A New Approach to Free Entry Markets in Mixed Oligopolies: Welfare Implications[*]

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Abstract

This study formulates a new model of mixed oligopolies in free entry markets. A state-owned public enterprise is established before the game, private enterprises enter the market, and then the government chooses the degree of privatization of the public enterprise (Entry-then-Privatization Model). We find that under general demand and cost functions, the timing of privatization does not affect consumer surplus or the output of each private firm, while it does affect the equilibrium degree of privatization, number of entering firms, and output of the public firm. The equilibrium degree of privatization is too high (low) for both domestic and world welfare if private firms are domestic (foreign).

JEL classification   H42, L13

Keywords   timing of privatization, commitment, state-owned public enterprises; foreign competition

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Highlights

We investigate a mixed oligopoly with free entries of private firms.

A new model of privatization (entry-then-privatization) is presented.

The timing of privatization does not affect consumer surplus but affects welfare.

The degree of privatization is excessive with domestic private firms.

The opposite result is obtained with foreign private firms.

1 Introduction

For more than 30 years, we have observed a worldwide wave of the privatization of state-owned public enterprises. Nevertheless, many public and semipublic enterprises (i.e. firms owned by both public and private sectors) are still active in planned and market economies in developed, developing, and transitional countries.[1] Further, while some public enterprises are traditional monopolists in natural monopoly markets, a considerable number of public (including semipublic) enterprises compete with private enterprises in a wide range of industries.[2] The optimal privatization policies in such industries have attracted extensive attention from economics researchers in such fields as industrial organization, public economics, financial economics, and development economics.[3]

Owing to recent deregulation and liberalization, entry restrictions in mixed oligopolies have significantly weakened. As a result, private enterprises have newly entered many mixed oligopolies such as the banking, insurance, telecommunications, and transportation industries. The literature on mixed oligopolies has intensively investigated optimal privatization policy in free entry markets. By using a monopolistic competition framework, for example, Anderson et al. (1997) showed that privatization may improve welfare when private competitors are domestic, and Matsumura *et al.* (2009) showed that privatization is more likely to improve welfare when private enterprises are foreign. Matsumura and Kanda (2005) adopted the partial privatization approach formulated by Matsumura (1998) and showed that the optimal degree of privatization is zero when private competitors are domestic, while Cato and Matsumura (2012) showed that it is strictly positive when they are foreign and that this is increasing in the foreign ownership share in private firms. Fujiwara (2007) showed a non-monotonic relationship between the degree of product differentiation and

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[1] In addition, many private enterprises facing financial problems such as Federal Home Loan Mortgage Corporation, General Motors, American International Group, Daewoo Shipbuilding & Marine Engineering, Japan Airlines, Tokyo Electric Power Corporation, Anglo Irish Bank, Northern Rock, and Bradford & Bingley have been nationalized, either fully or partially and by either direct or indirect investment through public financial institutions.

[2] Examples include United States Postal Service, Deutsche Post AG, Areva, Nippon Telecom and Telecommunication (NTT), Japan Tobacco (JT), Volkswagen, Renault, Electricite de France, Japan Postal Bank, Kampo, Korea Development Bank, and Korea Investment Corporation.

[3] For examples of mixed oligopolies and recent developments in this field, see Ishibashi and Matsumura (2006), Ishida and Matsushima (2009), Heywood and Ye (2009a), and the works cited therein.

optimal degree of privatization. Cato and Matsumura (2015) discussed the relationship between optimal trade and privatization policies and showed that a higher tariff rate reduces the optimal degree of privatization as well as that the optimal tariff rate can be negative. Cato and Matsumura (2013a) showed the privatization neutrality theorem originally discussed by White (1996) in a duopoly. Fujiwara (2015) showed that in a general equilibrium model, privatization has no effect on welfare and labour income.

