University Competition: An Application to Brazil

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Abstract

We extend the model in Fernandez and Gali (1999) and Fernandez (2008) to allow for strategic interaction between a high quality public university and a lower quality private university. Our aim is to design a framework that reproduces the main characteristics of the Brazilian higher education system in order to analyze the different policies thereby proposed to increase access.

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1 Introduction

As many developing countries, Brazil has recently experienced a surge in the number of individuals enrolled in higher education. According to INEP,[1] the number of students enrolled in higher education went from 3.8 million in 2003 to 7.3 million in 2013. This is not only a consequence of the increased number of students completing secondary education, but also the result of several public policies meant to increase access to university education.

Similarly to most countries, Brazil's higher education system is characterized by the existence of both public and private universities. Around 25% of students are enrolled in public universities, which are considered to be of higher quality. Some of these universities apply entrance exams, which can be very competitive, or use the ENEM (end of high school national test) as a selection mechanism. Public universities do not charge tuition fees.

Private universities in Brazil are mostly characterized by lower education quality, in part because they tend to attract weaker students. While they may use ENEM or other entrance exams, the requirements needed for access are typically quite low. These universities charge tuition fees.

Since public universities require higher scores for admission, they tend to attract students coming from more privileged backgrounds, who have completed high school in a private institution. Until recently, students from poor backgrounds were practically excluded from university attendance since they could neither afford to attend high school at a private institution or pay tuition fees at private universities. Therefore, the system was

[1] *Instituto Nacional de Estudos e Pesquisas Educacionais Anisio Teixeira*, National Institute for Education Studies and Research Anisio Teixeira, www.inep.gov.br, a federal institution related to the Ministry of Education

seen as unfair and several policies have been implemented to increase participation in the higher education system, such as PROUNI and FIES.

PROUNI provides scholarships in private universities for students coming from public high schools and low income households who had a sufficiently high grade at ENEM. In 2015, 213,113 scholarships were granted.

FIES is a low-interest rate loan program granted to around 20% of higher education students. Until recently, FIES did not require a minimum grade at ENEM, but there was a requirement in terms of household income to participate in the program.[2]

Finally, most public universities nowadays adopt some type of affirmative action policy. These programs typically target students from public high schools (a proxy for income) and/or from disadvantaged racial groups. While most of the programs are based on quotas, some state universities adopted a bonus system where targeted students receive additional points at the entrance exam.

These programs may potentially affect the higher education market partition between public and private universities. However, little is known about the interaction between the public and private higher education system, as it has been under-exploited in the literature (Del Rey, 2009). As higher education systems worldwide are typically characterized by the presence of public and private universities and government intervention in the form of regulations, scholarships or loans, the lessons learnt from the Brazilian case can apply to other settings as well.

In order to analyze the impact of such policies in the Brazilian higher education market, we extend the model in Fernandez and Gali (1999) and Fernandez (2008) to allow for

[2] In 2015, the government is reviewing the eligibility rules and has announced a mininum grade at ENEM to participate in the program.

strategic interaction between public and private universities.

The assumptions in our model are meant to reproduce the main characteristics of the Brazilian higher education system. We consider the existence of a high quality public university and a low quality private university. We start by assuming that these quality levels, which may be thought of as reputation, are exogenous. There is a continuum of students characterized by an ability level and an income endowment, assumed to be unformly distributed and independent.

All universities can in principle charge tuition fees and/or organize an admission exam to select all students that obtain at least a given score. To comply with the Brazilian case, we assume also that, initially, the public university is more selective and charges no fees. The public university has fixed capacity and is financed by public funds. The private university has no fixed capacity and receives no public funding. The private university is for profit and wants to maximize revenue. Therefore, it will not use selective exams, because they limit enrolments without generating revenue. We later depart from this situation and check ex-post whether it constitutes an equilibrium by verifying whether the public and private universities have incentives to set tuition fees and organize admission exams, respectively.

The admission exam technology transforms ability and expenditures into a score. Unlike Del Rey (2009) and Romero and del Rey (2004) we assume that this technology is not able to perfectly reveal the individual ability. In contrast, and like in Fernandez and Gali (1999), the expenditure required to pass the exam is decreasing in ability and increasing in the minimum required score. As a first aproximation, we do not consider peer group effects (see Cremer and Maldonado (2013) for a model of mixed oligopoly in education with peer effects).

Individuals obtain utility from attending university, which depends on the human capital obtained, tuition fees paid and expenditures made in order to pass the admission exam. In the absence of perfect capital markets, the individual is subject to a revenue constraint, and can only afford university provided that her initial income endowment can pay for tuition fees or entrance exam related expenditures. Thus, individuals have in principle three choices: attend public or private university, or no higher education. We can define the ability thresholds that separate individuals according to their preferred option. For both universities, effective demand is simultaneously determined by willingness to pay and ability to pay the costs required to be admitted.

