standard literature by defining the inefficient provision of public goods an allocation characterized by the inequality between the marginal rate of substitution MRS^i and the marginal rate of transformation $\text{MRT}^i = 1$.

The two first assumptions assume that households do have a preference for one of the substitutions that appear in equation (2): $\text{MRS}_{Q_i.C_i} = 0$, or $\text{MRS}_{G_i.C_i} = 0$. The third assumption is a more global approach: $\text{MRS}_{G_i.C_i} \neq 0$, $\text{MRS}_{Q_i.C_i} \neq 0$. We propose to formally present our assumptions before resolving our program in each of these assumptions.

•**Assumption** A_1

In assumption A_1 we consider that private goods and the quality of public goods are perfect complements, there is thus no substitution. Hence, we have $\text{MRS}_{Q_i.C_i} = 0$. This behaviour corresponds to expenses that the local government undertake for increasing the quality of public goods that is latent. The term latent is interpreted by the idea in which the quality of public good remains hidden, but might become visible at some point during adaptation in supply of this specific good. Under this assumption the equation (2) becomes

$$\text{MRS}_{G_i.C_i} \equiv -\frac{dC_i}{dG_i} = -\frac{U_{C_i}(dC_i, ., .)}{U_{G_i}(., dG_i, .)} \neq 0 \qquad \textbf{(2a)}$$

This is exactly the same result that we obtain in the context of fiscal competition with only private and public goods without quality. It is common (easy) to find examples of firms that have specific needs for public goods. Let us consider a transportation company will need roads of good quality, whearas a call center and as well as the employees that work in both firms well need a good communication infrastructure, but they are less willing to pay for the quality. These examples are essential for the argumentation of the existance of latent quality lies in the underlying assumption of private goods and quality of public goods as "perfect complements".

•**Assumption** A_2

On the other hand, under assumption A_2 we consider that the private goods and public goods are perfect complements, $\text{MRS}_{Q_i.C_i} = 0$ (given the same utility level, there is no exchange between quantity of private goods and the quantity of public goods). Consequently, according to equation (2), we obtain

$$\text{MRS}_{Q_i.C_i} \equiv -\frac{dC_i}{dQ_i} = -\frac{U_{C_i}(dC_i, ., .)}{U_{Q_i}(., ., dQ_i)} \neq 0 \qquad \textbf{(2b)}$$

As a result to the second assumption we see that the households are sensitive to the quality of the public goods, and they are consider the expenses the government pay a for the quality of public goods. The quality of the public goods is no longer latent, but a consequence of expenses done in the quality of public goods. In this case, an improvement of the transportation network induces a better efficiency the for employees in terms of productivity, improved efficiency of market transactions, as well as a better legal context has positive impact on both firms and employees.

•**Assumption** A_3

In hypothesis A_3 we considera more global approach, where the households substitute (freely) between private goods and public goods according to $\text{MRS}_{G_i.C_i} \neq 0$ or private goods and quality of public goods $\text{MRS}_{Q_i.C_i} \neq 0$. We obtain thus form $\text{MRS}_{G_i.C_i}(., .) \equiv ....$ From equation (2), we obtain the following general form[9]

$$\text{MRS}_{G_i.C_i} \equiv -\left[\frac{dC_i}{dG_i}\right] + \text{MRS}_{Q_i.C_i}\left[\frac{dQ_i}{dG_i}\right] \qquad \textbf{(3)}$$

[9] See Annex 2 for the result of the equation (3).

The quation (3) is composed of two parts. Firstly, the part related to the quantity of the public good, denoted $-dC_i/dG_i$ and secondly, the component related to the quality of contribution to the public good, denoted $\text{MRS}_{Q_i.C_i}(.;.)$. According to the underlying assumption, the solution of model depends on the fiscal choice made by the government.[10] The Assumptions A_1 and A_2 require the households to have a preference for "substitutions" that appears in equation (2), *i.e.*, $\text{MRS}_{G_i.C_i}$ or $\text{MRS}_{Q_i.C_i}$.

3.2 HYPOTHETICAL RESOLUTION OF MODEL

The objective of the local government is to maximize the welfare of its residents, which solves the maximization program under the budget constraint. By using assumption A_1, we calculate the marginal rate of substitution between the quantity of private-good and the quantity of the public-goods $\text{MRS}_{G_i.C_i}$, we use 1-(a) and 1-(b) in the welfare of residents (households) program (1). By using the relation $\text{MRS}_{G_i.C_i} = -dC_i/dG_i$ and remplacing dC_i and dG_i by its values, we obtain

$$\text{MRS}_{G_i.C_i} = -\frac{F_{K_i}dK_i - \rho dK_i - t_i dK_i - K_i dt_i - dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i\left[t_i dK_i + K_i dt_i + dH_i\right]} \tag{4}$$

We substitute the return of net capital condition, $F_{K_i} - t_i = \rho$, in (4) and we obtain

$$\text{MRS}_{G_i.C_i} = -\frac{\rho dK_i + t_i dK_i - \rho dK_i - t_i dK_i - K_i dt_i - dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i\left[t_i dK_i + K_i dt_i + dH_i\right]} \tag{4a}$$

After simplification we have

$$\text{MRS}_{G_i.C_i} = \frac{-\rho dK_i - t_i dK_i + \rho dK_i + t_i dK_i + K_i dt_i + dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i\left[t_i dK_i + K_i dt_i + dH_i\right]} \tag{4b}$$

$$\text{MRS}_{G_i.C_i} = \frac{K_i dt_i + dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i\left[t_i dK_i + K_i dt_i + dH_i\right]} \tag{4c}$$