All of these studies assumed that the government chooses its privatization policies before the entries of private enterprises.[4] This assumption, however, is not always realistic. For example, in the Japanese banking and life insurance industries, entry restrictions have been gradually deregulated since the 1970s. On the contrary, the privatization of public financial institutions has only recently become prominent. For instance, Japan Post, which owns part of Postal Bank, the largest bank in Japan, and Kampo, a large life insurance company, were first privatized in 2015, and the government still holds more than 80% of their shares. Although the government announced its intention to privatize the Development Bank of Japan, its privatization has been repeatedly postponed and the government still holds 100% of its shares. Further, while the government opened up the Japanese telecommunications market in 1985, it continued to sell shares in NTT, a state-owned public monopolist until 1985, from 1986 to 2016. Moreover, the government still holds a more than 30% share in NTT. These examples suggest that the timeline in existing works (i.e. privatization then entry) may be unrealistic.

Moreover, even when privatization occurs before the entry of private enterprises, the above timeline may still be problematic. As noted above, the Japanese government reduced its ownership of NTT gradually over more than 10 years. Thus, even if the government partially privatizes a firm before opening up the market, it may additionally sell shares in partially privatized enterprises

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[4]Xu *et al.* (2015) is one exception, as it discussed the timing of privatization, showing that earlier privatization is better for domestic and world social welfare. Regarding the timing of commitment, Ino and Matsumura (2012) discussed two Stackelberg models in private oligopolies. In the strongly (weakly) persistent leadership model, Stackelberg leaders produce their outputs before (after) the entry of followers. However, they showed that the two models yield similar welfare results (i.e. leadership improves welfare in both models).

after observing the entries of private enterprises.[5] In other words, the government may be unable to commit to a privatization policy before such entries, especially when it has an incentive to increase the degree of privatization (to reduce its public ownership share) after private enterprises have incurred their sunk entry costs.[6]

In this study, we consider a different timeline in free entry mixed oligopolies. We assume that private enterprises first choose whether to enter the market and then the government chooses the degree of privatization in the public enterprise. We show that the equilibrium degree of privatization is too high (low) from the viewpoint of welfare when private enterprises are domestic (foreign). This result has an important policy implication. When competitors are private (foreign), committing to not increasing (decreasing) the degree of privatization is welfare improving. If it is more difficult for the government to increase its ownership share in the partially privatized enterprise than to decrease it, the government can more easily implement an efficient privatization policy when private enterprises are foreign.[7]

2 Model

Consider a market in which one (partially privatized) domestic state-owned public firm, firm 0, competes against n private firms.[8] Following the standard formulation in the literature on mixed oligopolies, we assume that firm 0 maximizes the weighted average of social welfare and its own profit, whereas private firms maximize their own profits (Matsumura, 1998).

Firms produce perfectly substitutable commodities for which the inverse demand function is denoted by $p(Q)$, where p is the price and Q is the total output. We assume that p is twice continuously differentiable and $p' < 0$ as long as $p > 0$. Firm 0's cost function is $c_0(q_0) + K_0$ where

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[5]Moreover, the government plans to sell its share in Japan Post, too.

[6]In Japan, the government rarely increases its ownership of partially privatized enterprises except when they face financial problems. However, this is not the case in France. For example, the French government increased its ownership of Renault from 15% to 19.4% in 2015.

[7]For discussions on the nationality of private enterprises in mixed oligopolies, see the literature starting with Corneo and Jeanne (1994) and Fjell and Pal (1996). See also Pal and White (1998) and Heywood and Ye (2009b).

[8]Our result hold true even when multiple public firms exist. For a discussion on multiple public firms, see Matsumura and Shimizu (2010).

q_0 is the output of firm 0. Each private firm $i \; (= 1, \ldots, n)$ has an identical cost function, $c(q_i) + K$, where q_i is the output of firm private firm i, $c(q_i)$ is the production cost, and K is the entry cost.[9] We assume that c_0 and c are twice continuously differentiable. To ensure an interior solution, we further assume that $c_0', c' \geq 0$, $c_0'', c'' > 0$, $c_0'(0) = c'(0) = 0$, and $\lim_{q \to \infty} c_0'(q), c'(q) = \infty$.

