Workers' Remittances and Borrowing Constraints in Recipient Countries *

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Abstract

This paper focuses on the role played by remittances in constrained economies. We consider an overlapping generations economy in which households have access to International Capital Market and the possibility to borrow to finance children education in order to receive remittances. Following the literature, we assume that remittances relax borrowing constraints. These inflows may reduce or increase domestic savings and capital accumulation, according to the level of capital inflows constraint. Because of the OLG structure, the country may either be constrained or unconstrained in the long run. Remittances may make the initially constrained economy converging to the unconstrained steady state.

Keywords: Remittances; Overlapping generations; Capital inflows constraint; Capital accumulation.

JEL classification: O11; F24; F41; C62.

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1 Introduction

An issue of globalization and migratory flows is the development of workers' remittances. These currency transfers sent by migrants to their family stayed in the home country are exponentially growing and have reached significant levels, particularly at the scale of GDP in developing countries. According to Chami and Fullenkamp (2013) [14], remittances represent more than 1% of GDP for 108 countries, at least 5% of GDP for 44 countries and more than 5% for 22 countries. Barajas *et al.* (2008) [6] argue that the relative amount attains one quarter of GDP in some countries, and even nearly half of GDP for other as Tajikistan for instance. The World Bank evaluates the amount of remittances to more than 500 billion of US Dollars. In 2000, this amount was 100 billion against to only 10 billion in the seventies. Each country is receiving more and more inflows, and more and more countries are receiving them. According to Acosta *et al.* (2009) [1] workers' remittances represent two-third of foreign direct investments (FDI) in average and 2% of GDP in the developing world. These flows exceed official aids, and even FDI for some countries. Moreover, according to Ratha (2005) [27] the real amounts including unofficial flows would be at least 50 percent larger.

Empirical studies point out the great impact of remittances on poverty reduction, on health, and on mortality (see Adams and Page (2005) [2], Ratha (2013) [29]). They argue that remittances are flows, which have the greater impact on poverty reduction of households. Positive effects on education are also underlined. Remittances increase the number of children attending school, but also the length of studies. This is the result of Edwards and Ureta (2003) [19] for El Salvador. They estimate that the median amount of remittances falls the hazard that a child will not attain school by 25% in rural areas. Moreover, according to Calero *et al.* (2009) [11], remittances raise the school enrollment in Ecuador. For them, the average impact of remittances on education is an increase of 2.59 percentage points of the school enrollment. Furthermore, the impact is particularly important in rural areas for girls. Zhunio *et al.* (2012) [32] also reveal positive effects on education for a sample of 69 low and middle income countries. A 1% increase of inflows implies a 0.12% raise in the enrollment rate in the secondary school and a 0.09% increase for the primary.

Remittances are mainly used to consume and not much to invest, particularly so due to the low volatility. The positive effect on income induces a negative correlation between remittances and saving. However, as we will explain in the next paragraph, remittances are a source for the financial development in recipient countries with a positive effect on investment and saving. Consequently, the impact on saving, is ambiguous and country-specific. Studies estimate positive or negative effects on saving according to the country and the period. For instance, Morton et al. (2010) [25] find a negative correlation between remittances and saving. Some econometric studies, like Athukorala and Sen (2004) [4] for India, and Hossain (2014) [23] for 63 developing countries, find a negative impact. These two studies are based on the life-cycle model with different estimation methods. In the first, a 1 percentage point rise of remittances relative to the Gross National Disposable Income decreases the saving in the long-run by 0.71 percentage point. For Hossain, an increase of 1 percentage point of remittances reduces the saving by 1.215 percentage point. However, other studies show that remittances have a positive impact on saving. This is the result shown by Baldé (2011) [5] in Sub-Saharan Africa. A 10% increase of remittances in these sub-Saharan countries raises the saving by 7%. Finally, Ziesemer (2012) [33] argues that the total effect of remittances on investment is positive. The point here, for the positive effect is that remittances increase the propensity to invest and reduce the propensity to consume. Even if consumption is raised, an increase of investment may reduce the propensity to

consume. Furthermore, saving reduces the cost of capital which increases the level of investment in the economy. The ratio credits over GDP can increase in this case.

According to the literature, remittances foster the financial development by means of access to the international financial market. For instance, Barajas et al. (2008) [6] argue that with remittances, the banking system is more developed. Brown et al. (2013) [10] explain the common idea of the "induced financial literacy hypothesis" whereby households receiving remittances are more willing to make use of financial services. Thus, remittances improve the financial development when households deposit them from abroad into banks. This creates an increase in deposits but also in the market for credit with the development of the financial sector in the country. The principal consequence of the financial development is indeed the reduction of borrowing constraints, a major obstacle to investment (with also the lack of infrastructures). Moreover, since remittances are considered as an increase in income, lenders detect a lower risk of default, and lend to allow households to invest. This is due to the low volatility. By the way, according to Barajas et al. (2008) [6], the official development aid is 3 times more volatile than remittances, FDI are 22 times more volatile and exports, 74 times. Ratha (2013) [29] argues that during the last global crisis, FDI have been more impacted than remittances. According to him, between 2009 and 2011, FDI have increased by less than one percent, while remittances have continued to increase (even if they have decreased at the beginning of the crisis). Remittances are more stable than other flows when shocks appear but particularly, they are contra-cyclical. By reducing credit constraints, remittances can increase investment in physical capital. Moreover, with more credit, entrepreneurs can exploit economies of scales. Hence, according to empirical studies, the ratio credit over GDP increases with remittances (see Aggarwal et al. (2011) [3]). For this reason we will assume in our model that remittances relax the credit constraints in order to analyze their impact in recipient countries.

Following empirical studies, we theoretically focus on the effects of remittances on capital accumulation. The framework is more or less based on Obstfeld and Rogoff (1996) [26] where economies have access to the international capital market with a constraint on inflows. Nevertheless, we add education and remittances in the standard model¹. The agent has available a new asset which has the particularity of directly relax the borrowing constraint.

We show that, in our framework, the education of children could be perceived as a substitute for usual saving, because future remittances will provide for the last period of consumption. Therefore they could decrease the saving and the capital stock through a wealth effect. Nevertheless, by taking into account the investment possibility due to the slackening of credit constraints allowed by remittances, they could increase saving and investment in recipient countries. Hence, we show the role of borrowing constraints on the impact of remittances. According to the level of the constraint, remittances can increase or decrease the capital stock. If the part of remittances used as collateral is large enough, remittances have a positive impact on the capital-labor ratio. On the contrary, they have a negative impact if the part used as collateral is low enough. Furthermore, the effect tends to be negative in economies where the credit constraints are low even without remittances. We also determine how these inflows can bring developing economies to the well-known small open economy framework in the long-run with a perfect financial integration. We make comparisons between recipient and non-recipient countries.

This model allows to understand the fact that remittances have a positive impact in economies with an initially less developed financial system. Indeed, in our model, the more an economy would be constrained without remittances, the more remittances tend to have a positive impact.

¹Parents educate their children in order to receive remittances when they have migrated.

However, the less an economy would be constrained without remittances, the more remittances tend to have a negative impact. This result is empirically consistent. Indeed, Giuliano and Ruiz-Arranz (2009) [22] show how the financial sector influences the effect of remittances in the economy (data for 100 developing countries). According to them the impact is better in an economy with a less-developed financial system. Indeed, in such an economy, remittances provide a way to invest. Moreover, with a threshold estimation, their results show a positive effect in economies with a low financial system and particularly a negative effect in financially integrated economies. Sobiech (2015) [30] also argues that the impact of remittances decreases with the level of financial development and can be negative. According to this author, remittances and financial development without remittances is large enough. Indeed, in these countries, remittances can reduce the labor supply and the saving because households can invest even without remittances with low credit constraints. Nevertheless, in countries with a low level of financial development, the relax of credit constraints due to remittances allows to finance growth-enhancing activities.

This paper is organized as follows: the next section sets up the simple model based on Obstfeld and Rogoff for the recipient economy. Section 3 proves the existence of a unique steady state and shows the impact of entering remittances on capital accumulation through access to international capital market. This section also demonstrates how remittances can bring a constrained economy to a small open economy framework. Finally the last section contains concluding remarks.

2 The model

The model is a variant of Obstfeld and Rogoff (1996) [26] who consider a small open overlapping generation economy facing a constraint on capital inflows. Nevertheless, we introduce remittances to analyze their effects in a recipient economy with access to international capital market.

The economy is composed by households (modeled by a representative agent) and a production sector (modeled by a representative firm). Decisions are taken in each point of discrete time, with an initial condition in period t = 0. The produced output can be consumed or invested as physical capital.

2.1 The households

In the recipient economy, households live for three periods: the childhood, the adulthood and the retirement period. Nevertheless, during the first period education is the unique decision variable, controlled by the parents.

