

# Intergenerational transfers, tax policies and public debt

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## Abstract

This paper studies the impact of the tax system on intergenerational family transfers in an overlapping generation model of a closed economy, with endogenous human capital growth. We limit ourselves to simple tax structures with labor and inheritance taxes. When public debt is an available instrument for the government, we show that the fiscal policy used to achieve the long run optimal endogenous growth improves the individuals' consumption of the first generations. In this case, the government reduces the tax burden on labor, encourages human capital development and puts in place a redistributive policy. If the public debt is not available, the government does not pursue a redistributive policy, both tax rates implemented are higher and the long run human capital growth is greater as well. In all cases, the optimal inheritance tax rate is higher than the optimal tax rate on labor income.

**Keywords:** family transfers, debt, altruism, growth, optimal taxation.

**JEL codes:** D64, H21, H23, H63, I31.

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# 1 Introduction

A debt crisis has affected in 2010 a number of European countries. At different levels, each state has been later confronted to an economic crisis and prompted to ensure the public debt sustainability. Some governments established a budgetary balance in their countries, such as Germany, resulting in the capacity to reduce their budgetary deficits or improve their tax revenues. The severity of this crisis highlights the importance of the public debt availability on growth and on the economic policies used.

The nature of the relationship between public debt and economic growth is related to their own characteristics. A part of the economic literature demonstrates that public capital raises the long run economic growth assuming a balanced government budget constraint. For instance, Futagami et al. (1993) extend the simple approach of Barro (1990) by presenting an endogenous growth model with productive public capital which encouraged per-capital growth in the long run. Furthermore, according to the literature, a public debt caused by budgetary deficit, has a negative effect in the long run economic growth in the sense that the taxes needed to finance the interest payments, reduce individuals' incomes and savings and thus the capital stock. This negative effect has been shown in endogenous growth model by several authors (see Futagami et al. (2008), Greiner (2012) and Yakita (2008)).

In most cases, the economic growth is characterized through the physical capital accumulation. However, a certain number of authors, such as Romer (1986) and Lucas (1988), highlighted the crucial importance of human capital investment in contributing to economic growth. Considering that the developed economies value knowledge and given that the knowledge-based economy will be a key parameter of future economic growth, governments must consider this growth's feature in their fiscal policies.

Regarding the budgetary balance rule which results from the European debt crisis, the European governments have managed to reduce their budget deficits and modified their tax policies. As regards the tax structure, governments are confronted with an arbitrage between economic growth and the way taxes are combined and designed to generate revenues. Indeed, each tax has its own features and affect differently the evolution of the market economy and the redistributive issues.

Concerning the redistribution problem, Piketty (2011) observes an increase in the intergenerational inequalities since the beginning of the eighties. Currently, he states that the average wealth at death is more than twice than that of the current living. Furthermore, Piketty (2011) notes that the inheritance transfers represent a large part of the average wealth at death and this is increasing over time. As a result, the inheritance transfers are probably the leading of the increase in intergenerational inequalities. In most developed countries, however, the tax revenues generated by wealth transfer taxation have been especially low and decreasing since the seventies (see Arrondel and Masson (2013)). In the public debate, the main arguments in favour of a low inheritance tax level are that this taxation might discourage wealth accumulation coming from family altruism and the resulting tax revenue loss are lower using other taxes. In the economic literature, two famous theoretical models from Atkinson and Stiglitz (1976) and Chamley (1986) and Judd (1985) postulate that the optimal tax rate on capital is equal to zero in the long run. According to them, it would be optimal to exclude capital income from the

tax structure whatever the capital distribution across agents. Over the past few years, however, some studies have shown that we can overturn this famous zero capital tax result by relaxing some of these assumptions. Indeed, this result happens because they consider inheritance as the life cycle savings from agents or dynasties depending on the models. In the new literature, Piketty and Saez (2013) or Cremer and Pestieau (2011) for instance, analyze the bequest behavior independently to the one of savings and its impact on the long run optimal tax structure. Another strand of this literature studies the optimal capital taxation taking into account human capital investments. Jacobs and Bovenberg (2010) find a positive optimal capital income tax due to the distortionary impact of labor income tax on the human capital formation. Lastly, we can also relax the optimal zero capital income tax result by considering two types of intergenerational family transfers. Indeed, most models of intergenerational family transfers focus on inheritance whereas it is not the only one. We can moreover consider the parents' investment in their children's education attainment. The intergenerational transmission of education through, for example, the parents' investment in child's education improves the well-being of children. An extensive empirical literature argues that higher education has a positive impact on income, health and lifespan. Considering the education transfers, the individuals' trade off across both transfers creates inequalities across agents and generations, depending on their differences with respect to their lifetime resources. Therefore, the concept of education family transfers involves that optimal fiscal policy changes in order to solve the intergenerational inequalities issue.

The objective of this paper is to present a theoretical model where intergenerational redistribution and public debt availability are considered together. By limiting ascendant public transfers, we explore how a budgetary balance rule such as the European one, affects the households' intergenerational family transfers distribution and consequently, the economic growth and the agents' welfare. In our analysis, we focus on human capital economic growth to underline the increasing proportion of economic growth based on this. We assume that two types of intergenerational family transfers, the parent's education spending and the inheritance, are used by parents to improve the resources of their children. The two taxes available are the ones affecting directly the agents' trade off across both transfers. For these reasons, we consider a dynamics overlapping generation model of a closed economy, with endogenous human capital growth where the labor income tax, the inheritance tax and the public debt have to finance a public spending.

Our main findings is that public debt is required to achieve the long run optimal human capital growth and implement an intergenerational redistribution policy. Thanks to the public debt, the tax burden on both labor and capital can be adjusted and governments can promote human capital development and improve the individuals' consumption from the present generations. When the public debt is not available, we show that the human capital growth is higher as well as the two tax instruments. In this case, the government internalizes the positive human capital externality and pursues a redistributive policy only using the inheritance tax and the labor one.

In section 2 the framework of the model is developed. Section 3 analyzes the long run optimal tax rates according to the public debt availability. Numerical illustration is used to show that the transition dynamics quickly converges towards the optimal steady state depending on

the availability of the public debt. Finally, in the last section we conclude this study.

## 2 The framework

We consider a dynamic overlapping generation model in a closed economy wherein each generation lives two periods. We assume a representative agent within generation as households have homogeneous preferences. In the first period of life, he gets an education. Then, the agent works, consumes and leaves intergenerational transfers to his offspring in the second period. Population size is constant over time and we assume that individuals will become parents for sure.

This model is a simplified version of the traditional two overlapping generation economy, in which the inheritance is the only source of saving. We have two types of intergenerational family transfers: the inheritance  $x_{t+1}$  and the education expenditure  $e_{t+1}$  left from parent belonging to generation  $t$  to his child from generation  $t + 1$ .

During his schooling period, the child improves his human capital level using the parent's education spending. In the next period, the parent's investment in education affects positively the child's wage  $w_{t+1}$  through his human capital level  $H_{t+1}$ . Moreover, the representative household recovers from the dynasty, the accumulated knowledge overtime. The use of parental knowledge as a child's human capital factor is consistent with a number of studies, such as the empirical approach from Hertz et al. (2007). Thus, we concentrate on private education regime where the child's learning relies on the parent's stock of human capital and his child's education expenditure decision:

$$H_{t+1} = G(e_{t+1}, H_t), t \geq 0 \quad (1)$$

Furthermore, the parent transmits the stock of human capital from the dynasty by investing first in child's education. He has incentive to invest in education in order to provide the accumulated knowledge of the dynasty. A Cobb Douglas human capital function is used to describe these features of the agent's human capital:

$$G(e_{t+1}, H_t) = B(e_{t+1})^\delta (H_t)^{1-\delta}$$

where  $B$  is a strictly positive technological parameter,  $\delta$  represents the responsiveness of child's human capital to a change in parent's education spending. We assume  $0 < \delta < 1$ . The agent's human capital function is continuously differentiable. The child's human capital is a strictly increasing function of education expenditure and of human capital transmitted from parent. The two factors of human capital are imperfect substitute as the parent needs to invest in child's education to transfer the stock of human capital from the dynasty. Furthermore, the higher is the accumulated knowledge transferred to the offspring, the larger is the necessary education investment to improve marginally the child's human capital level. We note that invest in basic knowledges is less costly than the next learning.

