Public communication policies in an international economy: What should policymakers reveal?

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Abstract

We study non-cooperative communication games being played by policymakers in an international economy. Each policymaker receives signals on the real idiosyncratic shocks which affect the country economies. It has the choice of revealing or not the received signals. The model is characterized by a beauty contest argument in the utility function and cross-border real spillovers. The non-cooperative equilibrium is never characterized by no revelation. A full transparency outcome may be the equilibrium outcome and is then Pareto-optimal. From a normative point of view, no revelation may be Pareto-optimal: the social value of public information may be negative in international economies as well as in closed economies. Partial revelation schemes are possible outcomes but never Pareto-optimal. We apply our result to an international monetary game.

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1 Introduction

- We address the Morris & Shin issue ("What is the social value of public information ?", or "Should the policymaker reveal what it knows?") in a game-theoretical framework where the decisions about revelation are taken non-cooperatively.
- The broad issue: What should the policymakers reveal about their information to the public, given that they decide non-cooperatively and the public is subject to a "beauty contest" with cross-border spillovers?
- We study a two-country economy. A "beauty contest" feature in the payoff function of private individuals. Existence of technological spillovers between countries. In each country, a policymaker and private agents.
- What is the public communication non-cooperative game:
  - Each policymaker receives signals about the two country-specific fundamentals and chooses non-cooperatively its information revealing policy: to emit both of the signals, one of them or none.
  - A "$(0, 1)$" decision: No addition of noise to signals. Either true revelation, that we call "transparency" or no revelation, that we call "opacity".
- Aims:
  1. Compare the outcomes of the various possible configurations.
  2. Find the non-cooperative Nash equilibrium of this game played by the policymakers.
  3. Discuss the equilibrium. characterize it: is it of a prisoner's dilemma variety or not? A battle of sex game? A coordination game? analyze its properties.
  4. Under which conditions is the equilibrium of the game Pareto-optimal? Under which conditions is full opacity or full transparency Pareto-superior to the equilibrium? (Equivalently, what is the social value of public information in the context of information policy games?
  5. Apply to an international monetary policy game.
- Results:
  1. Total absence of public information is not an equilibrium.
  2. Total public information may not be an equilibrium.
  3. Consequently partial information can be an equilibrium.

4. The equilibrium depends on the strength of the real and beauty contest parameters.
5. Partial revelation is never Pareto-optimal. But full opacity of full transparency can be. Which one is optimal depends on the real and beauty contest spillover parameters.
6. The total revelation equilibrium can be Pareto-optimal if the real parameter is in an intermediate range. No other equilibrium can be Pareto-optimal.

2 Set-up

2.1 The model

Two countries indexed by j. Agents in a country indexed by i. Fundamental (real) shocks θ^j. Countries of equal size: (n^1, n^2) with $n^1 = n^2 = 1/2$. Total size normalized by 1.

Fundamentals. The composite variable characterizing j:

$$\Theta^j = \phi\theta^j + (1-\phi)\,\theta^{-j} \tag{1}$$

The regional fundamental:

$$\theta^j \sim N\left(\mu, \sigma_\theta^2\right)$$

- ϕ: a cross-border "fundamental" (real) spillover parameter. If $\phi = 1$, no cross-border real spillover; If $\phi = 0$, full cross-border real spillover.

Private signals. Each private agent receives a signal on the fundamental θ^j.

$$\begin{aligned} x_i^j &= \theta^j + \varepsilon_i \\ \varepsilon_i &\sim i.i.d.\left(0, \sigma_x^2\right) \end{aligned} \tag{2}$$

Precision of private signal σ_x^{-2}.

Public signals. Each policymaker j receives a comprehensive signal $\left(y_j^1, y_j^2\right)$ on the fundamentals (θ^1, θ^2)

$$\begin{aligned} y_j^k &= \theta^k + \eta_j^k, k = 1, 2 \\ \eta_j^k &\sim i.i.d.\left(0, \sigma_{y,k,j}^2\right) \end{aligned} \tag{3}$$

The precision of the signal about θ^j received by policymaker j is equal to $\sigma_{y,j,j}^2 = \sigma_{y,h}^{-2}$ – this is precision of home information. Precision of the signal about θ^{-j} received by policymaker j

is equal to $\sigma^2_{y,-j,j} = \sigma^{-2}_{y,f}$ – this is precision of foreign information. Without loss of generality, we assume that $\sigma^{-2}_{y,h} \geq \sigma^{-2}_{y,f}$. In other words, home information cannot be less precise than the foreign information. Moreover, we assume that $\frac{\sigma_\theta^{-2}+\sigma^{-2}_{y,f}}{\sigma_x^{-2}} > 1$. This assumption amounts to say that even the foreign public information about the fundamental shock θ^j is better than the information received by private agents. This is justified by the fact that policymakers have at their disposal a professional body of statistical agencies and therefore a superior capacity to observe shocks.

Assumption 1. $\frac{\sigma_\theta^{-2}+\sigma^{-2}_{y,f}}{\sigma_x^{-2}} > 1$.

Private loss function.

The private loss function of i in country j:

$$l_i^j = \left(\frac{1-r}{2}\right)\left(a_i^j - \Theta^j\right)^2 + \frac{r}{2}\left(L_i - \overline{L}\right) \tag{4}$$

with $L_i = \int_0^1 (a_k - a_i)^2 \, \mathrm{d}k$ and $\bar{L} = \int_0^1 L_k \, \mathrm{d}k$. International beauty contest.

- r: "beauty contest" spillover parameter. If r is 0, no "beauty contest effect". We assume $0 \leq r < 1$.

We rewrite (4) to obtain the following private loss function (like AP2007):

$$l_i^j = \frac{1-r}{2}\left(a_i^j - \Theta^j\right)^2 + \frac{r}{4}\left[\left(\overline{a}^j - a_i^j\right)^2 + \left(\overline{a}^{-j} - a_i^j\right)^2 - \sigma^2_{a^j} - \sigma^2_{a^{-j}} - \left(\overline{a}^j - \overline{a}^{-j}\right)^2\right] \tag{5}$$

Public loss function

The loss of policymaker j is the sum of private loss in region j: $L_P^j = \int_{i \in S^j} l_i^j \, \mathrm{d}i$. Taking into account (5), we get the loss function of the policymaker in country j:

$$L_P^j = \frac{1-r}{2}\int_{i \in S^j}\left(a_i^j - \Theta^j\right)^2 \mathrm{d}i + \frac{r}{8}\left[\sigma^2_{a^j} - \sigma^2_{a^{-j}}\right] \tag{6}$$

2.2 Expectations

Thus, the signal sent:

$$\begin{aligned} s_j^k &= \theta^k + \psi_j^k, k = 1, 2 \\ \psi_j^k &\sim i.i.d.\left(0, \sigma^2_{s,k,j}\right) \end{aligned} \tag{7}$$

where $\psi_j^k = \eta_j^k + \nu_j^k$. In binary version $\sigma^2_{\nu,k,j} \in \{0, \infty\}$ and $\sigma^2_{s,k,j} \in \left\{\sigma^2_{y,k,j}, \infty\right\}$. Precision of signal s_j^k is denoted by $\sigma^{-2}_{s,k,j}$. If transparency, $\sigma^{-2}_{s,k,j}$ is equal to $\sigma^{-2}_{y,k,j}$. If opacity, $\sigma^{-2}_{s,k,j}$ is equal to 0.

Thus, there are two public cumulative signals s^k:

$$s^k = \frac{\sigma^{-2}_{s,k,j} s_j^k + \sigma^{-2}_{s,k,-j} s^k_{-j}}{\sigma^{-2}_{s,k,j} + \sigma^{-2}_{s,k,-j}}$$

Precision of public signal s^k is equal to $\sigma^{-2}_{s,k} = \sigma^{-2}_{s,j,j} + \sigma^{-2}_{s,j,-j}$.

If both are transparent, $\sigma^{-2}_{s,k} = \sigma^{-2}_{y,h} + \sigma^{-2}_{y,f}$. If both are opaque, $\sigma^{-2}_{s,k} = 0$. If there is home transparency and foreign opacity, $\sigma^{-2}_{s,k} = \sigma^{-2}_{y,h}$. If there is home opacity and foreign transparency, $\sigma^{-2}_{s,k} = \sigma^{-2}_{y,f}$.