We denote as N_{t-1} the number of individuals born in period t-1. After the childhood, they can migrate in a foreign country or remain in the home country to work at a competitive wage rate. We assume that an exogenous faction p of children in each family is successfully migrating per period. The number of workers in period t, denoted by N_t^w , satisfies $N_t^w = (1-p) N_{t-1}$. Each of them gives birth to 1 + n children in period t, with n positive and constant over time. Therefore, p(1+n) children migrate in each family and remit to their parents. The number of workers in period t + 1 is $N_{t+1}^w = (1 + n) (1 - p)^2 N_{t-1}$. Hence, the evolution of workers between periods is given by:

$$\frac{N_{t+1}^w}{N_t^w} = (1+n)\,(1-p)$$

Following Obstfeld and Rogoff, national savings can be held into the capital stock and net foreign assets. The recipient economy faces a constraint on capital inflows.

Assumption 1. The net foreign assets (NFA) satisfy:

$$F_{t+1} \ge -\eta N_t^w w_t - \omega N_t^w p(1+n) B_{t+1}^e$$

with $\eta \in [0;1]$, $\omega \in [0;1]$ and B_{t+1}^e is the expected amount of sent money by each emigrated child.

Under the assumption 1, the return of capital in the domestic country R_t may be higher than the world interest rate R_* . Since return of each stock may be different, the agent chooses the amount of national savings s_t and the allocation between the two types of assets. The agents can also borrow from the worldwide market. A spread between the domestic and the foreign return is a source of income. In other terms, they can borrow from international to lend for physical capital if $R_t > R_*$. Hence, they realize a capital gain. Nevertheless, the households face a borrowing constraint, banks cannot lend more than a certain amount at the world market interest rate. In recipient economies, the households can borrow to finance education and receive more remittances. Moreover, as remittances are considered as stable flows, they can reduce the constraints on credits for individuals. Therefore, they can borrow more if they will receive remittances. If the constraint binds, we have $R_{t+1} > R_*$.

The parameters η and ω express the ease of access to the international capital market. This ease may depend on institutional features like restrictions to capital due to capital market imperfection or sovereign risks for instance. Following Obstfeld and Rogoff, without remittances, the domestic country can borrow a proportion η of wages at the international². The greater η , the less constrained is the economy. Nevertheless, we introduce the fact that the recipient economy can borrow more. Therefore, the lower ω , the more constrained the economy is to capital inflows.

This assumption whereby agents can borrow more if they receive remittances is based on the empirical literature. First of all, there is a consensus on the fact that remittances develop the financial system and slack the credit constraints. If they are deposited in domestic banks, the households can take advantage of the financial activities. According to Ratha (2005) [27], receiving remittances is a way to facilitate the access to international capital market, particularly for the poor countries with an improvement of the credit rating from agencies³. The author argues that with remittances, inflows from the international capital market can raise by 9 billion dollars in developing countries whose 3 for low-income countries. This necessarily increases the ratio of credit over GDP⁴. Secondly, we use the defined form for F_{t+1} in recipient countries due to the fact that even future remittances can be used as collateral to borrow on the international capital market. Ratha (2005) [28], explains this phenomenon which is due to the very low

²If there is no remittances, the constraint becomes $F_{t+1} \ge -\eta N_t w_t$, see Gente (2006) [20] for more explanations. ³According to computations, Ratha argues that the rating of Haiti could increase from CCC to B- thanks to remittances. The rating could increase from B- to B+ in Lebanon.

⁴Aggarwal *et al.* (2011) [3], argue that an increase of one percentage point of remittances over GDP raises the ratio of deposits over GDP by 0.35-037 percentage point and the ratio of credit over GDP by 0.29 percentage point. Their dataset covers the period 1975-2007 for 109 developing countries.

volatility of these flows⁵. However, according to Brown *et al.* (2013) [10] remittances could at the micro-economic level reduce the need to opening a bank account. The positive impact on credit can be reduced⁶.

We assume that developing economies are initially constrained. The gap on interest rates can be interpreted as a risk premium. Indeed, the access to international capital market is not perfect in many developing countries.

In his home country, an agent born in period t - 1, who does not migrate, draws utility from consumption c_t in period t when middle-aged, and consumption d_{t+1} in period t + 1 when old. He supplies inelastically labor in period t and receives a wage w_t , which is dedicated to consumption, investment in physical capital for the next period $^7 (1-p) (1+n) k_{t+1}$, investment in NFA⁸ $(1-p) (1+n) k_{t+1}$ and education of children who can emigrate in another country with more favorable economic conditions $p (1+n) e_t$. The last period is a retirement period where households spend the return of the saving invested in physical capital at the rate $R_{t+1} = 1+r_{t+1}$, and in foreign assets at the rate R_* , plus the amount of remittances $p(1+n)B_{t+1}^*$, (with B_{t+1}^* the amount that each child remits) to consume.

Assumption 2. The remitted amount by each emigrated child is defined by $B_{t+1}^* = \gamma \alpha e_t^{\lambda}$.

This assumption is the result of the utility maximization of each emigrated child under each period budget constraint⁹. The parameter γ reflects the ascendant altruism for the emigrated child toward family. The higher γ , the higher remittances are. The term αe_t^{λ} is the foreign wage for migrant. As there is p(1+n) children who migrate in each family, the total remitted amount is therefore $p(1+n) \gamma \alpha e_t^{\lambda}$.

The life-cycle income for agents is the wage, the return of education though remittances and the capital gain by borrowing at world rate to invest in domestic capital whose higher return in constrained economies.

$$\begin{array}{ll} \underset{c_{t+1}^{*},B_{t+1}^{*},s_{t+1}^{*},d_{t+2}^{*}}{\text{Max}} & \sigma \ln c_{t+1}^{*} + \beta \ln d_{t+2}^{*} + \gamma \ln B_{t+1}^{*} \\ \text{s.t.} & w_{t+1}^{*}(e_{t}) = c_{t+1}^{*} + B_{t+1}^{*} + s_{t+1}^{*} \\ & s_{t+1}^{*}R_{t+2}^{*} = d_{t+2}^{*} \\ & \sigma + \beta + \gamma = 1 \end{array}$$

The parameter $\sigma \in [0; 1]$ is the weight of consumption in first period of life. The parameter $\beta \in [0; 1]$ is the weight of second period consumption, and the parameter $\gamma \in [0; 1]$ represents the altruism of the migrant toward family. The sent amount by each emigrated child is therefore a fraction of his wage.

 $^{^{5}}$ Some banks (in Brazil for instance) allow for securitization of future remittances in order to develop the external financing according to Ratha.

 $^{^{6}}$ Brown *et al.* find a negative correlation between remittances and the likelihoud to open a bank account in Azerbaijan. However, the effect is positive for Kyrgyzstan.

⁷The stock of capital satisfies: $K_{t+1} = k_{t+1}l_{t+1} \Leftrightarrow K_{t+1} = k_{t+1}N_t^w (1-p)(1+n).$

⁸The Net Foreign Assets satisfy: $F_{t+1} = f_{t+1}N_{t+1}^* \Leftrightarrow F_{t+1} = f_{t+1}N_t^w (1-p)(1+n).$

⁹Each emigrated child draws utility from consumption when middle-aged c_{t+1}^* and old d_{t+2}^* , but also from the amount of remittances B_{t+1}^* . When middle-aged, the wage is defined by $w_{t+1}^*(e_t) = \alpha e_t^{\lambda}$ with $0 < \lambda < 1$. It positively depends on education with decreasing returns, and is used to consume, save and remit. In the last period, the return of saving allows for consumption. The program of each child is:

Let us assume that at the equilibrium $B_{t+1}^e = B_{t+1}^*$. The agent maximizes the following program:

$$\begin{split} \underset{c_{t},e_{t},k_{t+1},f_{t+1},d_{t+1}}{\text{Max}} & (1-\delta)\ln c_{t} + \delta \ln d_{t+1} \\ \text{s.t.} & w_{t} = c_{t} + (1-p)\left(1+n\right)k_{t+1} + (1-p)\left(1+n\right)f_{t+1} + p\left(1+n\right)e_{t} \\ & p\left(1+n\right)\gamma\alpha e_{t}^{\lambda} + (1-p)\left(1+n\right)k_{t+1}R_{t+1} + (1-p)\left(1+n\right)f_{t+1}R_{*} = d_{t+1} \\ & f_{t+1} \geqslant -\frac{\eta w_{t}}{(1-p)\left(1+n\right)} - \frac{\omega p\left(1+n\right)\gamma\alpha e_{t}^{\lambda}}{(1-p)\left(1+n\right)} \end{split}$$

where the parameter $\delta \in [0; 1]$ is the weight of second period of consumption in the life-cycle utility function. It expresses the actualization factor.

2.2 The firms

The representative firm produces a unique output good at each period using physical capital (K) and labor (L) with a neoclassical production function $F(K_t, L_t)$. The part of national saving accumulated in order to invest in physical capital during a period determines the stock of capital in the next period with a fully capital depreciation across period. We consider a Cobb-Douglas production function (increasing for each argument, concave over \mathbb{R}_{++} and homogeneous of degree one) defined in period t by:

$$F(K_t, L_t) = AK_t^s L_t^{1-s}$$

The parameter A is the total factor productivity level and s is the elasticity of revenue with respect to capital stock.

Assumption 3. s < 1/2

This assumption is empirically relevant. It follows empirical studies (see: Bernanke and Gürkaynak (2002) [9], and Caselli (2005) [12]).