By using assumption A_2, the government offers the quality of public good. By using the relation $\text{MRS}_{Q_i.C_i} = -dC_i/dQ_i$ and remplacing dC_i and dQ_i by its values, we obtain

$$\text{MRS}_{Q_i.C_i} = \frac{K_i dt_i + dH_i}{t_i dK_i + K_i dt_i + dH_i - dG_i} \tag{4d}$$

The general case where the government provides the public good in quantity and quality. Under Assumption A_3, we have[11]

$$\text{MRS}_{G_i.C_i} = \frac{K_i dt_i + dH_i - \text{MRS}_{Q_i.C_i}\left[t_i dK_i + K_i dt_i + dH_i - dG_i\right]}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i\left[t_i dK_i + K_i dt_i + dH_i\right]} \tag{4e}$$

In the following, our analysis will be based on the equations (2), (4c), (4d), and (4e), to see under what conditions the rule of Samuelson is respected and the different interpretations that we can give for each assumption.

3.3 LUMP-SUM TAX AND PROVISION OF PUBLIC-GOODS

We begin by considering that the local governments entirely finance the provision of public-goods in respecting the standards of quality with a lump-sum tax $dH_i \neq 0$, without taxation of capital income $dt_i = 0$ as illustrated in diagram 1, option-(a). In this case, we provide the following result

[10] The first two assumptions be applied to the realities.

[11] See Annex 2 for the result of the equation (4e).

Proposition 1. *When a local governement uses a lump-sum tax* $dH_i \neq 0$ *to finance public-goods in quantity and quality, without any variation in the taxation of capital* $dt_i = 0$, *the Rule of Samuelson* $\text{MRS}^i = \text{MRT}^i = 1$ *is respected according to the following assumptions:*

(i) *If* A_1 *holds, the* $\text{MRS}_{G_i.C_i} = 1$ *is optimal if and of only if* $d\varepsilon_i = 0$*;*
(ii) *If* A_2 *holds, the* $\text{MRS}_{Q_i.C_i} = 1$ *is optimal if and only if* $\varepsilon_i \rightarrow 1$*;*
(iii) *If* A_3 *holds, the* $\text{MRS}_{G_i.C_i} = 1 - \text{MRS}_{Q_i.C_i}.(\varepsilon_i)$ *is optimal if and only if* $d\varepsilon_i = 0$.

Proof. If the local government uses the lump-sum tax to finance the provision of public-goods into quantity and quality, then it can increase the tax on households H_i, but does not have the possibility of increasing the tax on capital. From A_1, and using equation (4c), we obtain $\text{MRS}_{G_i.C_i} = 1/(1 - d\varepsilon_i)$, that we can rewrite as follows

$$\text{MRS}_{G_i.C_i} = \begin{cases} 1 & \text{if } d\varepsilon_i = 0; \\ \frac{1}{1-d\varepsilon_i} & \text{if } 0 \leq d\varepsilon_i < 1; \\ +\infty & \text{if } d\varepsilon_i \rightarrow 1. \end{cases} \qquad \textbf{(4d')}$$

From equation (4d') we have three possibilities for the provision of the public-goods that depends on $d\varepsilon_i$. If $d\varepsilon_i = 0$, this means that the quantity of public-goods the government offers is optimal (the funding the quality is close to its optimum level). This condition equalizes the marginal rate of substitution between the quantity of public and private goods and the marginal rate of transformation such that $\text{MRS}_{G_i.C_i} = \text{MRT}_{G_i.C_i} = 1$. In contrast, if $0 \leq d\varepsilon_i < 1$, then $\text{MRS}_{G_i.C_i} = 1/(1 - d\varepsilon_i) > 1$, the provision of public-goods is not optimal because the $\text{MRS}_{G_i.C_i} > \text{MRT}_{G_i.C_i} = 1$ because there is the marginal cost related to the financing of the quality of the public goods. Finally, if $d\varepsilon_i$ tending toward unity $d\varepsilon_i \rightarrow 1$, then the variation in choice of funding by the government is at its maximum level so $\lim_{d\varepsilon_i \rightarrow 1} \frac{1}{1-d\varepsilon_i} = +\infty$ this implies economically that the two goods are perfect complements and hence we have the result $\text{MRS}_{G_i.C_i} = +\infty$.

Under A_2, from equation (2), and according to the equation (4d). The local government finances the quality of public-good provision in taxing the households $dH_i \neq 0$, without varying the tax rate on capital, $dt_i = 0$ this implies that $dK_i = 0$, we obtain the form of $\text{MRS}_{Q_i.C_i} = dH_i/dH_i - dG_i$ that we can write the following $\text{MRS}_{Q_i.C_i} = \frac{1}{1-dG_i/dH_i}$, with $\frac{dG_i}{dH_i} = 1 - \varepsilon_i$. The form of $\text{MRS}_{Q_i.C_i} = \frac{1}{\varepsilon_i}$, that we can rewrite as follows

$$\text{MRS}_{Q_i.C_i} = \begin{cases} 1 & \text{if } \varepsilon_i \rightarrow 1; \\ \frac{1}{\varepsilon_i} & \text{if } \varepsilon_i \neq 1. \end{cases} \qquad \textbf{(4e)}$$

According to the equation (4e), the provision of the quality of public-goods depends essentially of to the part ε_i of the total tax revenues that the government allocates of its total budget to finance the quality.[12] If the levy reaches to the maximum level then $\varepsilon_i \rightarrow 1 \Rightarrow \text{MRS}_{Q_i.C_i} = 1$, i (funding the quality by government is at his maximum level). In contrast, for a value $\varepsilon_i \neq 1$, the ε_i is to finance the required quality is not efficient, $\text{MRS}_{Q_i.C_i} > 1$. From A_3, however using equation (4e), we have

$$\text{MRS}_{G_i.C_i} = \frac{1}{1 - d\varepsilon_i} \left\{1 - \text{MRS}_{Q_i.C_i} \times \varepsilon_i\right\} \qquad \textbf{(4)}$$

