The failure of stabilization policy: fiscal rules in the presence of incompressible public expenditures

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Abstract

We consider a Ramsey model with possible increasing returns to scale and a government. The government balances its budget at each point in time and issues (i) a tax on income in order to finance unavoidable public expenditures, and (ii) further uses a tax rate rule with the purpose of stabilizing the economy. We show that insulating this economy from belief driven fluctuations is not possible if the government needs to raise a fixed amount of tax revenues to finance incompressible public expenditures. In this case, we always have steady state multiplicity (exactly two steady states) and global indeterminacy, while local indeterminacy is also possible. More precisely, even if a sufficiently procyclical tax rate is still able to eliminate local indeterminacy, two saddle steady states prevail, so that, depending on expectations, the economy may either converge to the low steady state or to the high steady state. This implies that a regime switching rational expectation equilibrium, where the economy switches between paths converging to the two different steady states, easily arises. As expectations are able to influence long run outcomes, our model is able to generate large and sudden boom and boost cycles in response to expectation shocks. Therefore, incompressible public expenditures may also be responsible for the sharp and sudden recession observed in the last decade.

1 Introduction

In recent years we have observed a revival of interest of macroeconomics in fiscal policy. For example, Feldstein (2009) discusses the recent rise of fiscal activism and Taylor (2011) asesses the size of the fical multipliers associated with the US stimulus packages of the period 2001-2009. While fiscal multipliers measure the impact of discretionary fiscal policy on output levels, another related strand of the literature studies instead the stabilization role of fiscal policy. Among those see Moldovan (2010) and Mckay and Reis (2016) who revisit the role of macroeconomic stabilizers using modern macrodynamic models. They consider a model with a unique determinate equilibrium and focus on the impact of stabilizers on the volatility of endogenous variables, due to exogenous shocks in fundamentals.

In this paper, we consider a non-monetary general equilibrium dynamic model where the government balances its budget. Each period the government must raise a fixed minimum amount of tax revenues in order to finance unavoidable public expenditures, which should remain constant along business cycles.[1] This implies a countercyclical income tax, which creates steady state multiplicity and may lead to the emergence of indeterminacy. We then discuss whether, in this context of incompressible public expenditures, procyclical tax rates are able to stabilize endogeneous business cycle fluctuations driven by volatile self fulfilling expectations.

Conventional wisdom states that procyclical/progressive tax rates have stabilizing effects, which help smooth out business cycle fluctuations due either to exogenous shocks to fundamentals or to volatile expectations (sunspots).[2] Friedman (1948) was one of the first to advocate a progressive tax system, which places primary reliance on the income tax, in order to attain both long run goals and short run stability.[3] Here, we show that in the presence of a minimum level of tax revenues, the stabilization ability of a procyclical tax rate rule is lost.

But is the presence of incompressible public expenditures an empirically relevant issue? The answer is a resounding yes. They correspond to expenditures associated with the basic functions of government (public safety, defense and general public services)

[1] They reflect the views of society on the appropriate size of expenditures associated with the basic functions of government.

[2] See, for example, Kletzer (2006) and Moldovan (2010) for the case with exogenous shocks to fundamentals and Guo and Lansing (1998), Dromel and Pintus (2008) for the case of shocks to expectations.

[3] Friedman (1948) also defended that government spending should be stable and determined by the needs of society, which already suggests incompressible public expenditures. Moreover in his proposal the budget should on average be balanced and the government should not issue interest-bearing securities to the public. Note that in our non monetary setup this implies that the government should balance its budget at all periods.

and have been remarkably constant for most developed countries, for very long time horizons.

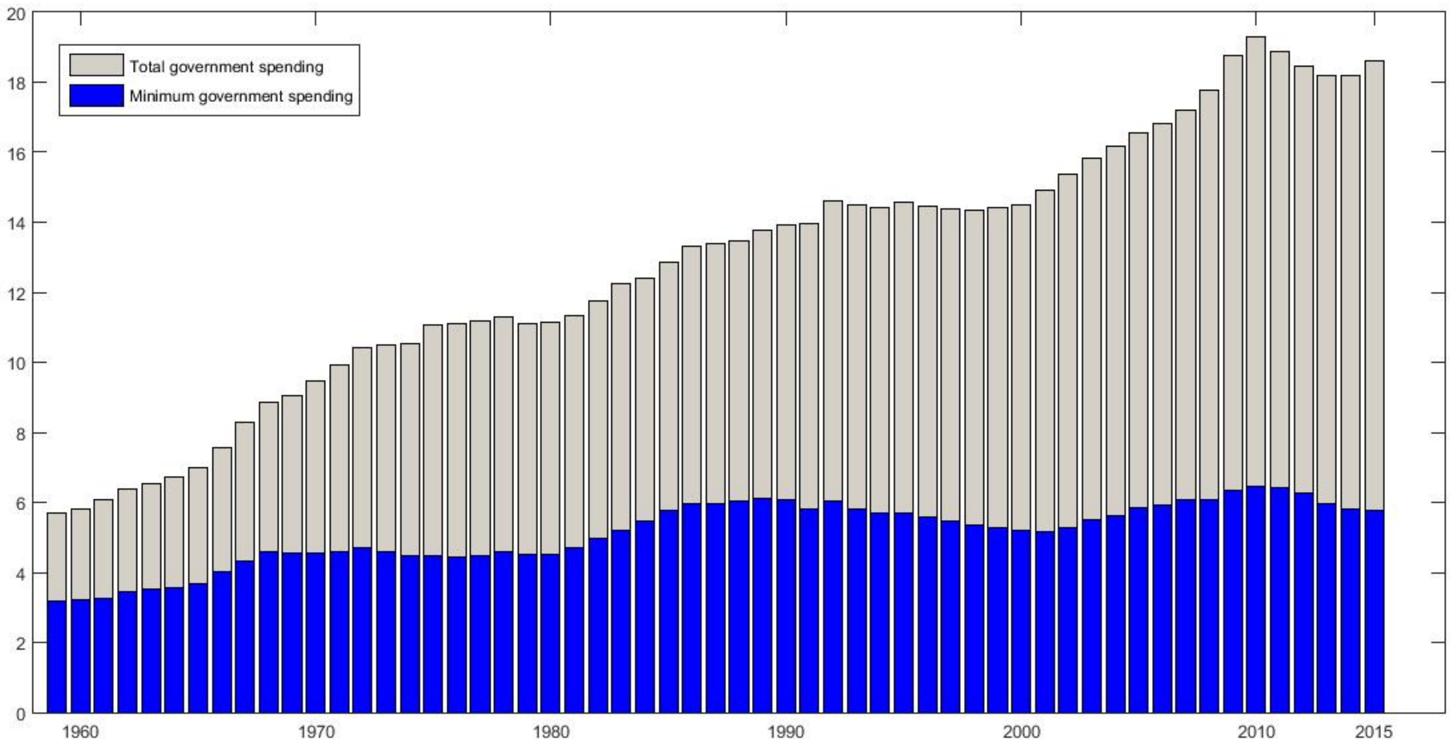

Figure 1: minimum government spending (blue) vs total government spending per capita in the U.S. (in thousands of dollars)

In figure 1 we present the evolution of these public expenditures and total public expenditures for the USA, in per capita terms, from 1959 to 2015 in constant prices of 2015. We can see that while total per capita government spending increased steadily in time, the level of per capita expenditures associated with the basic functions of government did not change much over such a long period, which implies that they do not follow business cycles. Also the relative standard deviation of these expenditures with respect to GDP is 10%, representing only 23% of the standard deviation of total public expenditures (see Appendix 1).

Note that the per capita level of these expenditures may vary across countries, reflecting the views of each society on how much should be spent on the basic functions of government. However, we expect them to be relatively stable in time for each country, regardless of the time evolution of total government spending. This is indeed the pattern we find across European countries from 2002 to 2014, a particularly turbulent period in macroeconomic terms. See figure 2. In Appendix 1 we also provide the standard

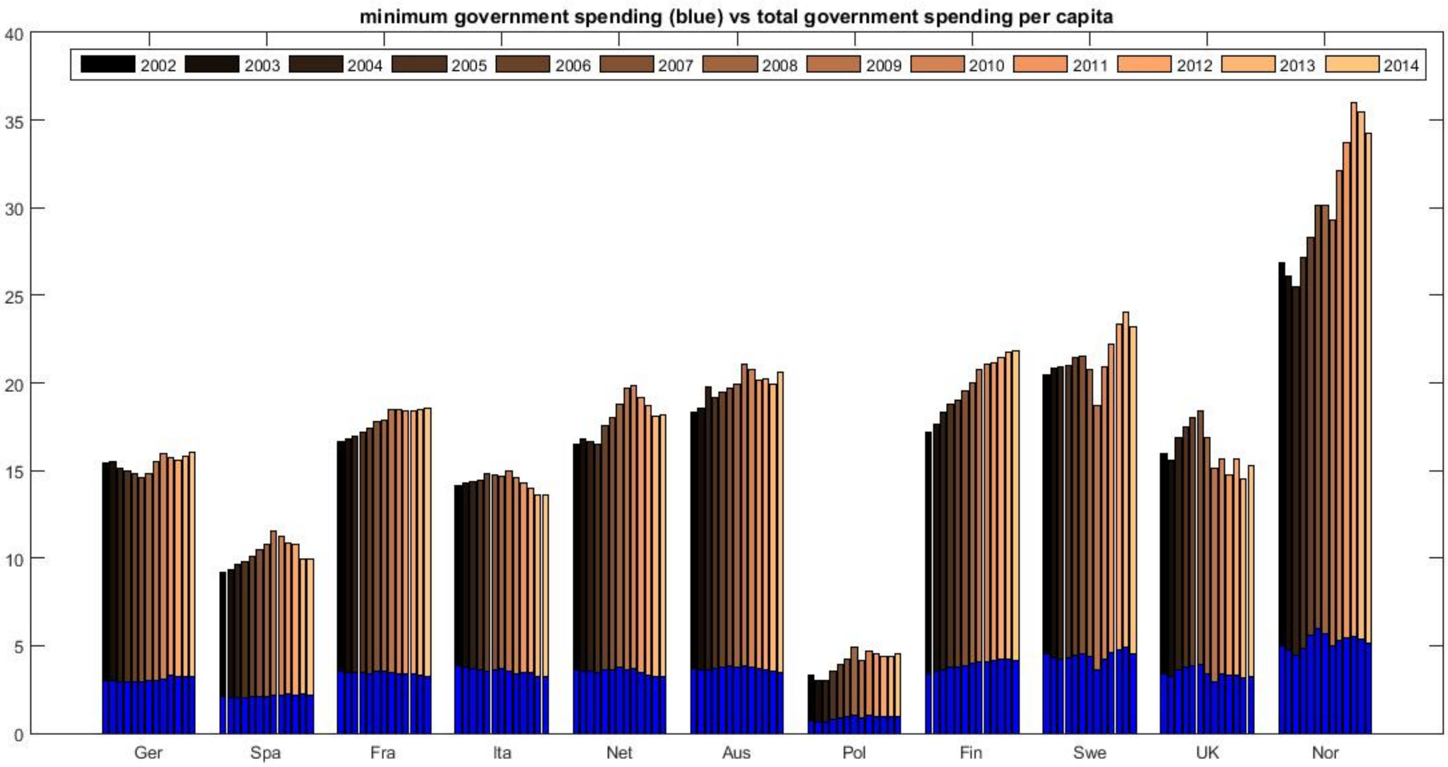

Figure 2: Minimum government spending (blue) vs total government spending per capita in others countries (in thousands of euros)

deviation of these incompressible government expenditures relative to that of GDP, together with the relative standard deviation of total government spending. Again, we obtain small relative standard deviations of incompressible public expenditures that represent, in most cases, less than 1/6 of the volatility of total government spending.