The firm profit is written, by normalizing the price to one, according:

$$\Pi_t = F(K_t, L_t) - w_t L_t - R_t K_t$$

Let us denote $f(k_t) = Ak_t^s$, the production function expressed in its intensive form with $k_t = \frac{K_t}{L_t}$. Therefore, the maximization of profit trough a competitive framework with respect to labor and capital per period determines the competitive wage and the interest return which satisfy:

$$w_t = (1-s) Ak_t^s \tag{1}$$

$$R_t = sAk_t^{s-1} \tag{2}$$

2.3 The temporary equilibrium in the constrained case

Let us first assume that the constraint initially binds¹⁰ with:

$$f_{t+1} = -\frac{\eta w_t}{(1-p)(1+n)} - \frac{\omega p(1+n) \gamma \alpha e_t^{\lambda}}{(1-p)(1+n)}$$
(3)

¹⁰A constrained temporary equilibrium is such that $R_{t+1} > R_*$ in the recipient developing country. The agent is constrained on the borrowing amount.

Then, the constrained maximization problem gives:

$$e_t = \left(\frac{\gamma \alpha \lambda \left(1 + \omega \left(R_{t+1} - R_*\right)\right)}{R_{t+1}}\right)^{\frac{1}{1-\lambda}} \tag{4}$$

$$k_{t+1}(1-p)(1+n) = \frac{w_t \left(\delta R_{t+1}(1+\eta) + (1-\delta)\eta R_*\right)}{R_{t+1}} - p(1+n) \times \left(\frac{\gamma \alpha \lambda \left(1 + \omega \left(R_{t+1} - R_*\right)\right)}{R_{t+1}}\right)^{\frac{1}{1-\lambda}} \left(\delta - \frac{\omega \delta R_{t+1} - (1-\delta)(1-\omega R_*)}{\lambda \left(1 + \omega \left(R_{t+1} - R_*\right)\right)}\right)$$
(5)

These equations determine the partial equilibrium.

Assumption 4. $\omega R_* < 1$

To be in line with the literature (see: Becker and Tomes (1976) [8], Edwards and Ureta (2003) [19]) the assumption 4 means that education is increasing with k and therefore with the parental wage¹¹. Furthermore, under this assumption, the amount of education is decreasing with the domestic interest rate. Hence, there is a trade-off between investment in education (through remittances) and investment in physical capital. Parents equalize the marginal return of education with the marginal return of the saving. They can use the education as a substitute for saving. We can also remark that under this assumption, the repaying of the loan by the agent is lower than the amount of remittances. It is a way to consider remittances as a collateral.

Therefore, the education decreases with the domestic interest rate R_{t+1} under the assumption 4. Indeed, when the domestic interest rate increases, the investment in physical capital becomes more profitable, and households spend less on education. Moreover, the education is also decreasing with the world interest rate R_* . An increase in this rate implies an increase of the credit cost. However, the amount of education raises, ceteris paribus, with the altruism parameter γ , with the part of remittances the agent can borrow ω , and with foreign wage parameters α and λ . The more the agent can receive remittances, the more he will invest in education. Following this point, the borrowed amount from the international capital market, given by the equation (3) grows with k. Hence, an increase in capital per capita relaxes the constraint. The education is positively related to the proportion of remittances, ω that agents can borrow, because as remittances depend on education and loan depends on remittances, households educate more to receive more remittances and borrow more.

The domestic saving invested in physical capital is increasing with the wage w_t and with the parameters η and δ . Nevertheless, in our framework, the investment in capital grows with the child altruism γ , only if ω is large enough¹². Moreover the investment in capital is also increasing with the parameter ω only if it is sufficiently large¹³.

At each period, two temporary conditions hold to define the capital market equilibrium. The first defines the capital inflows from the international capital market and comes from the equation (3). The second condition determines the stock of capital per worker.

¹¹By inserting the expression of the interest rate given by the equation (2) in the expression of education given by the equation (4), this last expression becomes $e_t = \left(\frac{\gamma \alpha \lambda \left(k_{t+1}^{1-s}(1-\omega R_*)+\omega sA\right)}{sA}\right)^{\frac{1}{1-\lambda}}$. We directly determine that
$$\begin{split} \frac{\partial e_t}{\partial k_{t+1}} &> 0 \text{ if } \omega R_* < 1. \end{split}$$
 $\overset{}{\overset{12}{}} \text{The investment in physical capital increases with } \gamma \text{ if } \omega > \frac{\lambda \delta + 1 - \delta}{\lambda \delta (R_{t+1} - R_*) + \delta R_{t+1} + (1 - \delta) R_*}. \\ \overset{13}{} \text{The investment in physical capital increases with } \omega \text{ if } \omega > \frac{\lambda (R_{t+1} - R_*) - (1 - \delta)(1 - \lambda) - \delta(1 - \lambda) R_{t+1}}{(R_{t+1} - R_*) - (1 - \lambda)(1 - \lambda) - \delta(1 - \lambda) R_{t+1}}. \end{split}$

By using the equations (1) to (5), we get the dynamical equation of the macroeconomic equilibrium:

$$(1+n)(1-p)k_{t+1} - \frac{(1-s)k_t^s \left(\delta sAk_{t+1}^{s-1}(1+\eta) + (1-\delta)\eta R_*\right)}{sk_{t+1}^{s-1}} + p(1+n)\left(\frac{\gamma\alpha\lambda\left(k_{t+1}^{1-s}(1-\omega R_*) + \omega sA\right)}{sA}\right)^{\frac{1}{1-\lambda}} \left(\delta - \frac{\omega\delta sAk_{t+1}^{s-1} - (1-\delta)(1-\omega R_*)}{\lambda\left(1+\omega\left(sAk_{t+1}^{s-1} - R_*\right)\right)}\right) = 0$$
(6)

Following Gente (2006) [21] and Christopoulos *et al.* (2012) [16] the initially constrained economy may converge, in the long-run, to a constrained steady state or an unconstrained. We will determine how remittances may affect the nature (constrained of unconstrained) of the steady state of this economy.

3 The long-run equilibrium

As Christopoulos *et al.* (2012) [16], our analysis is based on three stages. We firstly determine the constrained steady state to then describe the unconstrained steady state and finally the conditions on the credit constraint whereby the economy converges to the unconstrained longrun equilibrium.

3.1 The constrained steady state

We focus on the constrained stage in order to describe the effects of remittances in the longrun in a constrained economy. We compare the constrained steady state¹⁴ with and without remittances.

3.1.1 The benchmark: equilibrium without remittances

We first describe the long-run equilibrium in an economy without remittances. We provide an analytic explanation in the benchmark case, using the equation (6) when the parameter γ is equal to zero. As parents do not receive remittances, they do not educate their children who leave the home country. This dynamical equation becomes:

$$(1+n)(1-p)k_{t+1} - \frac{(1-s)k_t^s \left(\delta s A k_{t+1}^{s-1}(1+\eta) + (1-\delta)\eta R_*\right)}{s k_{t+1}^{s-1}} = 0$$
(7)

Let us denote:

$$X = k^{1-s} \tag{8}$$

and

$$\Gamma(X) = \left((1+n)(1-p) - \frac{(1-s)(1-\delta)\eta R_*}{s} \right) X^{\frac{1}{1-s}} - \delta(1-s) A(1+\eta) X^{\frac{s}{1-s}}$$
(9)

 $^{14}\mathrm{We}$ denote a constrained steady state (where the constraint always binds) with an over-bar.

A steady state \overline{k} is a stationary capital stock per capita satisfying:

$$\begin{cases} \Gamma(\overline{X}) = 0 \tag{10}$$

$$\int \overline{k} = \overline{X}^{\frac{1}{1-s}} \tag{11}$$

Proposition 3.1. Under assumptions 1 and 3, there exist in the benchmark case without remittances (when $\gamma = 0$), a trivial unstable steady state with no capital accumulation, \overline{k}_0 , and one stable constrained steady state with a positive stock of capital, defined by:

$$\overline{k} \equiv \left(\frac{\delta\left(1-s\right)A\left(1+\eta\right)}{\left(1+n\right)\left(1-p\right) - \frac{\left(1-s\right)\left(1-\delta\right)\eta R_{*}}{s}}\right)^{\frac{1}{1-s}}$$

Proof. See Appendix A.1.

The assumption of a constrained steady state necessary implies a positive domestic interest factor and therefore $\overline{k} > 0$. This creates the following condition: we assume that $\eta < \frac{s(1+n)(1-p)}{(1-s)(1-\delta)R_*}$. The steady state value of capital stock per head is increasing with δ , η , R_* and p. It is decreasing with s and n. The fact that the agent can borrow more has a positive impact on investment¹⁵. Moreover, a rise in the number of migrants increases the long-run capital-labor ratio, by decreasing the number of workers in the country. If the world interest rate increases, the agent needs to save more to repay the interests.

Therefore, there exists one unique sustainable constrained long-run equilibrium, with a positive capital stock and a monotonous convergence. We provide the same analysis when $\gamma > 0$ implying that the economy is receiving remittances.