The features of the human capital function imply an endogenous human capital growth. We assume that everybody works one unit of time (implying an inelastic labor supply) and that the

initial endowments of financial wealth and human capital received by agent from period  $t = 0$  are given. The representative agent from the first period receives a positive amount of human capital  $h_0$  and inheritance  $x_0$ .

## 2.1 Production

We consider a representative firm which produces a homogeneous good at each generation by using physical capital  $K_t$  and human capital  $H_t$  production factors and which behaves competitively. From the maximization program of the firm, the returns of production's factors  $w_t$  and  $R_t$  are equal to their marginal products. We assume a total depreciation of physical capital. We use a Cobb Douglas production function with the following form:

$$F(K_t, H_t) = AK_t^\alpha H_t^{1-\alpha}$$

where  $A$  is a strictly positive technological parameter and  $0 < \alpha < 1$ .

## 2.2 Government

At each period, the government faces a public spending amount which corresponds to a share of production  $\Gamma$ . Thus, the public spending at time  $t$  is equal to  $\Gamma F(K_t, H_t)$ . The share of production devoted to public sector is assumed to be lower than the amount produced,  $0 \leq \Gamma < 1$ . The fiscal instruments available to finance the public spending are the inheritance tax  $\tau_t^B$ , the labor income tax  $\tau_t^L$  and the public debt  $\Delta_t$ . The government's budget constraint at time  $t$  is characterized as follow:

$$\Gamma F(K_t, H_t) + R_t \Delta_{t-1} = \tau_t^B R_t K_t + \tau_t^L w_t H_t + \Delta_t \quad (2)$$

Since we have a Cobb Douglas production function and using equation (2), we obtain the following result for the share of production devoted to public sector when the public debt is not available:

$$\Gamma = \tau_t^B \alpha + \tau_t^L (1 - \alpha) \quad (3)$$

## 2.3 Consumers

Each household maximizes his own utility subject to his budget constraint. The individual's resources comes from two channels: work and inheritance. The total after tax lifetime income is represented by  $\Omega_t$  and equals to:

$$\Omega_t = (1 - \tau_t^B) R_t x_t + (1 - \tau_t^L) w_t H_t \quad (4)$$

where  $(1 - \tau_t^B) R_t x_t$  is the capital income after the inheritance tax and  $(1 - \tau_t^L) w_t H_t$  corresponds to the labor income after the labor tax. These individual's resources are allocated to consumption

$c_t$ , inheritance  $x_{t+1}$  and education expenditure for the child  $e_{t+1}$ .

$$\Omega_t = c_t + e_{t+1} + x_{t+1} \quad (5)$$

Cremer and Pestieau (2011) show the connection between the households' bequest motives and the optimal fiscal policy. In our framework, we only consider family altruism such that parent derives utility from the future offspring's resources. This household's bequest motive can be viewed as an intermediate situation between the paternalistic bequest and the pure altruistic one. As a result, the individual's preferences are given by the sum of the agent's consumption utility and the utility related to his offspring's resources. We used a particular logarithmic utility function to describe these features:

$$u_t = (1 - \gamma) \ln c_t + \gamma \ln \Omega_{t+1} \quad (6)$$

where  $0 < \gamma < 1$ . The parent arbitrates between the child's resources  $\Omega_{t+1}$ , his own consumption and both family transfers used to improve the offspring's incomes. However, the agent does not consider the full welfare consequence of his decision. Each extra unit given by the parent to his offspring delivers utility to the parent, the child and all future generations of the dynasty. By increasing the child's resources, the parent improves in the same way the grand child's resource through the child's intergenerational transfers and through the impact of his human capital level on the grand child's human capital. The allocation of the intergenerational family transfers also plays an important role for the child's transfers decision, and for the grand child human capital level. Indeed, an additional unit of parent's education spending, improves the grand child human capital and increases the level of child's education expenditure required to obtain the same grand child's human capital objective than before. The agent's transfers decision involves a positive human capital externality on future generations.

The individuals maximize their utilities choosing the best allocations of their resources. To replace the agent's consumption and the child's resources by their expression given through (4) and (5), we obtain the first order conditions:

- with respect to  $e_{t+1}$ , for  $t \geq 0$ ,

$$-\frac{1 - \gamma}{c_t} + (1 - \tau_{t+1}^L)w_{t+1} \frac{\partial G}{\partial e}(e_{t+1}, H_t) \frac{\gamma}{\Omega_{t+1}} = 0 \quad (7a)$$

- with respect to  $x_{t+1}$ , for  $t \geq 0$ ,

$$-\frac{1 - \gamma}{c_t} + (1 - \tau_{t+1}^B)R_{t+1} \frac{\gamma}{\Omega_{t+1}} = 0 \quad \text{if } x_{t+1} > 0, \leq 0 \text{ otherwise} \quad (7b)$$

Equation (7a) suggests that parent's education spending is always positive. In fact, the after tax marginal return of education expenditure is decreasing, concave and close to infinite when the education level is equal to zero. We can not have a positive amount of inheritance without investing first in the child's education. Indeed, agent primarily invests in human capital until the marginal return of education expenditure corresponds to the one of inheritance. Thereafter, the focus is exclusively on the interior solution with positive amount of parent's capital transfer.

Then, the parent uses both types of bequests to improve the child's welfare and the following condition is verified:

$$(1 - \tau_{t+1}^B)R_{t+1} = (1 - \tau_{t+1}^L)w_{t+1} \frac{\partial G}{\partial e}(e_{t+1}, H_t) \quad (8)$$

Using equation (8), the agent's education spending with respect to his human capital level is characterized by:

$$\frac{e_{t+1}}{h_t} = (\rho_{t+1}B\delta)^{\frac{1}{1-\delta}} \quad (9)$$

We define  $\rho_{t+1} = \frac{1-\tau_{t+1}^L}{1-\tau_{t+1}^B} \frac{w_{t+1}}{R_{t+1}} = \frac{1-\tau_{t+1}^L}{1-\tau_{t+1}^B} \frac{1-\alpha}{\alpha} \frac{K_{t+1}}{H_{t+1}}$  as the after tax ratio of factor prices. This describes the agent's incentive to invest in education rather than in inheritance. We deduce that the human capital growth at a given generation is equivalent to:

$$\frac{H_{t+1}}{H_t} = B (\rho_{t+1}B\delta)^{\frac{\delta}{1-\delta}} \quad (10)$$

Furthermore, the agent's inheritance level corresponds to:

$$x_{t+1} = \gamma [\Omega_t - e_{t+1}] - (1 - \gamma) \rho_{t+1} H_{t+1} \quad (11)$$

## 2.4 The dynamics

The capital market equilibrium is characterized by:

$$\Delta_t + K_{t+1} = x_{t+1} \quad (12)$$

Thus, the capital dynamics rely on the public debt availability and on the agent's family transfers allocation. As argued in the previous section, our focus is now on the interior solution where every representative agent leaves both transfers to his offspring. Therefore, the representative dynasty has to satisfy the following inheritance condition overtime.

$$x_{t+1} > 0 \Leftrightarrow (1 - \tau_t^B) R_t \left( \rho_t + \frac{x_t}{H_t} \right) > \left( 1 + \frac{1-\gamma}{\gamma\delta} \right) (\rho_{t+1}B\delta)^{\frac{1}{1-\delta}} \quad (13)$$

In Appendix, we show that agents always satisfy the inheritance condition (13) at steady state assuming a human capital growth higher than the after tax return of inheritance. Moreover, this condition is also verified when the public debt is not available along the dynamics. In both situations, we need that tax instruments are constants between generations.