Until now the literature considered either fully flexible government expenditures or a totally constant public spending. In this paper, in line with empirical evidence, we address simultaneously the existence of a fully flexible total government spending which includes one fixed, incompressible, component. We find that, when the government needs to raise a minimum fixed amount of tax revenues in order to finance incompressible public expenditures, two steady states always emerge, one being always a saddle. Focusing on local dynamics is therefore not enough for stabilization purposes. Indeed, in contrast to standard previous results[4] we find that, although a procyclical tax rate policy is able to stabilize locally the indeterminate steady state, it will not eliminate steady state multiplicity and global indeterminacy as the economy may switch from

[4]Guo and Lansing (1998) find that a procyclical tax policy removes both local indeterminacy and steady state multiplicity.

one equilibrium path to the other. Therefore, in the presence of incompressible public expenditures a procyclical tax rate rule is no longer able to insulate the economy from belief driven fluctuations. In this context the management of expectations is crucial to guarantee that the economy remains on the path converging to the high output equilibrium.

However, if the government is not able to control expectations, the existence of multiple equilibria, associated with different expectations about the state of the economy, implies that a regime switching rational expectation equilibrium easily arises. In this equilibrium the economy switches between paths converging to the two different steady states, according to a sunspot variable. This implies that in our framework expectations are able to influence the long run outcomes of the economy, and not just the choice of the convergence path to one steady state. Therefore, in addition to small fluctuations around a locally indeterminate steady state, we are able to account for large fluctuations generated by a regime switching sunspot process. Indeed we show that our model is able to generate large and sudden boom and boost cycles in response to expectation shocks. We conclude that the widespread existence of incompressible public expenditures in developped countries, not only implies the failure of traditional tax stabilization policies, but may also be responsible for the sharp and sudden recession observed in the last decade.

The rest of the paper is organized as follows. In the next section, we present the model considered and obtain the perfect foresight equilibria. In section 3, we study steady state existence and multiplicity. Section 4 is devoted to the study of local dynamics, while section 5 examines global dynamics, emphasizing in both sections the consequence of using the income tax as a stabilizing tool. In section 6 we develop an augmented version of the model in which agent's expectations about future economic activity (output) follow a Markov switching process and provide a numerical illustration of the effects of expectation shocks. In section 7 we consider more general tax rules, and show that our results are robust. Finally, in section 8 we provide some concluding remarks. Mathematical proofs are relegated to the Appendix.

2 The model

We consider an infinite-horizon Ramsey model where a government balances the budget at each point in time and issues (i) a tax on income with the purpose of raising a fixed amount of tax revenues needed to finance uncompressible public expenditures, and (ii) further uses a tax rate rule (that may be constant, procyclical or countercyclical) with the purpose of stabilizing the economy. Households are infinitely-lived and have a logarithmic utility function in consumption and a perfectly elastic labor supply. Firms have access to a Cobb-Douglas technology, which may exhibit increasing returns to scale, and use labor and capital to produce a single good which is consumed or invested. This section describes such an economy.

2.1 Government

The government levies a fixed amount of tax revenues $\bar{T} \geq 0$ according to an income tax $\tau_y(y_t) \in [0, 1)$, such that:

$$\tau_y(y_t) = \frac{\bar{T}}{y_t} \tag{1}$$

where y_t is aggregate output,

According to (1), the tax rate τ_y is countercyclical, i.e. it decreases when output increases. Remark that $\bar{T}$ can also be viewed as the minimum size of government spending. The government also uses, with stabilization purposes, another income tax $\tau(y_t) \in [0, 1)$ which is variable with respect to aggregate income *à la* Lloyd-Braga et al. (2008):

$$\tau(y_t) = \mu y_t^{\phi} \tag{2}$$

The parameters $\mu \geq 0$ and $\phi \in \Re$, govern respectively the level of the tax rate and the response of the tax rate to output. When $\phi < 0$, the tax rate decreases when output expands, i.e. the tax rate is countercyclical. The case $\phi > 0$ corresponds to the case where the tax rate increases with output, i.e., a procyclical tax rate. A constant tax μ is considered when $\phi = 0$.

The total tax rate on income is then given by $(\tau_y(y_t) + \tau(y_t))$. Tax revenues finance wasteful[5] public expenditures G_t and the government budget is balanced at each point in time, i.e. we have that:

$$G_t = \tau(y_t)y_t + \bar{T}. \tag{3}$$

2.2 Households' behavior

We consider an economy populated by a large number of identical infinitely-lived agents. We assume without loss of generality that the total population is constant and normalized to one. At each period an agent has a perfectly elastic labor supply l_t with $l_t \in [0, \bar{l}]$ and $\bar{l} > 1$ his time endowment. She derives utility from consumption, c_t, and disutility from labor, l_t, according to the instantaneous utility function $U(c_t, l_t)$:

$$U(c_t, l_t) = ln(c_t) - \frac{l_t}{B} \tag{4}$$

[5] Externalities of fixed public spending do not play any role.

where $B > 0$ is a scaling parameter.

Households, when choosing c_t and l_t, face the following budget constraint:

$$\dot{k}_t + c_t = z(y_t)[w_t l_t + r_t k_t] - \delta k_t, \tag{5}$$

where k_t is the capital stock at time t, w_t the wage rate, r_t the rental rate of capital and $\delta > 0$ the depreciation rate of capital. The fiscal wedge, $z(y_t) \in (0, 1]$, is given as follows:[6]

$$z(y_t) \equiv 1 - \tau(y_t) - \tau_y(y_t) = 1 - \mu y_t^{\phi} - \frac{\bar{T}}{y_t}. \tag{6}$$

The intertemporal maximization problem of the representative household is given below:

$$\begin{aligned} &\max_{c_t, k_t, l_t} \quad \int_{t=0}^{+\infty} e^{-\rho t} U(c_t, l_t)\, dt \\ &s.t. \quad (5) \end{aligned} \tag{7}$$

where $\rho > 0$ is the discount factor. Note that households take as given the total tax rate i.e. $1 - z(y_t)$ when maximizing intertemporal utility.

Denoting by $\lambda(t)$ the shadow price of capital, the current-value Hamiltonian writes:

$$U(c_t, l_t) + \lambda_t \left[z(y_t)[w_t l_t + r_t k_t] - \delta k_t - c_t\right] \tag{8}$$

The first-order conditions are:

$$c_t^{-1} = \lambda_t \tag{9}$$

$$1 = B \lambda_t z(y_t) w_t \tag{10}$$

$$\frac{\dot{\lambda}_t}{\lambda_t} = -[z(y_t) r_t - (\rho + \delta)] \tag{11}$$

Any solution needs also to satisfy the transversality condition:

$$\lim_{t \to +\infty} e^{-\rho t} \lambda_t k_t = 0 \tag{12}$$

2.3 The production structure

We consider a competitive environment in which a continuum of measure one of identical firms produce a single good y_t using capital k_t and labor l_t. The firms' technology displays constant returns to scale at the private level according to a Cobb-Douglas specification $y_t = F(k_t, l_t, \bar{k}_t, \bar{l}_t) = e(\bar{k}_t, \bar{l}_t) k_t^s l_t^{1-s}$ with $e(\bar{k}_t, \bar{l}_t) \equiv (\bar{k}_t^s \bar{l}_t^{1-s})^{\gamma}$, $\gamma \geq 0$, a learning-by-doing externality, $\bar{k}_t$, $\bar{l}_t$ being respectively the average-wide stock of capital and hours worked, which are taken as given by individual firms. Since at the aggregate

[6] The fiscal wedge is the ratio between net (of taxes) and gross income.

level we have $\bar{k}_t = k_t$ and $\bar{l}_t = l_t$, the technology displays increasing returns to scale such that $y_t = k_t^{\alpha} l_t^{\beta}$ with $\alpha = s(1+\gamma)$, $\beta = (1-s)(1+\gamma)$.

From the profit maximisation of the firm, we obtain the real wage rate w_t and the real rental rate of capital r_t as:

$$r_t = se(\bar{k}_t \bar{l}_t) \left(\frac{k_t}{l_t}\right)^{s-1} = \frac{sy_t}{k_t} \equiv r(k_t, y_t) \tag{13}$$

$$w_t = (1-s)e(\bar{k}_t, \bar{l}_t) \left(\frac{k_t}{l_t}\right)^{s} \equiv \frac{(1-s)y_t}{l_t} \equiv w(l_t, y_t) \tag{14}$$

Hence, profits are zero and $y_t = w_t l_t + r_t k_t$.

In what follows, we assume that s is small, i.e., $s < 0.5$, as usually done in the literature. Moreover, in order to avoid endogenous growth, we consider not too strong productive externalities, i.e. we assume that $\gamma < \frac{1-s}{s}$, so that $\alpha < 1$. Together these two assumptions imply that $\beta > \max\{\alpha, \gamma\}$. All these assumptions are summarized below in Assumption 1 and we consider them satisfied in the rest of the paper.

Assumption 1. $s < 0.5$ *and* $\gamma < \frac{1-s}{s}$ *so that* $\alpha < 1$ *and* $\beta > \max\{\alpha, \gamma\}$.

2.4 Intertemporal equilibrium

In this section, we define the intertemporal perfect foresight equilibrium of this economy. From the aggregate production function we can write $l_t = l(k_t, y_t) \equiv y_t^{1/\beta} k_t^{-\alpha/\beta}$ which implies that, using (14), we can express the wage as a function of k_t and $l(k_t, y_t)$ so that:

$$w_t = w(k_t, y_t) \equiv (1-s)\, k_t^{\alpha/\beta} y_t^{(\beta-1)/\beta} \tag{15}$$

Substituting (9) and (15) in (10), we solve this equation with respect to c_t and obtain:

$$c_t = c(k_t, y_t) \equiv B(1-s) z(y_t) k_t^{\alpha/\beta} y_t^{(\beta-1)/\beta} \tag{16}$$

Below, we provide the elasticities of the latter expression:

$$\varepsilon_{cy} = \tfrac{[\beta(1+\varepsilon_z(y_t)-1]}{\beta} \quad \varepsilon_{ck} = \tfrac{\alpha}{\beta} \tag{17}$$

$$\text{where } \varepsilon_z(y_t) \equiv \frac{z'(y_t)y_t}{z(y_t)} = \frac{\bar{T} - \phi\mu y_t^{1+\phi}}{y_t - \bar{T} - \mu y_t^{1+\phi}} \tag{18}$$

Differentiating equation (16) with respect to time, we obtain:

$$\frac{\dot{c}_t}{c_t} = \frac{\alpha}{\beta}\frac{\dot{k}_t}{k_t} + \frac{\beta(1+\varepsilon_z(y_t)) - 1}{\beta}\frac{\dot{y}_t}{y_t}. \tag{19}$$

Subtituting (9) and (13) in (11) we have

$$\frac{\dot{c}_t}{c_t} = \frac{sz(y_t)y_t}{k_t} - (\rho + \delta). \tag{20}$$

Equating now (19) and (20) and rearranging terms we finally obtain:

$$\frac{\dot{y}_t}{y_t} = \frac{s\beta z(y_t)y_t - (\rho+\delta)\beta k_t - \alpha \dot{k}_t}{k_t\left[\beta(1+\varepsilon_z(y_t)) - 1\right]} \tag{21}$$

with $z(y_t) > 0$ given in (6), and $\varepsilon_z(y_t)$ given in (18).

Substituting now (16) in the the households' budget constraint (5), we obtain the law of motion of the capital stock:

$$\dot{k}_t = z(y_t)y_t - \delta k_t - c(k_t, y_t) \tag{22}$$

Definition 1. *An intertemporal perfect foresight equilibrium is a path $\{k_t, y_t\}_{t\geq 0}$ satisfying equations (21)-(22) and the transversality condition (12), with $z(y_t) \in (0,1]$ given in (6), $\varepsilon_z(y_t)$ given in (18) and $c(k_t, y_t)$ given in (16).*

Note that if $\beta(1+\varepsilon_z(y_t)) - 1 = 0$, at some point in time t we say that we have a *singularity*, and equation *(21)* is not properly defined. We will discuss later the implications of the existence of singularities on the study of the dynamics of the model.