3.1.2 The constrained equilibrium with remittances

The dynamical equation is given by the equation (6) in the recipient economy. Let us denote:

$$\Theta(X) = \left((1+n) (1-p) - \frac{(1-s) (1-\delta) \eta R_*}{s} \right) X^{\frac{1}{1-s}} - \delta (1-s) A (1+\eta) X^{\frac{s}{1-s}} + p (1+n) \left(\frac{\gamma \alpha \lambda \left(X (1-\omega R_*) + \omega s A \right)}{s A} \right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{(1-\delta) (1-\omega R_*) X - \omega \delta s A}{\lambda \left(X (1-\omega R_*) + \omega s A \right)} \right)$$
(12)

In a recipient economy, a steady state \overline{k} is a stationary capital stock per capita satisfying:

$$\int \Theta(\overline{X}) = 0 \tag{13}$$

$$\int \overline{k} = \overline{X}^{\frac{1}{1-s}} \tag{14}$$

Proposition 3.2. Under assumptions 1, 2, 3 and 4, there exists a unique stable positive steady state $\bar{k}_R > 0$, in the constrained economy receiving workers' remittances used as collateral to borrow from the international capital market.

Proof. See Appendix A.2.

 $^{{}^{15}\}overline{k}$ is increasing with η . That is why there is a condition on η because a too large η would involve a too low interest return which could be lower than the worldwide. This could imply a negative capital accumulation which is not sustainable.

There is no trivial steady state since households can always borrow by using remittances as collateral, to educate the children and invest in physical capital¹⁶.

In the remain of the subsection, we analyze the impact of remittances on the long-run constrained capital-labor ratio.

Proposition 3.3. Let us denote:

$$\underline{\omega} \equiv \frac{(1-s)(1+\eta)(\lambda\delta + 1 - \delta)}{(1-\lambda)(s(1+n)(1-p) - (1-s)(1-\delta)\eta R_*) + (1-s)(1+\eta)(\lambda\delta + 1 - \delta)R_*} > 0$$

Under the assumptions 1, 2, 3 and 4, the impact of remittances on the unique positive constrained steady state is summarized by the following conditions:

- If $\omega < \underline{\omega}$, remittances have a negative impact on capital accumulation through access to the international capital market. Moreover, the parameter γ declines \overline{k}_R .
- If $\omega > \underline{\omega}$, remittances have a positive impact on capital accumulation through access to the international capital market. Moreover, the parameter γ raises \overline{k}_R .

Furthermore, the impact of remittances is positive for a smaller range of ω when η increases.

Proof. See Appendix A.3.

Remark 3.1. $0 < \underline{\omega} < \frac{1}{R_*}$ which implies that the three cases can appear under the assumption 4. Remittances can have a negative impact on k if ω is small enough or a positive if ω is large enough.

Therefore, $\frac{d\bar{k}_R}{d\gamma}\Big|_{\omega<\omega} < 0$ and $\frac{d\bar{k}_R}{d\gamma}\Big|_{\omega>\omega} > 0$. The impact of the studied inflows on the long-run capital-labor ratio depends on the possibility to borrow and to invest. If the part of remittances used as collateral is low, then remittances decrease the capital stock through a wealth effect. Indeed households need to save less if they will receive remittances. There is an education-saving trade-off. Nevertheless, if the part of remittances used as collateral is large enough, then the positive effect on investment, due to foreign capital inflows, compensates the decrease of saving due to the wealth effect. The Figure 3.1 summarizes these results¹⁷. Therefore, there are two opposite effects. First of all, workers' remittances act like a new financial asset different from usual saving. The agent finances education to the retirement period. In this configuration, remittances could have negative effects on the saving in the long-run being a substitute for saving. Nevertheless, by relaxing the credit constraints, remittances allow to invest more and could have beneficial effects on the capital accumulation. The sign of the effect depends on the amount agent can borrow.

In the last part of the proposition 3.3, we compare the impact of remittances relative to the parameter η . In other terms, we determine the impact of remittances according to the borrowing constraint in the benchmark without remittances. For an exogenous ω it tends to be positive in more financially constrained economy without remittances and tends to be negative in less financially constrained economies without remittances. This theoretical result follows empirical studies like Giuliano and Ruiz-Arranz (2009) [22]. Using the System Generalized Method of

¹⁶Without remittances the agent could not borrow if k = 0 and could not invest in physical capital.

 $^{^{17} \}rm{The}$ computations for the Figure 3.1 are given by the lemmas A.1 and A.2 in Appendix A.2 and by the Appendix A.3.



Figure 3.1: Representations of equilibria according to the parameter ω .

Moments regressions they determine a negative relation between the impact of remittances and the financial development. In other terms, the impact of remittances is better in countries with less developed financial system and decreases with the level of financial development. Moreover, they argue that remittances can have a negative impact on growth in countries with a developed financial system. This result is found using a threshold level estimation. Sobiech (2015) [30] finds a similar result for growth but also for the output in the long-run. The effect of remittances is positive in countries with a low level of finance and can be negative if the level of financial development is large enough. This is our result. The larger η , the larger ω is, and the more we tend to get $\omega < \underline{\omega}$ (negative effects of remittances) for an exogenous ω . Hence, remittances have a negative impact if η is large enough. This result occurs because in more financially developed economies, the agent could already borrow enough to invest without remittances. In other terms, the mechanism behind this point is that if the credit constraints are low, the households do not need to overcome these constraints to invest. Hence, remittances in this case serve to consume and to reduce the saving, which implies a negative impact on the capital stock in the long-run. Nevertheless, if the households are constrained to borrow, the relax of the credit constraints due to remittances allow to invest in physical capital with a positive impact on the long-run output.

Concerning the parameters η and ω (determining the level of the financial constraint), the capital-labor ratio in the long-run is increasing with the parameter η , and increasing with the parameter ω only if it is not too low¹⁸. Indeed, if ω is low enough, an increase in omega raises the borrowing but also the education. Nevertheless, in this case, there is to opposite effects. Firstly, the increase in borrowing tends to increase the investment in capital. Secondly, the increase of education raises remittances, but if ω is small enough, remittances bring a negative impact on the capital stock. The negative impact dominates, implying that an increase in ω reduces \overline{k}_r . However, if ω is higher, the positive effect dominates. Finally, if $\omega > \omega$, the effect of education on \overline{k}_r is positive, as the effect of the borrowing, therefore, the general impact of ω is necessarily positive.

Following Obstfeld and Rogoff (1996) [26], another long-run equilibrium can appear: an unconstrained steady state. It is called unconstrained because the borrowing constraint will not bind, the economy is totally financially integrated. We then recover the small open economy setting.

3.2 The unconstrained steady state

The unconstrained steady state¹⁹ is the standard steady state that occurs in a small open economy model. In such an equilibrium, there is a perfect access to the international capital market. The domestic return on capital converges to the world one $\overline{R} = R_*$.

By using the equation (2) we get the unconstrained long-run capital-labor ratio:

$$k = \left(\frac{sA}{R_*}\right)^{\frac{1}{1-s}} \equiv k_* \tag{15}$$

¹⁸By using the implicit function theorem, we determine that an increase in ω increases \overline{X}_r and therefore \overline{k}_R if:

$$\omega > \frac{\left(SA - \overline{X}_{r}R_{*}\right)\lambda\overline{X}_{r} - (1 - \lambda)\left((1 - \delta)\overline{X}_{r}R_{*} + \delta sA\right)\overline{X}_{r}}{\left(SA - \overline{X}_{r}R_{*}\right)\left((1 - \lambda)\left((1 - \delta)\overline{X}_{r}R_{*} + \delta sA\right) + \lambda\overline{X}_{r}R_{*}\right)}$$

¹⁹We denote an unconstrained steady state by a star.

Knowing the properties of the domestic interest rate, the capital-labor ratio is greater than the one in the constrained economy.

In the last subsection, we detail the conditions implying that the economy is constrained or unconstrained in the long-run in order to explain the impact of remittances in economic opening framework.

3.3 The threshold levels

With the globalization, the interest rate can converge to the unconstrained case when the stock of capital is large enough. The parameters η and ω are exogenous. They represent the restriction on financial inflows. If the restriction is large (η and ω low), the economy converges to the constrained steady state (so remains constrained in the long run), because as credit is low, the capital accumulation is also low. If the restriction is low enough (η and ω large enough) the economy converges to the unconstrained steady state. We determine the threshold level of the borrowing constraint such that the economy converges to the unconstrained steady state k_* as in the small open economy framework.

3.3.1 The threshold level without remittances

We will derive a condition to determine the cases in which the steady state is constrained or not. Remind that in the benchmark without remittances, $f_{t+1} \ge -\frac{\eta w_t}{(1-p)(1+n)}$.