Let z^j denote a common posterior of θ^j given only public information:

$$z^j = E\left(\theta^j \middle| s_j^j, s_{-j}^j\right) = \omega^j s^j + \left(1 - \omega^j\right)\mu \tag{8}$$

Where $\omega^j = \frac{\sigma^{-2}_{s,j}}{\sigma^{-2}_{s,j} + \sigma_\theta^{-2}}$.

Precision of this common posterior is equal to $\sigma^{-2}_{z,j} = \sigma_\theta^{-2} + \sigma^{-2}_{s,j}$.

The relative precision of z^j in comparison to private information:

$$\zeta^j = \frac{\sigma^{-2}_{z,j}}{\sigma_x^{-2}} = \frac{\sigma_\theta^{-2} + \sigma^{-2}_{s,j}}{\sigma_x^{-2}} \tag{9}$$

We denote by $\zeta^j \equiv \frac{\sigma_\theta^{-2} + \sigma^{-2}_{s,j,j} + \sigma^{-2}_{s,j,-j}}{\sigma_x^{-2}}$ the relative precision of public information about fundamental shock θ^j. With the assumption 1 made before, we assume the following

Assumption 2. $\zeta^j \equiv \frac{\sigma_\theta^{-2} + \sigma^{-2}_{s,j,j} + \sigma^{-2}_{s,j,-j}}{\sigma_x^{-2}} > 1$

Expectations of i in region j:

$$E\left(\theta^{-j} \middle| z^{-j}\right) = z^{-j}$$

$$E\left(\theta^j \middle| z^j, x_i^j\right) = \frac{\zeta^j}{1+\zeta^j} z^j + \frac{1}{1+\zeta^j} x_i^j$$

3 A non-cooperative game on public information.

We restrict the analysis to the choice between opacity versus transparency:

- If the policymaker is home transparent, precision $\sigma^{-2}_{s,j,j}$ is equal to $\sigma^{-2}_{y,h}$ and the relative precision of public information is precision $\zeta^j = \frac{\sigma_\theta^{-2} + \sigma^{-2}_{y,h} + \sigma^{-2}_{s,j,-j}}{\sigma_x^{-2}}$.

- If the policymaker is home opaque, precision $\sigma_{s,j,j}^{-2}$ is equal to 0 and $\zeta^j = \frac{\sigma_\theta^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}$.
- If the policymaker is foreign transparent, precision $\sigma_{s,-j,j}^{-2}$ is equal to $\sigma_{y,f}^{-2}$. Thus, for given $\sigma_{s,-j,-j}^{-2}$, the relative precision of foreign public information is $\zeta^{-j} = \frac{\sigma_\theta^{-2}+\sigma_{y,f}^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}$.
- If the policymaker is foreign opaque, precision $\sigma_{s,-j,j}^{-2}$ is equal to 0. Thus, for given $\sigma_{s,-j,-j}^{-2}$, the relative precision of foreign public information is $\zeta^{-j} = \frac{\sigma_\theta^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}$.

3.1 A sequential game.

The sequence of the game:

Step 1. Each policymaker decides non-cooperatively what it will reveal from what it knows. Here are the four possible decisions considered by policymaker j:

1. $(0,0)$. Full opacity.
2. $\left(y_j^1, y_j^2\right)$. Full transparency.
3. $\left(0, y_j^2\right)$. Domestic opacity / foreign transparency.
4. $\left(y_j^1, 0\right)$. Domestic transparency / foreign opacity.

Step 2. Each agent receives his / her specific information.

Step 3. Public signals are emitted in accordance with decision of Step 1. The signals emitted by policymakers are universally received.

Step 4. Expectations of private agents are computed: $E\left[\ldots \left| x_i^j,\, s^j,\, s^{-j},\, \sigma_{s^j}^2,\, \sigma_{s^{-j}}^2\right.\right]$. Private actions $\left(a_i^j\right)$ are chosen non-cooperatively so as to minimize expected loss.

Step 5. The shocks are realized: (θ^j, θ^{-j}). Losses are computed: $\left(l_i^j,\, L_P^j, L_W^j\right)$.

Therefore, given that each policymaker has 4 decision possibilities, there are 16 possible outcomes for this game.

3.2 Private actions (step 4)

The optimal choice of a representative private agent i living in country j solves is as follows:

$$a_i^j = \arg\min_{a_i'} \frac{1-r}{2}\left(a_i' - \Theta^j\right)^2 + \frac{r}{4}\left[\left(\overline{a}^j - a_i'\right)^2 + \left(\overline{a}^{-j} - a_i'\right)^2 - \sigma_{a^j}^2 - \sigma_{a^{-j}}^2 - \left(\overline{a}^j - \overline{a}^{-j}\right)^2\right] \quad (10)$$

The first order condition is:

$$a_i^j = E\left[(1-r)\left(\phi\theta^j + (1-\phi)\,\theta^{-j}\right) + \frac{r}{2}\left(\overline{a}^j + \overline{a}^j\right)\Big|\, I_i^j\right] \quad (11)$$

As we can see from (11), private actions are defined by expected fundamentals and expected average actions in both the regions, according to information set I_i^j of the agent.

We assume the following equilibrium private linear strategy:

$$a_i^j = b^j x_i^j + c^j z^j + d^j z^{-j} \quad (12)$$

Solving for the equilibrium of this subgame, gives us the following solutions:

$$b^j = \frac{(1-r)\,\phi}{(1-{}^r\!/_2) + \zeta^j} \quad (13)$$

$$c^j = {}^r\!/_2 + \frac{(1-r)\,\phi\left[\zeta^j - {}^r\!/_2\right]}{(1-{}^r\!/_2) + \zeta^j} \quad (14)$$

$$d^j = (1-r)(1-\phi) + {}^r\!/_2 \quad (15)$$

Notice that:

$$b^j + c^j = \phi + \frac{r}{2}(1-2\phi) = \frac{r}{2} + (1-r)\,\phi$$

$$b^j + c^j + d^j = 1$$

Otherwise, we can rewrite the strategy of private agents in terms of signals:

$$\tilde{a}_i^j = b^j x_i^j + \tilde{c}^j s^j + \tilde{d}^j s^{-j} + \tilde{e}^j \mu,$$

where $\tilde{c}^j = \omega^j c^j$, $\tilde{d}^j = \omega^{-j} d^j$ and $\tilde{e}^j = (1-\omega^j)\,c^j + (1-\omega^{-j})\,d^j$. If $\mu = 0$, we get our usual representation:

$$\tilde{a}_i^j = b^j x_i^j + \tilde{c}^j s^j + \tilde{d}^j s^{-j}, \tag{16}$$

with $b^j + \tilde{c}^j + \tilde{d}^j < 1$.

3.3 Public actions (step 3)

With the help of private strategy (12), we rewrite public loss (6):

$$E\left(L_P^j\right) = \frac{1}{4}\left[\rho_j^j\left(\zeta^j\right) + \rho_j^{-j}\left(\zeta^{-j}\right)\right], \tag{17}$$

where $\rho_j^j\left(\zeta^j\right)$ is the "home" informational loss component, which depends on the information about fundamental θ^j, and $\rho_j^{-j}\left(\zeta^{-j}\right)$ is the "foreign" informational loss component, which depends on the information about fundamental θ^{-j}. The "home" (informational) loss component in region j can be expressed as follows:

$$\rho_j^j\left(\zeta^j\right) = (1-r)\left(b^j + \omega^j c^j - \phi\right)^2 \sigma_\theta^2 + [1 - r/2]\left(b^j\right)^2 \sigma_x^2 + \left(\omega^j\right)^2 (1-r)\left(c^j\right)^2 \sigma_{s,j}^2. \tag{18}$$

Precision $\sigma_{s,j,j}^{-2}$ influences the relative precision $\zeta^j = \frac{\sigma_\theta^{-2} + \sigma_{s,j,j}^{-2} + \sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}$ and consequently, the weights b^j, c^j and coefficient ω^j.