Assuming that public debt is an available instruments and using the agent's inheritance level (11) as well as the individual's education spending amount (9) and the human capital growth (10), the capital market equation (12) corresponds to:

$$d_{t+1} + g_{t+1}^H k_{t+1} = \gamma (1 - \tau_t^B) A \alpha k_t^{\alpha-1} \left( \rho_t + \frac{d_t}{g_t^H} + k_t \right) - \left( \gamma + \frac{1-\gamma}{\delta} \right) (\rho_{t+1}B\delta)^{\frac{1}{1-\delta}} \quad (14)$$

where  $g_{t+1}^H$  corresponds to the human capital growth (10),  $d_{t+1}$  is the ratio of public debt with respect to human capital from the previous period  $\frac{\Delta_t}{H_t}$  and  $k_{t+1}$  is the capital-labor ratio  $\frac{K_{t+1}}{H_{t+1}}$

from period  $t + 1$ . Besides, the government budget constraint (2) looks as follows using the Cobb Douglas production function:

$$d_{t+1} = (1 - \tau_t^B) A \alpha k_t^{\alpha-1} \frac{d_t}{g_t^H} + A k_t^\alpha [\Gamma - \tau_t^B \alpha - \tau_t^L (1 - \alpha)] \quad (15)$$

Since the after tax ratio of factor prices is linear with respect to capital labor ratio, it is equivalent to analyze its dynamics. Considering that tax instruments are constants over time and from equations (14) and (15), the dynamics of the after tax ratio of factor prices and of the public debt with respect to human capital are characterized by:

$$(\rho_{t+1} B \delta)^{\frac{1}{1-\delta}} = \frac{\gamma \delta A (1 - \Gamma) \left( \frac{1 - \tau^B}{1 - \tau^L} \frac{\alpha}{1 - \alpha} \right)^\alpha}{\frac{1 - \tau^B}{1 - \tau^L} \frac{\alpha}{1 - \alpha} + \gamma \delta + 1 - \gamma} \rho_t^\alpha \left[ 1 - \Gamma - \frac{1 - \gamma}{\gamma} \frac{d_t (1 - \tau^L) (1 - \alpha)}{g_t^H \rho_t} \right] \quad (16a)$$

$$d_{t+1} = A \left( \frac{1 - \tau^B}{1 - \tau^L} \frac{\alpha}{1 - \alpha} \rho_t \right)^\alpha \left[ \frac{d_t (1 - \tau^L) (1 - \alpha)}{g_t^H \rho_t} + \Gamma - \tau^B \alpha - \tau^L (1 - \alpha) \right] \quad (16b)$$

The system of equation (16) illustrates the dynamics of both variables, with  $g_t^H = B (\rho_{t+1} B \delta)^{\frac{\delta}{1-\delta}}$ . When the public debt is not available  $d_{t+1} = d_t = 0$ , the dynamics of the after tax ratio of factor prices converge towards one of the two achievable steady states, since the right hand side of equation (16a) is increasing and concave whereas the left hand side is increasing and convex. The first one corresponds to a situation where the stationary value is strictly positive. In the other one, the steady state realized, is equal to zero. However, the human capital growth depends on the after tax ratio of factor prices because it affects the agent's family transfers decision. Indeed, parent is more incentive to invest in human capital when it increases. The case where the human capital growth tends towards zeros is not desirable for any agents assuming this is always beneficial to individuals. As a result, the only feasible situation is when the stationary state  $\rho$  is strictly positive. Thus, the dynamics are monotonous, strictly increasing across periods and converge towards  $\rho$ . As the after tax ratio of factor prices is linear in relation to the capital-labor ratio, its dynamics also converge to a positive and constant stationary state. The next section focuses on the public debt availability impact on the long run optimal fiscal policy and the long run optimal human capital growth.

### 3 The optimal tax rates and public debt availability

The social planner cares about the social welfare of everyone in the same way, equivalently. Considering the particular nature that characterises the agent's altruism, there are at least two feasible social criteria. Indeed, the approaches is different whether or not, the government takes into account the agent's utility related to his child's resources. We concentrate our analysis on a social welfare function where the government focuses on optimizing the consumption of each generation. In this case, the social planner does not think about parent's altruism and maximizes the agent's life cycle utility. Using the logarithmic agent's utility function, the government's



social welfare function is characterized by:

$$SWF = \sum_{t=0}^{+\infty} \beta^t (1 - \gamma) \ln(c_t) \quad (17)$$

where  $\beta^t$  is the time preference from period  $t$ . In this section, we analyze the impact of the public debt availability on the long run optimal human capital growth arising from the fiscal policy used. We assume that the social planner faces a positive public debt constraint, taking into account the social welfare function (17). First, we concentrate on the first best social optimum when the public debt is an available instrument for the government. Second, we assume a positive public debt with a zero public debt constraint and we analyze its effect on the long run economic growth and on the fiscal policy chosen. Finally, we make a numerical illustration to study the transition dynamics towards the optimal steady state and analyze the optimal stationary results obtained.

### 3.1 The first best optimal solutions

The first best social planner maximization problem is characterized by the social welfare function (17) with respect to the sequence  $(c_t, e_{t+1}, H_{t+1}, K_{t+1})_{t \geq 0}$  subject to the human capital technology (1) and the resource constraint:

$$K_{t+1} = (1 - \Gamma) F(K_t, H_t) - c_t - e_{t+1} \quad (18)$$

From the optimality conditions (36) and (39) given in Appendix, we obtain the agent's optimal consumption growth level:

$$\frac{c_{t+1}^*}{c_t^*} = \beta \left( (1 - \Gamma) F'_K(K_{t+1}^*, H_{t+1}^*) \right) \quad (19)$$

Using (36), (37) and (38) in Appendix, we deduce:

$$\frac{\beta^t}{c_t} = \frac{\beta^{t+1}}{c_{t+1}} \frac{\partial G}{\partial e}(e_{t+1}, H_t) \left[ (1 - \Gamma) F'_H(K_{t+1}, H_{t+1}) + \frac{\frac{\partial G}{\partial H}(e_{t+2}, H_{t+1})}{\frac{\partial G}{\partial e}(e_{t+2}, H_{t+1})} \right]$$

From the two last equations, one gets:

$$(1 - \Gamma) F'_K(K_{t+1}^*, H_{t+1}^*) = \frac{\partial G}{\partial e}(e_{t+1}^*, H_t^*) \left[ (1 - \Gamma) F'_H(K_{t+1}^*, H_{t+1}^*) + \frac{\frac{\partial G}{\partial H}(e_{t+2}^*, H_{t+1}^*)}{\frac{\partial G}{\partial e}(e_{t+2}^*, H_{t+1}^*)} \right] \quad (20)$$

The equation (20) shows that the optimal level of agent's education expenditure is obtained when the marginal social returns of both transfers are equal. However, the social return of an additional unit of education spending (right side of equation (20)) is different to the return received by the parent (right side of equation (8)). Indeed, there are two different effects in the social return of an extra unit of education spending. The direct effect on the child's wage and the indirect effects on the future generations with respect to their human capital levels and their family transfers decisions. Therefore, the social planner satisfies his social criterion by taking

into account the positive human capital externality coming from the parent's behavior. Indeed, the individual's bequest motive is to improve the lifetime resources of the donee and not to increase his welfare and, given the agent's human capital function, there is a positive human capital externality on all the next generations. In addition, the return of each transfers improves the total income of each generation compare to the previous one. Thus, the government should make a redistributive policy in order to maximize the individuals' life cycle utilities.