3 Steady state analysis

A steady state is a 4-tuple (k, l, c, y), satisfying:

$$\begin{aligned} y &= k^\alpha l^\beta && (23)\\ cl &= B(1-s)z(y)y && (24)\\ sz(y)y &= (\rho+\delta)k && (25)\\ c &= z(y)y - \delta k && (26)\\ z(y) &> 0 && (27) \end{aligned}$$

Using this system of equations leads to:

$$\begin{gathered} k = \tfrac{sz(y)y}{(\rho+\delta)} \\ c = \tfrac{[\rho+(1-s)\delta]z(y)y}{(\rho+\delta)} \\ l = \left[\tfrac{[B(1-s)(\rho+\delta)]}{[\rho+(1-s)\delta]}\right] \\ \underbrace{(z(y)y)^\alpha y^{-1}}_{\equiv H(y)} = \underbrace{\left(\frac{s}{(\rho+\delta)}\right)^{-\alpha}\left(\frac{(1-s)(\rho+\delta)B}{\rho+(1-s)\delta)}\right)^{-\beta}}_{\equiv \bar{H}} \\ z(y) > 0 \end{gathered} \tag{28}$$

3.1 Existence and Multiplicity

Steady state existence and multiplicity are determined by the solutions of $H(y) = \bar{H}$. Note that we restrict $y \in (\underline{y}, \bar{y})$ with $\underline{y} > 0$ and $\bar{y} \in (\underline{y}, +\infty)$ to ensure that $z(y) = (1 - \mu y^{\phi} - \frac{\bar{T}}{y}) > 0$. See Appendix 8.1. We use the scaling parameter $B > 0$ to ensure the existence of a normalized steady state (NSS), $y = 1$. Hence,

Proposition 1. *The solution* $(k_{nss}, l_{nss}, c_{nss}, 1)$ *of system (28) where*

$$\begin{aligned} k_{nss} &= \frac{sz(1)}{(\rho+\delta)} \\ l_{nss} &= \left[\frac{[B(1-s)(\rho+\delta)]}{[\rho+(1-s)\delta]}\right] \\ c_{nss} &= \frac{[\rho+(1-s)\delta]\, z(1)}{(\rho+\delta)} \\ y_{nss} &= 1 \end{aligned}$$

is a NSS if and only if $B = B^*$ *with:*

$$B^* = \frac{\frac{[\rho+(1-s)\delta]}{(\rho+\delta)}}{\left(\frac{s}{(\rho+\delta)}\right)^{\frac{\alpha}{\beta}}(1-s)z(1)} \tag{29}$$

$$z(1) = 1 - \bar{T} - \mu > 0 \tag{30}$$

Remark that, since B^* does not depend on ϕ, existence of the normalized steady state is persistent and always ensured as ϕ varies.

To study steady state multiplicity, we must characterize the sign of $\varepsilon_H(y) \equiv \frac{H'(y)y}{H(y)}$. Using (28) we have that

$$\varepsilon_H(y) = [\alpha(1 + \varepsilon_z(y)) - 1] \tag{31}$$

where $\varepsilon_z(y)$ is given in (18). We can easily show that, in the presence of any form of countercyclical tax rates, $H(y) > 0$ is first increasing and then decreasing in $y \in (\underline{y}, \bar{y})$, i.e. $\varepsilon_H(y)$ changes sign once. Indeed we have that:

Proposition 2. *Under Assumption 1 and Proposition 1, for* $\bar{T} > 0$, $H(y) > 0$ *is always single-peaked and there are exactly two steady states. Moreover, when* $\bar{T} = 0$, $H(y) > 0$ *is single peaked and there are exactly two steady states if and only if* $\phi < 0$. *In contrast, when* $\bar{T} = 0$, *and* $\phi > 0$ *the steady state is unique.*

Proof. See the Appendix. □

Corollary 1. *A strictly positive* $\bar{T}$ *is sufficient for steady state multiplicity. Denoting the low output steady state by* y_l *and the high output steady state by* y_h, *we have* $(1 + \varepsilon_z(y_l)) > \frac{1}{\alpha}$ *and* $(1 + \varepsilon_z(y_h)) < \frac{1}{\alpha}$.

Proposition 2 and Corollary 1 tell us that steady state multiplicity can not be eliminated by the use of cyclical stabilization policy ($\phi \neq 0$) if the government needs to finance a fixed minimum amount of spending ($\bar{T} > 0$). Hence, in contrast to Guo and Lansing (1998), multiplicity of steady state remains, even in the presence of a sufficiently procyclical tax rate, i.e., with a ϕ sufficiently positive, provided $\bar{T} > 0$. Indeed, a necessary condition for steady state multiplicity is that $\varepsilon_H(y)$ changes sign at least once, which requires that $\varepsilon_z(y)$, the elasticity of the tax wedge, varies with y. See (31). From (18) we can see that this will happen if and only if we have $\bar{T} > 0$ and or $\phi \neq 0$. However this is not sufficient for steady state multiplicity since when $\bar{T} = 0$ and $\phi > 0$ steady state uniqueness is always obtained.[7]

4 Local Analysis

We now characterize the local stability properties of our dynamic system around a steady state. We start by linearizing system (21)-(22) around a steady state obtaining:

$$\begin{pmatrix} d\dot{y}(t) \\ d\dot{k}(t) \end{pmatrix} = J \begin{pmatrix} dy(t) \\ dk(t) \end{pmatrix}. \tag{32}$$

The local stability properties of the model are determined by the eigenvalues of the Jacobian matrix J (given in Appendix 8.2) or, equivalently, by its trace, Tr, and determinant, D, which correspond respectively to the product and sum of the two roots (eigenvalues) of the associated characteristic polynomial $Q(\lambda) = \lambda^2 - Tr\lambda + Det$ with:

$$Tr = \rho + \frac{(\rho + \delta)\left[1 - (1 + \varepsilon_z)(\alpha + \beta)\right]}{\{\beta(1 + \varepsilon_z) - 1\}} \tag{33}$$

$$D = \frac{(\rho + \delta)^2 \left[1 - \alpha(1 + \varepsilon_z)\right]}{s\,\{\beta(1 + \varepsilon_z) - 1\}} \tag{34}$$

with $\varepsilon_z \equiv \varepsilon_z(y)$. Necessary and sufficient conditions to obtain local indeterminacy (a sink) are $D > 0$ and $Tr < 0$, while the necessary and sufficient condition to get local saddle-path stability is $D < 0$. Finally, the steady state is locally a source if and only if $D > 0$ and $Tr > 0$.

Proposition 3. *Under Assumption 1 and Proposition 1, the high output steady state is locally indeterminate (a sink) if and only if* $(1 + \varepsilon_z(y_h)) \in (\frac{1}{\beta}, \frac{1}{\alpha})$ *and is locally determinate (a saddle) if and only if* $(1 + \varepsilon_z(y_h)) < \frac{1}{\beta}$. *Furthermore, the low output steady state is always locally determinate (a saddle).*

[7] This is the case explored in Guo and Lansing (1998). In the light of our analysis it is easy to understand their results since in their case $\bar{T} = 0$ and the specification chosen for their tax rate is such that $\varepsilon_z(y)$ is constant.

Proof. Note that the numerator of the determinant at the high output steady state y_h is positive as the condition $1+\varepsilon_z(y_h) < \frac{1}{\alpha}$ holds. Local indeterminacy requires therefore a positive denominator of both trace and determinant which implies $1 + \varepsilon_z(y_h) < \frac{1}{\beta}$. Since $\alpha < \beta$ under Assumption 1, the latter condition also leads to a negative trace, so that the necessary and sufficient conditions to get local indeterminacy around the high output steady state are $1 + \varepsilon_z(1) \in (\frac{1}{\beta}, \frac{1}{\alpha})$. The rest of the proposition follows since at the low output steady state $(1 + \varepsilon_z(y_l)) > \frac{1}{\alpha} > \frac{1}{\beta}$, so that the determinant is always negative. □

Our local indeterminacy mechanism is not new and, as in the seminal works of Schmitt-Grohe and Uribe (1997) and Benhabib and Farmer (1994), is once again related with the labor market "wrong slopes" condition. Noting that the slope of the MPL (marginal productivity of labor) curve is $-(1-\beta)$, while the slope of the inverse labor supply curve is $-\beta\varepsilon_z$, [8] it is easy to see that our indeterminacy condition, $\beta(1+\varepsilon_z(y_h)) > 1$, requires (i) either a negatively sloped inverse labor supply schedule ($\varepsilon_z > 0$) steeper than the (also negatively sloped) MPL curve ($\beta < 1$), or (ii) a positively sloped MPL curve ($\beta > 1$), steeper than the (also positively sloped) inverse labor supply schedule ($\varepsilon_z < 0$), or (iii) a positively sloped MPL curve and a negatively sloped inverse labor supply schedule.

Our work encompasses several related papers, which can be recovered as particular cases of our framework. Indeed, in the absence of productive externalities and cyclical taxation our indeterminacy condition collapses into the Schmitt-Grohe and Uribe´s (1997) indeterminacy condition $s < \frac{\bar{T}}{y} < 1 - s$, whereas in the absence of government we recover Benhabib and Farmer´s (1994) indeterminacy condition $\alpha < 1 < \beta$.

4.1 Stabilizing locally

We consider now Proposition 1 satisfied, so that the normalized steady state exists, and choose a parameterization such that the NSS is locally indeterminate in the absence of an active stabilization policy ($\phi = 0$).[9] In Proposition 4 below we state the necessary and sufficient conditions for this to happen:

Proposition 4. *Under Assumption 1 and Propositions 1 and 3, consider that the government does not pursue an active stabilization policy* ($\phi = 0$)*. Then the NSS is locally indeterminate if and only if* $(1-\beta)(1-\mu) < \bar{T} < (1-\alpha)(1-\mu)$*.*

[8] At the general equilibrium level where $\bar{l} = l$ we can rewrite the MPL shedule (1) as $d \log w_t = \alpha d \log k_t - (1-\beta) d \log l_t$. In what concerns the inverse of the labor supply schedule from (10), considering a constant λ, $z(y)$ given by (6) with $y_t = k_t^{\alpha} l_t^{\beta}$, we obtain $d \log w_t = -\beta\varepsilon_z d \log l_t - \alpha\varepsilon_z d \log k_t$.

[9] Under Proposition 3 this implies that the NSS is the high output steady-state.

Proof. Note that, since $\varepsilon_z(1) = \frac{\bar{T}-\phi\mu}{1-\bar{T}-\mu}$, when $\phi = 0$, from Proposition 3, we obtain immediately the condition above. □

As $\alpha = (1+\gamma)s$ and $\beta = (1+\gamma)(1-s)$, we conclude that in the absence of production externalities, $\gamma = 0$, indeterminacy requires a strictly positive $\bar{T}$. However, with production externalities, since under Assumption 1 $\alpha < 1$, this inequality can only be verified for $\bar{T} = 0$ if $\beta > 1$. We conclude that sufficiently strong production externalities and/or a strictly positive $\bar{T}$ are required for the NSS to be indeterminate in the absence of cyclical taxation.[10]

Assume now that the government wants to insulate the economy from belief driven fluctuations around the NSS. This is done by eliminating local indeterminacy. In Proposition 5 below we state how the government can eliminate local indeterminacy using cyclical taxation.

Proposition 5. *Under Assumption 1 and Propositions 1 and 3, assume that* $(1-\beta)(1-\mu) < \bar{T} < (1-\alpha)(1-\mu)$. *Then, local indeterminacy of the NSS is eliminated, and local saddle path stability of the normalized steady state is achieved with a sufficiently procyclical income tax rate such that* $\phi > \frac{\bar{T}-(1-\beta)(1-\mu)}{\mu\beta} > 0$.