Similarly the "foreign" (informational) loss component in region j, which depends on $\zeta^{-j} = \frac{\sigma_\theta^{-2} + \sigma_{s,-j,j}^{-2} + \sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}$, can be expressed as follows:

$$\rho_j^{-j}\left(\zeta^{-j}\right) = (1-r)\left(\omega^{-j} d^j - (1-\phi)\right)^2 \sigma_\theta^2 - r/2\left(b^{-j}\right)^2 \sigma_x^2 + \left(\omega^{-j}\right)^2 (1-r)\left(d^j\right)^2 \sigma_{s,-j}^2. \tag{19}$$

Precision $\sigma_{s,-j,j}^{-2}$ influences the relative precision $\zeta^{-j} = \frac{\sigma_\theta^{-2} + \sigma_{s,-j,j}^{-2} + \sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}$ and consequently, weights b^{-j}, c^{-j} and coefficient ω^{-j}.

Thus, the optimal value of $\sigma_{s,j,j}^{-2}$ is defined independently from the equilibrium value of $\sigma_{s,-j,j}^{-2}$.

3.4 Existence and unicity of equilibrium

Definition 1. The equilibrium in a policy game is the pair of strategies (P_1^*, P_2^*), where vector $P_j^* = \left(\left(\sigma_{s,j,j}^{-2}\right)^*, \left(\sigma_{s,-j,j}^{-2}\right)^*\right)$ is such that

1. $\left(\sigma_{s,j,j}^{-2}\right)^* = \arg \min\limits_{\sigma_{s,j,j}^{-2} \in \left\{0, \sigma_{y,h}^{-2}\right\}} \tilde{\rho}_j^j\left(\frac{\sigma_\theta^{-2} + \sigma_{s,j,j}^{-2} + \left(\sigma_{s,-j,j}^{-2}\right)^*}{\sigma_x^{-2}}\right)$

2. $\left(\sigma_{s,-j,j}^{-2}\right)^* = \arg \min_{\sigma_{s,-j,j}^{-2} \in \left\{0, \sigma_{y,f}^{-2}\right\}} \tilde{\rho}_j^{-j}\left(\frac{\sigma_\theta^{-2}+\sigma_{s,-j,j}^{-2}+\left(\sigma_{s,-j,-j}^{-2}\right)^*}{\sigma_x^{-2}}\right)$

Definition 2. The symmetric equilibrium is an equilibrium such that $P_1^* = P_2^*$.

For given $\sigma_{s,j,-j}^{-2}$, if the loss difference $\Delta_j^j\left(\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}, \frac{\sigma_\theta^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}\right)$ is positive, the policymaker chooses home opacity. Vice versa, if the loss difference $\Delta_j^j\left(\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}, \frac{\sigma_\theta^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}\right)$ is negative, the policymaker chooses home transparency. For given $\sigma_{s,-j,-j}^{-2}$, if the loss difference $\Delta_j^{-j}\left(\frac{\sigma_\theta^{-2}+\sigma_{y,f}^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}, \frac{\sigma_\theta^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}\right)$ is positive, the policymaker chooses foreign opacity. Vice versa, if the loss difference $\Delta_j^{-j}\left(\frac{\sigma_\theta^{-2}+\sigma_{y,f}^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}, \frac{\sigma_\theta^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}\right)$ is negative, the policymaker chooses foreign transparency.

Proposition 1. *For any $\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$ an equilibrium exists. For almost all ϕ, the equilibrium is unique and symmetric.*

Remark 1. We clarify 'for almost all ϕ': as we show later, for any $\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$ there exist $\underline{\phi}$ and $\overline{\phi}$. If $\phi = \underline{\phi}$, each policymaker is indifferent between home opacity and home transparency. Thus, we have 4 equilibria in pure strategies. If $\phi = \overline{\phi}$, each policymaker is indifferent between foreign opacity and foreign transparency. Thus, there are 4 equilibria in pure strategies. For any $\phi \neq \left\{\underline{\phi}, \overline{\phi}\right\}$, equilibrium is unique and symmetric.

4 Properties of the equilibrium

4.1 The impact of the technological spillover on equilibrium

We get the following Proposition about the equilibrium in policy game:

Proposition 2. *For given $\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$, there exist $\underline{\phi}$ and $\overline{\phi}$ such that*

1. *if $\phi < \underline{\phi}$, the equilibrium strategy is $P_j^* = \left(0, \sigma_{y,f}^{-2}\right)$ – home opacity, foreign transparency.*
2. *if $\underline{\phi} < \phi < \overline{\phi}$, the equilibrium strategy is $P_j^* = \left(\sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}\right)$ – home transparency, foreign transparency.*
3. *if $\overline{\phi} < \phi$, the equilibrium strategy is $P_j^* = \left(\sigma_{y,h}^{-2}, 0\right)$ – home transparency, foreign opacity.*

Proof. See Appendix. □

Remark 2. $\underline{\phi}$ and $\overline{\phi}$ are functions of $\left(\sigma_{\theta}^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_{x}^{-2}, r\right)$. As $\frac{\sigma_{\theta}^{-2}+\sigma_{y,f}^{-2}}{\sigma_{x}^{-2}} \geq 1$, the following is true.

Proposition 3. *1. Properties of* $\underline{\phi}$*:*

(a) *Precision of prior information and policymakers information lowers* $\underline{\phi}$*:* $\frac{\partial \underline{\phi}}{\partial \sigma_{\theta}^{-2}} < 0$, $\frac{\partial \underline{\phi}}{\partial \sigma_{y,h}^{-2}} < 0$, $\frac{\partial \underline{\phi}}{\partial \sigma_{y,f}^{-2}} < 0$

(b) *Precision of private information increases* $\underline{\phi}$*:* $\frac{\partial \underline{\phi}}{\partial \sigma_{x}^{-2}} > 0$

(c) $\underline{\phi}$ *is monotonously increasing in* r*. If* $r = 0$, $\underline{\phi} = 0$*. If* $r = 1$, $\underline{\phi} = 1/4$.

2. *Properties of* $\overline{\phi}$*:*

(a) *Precision of prior information and policymakers information lowers* $\overline{\phi}$*:* $\frac{\partial \overline{\phi}}{\partial \sigma_{\theta}^{-2}} < 0$, $\frac{\partial \overline{\phi}}{\partial \sigma_{y,h}^{-2}} < 0$, $\frac{\partial \overline{\phi}}{\partial \sigma_{y,f}^{-2}} < 0$

(b) *Precision of private information increases* $\overline{\phi}$*:* $\frac{\partial \overline{\phi}}{\partial \sigma_{x}^{-2}} > 0$

(c) $\overline{\phi}$ *is monotonously decreasing in* r*. If* $r = 0$, $\overline{\phi} = 1$*. If* $r = 1$, $\overline{\phi} = 3/4$.

3. *Propperties of* $\left(\overline{\phi} - \underline{\phi}\right)$*:*

(a) *Precision of public information enlarges the region of transparency:* $\frac{\partial\left(\overline{\phi}-\underline{\phi}\right)}{\partial \sigma_{\theta}^{-2}/\sigma_{x}^{-2}} > 0$, $\frac{\partial\left(\overline{\phi}-\underline{\phi}\right)}{\partial \sigma_{y,h}^{-2}/\sigma_{x}^{-2}} > 0$, $\frac{\partial\left(\overline{\phi}-\underline{\phi}\right)}{\partial \sigma_{y,f}^{-2}/\sigma_{x}^{-2}} > 0$

Proof. See Appendix. □

4.2 The impact of the beauty contest spillover

Similarly the impact of the beauty contest spillover r on equilibrium is described in the following:

Proposition 4. *For given* $\left(\sigma_{\theta}^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_{x}^{-2}\right)$,

1. *if* $\phi > 3/4$*, there exists a threshold value* $\overline{r}(\phi)$ *such that for* $r < \overline{r}(\phi)$ *equilibrium is* $P_j^* = \left(\sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}\right)$ *– home transparency, foreign transparency. For* $r > \overline{r}(\phi)$*, equilibrium is* $P_j^* = \left(\sigma_{y,h}^{-2}, 0\right)$ *– home transparency, foreign opacity*

2. *if* $1/4 < \phi < 3/4$*, the equilibrium strategy is* $P_j^* = \left(\sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}\right)$ *– home transparency, foreign transparency for any* r *.*

