

Public Investment and Golden Rule of Public Finance in an Overlapping Generations Model

Akira Kamiguchi^{*a} Toshiki Tamai^{*b}

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Abstract

This paper develops an overlapping generations model with debt-financing public investment. The model assumes that the government is subject to the Golden Rule of Public Finance, and that households are Yaari-Blanchard type, which is enable to compare the model with Ramsey type households. It is shown in the paper that the growth-maximizing and utility-maximizing tax rate do not satisfy Barro tax rule. Furthermore, we show that both tax rates positively depend on the longevity and therefore population aging increases debt per GDP. This result captures a tendency of increasing debt per GDP under population aging.

Keywords: Public capital; Golden rule of public finance; Economic growth

JEL classification: H54; H60; O40

^a School of Economics, Hokusei Gakuen University, Oyachi-Nishi, Atsubetsu-ku, Sapporo, Hokkaido, 004-8631, Japan. E-mail: a-kamiguchi@hokusei.ac.jp.

^b Graduate School of Economics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, 464-8601, Japan. Tel: +81-52-789-4933. E-mail: tamai@soec.nagoya-u.ac.jp.

1. Introduction

Numerous studies have examined the economic effects of public investment since the outstanding study by Arrow and Kurz (1970). In particular, Barro (1990) and Futagami et al. (1993) analyzed the macroeconomic effects of public investment using endogenous growth models. These models assumed that public capital enters into the production function as one of inputs.¹ They showed that the growth-maximizing tax rate is equal to the output elasticity of public capital. Furthermore, Futagami et al. (1993) demonstrated that the welfare-maximizing does not coincide with the growth-maximizing in the model with public capital accumulation, although Barro (1990) found the compatibility of growth- and welfare-maximizing.²

In recent years, the economic impacts of fiscal deficit, fiscal rules, and sustainability of public debt are studied using extended models of Barro (1990) and Futagami et al. (1993), and fiscal deterioration of national budget in major countries are behind it.³ The productivity effects of public investment financed by public debt and the significance of fiscal institutions have been widely recognized since the global financial crisis (IMF, 2014, Ch. 3). Indeed, many studies investigated the macroeconomic effects of fiscal policy under various fiscal rules (e.g., Greiner and Semmler 2000; Ghosh and Mourmouras 2004; Greiner 2007, 2010; Minea and Villieu 2009; Groneck 2011; Kamiguchi and Tamai 2012; Tamai 2014, 2016).

Minea and Villieu (2009) analyze an economy where the government adopts the golden rule of public finance. Their study shows that the fiscal policy under the golden rule of public finance worsens the long-run economic growth, but however there is a possibility to improve the intertemporal welfare compared with the balanced-budget rules. Groneck (2011) also uses the Ramsey-type growth model to analyze the role of the golden rule of public finance on the economic growth rate and welfare. He shows that the government's policy would have positive effects on both of the long-run growth and welfare, if the government spending benefits to the individuals' utility and raises the marginal productivity of private capital. He mentioned that the positive growth effects are observed only when public consumption expenditures are lowered in the long-run.

On the other hand, the effects of debt-financing public investment on intergenerational welfare have not been studied enough to clarify their properties.⁴ This is because these previous studies assumed that the representative household is infinitely lived. Public debt has a role of intergenerational transfer, if the government issues public debt to invest on the public capital, the benefit is provided not only to the current individuals but to the future ones. To capture this mechanism clearly, we use the

¹ Aschauer (1989) is the pioneering study of empirical analysis on the productivity effects of public capital. See Ligthart and Suárez (2011), Pereira and Andrzej (2013), and Bom and Ligthart (2014) for the issues of recent empirical studies.

² This property holds if and only if the production function has constant elasticities of output with respect to inputs. Misch et al. (2013) examined this issue using the CES production function.

³ See Irmen and Kuehnel (2009) for a survey of this literature.

⁴ Some studies examined this issue (See Saint-Paul 1992; Tanaka 2003; Tamai 2009).

overlapping generations model. Further, our study incorporates the probability of death, and thus it is possible to analyze how the longevity affects the economic circumstances and the government's policy under the population aging.

Some recent studies investigated the effects of public investment using the Diamond type of OLG model (Yakita 2008; Arai 2011; Teles and Mussolini 2014). However, these studies do not take into account GRPF, and therefore do not focus on the issue of intergenerational effect of debt-financing public investment. As shown in Tamai (2016), GRPF can actualize the first-best equilibrium. It needs to develop the model, which is able to compare the existing model widely used. Therefore, this paper examined the macroeconomic effects of debt-financing public investment under GRPF using an OLG model developed by Yaari (1965) and Blanchard (1985).

This paper shows the following results. First, Barro tax rule does not hold; the growth-maximizing tax rate is less than the output elasticity of public capital. Second, the growth-maximizing is not equivalent to the utility-maximizing although there is no transitional dynamics. Third, growth- and utility-maximizing tax rate are positively associated with longevity. Based on this relation, population aging increases the equilibrium tax rate by majority voting and the ratio of debt to GDP.

The remainder of this paper is organized as follows. Next section explains a basic setup of our model and characterizes the dynamic equilibrium. Section 3 considers the growth and welfare effects of debt-financing public investment, and it characterizes the intergenerational effects of public investment financed by public debt. Section 4 examines the relation between fiscal policy and longevity, and provides some results based on numerical analysis. Finally, Section 5 concludes this paper.