As a result, the social planner's fiscal policy objectives are to finance the public spending, to take into account the externality and to pursue a redistributive policy. In the next section, we analyze the fiscal policy used to decentralize the optimum solutions (19) and (20) such that the individuals make the optimal choices regarding their transfers decisions.

### 3.2 Decentralization of the first best optimal solutions and fiscal policy

In the equilibrium situation, the amount of both transfers from the agent's transfers decision are not equal to their optimal levels. For this purpose, the government determines the fiscal policy which decentralized the optimum solutions (19) and (20), from the equilibrium situation. The resource constraint (18) and the human capital function (10) are satisfied in both cases. Using the equilibrium condition (8) which determines the agent's education transfer and the optimal solution (20), we obtain:

$$\frac{1 - \tau_{t+1}^{L*}}{1 - \tau_{t+1}^{B*}} = 1 + \frac{1}{(1 - \Gamma) F'_H(K_{t+1}^*, H_{t+1}^*)} \left[ \frac{\frac{\partial G}{\partial h}(e_{t+2}^{i*}, h_{t+1}^{i*})}{\frac{\partial G}{\partial e}(e_{t+2}^{i*}, h_{t+1}^{i*})} \right] \quad (21)$$

Equation (21) shows that the optimal inheritance tax is positive and higher than the labor one and the share of public spending in the production sector. The gap across the two tax rates is used to internalize the positive human capital externality in the agent's intergenerational transfers decision. Having an inheritance tax higher than the labor tax, gives incentive to households to invest more in education and less in inheritance. At each period, the government uses the following fiscal policy to decentralize the optimal solution. From equation (14), we get the following optimal level of public debt:

$$d_{t+1}^* = \frac{1}{1 - \gamma} \left[ \gamma(1 - \Gamma) F(k_t^*, 1) - g_{t+1}^* k_{t+1}^* - \left( \gamma + \frac{1 - \gamma}{\delta} \right) \eta_{t+1}^* \right] \quad (22)$$

where  $\eta_{t+1} = \frac{e_{t+1}}{H_t}$ . Using the government budget constraint (15), we deduce the optimum tax rates at each period. Furthermore, the degree of agents' altruism has an impact on the fiscal policy implemented to decentralize the optimum solutions. Indeed, the optimal amount of public debt depends on it. The government uses the public debt to implement the first best redistributive policy and to support the tax instruments which internalize the positive human capital externality. Therefore, the availability of public debt plays an important role to reach the first best optimal solution. We thereafter analyze the impact of the public debt availability on the government's fiscal policy implemented to decentralize the optimum at steady state.

### 3.3 The first best optimal fiscal policy at steady state

At steady state, the capital-labor ratio is strictly positive and constant  $k_{t+1} = k$ . Then, the resource constraint (18) is equivalent to:

$$k = \frac{(1 - \Gamma) F(k, 1)}{g^H} - \frac{1}{g^H} \left( \frac{c}{H} \right) - \frac{1}{g^H} \eta \quad (23)$$

The agent's human capital growth (10) and the individual's ratio of education expenditure relative to human capital (9) are constant at steady state since the after tax ratio of factor prices is stationary. Furthermore, the share of production devoted to the private sector is also stationary and equation (23) shows that  $\frac{c_t}{H_t}$  is constant. As a result, the agent's optimal consumption growth and the agent's human capital growth are the same for every generation and are equivalent to each other. They are characterized by the following equation obtained with the optimal solution (19):

$$g^{H*} = B(\eta)^\delta = \beta(1 - \Gamma) F'_K(k^*, 1) \quad (24)$$

Using the optimum solutions (20) and (24), we get the stationary value of the agent's education spending with respect to his human capital and thus we deduce the first best steady state capital-labor ratio:

$$k^* = \left[ \frac{(1 - \beta(1 - \delta))^\delta (\beta(1 - \Gamma)\alpha A)^{1-\delta}}{B\left(\frac{1-\alpha}{\alpha}\delta\right)^\delta} \right]^{\frac{1}{1-\alpha+\alpha\delta}} \quad (25)$$

and also the first best optimal long run human capital growth:

$$g^{H*} = \left[ B^{1-\alpha} \left( \frac{(1 - \alpha)\delta}{\alpha(1 - \beta(1 - \delta))} \right)^{(1-\alpha)\delta} (\beta(1 - \Gamma)\alpha A)^\delta \right]^{\frac{1}{1-\alpha+\alpha\delta}} \quad (26)$$

Both optimum ratios are positively affected by the discount factor  $\beta$ . Indeed, the human capital growth is higher when the social planner gives more weight to future generations. In order to achieve the first best optimum (25) and (26), the government uses the following fiscal policy from the optimum solutions (21) and (24):

$$\frac{1 - \tau^{L*}}{1 - \tau^{B*}} = \frac{1}{1 - \beta(1 - \delta)} \quad (27)$$

Equation (27) describes the optimal tax ratio which is positive with an inheritance tax higher than the labor one. The effect of internalize the positive human capital externality into the agents behavior leads to increase the inheritance tax rate and decrease the labor income tax rate. Furthermore, the inheritance tax is positively affected by an higher  $\beta$ , while the impact is negative with respect to the labor income tax. The higher the time preference is, the higher is the gap across the two taxes. Thanks to the optimal tax policy (27), the households have incentive to invest more in education expenditures in order to reach the long run optimal human capital growth (26).

At steady state, the optimal level of public debt (22) using the optimal solutions (20) and

(24) is equal to:

$$d^* = \frac{\gamma - \beta}{1 - \gamma} \left( 1 + \frac{(1 - \alpha) \beta (1 - \delta)}{1 - \beta (1 - \delta)} \right) (1 - \Gamma) A k^{\alpha*} \quad (28)$$

Equation (28) illustrates that the sign of the public debt depends on the time preference value and the degree of agent's altruism.

In the particular situation with  $\beta = \gamma$ , the government only uses the two tax instruments to decentralize the first best optimum. From the budget constraint (3), the first best optimum levels of tax instruments are equal to:

$$\begin{aligned} \tau^{B*} &= \Gamma + \frac{(1 - \alpha) \beta (1 - \delta) (1 - \Gamma)}{1 - \alpha \beta (1 - \delta)} \\ \tau^{L*} &= \Gamma - \frac{\alpha \beta (1 - \delta) (1 - \Gamma)}{1 - \alpha \beta (1 - \delta)} \end{aligned}$$

In this case, the inheritance tax is always positive whereas the labor income tax can be either positive or negative. The optimal levels of tax instruments depend on the amount of public spending and the government's policy which encourages the agents to transfer their optimal levels of intergenerational transfers. Furthermore, there are two other factors which influence differently the two taxes. On the one hand, when the share of capital into the production factor increases, this affects negatively both optimal tax rates. On the other hand, the responsiveness of agent's human capital to an increase of the accumulated knowledge ( $1 - \delta$ ), affects both taxes but in an opposite way.