3. *if* $\phi < 1/4$*, there exists a threshold value* $\underline{r}(\phi)$ *such that for* $r < \underline{r}(\phi)$ *equilibrium is* $P_j^* = \left(\sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}\right)$ *– home transparency, foreign transparency. For* $r > \underline{r}(\phi)$*, equilibrium is* $P_j^* = \left(0, \sigma_{y,f}^{-2}\right)$ *– home opacity, foreign transparency.*

Proof. See Appendix. □

5 Optimal policy

To find the socially optimal policy, we derive the social losses, which are the sum of losses of all the agents in the economy. These losses are the following:

$$E\left(L_S\right)=\frac{\left(1-r\right)\sigma_x^2}{2}\left[\tilde{\rho}_S^j\left(\zeta^j\right)+\tilde{\rho}_S^{-j}\left(\zeta^{-j}\right)\right], \tag{20}$$

where $\tilde{\rho}_S^j\left(\zeta^j\right)=\tilde{\rho}_j^j\left(\zeta^j\right)+\tilde{\rho}_{-j}^j\left(\zeta^j\right)$ is the component which depends on the precision of information about fundamental θ^j and $\tilde{\rho}_S^{-j}\left(\zeta^{-j}\right)=\tilde{\rho}_j^{-j}\left(\zeta^{-j}\right)+\tilde{\rho}_{-j}^{-j}\left(\zeta^{-j}\right)$ is the component which depends on the precision of information about fundamental θ^j. Due to symmetry, both the functions ρ_S^j and ρ_S^{-j} are as follows:

$$\tilde{\rho}_S^j=\frac{\left(4r^2\phi^2-2r^2\left(2-r\right)^2\left(1-2\phi\right)^2+4\left(2-r\right)^2\left(1-\phi\right)^2\right)}{4\left(2-r\right)^2\zeta^j}+\frac{4\phi^2\left(1-r\right)}{\left(2-r\right)^2\left(\left(1-{}^{r}\!/\!{}_{2}\right)+\zeta^j\right)}- \tag{21}$$
$$-\frac{\left(1-r\right)^2\phi^2 r}{\left(2-r\right)\left(\left(1-{}^{r}\!/\!{}_{2}\right)+\zeta^j\right)^2}$$

Definition 3. The social optimum is the vector $\left(\tilde{\sigma}_{s,1}^{-2},\tilde{\sigma}_{s,2}^{-2}\right)$ such that

$$\tilde{\sigma}_{s,j}^{-2}=\arg\min_{\sigma_{s,j}^{-2}\in\left\{0,\sigma_{y,f}^{-2},\sigma_{y,h}^{-2},\sigma_{y,f}^{-2}+\sigma_{y,h}^{-2}\right\}}\tilde{\rho}_S^j\left(\frac{\sigma_\theta^{-2}+\sigma_{s,j}^{-2}}{\sigma_x^{-2}}\right),\ j\in\{1,2\}$$

We first study the properties of the public loss function, which are summarized in the following lemma:

Lemma 1. *For given r, there exists $\tilde{\phi}\left(r\right)=\min\left\{1,\frac{\left(2-r\right)\left(2-r^2\right)}{\left(2-r\right)^2\left(1+r\right)-r^2\left(3-r\right)}\right\}$, such that*

1. *if $\phi<\tilde{\phi}$, loss component $\tilde{\rho}_S^j\left(\zeta^j\right)$ is monotonously decreasing in ζ^j .*
2. *if $\phi>\tilde{\phi}$, loss component $\tilde{\rho}_S^j\left(\zeta^j\right)$ has an inverted U-form.*

Proof. See Appendix. □

Lemma 1 tells us that intermediate transparency is never socially optimal. Either full opacity or full transparency is optimal. From here we can conclude that for $\phi<\underline{\phi}$, where we have home opacity and foreign transparency in equilibrium, equilibrium is not optimal. If $\phi>\overline{\phi}$, we have home transparency and foreign opacity in equilibrium. This equilibrium is also not optimal. To conclude about the optimality for $\phi\in\left[\underline{\phi},\overline{\phi}\right]$ and to describe the social optimum, we proceed with its characteristics.

Using Lemma 1 and focusing on ϕ, we immediately come to Proposition 5 with the characteristics of social optimum:

Proposition 5. *For r given,*

1. If $\phi < \tilde{\phi}(r)$, full transparency ($\tilde{\sigma}_{s,j}^{-2} = \sigma_{y,f}^{-2} + \sigma_{y,h}^{-2}$) is socially optimal for all $\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}\right)$.
2. If$\phi > \tilde{\phi}(r)$, there exist $\underline{\zeta}(\phi, r)$ and $\overline{\zeta}(\phi, r)$ such that:
   (a) if $\frac{\sigma_\theta^{-2}}{\sigma_x^{-2}} < \underline{\zeta}(\phi, r)$, opacity ($\tilde{\sigma}_{s,j}^{-2} = 0$) is socially optimal for all $\left(\sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}\right)$.
   (b) if $\frac{\sigma_\theta^{-2}}{\sigma_x^{-2}} > \overline{\zeta}(\phi, r)$, transparency ($\tilde{\sigma}_{s,j}^{-2} = \sigma_{y,f}^{-2} + \sigma_{y,h}^{-2}$) is socially optimal for all $\left(\sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}\right)$
   (c) if $\underline{\zeta}(\phi, r) < \frac{\sigma_\theta^{-2}}{\sigma_x^{-2}} < \overline{\zeta}(\phi, r)$, there exist $\hat{\sigma}_y^{-2}$ such that
      i. if $\sigma_{y,h}^{-2} + \sigma_{y,f}^{-2} < \hat{\sigma}_y^{-2}$, opacity ($\tilde{\sigma}_{s,j}^{-2} = 0$) is socially optimal
      ii. if $\sigma_{y,h}^{-2} + \sigma_{y,f}^{-2} = \hat{\sigma}_y^{-2}$, society is indifferent between opacity and transparency.
      iii. if $\sigma_{y,h}^{-2} + \sigma_{y,f}^{-2} > \hat{\sigma}_y^{-2}$, transparency ($\tilde{\sigma}_{s,j}^{-2} = \sigma_{y,f}^{-2} + \sigma_{y,h}^{-2}$) is socially optimal

Proof. See Appendix. □

Using Lemma 1, and focusing now on r we get a similar Proposition:

Proposition 6. *1. if $r > 1 - \left(\sqrt{2} - 1\right)$, full transparency is socially optimal for all $\left(\sigma_\theta^{-2}, \sigma_{y,j,j}^{-2}, \sigma_{y,j,-j}^{-2}, \sigma_x^{-2}, \phi\right.$*

2. if $r < 1 - \left(\sqrt{2} - 1\right)$ and $\phi < \tilde{\phi}(r)$, full transparency is socially optimal for all $\left(\sigma_\theta^{-2}, \sigma_{y,j,j}^{-2}, \sigma_{y,j,-j}^{-2}, \sigma_x^{-2}\right)$.

3. if $r < 1 - \left(\sqrt{2} - 1\right)$ and $\phi > \tilde{\phi}(r)$, there exist $\underline{\zeta}(\phi, r)$ and $\overline{\zeta}(\phi, r)$ such that

   (a) $\frac{\sigma_\theta^{-2}}{\sigma_x^{-2}} < \underline{\zeta}(\phi, r)$, opacity ($\tilde{\sigma}_{s,j}^{-2} = 0$) is socially optimal for all $\left(\sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}\right)$
   (b) $\frac{\sigma_\theta^{-2}}{\sigma_x^{-2}} > \overline{\zeta}(\phi, r)$, transparency ($\tilde{\sigma}_{s,j}^{-2} = \sigma_{y,f}^{-2} + \sigma_{y,h}^{-2}$) is socially optimal for all $\left(\sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}\right)$.
   (c) if $\underline{\zeta}(\phi, r) < \frac{\sigma_\theta^{-2}}{\sigma_x^{-2}} < \overline{\zeta}(\phi, r)$, there exist $\hat{\sigma}_y^{-2}(\phi, r)$ such that:
      - *if $\sigma_{y,h}^{-2} + \sigma_{y,f}^{-2} < \hat{\sigma}_y^{-2}$, opacity ($\tilde{\sigma}_{s,j}^{-2} = 0$) is socially optimal;*
      - *if $\sigma_{y,h}^{-2} + \sigma_{y,f}^{-2} = \hat{\sigma}_y^{-2}$, society is indifferent between opacity and transparency;*
      - *if $\sigma_{y,h}^{-2} + \sigma_{y,f}^{-2} > \hat{\sigma}_y^{-2}$, transparency ($\tilde{\sigma}_{s,j}^{-2} = \sigma_{y,f}^{-2} + \sigma_{y,h}^{-2}$) is socially optimal.*

Proof. See Appendix.

