When the Going Gets Tough, the Tough Get Going

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July, 2016

Abstract

This paper uses a citizen-candidate model to analyse how the character-
istics of the political office drive the valence of the political candidates. We
set up a model with a political job composed of several tasks, with random
outcomes. Voters observe the quality of the different tasks but not the politi-
cian's valence. The complexity of the political office, measured by the number
of tasks to be undertaken, each task's difficulty, and their variability, change
the valence signal conveyed by the politician's performance in office. This has
an impact on reelection probabilities which change the relative attractiveness
of the political job for high and low-valence individuals politicians. We show
that the quality of the polity depends on the interaction between self-selection
into politics and the screening mechanism of reelections. We then introduce
incumbency advantage, and show that the screening effect becomes less pow-
erful. This leads to a lower quality of the polity. We characterise the possibly
multiple equilibria of the model with incumbency advantage and show that
the only stable equilibrium is the one with highest valence.

Keywords: Endogenous candidates, political accountability, incumbency
advantage

JEL Classification:

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1 Introduction

The quality of politicians matters a lot for the prosperity of countries and their citizens. Besley et al. (2011), using education as a proxy of politician's quality, show that the departure of an educated leader leads to a 0.713 percentage point reduction in growth per annum, contrasting with just 0.05 percentage points after the death of leader without a post-graduate qualification. The key question is then: what determines politicians' quality? Electoral competition is part of the answer: democracies are around 20% more likely to select educated leaders than autocracies (Besley and Reynal-Querol, 2011).[1] Galasso and Nannicini (2009) show that more contestable districts in elections for members of parliament in Italy attract better quality politicians. Using a unique and comprehensive dataset of Swedish municipal candidates and elected politicians, that includes pre-politics wages and measures of cognitive abilities, Dal Bó et al. (2015) show that politicians are, on average, *significantly smarter and better leaders* than the average citizen. However, as Dal Bó et al. (2015) put it, *"Economic models of politics suggest that the less able have a comparative advantage at entering public life due to free-riding and lower opportunity costs"*. We develop one of the first theoretical analysis of the determinants of politician's quality that sheds light on why *smarter and better* individuals decide to enter the political market.

We use an overlapping generations citizen-candidate model with two types of individuals – the high and low valence –, who may become political candidates. Valence (Stokes, 1963) refers to fundamental characteristics of politicians that all voters value, independently of ideology. Some valence features, such as charisma and rhetorical skills are observable before election. We do not focus on these ones. Rather, we follow Bernhardt et al. (2011) and focus on valence dimensions which are signalled during the politician's tenure in office – more specifically, efficiency in public service delivery.

We provide a positive theory about the quality of the polity, based on the idea that a good political system should induce a self selection of good citizen candidates, select the best candidates into office, and be able to to get rid of a poorly performing elected politician. Individuals live for two periods and make a candidacy decision at the beginning of their lives. All the candidates face the same probability of election; if elected, they are up for reelection at the end of the first term in office. The incumbent performs a political job with several tasks, whose expected outcome increases with valence. The voters observe the outcome of each task and compute the updated probability that the incumbent's valence is high. Therefore, the politician's performance while in office conveys a signal to the voters, the *valence signal*. Voters reelect the incumbent if the updated probability of her being high-valence is higher than the average quality of the polity, which results from the endogenous candidacy decisions. Hence, high-quality individuals face higher reelection prospects.

Our main result is that the design of the political office drives the quality of politicians, which will be a high-quality selection of the overall population. Therefore, our model explains the empirical evidence in Dal Bó et al. (2015) that politicians are smarter than the average citizen. This bias is driven by two effects. The first is *self-selection*:

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[1]Elections exist to ensure that the polity is not a representative selection of the population. If societies wanted to be ruled by the *average citizen*, then a more economical way to select rulers would be picking them at random amongst the citizens. This practice was followed in ancient Greece (Manin, 1997, cited by Besley, 2005).

candidates with high valence have a comparative advantage in reelection because of the valence signal, and are thus more likely to become candidates. The second is *screening*, i.e., the fact that the valence signal allows the voters to oust low-valence incumbents. We analyse the interaction of candidate selection and screening via the reelection mechanism. The two effects reinforce each other. Self-selection works because of screening, and better self-selection increases the advantage of screening, because it improves the pool of available candidates to replace an ousted incumbent.

The power of the valence signal increases with the job's *difficulty*, i.e., the difference in value delivery of high and low-valence politicians, and the job's *complexity*, i.e., the number of tasks it entails, while it decreases with the randomness of political performance. The power of the valence signal is the discrepancy between reelection probabilities of high and low-valence politicians. Therefore, when the valence signal improves, the share of high-valence candidates increases. We characterise the equilibrium share of high-valence candidates in politics as a function of the valence signal, the ego-rents and type-specific private wages. INCUMBENCY ADVANTAGE Finally, we derive an expression for the (two-period) voter life-time expected quality of political tasks in the stationary equilibrium which is, not surprisingly, increasing in the share of high-valence candidates.