In the other situations with  $\gamma \neq \beta$ , the public debt is required to achieve the first best optimum. When  $\gamma > \beta$ , the public debt is positive and negative otherwise. The first best optimum tax policy with non zero public debt is deduced by using the steady state government budget constraint (15), the optimal solution (24), the optimal tax ratio (27) and the long run optimal public debt (28). On the one hand, the first best optimal labor income tax at steady state corresponds to:

$$\tau^{L*} = \Gamma - (1 - \Gamma) \left[ C + \frac{D}{1 + \frac{\gamma - \beta}{1 - \gamma} \frac{1}{\beta}} \right]$$

where  $C = \frac{\beta}{1 - \beta(1 - \delta)} - 1$  and  $D = \frac{1}{1 - \alpha\beta(1 - \delta)} - \frac{\beta}{1 - \beta(1 - \delta)}$ . The labor income tax is positive or negative depending on the parameters values. On the other hand, the first best optimal inheritance tax is equal to:

$$\tau^{B*} = \Gamma + (1 - \Gamma) \left[ E + \frac{F}{1 + \frac{\gamma - \beta}{1 - \gamma} \frac{1}{\beta}} \right]$$

where  $E = 1 - \beta$  and  $F = \beta - \frac{1 - \beta(1 - \delta)}{1 - \alpha\beta(1 - \delta)}$ . As  $0 \leq \beta < 1$ , we have  $E \geq 0$  and the sign of  $F$  is the opposite sign of  $D$ .

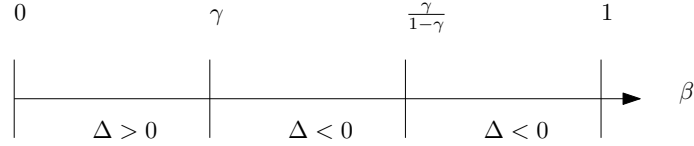
The degree of agent's altruism affects the optimal tax levels in the same way. Indeed, the effect of  $\gamma$  on both optimal tax values is either positive or negative depending on the sign of  $D$  which is ambiguous as it relies on the time preference.<sup>1</sup> For given values of time preference and of

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<sup>1</sup>In the case where  $\beta \leq \frac{1}{2}$ , the sign of  $D$  is always positive and both tax instruments increase with the degree of agent's altruism. When  $\beta > \frac{1}{2}$ , the effect is more ambiguous and depends on the parameters values.

individual's altruism degree, the fiscal policy implemented by the social planner to decentralize the optimal human capital growth (26) is different. The situations are represented in the next figure.

Figure 1: Optimal public debt according to the time preference value  $\beta$



First, when  $\gamma > \beta$ , the representative agent's over invests in child's resources such as he penalizes his own consumption. The social planner promotes a low human capital growth (26) in order to increase the agent's consumption of the current generation by using a positive public debt and both tax instruments. The government must also consider the externality issue and finance the public spending. For these purposes, the labor income tax is still less than the inheritance one to take into account the positive externality and both taxes are higher in order to reduce the individual's incentive to transfer to his child. Thus, this fiscal policy encourages agents to consume rather than invest for the future generations by reducing the incentive to pass on resources to the next generation using higher tax rates and positive public debt.

When the gap across the time preference and the degree of agent's altruism is reduced, the one across both tax rates is higher and the public debt tends to zero. When  $\gamma = \beta$ , the agents still transfer too much to their child but the government uses only the two tax instruments to decentralize the first best solution. In this particular case, the first best optimal fiscal policy which ensure the optimal human capital growth corresponds to the one which promotes the highest human capital growth from the equilibrium case.

Then, when  $\gamma < \beta < \frac{\gamma}{1-\gamma}$ , the agents still over invest in their child resources. However, the long run human capital growth as well as the consumption growth are too low compared to their first best optimal levels which are equivalent. Hence the government wants to increase the future generations' consumptions, but it can not internalize the positive human capital externality only using the two tax instruments. By improving the gap across the two taxes, the government increases the agent's incentives to invest in education expenditure rather than inheritance which affects negatively the capital-labor ratio. Thus, a negative public debt is necessary to achieve the optimal human capital growth (26). The negative public debt implies a public capital accumulation which improves the capital labor ratio and removes the negative impact on the capital-labor ratio. Thus, the social planner achieves an higher human capital growth using a higher gap across the two taxes and a negative public debt.

In the last case, when the agents are selfish in the way that they do not enough consider their child,  $\beta > \frac{\gamma}{1-\gamma}$ . They do not give enough to their offspring in order to decentralize the first best optimal solution only by using the two tax instruments. The social planner needs a negative public debt to increase the future generations' consumptions. In addition, when the gap across the time preference and the degree of households' altruism is sufficiently large and the amount of spending is rather low, the negative public debt is attended with negative tax instruments. Indeed, the inheritance tax is always negative when  $\gamma$  is low and  $\beta$  is close to one.

Thus, the labor tax is also negative as the inheritance tax is always higher than the labor one. The reduction of the tax rates encourages the representative agent to transfer more to his child in order to internalize the positive human capital externality.

Thanks to the public debt and the tax instruments, the social planner pursues the redistributive policy between generations and internalizes the positive human capital externality in order to maximize the welfare of each generation. However, the government does not achieve the first best optimal solution when the public debt is not available. In the next section, we analyse the impact of a positive public debt constraint on the human capital growth and on the fiscal policy used.

### 3.4 The second best social planner maximization problem

The government adopts as social criteria the discounted sum of generational consumption's utility (17) as in previous section. However, the positive public debt is not available and the government sets the optimal tax instruments. The physical capital is the only way to invest inheritance. From the agent's first order conditions (7), the agent's consumption when the public debt is not available, corresponds to:

$$c_t = \frac{1-\gamma}{\gamma} \left( 1 + \frac{1-\alpha}{\alpha} \phi_{t+1} \right) K_{t+1} \quad (29)$$

with  $\phi_{t+1} = \frac{1-\tau_{t+1}^L}{1-\tau_{t+1}^B}$ . Using the equilibrium condition (8), the amount of agent's education expenditure is equal to:

$$e_{t+1} = \delta \frac{1-\alpha}{\alpha} \phi_{t+1} K_{t+1} \quad (30)$$

From equation (30), we rewrite the human capital growth (10) as:

$$\frac{H_{t+1}}{H_t} = B^{\frac{1}{1-\delta}} \left( \delta \frac{1-\alpha}{\alpha} \phi_{t+1} k_{t+1} \right)^{\frac{\delta}{1-\delta}} \quad (31)$$

Using the agent's consumption level (equation (29)) and his education expenditure (equation (30)), the resource constraint (18) corresponds to:

$$K_{t+1} \left[ 1 + \frac{1-\gamma}{\gamma} \left( 1 + \frac{1-\alpha}{\alpha} \phi_{t+1} \right) + \delta \frac{1-\alpha}{\alpha} \phi_{t+1} \right] = (1-\Gamma) F(K_t, H_t)$$

As a result, the social planner maximizes the social welfare function (17) with respect to the sequence  $(\phi_{t+1}, H_{t+1}, K_{t+1})_{t \geq 0}$  subject to the resource constraint (18), the human capital technology (1) and taking into account the agent's consumption level (29) and his education expenditure (30). The details concerning the optimality conditions are included in the Appendix.

### 3.5 The second best optimal fiscal policy at steady state

From the first order conditions given in Appendix, we get the following proposition:

**Proposition:** When  $\gamma > \beta$  at steady state, (a) *the second best after tax ratio is higher than the first best one such that  $\phi^{**} > \phi^*$ .*

- (b) The second best capital-labor ratio is higher than the first best one such that  $k^{**} < k^*$  .  
(c) the second best economic growth is higher than the first best one.