□

Remark 3. Proposition 5 is too bulky, isn't it? What if we rewrite it in the following way:

i) If $\phi < \tilde{\phi}(r)$, full transparency ($\tilde{\sigma}_{s,j}^{-2} = \sigma_{y,f}^{-2} + \sigma_{y,h}^{-2}$) is socially optimal for all $\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}\right)$

ii) There exists $\overline{\zeta}$ such that for any $\frac{\sigma_\theta^{-2}}{\sigma_x^{-2}} > \overline{\zeta}$, transparency ($\tilde{\sigma}_{s,j}^{-2} = \sigma_{y,f}^{-2} + \sigma_{y,h}^{-2}$) is socially optimal for all $\left(\sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \phi, r\right)$. [1]

[1] *Remark* 4. $\overline{\zeta} = \arg\max_{\zeta^j} \tilde{\rho}_S^j\left(\zeta^j\right)\Big|_{\phi=1}$

iii) For any $\frac{\sigma_\theta^{-2}+\sigma_{y,f}^{-2}+\sigma_{y,h}^{-2}}{\sigma_x^{-2}} > 1$ and $\phi > \tilde{\phi}(r)$, there exist $\underline{\zeta}(\phi, r) < \overline{\zeta}$ such that if $\frac{\sigma_\theta^{-2}}{\sigma_x^{-2}} < \underline{\zeta}(\phi, r)$, opacity ($\tilde{\sigma}_{s,j}^{-2} = 0$) is socially optimal, if $\frac{\sigma_\theta^{-2}}{\sigma_x^{-2}} > \underline{\zeta}(\phi, r)$, transparency ($\tilde{\sigma}_{s,j}^{-2} = \sigma_{y,f}^{-2} + \sigma_{y,h}^{-2}$) is socially optimal.

Moreover, $\frac{\partial \underline{\zeta}(\phi,r)}{\partial \phi} > 0$. All this means that an increase in technological spill-over increases the chance for opacity to be socially optimal.

If we agree on this, we can also rewrite Proposition 6.

5.1 Comparison of equilibrium with social optimum.

Proposition 7. *The non-cooperative Nash Equilibrium is socially optimal if and only if* $\phi \in \left[\underline{\phi}, \overline{\phi}\right]$.

Proof. See Appendix. □

Proposition 7 states that if we have transparency in equilibrium, this equilibrium coincides with the social optimum. If we have intermediate transparency (either home transparency and foreign opacity or home opacity and foreign transparency), this is never socially optimal.

Proposition 8. *For given* $\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$ *and* $\phi \notin \left[\underline{\phi}, \overline{\phi}\right]$, *the social optimum Pareto-dominates the non-cooperative Nash equilibrium.*

Proof. See Appendix. □

6 An application: International monetary policy games.

To be added.

7 Conclusion.

Is the result that the social value of public information may be negative sustained in an international environment? More broadly how to understand the communication policies designed by public policymakers in such an environment?

Reflecting in an international environment (more largely, in a multi-jurisdictional environment) considerably complicates the matter. Not only multiple sources of information but also multiple policymaker deciding on their communication policy must be taken into account. This creates a strategic dimension which is absent in the simple one-policymaker studied by Morris and Shin.

In turn, this strategic environment generates two issues:

1. assuming that these policymakers act non-cooperative for the sake of their own country, what is the equilibrium of the non-cooperative game they play? Is it unique?

2. how to evaluate this equilibrium (or possibly, equilibria) with respect to a normative criterion such as the Pareto criterion?

We address this issue by means of solving a communication non-cooperative game played between the country policymakers where these players have to decide upon which information in their possession to reveal to the public.

The multi-country model we use displays three types of spillovers: a real shock spillover, a “beauty contest effect à la Morris and Shin and the informational spillovers due to the fact that the information bits revealed by policymakers are free and reach the entire set of private agents in the whole economy. Policymakers can neither modify the information they reveal nor target a subset of agents benefiting from their information policy.

The results reached in this paper shed some light on the two questions mentioned above:

There exists a unique equilibrium. This equilibrium always involves some revelation by the policymakers. In other words, full opacity is never a solution. Which does not imply that it cannot be a superior policy. Actually we prove that for some subset of the parameter space, it is Pareto-dominant. This vindicates the Morris and Shin claim: in international environment the social value of public information may be negative. This is likely to occur when the beauty contest parameter is large relative to the real spillover parameter.

On the contrary the full transparency equilibrium can be obtained for intermediate values of the real spillover parameter and it is the Pareto-dominant solution.

Partial communication solutions can be the equilibrium outcome but can never be optimal.

These results can be readily applied to a standard international monetary game. Combining the communication tools with standard policy tools appears to be a challenging but intriguing task which is left to further research.

A Appendix

A.1 Choice between home transparency and home opacity

We rewrite the loss component ρ_j^j, which depends on $\zeta^j = \frac{\sigma_\theta^{-2}+\sigma_{s,j,j}^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}$:

$$\rho_j^j\left(\zeta^j\right) = (1-r)\left(\tilde{\rho}_j{}^j\left(\zeta^j\right)\sigma_x^2 + \frac{r}{2}(1-2\phi)\,\sigma_\theta^2\right),$$

where

$$\tilde{\rho}_j{}^j\left(\zeta^j\right) = \frac{r^2\left(4\phi^2-(2-r)^2(1-2\phi)^2\right)}{4(2-r)^2\zeta^j} + \frac{4\phi^2(1-r)}{(2-r)^2\left((1-r/2)+\zeta^j\right)} + \frac{(1-r)\,\phi^2r^2}{2(2-r)\left((1-r/2)+\zeta^j\right)^2} \tag{22}$$

$\tilde{\rho}_j{}^j\left(\zeta^j\right)$ is a monotonic transformation of loss ρ_j^j and $\zeta^j = \frac{\sigma_\theta^{-2}+\sigma_{s,j,j}^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}$ is the relative precision of public information about fundamental θ^j.

Let $\Delta_j^j\left(\zeta_1^j,\zeta_2^j\right)$denote the loss difference for two positive levels of relative precision ζ_1^j and ζ_2^j:

$$\Delta_j^j\left(\zeta_1^j,\zeta_2^j\right) = \tilde{\rho}_j{}^j\left(\zeta_2^j\right) - \tilde{\rho}_j{}^j\left(\zeta_1^j\right) \tag{23}$$

If $\Delta_j^j\left(\zeta_1^j,\zeta_2^j\right) > 0$, the policymaker prefers ζ_1^j over ζ_2^j. If $\Delta_j^j\left(\zeta_1^j,\zeta_2^j\right) < 0$, the policymaker prefers ζ_2^j over ζ_1^j. If $\Delta_j^j\left(\zeta_1^j,\zeta_2^j\right) = 0$, the policymaker is indifferent between ζ_1^j and ζ_2^j.