Proof. From Proposition 3, it is easy to see that the government can eliminate local indeterminacy, obtaining saddle path stability of the NSS, by increasing ϕ, so that $\varepsilon_z(1) = \frac{\bar{T}-\phi\mu}{y_t-\bar{T}-\mu}$ decreases, satisfying the inequality $(1+\varepsilon_z(1)) < \frac{1}{\beta}$, that we can rewrite as $\phi > \frac{\bar{T}-(1-\beta)(1-\mu)}{\mu\beta} > 0$. □

Guo and Lansing (1998) in a Ramsey model have also shown that sufficiently procyclical (or progressive) tax rates on income eliminate local indeterminacy caused by productive externalities, whereas Guo (1999) considering only progressive labor income taxation obtained a similar result. In our framework the same is true, and the NSS becomes then a saddle, which eliminates the existence of local sunspot fluctuations. However, we know from our previous analysis, that another steady state with a lower level of output also exists. Both steady states are saddles and therefore locally determinate. Nevertheless, as will be shown in the next section there is global indeterminacy. Indeed, in the presence of multiple steady states, ensuring that all of them are locally determinate does not eliminate global indeterminacy and sunspots. To adress these issues we must analyse the global dynamics of the model

[10]Note that when $\gamma = 0$ and $\bar{T} = 0$ a sufficiently negative ϕ also guarantees local indeterminacy.

5 Global Analysis

5.1 Phase diagram

Substituting (22) in (21) our dynamic sytem (21)-(22) can be rewritten in the following way:

$$\begin{bmatrix} \dot{k}_t \\ \dot{y}_t \end{bmatrix} = \begin{bmatrix} f_1(k_t, y_t) \\ \frac{(1+\gamma) f_2(k_t, y_t)}{g(y_t)} \frac{y_t}{k_t} \end{bmatrix} \tag{35}$$

where the vector in the RHS is the vector field of sytem *(35)* and:

$$\begin{aligned} f_1(k_t, y_t) &\equiv z(y_t)y_t - \delta k_t - c(k_t, y_t) \\ f_2(k_t, y_t) &\equiv s\left[c(k_t, y_t) - sz(y_t)y_t\right] - \left[\rho(1-s) + \delta(1-2s)\right]k_t \\ g(y_t) &\equiv \beta(1 + \varepsilon_z(y_t)) - 1 \end{aligned}$$

with $c(k_t, y_t)$, $z(y_t)$ and $\varepsilon_z(y_t)$ given respectively by (16), (6) and (18).

In order to analyze global dynamics, in figures 3 and 4, we depict in the space (y, k) the k and the y nullclines and the arrows that represent the vector field. The $k-$nullcline satisfies $f_1(k_t, y_t) = 0$, and the $y-$nullcline satisfies $f_2(k, y) = 0$. Of course these two schedules cross twice, respectively at the low and high output steady states. In the Appendix we show that along the $k-$nullcline we have $dk/dy > 0$, and that the slope of the $y-$nullcline will change sign at most two times. Moreover, for $k = 0$, i.e., at the intersection between the $y-$nullcline and the line $k = 0$, the slope of the $y-$nullcline is positive and above unity.[11] As both k and y increase, the slope of the $y-$nullcline decreases, and the nullcline reaches a maximum when its slope becomes zero. As y further increases its slope becomes negative, reaching $-\infty$, so that the $y-$nullcline becomes vertical. In the Appendix we show that for reasonable values of the parameters we obtain a correspondence, i.e., after becoming vertical the nullcline bends inwards, as depicted in figures 3 and 4. It is also easy to show[12] that when $k = 0$ the $y-$nullcline is located on the right of the $k-$nullcline, as represented in figures 3 and 4.

In the Appendix we also show that above the $k-$nullcline we have $\dot{k} < 0$, i.e., above (below) the $\dot{k} = 0$ line the vertical arrows that represent the vector field of $\dot{k}_t$ point downwards (upwards). Before determining the directions of the horizontal arrows that represent the vector field of $\dot{y}_t$ it is important to note that our model exibits a singularity for $y = y^s$ such that $g(y^s) = \beta(1 + \varepsilon_z(y^s)) - 1 = 0$. In the space (y, k), $y = y^s$ defines a vertical line. This line partitions the space (y, k) into two subsets of regular points:

[11] It is equal to $1/\alpha$.

[12] See the Appendix.

one set, which we denote by Ω_+, where $g(y) > 0$, i.e., where the necessary condition for indeterminacy is satisfied, and another, denoted by Ω_-, where this condition is not satisfied, i.e. $g(y) < 0$. Of course, on different sides of the vertical line $y = y_s$ horizontal arrows point in opposite directions. The full determination of the direction of the horizontal arrows, depicted in Figures 3 and 4, is also provided in the Appendix.

In the following, we will restrict our analysis to equilibrium regular paths that converge to a steady state.[13]

5.2 Global Dynamics when the NSS is a sink

We will start by addressing the situation where the NSS is a sink, that is depicted in figure 3. As explained above the NSS is the high output steady-state, which coexists with a lower output steady state which is a saddle. Since at both steady states $g(y) = \beta(1+\varepsilon_z(y))-1 > 0$, they are both located on the same side of the singularity so that $y_s > 1$. All deterministic trajectories starting on the left of the saddle path diverge to either $k = 0$ or to $y = \underline{y}$ and cannot be equilibrium paths. Otherwise, all other deterministic trajectories converge to the higher output sink steady state, with the exception of those starting precisely on the stable arm of the saddle, which converge to the lower output steady state. This means that, for the same initial given value of the predetermined variable, the capital stock, there are several different equilibrium trajectories that converge to different steady states.[14] The equilibrium trajectory obtained depends on the value of the non predetermined variable chosen, which is expected output. This means that we have global indeterminacy[15]. Also, since the NSS is a sink there exist local stochastic endogenous fluctuations (sunspots) around it. See Grandmont et al (1998). We can therefore state the following:

Proposition 6. *Under Assumption 1 and Propositions 1 and 3, when* $(1 + \varepsilon_z(1)) \in (\frac{1}{\beta}, \frac{1}{\alpha})$ *there are exactly two steady states: the NSS, which is the high output steady and a sink, coexists with the low output steady state which is a saddle. In this case there is global indeterminacy. Furthermore we have local indeterminacy of the NSS steady state and there exist local stochastic endogenous fluctuations (sunspots) around it.*

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[13] We define equilibrium regular paths as solutions of (35) that do not collide with $y = y_s$ and verify the initial and transversality conditions. For an analysis of singular dynamics paths see Brito et al. (2016).

[14] Note however, that since all equilibrium trajectories, with the exception of the one that converges to the low output steady state, end up at the high output steady state, the likelihood of reaching the low output steady state is low.

[15] See Raurich (2000) for a definition and a clear cut discussion about global indeterminacy issues.

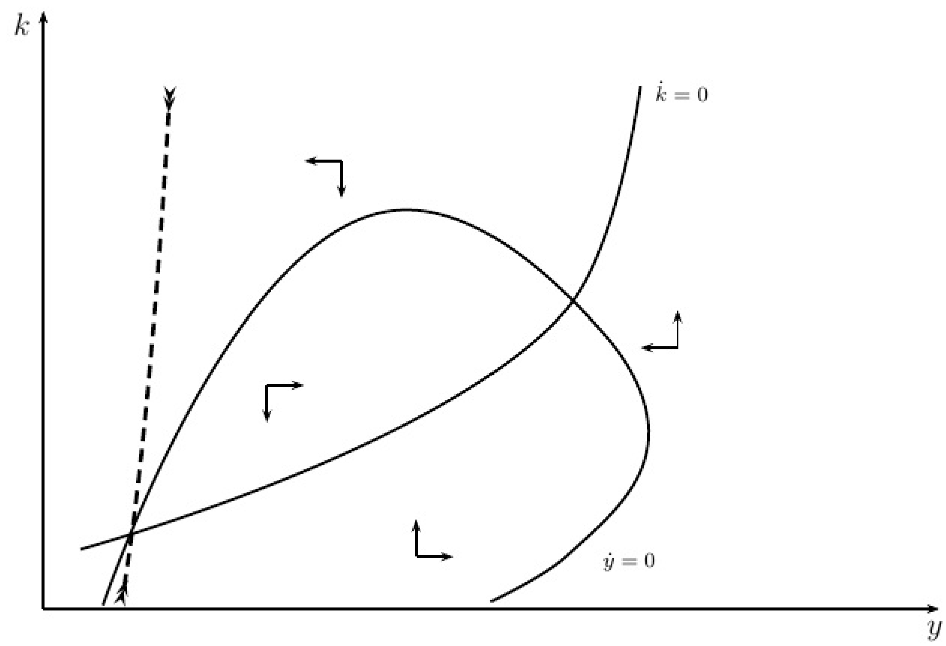

Figure 3: Sink-saddle configuration

5.3 Global Dynamics with two saddles

Now, if the government decides to stabilize locally the NSS, it can, as described above, make it saddle-stable by increasing ϕ so that $g(1) = \beta(1+\varepsilon_z(1)) - 1 < 0$. Both steady states are now locally saddle-stable, but in the low-output steady state $g(y_l) = \beta(1+\varepsilon_z(y_l))-1 > 0$. This means that we have $y_l < y_s < 1$ so that the situation depicted in figure 4 emerges. In this case, although local indeterminacy and sunspots no longer exist, the problem of global indeterminacy remains. Again, for a given initial value of the capital stock, the model admits equilibria that converge either to the lower steady state or to the NSS.[16] These equilibria differ with respect to the agents' expectations about future output. This implies that expectations about future output, determine the long-run outcomes of the economy, i.e. we also have global indeterminacy.

Proposition 7. *Under Assumption 1 and Propositions 1 and 3, when* $1+\varepsilon_z(1) < \frac{1}{\beta} < \frac{1}{\alpha}$ *there are exactly two steady states: the NSS, which is the high output steady and a saddle, coexists with the low output steady state which is a saddle. In this case there exist two distinct saddle paths and hence there is global indeterminacy.*

We conclude that in our model, and in contrast to previous results, procyclical tax

[16]For all other values of y we obtain divergent trajectories.

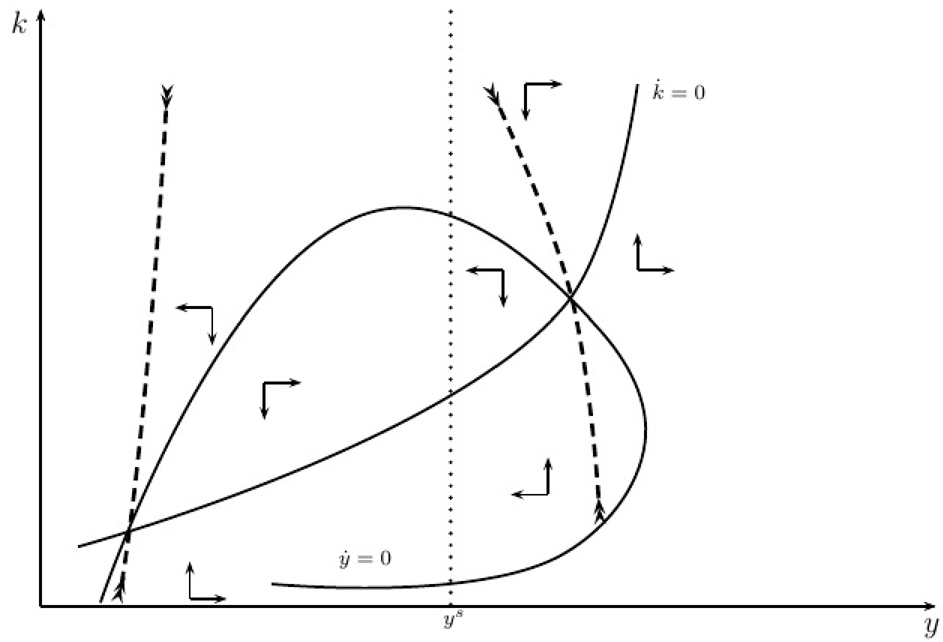

Figure 4: Saddle-saddle configuration

rates are not able to insulate the economy from belief driven fluctuations. Furthermore, since these fluctuations are due to the existence of global (and not local) indeterminacy, the current indeterminacy mechanism is able to generate (or account for) sharp fluctuations in output as, if agents' expectations are revised downwards, the economy is displaced from the upper to the lower stable arm, making the economy converge to the lower output steady state.[17] Hence, a tax policy that locally stabilizes, eliminating small fluctuations,is not able to prevent (big) fluctuations caused by changes in agents' expectations. In this context the management of expectations is crucial to guarantee that the economy remains on the right path, avoiding sharp belief driven fluctuations.