The equation (4), we can rewrite in the following form

[12] If $\lim_{\varepsilon_i \rightarrow 1} \frac{1}{\varepsilon_i} = 1$ this implies that the $\text{MRS}_{Q_i.C_i} = 1$.

$$\text{MRS}_{G_i.C_i}= \begin{cases} 1-\text{MRS}_{Q_i.C_i}.\varepsilon_i & \text{if } d\varepsilon_i=0; \\ \frac{1}{1-d\varepsilon_i}\left\{1-\text{MRS}_{Q_i.C_i}.\varepsilon_i\right\} & \text{if } 0\leq d\varepsilon_i<1; \\ +\infty & \text{if } d\varepsilon_i\rightarrow 1. \end{cases} \qquad \textbf{(4a)}$$

From equation (4a), the optimality of public-goods depends on variation of funding the quality $d\varepsilon_i$. If $d\varepsilon_i= 0$, then $\text{MRS}_{G_i.C_i}= 1-\text{MRS}_{Q_i.C_i}.\varepsilon_i$ (the public-goods provision is optimal, because we are in a more global approach) then $\text{MRS}_{G_i.C_i}= 1-\text{MRS}_{Q_i.C_i}.\varepsilon_i$ is equal to unity less the marginal contribution of the quality to public-goods $\text{MRS}_{Q_i.C_i} \times \varepsilon_i$. We recall that financing this quality can be determined by $0 \leq d\varepsilon_i< 1$. However, if $d\varepsilon_i\neq 0$, this indicates a choice of ineffective financing of the quality by the government. Consequently, we obtain the Samuelson condition with a marginal cost of public funds is equal $1/(1 - d\varepsilon_i)> 1$ related to the financing of the quantity. Finally, a value of $d\varepsilon_i$ which tend toward: $d\varepsilon_i\rightarrow 1$, meaning that funding this quality by the government is at its maximum level. The economic intuition is as follows : the two goods (G_i, C_i) are perfect complements.■

The idea behind Proposition 1 is that the government can achieve the optimum if and only if the tax lump-sum H_i is used to its maximum level $\bar{H}$.

From the following equation: $C_i= F_i(K_i) - (\rho + t_i)K_i+\rho(\bar{K}/M) - H_i$. We will determine the reaction of the private consumption in the tax lump-sum: $\partial C_i/\partial H_i= -1 < 0$, and $\partial C_i/\partial \bar{H} = 0$, it is affected by an increase in lump-sum tax H_i, and it will decreased due to the increase in the lump-sum tax to its maximum level $H_i\rightarrow \bar{H}$. This decreases the welfare of individuals. The table 1 a synthesis of this approach when $dH_i\neq 0$ and $dt_i= 0$.

3.4 TAX ON CAPITAL AND PROVISION OF PUBLIC-GOODS

Now, we assume that the local governement no longer has the possibility to finance the public-goods by tax on households. The government funds the provision of the public-goods by a tax rates on mobile capital therefore $dt_i\neq 0$, as illustrated in diagram 1, option-(b). In this case, we provide the following proposition

Proposition 2. *Let $dt_i\neq 0$ and $dH_i= 0$; if the local gouvernemet increases the tax rates on mobile capital and the assumptions A_1, A_2 and A_3 hold, the rule Samuelson is respected under certain conditions that depend on $e_{K_i.t_i}$, and ε_i:*

(i) *Under A_1, if a public good is optimal $\text{MRS}_{G_i.C_i}= 1$, then $d\varepsilon_i= e_{K_i.t_i}/(e_{K_i.t_i}+1)$;*
(ii) *Under A_2, if a quality is optimal $\text{MRS}_{Q_i.C_i}= 1$, then; $\varepsilon_i= 1 - e_{K_i.t_i}$;*
(iii) *Under A_3, if $\text{MRS}_{G_i.C_i}= 1$, is optimal if and only if $d\varepsilon_i=\frac{K_i dt_i}{2}\left[\text{MRS}_{Q_i.C_i}\times(\varepsilon_i+e_{K_i.t_i})-1\right]$.*

Proof. Let $dt_i\neq 0$, and $dH_i= 0$, hence the local government can not impose households with a sufficiently high level $dH_i= 0$, and in this case it must tax rates on mobile capital. However, the fact to impose mobile capital instead to the immobile households generates a strong distortion because the government is subject in the double constraint: the financing of the quality ε_i and lump-sum tax $dH_i= 0$. Under A_1, and using the equation (4c), we obtain the following form[13]

$$\text{MRS}_{G_i.C_i}=\frac{1}{[e_{K_i t_i}+1]\,(1 - d\varepsilon_i)}.\text{MRT}_{G_i.C_i} \qquad \textbf{(5)}$$

[13]See Appendix 3 for the result of equation (5).