The profit of firm 0 is given by $\pi_0 = p(Q)q_0 - c_0(q_0) - K_0$, and that of firm $i \; (= 1, \ldots, n)$ is given by $\pi_i = p(Q)q_i - c(q_i) - K$. Domestic social welfare is defined as

$$W = \int_0^Q p(q)dq - pQ + \pi_0 + (1 - \theta)\sum_{i=1}^n \pi_i, \tag{1}$$

where private firms are foreign (domestic) when $\theta = 1 \; (\theta = 0)$.

Firm 0 maximizes $(1 - \alpha)W + \alpha\pi_0 \; (\alpha \in [0, 1])$. In the case of full nationalization ($\alpha = 0$), it maximizes social welfare. In the case of full privatization ($\alpha = 1$), it maximizes its own profit. The degree of privatization is measured by α.

The three-stage game runs as follows. In the first stage, each private firm chooses whether to enter the market. In the second stage, the government chooses the degree of privatization, α. In the third stage, firms entering the market compete in quantities. We use the subgame perfect Nash equilibrium as the equilibrium concept. Throughout this study, we restrict our attention to the case where the number of entering private firms, n, is larger than one.

3 Equilibrium

We solve the game by backward induction. In the third stage, each firm chooses its output simultaneously. The first-order condition of firm 0 is

$$p(Q) + (1 - (1 - \alpha)(1 - \theta))p'(Q)q_0 - c_0'(q_0) - (1 - \alpha)\theta p'(Q)Q = 0. \tag{2}$$

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[9]In this study, we allow a cost difference between public and private firms, although we do not allow a cost difference among private firms. While some readers might think that the public firm must be less efficient than the private firm, not all empirical studies support this view. See Megginson and Netter (2001) and Stiglitz (1988). In addition, Martin and Parker (1997) suggested that corporate performance can either increase or decrease after privatization, based on their study in the United Kingdom. See Matsumura and Matsushima (2004) for a theoretical discussion of the endogenous cost differences between public and private enterprises.

The first-order conditions of the private firms are

$$p(Q) + p'(Q)q_i - c'(q_i) = 0 \text{ for } i = 1, \ldots, n. \tag{3}$$

We assume that the second-order conditions,

$$(1 + \alpha)p' + (1 - (1 - \alpha)(1 - \theta))p''q_0 - (1 - \alpha)\theta p''Q - c_0'' < 0 \tag{4}$$

and

$$2p' + p''q - c'' < 0, \tag{5}$$

are satisfied. A sufficient but not necessary condition is that c_0'' and c'' are sufficiently large. We also assume

$$p' + p''q < 0. \tag{6}$$

This implies that that the strategies of private firms in the quantity competition stage are strategic substitutes.[10] A sufficient but not necessary condition is $p'' \leq 0$. These are standard assumptions in the literature.

Henceforth, we focus on the symmetric equilibrium wherein all private firms produce the same output level q (i.e. $q_i = q_j = q$ for all $i, j = 1, \ldots, n$). Solving equations (2), (3), and the following equation (7) leads to the equilibrium outputs in the third stage, given α and n:

$$Q = q_0 + nq. \tag{7}$$

Let $q_0(\alpha, n)$, $q(\alpha, n)$, and $Q(\alpha, n) := q_0(\alpha, n) + nq(\alpha, n)$ be the equilibrium output of firm 0, that of each private firm, and the equilibrium total output.

Result 1 *(i) $q_0(\alpha, n)$ is decreasing in α, (ii) $q(\alpha, n)$ is increasing in α, and (iii) $Q(\alpha, n)$ is decreasing in α.*

Proof See the Appendix.

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[10]We do not assume that the strategy of the public firm is a strategic substitute because the public firm can be a strategic complement under plausible assumptions when private firms are foreign. See Matsumura (2003).