It is clear from the above discussion that steady state multiplicity is responsible for these results. Also, as explained above, in our model steady state multiplicity is pervasive, due to the existence of a fixed amount of minimum government expenditures. It follows, that when the government is not able to manage private agents' expectations, abandoning the view that government expenditures, even in recessions, can not fall below a fixed minimum level, is the only way to avoid the perils of stabilization. Indeed, making government spending fully flexible, i.e., eliminating $\bar{T}$, is the only way to obtain

[17]Furthermore, such major crisis is potentielly long lasting if the revision in agents' expectations is persistent.

simultaneously saddle path stability and steady state uniqueness, and hence global determinacy, fully restoring the ability to stabilize of a procyclical tax rate policy.

However, if the government is not able to eliminate $\bar{T}$ or to manage expectations we obtain multiple equilibrium trajectories associated with different expectations about future output. Note however that in each equilibrium agent's expectations are correct. Indeed, when agents' confidence falls and the economy lands on the low output trajectory, the output that materializes is the one expected by the agents. Similarly, when confidence is restored and agents are optimistic, the economy switches to the high output trajectory. Again the output that materializes is the one expected by agents.

6 Expectations Shocks

In this section, we discuss and illustrate the effects of expectations shocks. We start by providing a version of the model in which the agents' expectations about long-run output follow a simple two state Markov switching process, allowing y_t to jump between trajectories. We then use the augmented model to illustrate the effect of shocks to the agents' expectations about long-run output. Finally, we discuss the economic mechanism responsible for the emergence of global indeterminacy and regime switching sunspots.

6.1 Modeling Expectation Shocks

In this section we follow closely Kaplan and Menzio (2016),[18] borrowing their definition of a Markov switching rational expectation equilibrium. We introduce a sunspots variable, S_t, which takes two values, 0 or 1. We assume that $S_t = 1$ is associated with the belief that the economy is in a trajectory converging to the high output steady state (conditional on remaining in the same optimistic state), whereas $S_t = 0$ is associated with the belief that the economy is on a trajectory converging to the low output steady state (conditional on remaining in the same pessimistic state). Agents' expectations switch from optimistic to pessimistic with probability p, i.e., the probability that S_t changes from unity to zero in a short interval is p. In this case output falls by $D_{1,0}(k, y)$. Similarly the agents' expectations switch from pessimistic to optimistic with probabilty q, in which case output increases by $D_{0,1}(k, y)$.

In the optimistic state, the evolution of the economy is described by (22) and

$$\frac{\dot{y}_t}{y_t} = -\frac{\alpha}{\left[\beta(1+\varepsilon_z(y_t^h)) - 1\right]}\frac{\dot{k}_t}{k_t} + \beta\frac{z(y_t)y_t - (\rho+\delta)k_t}{k_t\left[\beta(1+\varepsilon_z(y_t^j)) - 1\right]} + pD_{1,0}(k_t, y_t). \tag{36}$$

[18] See also Benhabib et al. (2016).

The term in the LHS represents the change in output conditional on the economy remaining in the optimistic state. The first two terms on the RHS correspond to the RHS of (21), while the last term is the probability that the economy switches to the pessimistic state, p, times the resulting change in output conditional on the economy switching states, $D_{1,0}(k, y)$. Similarly, in the pessimistic state the behavior of the economy is described by (22) and

$$\frac{\dot{y}_t}{y_t} = -\frac{\alpha}{\left[\beta(1 + \varepsilon_z(y_t^l)) - 1\right]} \frac{\dot{k}_t}{k_t} + \beta \frac{z(y_t)y_t - (\rho + \delta)k_t}{k_t \left[\beta(1 + \varepsilon_z(y_t)) - 1\right]} + qD_{0,1}(k_t, y_t). \tag{37}$$

The term in the LHS represents the change in output conditional on the economy remaining in the pessimistic state. The first two terms on the RHS correspond, as in the optimistic case, to the RHS of (21), while the last term is the probability that the economy switches to the optimistic state, q, times the resulting change in output conditional on the economy switching states, $D_{0,1}(k, y)$.

Since expectations must be rational we need to impose the following conditions. First, when the economy switches from the optimistic to the pessimistic state, the value of output must land on y_l^S, where y_l^S denotes the stable manifold associated with the low-output steady state, y_0^l. This guarantees that, if the economy then remains in the pessimistic state forever, it will converge to the low output steady state y_0^l. Second, when the economy switches from the pessimistic to the optimistic state, the value of output must fall on y_h^S the stable manifold associated with the high-output steady state if this steady state is a saddle, or its basin of attraction if it is a sink. This guarantees that, if the economy then remains in the optimistic state forever, it will converge to the high output steady state y_1^h. Finally, if the initial state of the economy is optimistic, the initial value of output must be on the stable manifold associated with the high-output steady state or in its bassin of traction, while if the initial state of the economy is pessimistic, the initial value of output must be on the stable manifold associated with the low output steady state.

Let S denote a history of realizations of S_t and $t_n(S)$ the n^{th} time at which the state of the process switches in history S. Then, following Kaplan and Menzio (2016) we define:

Definition 2. *A Markov switching rational expectation equilibrium is a history-dependent path $\{k_t(S), y_t(S)\}$ such that, for every S, the following conditions are satisfied: (i) For all $t \in [t_n, t_{n+1})$ with $S_{t_n} = 1$, $\{k_t, y_t\}$ satisfies (22) and (36). (ii) For all $t \in [t_n, t_{n+1})$ with $S_{t_n} = 0$, $\{k_t, y_t\}$ satisfies (22) and (37). (iii) For any k and any $y \in y_h^S(k)$, $y + D_{1,0}(k, y) \in y_l^S(k)$. For any k and any $y \in y_l^S(k)$, $y + D_{0,1}(k, y) \in y_h^S(k)$. (iv) $y_0 \in y_h^S(k_0)$ if $S_0 = 1$, and $y_0 \in y_l^S(k_0)$ if $S_0 = 0$.*

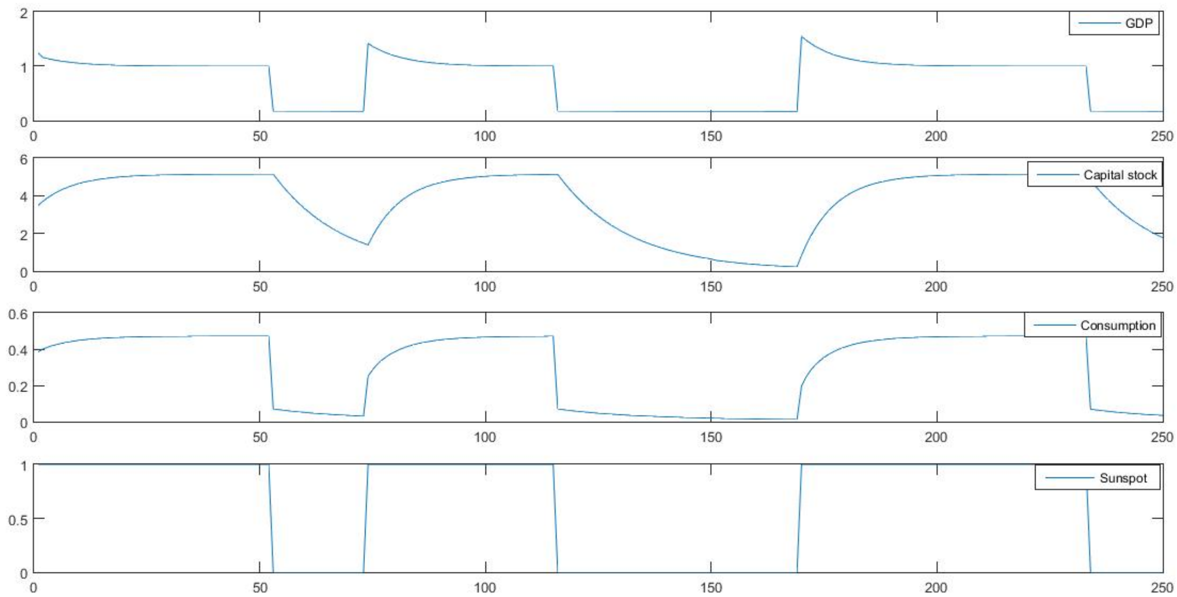

Figure 5: Simulation of a path-switching sunspot process, $\rho = 0.01, \delta = 0.025, s = 0.3, \gamma = 0.35, \mu = 0.25, \phi = 0.75, \bar{T} = 0.15$

Note that when $p = q = 0$ the solution of (36) and (22) is any equibrium path which converges to the high output steady state y_h, depicted in figure 3, if this steady state is a sink, or the saddle path towards the high output steady state y_h depicted in figure 4 if this steady state is a saddle. Similarly, when $p = q = 0$ the solution of (37) and (22) is the saddle path towards the low output steady state y_l, depicted in figures and 3 and 4. By continuity these functions exist for small values of p and q and solve respectively (36) and (22), and (37) and (22). In the Appendix we discuss the approximation used.

6.2 Illustrating the effects of an expectation shock

We now illustrate the behaviour of the model economy under global indeterminacy and sunspot shocks. In order to better illustrate the limits and perils of stabilization policy, we consider the case where the two steady states are both saddles and where the economy starts in the optimistic state, being therefore described by equations (36) and (22). However, our economy can be hit by a severe and persistent crisis triggered by a sudden loss in confidence, which brings it to the lower equilibrium trajectory. The next figure depicts such a numerical exercise where we assume $p = 0.02$ and $q = 0.04$. These values imply that the model economy will remain in the optimistic state with probability 0.666.

The economy starts in the optimistic state and remains there for 52 periods, reaching the high output steady-state. Then, in period 53, as S_t drops from 1 to 0, agents' expectations about future output become pessimistic and the economy jumps to the trajectory converging to the low output steady-state. We observe an immediate drop in output and consumption. However, as expected, the fall in capital is slower, which is a nice feature of our model. The model economy stays in recession for 17 periods, reaching the low output steady state. Then, as S_t jumps from 0 to 1 in period 61, agents become optimistic again, and the economy jumps to the saddle path converging to the high output steady state. The same pattern repeats itself two more times with jumping events at period 112 (recession), 168 (boom) and 243 (recession). Note that, since all these movements are generated by switches between trajectories converging to quite different long run output levels, the ups and downs we observe are considerably larger than fluctuations around one single trajectory, like the ones generated by exogenous productivity shocks or local sunspots in the case of an indeterminate steady state. Furthermore, output always overshoots when jumping from the lower to the upper saddle-path, and the longer the time spent in the low regime the higher the overshoot.[19] This is due to the relative slopes of the lower and the upper saddle-paths: the former is positively slopped while the latter has a negative slope (see figure 4).