Proposition 3.4. Under assumptions 1 and 3, the economy without remittances converges to the constrained steady state when:

$$\eta < \frac{s(1+n)(1-p) - \delta(1-s)R_*}{(1-s)R_*} \equiv \hat{\eta}$$

Proof. The economy remains constrained in the long-run if:

$$\overline{k} < k_* \Leftrightarrow \left(\frac{\delta\left(1-s\right)A\left(1+\eta\right)}{\left(1+n\right)\left(1-p\right) - \frac{\left(1-s\right)\left(1-\delta\right)\eta R_*}{s}}\right)^{\frac{1}{1-s}} < \left(\frac{sA}{R_*}\right)^{\frac{1}{1-s}} \Leftrightarrow \eta < \widehat{\eta}$$

We assume that the economy is initially constrained. Moreover, if $\eta = 0$, the economy would be so constrained that $f_t = 0$. This case corresponds to a closed economy. Hence, this assumption implies $\hat{\eta} > 0$. We therefore assume that $\delta < \frac{s(1+n)(1-p)}{(1-s)R_*}$ to guaranties $\hat{\eta} > 0$. Under this assumption, a constrained economy can remain constrained in the long-run or converge to the unconstrained steady state.

The Figure 3.2 illustrates this threshold level $\hat{\eta}$ in the benchmark and the area where the economy is constrained or unconstrained in the long-run. If $\eta < \hat{\eta}$ the economy converges to the constrained steady state \bar{k} , and remains constrained in the long-run. Nevertheless, if $\eta \geq \hat{\eta}$, the economy converges to the unconstrained steady state k_* .



Figure 3.2: Representation of the threshold level in the benchmark.

In this model, remittances improve the credit-worthiness of the agent. We determine the threshold on the credit constraint in recipient economies.

3.3.2 The threshold level with remittances

As previously, a recipient economy would be constrained in the long-run if the capital-labor ratio is lower than in the standard open economy setting. Remind that in a recipient economy, $f_{t+1} \ge -\frac{\eta w_t}{(1-p)(1+n)} - \frac{\omega p(1+n)\gamma \alpha e_t^{\lambda}}{(1-p)(1+n)}.$

Proposition 3.5. Under assumptions 1, 2, 3 and 4 the recipient economy is constrained in the long-run when:

$$\omega < \frac{\lambda A \left(\frac{sA}{R_*}\right)^{\frac{s}{1-s}} \left(\frac{s(1+n)(1-p)}{R_*} - (1-s)\left(\eta + \delta\right)\right)}{p\left(1+n\right) \left(\frac{\alpha\gamma\lambda}{R_*}\right)^{\frac{1}{1-\lambda}} R_*} + \frac{\lambda\delta + 1 - \delta}{R_*} \equiv \widehat{\omega}$$

Moreover, the recipient economy is constrained for a smaller range of ω when η increases.

Proof. $\overline{k}_R < \overline{k}_* \Leftrightarrow \Theta(\overline{k}_*) > 0 \Leftrightarrow \omega < \widehat{\omega}$ To prove the second part, we compute:

$$\frac{\partial \widehat{\omega}}{\eta} = \frac{-\lambda A \left(\frac{sA}{R_*}\right)^{\frac{s}{1-s}}}{p \left(1+n\right) \left(\frac{\alpha \gamma \lambda}{R_*}\right)^{\frac{1}{1-\lambda}} R_*} < 0$$

Therefore if $\omega < \hat{\omega}$, the recipient economy converges to the constrained steady state $\overline{k}_R < \overline{k}_*$. Otherwise, if $\omega \ge \hat{\omega}$, the recipient economy converges to the unconstrained steady state \overline{k}_* . The greater η is, the lower $\hat{\omega}$ is, the more the economy could be unconstrained for an exogenous ω .

Remark 3.2. Let us denote:

$$\widetilde{\eta}_{R} \equiv \frac{s\left(1+n\right)\left(1-p\right)-\delta\left(1-s\right)R_{*}}{\left(1-s\right)R_{*}} + \frac{p\left(1+n\right)\left(\frac{\alpha\gamma\lambda}{R_{*}}\right)^{\frac{1}{1-\lambda}}\left(\frac{\lambda\delta+1-\delta}{R_{*}}\right)}{\lambda\left(\frac{sA}{R_{*}}\right)^{\frac{s}{1-s}}\left(1-s\right)A}$$

If $\eta > \widetilde{\eta}_R$ then $\widehat{\omega} < 0$, the recipient economy would be necessarily unconstrained in the long-run for all $\omega \in \left[0, \frac{1}{R_*}\right]$.

The intuition behind this remark is that if η is heavily large, the recipient economy would be necessary unconstrained even if remittances would have negative effects ($\omega < \underline{\omega}$) because η is large enough to compensate the negative effect of remittances.

Corollary 1. Under assumptions 1, 2, 3 and 4 the recipient economy is constrained in the long-run when:

$$\eta < \frac{s\left(1+n\right)\left(1-p\right) - \delta\left(1-s\right)R_*}{\left(1-s\right)R_*} + \frac{p\left(1+n\right)\left(\frac{\alpha\gamma\lambda}{R_*}\right)^{\frac{1}{1-\lambda}}\left(\frac{\lambda\delta+1-\delta-\omega R_*}{\lambda}\right)}{\left(\frac{sA}{R_*}\right)^{\frac{s}{1-s}}\left(1-s\right)A} \equiv \widehat{\eta}_R$$

Moreover, the recipient economy is constrained for a smaller range of η when ω increases.

Knowing that the impact of η on the capital-labor ratio is positive, the recipient economy is constrained if η is small enough and unconstrained if η is large enough. We compare the two thresholds corresponding to the situations without and with remittances.

Lemma 3.1.
$$\frac{\partial \widehat{\eta}_R}{\partial \omega} < 0.$$
 Moreover, $\omega \leq \frac{\lambda \delta + 1 - \delta}{R_*} \Leftrightarrow \widehat{\eta}_R \geq \widehat{\eta}.$ Finally, $\frac{\partial \widehat{\eta}_R}{\partial \gamma} > 0 \Leftrightarrow \omega < \frac{\lambda \delta + 1 - \delta}{R_*}$
Proof. $\widehat{\eta}_R > \widehat{\eta} \Leftrightarrow \frac{\lambda \delta + 1 - \delta - \omega R_*}{\lambda} > 0 \Leftrightarrow \omega < \frac{\lambda \delta + 1 - \delta}{R_*}$

The threshold value with remittances is decreasing with the parameter ω . When it increases, remittances tend to have positive effects on \overline{k}_R and necessarily, the threshold $\widehat{\eta}_R$ decreases. Indeed, the more ω increases, the more \overline{k}_R would increase and the more the economy could be unconstrained for an exogenous η . However, $\widehat{\eta}_R$ is increasing with respect to the altruism parameter only if ω is low enough. The intuition is such that if ω is small enough, remittances decrease \overline{k}_R . Therefore, if γ increases, the recipient economy would tend to be constrained in the long-run, for an exogenous η . However, if ω is large enough, remittances increase \overline{k}_R . In this configuration, if γ increases, the recipient economy would tend to be unconstrained for an exogenous η .

Remark 3.3. Let us denote:

$$\widetilde{\omega} \equiv \frac{\lambda \widehat{\eta} \left(\frac{sA}{R_*}\right)^{\frac{s}{1-s}} (1-s)A}{p\left(1+n\right) \left(\frac{\alpha\gamma\lambda}{R_*}\right)^{\frac{1}{1-\lambda}} R_*} + \frac{\lambda\delta + 1-\delta}{R_*}$$

If $\frac{1}{R_*} > \omega > \widetilde{\omega}$ then $\widehat{\eta}_R < 0$, the recipient economy would be necessarily unconstrained in the long-run for all $\eta \in [0, 1]$.

As previously, the intuition behind this remark is that if ω is heavily large, the recipient economy would be necessary unconstrained even if η is low, because the positive effect of remittances through an increase of saving due to the borrowing at the international, is sufficient for the economy to converge to the unconstrained steady state.

Lemma 3.2. $\eta \gtrless \widehat{\eta} \Leftrightarrow \underline{\omega} \gtrless \frac{\lambda \delta + 1 - \delta}{R_*}$

Proof. First, if $\eta = \hat{\eta}$ then $\underline{\omega} = \frac{\lambda \delta + 1 - \delta}{R_*}$. Moreover, we get from previous computations $\frac{\partial \omega}{\eta} > 0$ (See Appendix A.3).

These two lemmas allow for the next proposition, determining the conditions to get a constrained or unconstrained economy in the long-run, but also the impact of remittances on the capital-labor ratio.