Result 1 is intuitive. A decrease in α makes the public firm, firm 0, more aggressive because it is more concerned with consumer surplus. Although the objective of each private firm is not related to α, a decrease in α reduces the output of each private firm through the strategic interaction. Note that we assume that private firms' strategies are strategic substitutes. The first direct effect dominates the second indirect strategic effect, and thus a decrease in α increases the total output.[11]

In the second stage, given n, the government maximizes W with respect to α. The first-order condition is

$$\frac{dW}{d\alpha} = \left(\frac{dq_0}{d\alpha}\right)(-p'Q + p + p'q_0 - c_0' + (1-\theta)np'q) + n\left(\frac{dq}{d\alpha}\right)(-p'Q + p'q_0) = 0, \qquad (8)$$

where we use (3). We assume that the second order-condition is satisfied. Let $\alpha(n)$ be the optimal degree of privatization, given n.

Result 2 *(i) $\alpha(n) > 0$ regardless of θ. (ii) $\alpha(n) < 1$ if $\theta = 1$.*

Proof See the Appendix.

Note that in Result 2(ii), $\theta = 1$ is a sufficient, but very far from necessary for $\alpha(n) < 1$. Even when $\theta = 0$, $\alpha(n) < 1$ if both public and private firms have the same cost function (Matsumura, 1998).

We now present an important property on the behaviour of the public firm.

Result 3 *Suppose that n is given exogenously and that the government chooses $\alpha = \alpha(n)$. Then, $p - c' > \ (<) \ 0$ if $\theta = 0$ (1).*

Proof See the Appendix.

Again, the result with $\theta = 0$ is shown in Matsumura (1998) in duopolies and that with $\theta = 1$ is shown by several studies such as Matsushima and Matsumura (2006) and Lin and Matsumura (2012) under specific demand and cost functions.

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[11]This result is not new in the literature on mixed oligopolies. Matsumura (1998) showed this result in duopolies and Matsumura and Kanda (2005) showed it in oligopolies in the case of $\theta = 0$.

Suppose that private firms are domestic. Given the outputs of private firms, $p = c_0'$ leads to the best outcome for welfare and $\alpha = 0$ yields this public firm's behaviour. However, by expecting such aggressive behaviour by the public firm, each private firm chooses a small output, resulting in a loss of welfare. Under these conditions, choosing a strictly positive α induces a larger output for each private firm, which improves welfare. This is why $\alpha(n) > 0$, resulting in $p - c_0' > 0$.

Suppose that private firms are foreign. After the entries of private firms, aggressive behaviour by the public firm reduces the price and thus the outflow of the domestic surplus to foreign firms. Thus, the government chooses α to induce more aggressive behaviour by the public firm than that under marginal cost pricing. Note that from (2) we obtain $p - c_0' < 0$ when $\theta = 1$ in contrast to the case with $\theta = 0$.

In the first stage, the number of entering firms is determined by the zero-profit condition (i.e. each firm obtains only normal profits). Note that the cost includes capital costs. Let α^* be the equilibrium degree of privatization, q_0^* be the equilibrium output of the public firm, q^* be that of the private firm, n^* be the equilibrium number of private firms, and Q^* be the equilibrium total output. The following equation system determines the equilibrium outcomes:

$$\alpha^* = \alpha(n^*), \tag{9}$$

$$p(Q^*) + (1 - (1 - \alpha^*)(1 - \theta))p'(Q^*)q_0^* - c_0'(q_0^*) - (1 - \alpha*)\theta p'(Q^*)Q^* = 0 \tag{10}$$

$$p(Q^*) + p'(Q^*)q^* - c'(q^*) = 0, \tag{11}$$

$$p(Q^*)q^* - c(q^*) - K = 0, \tag{12}$$

$$Q^* = q_0^* + n^*q^*. \tag{13}$$

4   Result

We now consider the following situation as a benchmark. In the second stage, the government is forced to choose $\alpha = \alpha^{**}$. In other words, $\alpha = \alpha^{**}$ is given exogenously and is common knowledge. Given $\alpha = \alpha^{**}$, the following equation system determines the equilibrium outcomes of

this benchmark case:

$$p(Q^{**}) + (1 - (1 - \alpha^{**})(1 - \theta))p'(Q^{**})q_0^{**} - c_0'(q_0^{**}) - (1 - \alpha^{**})\theta p'(Q^{**})Q^{**} = 0, \quad (14)$$

$$p(Q^{**}) + p'(Q^{**})q^{**} - c'(q^{**}) = 0, \quad (15)$$

$$p(Q^{**})q^{**} - c(q^{**}) - K = 0, \quad (16)$$

$$Q^{**} = q_0^{**} + n^{**}q^{**}. \quad (17)$$

We now discuss the relationship between α^{**} and the equilibrium outcomes.