6.3 The economic mechanism behind regime switching sunspots

Below, we describe the economic mechanism behind the emergence of regime switching sunspots in the case with two saddles. Note first that in the absence of distortions, i.e., without productive externalities ($\gamma = 0$) and without government ($\bar{T} = 0$ and $\phi = 0$), we recover the results of the classical Ramsey model: the steady state is unique and saddle-stable. Neither local nor global indeterminacy are possible, so that endogenous fluctuations are ruled out. Furthermore, consumption is a decreasing function of output.[20] When we introduce sufficiently strong productive externalities, but no taxes, the steady state is still unique but indeterminate (a sink). Local sunspots fluctuations are therefore possible. Also in this case consumption is increasing in income.[21] When we consider taxation ($\bar{T} > 0$ and $\phi \neq 0$), and even without externalities, we always have steady state multiplicity.[22] A low output steady state, where $[\alpha(1 + \varepsilon_z(y_l)) - 1] > 0$, ap-

[19] A similar pattern is obtained by Benhaib *et al.* (2016).

[20] Note that with an infinitely elastic labor supply the income effect is constant and equal to 1 and there is no substitution effect. Indeed using (9) and (10) we obtain $c_t = Bw_t$ so that $\frac{dc_t}{c_t} = \frac{dw_t}{w_t}$. Substituting now (15) in the previous expression we have $c_t = c(k_t, y_t) \equiv B(1-s)k_t^{\alpha/\beta} y_t^{(\beta-1)/\beta}$ so that equilibrium consumption is decreasing in y and increasing in k since $\beta = (1-s) < 1$.

[21] As before $\frac{\partial C_t}{\partial y_t} = \frac{(\beta-1)}{\beta} y_t^{-\frac{1}{\beta}}$, but now $\beta = (1-s)(1+\gamma) > 1$.

[22] The function $\varepsilon_H(y) = [\alpha(1 + \varepsilon_z(y)) - 1]$, which without any form of countercyclical taxation is always negative, now changes sign once. See Proposition 2.

pears and coexists with a high output steady state where $[\alpha(1+\varepsilon_z(y_l))-1]<0$. When the two steady states are saddles, we have $\varepsilon_z(y_h)<\frac{1-\beta}{\beta}<\frac{1-\alpha}{\alpha}$, while at the low output steady state $\varepsilon_z(y_l)>\frac{1-\alpha}{\alpha}>\frac{1-\beta}{\beta}$. This means that the function $[\beta(1+\varepsilon_z(y))-1]=\frac{\partial C_t}{\partial y_t}$ (see (17)) is positive for values of $y<y_s$, is zero at $y=y_s$ and becomes negative when $y>y_s$. We conclude, that for a given value of capital, consumption is a single peaked function of income. Therefore, for a given value of capital, there are two values of output on different sides of the singularity, $y_1<y_s$ and $y_2>y_s$ that sustain the same level of consumption, i.e., from (16) we have $z(y_1)y_1^{(\beta-1)/\beta}=z(y_2)y_2^{(\beta-1)/\beta}$. We know that when $\bar{T}>0$, provided ϕ is not too positive, $z(y)$ is increasing in y,[23] so that tax rates, $1-z(y)$, are decreasing in income (countercyclical), i.e., lower values of output are associated with higher tax rates.

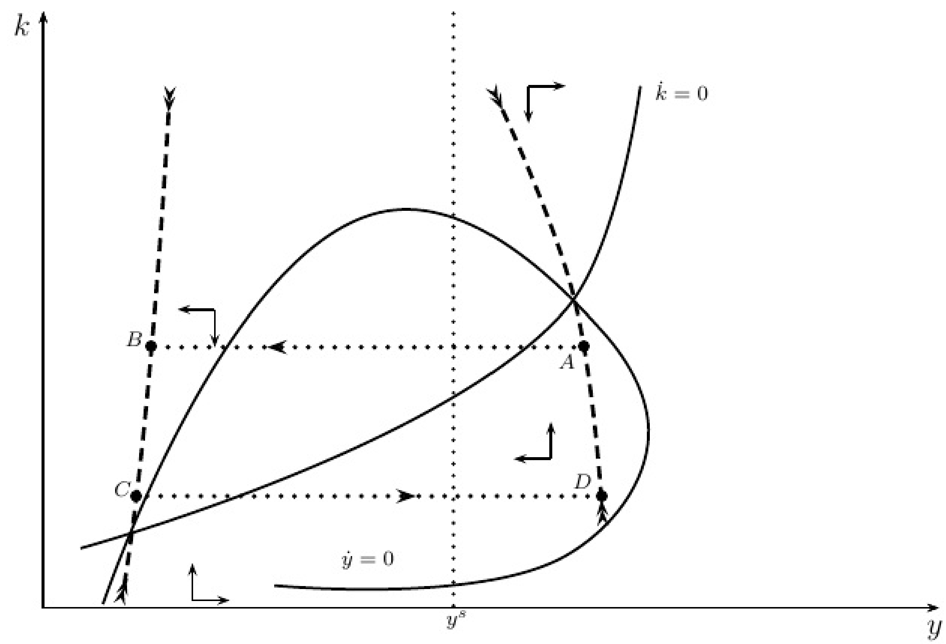

Figure 6: Regime-Switching Expectations.From point A to B: pessimistic expectation. From point C to D: optimistic expectation

Consider now the following. Departing from a situation where expectations are optimistic, so that the economy is on the saddle path converging to the high output steady state, we observe a sudden drop in confidence. Agents become pessimistic about the future of the economy and expect a simultaneous fall in consumption, capital and output. As along the saddle path converging to the high output steady state we have a negative

[23]Assuming $\beta<1$, we have $y_2^{(\beta-1)/\beta}<y_1^{(\beta-1)/\beta}$.

relation between capital and income, it is easy to see that for these expectations to be self-fulfilling, the economy has to switch to the saddle path on the left of the singularity, where the existing relation between capital and output is positive. Indeed, for the same value of capital, the economy jumps from point A to point B and starts moving downwards along the new saddle path, in the direction of the low output steady state. We observe therefore a simultaneous decrease in output and capital. As consumption increases with capital and, on this side of the singularity, increases with output, consumption also falls unambiguously. Expectations are therefore self-fulfilling. Consider now the situation where agents, while on the path converging to the low output steady state, become optimistic, expecting an increase in output, capital and consumption. Again, in order for the expectations to be self fulfilling, the economy must jump to the saddle path converging to the high output steady state. For the same level of capital the economy jumps from point C to point D, where consumption is identical. The economy then starts moving upwards along the (negatively sloped) saddle path on the right of the singularity, converging to the high output steady state. As discussed above, the initial jump in output is such that, along the higher output saddle path, output always exceeds its steady state value, i.e output overshoots. Moreover, as along the high output saddle path, capital increases and income decreases, since on this side of the singularity consumption decreases with y, we obtain an unambiguous increase in consumption. Therfore again expectations are self fulfilling.

7 Robustness

In this section, we assess through numerical exercises the robustness of our conclusions by relaxing the assumption of an identical tax rate for labor and capital income. We maintain the assumption that the fixed minimum level of tax revenue is given by $\bar{T} = \tau_y(y)y$ but now we assume that the government has two different tools that can be used to stabilize: a variable labor income tax τ_l and a variable capital income tax τ_k. These two tax instruments may differ in level and in their response to aggregate output, according the previously considered functional form. We thus get:

$$\tau_j(y) = \mu_j y^{\phi_j}, \quad j = k, l \tag{38}$$

The disposable income of households is therefore given now by $z_l(y)wl + z_k(y)rk$ with $z_j(y) = (1 - \tau_j(y) - \frac{\bar{T}}{y})$, $j = k, l$.

It is obvious that our conclusions on the existence and the multiplicity of steady states remain, since they rely only on the presence of $\bar{T}$ in the after-tax labor and capital income. We focus therefore on the local and global properties of the extended model.

We consider calibrated values of the parameters for a quarterly frequency. In particular, we set $(\rho, \delta, s) = (0.01, 0.025, 0.3)$. The first two values were chosen in order to target a 0.035 steady-state interest rate, while the value considered for the share of capital income in national income is standard in the macroeconomic literature. In addition, we set the size of the learning-by-doing externality at $\gamma = 0.35$, which falls in $(0, 0.44)$, the interval of estimated values for increasing return to scale.[24] With these values, without countercyclical tax rates, we do not get local inderminacy around the NSS. However, local indeterminacy around the NSS will emerge for any $\bar{T}$ higher than 0.055. We set therefore $\bar{T} = 0.15$. Regarding the tax rates on labor and capital income, we set $(\mu_k, \mu_l) = (0.2, 0.35)$ according to the estimates provided by Trabandt and Uhlig (2011) for countries in the European Union.[25]

In the following numerical exercises different sets of values for ϕ_l and ϕ_k were chosen to allow for different choices of government fiscal policy. We observe that the stabilizing power of the two tools is dramatically different. While the labor income tax rate can be used to successfully stabilize locally the economy, the same is not true for the capital income tax rate. Indeed, a value of $\phi_l > 0.345$, fully prevents the economy from stationary expectation-driven fluctuations, regardless of the value chosen for ϕ_k. Such conclusions confirm and complement the contribution of Guo (1999) that a progressive labor income tax (only) is stabilizing. We also observe that the likelihood of local indeterminacy increases as the capital income tax becomes more procyclical, since in this case a wider range of tax rates leads to local ineterminacy.

We now study global dynamics by choosing values of ϕ_l and ϕ_k corresponding to either a locally destabilizing or stabilizing fiscal policy. Figure 7 illustrates a case where the government sets $\phi_l = -0.75$ and $\phi_k = -0.25$, which leads to a sink-saddle configuration.

The first conclusion is that this figure is quite similar to figure 3, which supports the robustness of our results. The solid lines represent the $k-$nullcline and the $y-$nullcline, respectively in red and black. As in figure 3, the upper steady state (NSS) is locally indeterminate and therefore, for a given $k_{0,}$ there are an infinite number of initial values of output y that converge to this steady state. The dashed-dotted lines depict this kind of trajectories. We can observe that these equilibrium paths converge in a non-monotonic way, which implies therefore an endogenous propagation mechanism.

[24] See Basu and Fernald (1997) and Burnside *et al.* (1995) for a discussion about the size of increasing returns

[25] According to Trabandt and Uhlig (2011) in most countries in the European Union a quite high labor income tax rate coexists with a lower capital income tax rate. On the other hand, in North America, evidence points to the opposite: a low labor income tax and a rather high capital income tax rate.

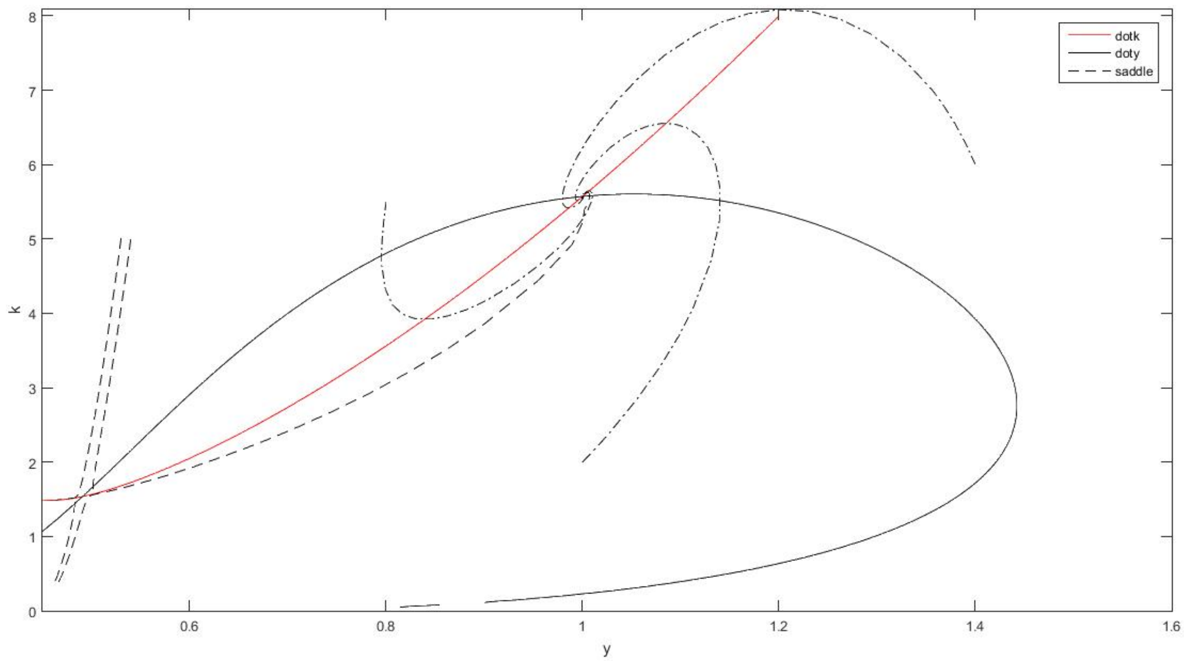

Figure 7: Sink-saddle configuration

We also plot two trajectories, in dashed lines, surrounding the nonlinear saddle-path that converges to the lower steady state. In contrast, this equilibrium path, for a given k_0, admits a unique initial value for y compatible to convergence to the lower steady state. However, as in the case with just a unique income tax rate we have global indeterminacy. For initial values of k there are different values of y compatible with convergence to the lower or to higher steady state.