Under these conditions, we are able to distinguish and characterize two cases. In the first case (that encompasses cases A and B below), all individuals prefer to go to the public university that acts as a market leader. The private university can only exist if it is able to charge a sufficiently low tuition fee. If it exists, it only attracts those students who could not be admitted at the public university. This case is more likely to hold when the quality of the private university is relatively low with respect to the quality of public university and with respect to the fees required at the private institution. Otherwise (Case C), some students will prefer to go to the public university, while other students would rather go to private universities. This happens when the public university is more selective, so it may be optimal for some lower ability students to pay for tuition fees rather than paying the cost related to the admission exam.

This preliminary note presents the benchmark model and shows some of the tradeoffs involved in the public university's decision related to its selectivity and the private university's choice of tuition fees. Next, the model will be calibrated to account for important elements of the Brazilian higher education system. Then we plan to simulate the different policies to investigate its effects on the market partition, as well as selectivity level and tuition fees charged by public and private universities, respectively.

2 The model

In this section, we adapt the framework by Fernandez (2008) to a context where the education market is characterized by a high quality university θ_H and a low quality private university θ_L.

We start by assuming that the quality levels θ are exogenous. There is a continuum of students characterized by an ability level $a \in [0, 1]$ and an income endowment $w \in [0, \bar{w}]$. The joint distribution of ability and wealth is given by $f(a, w)$ and the cumulative distribution $F(a', w') = \int_0^{a'} \int_0^{w'} f(a, w) dw da$. We assume that ability and wealth are uniformly distributed and independent.

All universities can in principle charge tuition fees, p_j, and/or organize an admission exam to select all students that obtain at least score v. To comply with the Brazilian case, we assume also that, initially, the public university is more selective and charges no fees. The public university has capacity κ and is financed by public funds. The private university has no fixed capacity and receives no public funding. The private university is for profit and wants to maximize revenue. Therefore, it will not use selective exams, because they limit enrolments without generating revenue. Thus, we assume initially that $p_H = 0$ and $v_L = 0$. We depart from this situation and check ex-post whether it constitutes an equilibrium by verifying whether the public and private universities have incentives to increase p_H and v_L, respectively.

The admission exam function $V(a,e) = v$ (with $V_a \geq 0$, $V_e \geq 0$, $V_{ae} \geq 0$) transforms ability and expenditures e into a score v. Thus, the expenditure level required by an individual of ability a to obtain score $V(a,e)$ is implicitly defined by $V(a, e(v,a)) = v$. As Fernandez (2008) and Fernandez and Gali (1999), we work with the dual of $V(a, e(v,a))$, $E(a,v)$. We also assume that the expenditure required to pass the exam is decreasing in ability and increasing in the minimum score, i.e., $E_a(a,v) < 0$ and $E_v(a,v) > 0$. The marginal expenditure required to pass a given score is decreasing in ability, i.e., $E_{av}(a,v) \leq 0$. In order to simplify our presentation of results, we further assume that $E(1,v) = 0 \quad \forall v$ and $E(0,v) \to \infty \quad \forall v$, that is, the most able individual incurs no cost to be admitted for any score required and the least able individual cannot be admitted even with high investment. The slope of $E(a,v)$ in the (w,a) space is negative and decreasing in v : the more selective the university the steeper the function.

The utility of individual i attending university $j \in [H, L]$ is given by:

$$U_{ij} = w_i + h(a_i, \theta_j) - p_j - E(a_i, v_j), \tag{1}$$

where human capital $h(a_i, \theta_j)$ is a function of individual utility a_i and university quality θ_j; w_i is the individual's income endowment; $p_j \geq 0$ is the tuition fee and $E(a_i, v_j)$ are the resources spent by an individual of ability a_i to pass the admission test at university j. We assume that $h_a(a_i, \theta_j) > 0$ and $h_{aa}(a_i, \theta_j) \leq 0$, that is, human capital in increasing in the individual's ability at a non-increasing rate. We also assume that there is complementarity between ability and university quality, i.e., $h_{a\theta}(a_i, \theta_j) \geq 0$. In the absence of perfect capital markets, the individual is subject to a revenue constraint, and can only afford university

j provided that:

$$w_i \geq p_j + E(a_i, v_j) \tag{2}$$

Let $\hat{a}$ be the ability of the individual indifferent between the public and the private institution:

$$h(\hat{a}, \theta_H) - h(\hat{a}, \theta_L) = E(\hat{a}, v_H) + p_H - E(\hat{a}, v_L) - p_L. \tag{3}$$