Our analysis complements existing citizen-candidate models that are mostly focused on explaining the bad quality of the polity. In his seminal book, Besley (2005) identifies the following determinants of political selection: the attractiveness ratio, the opportunity cost ratio, the accountability ratio, and the success ratio, each about the type-specific pure motivation of holding office, private wages, and reelection and election probabilities, respectively. This paper focuses on the first three, by assuming away any information revelation of political campaigns. Our analysis is, in this sense, complementary to Caselli and Morelli (2004), who focus on election, instead of reelection probabilities, together with attractiveness and opportunity costs. We do not contend that campaigning is not informative. However, by shutting down that mechanism, we are able to identify the features of the political office that determine the degree of information voters can extract from politician's performance. We characterise a stationary equilibrium where the endogenous average quality of the candidates is the same in every period. Caselli and Morelli (2004) highlight the comparative advantage of bad politicians in the political market, given their lower market wage. This is traded-off against a higher election probability of good candidates, assuming that voters extract information from electoral campaigns. Moreover, if ego-rents from holding office depend on the quality of the polity, it is conceivable that the economy gets stuck in a *bad politicians* equilibrium whereby the ego-rents are low because politicians are bad, and good individuals do not enter politics because of the low returns from holding office. We depart from Caselli and Morelli (2004) in that *(i)* we explicitly model the valence signal, that depends on the politician's performance in office, and *(ii)* we use a dynamic model with retrospective voting. This allows us to characterise the quality of the polity as the result of the interaction between self-selection and screening and show how the valence signal may avoid the *bad politicians* equilibrium in Caselli and Morelli (2004). Messner and Polborn (2004) analyse another mechanism for bad candidates to prevail in equilibrium: free-riding by good individuals. They abstract from running costs, but rather posit a cost to perform the public service, which may lead good candidates to stay away from politics, thus free-riding on the willingness of bad ones to fill the ruling job. Importantly, the quality of the candidates is known by all the individuals in the model and, in this sense, Messner and Polborn (2004) analysis fits an election of a

representative in a small organisation, more than large elections. These two papers use citizen-candidate frameworks, in which the decision to run for politics is not mediated by political parties, whose role in political selection has been analysed by Poutvaara and Takalo (2007) and Carrillo and Mariotti (2001). There are a few papers which analyse valence as an observable characteristic (Ansolabehere and Snyder, 2000, Aragones and Palfrey,2002, Groseclose, 2001). Bernhardt et al. (2011) analyse a model with politicians who differ both in ideology and valence, which is initially private information of the incumbent, but is revealed without noise during her tenure in office. This is in contrast to our case, in which the political performance sends only an imperfect signal to the voters about the politician's valence.

The remainder of the paper is organised as follows. In Section 2 we present the model; Section 3 discusses the determinants of the quality of the polity; Section 4 computes the expected life-time utility of a voter. Finally, Section 5 concludes.

2   The Model

The economy is populated by κ individuals of each type, the high- and low-valence, denoted $i = 1, 2$. Individuals decide whether to run for office at the beginning of their lives; they live for two periods and do not discount the future. The valence is private information. The quality of each task is a normal random variable with variance σ^2 and expectation λ_i, $i = 1, 2$, with $\lambda_1 \geq \lambda_2$, for high and low-valence politicians. Therefore, valence changes the expected quality of a task, but not its variability. The impact of valence on the quality of a given task depends on its nature. Simpler tasks do not suffer much from being undertaken by low-valence politicians – therefore, $\lambda_1 - \lambda_2$ measures the task's *difficulty*.

In all the periods in which an individual is not serving as the elected politician, she earns a type-specific private market wage, $w_1 \geq w_2$ – that is, the two periods for individuals who either do not enter the political market or enter, but are not elected, and the second period for non-reelected incumbents. The elected politicians enjoys an ego-rent, or political wage, of $\mu > w_1$.

Individuals pay an idiosyncratic campaign cost to contest the political job, given by γ, between 0 and 1, with exactly one individual of each type $j = g, b$ with $\gamma = 0, 1/\kappa, 2/\kappa, \ldots, 1$. Note that the two individuals – one of each type – with $\gamma = 0$ always become candidates. When κ is high enough, which we assume hereafter, the number of individuals with an entry cost below a given level γ can be approximated by the uniform distribution $F(\gamma) = \gamma$. A single candidate need not campaign and pays no entry cost. The political campaign may convey some information about the political valence; however, it is reasonable to assume that one's record as a politician is a much better signal of one's valence than the campaign. We capture this feature by assuming that the campaign is uninformative about valence; hence, all non-incumbent candidates face an equal chance of winning the election, which we denote q. The entry process generates an endogenous proportion of high-valence candidates, denoted β.[2]

The political office comprises n tasks, indexed by τ; a higher number of tasks implies a greater complexity of the office. Voters observe a vector of task qualities $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$. Given the normality assumption, the probability that a politician with