2. The model

This section describes the basic setup of our mathematical model. Following Futagami et al. (1993), final goods are produced using private and public capital. The production function is specified as

$$Y(t) = \phi K(t)^{1-\alpha} G(t)^\alpha, \quad (1)$$

where $Y(t)$ is total output of final goods, $K(t)$ the private capital, and $G(t)$ the public capital. The parameters in equation (1) satisfy $0 < \alpha < 1$ and $\phi > 0$. After-tax factor prices are given by

$$r(t) = (1 - \tau)(1 - \alpha) \frac{Y(t)}{K(t)}, \quad (2a)$$

$$w(t) = (1 - \tau)\alpha Y(t), \quad (2b)$$

where τ denotes the tax rate on output.⁵

Regarding the setting of households, we follow Yaari (1965) and Blanchard (1985). Each household

⁵ Tamai (2009) examined how the difference output tax and income tax affect the results of equilibrium analysis using the OLG model presented by Yaari (1965) and Blanchard (1985).

faces the probability of death, $\lambda \geq 0$, at any moment. Without loss of generality, we assume that the size of a new cohort is also equal to λ . Then, the population size is

$$N(t) = \int_{-\infty}^t \lambda e^{-\lambda(t-v)} dv = 1.$$

We assume that the instantaneous utility function is a logarithmic function of private consumption. Then, the expected lifetime utility of a household born at time s is

$$EU = E \left[\int_t^{\infty} \log c(v, s) e^{-\rho(v-t)} dv \right],$$

where $c(v, s)$ denotes the private consumption at time v for a household born at time s ($s \leq t$). The probability of being alive at time v is $e^{-\lambda(v-t)}$. Therefore, the expected lifetime utility function is rewritten as

$$EU = \int_t^{\infty} \log c(v, s) e^{-(\rho+\lambda)(v-t)} dv. \quad (3)$$

The budget constraint for a household born at time s ($s \leq v$) is

$$\frac{da(v, s)}{dv} = (r(v) + \lambda)a(v, s) + w(v) - c(v, s), \quad (4)$$

$$a(s, s) = 0,$$

where $a(v, s)$ denotes the financial asset at time v of household born at time s ($s \leq t$). Households choose their private consumption to maximize equation (3) subject to equation (4). Solving the maximization problem, we obtain

$$\frac{dc(t, s)}{dt} = (r(t) - \rho)c(t, s), \quad (5a)$$

$$\lim_{v \rightarrow \infty} a(v, s) \exp \left[- \int_t^v \{r(z) + \lambda\} dz \right] = 0. \quad (5b)$$

Using (4), (5a) and (5b), the consumption function of generation s is

$$c(t, s) = (\rho + \lambda)[a(t, s) + h(t)], \quad (6a)$$

where $h(t)$ is the present value of labor income such as

$$h(t) \equiv \int_t^{\infty} w(v) \exp \left[- \int_t^v \{r(z) + \lambda\} dz \right] dv. \quad (6b)$$

Aggregation. We now consider aggregate variables and its dynamics. By the definition, the aggregate variables are

$$C(t) \equiv \int_{-\infty}^t c(t, s) \lambda e^{-\lambda(t-s)} ds. \quad (7a)$$

$$A(t) \equiv \int_{-\infty}^t a(t, s) \lambda e^{-\lambda(t-s)} ds. \quad (7b)$$

$$H(t) \equiv \int_{-\infty}^t h(s) \lambda e^{-\lambda(t-s)} ds = h(t). \quad (7c)$$

Using (6a)-(7c), we obtain the aggregate consumption function such as

$$C(t) = (\rho + \lambda)[A(t) + H(t)]. \quad (8)$$

Differentiation of (7b) and (7c) with respect to t provide

$$\dot{A}(t) = r(t)A(t) + w(t) - C(t), \quad (9a)$$

$$\dot{H}(t) = [r(t) + \lambda]H(t) - w(t). \quad (9b)$$

Using (8)-(9b), we have

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho - (\rho + \lambda)\lambda \frac{A(t)}{C(t)}. \quad (9c)$$

In (9c), the final term is based on the *generation replacement effects*. New coming households have no financial wealth but the existing households have some amount of financial assets. At each moment, some households are died and replaced by new comers. Therefore, the last term has a negative effect on consumption growth. This term will play important roles in dynamic analysis of fiscal policy.

The government taxes total sales and issues bonds, and allocates its revenue to interest payments and public investment. Then, the budget constraint for the government is

$$\dot{B}(t) + \tau Y(t) = r(t)B(t) + \dot{G}(t) + C_g(t), \quad (10)$$

where $B(t)$ represents the government bonds and $C_g(t)$ denotes the unproductive government spending that does not affect the utility and production of private sector. The unproductive government spending linearly depends on the aggregate output: $C_g(t) = \theta Y(t)$.

We assume that the government adopts the golden rule of public finance (GRPF). GRPF requires that the government bond issuance is permissible within financing public investment. Therefore, we have $\dot{B}(t) = \dot{G}(t)$. To fulfill the government's budget constraint, $B(0) = G(0)$ is required (See Appendix A.1). Then, we obtain

$$B(t) = G(t). \quad (11)$$

Using (10) and (11), we arrive at

$$(\tau - \theta)Y(t) = r(t)B(t).$$

Above equation, (1) and (2a) provide

$$b(t) \equiv \frac{B(t)}{K(t)} = \frac{(\tau - \theta)}{(1 - \tau)(1 - \alpha)}, \quad (12a)$$

$$g(t) \equiv \frac{G(t)}{K(t)} = \frac{(\tau - \theta)}{(1 - \tau)(1 - \alpha)}. \quad (12b)$$

We now consider the equilibrium conditions. The clearing condition for financial market is

$$A(t) = K(t) + B(t). \quad (13)$$

Using equations (1)-(2b), (9a), (10), (11) and (13), we arrive at the resource constraint of this economy:

$$\dot{K}(t) = Y(t) - C(t) - C_g(t) - \dot{G}(t) = Y(t) - C(t) - C_g(t) - \dot{B}(t). \quad (14a)$$