Proposition 3.6. Under assumptions 1, 2, 3 and 4, the impact of remittances through borrowing constraints is summarized by the following conditions:

- 1. Let us first consider $\eta < \hat{\eta}$ (i.e. the benchmark economy without remittances is constrained in the long-run, the capital-labor ratio converges to $\overline{k} < k_*$):
 - (a) If $\omega < \frac{\lambda \delta + 1 \delta}{R_*}$, then $\eta < \hat{\eta} < \hat{\eta}_R$. The recipient economy is constrained in the long-run $(\overline{k}_R < k_*)$.
 - The effect of remittances on k is negative if $\omega < \underline{\omega} < \frac{\lambda \delta + 1 \delta}{R_*}$.
 - The effect of remittances on k is positive if $\underline{\omega} < \omega < \frac{\lambda \delta + 1 \delta}{B_{\star}}$.
 - (b) If $\omega > \frac{\lambda \delta + 1 \delta}{R_*}$, then $\widehat{\eta}_R < \widehat{\eta}$.
 - *i.* The recipient economy is constrained in the long-run if $\eta < \hat{\eta}_R < \hat{\eta}$.
 - The effect of remittances on k is positive.
 - ii. The recipient economy is unconstrained in the long-run if $\hat{\eta}_R < \eta < \hat{\eta}$.
 - The effect of remittances on k is positive.
- 2. Let us now consider $\eta \geq \hat{\eta}$ (i.e. the benchmark economy without remittances is unconstrained in the long-run, the capital-labor ratio converges to k_*):
 - (a) If $\omega < \frac{\lambda \delta + 1 \delta}{R_*}$, then $\widehat{\eta} < \widehat{\eta}_R$.
 - *i.* The recipient economy is constrained in the long-run if $\hat{\eta} \leq \eta < \hat{\eta}_R$.
 - The effect of remittances on k is negative.
 - ii. The recipient economy is unconstrained in the long-run if η̂ < η̂_R ≤ η.
 Remittances have no impact on k.
 - (b) If $\omega > \frac{\lambda \delta + 1 \delta}{R_*}$, then $\widehat{\eta}_R < \widehat{\eta} \leq \eta$. The recipient economy is unconstrained.
 - Remittances have no impact on k.

With:

$$\widehat{\eta} = \frac{s(1+n)(1-p) - \delta(1-s)R_*}{(1-s)R_*}$$

$$\hat{\eta}_{R} = \frac{s\left(1+n\right)\left(1-p\right) - \delta\left(1-s\right)R_{*}}{\left(1-s\right)R_{*}} + \frac{p\left(1+n\right)\left(\frac{\alpha\gamma\lambda}{R_{*}}\right)^{\frac{1}{1-\lambda}}\left(\frac{\lambda\delta+1-\delta-\omega R_{*}}{\lambda}\right)}{\left(\frac{sA}{R_{*}}\right)^{\frac{1}{1-s}}\left(1-s\right)A}$$
$$\underline{\omega} = \frac{\left(1-s\right)\left(1+\eta\right)\left(\lambda\delta+1-\delta\right)}{\left(1-\lambda\right)\left(s\left(1+n\right)\left(1-p\right) - \left(1-s\right)\left(1-\delta\right)\eta R_{*}\right) + \left(1-s\right)\left(1+\eta\right)\left(\lambda\delta+1-\delta\right)R_{*}}$$

Proof. This proposition comes from previous proposition and computations.

The figures 3.3 and 3.4 illustrate each case of the proposition 3.6. We graphically see if a recipient economy would be constrained or unconstrained in the long-run according to parameters but also if this situation would differ with respect to the benchmark without remittances. Threshold level without remittances



Threshold level with remittances

Note:

In this configuration, a constrained economy (in the longrun) remains constrained with remittances.

Case 1.(a):
$$\omega < \frac{\lambda \delta + 1 - \delta}{R_*}$$
.

Threshold level without remittances



Threshold level with remittances

Note:

In this configuration, a constrained (in the long-run) economy remains constrained with remittances.

Case 1.(b).i:
$$\omega > \frac{\lambda \delta + 1 - \delta}{R_*}$$
 and $\eta < \hat{\eta}_R < \hat{\eta}_R$

Threshold level without remittances



Threshold level with remittances

<u>Note:</u> In this configuration, a constrained economy (in the longrun) becomes unconstrained with remittances.

Case 1.(b).*ii*:
$$\omega > \frac{\lambda \delta + 1 - \delta}{R_*}$$
 and $\widehat{\eta}_R < \eta < \widehat{\eta}$.

Figure 3.3: Comparison between the threshold levels without and with remittances when the economy would be constrained without remittances $(\eta < \hat{\eta})$.

Threshold level without remittances



Threshold level with remittances

Note:

In this configuration, an unconstrained economy (in the long-run) becomes constrained with remittances.

Case 2.(a).*i*:
$$\omega < \frac{\lambda \delta + 1 - \delta}{R_*}$$
 and $\hat{\eta} < \eta < \hat{\eta}_R$.

Threshold level without remittances



Threshold level with remittances

Note:

In this configuration, an unconstrained economy (in the long-run) remains unconstrained with remittances.

Case 2.(a).*ii*:
$$\omega < \frac{\lambda \delta + 1 - \delta}{R_*}$$
 and $\hat{\eta} < \hat{\eta}_R < \eta$.

Threshold level without remittances



Threshold level with remittances

Note:

In this configuration, an unconstrained economy (in the long-run) remains unconstrained with remittances.

Case 2.(b):
$$\omega > \frac{\lambda \delta + 1 - \delta}{R_*}$$
.

Figure 3.4: Comparison between the threshold levels without and with remittances when the economy would be unconstrained in the long-run without remittances $(\eta \ge \hat{\eta})$.

Therefore, according to the value of ω , remittances can increase or decrease the long-run capital-labor ratio. They raise the level of capital inflows from the international capital market by allowing the agent to borrow more. Nevertheless, if the part used as collateral is low enough, the increase of borrowing to invest does not compensate the negative effect of remittances through an increase of wealth (decrease of saving needed to consume in the last period of life). However, if the part used as collateral is large enough, the negative impact of remittances on saving due to the wealth effect is offset by the increase of investment in physical capital due to the possibility to borrow from the international capital market.

Remittances, through its impact on the credit constraint makes the country constrained or unconstrained at steady state. On one hand, an economy constrained without remittances may remain constrained if remittances have a negative effect on k or a low positive (case 1.(a)). This economy may also remain constraint if the impact of remittances is positive but the parameter η is low (case 1.(b).i). Nevertheless, if the impact of remittances is positive enough and the parameter η is large enough, the recipient economy may become unconstrained in the long-run (case 1.(b).ii). In this configuration they bring the constrained economy to the small open economy framework with a total financial integration. On the other hand, an economy unconstrained without remittances may become constrained because of remittances if ω and η are small enough (case 2.(a).i). In the other cases, the unconstrained economy remains unconstrained in the longrun with remittances (cases 2.(a).i and 2.(b)).

4 Concluding remarks

Worker's remittances are a transfer of money from a migrant to his family living in the home country. During the last forty years, these amounts have increased substantially. The growth is considered as exponential and remittances are more important than flows of development aids. Many economists classify them as a development resource like saving and private investment. Remittances affect the behavior of economic agents, with an important effect on consumption. The challenge for economists is to evaluate the macroeconomic impact of this private transfer of money. Empirically, there is no consensus yet on this question. Results are country-specific and may depend on the different methodologies used for empirical studies. We theoretically prove country-specific results in this paper. We show that the impact depends on the consequences of remittances on investment. We predict a positive impact on the capital stock if the households have the possibility to use a large enough part as collateral in order to invest. Nevertheless, we predict a negative impact in other cases, when the increase in wealth due to remittances implies a decrease of saving. Hence, the effects of remittances tend to be negative if agents only consider them as an increase of income, and tend to be positive if it is easier to make loans and investment. Furthermore, remittances tend to have a positive impact in countries where the individuals are more constrained to borrow without remittances. The theoretical results follow empirical studies like Giuliano and Ruiz-Arranz or also Sobiech. Remittances have a positive impact in less financially developed economies and a negative impact in more financially developed economies. We also provide conditions determining the impact of remittances on the fact that an economy would be financially constrained or not in the long-run.

A Appendix

A.1 Proof of Proposition 3.1

We immediately deduce the existence of the steady states from the equations (10) and (11). Moreover, the assumption of a constrained configuration whereby $R_t > R_* > 0 \forall t$, necessarily implies $\overline{k} > 0$. More precisely, we get for the unique stable constrained steady state:

$$\overline{k} > 0 \Leftrightarrow (1+n) (1-p) > \frac{(1-s) (1-\delta) \eta R_*}{s} \Leftrightarrow \eta < \frac{s (1+n) (1-p)}{(1-s) (1-\delta) R_*} \equiv \eta^{max}$$

Hence, the assumption $R_t > R_* > 0$ guaranties $\overline{k} > 0$ and implies $\eta < \eta^{max}$.

To determine the stability, we use the dynamical equation (7) expressed in X instead of k:

$$(1+n)(1-p)X_{t+1}^{\frac{1}{1-s}} - \delta(1-s)A(1+\eta)X_t^{\frac{s}{1-s}} - \frac{(1-s)(1-\delta)\eta R_*}{s}X_t^{\frac{s}{1-s}}X_{t+1} = 0$$

It follows for the positive steady state $\overline{X} = \overline{k}^{1-s}$:

$$\frac{\mathrm{d}X_{t+1}}{\mathrm{d}X_t}\Big|_{\overline{X}} = \frac{s\,(1+n)\,(1-p)}{(1+n)\,(1-p) - \frac{(1-s)^2(1-\delta)\eta R_*}{s}}$$

Remind that under the assumption that $\eta < \eta^{max}$, we get $(1+n)(1-p) > \frac{(1-s)(1-\delta)\eta R_*}{s}$. Therefore, we directly get $(1+n)(1-p) - \frac{(1-s)^2(1-\delta)\eta R_*}{s} > 0$. Hence $\frac{dX_{t+1}}{dX_t}\Big|_{\overline{X}} > 0$. We check:

$$\frac{\mathrm{d}X_{t+1}}{\mathrm{d}X_t}\Big|_{\overline{X}} < 1 \Leftrightarrow (1+n)(1-p) > \frac{(1-s)(1-\delta)\eta R_*}{s}$$

Therefore,

$$\frac{\mathrm{d}X_{t+1}}{\mathrm{d}X_t}\Big|_{\overline{X}} \in \left]0;1\right[$$

For the trivial steady state, we compute:

$$\lim_{X \to \overline{X}_0} \left. \frac{\mathrm{d}X_{t+1}}{\mathrm{d}X_t} \right|_X = +\infty$$

Hence, the unique positive stationary capital-labor ratio is stable. The trivial is unstable.