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[2]We show below that β is time-invariant.

valence $i = 1, 2$ generates vector $\boldsymbol{x}$ is

$$v\left(\boldsymbol{x}, \lambda_i\right) = \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(x_j - \lambda_i\right)^2}{2\sigma^2}} = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^{n} e^{-\sum_{j=1}^{n} \frac{\left(x_j - \lambda_i\right)^2}{2\sigma^2}} \tag{1}$$

2.1 The reelection stage

Politicians are term-limited, and can only be re-elected once. We assume that there are unforeseen events that are not correlated with one's performance in office, e.g. a corruption or political scandal, that make the voters oust the incumbent or lead to an early voluntary retirement from politics. This happens with probability α. In the remaining cases, at the end of the first period, the voters compute the posterior probability of a high-valence politician, given the observed performance, $p(\boldsymbol{x})$ – the *valence signal* –, and reelect her if it is higher than the prior, β, i.e.,

$$p(\boldsymbol{x}) = \frac{\beta v\left(\boldsymbol{x}, \lambda_1\right)}{\beta v\left(\boldsymbol{x}, \lambda_1\right) + (1 - \beta) v\left(\boldsymbol{x}, \lambda_2\right)} \geq \beta \tag{2}$$

For $0 \leq \beta \leq 1$, the updated probability of a facing a high-valence politician is greater than the prior if and only if

$$v\left(\boldsymbol{x}, \lambda_1\right) > v\left(\boldsymbol{x}, \lambda_2\right)$$

which, after straightforward simplification, becomes

$$\frac{\sum_{i=1}^{n} x_i}{n} > \frac{\lambda_1 + \lambda_2}{2} \tag{3}$$

Using (3), the probability that a competent politician is re-elected is given by $\rho_1 = (1 - \alpha)P_1$, where

$$P_1 = P\left(\frac{\sum_{j=1}^{n} x_j}{n} > \frac{\lambda_2 - \lambda_1}{2}\right) = 1 - \Phi\left(\frac{\sqrt{n}}{\sigma} \frac{\lambda_2 - \lambda_1}{2}\right) = \Phi\left(\frac{\sqrt{n}}{\sigma} \frac{\lambda_1 - \lambda_2}{2}\right) \tag{4}$$

Where $\Phi(\cdot)$ is the distribution function of the standardised normal distribution.[3] Analogously, the probability that an incompetent politician is re-elected is $\rho_2 = (1 - \alpha)P_2$, with

$$P_2 = 1 - \Phi\left(\frac{\sqrt{n}}{\sigma} \frac{\lambda_1 - \lambda_2}{2}\right) \tag{5}$$

From (4) and (5), it is clear that the power of the signal conveyed by the political office is captured by s, defined as follows

$$s = \frac{\sqrt{n}}{\sigma} \frac{\lambda_1 - \lambda_2}{2}.$$

The power of the signal increases with the complexity of the political office, n, and the tasks' difficulty $\lambda_1 - \lambda_2$; conversely, it decreases with the randomness of task quality, σ.

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[3]We use the fact that the distribution of the sample average follows a normal distribution with expectation λ_i and variance σ^2/n.

A few interesting properties are apparent from (4) and (5). Firstly, using the symmetry of the normal distribution,

$$P_1(s) = 1 - P_2(s)$$

Secondly, the fact that $\lambda_1 \geq \lambda_2$ ensures that high-valence politicians face a higher probability of being re-elected than not, while the reverse happens for the low-valence ones, i.e.

$$P_1(s) \geq \frac{1}{2} \geq P_2(s)$$

Thirdly, increasing the power of the signal increases the reelection chances of the high-valence politician, while it decreases those of the low-valence one. Indeed,

$$\frac{\partial P_1(s)}{\partial s} = -\frac{\partial P_2(s)}{\partial s} = \phi(s) > 0$$

where $\phi(\cdot)$ stands for the standardised normal density. This property ensures that more complex political jobs (i.e., with higher number of tasks n), less random, and/or more difficult (i.e., with higher $\lambda_1 - \lambda_2$), are relatively more attractive for high-valence individuals, as they convey a more powerful signal s.

2.2 The entry decision

The expected utility of running for office for a candidate of valence $i = 1, 2$ is $q(1 + \rho_i(s))(\mu - w_j) - \gamma$, $i = 1, 2$ yielding a type-specific cut-off entry cost of

$$\hat{\gamma}_j(q; s) = q(1 + \rho_i(s))(\mu - w_i), \ i = 1, 2 \tag{6}$$

which is increasing in both the election and reelection probabilities, i.e., the individuals are willing to pay a higher cost to enter the political market if they face better election prospects. Notice the fundamental trade-off between the outside option and political signalling. On the one hand, high-valence individuals have higher market wages, which decreases the cut-off campaign cost. On the other hand, their reelection probabilities are higher because $\rho_1 > \rho_2$, hence increasing their cut-off campaign cost.