Equation (1), (12a), (12b) and (14a) lead to

$$\frac{\dot{K}(t)}{K(t)} = \frac{(1 - \theta)\phi g^\alpha - z(t)}{1 + g}, \quad (14b)$$

where $z(t) \equiv C(t)/K(t)$. Using equations (9c), (12a), (12b), (13), and (14b), we obtain

$$\frac{\dot{z}(t)}{z(t)} = \frac{\dot{C}(t)}{C(t)} - \frac{\dot{K}(t)}{K(t)} = (1 - \tau)(1 - \alpha)\phi g^\alpha - \rho - \frac{(1 + g)(\rho + \lambda)\lambda}{z(t)} - \frac{(1 - \theta)\phi g^\alpha - z(t)}{1 + g}. \quad (15)$$

We now define the balanced growth equilibrium (BGE) as the equilibrium that satisfies

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{K}(t)}{K(t)} = \frac{\dot{G}(t)}{G(t)} = \frac{\dot{B}(t)}{B(t)}.$$

BGE requires $\dot{z}(t) = 0$. Equation (15) has the properties:

$$\frac{d}{dz(t)} \left(\frac{\dot{z}(t)}{z(t)} \right) = \frac{(1 + g)(\rho + \lambda)\lambda}{z(t)^2} + \frac{1}{1 + g} > 0,$$

$$\lim_{z(t) \rightarrow 0} \left(\frac{\dot{z}(t)}{z(t)} \right) = -\infty, \quad \lim_{z(t) \rightarrow \infty} \left(\frac{\dot{z}(t)}{z(t)} \right) = \infty.$$

Therefore, we obtain the unique value of $z(t)$ that satisfies BGE condition from equation (15) with $\dot{z}(t) = 0$. Defined γ as the equilibrium growth rate. The equilibrium growth rate is given as

$$\gamma = (1 - \tau)(1 - \alpha)\phi g^\alpha - \rho - \frac{(1 + g)(\rho + \lambda)\lambda}{z}. \quad (16)$$

3. Growth and welfare effects of fiscal policy

3.1 Growth effect of fiscal policy

In this subsection, we examine the growth effect of fiscal policy. The unproductive government spending does not affect the utility and production of private sector. Then, it is obvious that the unproductive government spending have negative effects on economic growth and welfare. Therefore, we set $\theta = 0$ in the theoretical part of this paper.

We begin our analysis to derive the effect of fiscal policy on the ratio of public to private capital. Differentiating (12b) with respect to τ , we obtain

$$\frac{dg}{d\tau} = \frac{1}{(1 - \alpha)(1 - \tau)^2} > 0. \quad (17)$$

A rise in τ increases the tax revenue and decreases the interest payment through after-tax interest rate declines. Then, the government enables to issue public bonds to finance public investment. Therefore, a rise in τ increases the ratio of public to private capital.

Differentiating (2a) with respect to τ and using (17), we have

$$\frac{dr}{d\tau} = -(1-\alpha)\phi g^\alpha + (1-\tau)(1-\alpha)\alpha\phi g^{\alpha-1}\frac{dg}{d\tau} = (1-\alpha)\phi g^\alpha \left[\frac{\alpha}{\tau} - 1 \right] \geq 0 \Leftrightarrow \tau \leq \alpha. \quad (18)$$

This formula is a well-known result as Barro rule in the studies of fiscal policy and economic growth (e.g., Barro 1990; Futagami et al. 1993). However, in the OLG model, the growth-maximizing will not be attainable as if the government sets τ to α . This is because the equilibrium growth rate depends on the tax rate through not only the interest rate but also the specific term of the OLG model.

By differentiation of (16) with respect to τ , we arrive at the growth effect of fiscal policy:

$$\frac{d\gamma}{d\tau} = \frac{dr}{d\tau} - \frac{d}{d\tau} \left[\frac{(1+g)(\rho+\lambda)\lambda}{z} \right] = \frac{dr}{d\tau} - (\rho+\lambda)\lambda \frac{d\omega}{d\tau}, \quad (19)$$

where

$$\omega \equiv \frac{1+g}{z} = \frac{A(t)}{C(t)}.$$

The second term in (19) captures the effects of income tax through the generation replacement effects.

Using (15) and BGE condition, we can derive it as

$$\begin{aligned} (\rho+\lambda)\lambda \frac{d\omega}{d\tau} &= \frac{(\rho+\lambda)\lambda}{(\rho+\lambda)\lambda + \frac{1}{\omega^2}} \left[\frac{dr}{d\tau} - \frac{d}{d\tau} \left(\frac{\phi g^\alpha}{1+g} \right) \right] \\ &= \frac{(\rho+\lambda)\lambda}{(\rho+\lambda)\lambda + \frac{1}{\omega^2}} \left\{ \frac{dr}{d\tau} - \frac{\phi g^\alpha}{1+g} \left[\frac{\alpha - \frac{\tau}{1-\tau}}{(1+g)g} \right] \frac{dg}{d\tau} \right\}. \end{aligned} \quad (20a)$$

Note that

$$\left. \frac{d\omega}{d\tau} \right|_{\tau=\alpha} = \frac{\alpha^2}{(1-\alpha) \left[(\rho+\lambda)\lambda + \frac{1}{\omega^2} \right]} \frac{\phi g^\alpha}{1+g} \left. \frac{dg}{d\tau} \right|_{\tau=\alpha} > 0, \quad (20b)$$

$$\left. \frac{d\omega}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} = \frac{1}{(\rho+\lambda)\lambda + \frac{1}{\omega^2}} \left. \frac{dr}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} > 0. \quad (20c)$$

Without additional restrictions of parameters, we cannot identify the sign of equation (19). As described above, previous studies showed that the growth-maximizing tax rate is equal to the output elasticity of public capital (e.g., Barro 1990; Futagami et al. 1993). Therefore, we should evaluate equation (19) at $\tau = \alpha$. Inserting $\tau = \alpha$ into (19) and using (20b), we obtain

$$\left. \frac{d\gamma}{d\tau} \right|_{\tau=\alpha} = -(\rho+\lambda)\lambda \left. \frac{d\omega}{d\tau} \right|_{\tau=\alpha} = -\frac{(\rho+\lambda)\lambda\omega^2}{1 + (\rho+\lambda)\lambda\omega^2} \frac{\alpha^2}{1-\alpha} \frac{\phi g^{\alpha-1}}{(1+g)^2} \left. \frac{dg}{d\tau} \right|_{\tau=\alpha} < 0. \quad (21a)$$