In turn, let a_{0L} be the individual indifferent between the private lower quality university and not going to university and obtaining 0 human capital:

$$h(a_{0L}, \theta_L) = E(a_{0L}, v_L) + p_L. \tag{4}$$

Finally, a_{0H} will be the individual indifferent between no education and high quality education:

$$h(a_{0H}, \theta_H) = E(a_{0H}, v_H) + p_H. \tag{5}$$

If $v_L = 0$ and $p_H = 0$, these conditions reduce to:

$$h(a_{0L}, \theta_L) = p_L$$

$$h(a_{0H}, \theta_H) = E(a_{0H}, v_H)$$

3 Market partitions

Effective demand is simultaneously determined by willingness to pay and ability to pay the costs required to be admitted. With respect to willingness to pay, individual rationality implies that, either $\hat{a} < a_{0H} < a_{0L}$ or $a_{0L} < a_{0H} < \hat{a}$ (see Appendix 1). In the former case, it does not matter, for the configuration of the market partition, whether $\hat{a} \in (0, 1)$ or not (let us call this Case A). If $a_{0L} < a_{0H} < \hat{a}$, the corner solution $\hat{a} < 0$ implies that all the student body prefers to attend the public university, but only some prefer to attend the private if they do not get admitted at the public (let us call this Case B).[3] If $\hat{a} \in (0, 1)$, higher ability individuals prefer the public over the private university and if not admitted, some prefer the private rather than not studying (let us call this case C). Ability to pay depends on the choice of p and v by the university. Willingness and ability to pay determine enrollments.

In order to define enrolments at the private university and thus characterize full market partitions, it will be useful to let α be the level of a such that $p_H + E(\alpha, v_H) = p_L + E(\alpha, v_L)$. Graphically, this is the point where the revenue constraints limiting access to the two universities cross. Our assumption that $E(1, v) = 0, \forall v$ implies that $\alpha < 1$. Also, let β be the level of a such that $\bar{w} = E(\beta, v_H)$, with $\beta < 1$ given that $E(0, v) \to \infty, \forall v$.

3.1 Case A: $\hat{a} < a_{0H} < a_{0L}$

The preferred choice of individuals with $a > a_{0H}$ is the public university and students to the left of a_{0H} prefer not to enrol at any university. Only those to the right of a_{0L} would consider enrolling in the private university if they were not admitted to the public university.

The public university chooses v_H to fill capacity, κ. Enrolment at the public institution is given by:

$$\begin{cases} \int_{a_{0H}}^{1} \int_{E(a,v_H)}^{\bar{w}} f(w,a)dwda = \kappa \text{ if } \beta \leq a_{0H} \\ \int_{\beta}^{1} \int_{E(a,v_H)}^{\bar{w}} f(w,a)dwda = \kappa \text{ if } \beta > a_{0H} \end{cases} \tag{6}$$

In Figure 1, the first case would correspond to β_1 and the second to either β_2 or β_3. Given

[3] Note that we do not consider the corner $\hat{a} > 1$ in which all students prefer to attend the private university.

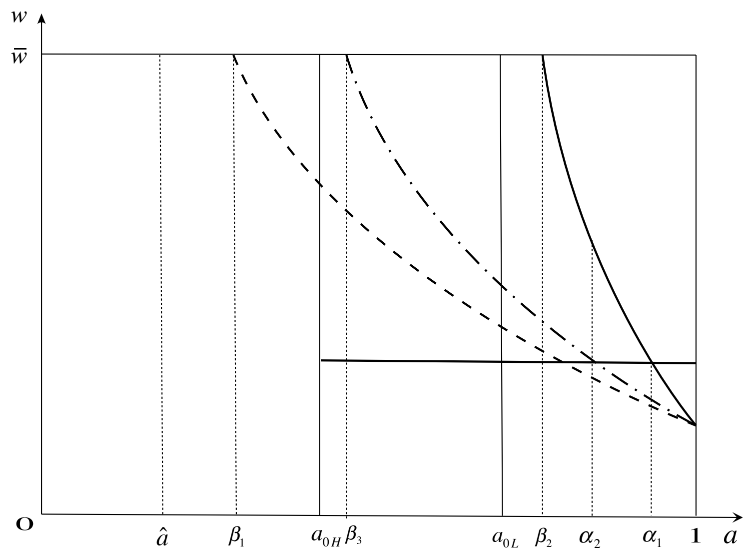

Figure 1: Case A: Market Partition

our assumption on $f(a, w)$, enrollment can be rewritten as:

$$\begin{cases} \frac{1}{\bar{w}} \int_{a_{0H}}^{1} (\bar{w} - E(a, v_H))\, da = \kappa \text{ if } \beta \leq a_{0H} \\ \frac{1}{\bar{w}} \int_{\beta}^{1} (\bar{w} - E(a, v_H))\, da = \kappa \text{ otherwise} \end{cases}$$

An exogenous increase in capacity leads the public university to lower its standard:

- If $\beta \leq a_{0H}$, differentiate totally $\int_{a_{0H}}^{1} \int_{E(a,v_H)}^{\bar{w}} f(w, a) dw da = \kappa$ with respect to v_H and κ to obtain

$$\left.\frac{dv_H}{d\kappa}\right|_{\beta \leq a_{0H}} = \frac{-1}{\frac{da_{0H}}{dv_H} (\bar{w} - E(a_{0H}, v_H)) + \int_{a_{0H}}^{1} E_v(a, v_H) da} < 0$$

- If $\beta > a_{0H}$, differentiate totally $\int_{\beta}^{1} \int_{E(a,v_H)}^{\bar{w}} f(w, a) dw da = \kappa$

$$\left.\frac{dv_H}{d\kappa}\right|_{\beta > a_{0H}} = \frac{-1}{\int_{\beta}^{1} E_v(a, v_H) da} < 0$$

Note that the choice of v_H by the public university is not influenced by the price charged by the private university: $\frac{dv_H}{dp_L} = 0$. This is due to the fact that all individuals prefer the public to the private university in this setting. Therefore, the public university acts as a first mover and the private university only exists if students not admitted to the public university with $a > a_{0L}$ are able to afford tuition fees, as will be shown below.

Indeed, if $\alpha < a_{0L}$ the private university **enrols no students**, since only those with $a > a_{0L}$ prefer to enrol at the private institution rather than getting no higher education. If $a_{0L} < \alpha < 1$ (as with α_1, α_2 in Figure 1) private university enrolments are given by:

$$\begin{cases} \int_{a_{0L}}^{\alpha} \int_{p_L}^{E(a,v_H)} f(w,a)dwda \text{ if } \beta \leq a_{0L} \\ \int_{a_{0L}}^{\beta} \int_{p_L}^{\bar{w}} f(w,a)dwda + \int_{\beta}^{\alpha} \int_{p_L}^{E(a,v_H)} f(w,a)dwda \text{ if } \beta > a_{0L} \end{cases} \tag{7}$$

In Figure 1, the former case is depicted by β_3 and the latter by β_2.

The private university chooses p_L to maximize $p \times D_L$. We start by calculating the impact of a change in p_L on a_{0L} and α. Applying the implicit function theorem, we obtain:

$$\begin{aligned} \frac{da_{0L}}{dp_L} &= -\frac{-1}{h_a(a_{0L},\theta_L)} > 0, \\ \frac{d^2a_{0L}}{dp_L^2} &= -\frac{h_{aa}(a_{0L},\theta_L)}{(h_a(a_{0L},\theta_L))^2} > 0, \\ \frac{d\alpha}{dp_L} &= -\frac{-1}{E_a(\alpha,v_H)} < 0, \text{and} \\ \frac{d\beta}{dp_L} &= 0 \end{aligned}$$

Similarly, we look at the impact of marginal changes in v_H on the different thresholds

$$\begin{aligned} \frac{d\alpha}{dv_H} &= -\frac{E_v(\alpha,v_H)}{E_a(\alpha,v_H)} > 0, \\ \frac{d\beta}{dv_H} &= -\frac{E_v(\beta,v_H)}{E_a(\beta,v_H)} > 0 \\ \frac{da_{0L}}{dv_H} &= 0 \end{aligned}$$

An increase in tuition fees charged by the private university decreases the proportion of individuals who prefer the private university to getting no education. It also decreases the ability level of the last student enrolled in the private university, α. Thus, an increase in p_L may eventually lead to $\alpha < a_{0L}$, a situation in which the private university enrolls no students as discussed above.

Assumption 1. *$\alpha > a_{0L}$ and $\beta \leq a_{0L}$ in Case A.*

Under Assumption 1, we know check the conditions for a maximum in the private's university problem:

$$\max_{p_L} p_L \frac{1}{\bar{w}} \int_{a_{0L}}^{\alpha} \left(E(a, v_H) - p_L\right) da.$$

The necessary condition for a maximum is given by:

$$\underbrace{\int_{a_{0L}}^{\alpha} \left(E(a, v_H) - p_L\right) da}_{+} \underbrace{- p_L \frac{da_{0L}}{dp_L} \left(E(a_{0L}, v_H) - p_L\right)}_{-} \underbrace{- p_L \left(\alpha - a_{0L}\right)}_{-} = 0$$

The condition above is sufficient if and only if:

$$\underbrace{-2 \frac{da_{0L}}{dp_L} \left(E(a_{0L}, v_H) - p_L\right)}_{-} \underbrace{- \left(p_L + 1\right)\left(\alpha - a_{0L}\right)}_{-}$$

$$\underbrace{- p_L \frac{d^2 a_{0L}}{dp_L^2} \left(E(a_{0L}, v_H) - p_L\right)}_{-} \underbrace{- p_L \frac{da_{0L}}{dp_L} \left(E_a \frac{da_{0L}}{dp_L} - 1\right)}_{+} \underbrace{- p_L \left(\frac{d\alpha}{dp_L} - \frac{da_{0L}}{dp_L}\right)}_{+} < 0$$