We now compute election probabilities. To do so, we must start by enumerating the possible strategic situations that one faces when running for office. In all periods after the initial one, there may be an incumbent that is ending her second term or an incumbent with a bad record – in both cases, the incumbent is not re-elected. Alternatively, there may be an incumbent with a good record who is up for reelection, and can be ousted with probability α. The entry decision varies slightly between the two cases. In the first one, the election probability for an individual is given by

$$q_1 = \frac{1}{\kappa \left(\hat{\gamma}_1 + \hat{\gamma}_2 \right)}, \tag{7}$$

In the second case, the election is lost for sure with probability $1 - \alpha$, and with probability α the election probability is given by (7), hence the expected election probability is

$$q_2 = \frac{\alpha}{\kappa \left(\hat{\gamma}_1 + \hat{\gamma}_2 \right)}, \tag{8}$$

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Finally, notice that the expected number of individuals in the political market is given by

$$\kappa \left(\hat{\gamma}_1 + \hat{\gamma}_2 \right)$$

Using (??) in (7) and (8), we get

$$q_1^* = \sqrt{\frac{1}{\kappa \left[(1 + \rho_1)(\mu - w_1) + (1 + \rho_2)(\mu - w_2) \right]}}, \text{ and } q_2^* = \sqrt{\alpha} q_1^* \qquad (9)$$

The election probability is given by $q = q_1^*$, or $q = q_2^*$, for each of the two possible cases. The election probability is smaller than one because the high and low-valence individuals with $\gamma = 0$ enter the market for sure. The share of high-valence candidates, β, is the same in both cases.

The election probability is decreasing in κ, reflecting the natural effect of a bigger population on the number of candidates in the market. The election probability also depends on the valence signal, s. This ultimately shapes the quality of the polity. We address these important topics in the next section.

3 Good politicians?

The share of high-valence candidates is given by

$$\beta(s) = \frac{\hat{\gamma}_g(s)}{\hat{\gamma}_g(s) + \hat{\gamma}_b(s)} \qquad (10)$$

Using (6) in (10), it can finally be written as

$$\beta(s) = \frac{1}{1 + \frac{1 + \rho_2}{1 + \rho_1} \frac{\mu - w_2}{\mu - w_1}}.$$

On the one hand, $\beta(s)$ increases with the signal of the political task, s. On the other hand, a higher difference between the private wages (i.e., a higher w_1 or a lower w_2) decreases the quality of the polity.

The fact that the *share* of high-valence candidates increases in s does not ensure that the number of high-valence candidates increases alike. The effect on the number of candidates of each type depends on the impact of the valence signal on the election probability, which is increasing, since $(1 + \rho_1)(\mu - w_1) + (1 + \rho_2)(\mu - w_2)$ is decreasing in s. Better signalling increases the attractiveness of politics for high-valence individuals and decreases it for low-valence ones. This effect is discounted by the difference between the political ego-rent and the private outside option and it is thus amplified for low-valence individuals. In fact, despite the higher election probability, increasing s decreases the number of low-valence candidates. Not surprisingly, since both the election and reelection prospects of high-valence individuals go up, more of them become candidates.

We summarise these results in the following proposition.

Proposition 1 *When the valence signal, s increases, the number of low-valence candidates decreases, while that of high-valence ones increases. Overall, less citizens become candidates, and the election probability increases.*

Proof Notice that

$$\frac{dq_1^*}{ds} = -\frac{1}{2}\left(\kappa\left((1+\rho_1)(\mu-w_1)+(1+\rho_2)(\mu-w_2)\right)\right)^{-3/2}\kappa(w_1-w_2)\frac{d\rho_2}{ds} > 0,$$

since $d\rho_2/ds < 0$.

Also,

$$\frac{d\gamma_2}{ds} = q_1^*(\mu-w_2)\frac{d\rho_2}{ds}\left(1-\frac{1}{2}q_1^{*2}\kappa(w_1-w_2)(1+\rho_2)\right)$$

$$= q_1^*(\mu-w_2)\frac{d\rho_2}{ds}\left(1-\frac{1}{2}\frac{(w_1-w_2)(1+\rho_2)}{(1+\rho_1)(\mu-w_1)+(1+\rho_2)(\mu-w_2)}\right) < 0,$$

where we use the fact that $\mu - w_2 \geq \mu - w_1$.

Moreover, $\frac{d\gamma_1}{ds} > 0$, given that both q_1^* and ρ_1 are increasing in s.

The computations for $q_2^* = \sqrt{\alpha}q_1^*$ are analogous, up to the multiplicative constant $\sqrt{\alpha}$. $\square$

This result shows that the characteristics of the political job that allow the voters to extract information about the politician's valence have a direct impact on the quality of the polity. More complex (i..e, with higher n), more difficult (i.e., with higher $\lambda_1 - \lambda_2$), and less random (i.e., with lower σ) political jobs attract more high-valence candidates.