Further, using (18), (19) and (20c), we have

$$\left. \frac{d\gamma}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} = \left. \frac{dr}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} - (\rho+\lambda)\lambda \left. \frac{d\omega}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} = \frac{1}{1 + (\rho+\lambda)\lambda\omega^2} \left. \frac{dr}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} > 0. \quad (21b)$$

Equations (21a) and (21b) show that further increase in the tax rate from $\tau = \alpha$ decreases the equilibrium growth rate while further increase in the tax rate from $\tau = \alpha/(1+\alpha)$ increases the equilibrium growth rate. Therefore, the following proposition holds:

Proposition 1. *Under GRPF, the growth-maximizing tax rate τ^* is less than the output elasticity of public capital α and satisfies*

$$\frac{\alpha}{1 + \alpha} < \tau^* < \alpha.$$

Proposition 1 implies that the Barro tax rule does not hold in the OLG model with GRPF. Using the OLG model, Tamai (2009) shows that Barro tax rule holds in the case of balanced budget with the output tax. However, it is not true in the OLG model with GRPF. When $\lambda = 0$, Barro tax rule is true under GRPF. However, in the OLG model with $\lambda > 0$, public investment financed by public bonds has an intergenerational redistributive effects and the presence of generation replacement effects strengthen it. Through these effects, a rise in the tax rate has additional negative effect on economic growth in the OLG model with GRPF.

Some previous studies showed that the difference in fiscal rules make the difference in equilibrium growth rates (e.g. Minea and Villieu 2009; Greiner 2010; Groneck 2011). However, OLG model has the generation replacement effects and implies the different effects of GRPF on economic growth. Through these effects, fiscal policy under GRPF has different welfare effects in compared with previous studies. We will examine this point in the next section.

3.2 Welfare effects of fiscal policy

This subsection analyzes welfare effects of fiscal policy. The indirect utility functions born at different times take different values. In particular, using (3) and (6a), that of newcomers (born at time t) and others born at time s ($s > t$) are

$$V(t, t) = \frac{r - \rho}{(\rho + \lambda)^2} + \frac{\log(\rho + \lambda)}{\rho + \lambda} + \frac{\log h(t)}{\rho + \lambda}, \quad (22a)$$

$$V(t, s) = \frac{r - \rho}{(\rho + \lambda)^2} + \frac{\log(\rho + \lambda)}{\rho + \lambda} + \frac{\log[a(t, s) + h(t)]}{\rho + \lambda}, \quad (22b)$$

The indirect utility depends on weighted terms of two effects: the growth effect represented by first terms of equation (22a) and (22b) and the consumption effect represented by second and third terms. The consumption effect is decomposed of the effect on marginal propensity to consume (second terms) and on the wealth holding by household (third terms). Therefore, fiscal policy affects the indirect utility level through these effects.

We now consider the wealth effect of fiscal policy. Fiscal policy does not affect stock level of

financial assets although it affects asset portfolio (i.e., $da(t, s)/d\tau = 0$).⁶ Then, we have

$$\frac{dK(t)}{d\tau} = -\frac{K(t)}{1+g} \frac{dg}{d\tau} < 0. \quad (23)$$

Equation (23) implies that a rise in the tax rate brings about the crowding out effect on private capital. By a rise in the tax rate, the government enable to issue the public bonds. However, it simultaneously means that private capital is crowded out.

Total wealth holding by household is composed of financial wealth and human wealth. Therefore, fiscal policy affects private consumption through the effect on human wealth. In equilibrium, human wealth becomes

$$H(t) = h(t) = \frac{w(t)}{r + \lambda - \gamma} = \frac{\alpha r K(t)}{(1 - \alpha)(\rho + \lambda)(1 + \lambda\omega)}, \quad (24)$$

Using equations (23) and (24), the effect of fiscal policy on human wealth is given by

$$\frac{1}{H(t)} \frac{dH(t)}{d\tau} = \frac{1}{r} \frac{dr}{d\tau} - \frac{1}{1+g} \frac{dg}{d\tau} - \frac{\lambda}{1+\lambda\omega} \frac{d\omega}{d\tau}. \quad (25a)$$

The crowding out effect (23) has a negative effect on labor income. On the other hand, a rise in the tax rate increases labor income for low tax rates through a productivity effect of public capital. Furthermore, a rise in the tax rate affects the discount rate through a change in the interest rate. Equation (25a) is composed of these effects and therefore the effects of a rise in the tax rate on human wealth are mixed. However, evaluation of (25a) at some tax rates using (20b) and (20c) give

$$\left. \frac{1}{H(t)} \frac{dH(t)}{d\tau} \right|_{\tau=\alpha} = -\frac{1}{1+g} \frac{dg}{d\tau} - \frac{\lambda}{1+\lambda\omega} \frac{d\omega}{d\tau} \Big|_{\tau=\alpha} < 0, \quad (25b)$$

$$\left. \frac{1}{H(t)} \frac{dH(t)}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} = -1 - \alpha - \frac{\lambda}{1+\lambda\omega} \frac{d\omega}{d\tau} \Big|_{\tau=\frac{\alpha}{1+\alpha}} < 0. \quad (25c)$$

Using equations (6a), (9c), (10), and (25a), we can derive the consumption effect of fiscal policy:

$$\frac{dc(t)}{d\tau} = (\rho + \lambda) \left[\frac{da(t, s)}{d\tau} + \frac{dh(t)}{d\tau} \right] = (\rho + \lambda) \frac{dh(t)}{d\tau}, \quad (26a)$$

$$\frac{dC(t)}{d\tau} = (\rho + \lambda) \left[\frac{dA(t)}{d\tau} + \frac{dH(t)}{d\tau} \right] = (\rho + \lambda) \frac{dH(t)}{d\tau}. \quad (26b)$$

A rise in the tax rate affects only human wealth and therefore the intuition of equations (26a) and (26b) are directly explained by that of equation (25a).