How is the private university influenced by the public's decisions? How does p_L change with v_H? Assuming that the second-order condition holds, we can apply the implicit function theorem to foc(p_L) (and use again $E(\alpha, v_H) - p_L = 0$) to obtain:

$$\frac{dp_L}{dv_H} = -\frac{\underbrace{\int_{a_{0L}}^{\alpha} E_v(a, v_H)da}_{+} \underbrace{-p_L \frac{da_{0L}}{dp_L} E_v(a_{0L}, v_H)}_{-} \underbrace{-p_L \frac{d\alpha}{dv_H}}_{-}}{SOC < 0}$$

An increase in the selectivity of the public university has an ambiguous effect on the tuition fee charged by the private university. On the one hand, an increase in v_H increases the relative price of the public university, leaving some room to the private university to increase tuition fees without loosing students. On the other hand, an increase in tuition fees would lead some of its students to prefer no education and would reduce the proportion of relatively high ability students choosing the private university.

Assumption 2. *$\alpha > a_{0L}$ and $\beta > a_{0L}$ in Case A.*

Under assumption 1 the maximization problem of the private university becomes:

$$\max_{p_L} \frac{p_L}{\bar{w}} \left((\beta - a_{0L}) (\bar{w} - p_L) + \int_{\beta}^{\alpha} (E(a, v_H) - p_L) \, da \right)$$

The first-order condition is given by:

$$(\beta - a_{0L}) (\bar{w} - p_L) + \int_{\beta}^{\alpha} (E(a, v_H) - p_L) \, da - p_L (\beta - a_{0L}) - p_L \frac{da_{0L}}{dp_L} (\bar{w} - p_L)$$

$$-p_L(\alpha - \beta) = 0$$

The second order condition is then

$$-2\frac{da_{0L}}{dp_L} (\bar{w} - p_L) - 2 (\beta - a_{0L}) - 2(\alpha - \beta) + 2p_L \frac{da_{0L}}{dp_L}$$

$$-p_L \frac{d^2 a_{0L}}{dp_L^2} (\bar{w} - p_L) - p_L \frac{d\alpha}{dp_L} < 0$$

As before we are interested in the effect that raising v_H may have in the private university fee:

$$\frac{dp_L}{dv_H} = -\frac{\frac{d\alpha}{dv_H}\left(E(\alpha, v_H) - 2p_L\right) + \int_\beta^\alpha E_{v_H}(a, v_H)da}{soc < 0}$$

Once again, the sign of $\frac{dp_L}{dv_H}$ is ambiguous. *The effect will however be positive provided that*

$$E(\alpha, v_H)\frac{d\alpha}{dv_H} + \int_\beta^\alpha E_v(a, v_H)da > 2p_L\frac{d\alpha}{dv_H}$$

Summing up, Case A is characterised by a situation where the public university enrols students who do not consider in general the private university as an alternative. Only some highly talented individuals prefer to enrol at the private university rather than remain uneducated. The private tuition fee cannot be too large in this case, since those highly talented students will only enrol at the private university provided that the expenses that they have done in order to prepare for admission at the public university have not been enough, i.e. they are not the wealthiest.

3.2 Case B: $a_{0L} < a_{0H} < \hat{a} < 0$

We now turn to the case where all individuals prefer to go to the public university, but if not admitted, they would rather go the private university than receive no higher education. We refer to this case as Case B (see Figure 3.2). As in Case A, the public university acts as a leader, requiring a score v_H for admission. Then, all students not admitted that could afford tuition fees would enroll in the private university.

As before, the public university chooses v_h in order to fill capacity. Enrollment at the public university is given by:

$$\int_\beta^1 \int_{E(a,v_H)}^{\bar{w}} f(w, a)dwda = \kappa$$

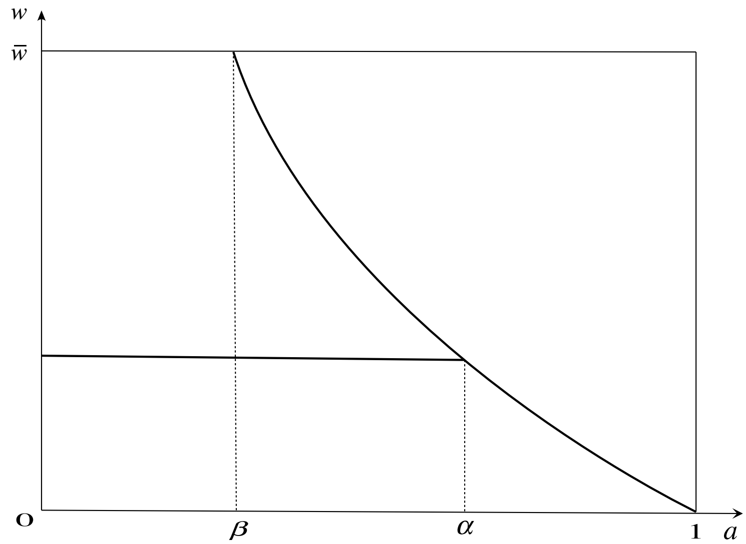

Figure 2: Case B: Market Partition

The private university then chooses p_L in order to maximize revenue. Its maximization problem is given by:

$$\max_{p_L} p_L \frac{1}{\bar{w}} \left[\beta \left(\bar{w} - p_L \right) + \int_{\beta}^{\alpha} \left(E(a, v_H) - p_L \right) da \right] \tag{8}$$

The first-order condition is given by:

$$\frac{1}{\bar{w}} \left[\beta \left(\bar{w} - p_L \right) + \int_{\beta}^{\alpha} \left(E(a, v_H) - p_L \right) da \right] - \frac{p_L}{\bar{w}} \alpha = 0$$

The second-order condition is always satisfied in this case (it reduces to $-2\alpha < 0$).

The impact of a change in v_H on the choice of p_L continues to be ambiguous in theory:

$$\frac{dp_L}{dv_H} = -\frac{\int_{\beta}^{\alpha} E_v(a, v_H) da - \frac{d\alpha}{dv_h}}{-2\alpha}$$

On the one hand, an increase in public university's selectivity would lead more students to choose the private university, leaving some room for tuition fees increases . On the other hand, it would enable the private university to attract relatively poor higher ability individuals who would attend university if fees were not so high. Therefore, the choice of p_L following an increase in v_H would balance these two effects.

3.3 Case C: $a_{0L} < a_{0H} < \hat{a}$

This case is characterized by a situation where some students, of higher ability, prefer the public university while others prefer the lower quality private institution. This may be due to the fact that, for higher ability individuals, it is less expensive to undergo the costs required to pass the admission exam at the public university. For lower ability individuals, paying the private university tuition fee can be a better option.

Differently from the other cases, there is a strategic interaction between the public and the private university in this case. Indeed, enrollment at the public university is affected by the tuition fee charged by the private university, as some individuals are indifferent between the public and the private university. Thus, we study the choice of v_h for a given p_L by the public university and the choice of p_L for a given v_H by the private university. At equilibrium, these choices must be compatible.

As depicted in Figure 3, Case C embodies three possibilities. Public university enrolments are given by:

$$\begin{aligned} \int_{\hat{a}}^{1} \int_{E(a,v_H)}^{\bar{w}} f(w,a)dwda &= \kappa \text{ if } \beta < \hat{a} \\ \int_{\beta}^{1} \int_{E(a,v_H)}^{\bar{w}} f(w,a)dwda &= \kappa \text{ if } \beta \geq \hat{a} \end{aligned}$$

Assumption 3. $\beta < \alpha < \hat{a}$ *in Case C.*

This ordering of thresholds implies that effective demand at the payoff of the private

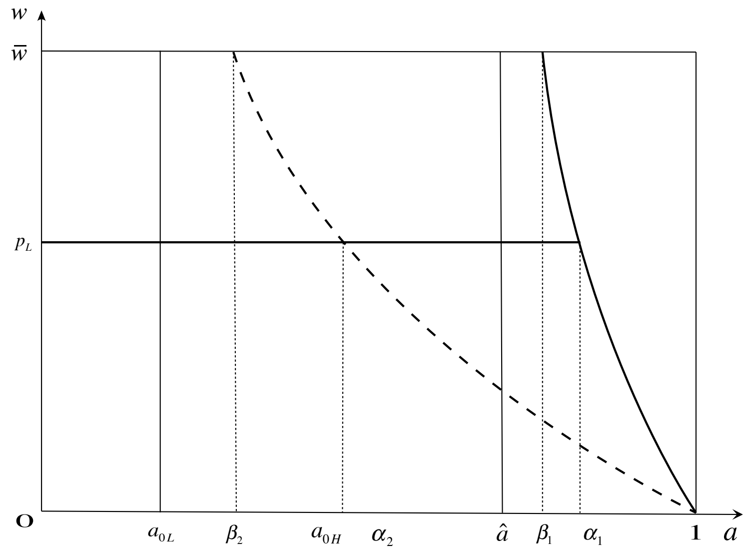

Figure 3: Case C: Market Partition

university will be given by:

$$\begin{cases} p_L \int_{a_{0L}}^{\hat{a}} \int_{p_L}^{\bar{w}} f(w,a)dwda \text{ if } \alpha < \hat{a} \\ p_L \int_{a_{0L}}^{\hat{a}} \int_{p_L}^{\bar{w}} f(w,a)dwda + p_L \int_{\hat{a}}^{\alpha} \int_{p_L}^{E(a,v_H)} f(w,a)dwda \text{ if } 1 > \alpha \geq \hat{a} \end{cases}$$