According to equation (5), we are faced with two parameters: (i) the variation in financing of the quality of public-goods $d\varepsilon_i$. (ii) the elasticity of capital to tax rates $e_{K_it_i}$. From equation (5), if the $\mathrm{MRS}_{G_i.C_i}=1$ this implies that the variation in financing of the quality $d\varepsilon_i$ is equal to the term $e_{K_it_i}/(e_{K_it_i}+1)$, the last term measures the pressure that capital market put on a decreasing as tax rate on the capital, thus the consequences on efficiency of provision of public goods, or the impact on the provision of public goods. Hence, the following condition is necessary $d\varepsilon_i=\frac{e_{K_it_i}}{e_{K_it_i}+1}$ with $e_{K_it_i}\neq -1$. This condition equalized the marginal rate of substitution $\mathrm{MRS}_{G_i.C_i}=1$ and the marginal rate of transformation $\mathrm{MRT}_{G_i.C_i}=1$. The most plausible explanation is as follows: the adjustment of the funding of quality by the government is here efficient because it confirms the rule Samuelson $\mathrm{MRS}_{G_i.C_i}=\mathrm{MRT}_{G_i.C_i}=1$. In contrast, if $\mathrm{MRS}_{G_i.C_i}\gtrless 1$, this implies that $d\varepsilon_i\gtrless\frac{e_{K_it_i}}{e_{K_it_i}+1}$, financing the quality shall be reduced because of "fiscal externality". Under A_2, therefore[14]

$$\mathrm{MRS}_{Q_i.C_i}=\frac{1}{e_{K_it_i}+\varepsilon_i}.\mathrm{MRT}_{Q_i.C_i} \qquad \textbf{(5a)}$$

According to equation (5a) the marginal rate of substitution $\mathrm{MRS}_{Q_i.C_i}$ depends on ε_i and on the elasticity of capital to tax rate $e_{K_it_i}$. The analysis focuses on the budget ε_i that goverment decides to allocate to effectively finance the quality of the public-goods. If the $\mathrm{MRS}_{Q_i.C_i}=1$, then $\varepsilon_i=1-e_{K_i.t_i}$. This difference $1-e_{K_i.t_i}$ is equal to the marginal rate of transformation (equal to unity) less the reaction of the capital tax rate. However, if $\varepsilon_i\lesseqgtr 1-e_{K_i.t_i}$ (high elasticity $e_{K_i.t_i}$, decreases, low elasticity $e_{K_i.t_i}$ increases ε_i respectivly and consequently the provision of quality of the public-goods). The underlying message is that the government depend mainly on the elasticity $e_{K_it_i}$, and less on ε_i, to finance the quality relative to marginal rate of transformation. Under A_3, and according to equation (4e), we obtain

$$\mathrm{MRS}_{G_i.C_i}=\frac{K_i dt_i}{2(1-d\varepsilon_i)}\left\{1-\mathrm{MRS}_{Q_i.C_i}(\varepsilon_i+e_{K_i.t_i})\right\} \qquad \textbf{(5b)}$$

Two explanations for the ineffciency of provison of quantity and quality of public goods can be identified when governments use tax on capital $dt_i\neq 0$, which is mobile. Whilst they increase the tax on capital by one unit, or consequently a negative fiscal extenality, (i) the households will support an additional cost since there will be capital outflows to other goverments (ii) The loss of fiscal revenues results in a reduction of provision of public goods in terms of both quality and quantity. This tax variation $dt_i\neq 0$ must be high enough to not only pay for the marginal resource cost of provision of public-goods but also to offset the negative impact of the capital outflow on tax revenue. Equation (5b), is a a generalized variant of the Samuelson rule, which is a modified version of the Samuelson rule. From equation (5b), if $\mathrm{MRS}_{G_i.C_i}=1$ we have $d\varepsilon_i=\frac{K_idt_i}{2}[(.,.)-1]$ or $d\varepsilon_i\gtrless\frac{K_idt_i}{2}[(.,.)\ -1]$.[15] ■

However, for a value of corresponds to a the loss or gain in terms of capital is equal to $|d\varepsilon_i|$ in absolute value. Consequently, the government decide either to lower or raise the quality of the public good as a function of quantity. Thus we have the following remark which confirms our analysis in terms of welfare. From equations (5), (5a) and (5b), the rate of capital taxation modifies the conditions of allocation of resources (in the sense of a larger choice in the provision of quality and quantity of public-goods). If the elasticity $e_{K_i.t_i}$ of capital tax rate is strong, the welfare in terms the provision of public-goods may decrease. Contrary, if the elasticity $e_{K_i.t_i}$ is low, then the welfare increases. The elasticity terms are explaining the variation in financing the quality of public-goods $d\varepsilon_i$ resulting from the adjustment of the rate of capital taxation $dt_i\neq 0$, equalling to the capital delocalized. With the equation of the private consumption $C_i=F_i(K_i)-(\rho+t_i)K_i+\rho(\bar{K}/M)-H_i$, we shall determine

[14] See Appendix 3 for the result of equation (5a).
[15] See Appendix 5 for the result of equation (5b).

the value of capital delocalised

$$K_i = -\frac{\left(C_i - F_i + H_i - \rho\left[\frac{\bar{K}}{M}\right]\right)}{\rho + t_i} < 0 \tag{6}$$

The variation of the capital to tax on capial rate (elasticity $e_{K_i.t_i}$)

$$\frac{\partial K_i}{\partial t_i} = \frac{-\bar{K}\rho + M\left(C_i + H_i - F_i\right)}{K_i M\left(\rho + t_i\right)^2} < 0 \tag{6a}$$

The variation of the private goods to tax on capital rate

$$\frac{\partial C_i}{\partial t_i} = -K_i < 0 \tag{6b}$$

Rate of tax on capital

$$t_i = \frac{\bar{K}\rho - M\left(\rho - C_i - F_i - H_i\right)}{M} > 0 \tag{6c}$$

The impact of the variation of the tax on capital $dt_i \neq 0$ for the private goods c_i is as follows: the representative resident own a fraction of the available capital stock K_i is in the economy, and $dt_i \neq 0$ implies that the capital K_i is deocalized to other local gouvernements and consequently decreases in the production of the private goods.