Result 4 *(i) Neither q^{**} nor Q^{**} depends on α^{**}. (ii) $q^{**} = q^*$ and $Q^{**} = Q^*$. (iii) q_0^{**} is decreasing in α^{**}. (iv) n^{**} is increasing in α^{**}. (v) $n^{**} \geq n^*$ if and only if $\alpha^{**} \geq \alpha^*$.*

Proof See the Appendix.

Result 5 *(i) Neither q^{**} nor Q^{**} depends on θ. (ii) If $\alpha^{**} = 1$, neither q_0^{**} nor n^{**} depends on θ. (iii) If $\alpha^{**} < 1$, q_0^{**} is increasing in θ and n^{**} is decreasing in θ.*

Proof See the Appendix.

An increase in α makes the public firm, firm 0, less aggressive because it is less concerned with consumer surplus. This is why q_0 is decreasing in α. This less aggressive behaviour by firm 0 makes the market more profitable for each private firm, given n. This stimulates the new entry of private firms, explaining why n is increasing in α. An increase in n makes the market less profitable. Eventually, additional new entries stop when the price falls to the level before the increase in α. For this reason, Q remains unchanged as α changes. Hence, the equilibrium price is equal to the average cost of each private firm and this condition yields the equilibrium output of each private firm. This is why q remains unchanged as α changes.

An increase in θ makes the public firm more aggressive because an increase in q_0 reduces the outflow to foreign investors. Thus, q_0 is increasing in θ. This aggressive behaviour makes the market less profitable, resulting in a reduction in n. Hence, n is decreasing in θ.

We now present our main result.

Proposition 1

$$\left.\frac{dW^{**}}{d\alpha^{**}}\right|_{\alpha^{**}=\alpha^*} < \ (>) \ 0$$

if $\theta = 0$ (1).

Proof See the Appendix.

Proposition 1 states that the equilibrium degree of privatization is excessive when private firms are domestic. By contrast, the equilibrium degree of privatization is insufficient when private firms are foreign. In other words, the commitment not to increase (decrease) the degree of privatization after the entry stage is required when private firms are domestic (foreign).

We now explain the intuition. Suppose that private firms are domestic. Given n, an increase in α stimulates the production of private firms. Because $p - c' > 0$ in the imperfectly competitive market, an increase in q improves welfare. Thus, the government has an incentive to raise α. A higher α makes the market more profitable, and expecting this, firms have more incentive to enter the market, resulting in an increase in n. However, an increase in n reduces welfare because of the business-stealing effect (Mankiw and Whinston, 1986).[12] Therefore, the equilibrium α is too high for social welfare.

Suppose that private firms are foreign. Given n, an increase in α reduces the total output and thus raises the price. This increases the outflow of the domestic surplus to foreign firms. Thus, the government has an incentive to lower α. A lower α makes the market less profitable, and expecting this, firms have less incentive to enter the market, resulting in a reduction in n. In the long run (in a free entry market), the equilibrium profit of each private firm is always zero, and thus the outflow of the surplus to foreign firms is zero. Because the lower α induces overproduction by the public firm (i.e. $p - c'_0 < 0$), this harms welfare. Therefore, the equilibrium α is too low for social welfare.[13]

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[12]Additional entry reduces welfare in various contexts because of this business-stealing effect. See Lahiri and Ono (1988, 1998), Etro (2004, 2006, 2007), and Cato and Matsumura (2013b).

[13]This finding implies that the number of entering firms is too small under the zero-profit condition. For discussions on insufficient entries for social welfare, see Ghosh and Morita (2007a,b).