We get the following Lemma:

Lemma 2. *For given r and for any $0 < \zeta_1^j < \zeta_2^j$, there exists $0 < \phi^* < \frac{2-r}{6-2r}$, such that*

1. *if $\phi < \phi^*$, $\Delta_j^j\left(\zeta_1^j,\zeta_2^j\right) > 0$ and policymaker prefers the lower precision of his home information.*
2. *if $\phi = \phi^*$, $\Delta_j^j\left(\zeta_1^j,\zeta_2^j\right) = 0$ and policymaker is indifferent between two precisions of home information.*
3. *if $\phi > \phi^*$, $\Delta_j^j\left(\zeta_1^j,\zeta_2^j\right) < 0$ and policymaker prefers the higher precision of his home information.*

Proof. Let Δ_j^j denote the difference between the loss under home transparency and home opacity:

$$\Delta_j^j = \tilde{\rho}_j{}^j\left(\sigma_{y,j,j}^{-2},\sigma_{s,j,-j}^{-2}\right) - \tilde{\rho}_j{}^j\left(0,\sigma_{s,j,-j}^{-2}\right) \tag{24}$$

We can rewrite this difference in the following way:

$$\Delta_j^j = \int_0^{\sigma_{y,j,j}^{-2}} \frac{\partial\tilde{\rho}_j{}^j\left(\sigma_{s,j,j}^{-2},\sigma_{s,j,-j}^{-2}\right)}{\partial\sigma_{s,j,j}^{-2}}\,\mathrm{d}\sigma_{s,j,j}^{-2} \tag{25}$$

We now use (22) to get the derivative:

$$\frac{\partial \tilde{\rho}_j{}^j}{\partial \sigma_{s,j,j}^{-2}} = -\frac{r^2\left(4\phi^2 - (2-r)^2(2\phi-1)^2\right)}{4(2-r)^2\left[\sigma_{z,j}^{-2}\right]^2} - \\ -\frac{4\phi^2(1-r)}{(2-r)^2\left[(1-{}^r\!/\!{}_2)\,\sigma_x^{-2} + \sigma_{z,j}^{-2}\right]^2} - \frac{\phi^2 r^2(1-r)\,\sigma_x^{-2}}{(2-r)\left[(1-{}^r\!/\!{}_2)\,\sigma_x^{-2} + \sigma_{z,j}^{-2}\right]^3} \tag{26}$$

Notice that if $\phi \in \left[\frac{2-r}{6-2r}, 1\right]$, the value $\left(4\phi^2 - (2-r)^2(2\phi-1)^2\right)$ is positive. In this case, all the terms in ((26)) are negative. This means that the loss ρ_j^j is decreasing in precision $\sigma_{s,j,j}^{-2}$ for all possible $\sigma_{z,j}^{-2}$. Thus, $\frac{\partial \tilde{\rho}_j{}^j}{\partial \sigma_{s,j,j}^{-2}}$ is negative for all $\sigma_{s,j,j}^{-2} \in \left(0, \sigma_{y,j,j}^{-2}\right)$ and their sum Δ_j^j (25) is negative.
If $\phi \in \left(0, \frac{2-r}{6-2r}\right)$, the value $\left(4\phi^2 - (2-r)^2(2\phi-1)^2\right)$ is negative. Thus, the first term in (26) is positive while two other are negative. To decide on the sign of Δ_j^j, let us take the derivative of (25) over ϕ:

$$\frac{\partial \Delta_j^j}{\partial \phi} = \int_0^{\sigma_{y,j,j}^{-2}} \frac{\partial^2 \tilde{\rho}_j{}^j\left(\sigma_{s,j,j}^{-2}, \sigma_{s,j,-j}^{-2}\right)}{\partial \sigma_{s,j,j}^{-2}\partial\phi}\, \mathrm{d}\sigma_{s,j,j}^{-2} \tag{27}$$

From (26) we get:

$$\frac{\partial^2 \tilde{\rho}_j{}^j}{\partial \sigma_{s,j,j}^{-2}\partial\phi} = \frac{r^2\left(2\phi(1-r)(3-r) - (2-r)^2\right)}{(2-r)^2\left[\sigma_{z,j}^{-2}\right]^2} - \frac{8\phi(1-r)}{(2-r)^2\left[(1-{}^r\!/\!{}_2)\,\sigma_x^{-2} + \sigma_{z,j}^{-2}\right]^2} - \frac{2\phi r^2(1-r)\,\sigma_x^{-2}}{(2-r)\left[(1-{}^r\!/\!{}_2)\,\sigma_x^{-2} + \sigma_{z,j}^{-2}\right]^3} \tag{28}$$

Easy to show that $\left(2\phi(1-r)(3-r) - (2-r)^2\right)$ is negative if $\phi \in \left(0, \frac{2-r}{6-2r}\right)$. Thus, all the terms in (28) are negative. This means that $\frac{\partial \Delta_j^j}{\partial \phi}$ is negative and the loss difference Δ_j^j is decreasing in ϕ. We have shown earlier that $\Delta_j^j < 0$ for $\phi = \frac{2-r}{6-2r}$. If ϕ is equal to 0, $\tilde{\rho}_j{}^j = -\frac{r^2(2-r)^2}{4(2-r)\sigma_{z,j}^{-2}}$ and $\Delta_j^j = -\frac{r^2(2-r)^2}{4(2-r)\left(\sigma_\theta^{-2} + \sigma_{y,j,j}^{-2} + \sigma_{s,j,-j}^{-2}\right)} + \frac{r^2(2-r)^2}{4(2-r)\left(\sigma_\theta^{-2} + \sigma_{s,j,-j}^{-2}\right)} > 0$. Thus, their exist a value $\phi^* \in \left(0, \frac{2-r}{6-2r}\right)$ such that :

- Δ_j^j is positive if $\phi < \phi^*$
- Δ_j^j is equal to 0 if $\phi = \phi^*$
- Δ_j^j is negative if $\phi > \phi^*$

From this, Lemma 2 comes immediately. □

Turning to the properties of $\phi^*\left(r, \zeta_1^j, \zeta_2^j\right)$, we get:

Lemma 3. *Properties of $\phi^*\left(r, \zeta_1^j, \zeta_2^j\right)$ for $\zeta_1^j > 1$:*

1. *ϕ^* is a decreasing function of ζ_1^j : $\frac{\partial \phi^*}{\partial \zeta_1^j} < 0$*

2. *ϕ^* is a decreasing function of ζ_2^j : $\frac{\partial\phi^*}{\partial\zeta_2^j} < 0$*

3. *For any $\left(\zeta_1^j, \zeta_2^j\right)$, $\phi^*\left(0, \zeta_1^j, \zeta_2^j\right) = 0$. For any $\left(\zeta_1^j, \zeta_2^j\right)$, $\phi^*\left(1, \zeta_1^j, \zeta_2^j\right) = 1/4$*

4. *ϕ^* is monotonously increasing in r.*

Proof. By definition, $\Delta_j^j\left(\phi^*, r, \zeta_1^j, \zeta_2^j\right) = 0$ is implicit function of ϕ^*. We use it to get the effect of r on ϕ^* for $\zeta_1^j > 1$: $\frac{\partial\phi^*}{\partial r} = -\frac{\partial\Delta_j^j\left(\phi,r,\zeta_1^j,\zeta_2^j\right)/\partial r}{\partial\Delta_j^j\left(\phi,r,\zeta_1^j,\zeta_2^j\right)/\partial\phi}$. We can easily get that $\partial\Delta_j^{-j}\left(\phi,r,\zeta_1^{-j},\zeta_2^{-j}\right)/\partial\phi < 0$ and $\partial\Delta_j^{-j}\left(\phi,r,\zeta_1^{-j},\zeta_2^{-j}\right)/\partial r > 0$, thus $\frac{\partial\phi^*}{\partial r} > 0$. □

A.2 Choice between foreign transparency and foreign opacity

We rewrite the loss component ρ_j^{-j}, which depends on ζ^{-j}:

$$\rho_j^{-j}\left(\zeta^{-j}\right) = (1-r)\left(\tilde{\rho}_j^{-j}\left(\zeta^{-j}\right)\sigma_x^2 - \frac{r}{2}(1-2\phi)\,\sigma_\theta^2\right),$$

where

$$\tilde{\rho}_j^{-j}\left(\sigma_{s,-j,j}^{-2}, \sigma_{s,-j,-j}^{-2}\right) = \frac{\left[(1-\phi)^2 - r^2/4\,(1-2\phi)^2\right]}{\zeta^{-j}} - \frac{\phi^2 r\,(1-r)}{2\left((1-r/2)+\zeta^{-j}\right)^2} \tag{29}$$

$\tilde{\rho}_j^{-j}\left(\zeta^{-j}\right)$ is a monotonic transformation of loss $\rho_j^{-j}$and $\zeta^{-j} = \frac{\sigma_\theta^{-2}+\sigma_{s,-j,j}^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}$ is the relative precision of public information about θ^{-j}.