Finally, we consider that a rise in the tax rate affects households' welfare through the dynamic effects such as (25a)-(26b). Taking into account (25a)-(26b), we arrive at the following formulas (See Appendix A.2):

⁶ Differentiating $A(t)$ w.r.t. the income tax rate, we obtain

$$0 = \frac{dA(t)}{d\tau} = \frac{dK(t)}{d\tau} + \frac{dB(t)}{d\tau} = (1+g) \frac{dK(t)}{d\tau} + K(t) \frac{dg}{d\tau}.$$

Using this equation, we obtain equation (24).

$$\begin{aligned}\frac{\partial V(t, s)}{\partial \tau} &= \frac{1}{(\rho + \lambda)^2} \frac{\partial r}{\partial \tau} + \frac{1}{(\rho + \lambda)[a(t, s) + h(t)]} \frac{\partial h(t)}{\partial \tau} \\ &= \frac{1}{(\rho + \lambda)\tau} \left[\frac{r}{\rho + \lambda r} \frac{\tau \partial r}{\partial \tau} + \beta(t, s) \frac{\tau}{h(t)} \frac{\partial h(t)}{\partial \tau} \right],\end{aligned}\quad (27a)$$

$$\frac{\partial V(t, t)}{\partial \tau} = \frac{1}{(\rho + \lambda)^2} \frac{\partial r}{\partial \tau} + \frac{1}{(\rho + \lambda)h(t)} \frac{\partial h(t)}{\partial \tau} = \frac{1}{(\rho + \lambda)\tau} \left[\frac{r}{\rho + \lambda r} \frac{\tau \partial r}{\partial \tau} + \frac{\tau}{h(t)} \frac{\partial h(t)}{\partial \tau} \right],\quad (27b)$$

where

$$\beta(t, s) \equiv \frac{h(t)}{a(t, s) + h(t)}.$$

Equation (27a) implies that the utility maximizing tax rate is equal to the output elasticity of public capital for the generation with $\beta = 0$; $\tau_{\beta=0}^* = \alpha > \tau^*$. Furthermore, equation (27b) can be derived from (27a) when $\beta = 1$.⁷

We focus on the relation between growth-maximizing and utility-maximizing fiscal policy. Inserting $\tau = \alpha$ into (27a) and (27b), we obtain

$$\left. \frac{\partial V(t, s)}{\partial \tau} \right|_{\tau=\alpha} = \frac{1}{(\rho + \lambda)[a(t, s) + h(t)]} \left. \frac{\partial h(t)}{\partial \tau} \right|_{\tau=\alpha} < 0,\quad (28a)$$

$$\left. \frac{\partial V(t, t)}{\partial \tau} \right|_{\tau=\alpha} = \frac{1}{(\rho + \lambda)h(t)} \left. \frac{\partial h(t)}{\partial \tau} \right|_{\tau=\alpha} < 0.\quad (28b)$$

Equations (28a) and (28b) imply that the utility-maximizing tax rates are less than the output elasticity of public capital. However, the signs of (27a) and (27b) evaluated at the growth-maximizing tax rate are ambiguous. Alternatively, we check the sign of (27b) evaluated at $\tau = \alpha/(1 + \alpha)$ (See Appendix A.2 for derivation of the following equation):

$$\left. \frac{\partial V(t, t)}{\partial \tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} = \frac{1}{(\rho + \lambda)} \left[-1 - \alpha + \frac{\frac{1}{(\rho + \lambda)\omega^2} + \frac{\lambda^2\omega}{1 + \lambda\omega}}{(\rho + \lambda)\lambda + \frac{1}{\omega^2}} (1 - \alpha)\alpha\phi g^\alpha \right].\quad (29a)$$

If λ and ρ are sufficiently small, $\lambda\rho$ and λ^2 are close to zero. Using this relation, equation (29a) becomes

$$\begin{aligned}\left. \frac{\partial V(t, t)}{\partial \tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} &= \frac{1}{(\rho + \lambda)} \left[-1 - \alpha + \frac{(1 - \alpha)^{1-\alpha} \alpha^{1+\alpha} \phi}{\rho + \lambda} \right] \gtrless 0 \\ &\Leftrightarrow \phi \gtrless \left(\frac{1 + \alpha}{\alpha} \right) \frac{(\rho + \lambda)}{(1 - \alpha)^{1-\alpha} \alpha^\alpha}.\end{aligned}\quad (29b)$$

A large (small) ϕ implies that the positive effect of a rise in the tax rate on marginal productivity of private capital is large (small). Then, the positive effect on utility through it is also large and the magnitude correlation between the growth- and welfare-maximizing tax rates might be reversible.

These results are summarized as the following proposition:

⁷ At time t , the generation born at time t has no financial asset; $a(t, t) = 0$. Thus, $\beta(t, t) = \beta = 1$.

Proposition 2. *Suppose that λ and ρ are small enough to satisfy $\lambda\rho \approx 0$ and $\lambda^2 \approx 0$. (i) If ϕ is sufficiently small, then the relation between growth-maximizing tax rate and utility-maximizing tax rate under GRPF satisfies*

$$\tau_{\beta=0}^* < \frac{\alpha}{1+\alpha} < \tau^* < \alpha = \tau_{\beta=1}^*.$$

(ii) *In contrast, if ϕ is sufficiently large, then the relation between growth-maximizing tax rate and utility-maximizing tax rate becomes*

$$\frac{\alpha}{1+\alpha} < \min[\tau_{\beta=0}^*, \tau^*] < \max[\tau_{\beta=0}^*, \tau^*] < \alpha = \tau_{\beta=1}^*.$$

This proposition shows that the share of financial asset to total asset is a key to derive the utility-maximizing tax rate and the relation between the growth-maximizing and utility-maximizing tax rate. By intuition, we can relate β to the life expectancy. If it is true, Proposition 2 also implies that there is the relation between growth-maximizing tax rate, utility-maximizing tax rate, and life expectancy. Therefore, we investigate this relation in the next section.