A.2 Proof of Proposition 3.2

We firstly determine the existence or not of a trivial steady state. As the equation (13) is not satisfied if $\overline{k} = \overline{X} = 0$, there is no trivial steady state. We then provide an analytic explanation for the non trivial, studying the function $\Theta(X)$ given by the equation (12).

For the simplicity of analysis, let us define:

$$\Theta(X) = \Gamma(X) + \Omega(X)$$

with:

$$\Omega(X) = p\left(1+n\right) \left(\frac{\gamma \alpha \lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}{s A}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A}\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A}\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A}\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A}\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A}\right)}\right)^{\frac{1}{1-\lambda}} \left(\delta + \frac{\left(1-\delta\right) \left(1-\omega R_*\right) X - \omega \delta s A}{\lambda \left(X \left(1-\omega R_*\right)+\omega s A}\right)}\right)^{\frac{1}{1-\lambda}}$$

Remark A.1. The equation $\Gamma(X) = 0$ determines the long-run equilibrium of the economy without remittances.

This function allows to graphically determine the equilibrium without remittances to then make a comparison with the situation with remittances.

Lemma A.1. The function $\Gamma(X)$, starts from 0, is decreasing and then increasing, tends to ∞ and is convex.

Proof. For the analysis of $\Gamma(X)$ we get:

$$\begin{split} \Gamma(0) &= 0 \qquad \lim_{X \to +\infty} \Gamma(X) = +\infty \\ \frac{\partial \Gamma(X)}{\partial X} &= \left(\frac{(1+n)\left(1-p\right)}{1-s} - \frac{(1-\delta)\eta R_*}{s}\right) X^{\frac{s}{1-s}} - s\delta A \left(1+\eta\right) X^{\frac{2s-1}{1-s}} \\ \frac{\partial \Gamma(X)}{\partial X} &> 0 \Leftrightarrow X > \frac{s\delta\left(1-s\right)A\left(1+\eta\right)}{(1+n)\left(1-p\right) - \frac{(1-s)(1-\delta)\eta R_*}{s}} < \overline{X} \\ \lim_{X \to 0^+} \frac{\partial \Gamma(X)}{\partial X} &= -\infty \qquad \lim_{X \to +\infty} \frac{\partial \Gamma(X)}{\partial X} > 0 \\ \frac{\partial^2 \Gamma(X)}{\partial X^2} &= \frac{s\left((1+n)\left(1-p\right) - \frac{(1-s)(1-\delta)\eta R_*}{s}\right)}{(1-s)^2} X^{\frac{2s-1}{1-s}} + \frac{(1-2s)s\delta A \left(1+\eta\right)}{1-s} X^{\frac{3s-2}{1-s}} > 0 \end{split}$$

Then, for the analysis of $\Omega(X)$ we get:

$$\Omega(0) = p (1+n) (\alpha \gamma \lambda \omega)^{\frac{1}{1-\lambda}} \frac{\delta (\lambda - 1)}{\lambda} < 0 \qquad \qquad \lim_{X \to +\infty} \Omega(X) = +\infty$$

$$\begin{split} \frac{\partial \Omega(X)}{\partial X} = & p\left(1+n\right) \left(\frac{\gamma \alpha \lambda \left(X\left(1-\omega R_{*}\right)+\omega s A\right)}{sA}\right)^{\frac{\lambda}{1-\lambda}} \frac{\gamma \alpha \left(1-\omega R_{*}\right)}{sA} \\ & \times \left(\frac{X\left(1-\omega R_{*}\right) \left(\lambda \delta+1-\delta\right)+\left(1-\delta\right) \left(1-\lambda\right) \omega s A}{\left(X\left(1-\omega R_{*}\right)+\omega s A\right) \left(1-\lambda\right)}\right) > 0 \end{split}$$

$$\begin{aligned} \frac{\partial^2 \Omega(X)}{\partial X^2} =& p\left(1+n\right) \left(\frac{\gamma \alpha \lambda \left(X \left(1-\omega R_*\right)+\omega s A\right)}{s A}\right)^{\frac{2\lambda-1}{1-\lambda}} \left(\frac{\gamma \alpha \lambda \left(1-\omega R_*\right)}{s A}\right)^2 \\ & \times \left(\frac{X \left(1-\omega R_*\right) \left(\lambda \delta+1-\delta\right)+\omega s A \left(1-\lambda\right) \left(2-\delta\right)}{\left(X \left(1-\omega R_*\right)+\omega s A\right) \left(1-\lambda\right)^2}\right) > 0 \end{aligned}$$

$$\Omega(X) > 0 \Leftrightarrow \delta + \frac{(1-\delta)(1-\omega R_*)X - \omega\delta sA}{\lambda(X(1-\omega R_*) + \omega sA)} > 0$$
$$\Leftrightarrow X > \frac{\omega\delta sA(1-\lambda)}{(1-\omega R_*)(\lambda\delta + 1 - \delta)} \equiv X_1$$

The function $\Omega(X)$ starts from a negative value, is increasing, convex and tends to $+\infty$. It is negative on $[0; X_1[$ and positive on $]X_1; \infty[$.

The properties of $\Gamma(X)$ and $\Omega(X)$ allows to directly obtain the following lemma.

Lemma A.2. Under assumptions 1, 2, 3 and 4, the function $\Theta(X)$ is convex, starts from a negative value, is convex, firstly decreasing, becomes increasing, and tends to $+\infty$. It cuts the axis of abscissa once (when it is increasing), which is the unique solution for the long-run constrained equilibrium with remittances.

Proof. One verifies:

$$\Theta(0) = \Omega(0) = p (1+n) (\alpha \gamma \lambda \omega)^{\frac{1}{1-\lambda}} \frac{\delta (\lambda - 1)}{\lambda} < 0 \qquad \lim_{X \to +\infty} \Theta(X) = +\infty$$
$$\lim_{X \to 0^+} \frac{\partial \Theta(X)}{\partial X} \Leftrightarrow \lim_{X \to 0^+} \frac{\partial \Gamma(X)}{\partial X} = -\infty \qquad \lim_{X \to +\infty} \frac{\partial \Theta(X)}{\partial X} = +\infty$$

Furthermore, $\Theta(X)$ is convex because the two functions $\Gamma(X)$ and $\Omega(X)$ are convex. Therefore, as there is no change of convexity, the function $\Theta(X)$ is only decreasing and then increasing. \Box

The equation $\Theta(X) = 0$ has one positive unique solution. Hence, there exists one unique positive steady state.

We determine the stability. The dynamical equation (6) expressed in $X = k^{1-s}$ in the recipient economy becomes:

$$(1+n)(1-p)X_{t+1}^{\frac{1}{1-s}} - \delta(1-s)A(1+\eta)X_t^{\frac{s}{1-s}} - \frac{(1-s)(1-\delta)\eta R_*}{s}X_t^{\frac{s}{1-s}}X_{t+1} + p(1+n)\left(\frac{\gamma\alpha\lambda(X_{t+1}(1-\omega R_*)+\omega sA)}{sA}\right)^{\frac{1}{1-\lambda}}\left(\delta + \frac{(1-\delta)(1-\omega R_*)X_{t+1}-\omega\delta sA}{\lambda(X_{t+1}(1-\omega R_*)+\omega sA)}\right) = 0 \Rightarrow X_{t+1}(X_t)$$

The analysis is dimension 1 again. By using the implicit function theorem, we obtain:

$$\frac{\mathrm{d}X_{t+1}}{\mathrm{d}X_t} = \frac{s\delta A \left(1+\eta\right) X_t^{\frac{2s-1}{1-s}} + (1-\delta) \eta R_* X_{t+1} X_t^{\frac{2s-1}{1-s}}}{\frac{(1+n)(1-p)}{1-s} X_{t+1}^{\frac{s}{1-s}} - \frac{(1-s)(1-\delta)\eta R_*}{s} X_t^{\frac{s}{1-s}} + \frac{\partial\Omega(X_{t+1})}{\partial X_{t+1}}}$$

It follows:

$$\frac{\mathrm{d}X_{t+1}}{\mathrm{d}X_t}\Big|_{\overline{X}_R} = \frac{s\delta A\left(1+\eta\right)\overline{X}_R^{\frac{2s-1}{1-s}} + (1-\delta)\,\eta R_*\overline{X}_R^{\frac{s}{1-s}}}{\overline{X}_R^{\frac{s}{1-s}}\left(\frac{(1+n)(1-p)}{1-s} - \frac{(1-s)(1-\delta)\eta R_*}{s}\right) + \frac{\partial\Omega(\overline{X}_R)}{\partial\overline{X}_R}}$$