In reality, it is more difficult to increase the public ownership share in (partially) privatized firms than to decrease it. Indeed, governments often gradually decrease their public ownership share in (partially) privatized firms. This fact suggests that the government should privatize the public firm before the entry of private firms when competitors are foreign because it can then commit to the optimal degree of privatization. After entry, the government has an incentive to decrease α (setting $\alpha = \alpha^*$); however, as mentioned above, it may be difficult for the government to decrease α for political reasons, and choosing $\alpha > \alpha^*$ could be a credible commitment. On the contrary, if competitors are domestic private firms, choosing α before entry may not be a credible commitment. After entry, the government has an incentive to increase α, and additionally selling the public ownership share to the private sector is easy.[14]

Because all the endogenous variables are continuous with respect to θ, a marginal increase in α^{**} from α^* improves (worsens) welfare when θ is sufficiently close to 1 (0). From these results, we speculate that there exists a threshold value of θ, $\bar{\theta}$ such that a marginal increase in α^{**} from α^* improves welfare if and only if $\theta > \bar{\theta}$. Unfortunately, we fail to derive this clear result under general demand and cost functions. However, we show that such $\bar{\theta} \in (0, 1)$ exists if demand is linear and costs are quadratic, which is a popular model formulation in the literature on mixed oligopolies.[15]

Such $\bar{\theta} \in (0, 1)$ exists if q_0^* is nondecreasing in θ.[16] Result 5 states that given α, q_0^* is nondecreasing in θ. However, the equilibrium α, α^*, depends on θ, and thus q_0^* is indirectly affected by θ through the change in α.

In non-free entry markets where n is an exogenous variable, Lin and Matsumura (2012) discussed the optimal degree of privatization in a model with linear demand and quadratic costs.

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[14]The government may commit to not reducing public ownership after entry by enacting a law with a minimal public ownership share obligation. For example, the government must hold more than one-third of shares in NTT by law. In the JT case, the government needed to hold a two-thirds share in JT until 2012; however, this was reduced to one-third thereafter. Thus, committing to not setting the public share in the future can be challenging.

[15]See De Fraja and Delbono (1989) and Matsumura and Shimizu (2010).

[16]In the proof of Proposition 1, we show that a marginal increase in α^{**} from α^* improves welfare if and only if $p - c_0' < 0$ when $\alpha^{**} = \alpha^*$. Because Q is independent of θ and $c_0'' > 0$, such $\bar{\theta} \in (0, 1)$ exists if q_0^* is increasing in θ.

They showed that the optimal degree of privatization is increasing in n and decreasing in θ. Result 5 states that in a free entry market, n is decreasing in θ. Thus, the equilibrium α in the free entry market is decreasing in θ. Moreover, q_0 is decreasing in α (Result 1(i)). Therefore, an increase in θ indirectly raises q_0 through the decrease in α.

Overall, through both the direct effect (the effect given α) and the indirect effect (the effect through the change in α), an increase in θ raises q_0. Therefore, such a threshold value exists if demand is linear and costs are quadratic.[17]

5   Concluding Remarks

In this study, we present an entry-then-privatization model and show that the equilibrium degree of privatization is too high when competitors are domestic and too low when competitors are foreign. This result suggests that the desirable policy depends on the nationality of private firms. The government should thus commit not to privatize further when competitors are domestic. One possible way in which to do this is to establish a legal obligation of a minimal share in semipublic firms. By contrast, when competitors are foreign, it is important for the government to commit not to renationalize semipublic firms. This might be easier because it is rare to renationalize semipublic firms for political reasons, unless the firms face financial problems.

In this study, we assume that the government sells shares in the public firm only once. However, we often observe governments selling them gradually over time, and our model does not capture this situation. Modelling step-by-step privatization is thus left for future research.

In this study, we assume that firms face quantity competition. As Matsumura and Ogawa (2012) showed, if firms can choose whether they compete on price or quantity, firms choose price competition in contrast to the private market (Singh and Vives, 1984).[18] Thus, we should consider price competition, too. However, discussing price competition under a general demand function is difficult even in private oligopolies, and this problem also remains for future research.