If $\alpha < \hat{a}$, the optimal fee for the private university is given by

$$\int_{a_{0L}}^{\hat{a}} \int_{p_L}^{\bar{w}} f(w,a)dwda + p_L \left(\frac{d\hat{a}}{dp_L} - \frac{da_{0L}}{dp_L} \right) (\bar{w} - p_L) - p_L (\hat{a} - a_{0L}) = 0$$

where

$$\frac{d\hat{a}}{dp_L} = \frac{-1}{h_a(\hat{a},\theta_H) - h_a(\hat{a},\theta_L) - E_a(\hat{a},v_H)} < 0,$$

since we assumed that ability and university quality are complements. The second order condition writes:

$$\left(\frac{d^2\hat{a}}{dp_L^2} - \frac{d^2 a_{0L}}{dp_L^2} \right) (\bar{w} - p_L) - 2 \left(\frac{d\hat{a}}{dp_L} - \frac{da_{0L}}{dp_L} \right) < 0$$

As for the effect of v_H on p_L :

$$\frac{dp_L}{dv_H} = -\frac{\frac{d\hat{a}}{dv_H}\left(\bar{w}-p_L\right)+p_L\frac{d^2\hat{a}}{dp_L dv_H}\left(\bar{w}-p_L\right)-p_L\frac{d\hat{a}}{dv_H}}{soc}$$

is ambiguous since:

$$\begin{aligned}\frac{d^2\hat{a}}{dp_L dv_H} &= \frac{-E_{av}(\hat{a},v_H)}{\left(h_a(\hat{a},\theta_H)-h_a(\hat{a},\theta_L)-E_a(\hat{a},v_H)\right)^2} > 0, \text{and} \\ \frac{d\hat{a}}{dv_H} &= -\frac{-E_v(\hat{a},v_H)}{h_a(\hat{a},\theta_H)-h_a(\hat{a},\theta_L)-E_a(\hat{a},v_H)} > 0.\end{aligned}$$

Thus, the private university may lower or raise tuition fees as a response to increased selectivity. On the one hand, an increase in v_H will tend to make students less sensitive to changes in tuition fees. On the other hand, it allows the private university to increase tuition fees without loosing too many students.

However, if $\bar{w}-2p_L > 0$, $\frac{dp_L}{dv_H} > 0$ and the private university will raise its price in response to an increase in the entry requirements at the public university.

Assumption 4. *$\beta < \hat{a} \leq \alpha < 1$ in Case C.*

The optimal fee for the private university is obtained by differentiating:

$$p_L \int_{a_{0L}}^{\hat{a}} \int_{p_L}^{\bar{w}} f(w,a)dwda + p_L \int_{\hat{a}}^{\alpha} \int_{p_L}^{E(a,v_H)} f(w,a)dwda$$

with respect to p_L . The first order condition for an optimal fee at the private university is:

$$\begin{aligned}&\int_{a_{0L}}^{\hat{a}} \int_{p_L}^{\bar{w}} f(w,a)dwda + \int_{\hat{a}}^{\alpha} \int_{p_L}^{E(a,v_H)} f(w,a)dwda + \\ &p_L\left(\frac{d\hat{a}}{dp_L}-\frac{da_{0L}}{dp_L}\right)\left(\bar{w}-p_L\right)-p_L\left(\alpha-a_{0L}\right)-p_L\frac{d\hat{a}}{dp_L}\left(E(\hat{a},v_H)-p_L\right) = 0\end{aligned}$$

Assuming the second order condition is satisfied

$$\frac{dp_L}{dv_H} = -\frac{\frac{d\hat{a}}{dv_H}\left(\bar{w} - E(\hat{a}, v_H)\right) + \int_{\hat{a}}^{\alpha} E_v(a, v_H)da + p_L \frac{d\hat{a}}{dp_L dv_H}\left(\bar{w} - E(\hat{a}, v_H)\right) - p_L \frac{d\hat{a}}{dp_L} E_v(\hat{a}, v_H) - p_L \frac{d\alpha}{dv_H}}{soc},$$

where $\frac{d\hat{a}}{dp_L dv_H} > 0$ (sign of $-E_{av}$), $\frac{d\hat{a}}{dp_L} > 0$, $\frac{d\hat{a}}{dv_H} > 0$ and $\frac{d\alpha}{dv_H} > 0$. The effects are similar to the ones discussed above, except that there is one further reason for not increasing tuition fees following an increase of v_H as the private university can benefit from attracting more students.