We know that $\frac{\partial \Omega(\overline{X}_R)}{\partial \overline{X}_R} > 0$ if $\overline{X}_R > 0$. Under the assumption whereby $\eta < \eta^{max}$ (see Appendix A.1), we know that $\frac{(1+n)(1-p)}{1-s} - \frac{(1-s)(1-\delta)\eta R_*}{s} > 0$. Therefore:

$$\frac{\mathrm{d}X_{t+1}}{\mathrm{d}X_t}\Big|_{\overline{X}_R} > 0$$

The trajectory is monotonous. We also compute:

$$\frac{\mathrm{d}X_{t+1}}{\mathrm{d}X_t}\Big|_{\overline{X}_R} < 1 \Leftrightarrow \left(\frac{(1+n)\left(1-p\right)}{1-s} - \frac{(1-\delta)\,\eta R_*}{s}\right) X^{\frac{s}{1-s}} - s\delta A\left(1+\eta\right) X^{\frac{2s-1}{1-s}} + \frac{\partial\Omega(\overline{X}_R)}{\partial\overline{X}_R} > 0$$

By taking the expression of $\frac{\partial \Gamma(\overline{X}_R)}{\partial \overline{X}_R}$, we remark:

$$\frac{\mathrm{d}X_{t+1}}{\mathrm{d}X_t}\bigg|_{\overline{X}_R} < 1 \Leftrightarrow \frac{\partial\Gamma(\overline{X}_R)}{\partial\overline{X}_R} + \frac{\partial\Omega(\overline{X}_R)}{\partial\overline{X}_R} > 0 \Leftrightarrow \frac{\partial\Theta(\overline{X}_R)}{\partial\overline{X}_R} > 0$$

Following the lemma A.2, we know that $\frac{\partial \Theta(\overline{X}_R)}{\partial \overline{X}_R} > 0$. Therefore,

$$\frac{\mathrm{d}X_{t+1}}{\mathrm{d}X_t}\Big|_{\overline{X}_R} \in (0,1)$$

Hence, the unique equilibrium stable with a monotonous convergence.

A.3 Proof of Proposition 3.3

Remittances have any impact on the capital per head (i.e. $\overline{X} = \overline{X}_R$) if the following equation holds:

$$\Theta(\overline{X}) = \Gamma(\overline{X}) = 0$$

This necessary implies:

$$\Omega(\overline{X}) = 0$$

The previous equation is only satisfied if:

$$\overline{X} = \frac{\omega \delta s A \left(1 - \lambda\right)}{\left(1 - \omega R_*\right) \left(\lambda \delta + 1 - \delta\right)} = X_1$$

By inserting the expression of \overline{X} , this equation becomes:

$$\frac{\delta\left(1-s\right)A\left(1+\eta\right)}{\left(1+n\right)\left(1-p\right)-\frac{\left(1-s\right)\left(1-\delta\right)\eta R_{*}}{s}} = \frac{\omega\delta sA\left(1-\lambda\right)}{\left(1-\omega R_{*}\right)\left(\lambda\delta+1-\delta\right)}$$
$$\Leftrightarrow \omega = \frac{\left(1-s\right)\left(1+\eta\right)\left(\lambda\delta+1-\delta\right)}{\left(1-\lambda\right)\left(s\left(1+n\right)\left(1-p\right)-\left(1-s\right)\left(1-\delta\right)\eta R_{*}\right)+\left(1-s\right)\left(1+\eta\right)\left(\lambda\delta+1-\delta\right)R_{*}} \equiv \underline{\omega}$$

Under the assumption that $\eta < \eta^{max}$, to guaranties $\overline{k} > 0$ in the benchmark configuration, we get $s(1+n)(1-p) - (1-s)(1-\delta)\eta R_* > 0$ (see Appendix A.1). Hence $\underline{\omega} > 0$.

To bring out a negative or positive impact, we compute:

$$\frac{\partial X_1}{\partial \omega} = \frac{(1-\lambda)\,\delta sA}{\left(\lambda\delta + 1 - \delta\right)\left(1 - \omega R_*\right)^2} > 0$$
$$\frac{\partial \overline{X}}{\partial \omega} = 0$$

- If $\omega > \underline{\omega}$ then $X_1 > \overline{X}$ which implies $\Omega(\overline{X}) < 0$ and $\Theta(\overline{X}) < 0$. By knowing the properties of $\Theta(X)$ we directly deduce $\Theta(\overline{X}_R) = 0 \Leftrightarrow \overline{X}_R > \overline{X}$.
- Obviously, if $\omega < \underline{\omega}$ then $X_1 < \overline{X}$ which implies $\Omega(\overline{X}) > 0$ and $\Theta(\overline{X}) > 0$. Therefore, $\Theta(\overline{X}_R) = 0 \Leftrightarrow \overline{X}_R < \overline{X}$.

To determine the impact of the ascendant altruism, we use the implicit function theorem related to the equilibrium equation defined here as $\Theta(\overline{X}_R, \gamma)$:

$$\frac{\mathrm{d}\overline{X}_R}{\mathrm{d}\gamma} = \frac{-\frac{\partial\Theta(X_R,\gamma)}{\partial\gamma}}{\frac{\partial\Theta(\overline{X}_R,\gamma)}{\partial\overline{X}^T}}$$

With:

$$\begin{split} \frac{\partial \Theta(\overline{X}_{R},\gamma)}{\partial \gamma} = & \frac{p\left(1+n\right)\alpha\lambda\left(\overline{X}_{R}\left(1-\omega R_{*}\right)+\omega sA\right)}{sA\left(1-\lambda\right)} \\ & \times \left(\frac{\gamma\alpha\lambda\left(\overline{X}_{R}\left(1-\omega R_{*}\right)+\omega sA\right)}{sA}\right)^{\frac{\lambda}{1-\lambda}}\left(\delta + \frac{\left(1-\delta\right)\left(1-\omega R_{*}\right)\overline{X}_{R}-\omega\delta sA}{\lambda\left(\overline{X}_{R}\left(1-\omega R_{*}\right)+\omega sA\right)}\right) \end{split}$$

$$\begin{split} \frac{\partial \Theta(\overline{X}_R, \gamma)}{\partial \overline{X}_R} = & \frac{(1+n)\left(1-p\right) - \frac{(1-s)(1-\delta)\eta R_*}{s}}{1-s} \overline{X}_R^{\frac{s}{1-s}} - s\delta A\left(1+\eta\right) \overline{X}_R^{\frac{2s-1}{1-s}} + \\ & p\left(1+n\right) \left(\frac{\gamma \alpha \lambda \left(\overline{X}_R\left(1-\omega R_*\right) + \omega s A\right)}{sA}\right)^{\frac{\lambda}{1-\lambda}} \frac{\gamma \alpha \left(1-\omega R_*\right)}{sA} \\ & \times \left(\frac{\overline{X}_R\left(1-\omega R_*\right)\left(\lambda \delta + 1-\delta\right) + \left(1-\delta\right)\left(1-\lambda\right)\omega s A}{\left(\overline{X}_R\left(1-\omega R_*\right) + \omega s A\right)\left(1-\lambda\right)}\right) \end{split}$$

First off all, under the lemma A.2, $\frac{\partial \Theta(\overline{X}_R, \gamma)}{\partial \overline{X}_R} > 0$. Secondly, we remark that $\frac{\partial \Theta(\overline{X}_R, \gamma)}{\partial \overline{\gamma}}$ has the same sign as $\Omega(X)$. If $\omega \geq \underline{\omega}$, we have proved that $\Theta(\overline{X}_R) = 0 \Leftrightarrow \overline{X}_R \geq \overline{X} \Leftrightarrow \Gamma(\overline{X}_R) \geq 0$. Knowing that $\Omega(\overline{X}_R) = -\Gamma(\overline{X}_R)$ therefore, $\omega \geq \underline{\omega} \Leftrightarrow \Omega(\overline{X}_R) \leq 0$. Hence, $\frac{d\overline{X}_R}{d\gamma}\Big|_{\omega \geq \underline{\omega}} \geq 0$. We remark that the impact and the proof are the same for the parameter α .

For the last part of the proposition, we compute:

$$\frac{\partial \underline{\omega}}{\eta} = \frac{\left(1-s\right)\left(\lambda\delta+1-\delta\right)\left(1-\lambda\right)\left(s\left(1+n\right)\left(1-p\right)+\left(1-s\right)\left(1-\delta\right)R_{*}\right)}{\left[\left(1-\lambda\right)\left(s\left(1+n\right)\left(1-p\right)-\left(1-s\right)\left(1-\delta\right)\eta R_{*}\right)+\left(1-s\right)\left(1+\eta\right)\left(\lambda\delta+1-\delta\right)R_{*}\right]^{2}}$$

Under the assumption that $\eta < \eta^{max}$, we have proved that $s(1+n)(1-p)-(1-s)(1-\delta)\eta R_* > 0$ 0 (See Appendix A.1). Therefore:

$$\frac{\partial \underline{\omega}}{\eta} > 0$$

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