**Proof.** By introducing equation (43) into the optimality condition (40), we obtain the following relationship between the second best optimal human capital growth  $g^{H**}$ , the second best optimal after tax ratio  $\phi^{**}$  and the second best capital-labor ratio  $k^{**}$ :

$$\frac{\delta}{\phi^{**} [1 - \beta (1 - \delta)]} + \frac{g^{H**}}{\beta \alpha (1 - \Gamma) \frac{F(k^{**}, 1)}{k^{**}}} (1 - \delta) = 1 \quad (32)$$

We note that the second best optimal after tax ratio is higher than the first best one when  $\phi^{**} > \frac{1}{1 - \beta(1 - \delta)}$ . In addition to this, we have the next relationship between the second best optimal human capital growth and the second best capital-labor ratio:

$$g^{H**} > \beta \alpha (1 - \Gamma) \frac{F(k^{**}, 1)}{k^{**}}$$

Using the resource constraint, equation (32) becomes:

$$\frac{\delta}{\phi^{**} [1 - \beta (1 - \delta)]} + \frac{\gamma}{\beta [\alpha + [1 - \gamma (1 - \delta)] (1 - \alpha) \phi^{**}]} (1 - \delta) = 1$$

From this equation, the second best after tax ratio is always higher than the first best one when the government uses positively the public debt. To analyse the public debt availability impact on the human capital growth and on the capital-labor ratio, we have to compare each of them in both cases. From the first best steady state capital-labor ratio (25), we obtain:

$$\frac{B^{\frac{1}{1-\delta}} (k^*)^{\frac{1-\alpha+\delta\alpha}{1-\delta}}}{\beta \alpha (1 - \Gamma) A} = \left( \alpha \frac{1 - \beta (1 - \delta)}{\delta (1 - \alpha)} \right)^{\frac{\delta}{1-\delta}} = \left( \frac{\alpha}{\delta (1 - \alpha) \phi^*} \right)^{\frac{\delta}{1-\delta}} \quad (33)$$

Using the equation (32) and the human capital growth (31), we get:

$$\Leftrightarrow \frac{B^{\frac{1}{1-\delta}} (k^{**})^{\frac{1-\alpha+\delta\alpha}{1-\delta}}}{\beta \alpha (1 - \Gamma) A} = \frac{1 - \delta \frac{\phi^*}{\phi^{**}}}{1 - \delta} \left( \frac{\alpha}{\delta (1 - \alpha) \phi^{**}} \right)^{\frac{\delta}{1-\delta}} \quad (34)$$

From equations (33) and (34), we obtain:

$$k^{**} < k^* \Leftrightarrow \frac{1 - \delta \frac{\phi^*}{\phi^{**}}}{1 - \delta} \left( \frac{\phi^*}{\phi^{**}} \right)^{\frac{\delta}{1-\delta}} < 1$$

This inequality is always satisfied when the second best after tax ratio  $\phi^{**}$  is different to the first best one  $\phi^*$ . Therefore, the second best capital-labor ratio is lower than the first best one regardless of the time preference value  $\beta$  and the degree of agent's altruism  $\gamma$ . Using the second best human capital growth (31) and assuming  $\gamma > \beta$ , we deduce that the second best economic growth is higher than the first best one:

$$g^{H**} = \beta \alpha (1 - \Gamma) \frac{F(k^{**}, 1)}{k^{**}} > \beta \alpha (1 - \Gamma) \frac{F(k^*, 1)}{k^*} = g^{H^*}, \forall \gamma > \beta$$

■

When government is facing on a positive public debt constraint, the social planner only uses the tax instruments to encourage agent's to invest in education rather than in inheritance, to finance the public spending and to reduce the incentive to transfer to the next generation. As a result, the government does not pursue its redistributive policy and the economic growth is higher than the first best one.

When the individuals consider their offsprings in the same way than the government, the availability of the pulic debt is not an issue to achieve the first best optimal human capital growth. In the other cases, the results coming from the unavailability of the public debt are more ambiguous and depend on the parameters values. However, these situations are less realistic than the case where the government uses positively the public debt. In each situation, the public debt plays an essential role to optimize the intergenerational family transfers between generations.

### 3.6 Numerical illustration

We use a numerical example to prove that dynamics converge quickly towards the optimal steady state. We also compare optimal solutions according to public debt availability and chosen parameter values appearing in all the functions given above. The basic parameter set is displayed in next table.

Table 1: Base-case parameter value

Parameter		Value	
Government			
Rate of time preference	$\beta$		0,39
Share of production devoted to public sector	$\Gamma$		0,1
Production function			
Technological parameter	$A$		2.5
Share parameter of physical capital	$\alpha$		0,33
Representative individual			
Agent's human capital production function			
Technological parameter	$B$		2.5
Share parameter of education spending	$\delta$		0,6
Taste			
Share of resources devoted to intergenerational transfers	$\gamma$		0,4

Most parameters values used, such that the share of production devoted to public spending, the technological and the physical capital parameters, are economically intuitive. However, we concentrate on situations where achieving the first best optimal solutions require a positive public debt. We calibrate the optimal transition dynamics with  $\beta < \gamma$ . Thus, we need to choose a time preference quite low such that the government cares more about the well-being of the present generations rather than the next one. This will result in a low  $\beta$  compared to usual estimate given that  $\gamma$  can not be reasonably too high. We assume  $\gamma = 0.4$  such that agent's is caring individual about his offspring. Furthermore, the elasticity of human capital to parent's



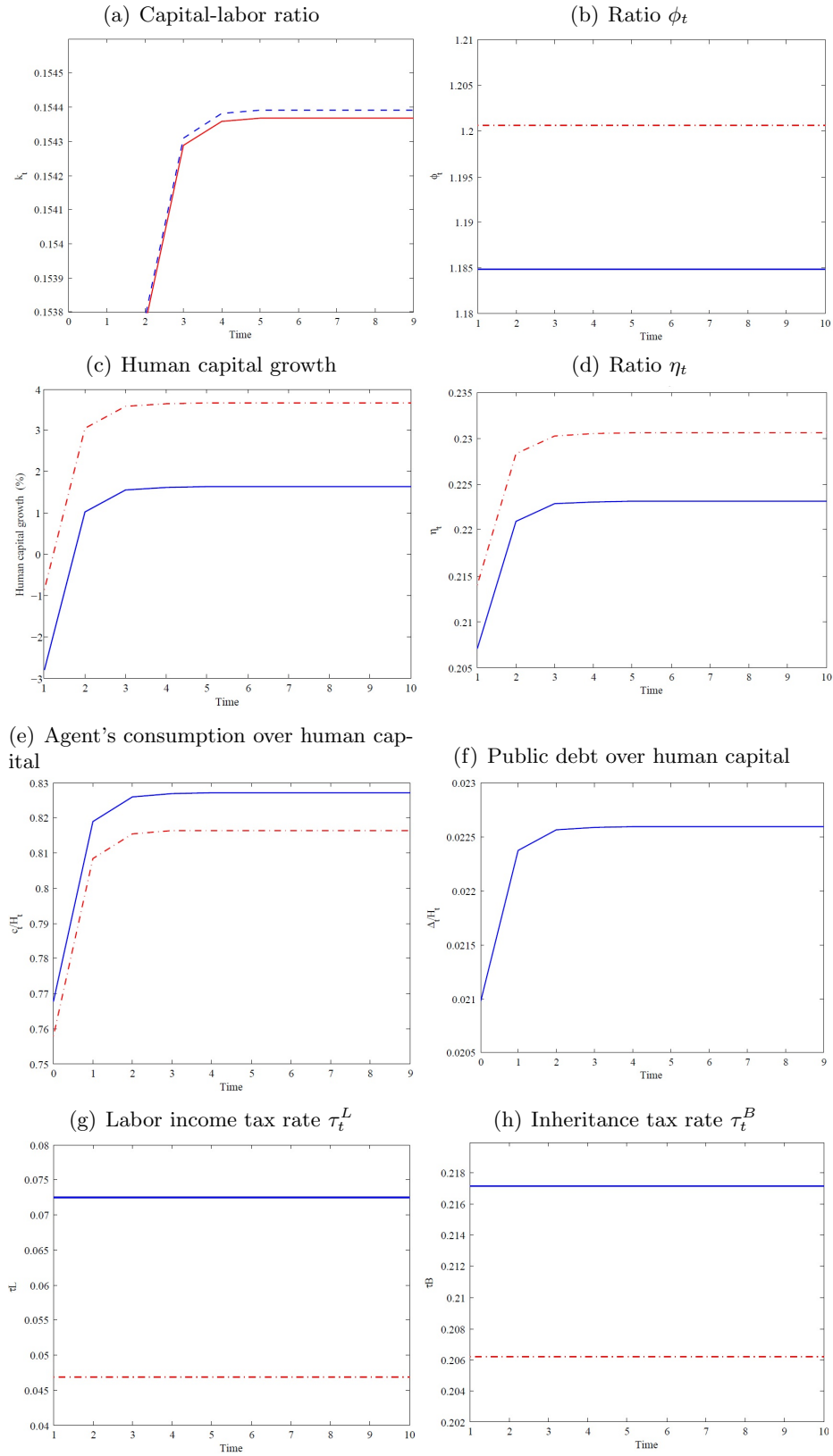
education spending is assumed to be relatively low. Indeed,  $\delta = 0.6$  implies that an increase in household's education expenditure is accompanied by a less than proportionate increase in child's human capital.