3.5 MARGINAL COST OF PUBLIC-FUNDS

When the government increases the tax rates on mobile capital $dt_i \neq 0$ or raises taxes on households lump-sum $dH_i \neq 0$ in order to increase the provision of public-goods, the ensuing a change in allocation of resources usually resulting in losses of efficiency of the term "public-goods" in the economy considered.

The cost of taxes paid by the private sector is in general higher than the fiscal revenues perceived by the goverments due to loss of efficacy related to taxation. This loss of efficacy is resulting from increases on the tax rate on capital can easily be measured by the marginal cost of public funds. The marginal cost of public funds (MCPF) measures the loss for the firm when the goverment increase by one monetery unit the fiscal revenue. For example, if the tax rate increaes with $dt_i = 10\%$ and the firms react by reducing the taxated activity by 2%, the fiscal revenues receieved by the goverment increases by 8% and not 10%.

The MCPF can be measured under assumption A_3 for the two approaches (tax on capital and lump-sum). We define the MCPF concept, which is derived from the model of Atkinson and Stern (1974) in which a single government uses a distorting tax on production factors (inputs). Atkinson and Stern (1974) demonstrated that the Samuelson (1954) rule for the optimum provision of public-goods needs to be modified to account for tax distortions. The optimal level of provision of public-goods should be lower if the marginal cost of public funds is higher.[16]

Definition 1. *A marginal cost of public funds measures the ratio between the marginal social cost of collecting additional resources denoted ω and the the social marginal value of private income denoted β, i.e.,* MCPF$=\frac{\omega}{\beta}$.

[16] For more details on this question, see the works of Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974), Ahmad and Stern (1984), Wildasin (1984), Mayshar (1991), Ahmed and Croushore (1995), Snow and Warren (1996), and Dahlby (1998), Sandmo (1998).

The aim is to compare the marginal cost of public funds between the tax on capital and the lump-sum tax approaches in order to identify the best welfare. We shall verify whether the result provided by the MCPF ratio corresponds to the above analysis based on assumption A_3. When considering definition 2, the marginal cost of public funds corresponds under A_3 with $dH_i \neq 0$ denoted MCPF^{H_i} and under A_3 when $dt_i \neq 0$ denoted MCPF^{t_i}. We provide the following proposition[17]

Proposition 3. *For a marginal cost of public funds MCPF, if assumption A_3 holds, then, we have the following assertions :*

(i) *If $dH_i \neq 0$ and $dt_i = 0$ so the marginal cost of public funds is equal to*

$$\text{MCPF}^{\text{H}_i} = \frac{1}{1 - d\varepsilon_i} > 1 \tag{7}$$

(ii) *If $dt_i \neq 0$ and $dH_i = 0$ then the marginal cost of public funds is equal to*

$$\text{MCPF}^{\text{t}_i} = \frac{K_i dt_i}{2\,(1 - d\varepsilon_i)} > 1 \tag{7a}$$

(iii) *If $K_i dt_i/2 < (1 - d\varepsilon_i)$ this implies that the $\text{MCPF}^{H_i} < \text{MCPF}^{t_i}$, then the welfare is better when the public-goods is financed by the lump-sum tax. In contrast, if $K_i dt_i/2 > (1 - d\varepsilon_i)$ we have the $\text{MCPF}^{H_i} > \text{MCPF}^{t_i}$, it is better to finance welfare by the tax rates on mobile capital.*

Proof. To determine the marginal cost of public funds for both cases under the general assumption A_3 we will use equations (4) and (5b). According to equation (4) we have the marginal rate of substitution equal $\text{MRS}_{G_i.C_i} = (1/1 - d\varepsilon_i)\,\{1 - \text{MRS}_{Q_i.C_i} \times \varepsilon_i\}$ allowing us to determine MCPF^{H_i} as following

$$\text{MRS}_{G_i.C_i} = \text{MCPF}^{\text{H}_i}\,[1 - \text{MRS}_{Q_i.C_i} \times \varepsilon_i] \tag{7b}$$

From equation (7b) the marginal cost of public funds is $\text{MCPF}^{H_i} = 1/(1 - d\varepsilon_i) \geq 1$. If $d\varepsilon_i = 0$ *i.e.,* the funding of the quality is at its optimum level, the provision of public goods is optimal according to Samuelson's rule because $\text{MRS}_{G_i.C_i} = 1 - \text{MRS}_{Q_i.C_i} \times \varepsilon_i$, or the $\text{MRT}_{G_i.C_i}$ equal to unity less the marginal contribution of the quality to public-goods equal to $\text{MRS}_{Q_i.C_i} \times \varepsilon_i$. However, if $d\varepsilon_i \neq 0$, then $\text{MCPF}^{H_i} > 1$, the provison of public is suboptimal ($\text{MRS}_{G_i.C_i} > 1$) if, and only if, $\text{MCPF}^{H_i} > 1/(1 - \text{MRS}_{Q_i.C_i} \times \varepsilon_i)$ and this is valid only if $1 - \text{MRS}_{Q_i.C_i} \times \varepsilon_i \neq 0$, which occurs when $\text{MRS}_{Q_i.C_i} > 1$ ($\text{MRS}_{Q_i.C_i} \neq \frac{1}{\varepsilon_i}$). We can conclude that, the funding of the quality of the public-goods is the reason for inefficiency. On the other hand, according to equation (5b) the marginal cost of public funds can found by the $\text{MRS}_{G_i.C_i}$ by using the following equation, where the$\text{MCPF}^{t_i} = K_i dt_i/2\,(1 - d\varepsilon_i)$.

$$\text{MRS}_{G_i.C_i} = \text{MCPF}^{\text{t}_i}\,[1 - \text{MRS}_{Q_i.C_i}(\varepsilon_i + e_{K_i t_i})] \tag{8}$$

The marginal cost of public funds is reflecting the distortionary effects of raising the marginal tax rate on capital $dt_i \neq 0$. This modification of the Samuelson rule focuses only on the distortionary effects raising from increasing the tax rate on capital and financing of the quality ε_i. If the equation (8): $\text{MRS}_{G_i.C_i} > 1$ if and of only if the following conditions are satisfaites: $\text{MCPF}^{t_i} > 1$ it is necessary that the $\text{MRS}_{Q_i.C_i} < 0$, and $-1 \leq e_{K_i t_i} \leq 0$ with $e_{K_i t_i} < \varepsilon_i$.