In order to better understand the result in Proposition 1, it is instructive to shut down the outside option effect by supposing that $w_2 = w_1$ and suppose that $\alpha \to 0$. Given that the ratio $(1+\rho_2)/(1+\rho_1)$ varies between $1/2$ (when s gets arbitrarily large) and 1 (when $s = 0$), the equilibrium quality of the polity varies between $1/2$ and $2/3$. The fact that there are on average more high than low-valence candidates is a result of the symmetry of the model. Indeed they are symmetric up to the reelection probability, which is increasing in valence. Hence, high-valence types are (weakly) more attracted by the political office than the low-valence ones. This shows the importance of the screening effect, which explains the difference between our results and those of Caselli and Morelli (2004). However, there is always a positive share of low-valence candidates, due the distribution of campaign costs. Changing the distribution of γ would have an impact on the quality of the polity but, importantly, $\hat{\beta}$ would still be increasing in s.

Allowing for different outside options, we may establish the following result.

Proposition 2 *Suppose that* $\dfrac{\mu - w_2}{\mu - w_1} > 2 - \alpha$. *Then, there are more low-valence than high-valence candidates, independently of the valence signal.*

This proposition shows that when the outside options are sufficiently different the economy is stuck in Caselli and Morelli's *bad politicians* equilibrium. When $\alpha \to 0$, the inequality

$$\frac{\mu - w_2}{\mu - w_1} > 2 - \alpha$$

becomes

$$w_1 - w_2 > \mu - w_1$$

The *bad candidates* equilibrium can only be avoided if the premium of joining the political market, for high-valence individuals, is higher than the skill premium in the private market.

However, there is a whole range of outside option values for which the signalling features of the political office may improve upon this undesirable equilibrium. We tackle this in the next proposition.

Proposition 3 (Good Politicians) *Suppose that* $\dfrac{\mu - w_2}{\mu - w_1} < 2 - \alpha$*. Then, if the valence signal is high enough, there are more high-valence than low-valence candidates.*

This may shed light on the empirical results in Dal Bó et al. (2015). The authors highlight that Sweden is a *quintessential advanced democracy* which has scored a perfect 10 in the -10 to 10 Polity-IV scale for a long period. In such a well established democracy, it is likely that the non-financial rewards from holding office are high, making μ high enough that politicians are, on average, quite good.

Our analysis identifies the following drivers of the quality of the polity. Higher ego-rents, which may also be interpreted as the politician's salary, increase the average quality of the politicians. The same happens when the private wages of the low valence increases or that of the high-valence decreases. Finally, the valence signal also increases the quality of the polity.

4   Incumbency advantage and the quality of the polity

Incumbency advantage has been shown to matter for electoral results (REFERENCE). The screening effect we consider has so far been purely bayesian, in the sense that the voters do not have any bias in their choice. As it will become clear, the introduction of incumbency advantage changes the screening effect and therefore has an impact on the quality of the polity.

We introduce an incumbency advantage in its simplest form, that is, we change (2) as follows. In the event that the incumbent is neither voluntarily retired nor ousted due to a scandal, she is re-elected if

$$p(\boldsymbol{x}) = \frac{\beta \upsilon\left(\boldsymbol{x}, \lambda_1\right)}{\beta \upsilon\left(\boldsymbol{x}, \lambda_1\right) + (1 - \beta)\upsilon\left(\boldsymbol{x}, \lambda_2\right)} \geq \beta - \delta, \tag{11}$$

where $0 < \delta < 1$ measures the incumbency advantage. Note that when $\beta \leq \delta$ the incumbent is always re-elected because the incumbency advantage outweighs the expected quality of the candidate who replaces the ousted incumbent. When $\beta > \delta$, straightforward simplification of (11), yields

$$\frac{\upsilon\left(\boldsymbol{x}, \lambda_2\right)}{\upsilon\left(\boldsymbol{x}, \lambda_1\right)} \leq 1 + \frac{\delta}{(1 - \beta)(\beta - \delta)} \tag{12}$$

Using (1), (12) yields

$$\frac{\sum_{j=1}^{n} x_j}{n} \geq \frac{\lambda_1 + \lambda_2}{2} - \frac{\sigma^2}{n(\lambda_1 - \lambda_2)} \ln\left(1 + \frac{\delta}{(1 - \beta)(\beta - \delta)}\right) \tag{13}$$

It is immediate that with $\delta = 0$, (13) boils down to (3). Using (13), we may now obtain the type-specific reelection probabilities, which are given by $\rho_i = (1-\alpha)P_i$, $i = 1, 2$, where

$$P_1 = \Phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_1 - \lambda_2}{2} + \mathcal{I}\right] \tag{14}$$

$$P_2 = \Phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_2 - \lambda_1}{2} + \mathcal{I}\right]$$