Results are reported in figure (2). All the variables quickly reach the optimal steady state in three periods. Therefore, the existence of an optimal transitional path which achieved promptly the steady state is confirmed. Thanks to optimal tax instruments, regardless of the public debt availability, individuals have incentives to modify their family transfers toward the optimal choices.

In addition to this, this illustration describes a quite realistic economy which is based on human capital as the capital-labor ratio is quite low in both cases (figure (5.a)). We have a reasonable human capital growth at steady state (figure (5.c)). This economy encourages agents to invest in human capital. This characteristics is also describe in the optimal tax instruments obtained. Figure (5.g) and (5.h) show the tax instruments values. The labor income tax is really low in both cases whereas the inheritance tax is around 20%. Concerning the effect of the public debt availability, this simulation illustrates results obtained previously. When the public debt is available, it is used positively to pursue the intergenerational redistributive policy. Otherwise, the results from the previous proposition are satisfied. The absence of public debt implies an higher level of human capital growth as the representative agent has no incentive to reduce his transfers to the next generation. Hence, the consumption's growth as well as the tax instruments are higher compared to the first best situation. Indeed, both taxes are used to correct the externality and to discourage agents from giving bequest to their child. The negative effect of increasing the tax policy on the capital-labor ratio can not be solved by the public debt. As a result, the second best fiscal policy is faced on a trade-off between reducing the agent's incentive to transfer to the next generation by increasing both taxes and the negative effect of this policy on the capital-labor ratio. Thus, the second best tax instruments are higher than the first best one in order to reduce the agent's transfers, however, it is not sufficient to achieve the first best redistributive policy.

Therefore, this numerical illustration describes the public debt availability issue correctly. It shows that the dynamics quickly converges towards a stationary steady state and that the tax instruments are higher when positive public debt is not available.

Figure 2: The optimal transition dynamics starting in the first period



Note: The first best optimal solutions: dashed line. The second best optimal solutions: bold line.

## 4 Conclusion

The long run optimal human capital growth that we are able to draw from this paper depends crucially on the public debt availability. The public debt sets the optimal households' family transfers distribution through which the first best optimal human capital growth is achieved. Thanks to the positive public debt, the government improves the consumption of the current generations without affecting the optimal human capital growth and uses both taxes to internalize the positive human capital externality. When the positive public debt is not available, the social planner can not completely satisfy these objectives such that the two taxes do not fully implement the intergenerational redistribution policy. For this reason, the tax rates chosen as well as the economic growth are higher than with public debt. Furthermore, the model reveals the necessity of public intervention to ensure that agents' decisions concerning their family transfers correspond to the long run optimal choices.

We focus on the intergenerational redistribution policy. We do not analyze the effect of intragenerational inequality on the optimal human capital growth which relies on the public debt availability. When we assume that agents have the same preferences, then being born in a good family is the only way to take advantage of this type of inequality. These disparities across agents should modify the optimal fiscal policy used and the long run human capital growth. In addition to this, study the influence of the tax competition across countries on the labor income tax and the inheritance one is an interesting question for further investigations.

## 5 Appendix

### 5.1 Interior solutions' conditions

The capital market equation (14) can be written as follow:

$$(\rho_{t+1}B\delta)^{\frac{1}{1-\delta}} = \frac{(1 - \tau_t^B) R_t \left[ \rho_t + k_t + \frac{d_t}{g_t^H} \right] - \frac{1}{\gamma} d_{t+1}}{\frac{1}{\gamma\delta} \frac{1-\tau_{t+1}^B}{1-\tau_{t+1}^L} \frac{\alpha}{1-\alpha} + 1 + \frac{1-\gamma}{\gamma\delta}} \quad (35)$$

Assuming that every agent leaves inheritance to their child, they must complete the inheritance condition (13) regardless of their reception of capital from their parents. Using equation (35), the inheritance condition (13) corresponds to the following inequality for every agent:

$$\begin{aligned} x_{t+1} > 0 &\Leftrightarrow (1 - \tau_t^B) R_t \left( \rho_t + \frac{x_t}{H_t} \right) > \left( 1 + \frac{1-\gamma}{\gamma\delta} \right) \frac{(1 - \tau_t^B) R_t \left[ \rho_t + k_t + \frac{d_t}{g_t^H} \right] - \frac{1}{\gamma} d_{t+1}}{1 + \frac{1-\gamma}{\gamma\delta} + \frac{1}{\gamma\delta} \frac{1-\tau_{t+1}^B}{1-\tau_{t+1}^L} \frac{\alpha}{1-\alpha}} \\ &\Leftrightarrow \frac{x_t}{H_t} > \left( \frac{1 + \frac{1-\gamma}{\gamma\delta} - \frac{1}{\gamma\delta} \frac{1-\tau_t^L}{1-\tau_t^B} \frac{1-\tau_{t+1}^B}{1-\tau_{t+1}^L}}{1 + \frac{1-\gamma}{\gamma\delta} + \frac{1}{\gamma\delta} \frac{1-\tau_{t+1}^B}{1-\tau_{t+1}^L} \frac{\alpha}{1-\alpha}} \right) k_t + \frac{1 + \frac{1-\gamma}{\gamma\delta}}{1 + \frac{1-\gamma}{\gamma\delta} + \frac{1}{\gamma\delta} \frac{1-\tau_{t+1}^B}{1-\tau_{t+1}^L} \frac{\alpha}{1-\alpha}} \left( \frac{d_t}{g_t^H} - \frac{d_{t+1}}{\gamma (1 - \tau_t^B) R_t} \right) \end{aligned}$$

Every agent verifies the inheritance condition (13) when  $\delta \leq 1$  and  $Z \equiv -\frac{d_{t+1}}{\gamma(1-\tau_t^B)R_t} + \frac{d_t}{g_t^H} \leq 0$ . On one hand, when the public debt is zero over time, there is an equilibrium where all agents

give inheritance to their offspring assuming that tax instruments are constant over time. On the other hand, when government uses public debt, we obtain from the government's budget constraint (15):

$$Z = -\frac{1-\gamma}{\gamma} \frac{d_t}{g_t^H} + \frac{Ak_t^\alpha [\Gamma - \tau^B \alpha - \tau^L (1-\alpha)]}{\gamma (1-\tau^B) R_t}$$

We can not ensure that all individuals satisfy the inheritance condition (13) along the dynamics. However, under certain conditions, the interior solution is satisfied at steady state. From the stationary government's budget constraint (15), we obtain:

$$d \left( 1 - \frac{(1-\tau^B) R}{g^H} \right) = Ak_t^\alpha [\Gamma - \tau^B \alpha - \tau^L (1-\alpha)]$$

Then, we get  $Z = \left( 1 - \frac{g^H}{\gamma(1-\tau^B)R} \right)$ . Assuming that  $g^H > \gamma(1-\tau^B)R$ , the inheritance condition (13) is verified. Furthermore, this condition which ensure an interior solution is not binding and the type of agent's altruism chosen in this model has an impact on it.