Comparing welfare of individuals in the two approaches, we know that the $\text{MCPF}^{H_i} = 1/(1 - d\varepsilon_i)$ with the variation of funding the quality $0 \leq d\varepsilon_i < 1$, that implies that the marginal cost of public funds, $\text{MCPF}^{H_i} > 1$. In contrast, the marginal cost of public fund when $dt_i \neq 0$ is equal to $\text{MCPF}^{t_i} \equiv K_i dt_i/2\,(1 - d\varepsilon_i)$. In this case we have two configurations which depend on $K_i dt_i$. If

[17] See Appendix 6 for the result of proposition 3.

$K_i dt_i > 2(1 - d\varepsilon_i)$ that implies that the MCPF$^{t_i}> 1$ with $(K_i dt_i < 2(1 - d\varepsilon_i) \Rightarrow$MCPF$^{t_i} < 1)$. The comparison of the marginal cost of public funds gives: if $K_i dt_i/2 < (1 - d\varepsilon_i) \Rightarrow$MCPF^{H_i} <MCPF^{t_i} or $K_i dt_i/2 > (1 - d\varepsilon_i) \Rightarrow$MCPF^{H_i} >MCPF^{t_i}.■

The classic approche provides the most general answer to the question of the optimal public goods supply in an economy with distortionary taxation. The sub-optimality of the provision of public goods is related to distortionary taxation. The local governments have a poor estimate of the marginal cost of public funds. This distortion is firstly related to the mobility of capital $e_{K_i t_i}$ following with an increase in the tax $dt_i \neq 0$, and is secondly relevant, to the loss of capital $-K_i < 0$ which causes a reduction of government fiscal revenues; hence a decreasing of funding for the quality of the public-goods.

4 CONCLUSION

We have built a model in order to study the behavior of local governments who face a financial choice in terms of quality of the public-goods. More specifically, we have examined whether it is interesting for governments to tax the mobile capital and under which requirements the Samuelson's condition for optimal allocation of resources between the private good and the public good is satisfied. This model teaches us several things on how to finance the supply of public-goods given some required standards on its quality.

The comparison of the results of different approaches confirm that when the public goods is financed by the lump-sun tax; the inefficiency of the provision of public-goods arises from the mode of financing the quality of the public-goods and it does not occur from the choice of household taxation (lump-sum tax). The secondly we show that taxation of mobile capital generates a strong distortion becauses the gouvernemet is subject in the double constraint: the financing of the quality and the elacitycity of tax on capital. The impact of the variation of the tax on capital for the private goods is as follows: if the gouvernemet increase in the tax $dt_i \neq 0$ then the capital will locate in other governments and this relocation decreases the tax revenue for the government.

The results in terms of welfare depend on the marginal cost of public funds. First, when the government uses a lump-sum tax to finance the quantity and quality of public-goods, welfare is better because of the marginal cost of public funds is lower compared to the second option, where $dt_i \neq 0$ resulting in a higher a marginal cost of public funding. When local governments decide not to tax capital, the optimal supply of public-goods is less constrained to the funding's choice of the quality (sedentary households). This appears crucial and strategic for the achievement of any optimum target. When capital is taxed, the Samuelson's condition is modified due to the existence of an additional cost born by households and due to a distorting tax. Nonetheless, our model suffers from a number of limitations. Is it realistic to assume that the only alternative to a tax on capital is a lump sum tax? It might be useful to complement with an income or VAT tax? Is there any difference if a local tax is used to finance the quality of public-goods or if the same local tax is imposed to finance a higher quantity of the public-goods? Specification of the utility and the production functions are needed to determine whether the results of our analysis would be affected.

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5 APPENDICES

•Appendix 1

To resolve the program of maximization for a households, we assume the following method with :

$$dU_i(C_i,G_i,Q_i)=0 \quad (1)$$

$$\Leftrightarrow U_{C_i}dC_i+U_{G_i}dG_i+U_{Q_i}dQ_i=0 \quad (2)$$

$$\Leftrightarrow \frac{U_{C_i}dC_i}{U_{C_i}dC_i}+\frac{U_{G_i}dG_i}{U_{C_i}dC_i}+\frac{U_{Q_i}dQ_i}{U_{C_i}dC_i}=0 \quad (3)$$

$$\Leftrightarrow 1+\frac{U_{G_i}dG_i}{U_{C_i}dC_i}+\frac{U_{Q_i}dQ_i}{U_{C_i}dC_i}=0 \quad (4)$$

$$\Leftrightarrow 1+\text{MRS}_{G_i.C_i}\frac{dG_i}{dC_i}+\text{MRS}_{Q_i.C_i}\frac{dQ_i}{dC_i}=0 \quad (5)$$

With $\frac{U_{G_i}}{U_{C_i}}\frac{dG_i}{dC_i}=\text{MRS}_{G_i.C_i}\frac{dG_i}{dC_i}$ and $\frac{U_{Q_i}}{U_{C_i}}\frac{dQ_i}{dC_i}=\text{MRS}_{Q_i.C_i}\frac{dQ_i}{dC_i}$.