Where

$$\mathcal{I} = \frac{\sigma}{\sqrt{n}(\lambda_1 - \lambda_2)}\ln\left(1 + \frac{\delta}{(1-\beta)(\beta - \delta)}\right)$$

is the noise introduced in the screening mechanism due to incumbency advantage. Interestingly, high-valence politicians still face higher reelection prospects than low-valence ones, but the difference between the two is now lower, since the incumbency advantage benefits more low-valence politicians.[4]

The term $\mathcal{I}$ is, not surprisingly, increasing in α. Less straightforward is the fact that it is non-monotonic in β, reaching a minimum when $\beta = \frac{1+\delta}{2}$. When $\beta = \delta$, the incumbency advantage is high and the incumbent is always re-elected. When $\beta = 1$, voters know that all the politicians are high-valence and therefore reelect for sure. Indeed, in both cases the expression gets arbitrarily large, and therefore $P_1 = P_2 = 1$. When $\beta > (1+\delta)/2$, the quality of the polity is high enough that the voters are easily convinced that the incumbent is high-valence. Therefore, the reelection probability increases with β. Conversely, when $\delta < \beta < (1+\delta)/2$, a higher value of β has only a minor impact on the valence signal and the reelection probabilities decrease with β. NEEDS TO BETTER EXPLAINED

Before proceeding, we take a closer look at the interaction of the self-selection and screening. Better screening increases the power of the valence signal, which improves the quality of the polity via self-selection. The quality of the candidates changes the pool from which nature draws a replacer for an ousted incumbent. This increases the standard against which voters evaluate the incumbent. One may expect that screening works better when self-selection is improved. This is not the case in Section 2, since the quality of the polity changes the valence signal, i.e., the posterior probability that the incumbent is high-valence, which is itself a function of the prior quality. Referring back to (2), it is immediate that the effect on the prior and on the posterior cancel each other out, and the quality of the polity, β, has no impact on screening. The introduction of the incumbency advantage creates room for this feedback effect.

The entry decision is the same as in the case without incumbency advantage, given by (10), up to the different reelection probabilities, i.e.,

$$\beta = \frac{1}{1 + \frac{1+\rho_2}{1+\rho_1}\frac{\mu - w_2}{\mu - w_1}}.$$

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[4]We show in the Appendix that

$$\frac{dP_2}{d\mathcal{I}} > \frac{dP_1}{d\mathcal{I}} > 0.$$

We show in the Appendix that the ratio $\frac{1+\rho_2}{1+\rho_1}$ is increasing in $\mathcal{I}$ and is, therefore, increasing in α, decreasing in β for $\beta < (\delta+1)/2$, and increasing in β otherwise.

We now turn to the analysis of the equilibrium, given by the simultaneous solution of $\rho_1 = (1-\alpha)P_1$, $\rho_2 = (1-\alpha)P_2$, as given by (14), and (10).

4.1 Equilibrium

##ceci doit aller en appendice. ici dans le texte il faut juste garder un graphique##

Let us begin by rewriting (10) as

$$G(\beta) = \frac{1-\beta}{\beta} \frac{\mu - w_1}{\mu - w_2}$$

We readily obtain that $G(\beta)$ is a decreasing convex function, with $G(0) \to \infty$, and $G(1) = \frac{\mu - w_1}{\mu - w_2} < 1$.

We may also write

$$\frac{1+\rho_2}{1+\rho_1} = F(\beta, \delta) = \begin{cases} \frac{1+\Phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_2 - \lambda_1}{2} + \mathcal{I}\right](1-\alpha)}{1+\Phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_1 - \lambda_2}{2} + \mathcal{I}\right](1-\alpha)} & \text{if} \quad \beta > \delta \\ 1 & \text{if} \quad \beta \le \delta \end{cases}$$

The function $F(\beta, \delta)$ is u-shaped in β, decreasing when $\beta < (\delta + 1)/2$, and increasing otherwise, with $F(\beta, \delta) = 1$, $\forall \beta \le \delta$ and $F(1, \delta) = 1$. The equilibrium is given by β^* such that $G(\beta^*) = F(\beta^*, \delta)$.

We have $F(0, \alpha) = 1 < G(0)$. Moreover, $F(1, \alpha) = 1 > G(0)$. Therefore, by continuity, the two functions cross at least once and an equilibrium exists.

Note also that when $\delta < \frac{\mu - w_1}{2\mu - w_2 - w_1}$, the two functions cross only once because $\frac{1+\Phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_2 - \lambda_1}{2} + \mathcal{I}\right](1-\alpha)}{1+\Phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_1 - \lambda_2}{2} + \mathcal{I}\right](1-\alpha)}$ is U shaped and $G(\delta) > 1$.

Note also that when $\delta \ge \frac{\mu - w_1}{2\mu - w_2 - w_1}$, the two functions cross at $\beta = \frac{\mu - w_1}{2\mu - w_2 - w_1}$,

Claim: *There is a unique equilibrium for all $\lambda_1, \lambda_2, \sigma$, iff there is a unique equilibrium when $\delta = \tilde{\delta} = \frac{\mu - w_1}{2\mu - w_2 - w_1}$.*

Proof : it is enough to show that if there is a unique equilibrium when $\delta = \frac{\mu - w_1}{2\mu - w_2 - w_1}$, the equilibrium is also unique for all the other δ.