## 5.2 First best social planner maximization problem

The first best social planner maximization problem is characterized by the social welfare function (17) with respect to the sequence  $(c_t, e_{t+1}, H_{t+1}, K_{t+1})_{t \geq 0}$  subject to the human capital technology (1) and the resource constraint (18). Let us denote by  $\lambda_t$  and  $\mu_t$  the respective lagrange multipliers of both constraints. Then the optimality conditions are:

- with respect to  $c_t$ , for  $t \geq 0$ ,

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Leftrightarrow \frac{\beta^t (1-\gamma)}{c_t} = \lambda_t \quad (36)$$

- with respect to  $e_{t+1}$ , for  $t \geq 0$ ,

$$\frac{\partial \mathcal{L}}{\partial e_{t+1}} = 0 \Leftrightarrow \mu_t \frac{\partial G}{\partial e} (e_{t+1}, H_t) = \lambda_t \quad (37)$$

- with respect to  $H_{t+1}$ , for  $t \geq 0$ ,

$$\frac{\partial \mathcal{L}}{\partial H_{t+1}} = 0 \Leftrightarrow \lambda_{t+1} (1-\Gamma) F'_H (K_{t+1}, H_{t+1}) + \mu_{t+1} \frac{\partial G}{\partial H} (e_{t+2}, H_{t+1}) = \mu_t \quad (38)$$

- with respect to  $K_{t+1}$ , for  $t \geq 0$ ,

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \Leftrightarrow \lambda_{t+1} (1-\Gamma) F'_K (K_{t+1}, H_{t+1}) = \lambda_t \quad (39)$$

### 5.3 Second best social planner maximization problem

From the second best social planner maximization problem, we get the following Lagrangien:

$$\sum_{t=0}^{+\infty} \beta^t [\ln c_t + \lambda_{t+1} [(1 - \Gamma) F(K_t, H_t) - c_t(K_{t+1}, \phi_{t+1}) - e_{t+1}(K_{t+1}, \phi_{t+1}) - K_{t+1}] + \mu_{t+1} [G(e_{t+1}(K_{t+1}, \phi_{t+1}), H_t) - H_{t+1}]]$$

with respect to the sequence  $(\phi_{t+1}, H_{t+1}, K_{t+1})_{t \geq 0}$ . Let us denote by  $\lambda_{t+1}$  and  $\mu_{t+1}$  the respective lagrange multipliers of both constraints. Taking into account the Cobb Douglas production function and the derivatives, we obtain the following first order conditions:

- with respect to  $H_{t+1}$ , for  $t \geq 0$ ,

$$-\frac{\mu_{t+1}}{\mu_{t+2}} + \beta \frac{\lambda_{t+2}}{\mu_{t+2}} (1 - \alpha) (1 - \Gamma) F(k_{t+1}, 1) + \beta (1 - \delta) \frac{H_{t+2}}{H_{t+1}} = 0 \quad (40)$$

- with respect to  $K_{t+1}$ , for  $t \geq 0$  and using also the optimality condition (42),

$$-1 + \beta \frac{\lambda_{t+2}}{\lambda_{t+1}} \alpha (1 - \Gamma) \frac{F(k_{t+1}, 1)}{k_{t+1}} + \left[ \frac{1}{\frac{1-\gamma}{\gamma} (1 + \frac{1-\alpha}{\alpha} \phi_{t+1}) \lambda_{t+1} K_{t+1}} - 1 \right] \frac{1 - \gamma}{\gamma} = 0 \quad (41)$$

- with respect to  $\phi_{t+1}$ , for  $t \geq 0$ ,

$$\left[ \frac{1}{\frac{1-\gamma}{\gamma} (1 + \frac{1-\alpha}{\alpha} \phi_{t+1}) \lambda_{t+1} K_{t+1}} - 1 \right] \frac{1 - \gamma}{\gamma} + \left[ \frac{\mu_{t+1}}{\lambda_{t+1}} \delta \frac{H_{t+1}}{e_{t+1}} - 1 \right] \delta = 0 \quad (42)$$

At steady state we have  $g^H = \frac{\lambda_{t+1}}{\lambda_{t+2}} = \frac{\mu_{t+1}}{\mu_{t+2}}$  and we define  $\frac{\lambda_{t+1}}{\mu_{t+1}} = L$  and  $\lambda_{t+1} K_{t+1} = M$ . As a result, under the optimality conditions (41) and (42), we get:

$$L = \frac{\delta^2 \frac{(g^H)}{\eta^{**}}}{\delta - 1 + \frac{\beta}{g^{H^{**}}} \alpha (1 - \Gamma) \frac{F(k^{**}, 1)}{k^{**}}} \quad (43)$$

## References

- Arrondel, L. and Masson, A. (2013). Taxing more (large) family bequests: why, when, where? *PSE Working Papers*, 17.
- Atkinson, A. B. and Stiglitz, J. E. (1976). The design of tax structure: direct versus indirect taxation. *Journal of public Economics*, 6(1):55–75.
- Barro, R. J. (1990). Government spending in a simple model of endogeneous growth. *Journal of political economy*, 98(5, Part 2):103–125.
- Chamley, C. (1986). Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica*, 54(3):607–622.

- Cremer, H. and Pestieau, P. (2011). The tax treatment of intergenerational wealth transfers. *CESifo Economic Studies*, 57(2):365–401.
- Futagami, K., Iwaisako, T., and Ohdoi, R. (2008). Debt policy rule, productive government spending, and multiple growth paths. *Macroeconomic Dynamics*, 12(04):445–462.
- Futagami, K., Morita, Y., and Shibata, A. (1993). Dynamic analysis of an endogenous growth model with public capital. *The Scandinavian Journal of Economics*, 94(4):607–625.
- Greiner, A. (2012). Public capital, sustainable debt and endogenous growth. *Research in Economics*, 66(3):230–238.
- Hertz, T., Jayasundera, T., Piraino, P., Selcuk, S., Smith, N., and Verashchagina, A. (2007). The inheritance of educational inequality: International comparisons and fifty-year trends. *The BE Journal of Economic Analysis & Policy*, 7(2):1–48.
- Jacobs, B. and Bovenberg, A. L. (2010). Human capital and optimal positive taxation of capital income. *International Tax and Public Finance*, 17(5):451–478.
- Judd, K. L. (1985). Redistributive taxation in a simple perfect foresight model. *Journal of public Economics*, 28(1):59–83.
- Lucas, R. E. (1988). On the mechanics of economic development. *Journal of monetary economics*, 22(1):3–42.
- Piketty, T. (2011). On the long-run evolution of inheritance: France 1820–2050. *The quarterly journal of economics*, 126(3):1071–1131.
- Piketty, T. and Saez, E. (2013). A theory of optimal inheritance taxation. *Econometrica*, 81(5):1851–1886.
- Romer, P. M. (1986). Increasing returns and long-run growth. *The journal of political economy*, 94(5):1002–1037.
- Yakita, A. (2008). Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. *Journal of Public Economics*, 92(3):897–914.