4. Longevity and fiscal policy

Making a preparation for analysis on the relation between longevity and fiscal policy, we investigate the effects of an increase in λ . Note that life expectancy is equal to $1/\lambda$. Total differentiation of (12a), (12b), (15) and (16) lead to

$$\frac{\partial g}{\partial \lambda} = \frac{\partial b}{\partial \lambda} = 0, \tag{30a}$$

$$\frac{\partial \omega}{\partial \lambda} = -\frac{(\rho + 2\lambda)\omega^3}{1 + (\rho + \lambda)\lambda\omega^2} < 0, \tag{30b}$$

$$\frac{\partial \gamma}{\partial \lambda} = -(\rho + 2\lambda)\omega - (\rho + \lambda)\lambda \frac{\partial \omega}{\partial \lambda} = -\frac{(\rho + 2\lambda)\omega}{1 + (\rho + \lambda)\lambda\omega^2} < 0. \tag{30c}$$

Equation (30a) shows that a change in λ does not affect the ratio of public to private capital and that of debt to private capital. GRPF requires that the interest payment is equal to the tax revenue. Then, the ratio of public to private capital and that of debt to private capital are independent of the ratio of private consumption to private capital, which is affected by λ . The result of equation (30b) is standard one. An increase in λ raises the marginal propensity to consume; private consumption increases and the growth rate of private capital declines. Therefore, an increase in λ increases the ratio of private consumption to private capital. Finally, equation (30c) shows that an increase in λ

decreases the equilibrium growth rate.

Let us denote $\tau_{s=t}^*$ and $\tau_{s<t}^*$ by the tax rate that maximizes the utility born at $s = t$ and that of $s > t$, respectively. Using (19), (27a), and (27b), we obtain the following results (See Appendix A.3):

$$\frac{d\tau^*}{d\lambda} < 0, \frac{d\tau_{s=t}^*}{d\lambda} < 0, \frac{d\tau_{s<t}^*}{d\lambda} < 0. \quad (31)$$

By Equation (31), the following proposition is true.

Proposition 3. *Suppose that the second-order conditions for growth-maximizing and utility-maximizing fiscal policy. An increases in the life expectancy raises the growth-maximizing tax rate and the utility-maximizing tax rates of each generation's utility.*

The interpretation of Proposition 3 is explained as follows. The equations for asset and the ratio of human wealth to total wealth are given as follows (Appendix A.4):

$$a(t, s) = \frac{\psi K(s)}{r - \rho - \gamma} [1 - e^{-(r-\rho-\gamma)(t-s)}] e^{(r-\rho)(t-s)}, \quad (32a)$$

$$\beta(t, s) = \frac{r - \rho - \gamma}{[e^{(r-\rho-\gamma)(t-s)} - 1] + r - \rho - \gamma}, \quad (32b)$$

where

$$\psi \equiv \frac{(r - \rho)(1 - \tau)\alpha\phi g^\alpha}{r + \lambda}.$$

Equations (32a) and (32b) imply that older generations have larger financial assets than younger generations, and the ratio of financial wealth to total wealth of older generations is also larger than that of younger generations. For expanding the life expectancy, all existing generations benefits from economic growth. Then, the generation replacement effects decreases. Therefore, all generations desire higher tax rate to maximize their utilities in compared with previous situation.

According to Proposition 3, we can consider the relation between the longevity and tax rate determined through majority voting. In the model, the median's voter age is $\log 2 / \lambda$. Therefore, a rise in the life expectancy increases the median's age and they will vote higher tax rate than the current tax rate. A rise in the tax rate increases the debt per GDP through fiscal rule. By Proposition 3 and this consideration, we obtain the following result:

Corollary. *When the tax rate is determined through majority voting, population aging raises the equilibrium tax rate and also increases the ratio of debt to GDP.*

5. Conclusion

This paper examined the growth and welfare effects of debt-financing public investment under GRPF using Yaari-Blanchard model. This paper showed that Barro tax rule does not hold, and that the growth- and utility-maximizing tax rates are increasing in the life expectancy. Population aging seriously affects the fiscal policy and economic performances: it increases the tax rate and debt per GDP in the voting equilibrium.

Finally, we discuss the future directions of this study. In this paper, we simplify the supply of labor and therefore, omit the possibility of labor retirement. Labor retirement issue is important to consider the intergenerational transfer and distribution of utility. Further, this paper assumed that the government taxes on only factor payments at a constant flat rate. However, this assumption should be relaxed to be differentiated tax rates and varied over time. This relaxation makes us to analyze dynamic equilibrium numerically. These extensions will be fruitful to consider policy implication for more realistic situation.

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Appendix

A.1. No Ponzi condition for the Government's budget

Equation (10) leads to

$$\dot{B}(t) + \tau Y(t) = r(t)B(t) + \dot{G}(t) \Rightarrow \dot{B}(t) = r(t)B(t) + \dot{G}(t) - \tau Y(t). \quad (\text{A1})$$

Multiplying $e^{-\int_0^t r(v)dv}$ to (A1) and integrating it with respect to t , we have

$$\begin{aligned} \int_0^T \dot{B}(t)e^{-\int_0^t r(v)dv} dt &= \int_0^T r(t)B(t)e^{-\int_0^t r(v)dv} dt \\ &+ \int_0^T \dot{G}(t)e^{-\int_0^t r(v)dv} dt - \tau \int_0^T Y(t)e^{-\int_0^t r(v)dv} dt. \end{aligned} \quad (\text{A2})$$