•Appendix 2

From equation (5), which defines the program of maximization for a households, we assume the following relation:

$$\text{MRS}_{G_i.C_i}\frac{dG_i}{dC_i}=-1-\text{MRS}_{Q_i/C_i}\frac{dQ_i}{dC_i} \quad (6)$$

$$\text{MRS}_{G_i.C_i}\equiv-\left[\frac{dC_i}{dG_i}\right]+\text{MRS}_{Q_i.C_i}\left[\frac{dQ_i}{dG_i}\right] \quad (7)$$

•Appendix 3

The government funds the provision of the public-goods by a tax on mobile capital therefore $dt_i\neq 0$, and $dH_i=0$. Under A_1 the calculation the marginal rate of substitution between the private good and the quantity of the public good $\text{MRS}_{G_i.C_i}$ requires the budget constraint:

$$C_i=F_i(K_i)-(\rho+t_i)K_i+\rho(\bar{K}/M)-H_i \quad \text{1-(a)}$$

$$G_i=(1-\varepsilon_i)[t_iK_i+H_i] \quad \text{1-(b)}$$

By using the formula $\text{MRS}_{G_i.C_i}=-\frac{dC_i}{dG_i}$ and replacing dC_i and dG_i by its values, we obtain:

$$\text{MRS}_{G_i.C_i}=-\frac{F_{K_i}dK_i-\rho dK_i-t_idK_i-K_idt_i-dH_i}{[t_idK_i+K_idt_i+dH_i]-d\varepsilon_i[t_idK_i+K_idt_i+dH_i]} \quad (9)$$

We substitute the capital net return condition, $F_{K_i}-t_i=\rho$, in (9) and we obtain

$$\text{MRS}_{G_i.C_i}=-\frac{\rho dK_i+t_idK_i-\rho dK_i-t_idK_i-K_idt_i-dH_i}{t_idK_i+K_idt_i+dH_i-d\varepsilon_i(t_idK_i+K_idt_i+dH_i)} \quad (10)$$

After simplification we have

$$\text{MRS}_{G_i.C_i}=\frac{-\rho dK_i-t_idK_i+\rho dK_i+t_idK_i+K_idt_i+dH_i}{t_idK_i+K_idt_i+dH_i-d\varepsilon_i(t_idK_i+K_idt_i+dH_i)} \quad (11)$$

$$\text{MRS}_{G_i.C_i}=\frac{K_idt_i+dH_i}{[t_idK_i+K_idt_i+dH_i]-d\varepsilon_i[t_idK_i+K_idt_i+dH_i]} \quad (12)$$

With $dH_i=0$, we obtain

$$\text{MRS}_{G_i.C_i}=\frac{K_idt_i}{t_idK_i+K_idt_i-d\varepsilon_i(t_idK_i+K_idt_i)} \quad (13)$$

$$\text{MRS}_{G_i \cdot C_i}=\frac{\frac{K_i dt_i}{K_i dt_i}}{\frac{t_i dK_i}{K_i dt_i}+\frac{K_i dt_i}{K_i dt_i}-d\varepsilon_i\left(\frac{t_i dK_i}{K_i dt_i}+\frac{K_i dt_i}{K_i dt_i}\right)} \tag{14}$$

$$\text{MRS}_{G_i \cdot C_i}=\frac{1}{e_{K_i.t_i}+1-d\varepsilon_i\left(e_{K_i.t_i}+1\right)} \tag{15}$$

The party $e_{K_i.t_i}+1-d\varepsilon_i\left(e_{K_i.t_i}+1\right)$ is written $\left(e_{K_i.t_i}+1\right)\left(1-d\varepsilon_i\right)$. We obtain

$$\text{MRS}_{G_i \cdot C_i}=\frac{1}{\left(e_{K_i.t_i}+1\right)\left(1-d\varepsilon_i\right)} \tag{16}$$

If $\text{MRS}_{G_i.C_i}=1 \Rightarrow (e+1)\left(1-d\varepsilon_i\right)=1$, Solution is: $d\varepsilon_i=\frac{e_{K_i.t_i}}{e_{K_i.t_i}+1}$ if $e_{K_i.t_i}\neq -1$.

•Appendix 4

Contrary to what happens under Assumption A_1, households are a conscious effort by the local government to improve the quality of the public good $\text{MRS}_{Q_i \cdot C_i}\neq 0$. We therefore consider that the private good C_i and the quantity of the public good G_i are of the goods of perfect complement $\text{MRS}_{G_i \cdot C_i}=0$. Under Assumption A_2 $\text{MRS}_{Q_i \cdot C_i}=-dC_i/dQ_i$

$$\text{MRS}_{Q_i \cdot C_i}=\frac{K_i dt_i+dH_i}{t_i dK_i+K_i dt_i+dH_i-dG_i} \tag{16a}$$

With $dt_i\neq 0$, $dK_i\neq 0$, and $dH_i=0$ we obtain

$$\text{MRS}_{Q_i \cdot C_i}=\frac{K_i dt_i}{t_i dK_i+K_i dt_i-dG_i} \tag{16b}$$

By dividing on $K_i dt_i$ we obtain

$$\text{MRS}_{Q_i \cdot C_i}=\frac{1}{\frac{t_i dK_i}{K_i dt_i}+1-\frac{dG_i}{K_i dt_i}}$$

With $\frac{t_i dK_i}{K_i dt_i}=e_{K_i.t_i}$ and $\frac{dG_i}{dt_i}=-K_i\left(\varepsilon_i-1\right)<0$. By replacing the latter value in MRS_{Q_i/C_i}, we obtain

$$\text{MRS}_{Q_i \cdot C_i}=\frac{1}{\varepsilon_i+e_{K_i.t_i}} \tag{17}$$

If $\text{MRS}_{Q_i \cdot C_i}=1 \Rightarrow \varepsilon_i+e_{K_i.t_i}=1 \Rightarrow \varepsilon_i=1-e$.