Assumption 5. *$\hat{a} < \beta < \alpha$ in Case C.*

Note that $\beta > \hat{a}$ implies that α cannot be smaller than $\hat{a}$.

$$D_L = \int_{a_{0L}}^{\beta} \int_{p_L}^{\bar{w}} f(w, a)dwda + \int_{\beta}^{\alpha} \int_{p_L}^{E(a,v_H)} f(w, a)dwda \text{ if } 1 > \alpha > \hat{a}.$$

The private university maximizes

$$p_L \int_{a_{0L}}^{\beta} \int_{p_L}^{\bar{w}} f(w, a)dwda + p_L \int_{\beta}^{\alpha} \int_{p_L}^{E(a,v_H)} f(w, a)dwda$$

The first order condition is given by:

$$\int_{a_{0L}}^{\beta} \left(\bar{w} - p_L\right) da + \int_{\beta}^{\alpha} \left(E(a, v_H) - p_L\right) da - p_L \frac{da_{0L}}{dp_L}\left(\bar{w} - p_L\right) + p_L \left(a_{0L} - \alpha\right) \; = 0$$

Finally,

$$sign \frac{dp_L}{dv_H} = sign \left[\frac{d\beta}{dv_H}\left(\bar{w} - E(\beta, v_H)\right) + \int_{\beta}^{\alpha} E_v(a, v_H)da - p_L \frac{d\alpha}{dv_H}\right]$$

4 Concluding comments and further steps

In this preliminary note, we have proposed a model that takes into account the interaction of a public and a private university. We have characterized market partitions that may arise depending on the model parameters. We assumed that the high quality public university defines a selectivity level and the lower quality private university sets tuition fees. We have shown that the interaction of these choices is typically ambiguous, which justifies the need to use calibration and numerical simulations to obtain further results. For this purpose, we believe that Brazil is an interesting case study. Brazil's higher education system is expanding and characterized by tuition free high quality public universities and lower quality private universities. Moreover, several public policies, including affirmative action, scholarships for private universities and loans for private universities have been recently adopted. Therefore, the next steps include calibrating the model for relevant parameters characterizing Brazilian's higher education system and simulating the effects of these different policies on market partitions.

5 Appendix 1

In this Appendix, we list the possible ordering of thresholds determining preferences for public and private university. We then analyze preferences ordering that do not violate transitivity of preferences. Figure 4 summarizes our results.

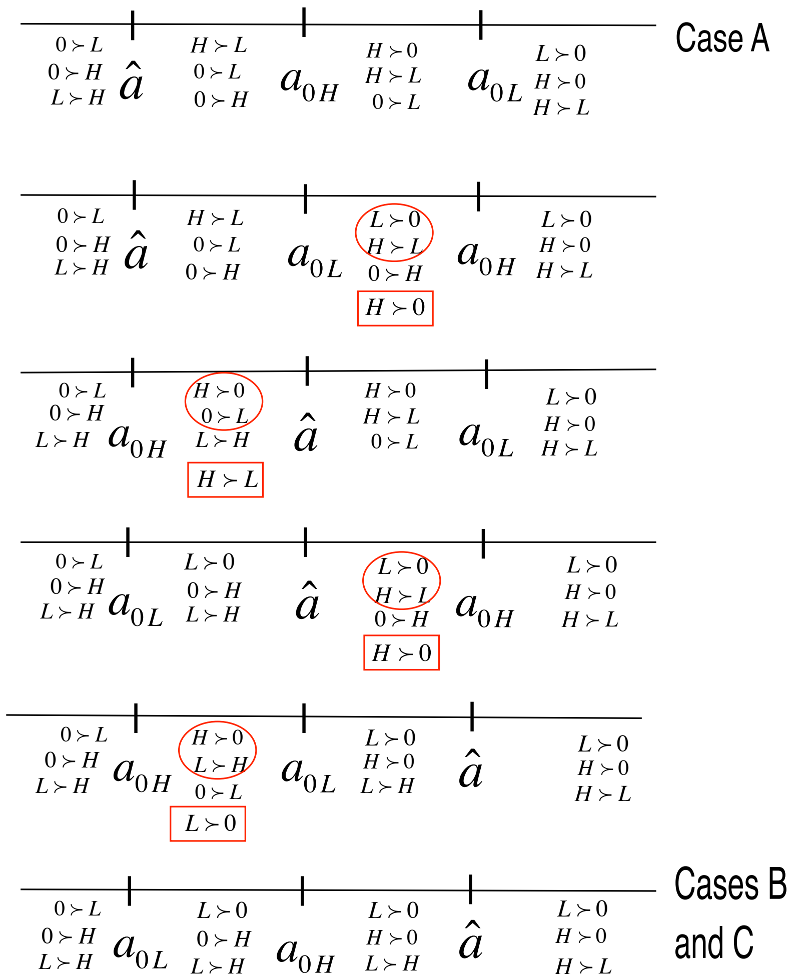

Figure 4: Preference ordering

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