Equation (A2) is reduced to

$$\left[B(t)e^{-\int_0^t r(v)dv} \right]_0^T = \left[G(t)e^{-\int_0^t r(v)dv} \right]_0^T + \int_0^T r(t)G(t)e^{-\int_0^t r(v)dv} dt - \tau \int_0^T Y(t)e^{-\int_0^t r(v)dv} dt.$$

When $t \rightarrow \infty$, above equation is

$$\begin{aligned} \lim_{t \rightarrow \infty} B(t)e^{-\int_0^t r(v)dv} - \lim_{t \rightarrow \infty} G(t)e^{-\int_0^t r(v)dv} \\ = \int_0^\infty r(t)G(t)e^{-\int_0^t r(v)dv} dt - \tau \int_0^\infty Y(t)e^{-\int_0^t r(v)dv} dt. \end{aligned} \quad (\text{A3})$$

The Golden rule leads to

$$B(t) - G(t) = B(0) - G(0). \quad (\text{A4})$$

Inserting (A4) into $G(t)$ of (A3),

$$\tau \int_0^\infty Y(t)e^{-\int_0^t r(v)dv} dt = \int_0^\infty r(t)B(t)e^{-\int_0^t r(v)dv} dt + [G(0) - B(0)] \int_0^\infty r(t)e^{-\int_0^t r(v)dv} dt. \quad (\text{A5})$$

Using (11) and $\tau Y(t) = r(t)B(t)$, equation (A5) is

$$[G(0) - B(0)] \int_0^\infty r(t)e^{-\int_0^t r(v)dv} dt = 0.$$

Therefore, we need $G(0) = B(0)$ to satisfy the balanced budget with (11).

A.2. Derivation of (29a)

$$\begin{aligned} \left. \frac{\partial V(t, t)}{\partial \tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} &= \frac{1}{(\rho + \lambda)} \left[\left. \frac{1}{\rho + \lambda} \frac{dr}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} - 1 - \alpha - \frac{\lambda}{1 + \lambda\omega} \left. \frac{d\omega}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} \right] \\ &= \frac{1}{(\rho + \lambda)} \left[\left. \frac{1}{\rho + \lambda} \frac{dr}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} - 1 - \alpha - \frac{\lambda}{1 + \lambda\omega} \frac{1}{(\rho + \lambda)\lambda + \frac{1}{\omega^2}} \left. \frac{dr}{d\tau} \right|_{\tau=\frac{\alpha}{1+\alpha}} \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{(\rho + \lambda)} \left[-1 - \alpha + \frac{\lambda + \frac{1}{(\rho + \lambda)\omega^2} - \frac{\lambda}{1 + \lambda\omega}}{(\rho + \lambda)\lambda + \frac{1}{\omega^2}} \frac{dr}{d\tau} \Big|_{\tau = \frac{\alpha}{1 + \alpha}} \right] \\
&= \frac{1}{(\rho + \lambda)} \left[-1 - \alpha + \frac{\frac{1}{(\rho + \lambda)\omega^2} + \frac{\lambda^2\omega}{1 + \lambda\omega}}{(\rho + \lambda)\lambda + \frac{1}{\omega^2}} (1 - \alpha)\alpha\phi g^\alpha \right].
\end{aligned}$$

A.3. Derivation of equation (31)

Total differentiation of equation (19) when $\partial\gamma/\partial\tau = 0$ is

$$\frac{\partial^2\gamma}{\partial\tau^2} d\tau + \frac{\partial^2\gamma}{\partial\lambda\partial\tau} d\lambda = 0 \Rightarrow \frac{d\tau^*}{d\lambda} = -\frac{\partial^2\gamma}{\partial\lambda\partial\tau} / \frac{\partial^2\gamma}{\partial\tau^2}. \quad (\text{A6})$$

Assume that the denominator of (A6) is negative (by second order condition). We have

$$\frac{\partial^2\gamma}{\partial\lambda\partial\tau} = \frac{\partial}{\partial\lambda} \left[\frac{\partial r}{\partial\tau} - (\rho + \lambda)\lambda \frac{\partial\omega}{\partial\tau} \right] = -(\rho + 2\lambda)\lambda \frac{\partial\omega}{\partial\tau} - (\rho + \lambda)\lambda \frac{\partial}{\partial\lambda} \left(\frac{\partial\omega}{\partial\tau} \right) < 0, \quad (\text{A7})$$

where

$$\frac{\partial}{\partial\lambda} \left(\frac{\partial\omega}{\partial\tau} \right) = \frac{1}{1 + (\rho + \lambda)\lambda\omega^2} \left[\frac{(\rho + 2\lambda)}{(\rho + \lambda)\lambda} - \frac{2}{\omega} \frac{\partial\omega}{\partial\lambda} \right] \frac{\partial\omega}{\partial\tau} > 0 \text{ for } \tau = \tau^*.$$

Equations (A6) and (A7) lead to the left inequality in equation (31).

Assume that the second order conditions are satisfied. Derivation of the middle and right inequalities in equation (31) are also derived by similar way. Total differentiation of equation (27b) when $\partial V(t, t)/\partial\tau = 0$ is

$$\frac{\partial^2 V(t, t)}{\partial\tau^2} d\tau + \frac{\partial^2 V(t, t)}{\partial\lambda\partial\tau} d\lambda = 0 \Rightarrow \frac{d\tau_{\beta=0}^*}{d\lambda} = -\frac{\partial^2 V(t, t)}{\partial\lambda\partial\tau} / \frac{\partial^2 V(t, t)}{\partial\tau^2}. \quad (\text{A8})$$