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[17]Direct proof of this result is available upon request.

[18]For the oligopoly version in mixed oligopolies, see Haraguchi and Matsumura (2016).

Appendix

In the following proofs, we suppress the arguments of functions.

Proof of Result 1

By differentiating (2), (3), and (7), we obtain

$$H \begin{pmatrix} dq_0 \\ dq_1 \\ dQ \end{pmatrix} = - \begin{pmatrix} (1-\theta)p'q_0 + \theta p'Q \\ 0 \\ 0 \end{pmatrix} d\alpha, \tag{18}$$

where

$$H := \begin{pmatrix} (1-(1-\alpha)(1-\theta))p' - c_0'' & 0 & (1-(1-\alpha)\theta)p' + (1-(1-\alpha)(1-\theta))p''q_0 - (1-\alpha)\theta p''Q \\ 0 & p' - c'' & p' + p''q \\ -1 & -n & 1 \end{pmatrix}.$$

From (6) and the second-order condition for q_0, we obtain

$$
\begin{aligned}
|H| &= (1-(1-\alpha)(1-\theta))p' - c_0'')(p' - c'') \\
&\quad + \Big((1-(1-\alpha)\theta)p' + (1-(1-\alpha)(1-\theta))p''q_0 - (1-\alpha)\theta p''Q \Big)(p' - c'') \\
&\quad + n(p' + p''q)((1-(1-\alpha)(1-\theta))p' - c_0'') \\
&= \Big((1+\alpha)p' + (1-(1-\alpha)(1-\theta))p''q_0 - (1-\alpha)\theta p''Q - c_0'' \Big)(p' - c'') \\
&\quad + n(p' + p''q)((1-(1-\alpha)(1-\theta))p' - c_0'') > 0.
\end{aligned}
$$

By applying Cramer's rule to (18), we obtain

$$
\begin{aligned}
\frac{dq_0}{d\alpha} &= -\frac{((1-\theta)p'q_0 + \theta p'Q)(p' - c'') + n(p' + p''q)((1-\theta)p'q_0 + \theta p'Q)}{|H|} < 0, \\
\frac{dq}{d\alpha} &= \frac{((1-\theta)p'q_0 + \theta p'Q)(p' + p''q)}{|H|} > 0, \\
\frac{dQ}{d\alpha} &= -\frac{((1-\theta)p'q_0 + \theta p'Q)(p' - c'')}{|H|} < 0.
\end{aligned}
$$

Q.E.D.

Proof of Result 2

By using (8), (2), and Result 1(ii), we obtain

$$\frac{dW}{d\alpha}\Big|_{\alpha=0} = n\Big(\frac{dq}{d\alpha}\Big)(-p'Q + p'q_0) > 0.$$

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This implies Result 2(i).

By substituting $\theta = 1$ into (8) and using (2), we obtain

$$\frac{dW}{d\alpha}\Big|_{\alpha=1} = \left(\frac{dq}{d\alpha}\right)(-p'Q) + n\left(\frac{dq}{d\alpha}\right)(-p'Q + p'q_0). \qquad (19)$$

From Result 1(iii), we obtain $dq_0/d\alpha + n(dq/d\alpha) < 0$. Because $p' < 0$ and $q_0 > 0$, we find that (19) is negative and thus $\alpha = 1$ is not an equilibrium outcome. Q.E.D.

Proof of Result 3

Substituting $\theta = 0$ into (8) yields

$$\frac{dq_0}{d\alpha}(p - c_0') - n\frac{dq}{d\alpha}p'(Q - q_0) = 0.$$

Because $dq_0/d\alpha < 0$, $dq/d\alpha > 0$ (Result 1(ii)) and $p' < 0$, we find that $p - c_0'$ is positive.