We already showed that there always exists a unique equilibrium when $\delta < \frac{\mu - w_1}{2\mu - w_2 - w_1}$. When $\delta \ge \frac{\mu - w_1}{2\mu - w_2 - w_1}$, $\beta = \frac{\mu - w_1}{2\mu - w_2 - w_1}$ is always an equilibrium as at that point $F(\beta, \delta) = G(\beta) = 1$. For this to be the unique equilibrium one need that $F(\beta, \delta) > G(\beta)$ for all $\beta > \delta$. As $F(\beta, \delta)$ is increasing in δ, if this condition is fulfilled for $\delta = \tilde{\delta}$, it will also be fulfilled for $\delta > \tilde{\delta}$.$\boxdot$

Claim a: For $\delta = \tilde{\delta}$, if $\left.\frac{dF(\beta, \tilde{\delta})}{d\beta}\right|_{\beta=\delta} < \left.\frac{dG(\beta)}{d\beta}\right|_{\beta=\delta}$, there are exactly two equilibria, $\beta = \tilde{\delta}$ and $\tilde{\beta} > \delta$. that is al then there are exactly two equilibria : the first $\tilde{\delta}$ with the second being, $\tilde{\beta} > \delta = \tilde{\delta}$.

Proof : The first equilibrium is obvious by definition of $\tilde{\delta}$. For the second, if $\frac{dF(\beta, \tilde{\delta})}{d\beta} < \frac{dG(\beta)}{d\beta}$, then there exists a neighborhood of $\beta > \tilde{\delta}$ such that $F(\beta, \delta) < G(\beta)$. As $F(\beta, \delta)$ and $G(\beta)$ are both continuous and as $F(1, \delta) > G(1)$, there exists $\tilde{\beta}$ such that $F(\tilde{\beta}, \delta) = G(\tilde{\beta})$.$\boxdot$

Claim: If $\left.\frac{dF(\beta,\tilde{\delta})}{d\beta}\right|_{\beta=\tilde{\delta}} < \left.\frac{dG(\beta)}{d\beta}\right|_{\beta=\tilde{\delta}}$, $\exists \tilde{\delta}_1 > \tilde{\delta}$, such that for $\delta \in]\tilde{\delta}, \tilde{\delta}_1[$, there are three equilibria, $\beta = \tilde{\delta}$, and $\beta_1(\delta)$ and $\beta_2(\delta)$ with $\beta_1(\delta) < \beta_2(\delta) < \tilde{\beta}$ and $\beta_1(\delta)$ unstable.

Proof : As $F(\beta,\delta)$ is increasing in δ, and as $F(\tilde{\delta},\tilde{\delta}) = G(\tilde{\delta})$, we have that $F(\tilde{\delta},\tilde{\delta}+\varepsilon) > G(\tilde{\delta})$.

As $F(\beta,\delta)$ is increasing in δ, and continuous in both β and δ we also have that when $\delta \geq \tilde{\delta}$, such that $F(\delta,\delta) > G(\delta)$. From claim a, when ε is small enough there exist $\beta \in]\tilde{\delta}, \tilde{\beta}[$ such that $F(\beta,\tilde{\delta}+\varepsilon) < G(\beta)$. This implies that there exists a $\tilde{\beta}_1$ such that $F(\tilde{\beta}_1,\tilde{\delta}+\varepsilon) = G(\tilde{\beta}_1)$.

Because of the continuity and the convexisty (to be shown) of $F(\beta,\delta)$ in β and because $F(1,\tilde{\delta}+\varepsilon) = 1 < G(1)$, there exists $\tilde{\beta}_2 > \tilde{\beta}_1$ such that $F(\tilde{\beta}_2,\tilde{\delta}+\varepsilon) = G(\tilde{\beta}_2)$. We also have that $\tilde{\beta}_2 < \tilde{\beta}$ as $F(\beta,\delta)$ is increasing in δ.

Claim final: $\left.\frac{dF(\beta,\tilde{\delta})}{d\beta}\right|_{\beta=\tilde{\delta}} = -\infty$

je dois revenir dessus, pcq les simulations me montrent que c'est bien $-\infty$ mais ma demonstration me mène à zero mais je suis confiant quant au fait que je vais pouvoir le montrer.