Let $\Delta_j^{-j}\left(\zeta_1^{-j}, \zeta_2^{-j}\right)$denote the loss difference for two positive levels of relative precision ζ_1^{-j} and ζ_2^{-j}:

$$\Delta_j^{-j}\left(\zeta_1^{-j}, \zeta_2^{-j}\right) = \tilde{\rho}_j^{-j}\left(\zeta_2^{-j}\right) - \tilde{\rho}_j^{-j}\left(\zeta_1^{-j}\right) \tag{30}$$

If $\Delta_j^{-j}\left(\zeta_1^{-j}, \zeta_2^{-j}\right) > 0$, the policymaker prefers ζ_1^{-j} over ζ_2^{-j}. If $\Delta_j^{-j}\left(\zeta_1^{-j}, \zeta_2^{-j}\right) < 0$, the policymaker prefers ζ_2^{-j} over ζ_1^{-j}. If $\Delta_j^{-j}\left(\zeta_1^{-j}, \zeta_2^{-j}\right) = 0$, the policymaker is indifferent between ζ_1^{-j} and ζ_2^{-j}.

We get the following Lemma:

Lemma 4. *For given r and for any $0 < \zeta_1^{-j} < \zeta_2^{-j}$, there exists $\frac{1}{2} < \phi^{**} < \frac{1+r/2}{1+r}$, such that*

1. *if $\phi < \phi^{**}$, $\Delta_j^{-j}\left(\zeta_1^{-j}, \zeta_2^{-j}\right) < 0$ and policymaker prefers the higher precision of foreign public information*

2. *if $\phi = \phi^{**}$, $\Delta_j^{-j}\left(\zeta_1^{-j}, \zeta_2^{-j}\right) = 0$ and policymaker is indifferent between two precisions of foreign public information.*

3. *if $\phi > \phi^{**}$, $\Delta_j^{-j}\left(\zeta_1^{-j}, \zeta_2^{-j}\right) > 0$ and policymaker prefers the lower precision of foreign public information.*

Proof. Let Δ_j^{-j} denote the difference between the loss under foreign transparency and foreign opacity:

$$\Delta_j^{-j} = \tilde{\rho}_j^{-j}\left(\sigma_{y,-j,j}^{-2}, \sigma_{s,-j,-j}^{-2}\right) - \tilde{\rho}_j^{-j}\left(0, \sigma_{s,-j,-j}^{-2}\right) \tag{31}$$

We can rewrite this difference in the following way:

$$\Delta_j^{-j} = \int_0^{\sigma_{y,-j,j}^{-2}} \frac{\partial \tilde{\rho}_j^{-j}\left(\sigma_{s,-j,j}^{-2}, \sigma_{s,-j,-j}^{-2}\right)}{\partial \sigma_{s,-j,j}^{-2}} \, \mathrm{d}\sigma_{s,-j,j}^{-2} \tag{32}$$

We now use (29) to get the derivative:

$$\frac{\partial \tilde{\rho}_j^{-j}}{\partial \sigma_{s,-j,j}^{-2}} = -\frac{\left[(1-\phi)^2 - {}^{r^2}\!/_4 (1-2\phi)^2\right]}{\left(\sigma_{z,-j}^{-2}\right)^2} + r\frac{\phi^2 (1-r)\,\sigma_x^{-2}}{\left((1-{}^{r}\!/_2)\,\sigma_x^{-2} + \sigma_{z,-j}^{-2}\right)^3} \tag{33}$$

Notice that if $\phi \in \left[\frac{1+r/2}{1+r}, 1\right]$, the value $\left[(1-\phi)^2 - {}^{r^2}\!/_4 (1-2\phi)^2\right]$ is negative. In this case, all the terms in (33) are positive. This means that the loss ρ_j^{-j} is increasing in precision $\sigma_{s,-j,j}^{-2}$ for all possible $\sigma_{z,-j}^{-2}$. Thus, $\frac{\partial \tilde{\rho}_j^{-j}}{\partial \sigma_{s,-j,j}^{-2}}$ is negative for all $\sigma_{s,-j,j}^{-2} \in \left(0, \sigma_{y,-j,j}^{-2}\right)$ and their sum Δ_j^{-j} (32) is positive. If $\phi \in \left(0, \frac{1+r/2}{1+r}\right)$, the value$\left[(1-\phi)^2 - {}^{r^2}\!/_4 (1-2\phi)^2\right]$ is positive. Thus, the first term in (33) is negative while the other is positive. To decide on the sign of Δ_j^{-j}, let us take the derivative of (32) over ϕ:

$$\frac{\partial \Delta_j^{-j}}{\partial \phi} = \int_0^{\sigma_{y,-j,j}^{-2}} \frac{\partial^2 \tilde{\rho}_j^{-j}\left(\sigma_{s,-j,j}^{-2}, \sigma_{s,-j,-j}^{-2}\right)}{\partial \sigma_{s,-j,j}^{-2} \partial \phi} \, \mathrm{d}\sigma_{s,-j,j}^{-2} \tag{34}$$

From (33) we get:

$$\frac{\partial^2 \tilde{\rho}_j^{-j}}{\partial \sigma_{s,-j,j}^{-2} \partial \phi} = -\frac{\left[(2\phi - 1)(1-r^2) - 1\right]}{\left(\sigma_{z,-j}^{-2}\right)^2} + r\frac{2\phi (1-r)\,\sigma_x^{-2}}{\left((1-{}^{r}\!/_2)\,\sigma_x^{-2} + \sigma_{z,-j}^{-2}\right)^3} \tag{35}$$

The coefficient $\left[(2\phi - 1)(1-r^2) - 1\right]$ depends positively on ϕ. If $\phi = 1$, this coefficient equals to $\left[1 - r^2 - 1\right] = -r^2$, thus, is negative. From here we can conclude that it is negative for all values of ϕ. Thus, the both terms in (35) are positive. Thus, the value $\frac{\partial \tilde{\rho}_j^{-j}}{\partial \sigma_{z,-j}^{-2}}$ is increasing in ϕ. We have shown earlier that Δ_j^{-j} is positive if $\phi \in \left[\frac{1+r/2}{1+r}, 1\right]$. For ϕ equal to $1/2$, Δ_j^{-j} is negative. Consequently, their exist a value $\phi^{**} \in \left(1/2, \frac{1+r/2}{1+r}\right)$ such that :

- Δ_j^{-j} is positive if $\phi > \phi^{**}$
- Δ_j^{-j} is equal to 0 if $\phi = \phi^{**}$
- Δ_j^{-j} is negative if $\phi < \phi^{**}$

From this, Lemma 2 comes immediately. $\square$

Turning to the properties of $\phi^{**}\left(r,\zeta_1^{-j},\zeta_2^{-j}\right)$, we get:

Lemma 5. *Properties of* $\phi^{**}\left(r,\zeta_1^{-j},\zeta_2^{-j}\right)$*:*

1. ϕ^{**} *is a decreasing function of* ζ_1^{-j} : $\frac{\partial\phi^{**}}{\partial\zeta_1^{-j}}<0$
2. ϕ^{**} *is a decreasing function of* ζ_2^{-j} : $\frac{\partial\phi^{**}}{\partial\zeta_2^{-j}}<0$
3. *For any* $\left(\zeta_1^{-j},\zeta_2^{-j}\right)$, $\phi^{**}\left(0,\zeta_1^{-j},\zeta_2^{-j}\right)=1.$. *For any* $\left(\zeta_1^{-j},\zeta_2^{-j}\right)$, $\phi^{**}\left(1,\zeta_1^{-j},\zeta_2^{-j}\right)=3/4.$
4. ϕ^{**} *is a decreasing function of* r: $\frac{\partial\phi^{**}}{\partial r}<0$

Proof. By definition, $\Delta_j^{-j}\left(\phi^{**},r,\zeta_1^{-j},\zeta_2^{-j}\right)=0$ is implicit function of ϕ^{**}. We use it to get the effect of r on ϕ^{**}: $\frac{\partial\phi^{**}}{\partial r}=-\frac{\partial\Delta_j^{-j}\left(\phi,r,\zeta_1^{-j},\zeta_2^{-j}\right)/\partial r}{\partial\Delta_j^{-j}\left(\phi,r,\zeta_1^{-j},\zeta_2^{-j}\right)/\partial\phi}$. We can easily get that $\partial\Delta_j^{-j}\left(\phi,r,\zeta_1^{-j},\zeta_2^{-j}\right)/\partial\phi>0$ and $\partial\Delta_j^{-j}\left(\phi,r,\zeta_1^{-j},\zeta_2^{-j}\right)/\partial r>0$, thus $\frac{\partial\phi^{**}}{\partial r}<0$. $\square$

A.3 Proof of Proposition 2

Comes immediately from Lemma 2 and Lemma 3.