Substituting $\theta = 1$ into (8) yields

$$\frac{dq_0}{d\alpha}(-p'Q + p - p'q_0 - c_0') - n\frac{dq}{d\alpha}(p'(Q - q_0)) = -\left(\frac{dq_0}{d\alpha} + n\frac{dq}{d\alpha}\right)(p'(Q - q_0)) + \frac{dq_0}{d\alpha}(p - c_0') = 0.$$

From Result 1(iii), we obtain $dq_0/d\alpha + n(dq/d\alpha) < 0$. From Result 1(i), we obtain $dq_0/d\alpha < 0$. Under these conditions, $p - c_0'$ must be negative. Q.E.D.

Proof of Result 4

In equations (15) and (16), there are only two unknown variables, Q^{**} and q^{**}. Thus, these two equations determine Q^{**} and q^{**}. Because neither α^{**} nor θ appears in these equations, Q^{**} and q^{**} must not depend on these two. This implies (i).

Because these two equations are common with equation system (9)–(13), $Q^{**} = Q^*$ and $q^{**} = q^*$. This implies (ii).

By differentiating (14), we obtain

$$\frac{dq_0^{**}}{d\alpha^{**}} = -\frac{p'n^{**}q^{**}}{(1 - (1 - \alpha^{**})(1 - \theta))p' - c_0''} < 0.$$

This implies (iii).

Because neither Q^{**} nor q^{**} depends on α^{**} and q_0^{**} is decreasing in α^{**}, (17) implies that n^{**} is increasing in α^{**}. This implies (iv).

Obviously, if $\alpha^* = \alpha^{**}$, then $n^* = n^{**}$. Thus, Result 4(iv) implies Result 4(v).   Q.E.D.

Proof of Result 5

(i) is proved in the proof of Result 4(i).

Because (15) and (16) determine q^{**} and Q^{**}, the remaining unknown variables q_0^{**} and n^{**} are determined by (14) and (16). By differentiating (14) and (17), we obtain

$$\begin{pmatrix} (1 - (1 - \alpha^{**})(1 - \theta))p' - c_0'' & 0 \\ -1 & q^{**} \end{pmatrix} \begin{pmatrix} dq_0^{**} \\ dn^{**} \end{pmatrix} = - \begin{pmatrix} (1 - \alpha^{**})(Q^{**} - q_0^{**}) \\ 0 \end{pmatrix} d\theta. \tag{20}$$

By applying Cramer's rule to (20), we obtain

$$\frac{dq_0^{**}}{d\theta} = \frac{(1 - \alpha^{**})(Q^{**} - q_0^{**})}{(1 - (1 - \alpha^{**})(1 - \theta))p' - c_0''} \geq 0, \tag{21}$$

$$\frac{dn^{**}}{d\theta} = \frac{(1 - \alpha^{**})(Q^{**} - q_0^{**})}{((1 - (1 - \alpha^{**})(1 - \theta))p' - c_0'')q} \leq 0, \tag{22}$$

and the equalities hold if and only if $\alpha^{**} = 1$. This implies (ii) and (iii).   Q.E.D.

Proof of Proposition 1

$$\begin{aligned}
\left. \frac{dW^{**}}{d\alpha^{**}} \right|_{\alpha^{**}=\alpha^*} &= \frac{dQ^{**}}{d\alpha^{**}} \frac{\partial W^{**}}{\partial Q} + \frac{dq_0^{**}}{d\alpha^{**}} \frac{\partial W^{**}}{\partial q_0} + \frac{dq^{**}}{d\alpha^{**}} \frac{\partial W^{**}}{\partial q} + \frac{dn^{**}}{d\alpha^{**}} \frac{\partial W^{**}}{\partial n} \\
&= \frac{dq_0^{**}}{d\alpha^{**}} (p - c_0') + \frac{dn^{**}}{d\alpha^{**}} n^{**}(1 - \theta)(pq^{**} - c - K) \\
&= \frac{dq_0^{**}}{d\alpha^{**}} (p - c_0'),
\end{aligned} \tag{23}$$

where we use Result 4 and (16). Thus, (23) is positive if and only if $p - c_0' < 0$. Result 3 implies Proposition 1.   Q.E.D.

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