5 Conclusion

Appendix

Let

$$\mathcal{R} = \frac{1+\rho_2}{1+\rho_1} = \frac{1 + \Phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_2-\lambda_1}{2} + \mathcal{I}\right](1-\alpha)}{1 + \Phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_1-\lambda_2}{2} + \mathcal{I}\right](1-\alpha)}$$

$$\frac{\partial\mathcal{R}}{\partial\delta} = \frac{\frac{\partial\Phi_2}{\partial x}(1+\rho_1) - \frac{\partial\Phi_1}{\partial x}(1+\rho_2)}{(1+\rho_1)^2}\frac{\partial\mathcal{I}}{\partial\delta}(1-\alpha)$$

where $\Phi_1 = \Phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_1-\lambda_2}{2} + \mathcal{I}\right]$, and $\Phi_2 = \Phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_2-\lambda_1}{2} + \mathcal{I}\right]$

Note that

$$\frac{\partial\Phi_1}{\partial x} = \phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_1-\lambda_2}{2} + \mathcal{I}\right] < \frac{\partial\Phi_2}{\partial x} = \phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_2-\lambda_1}{2} + \mathcal{I}\right],$$

where the inequality follows from

(i) if $\left(\frac{\sqrt{n}}{\sigma}\frac{\lambda_2-\lambda_1}{2} + \mathcal{I}\right) < 0$, we have that

$$\phi\left(\frac{\sqrt{n}}{\sigma}\frac{\lambda_1-\lambda_2}{2} + \mathcal{I}\right) < \phi\left(\frac{\sqrt{n}}{\sigma}\frac{\lambda_1-\lambda_2}{2}\right) = \phi\left(\frac{\sqrt{n}}{\sigma}\frac{\lambda_2-\lambda_1}{2}\right) < \phi\left[\frac{\sqrt{n}}{\sigma}\frac{\lambda_2-\lambda_1}{2} + \mathcal{I}\right],$$

recalling that $\lambda_1 - \lambda_2 > 0$ and the normal density $\phi(z)$ is increasing (resp., decreasing) when $z < 0$ (resp., $z > 0$).

(ii) if $\left(\frac{\sqrt{n}}{\sigma} \frac{\lambda_2 - \lambda_1}{2} + \mathcal{I} \right) > 0$, we have that

$\phi\left(\frac{\sqrt{n}}{\sigma} \frac{\lambda_2 - \lambda_1}{2} + \mathcal{I} \right) > \phi\left(\frac{\sqrt{n}}{\sigma} \frac{\lambda_1 - \lambda_2}{2} + \mathcal{I} \right)$, because the normal density $\phi(z)$ is decreasing when $z > 0$

It follows that $\frac{\partial \mathcal{R}}{\partial \delta}$ has the sign of $\frac{d\mathcal{I}}{d\delta} > 0$. A similar reasoning holds for $\frac{\partial \mathcal{R}}{\partial \beta} : \frac{\partial \mathcal{R}}{\partial \beta} < 0$ iff $\beta < (\delta + 1)/2$.

References

[1] Bernhardt, Dan, Odilon Câmara, and Francesco Squintani. "Competence and ideology." The Review of Economic Studies (2011): rdq019.

[2] Besley, Timothy and Stephen Coate, 1997. 'An Economic Model of Representative Democracy", Quarterly Journal of Economics, 112(1), 85-114.

[3] Besley, Timothy, 2004. "Joseph Schumpeter Lecture: Paying Politicians: Theory and Evidence." Journal of the European Economic Association: 193-215.

[4] Besley, Timothy, Jose G. Montalvo, and Marta Reynal-Querol, 2011. "Do Educated Leaders Matter?" The Economic Journal 121.554: F205-227.

[5] Besley, Timothy, and Marta Reynal-Querol, 2011. "Do Democracies Select More Educated Leaders?." American Political Science Review 105.03: 552-566.

[6] Besley, Timothy, and Michael Smart, 2007. "Fiscal restraints and voter welfare." Journal of Public Economics 91.3: 755-773.

[7] Carrillo, Juan D., and Thomas Mariotti, 2001. "Electoral Competition and Politician Turnover." European Economic Review 45.1: 1-25.

[8] Caselli, Francesco and Massimo Morelli, 2004, "Bad Politicians", Journal of Public Economics, 71(3), 829-53.

[9] Ferraz, C., and Finan, F. , 2009. "Motivating Politicians: The Impacts of Monetary Incentives on Quality and Performance". National Bureau of Economic Research DP 14906.

[10] Gagliarducci, S., and Nannicini, T., 2013. "Do Better Paid Politicians Perform Better? Disentangling Incentives from Selection". Journal of the European Economic Association, 11(2), 369-398.

[11] Kotakorpi, K., and Poutvaara, P., 2011. "Pay for Politicians and Candidate Selection: An Empirical Analysis". Journal of Public Economics, 95(7), 877-885.

[12] Osborne, Martin J., and Al Slivinski, 1996. "A model of political competition with citizen-candidates." The Quarterly Journal of Economics: 65-96.

[13] Persson, T., Tabellini, G., and Trebbi, F., 2003. "Electoral Rules and Corruption". Journal of the European Economic Association, 1(4), 958-989.

[14] Poutvaara, Panu, and Tuomas Takalo. "Candidate quality." International Tax and Public Finance 14.1 (2007): 7-27.