In figure 8, we illustrate the case where the government sets $\phi_l = 0.75$ and $\phi_k = 0.25$. In this case, with sufficiently procyclical labor and capital income tax rates, and as in figure 4, both steady states are locally determinate (saddles). As above, the nullclines are represented by the solid lines, while the location of the two nonlinear saddle-paths is given by two surrounding divergent trajectories represented by a dashed line. We also plot the two saddle paths of the linearized version of the modified model in dotted lines and the singularity that occurs at $y = 0.768$. One easily observes that, as in the case with just a unique income tax rate depicted in figure 4, for a given initial value of the capital stock k_0, there are two initial values of output y, each located on a different equilibrium trajectory on different sides of the singularity, i.e. we have global indeterminacy. It follows that, also in this case one may construct deterministic cycles and/or regime switching sunspot equilibria between the two saddle paths, which validates our previous results.

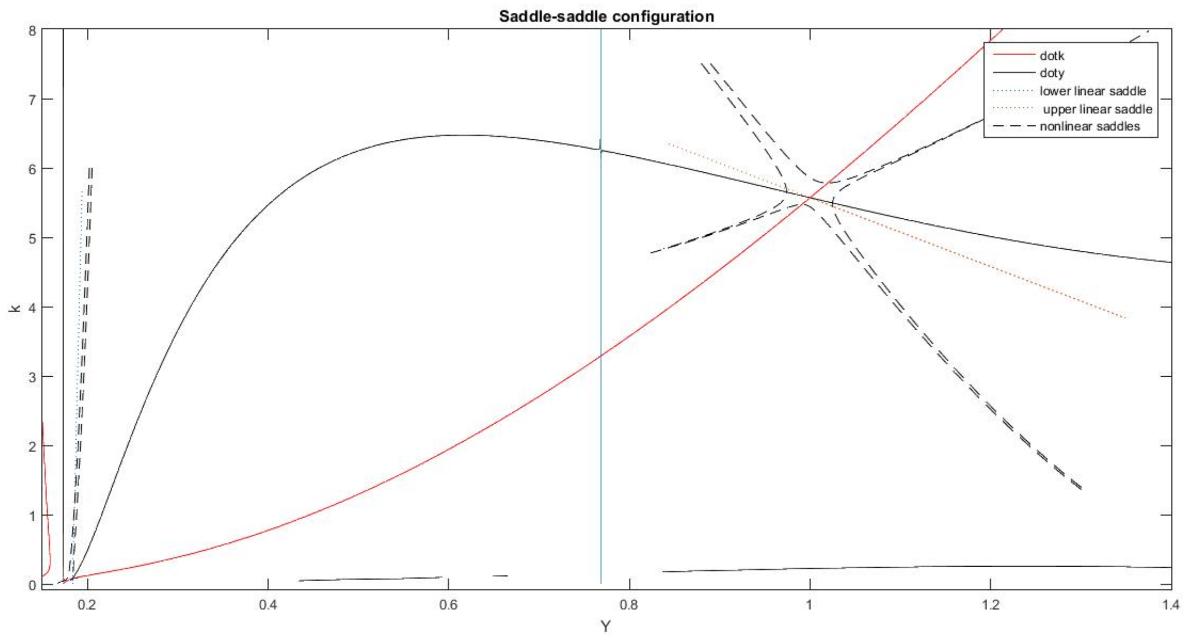

Figure 8: Saddle-saddle configuration

8 Concluding Comments

In this paper, we show that conventional stabilization policy recommendations are no longer valid in the presence of incompressible public expenditures, such as public safety, defense and general public services. Without such expenditures, procyclical tax rates are able to guarantee both local and global uniqueness of equilibrium, preventing expectation-driven fluctuations. In contrast, the need to raise a fixed amount of tax revenues in order to finance incompressible public expenditures, always generates global indeterminacy, due to the emergence of two steady states. We show that the low activity steady state is always saddle-path stable while the high activity one may be either a sink (locally indeterminate) or a saddle (locally determinate). A government, willing to eliminate local expectation-driven fluctuations around the high steady state, can do so by setting a procyclical income tax rate. But, as global indeterminacy persists, the economy remains exposed to large and persistent fluctuations based on a regime-switching sunspots process.

In this context, a government faces several trade-offs. The first is a "*welfare vs. stabilization*" trade-off. The only way to completely erradicate global indeterminacy and regime-switching fluctuations is to eliminate the incompressible property of expenditures associated with the basic functions of a State. In particular, these expenditures will have to follow the business cycle: increasing in a boom and decreasing in a recession. Of course, this option has severe political and social costs, especially in a recession, being therefore difficult to implement. The second trade-off has to do with the magnitude of the fluctuations. A government who wishes to maintain incompressible expenditures may chose to disregard and to endure "*small*" fluctuations around the high output (sink) steady state. Note that in this context regime-switching fluctuations are unlikely since this steady state acts as a global attractor. However, in the simulations performed, the existing multiple trajectories converging to the high output steady state were non-monotic and of long duration, which suggests non-negligible fluctuaction costs. Finally, a potential solution to simultaneously keep the incompressible expenditures while minimizing expectation-driven fluctuations is to successfully convince economic agents that the economy will remain in the high activity state. This requires a careful expectations' management which is uncertain and very difficult to implement.

We conclude that the existence of incompressible expenditures severely undermines the stabilization role of fiscal policy. However, our results were obtained using a stylized model, where the government balances its budget at each point in time. The consideration of public debt, breaking the link between incompressible expenditures and countercyclical tax rates, may attenuate some of the implications of incompressible public spending. Nevertheless, we conjecture that this problem remains in the long run, when considering a non-exploding public debt.

9 Appendix

9.1 Relative standard deviation of public spending for selected countries

country

relative standard deviations	US	Ger	Spa	Fra	Ita	Net
$\sigma_{\bar{G}}/\sigma_Y$	0.0964	0.1111	0.0483	0.1489	0.1134	0.0994
σ_G/σ_Y	0.4027	0.3664	0.4436	1.1139	0.2655	0.6860
$\sigma_{\bar{G}}/\sigma_G$	0.2393	0.3032	0.1089	0.1337	0.4269	0.1449
country	Aus	Pol	Fin	Swe	UK	Nor
$\sigma_{\bar{G}}/\sigma_Y$	0.0885	0.0873	0.1526	0.0961	0.0571	0.0455
σ_G/σ_Y	0.5786	0.4129	0.8607	0.4388	0.2490	0.3876
$\sigma_{\bar{G}}/\sigma_G$	0.1530	0.2115	0.1774	0.2191	0.2293	0.1174

9.2 Steady State Existence and Multiplicity

Existence and multiplicity are determined by the solutions of $H(y) = \bar{H}$. Note that we restrict $y \in (\underline{y}, \bar{y})$ with $\underline{y} > 0$ and $\bar{y} \in (\underline{y}, +\infty)$ to ensure that $z(y) = (1 - \mu y^{\phi} - \frac{\bar{T}}{y}) > 0$. In particular, due to the existence of the income tax $\tau_y = \frac{\bar{T}}{y}$, and independently of wether $\tau(y)$ is procyclical or countercyclical, there exists a $\underline{y} > 0$, such that $z(\underline{y}) = 0$. Similarly, with a procyclical $\tau(y)$, output must be bounded above so that a finite $\bar{y}$ such that $z(\bar{y}) = 0$ exists.[26] Note that when $\tau(y)$ is countercyclical, we have $\bar{y} = +\infty$. Below we consider separately the cases $\phi > 0$ and $\phi < 0$.

***a*) The case where** $\phi > 0$ Rewrite $H(y) = \bar{H}$ as:

$$(z(y)y)^{\alpha} = \bar{H}y \tag{39}$$

The left-hand side of (39) is lower than the right-hand side when $y = \underline{y}$ since $z(\underline{y}) = 0$. Similarly, $z(\bar{y}) = 0 < \bar{H}\bar{y}$. Hence, and since we already prove that a NSS exists, we have at least two stationary solutions when $\phi > 0$. Let us now show that there are exactly two solutions. While the right-hand side is linear in y, the first derivative of the left-hand side is given by:

$$\alpha(z(y)y)^{\alpha-1}\left(1 - \mu(1+\phi)y^{\phi}\right)$$

Notice that the first derivative may change sign only once. The second derivative is given by:

$$-\alpha(z(y)y)^{\alpha-2}\left[(1-\alpha)(1-\mu(1+\phi)y^{\phi})^2 + (1+\phi)\mu\phi y^{\phi} z(y)\right] \tag{40}$$

which is strictly negative so that the left-hand side of equation (39) is a concave function. Given that the right-hand side of this equation is linear in y, the two functions cross at most twice. Since we proved above that they also cross at least twice, it follows that there are exactly two solutions to this equation, and hence exactly two solutions to $H(y) = \bar{H}$, with $\phi > 0$.

Remark that when $\bar{T} = 0$, $\underline{y} = 0$ so that we obtain steady state uniqueness.

***b*) The case where** $\phi < 0$ Rewrite now $H(y) = \bar{H}$ as:

$$z(y)^{\alpha} = \bar{H}y^{1-\alpha} \tag{41}$$

When $y = \underline{y}$, the left-hand side of this equation is zero while it tends to unity when y tends to $+\infty$. In contrast, the right-hand side is positive and finite when $y = \underline{y}$ and

[26] For example for $\bar{T} = 0$ we have $\bar{y} = \left(\frac{1}{\mu}\right)^{\frac{1}{\phi}}$.

goes in infinity when y tends to infinity. As above, since we already prove that a NSS exists, we conclude that we have at least two solutions. The first derivative of the LHS is:

$$\alpha z(y)^{\alpha-1}\left[\frac{\bar{T}}{y^2}-\phi\mu y^{\phi-1}\right] \tag{42}$$

which is strictly positive with $\phi<0$. The second derivative is given by:

$$-\alpha z(y)^{\alpha-1}\left[(1-\alpha)\frac{\left(\frac{\bar{T}}{y^2}-\phi\mu y^{\phi-1}\right)^2}{1-\tau(y)-\frac{T}{y}}+\left(2\frac{\bar{T}}{y^3}+\mu\phi(\phi-1)y^{\phi-2}\right)\right] \tag{43}$$

This expression is negative for $\phi<0$. It is trivial to show that the right-hand side of (41) is also an increasing and concave function of y. Hence, equation (41) admits at most two solutions. Given that we also concluded that it has at least two solutions, it has exactly two solutions, and so has $H(y)=\bar{H}$ with $\phi<0$.