•Appendix 5

Sous A_3, the following equation that represents the general case, we obtain

$$\text{MRS}_{G_i \cdot C_i}=-\frac{dC_i}{dG_i}+\text{MRS}_{Q_i \cdot C_i}\frac{dQ_i}{dG_i}$$

$$\text{MRS}_{G_i \cdot C_i}=\frac{\frac{K_i dt_i}{K_i dt_i}+\frac{dH_i}{K_i dt_i}-\text{MRS}_{Q_i \cdot C_i}\left[\frac{t_i dK_i}{K_i dt_i}+\frac{K_i dt_i}{K_i dt_i}+\frac{dH_i}{K_i dt_i}-\frac{dG_i}{K_i dt_i}\right]}{\frac{t_i dK_i}{K_i dt_i}+\frac{K_i dt_i}{K_i dt_i}+\frac{dH_i}{K_i dt_i}-d\varepsilon_i\left[\frac{t_i dK_i}{K_i dt_i}+\frac{K_i dt_i}{K_i dt_i}+\frac{dH_i}{K_i dt_i}\right]}$$

$$\text{MRS}_{G_i \cdot C_i}=\frac{1+\frac{dH_i}{K_i dt_i}-\text{MRS}_{Q_i \cdot C_i}\left[e+1+\frac{dH}{Kdt}-\frac{dG}{Kdt}\right]}{e_{K_i t_i}+1+\frac{dH_i}{K_i dt_i}-d\varepsilon_i\left[e_{K_i t_i}+1+\frac{dH}{Kdt}\right]}$$

With $\frac{dG_i}{dt_i}=-K_i\left(\varepsilon_i-1\right)<0$, $\frac{dH_i}{K_i dt_i}=0$ and $\frac{K_i dt_i}{\frac{K_i dt_i}{t_i dK_i}+1}=\frac{1}{2}K_i dt_i$

$$\mathrm{MRS}_{G_i.C_i}=\frac{1-\mathrm{MRS}_{Q_i.C_i}\left[e_{K_it_i}+1-\frac{1}{K_i}\left(-K(\varepsilon_i-1)\right)\right]}{e_{K_it_i}+1-d\varepsilon\left(e_{K_it_i}+1\right)}$$

$$\mathrm{MRS}_{G_i.C_i}=\frac{-\left[\mathrm{MRS}_{Q_i.C_i}(e_{K_it_i}+\varepsilon_i)-1\right]}{\left(e_{K_it_i}+1\right)(1-d\varepsilon_i)}$$

$$\mathrm{MRS}_{G_i.C_i}=\left[\frac{1-\mathrm{MRS}_{Q_i.C_i}(e_{K_it_i}+\varepsilon_i)}{2(1-d\varepsilon)}\right]K_idt_i$$

$$\mathrm{MRS}_{G_i.C_i}=\left[1-\mathrm{MRS}_{Q_i.C_i}(e_{K_it_i}+\varepsilon_i)\right]\frac{K_idt_i}{2(1-d\varepsilon)}$$

With $\mathrm{MRS}_{G_i.C_i}=1$; solution is:

$$d\varepsilon=\frac{K_idt_i}{2}\left(\mathrm{MRS}_{Q_i.C_i}\left(e_{K_it_i}+\varepsilon_i\right)-1\right)+1 \text{ if } -K_idt_i\left(\mathrm{MRS}_{Q_i.C_i}\times\left(e_{K_it_i}+\varepsilon_i\right)-1\right)\neq 0$$

•Appendix 6.

From the standard MCPF definition, and without any variation in the taxation $dt_i=0$, the optimal rule for public-goods provision are characterized by

$$\mathrm{MRS}_{G_i.C_i}=\frac{1}{1-d\varepsilon_i}\left\{1-\mathrm{MRS}_{Q_i.C_i}\times\varepsilon_i\right\}$$

$$\Leftrightarrow\mathrm{MRS}_{G_i.C_i}=\mathrm{MCPF}^{\mathrm{H_i}}\left\{1-\mathrm{MRS}_{Q_i.C_i}\times\varepsilon_i\right\}$$

$$\Rightarrow\mathrm{MCPF}^{\mathrm{H_i}}\equiv\frac{1}{1-d\varepsilon_i}>1$$

The government funds the provision of the public good by a tax on mobile capital therefore $dt_i\neq 0$, the optimal rule for public-goods provision are characterized

$$\mathrm{MRS}_{G_i.C_i}=\frac{K_idt_i}{2\left(1-d\varepsilon_i\right)}\left\{1-\mathrm{MRS}_{Q_i.C_i}\times(\varepsilon_i+e_{K_i.t_i})\right\}$$

$$\Leftrightarrow\mathrm{MRS}_{G_i.C_i}=\mathrm{MCPF}^{t_i}\left\{1-\mathrm{MRS}_{Q_i.C_i}\times\left(e_{K_it_i}+\varepsilon_i\right)\right\}$$

By

$$\Rightarrow\mathrm{MCPF}^{t_i}\equiv\frac{K_idt_i}{2\left(1-d\varepsilon_i\right)}>1$$

Under the two generals approaches, we have the following result: (a) under A_3 with $dH_i\neq 0$, the financing of the quality of the public good ε_i is the reason for the effectiveness of the public good; (b) under A_3 with $dt_i\neq 0$, the tax rate $dt_i\neq 0$ and the financing of the quality of the public good ε_i are the reasons for inefficiency the provision of public good.