We can calculate the numerator of (A8) as

$$\frac{\partial^2 V(t, t)}{\partial\lambda\partial\tau} = -\frac{1}{\rho + \lambda} \left[\frac{1}{(\rho + \lambda)^2} \frac{\partial r}{\partial\tau} - \frac{\partial}{\partial\lambda} \left(\frac{1}{h(t)} \frac{\partial h(t)}{\partial\tau} \right) \right] < 0, \quad (\text{A9})$$

where

$$\begin{aligned}
\frac{\partial}{\partial\lambda} \left(\frac{1}{h(t)} \frac{\partial h(t)}{\partial\tau} \right) &= -\frac{\partial}{\partial\lambda} \left(\frac{\lambda}{1 + \lambda\omega} \frac{\partial\omega}{\partial\tau} \right) = -\frac{\left[\frac{\partial\omega}{\partial\tau} + \lambda \frac{\partial}{\partial\lambda} \left(\frac{\partial\omega}{\partial\tau} \right) \right] (1 + \lambda\omega) - \lambda \frac{\partial\omega}{\partial\tau} \left(\omega + \lambda \frac{\partial\omega}{\partial\lambda} \right)}{(1 + \lambda\omega)^2} \\
&= -\frac{\frac{\partial\omega}{\partial\tau} + (1 + \lambda\omega)\lambda \frac{\partial}{\partial\lambda} \left(\frac{\partial\omega}{\partial\tau} \right) - \lambda^2 \frac{\partial\omega}{\partial\tau} \frac{\partial\omega}{\partial\lambda}}{(1 + \lambda\omega)^2} < 0 \text{ for } \tau = \tau_{\beta \geq 0}^*
\end{aligned}$$

Using equations (A8) and (A9), we obtain the middle inequality in equation (31). In case of the right inequality in equation (31), we have

$$\frac{\partial^2 V(t, s)}{\partial \tau^2} d\tau + \frac{\partial^2 V(t, s)}{\partial \lambda \partial \tau} d\lambda = 0 \Rightarrow \frac{d\tau_{\beta > 0}^*}{d\lambda} = -\frac{\partial^2 V(t, s)}{\partial \lambda \partial \tau} / \frac{\partial^2 V(t, s)}{\partial \tau^2}, \quad (\text{A10})$$

$$\frac{\partial^2 V(t, s)}{\partial \lambda \partial \tau} = -\frac{1}{\rho + \lambda} \left[\frac{1}{(\rho + \lambda)^2} \frac{\partial r}{\partial \tau} - \frac{\partial \beta}{\partial \lambda} \frac{1}{h(t)} \frac{\partial h(t)}{\partial \tau} - \beta \frac{\partial}{\partial \lambda} \left(\frac{1}{h(t)} \frac{\partial h(t)}{\partial \tau} \right) \right] < 0 \quad (\text{A11})$$

where

$$\begin{aligned} \frac{\partial h(t)}{\partial \lambda} &= -\left[\frac{1}{\rho + \lambda} + \frac{\omega}{1 + \lambda\omega} \left(1 + \frac{\lambda}{\omega} \frac{\partial \omega}{\partial \lambda} \right) \right] h(t) < 0, \\ \frac{\partial \beta}{\partial \lambda} &= \frac{\beta(1 - \beta)}{h(t)} \frac{\partial h(t)}{\partial \lambda} < 0 \end{aligned}$$

Therefore, equations (A10) and (A11) provide the right inequality in equation (31).

A.4. Derivation of equations (32a) and (32b)

Equations (1), (2b), (4), (6a), (12b), and (21) lead to

$$\frac{da(v, s)}{dv} = (r - \rho)a(v, s) + \psi K(s)e^{\gamma v}. \quad (\text{A12})$$

The complementary solution to equation (A12) is

$$a(t, s) = X e^{(r-\rho)(t-s)}. \quad (\text{A13})$$

In (A13), X will be depend on t . Differentiating (A13) with respect to t , we obtain

$$\frac{da(t, s)}{dt} = \frac{dX}{dt} e^{(r-\rho)(t-s)} + X(r - \rho)e^{(r-\rho)(t-s)} = \frac{dX}{dt} e^{(r-\rho)(t-s)} + (r - \rho)a(t, s). \quad (\text{A14})$$

Using equations (A12)-(A14), we have

$$\frac{dX}{dt} = \psi K(s)e^{-(r-\rho-\gamma)(t-s)}. \quad (\text{A15})$$

Solving equation (A15) with respect to t , we derive

$$X = Z - \frac{\psi K(s)e^{-(r-\rho-\gamma)(t-s)}}{r - \rho - \gamma}. \quad (\text{A16})$$

Inserting equation (A16) into equation (A13),

$$a(t, s) = \left[Z e^{(r-\rho)(t-s)} - \frac{\psi K(s)e^{\gamma(t-s)}}{r - \rho - \gamma} \right]. \quad (\text{A17})$$

Since we have $a(t, t) = 0$, equation (A17) must satisfy

$$a(t, t) = Z - \frac{\psi K(s)}{r - \rho - \gamma} = 0 \Rightarrow Z = \frac{\psi K(s)}{r - \rho - \gamma}. \quad (\text{A18})$$

Using equations (A17) and (A18), we arrive at

$$a(t, s) = \frac{\psi K(s)}{r - \rho - \gamma} \left[e^{(r-\rho)(t-s)} - e^{\gamma(t-s)} \right] = \frac{\psi K(s)}{r - \rho - \gamma} \left[1 - e^{-(r-\rho-\gamma)(t-s)} \right] e^{(r-\rho)(t-s)}.$$

Above equation corresponds to equation (32a). Using (32a) and the definition of β , we obtain

$$\begin{aligned}\beta(t, s) &= \frac{(r - \rho - \gamma)K(t)}{[1 - e^{-(r-\rho-\gamma)(t-s)}]e^{(r-\rho)(t-s)}K(s) + (r - \rho - \gamma)K(t)} \\ &= \frac{r - \rho - \gamma}{[1 - e^{-(r-\rho-\gamma)(t-s)}]e^{(r-\rho)(t-s)}e^{\gamma(s-t)} + r - \rho - \gamma} \\ &= \frac{r - \rho - \gamma}{[e^{(r-\rho-\gamma)(t-s)} - 1] + r - \rho - \gamma}.\end{aligned}$$

Therefore, equation (32b) is derived.

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