A.4 Proof of Proposition 3.

Comes from Lemma 3 if $\underline{\phi}=\phi^{*}\left(\frac{\sigma_\theta^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}},\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}\right)$ and $\overline{\phi}=\phi^{**}\left(\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}}{\sigma_x^{-2}},\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}\right)$.

A.5 Proof of Proposition 4.

Comes from Proposition 2 and Proposition 3.

A.6 Proof of Lemma 1.

We can derive social loss (21) over $\sigma_{z,j}^{-2}$:

$$\frac{\partial \rho_S^j}{\partial \sigma_{z,j}^{-2}} = -\frac{\left(4r^2\phi^2 - 2r^2\left(2-r\right)^2\left(1-2\phi\right)^2 + 4\left(2-r\right)^2\left(1-\phi\right)^2\right)}{4\left(2-r\right)^2\left(\sigma_{z,j}^{-2}\right)^2} - \frac{4\phi^2\left(1-r\right)}{\left(2-r\right)^2\left(\left(1-{}^{r}\!/\!{}_{2}\right)\sigma_x^{-2} + \sigma_{z,j}^{-2}\right)^2} +$$

$$+\frac{2\left(1-r\right)^2\sigma_x^{-2}\phi^2 r}{\left(2-r\right)\left(\left(1-{}^{r}\!/\!{}_{2}\right)\sigma_x^{-2} + \sigma_{z,j}^{-2}\right)^3} = -\frac{\left(r^2\left(4\phi^2 - \left(2-r\right)^2\left(1-2\phi\right)^2\right) + \left(2-r\right)^2\left(4\left(1-\phi\right)^2 - r^2\left(1-2\phi\right)^2\right.\right.}{4\left(2-r\right)^2\left(\sigma_{z,j}^{-2}\right)^2}$$

$$-2\phi^2\left(1-r\right)\frac{2\left(\left(1-{}^{r}\!/\!{}_{2}\right)\sigma_x^{-2} + \sigma_{z,j}^{-2}\right) - \left(2-r\right)\left(1-r\right)\sigma_x^{-2} r}{\left(2-r\right)^2\left(\left(1-{}^{r}\!/\!{}_{2}\right)\sigma_x^{-2} + \sigma_{z,j}^{-2}\right)^3} =$$

$$= -\frac{2\left(\left(2-r\right) - 2\phi\left(1-r\right)\right)\left(\left(2-r\right)\left(2-r^2\right) - 2\phi\left(r^3 - 3r^2 + 2\right)\right)}{4\left(2-r\right)^2\left(\sigma_{z,j}^{-2}\right)^2} - 2\phi^2\left(1-r\right)\frac{2\sigma_{z,j}^{-2} + \left(2-r\right)\sigma_x^{-2} - \left(2-r\right)\left(\right.}{\left(2-r\right)^2\left(\left(1-{}^{r}\!/\!{}_{2}\right)\sigma_x^{-2} + \right.}$$

$$= -\frac{\left(\left(2-r\right) - 2\phi\left(1-r\right)\right)\left(\left(2-r\right)\left(2-r^2\right) - 2\phi\left(1-r\right)\left(3-\left(1-r\right)^2\right)\right)}{2\left(2-r\right)^2\sigma_{z,j}^{-2}} - 2\phi^2\left(1-r\right)\frac{2\sigma_{z,j}^{-2} + \left(2-r\right)\sigma_x^{-2}\left(1-\right.}{\left(2-r\right)^2\left(\left(1-{}^{r}\!/\!{}_{2}\right)\sigma_x^{-2}\right.} \tag{36}$$

$$= -\frac{\left(\left(2-r\right) - 2\phi\left(1-r\right)\right)\left(\left(2-r\right)\left(2-r^2\right) - 2\phi\left(1-r\right)\left(3-\left(1-r\right)^2\right)\right)}{2\left(2-r\right)^2\sigma_{z,j}^{-2}} - 2\phi^2\left(1-r\right)\frac{2\sigma_{z,j}^{-2} + \left(2-r\right)\sigma_x^{-2}\left(1-\right.}{\left(2-r\right)^2\left(\left(1-{}^{r}\!/\!{}_{2}\right)\sigma_x^{-2}\right.} \tag{37}$$

The second term in ((36)) is negative, the first term is negative if the numerator is positive. Expression $\left(\left(2-r\right) - 2\phi\left(1-r\right)\right)$ is positive, expression $\left(\left(2-r\right)\left(2-r^2\right) - 2\phi\left(1-r\right)\left(3-\left(1-r\right)^2\right)\right)$ is positive if $\phi < \frac{(2-r)\left(2-r^2\right)}{2(1-r)\left(3-(1-r)^2\right)}$. It is easy to show that $\frac{(2-r)\left(2-r^2\right)}{2(1-r)\left(3-(1-r)^2\right)}$ is greater than 1, if $r > 2-\sqrt{2}$. This means that for all possible values of ϕ expression $\left(\left(2-r\right)\left(2-r^2\right) - 2\phi\left(1-r\right)\left(3-\left(1-r\right)^2\right)\right)$ is positive and $\frac{\partial \rho_S^j}{\partial \sigma_{z,j}^{-2}}$ is negative for all values of $\sigma_{z,j}^{-2}$. Thus, the social loss is decreasing in the precision of public information.

If $r < 2-\sqrt{2}$, expression $\frac{(2-r)\left(2-r^2\right)}{2(1-r)\left(3-(1-r)^2\right)}$ is less than 1, thus there exist $\tilde{\phi} = \frac{(2-r)\left(2-r^2\right)}{2(1-r)\left(3-(1-r)^2\right)}$, such that for all $\phi < \tilde{\phi}$, the both terms in ((36)) are negative and the social loss is decreasing in the precision of public information for all values of $\sigma_{z,j}^{-2}$.

If $r < 2-\sqrt{2}$ and $\phi < \tilde{\phi}$, the first term in (36) is positive and the second term it is negative, from here 1 comes directly.

A.7 Proof of Proposition 5.

Comes from Lemma 1.

A.8 Proof of Proposition 6.

Comes from Lemma 1 and Proposition 5.

A.9 Proof of Proposition 7.

We can show that $\tilde{\phi} \geq \frac{1+r/2}{1+r} \geq \overline{\phi}$. From that, Proposition 7 derives immediately.

A.10 Proof of Proposition 8.

As the social optimum minimizes the sum of losses,

$$\Delta_j^j \left(\left(\sigma_{s,j,j}^{-2}\right)^* + \left(\sigma_{s,-j,j}^{-2}\right)^*, \tilde{\sigma}_{s,j}^{-2} \right) + \Delta_{-j}^j \left(\left(\sigma_{s,j,j}^{-2}\right)^* + \left(\sigma_{s,j,-j}^{-2}\right)^*, \tilde{\sigma}_{s,j}^{-2} \right) < 0.$$

Due to symmetry, $\Delta_{-j}^j \left(\left(\sigma_{s,j,j}^{-2}\right)^* + \left(\sigma_{s,j,-j}^{-2}\right)^*, \tilde{\sigma}_{s,j}^{-2} \right) = \Delta_j^{-j} \left(\left(\sigma_{s,-j,j}^{-2}\right)^* + \left(\sigma_{s,-j,-j}^{-2}\right)^*, \tilde{\sigma}_{s,-j}^{-2} \right)$. Thus,

$$\Delta_j^j \left(\left(\sigma_{s,j,j}^{-2}\right)^* + \left(\sigma_{s,-j,j}^{-2}\right)^*, \tilde{\sigma}_{s,j}^{-2} \right) + \Delta_j^{-j} \left(\left(\sigma_{s,-j,j}^{-2}\right)^* + \left(\sigma_{s,-j,-j}^{-2}\right)^*, \tilde{\sigma}_{s,-j}^{-2} \right) < 0.$$

This means that each policymaker gets a negative loss difference when moving from the equilibrium to the social optimum. Thus, the social optimum is Pareto-superior.