In this case when $\bar{T}=0$ a $\underline{y}>0$, such that $z(\underline{y})=0$ always exists, so that we still have two steady states.

All these arguments imply that $H(y)=\bar{H}$ has exactly two solutions. As $H(\underline{y})=H(\bar{y})=0<\bar{H}$, we conclude that $H(y)$ is single-peaked. Furthermore, differentiating $H(y)$, one can show that the lowest solution (i.e. the low output steady state y_L) is characterized by $1+\varepsilon_z(y_L)>\frac{1}{\alpha}$ while the high output steady state $y_H=1$ satisfies $1+\varepsilon_z(y_H)<\frac{1}{\alpha}$.

□

9.3 Matrix J

$$J=\begin{pmatrix}\frac{(\rho+\delta)\beta(1+\varepsilon_z)\frac{k}{y}-\alpha\left[(1+\varepsilon_z)z-\frac{\partial c(k,y)}{\partial y}\right]}{\beta(1+\varepsilon_z)-1} & \frac{-(\rho+\delta)\beta+\alpha\left[\delta+\frac{\partial c(k,y)}{\partial k}\right]}{\beta(1+\varepsilon_z)-1}\\ \\ (1+\varepsilon_z)z-\frac{\partial c(k,y)}{\partial y} & -(\delta+\frac{\partial c(k,y)}{\partial k})\end{pmatrix}$$

9.4 Derivation of the phase diagram

Consider equations (35). The $k-$nullcline satisfies $f_1(k_t,y_t)=0$ or equivalently:

$$\delta k_t=z(y_t)y_t-c(k_t,y_t) \tag{44}$$

The implicit solution $k=k_1(y)$ of the above relationship also satisfies the following relation (along the nullcline):

$$\frac{dk_1/k_1}{dy/y}=\frac{[\beta(1+\varepsilon_z(y))\delta k+c(k,y)]}{[\beta\delta k+\alpha]}>0 \tag{45}$$

Furthermore, we find that $f_1(0,y)>0$, $f_1(+\infty,y)<0$. It follows that for a fixed y, any given $k\in(0,k_1)$ implies $\dot{k}=f_1(k,y)>0$. See figures 1 and 2 where the we depict the $k-$ nullcline and the arrows that represent the vector field.

The $y-$nullcline satisfies $f_2(k,y)=0$ or:

$$s\left[c(k,y)-sz(y)y\right]-[\rho(1-s)+\delta(1-2s)]k(t)=0. \tag{46}$$

Let us first show that the relation between k and y derived from this expression, $k=k_2(y)$, may be multi-valued, i.e. for a fixed $y\in(\underline{y},\bar{y}))$, we may have zero, one or two values of k satisfying (46). Note that for a fixed $y\in(\underline{y},\bar{y})$, we have $\widetilde{f}_2(0)=f_2(0,y)<0$ and $\widetilde{f}_2(+\infty)=f_2(+\infty,y)<0$, since $c(k,y)$ is a concave function in k while the last term is linear in k. We also have:

$$\frac{\partial f_2(k,y)}{\partial k}=\frac{s^2}{(1-s)}\frac{c(k,y)}{k}-[(1-s)\rho+\delta(1-2s)]$$

Moreover, for $s<0.5$, as assumed along the paper, $\frac{c(k,y)}{k}=(1-s)Bz(y_t)y_t^{\frac{\beta-1}{\beta}}k_t^{\frac{\alpha-1}{\beta}}$ is strictly decreasing in k. Then, for a fixed $y\in(\underline{y},\bar{y})$, there is a critical value $\tilde{k}(y)$ such that $\frac{\partial f_2(k,y)}{\partial k}>(<)0$ if $k<(>)\tilde{k}(y)$, i.e. the function $\widetilde{f}_2(k)$ is first increasing and them decreasing in k. This implies that, for a given value of y, $\widetilde{f}_2(k)=0$ or equivalently equation (46) may have zero, one or two solutions in k satisfying $f_2(k,y)=0$. It follows that $k_2(y)$ is a two-valued function with an upper (lower) solution satisfying $\frac{\partial f_2(k,y)}{\partial k}<(>0)$. Notice now that, as $\frac{c}{k}$ evaluated at any steady state is independent of y, see (28), the sign of this derivative is identical for both steady states and is given by:

$$s[\rho+\delta(1-s)]-(1-s)[(1-s)\rho+\delta(1-2s)] \tag{47}$$

This means that both steady state are either on the upper or the lower solution $k_2(y)$. To simplify the exposition and without loss of generality, we will assume for the rest of this section that this expression is negative[27] which implies that both steady states are on the upper branch of $k_2(y)$.

We can now study the shape of the $y-$nullcline. We have that:

$$\begin{aligned}\frac{dk_2/k_2}{dy/y} &= \frac{sc(k,y)-\beta(1+\varepsilon_z(y))[\rho(1-s)+\delta(1-2s)]k}{\alpha\left[sc(k,y)-\frac{(1-s)}{s}[\rho(1-s)+\delta(1-2s)]k\right]} \\ &= \frac{s^2z(y)y-[\beta(1+\varepsilon_z(y))-1][\rho(1-s)+\delta(1-2s)]k}{\alpha\left\{s^2z(y)y-\frac{(1-2s)}{s}[\rho(1-s)+\delta(1-2s)]k\right\}}\end{aligned} \tag{48}$$

where the last equality has been derived using (46). It is easy to see that for $k=0$, i.e., at the intersection between the $y-$nullcline and the horizontal axis, the slope of

[27]For a standard parametrization $(\rho,\delta)=(0.01,0.025)$, this is satisfied for $s\in(0,0.39)$.

the $y-$nullcline is equal to $\frac{1}{\alpha} > 1$. Also, from (??), we can see that the slope of the $y-$nullcline will change sign at most two times and that the numerator is positive on the right-hand side of the singularity point y^s. Furthermore, when evaluated at the steady state, the slope of the nullcline is given by:

$$\frac{[s\,[\rho+\delta(1-s)]-(1-s)\,[(1-s)\rho+\delta(1-2s)]\,\alpha(1+\varepsilon_z(y_j))]}{\alpha\,[s\,[\rho+\delta(1-s)]-(1-s)\,[(1-s)\rho+\delta(1-2s)]]}, j=h,l \tag{49}$$

where by assumption the denominator is negative, see (47), so that the two steady states are located on the upper branch of the nullcline. Therefore, when the NSS is a saddle, i.e. located at the RHS of y^s, the slope of the $y-$nullcline is negative. This implies that the $y-$nullcline admits a maximum at a point $y^* < y^s$. We still need to characterize the slope around the lower steady state and around the upper steady state when it is a sink. Around the lower (upper) steady state, we have $(1+\varepsilon_z(yl)) > (<)\frac{1}{\alpha}$. Since by assumption we consider $\frac{s[\rho+\delta(1-s]}{(1-s)[(1-s)\rho+\delta(1-2s)]} < 1$, it follows that $\frac{dk}{dy} > 0$ around the lower steady state. In contrast, the sign of the numerator is left undetermined for the upper steady state when it is a sink i.e. it can be located on the increasing or decreasing part of the upper-solution of $k_2(y)$. Obviously, on the lower branch of the y-nullcline, the derivative has an oppositive sign. As a result, the y-nullcline is first increasing in y and then decreases until $\frac{s^2}{(1-s)}c(k,y) = (1-s)\rho+\delta(1-2s)]k$. It is therefore bended and goes back to the origin (without attaining it).

We now determine the directions of the arrows that represent the vector field of $\dot{y}_t$. Remember that our model exibits a singularity when $g(y^s) = \beta(1+\varepsilon_z(y^s)) - 1 = 0$, which in the space (y,k), defines a vertical line $y = y_s$. Of course, on different sides of the vertical line $y = y_s$ horizontal arrows point in opposite directions. Now consider a point (y_1, k_1) on the LHS of y_s and above the $y-$ nullcline. As we know that $\frac{\partial f_2(k,y)}{\partial k} = (1+\gamma)\left\{s^3 z(y)y - [\rho(1-s)+\delta(1-2s)](1-2s)k\right\}$ we know that moving from y_1 on the zero motion line, i.e. on $f_2(y_1,k) = 0$, vertically to (y_1,k_1), $f_2(y,k)$ is decreasing. Hence $\dot{y} > 0$ at (y_1,k_1), changing sign whenever, for the same k_1, we cross the $y-$nullcline or the $y = y_s$ line. The same reasoning applies to any fixed k on the LHS of y_s and above the $y-$nullcline. It follows that for any k on the LHS of y_s but below the the $y-$ nullcline $f_2(y,k)$ is decreasing, i.e., $\dot{y} < 0$, changing again sign whenever, for the same k, we cross the $y-$nullcline or the $y = y_s$ line. See figures 1 and 2.

It is also easy to show that when $k = 0$ the $y-$nullcline is located on the right of the $k-$nullcline as depicted in figures 1 and 2. Indeed, although $\lim_{k\to 0} c(k,y) = 0$, it is easy to see that k will tend to zero faster than $C(k,y)$. Therefore, rewriting *(44) and (46) respectively as:*

$$\begin{aligned} z(y_t)y_t &= c(k_t,y_t) + \delta k_t \\ z(y_t)y_t &= \frac{c(k_t,y_t)}{s} - \frac{[\rho(1-s)+\delta(1-2s)]k(t)}{s^2} \end{aligned}$$

when $k \to 0$, on the $\dot{k} = 0$ nullcline we have that $z(y)y = \lim_{k\to 0} c(k,y)$, while on the $\dot{y} = 0$ nullcline $z(y)y = \lim_{k\to 0} \frac{c(k,y)}{s} > \lim_{k\to 0} c(k,y)$. As $z(y)y$ is an increasing function of y, on the horizontal axis, the $y-$nullcline starts on the right hand side of the $k-$nullcline.

9.5 Obtaining the stable manifolds in the Markov switching rational expectation equilibrium with two saddles

We started by assuming p and q arbitrarily small (i.e. $p = q = 0$) and obtained the two saddle paths converging respectively to the high and the low output steady states. For simplicity we considered the saddle-path solutions of the linear approximations around the two steady-states of the dynamic system (21)-(22)[28]

$$\begin{aligned} y_t^j &= y_{ss}^j + (k_t - k_{ss}^j)\eta^j \qquad j = h,l \\ k_{t+1}^j &= k_{ss}^j + e^{\lambda^j}(k_t - k_{ss}^j) \qquad j = h,l \end{aligned} \tag{50}$$

where y_t^j, k_t^j are respectively the output and the capital stock on the saddle-path $j = h,l$, y_{ss}^j, k_{ss}^j the steady states, λ^j the stable eigenvalue associated to the saddle-path j and η^j the ratio of the elements of the eigenvector associated to the stable eigenvalue of the saddle-path j.

Given that the capital stock is predetermined and that for a given k_t two equilibrium values of output, $y^h(k_t, y_{ss}^h, k_{ss}^h)$ and $y^l(k_t, y_{ss}^l, k_{ss}^l)$, are feasible, observed output is obtained by randomizing across both saddle-paths using the sunspot variables S_t:

$$y_t = S_t y^h(k_t, y_{ss}^h, k_{ss}^h) + (1 - S_t) y^l(k_{t-1}, y_{ss}^l, k_{ss}^l) \tag{51}$$

where $S_t = (1-p)S_{t-1} + q(1 - S_{t-1})$. The realization of observed output in turn determines which saddle-path we are in. Hence, the capital stock $k_t(k_{t-1}, k_{ss}^s)$ is determined accordingly using (50).

□

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[28] Note that this system is obtained from:

$$\begin{aligned} y_t^j &= y_{ss}^j + e^{\lambda^j t}(k_0 - k_{ss}^j)\eta^j \\ k_t^j &= k_{ss}^j + e^{\lambda^j t}(k_0 - k_{ss}^j) \end{aligned}$$

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