Housing Taxation, Financial Regulation and Welfare

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Abstract

This paper studies the effects on macroeconomic aggregates of permanent changes in housing taxes and tax deductions, and in banking regulation through the lens of a multi-agent dynamic general equilibrium model. Specifically, the housing taxes that are examined consist in the property and land transfer taxes, and the tax deductions are the ones that are attached to the mortgage interest rate and imputed rental income. Our main result is that borrowing-constrained bankers play an important role for housing dynamics and for welfare improvements. On the contrary with tax deductions, policies that change housing taxes and banking requirements so that tax revenues are raised lead to a greater GDP. All policies are welfare-improving for homeowners, but welfare-diminishing for renters and bankers.


Keywords: Housing taxes, financial regulation, leverage ratio, dynamic general equilibrium.

1 Introduction

From a global perspective, housing is numbered among the most significant household assets. The value of housing assets in the US was around 28.5 trillion dollars in 2015 with a 1.1 trillion dollar increase from 2014 to 2015 according to the US census bureau.

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The significance of housing within the US economy becomes apparent when we compare it with other key elements of that economy. The US gross domestic product was 18.1 trillion dollars in the third quarter of 2015, about ten trillion less than the total value of the housing stock. In addition, the mentioned data has also confirmed that households spent around 535 billion dollars on renting in 2015 and 516 billion dollars in 2014. These figures prove that fluctuations within the housing market can dramatically impact the economy. Housing, therefore, is a subject of great significance within the field of macroeconomics.

Taxes are one of the main sources of government revenue. The share of taxes in the government revenue vary between countries depending on their tax code policies for instance in 2015 the tax revenue in the US was about 26% of all GDP. Figure 1 presents the share of income and property taxes in total tax revenue between 1980-2015 in the US. Income tax was about 38% of total tax revenue in 2015 and a part of this share is income revenue generated by owners renting houses. Housing related taxation is one of the greatest contributors to governmental revenue flow. Property tax was about 10% of total revenue tax in 2015 and it maintained this level in the subsequent decades. In addition, there are some deductible taxes in the US\(^1\). According to JTC 2010 the subsidy on mortgage interest rate and property tax amounted to around 114 billion dollars. This emphasises the significance of mortgages in the housing market (Sommer and Sullivan, 2013).

\[\text{Figure 1: Income and property tax over total tax revenue}\]

Figure 2 presents the value of mortgage over income(left) and housing price over rent(right) between 1995-2015(OECD data 2016). The figure shows the importance of mortgage in the budget of households, its recession during the 2008 housing crisis and post crisis upturn. Hence tax code alterations can easily have a tremendous impact on decisions of households on consumption, borrowing and investing (Glick et al., 2015).

\(^1\)https://www.irs.gov/publications/p530/ar02.html
Figure 2: Mortgage and house price

Using a dynamic general equilibrium model, this paper studies the impact of taxation on housing both indirectly, through mortgage deductions and imputed rental income interest rates, and directly, through rearranging property tax. This paper expands on existing research by contributing an intermediary agent in DGE model and analyzing both the impact of changes in tax codes and the financial regulation of banking system on agents’ decisions. Households are heterogeneous. Intermediary agents (bankers) as one type of household pool deposits from patient households and lend to impatient households as mortgages and to government as government loans. Another type of household is renters (hand to mouth) who only rent rental houses from the patient household thus consuming their wage. Both borrowers and bankers are constrained in borrowing and raising the liabilities. These financial frictions are are crucial to the model. Capital requirement and restricting banks to their assets are the basic principles of Basel Accord. Until now there were three versions of Basel issued by the Basel Committee on Banking Supervision in each of which there are different financial regulation structures (Angelini et al., 2015). Figure 3 shows the bank capital to assets ratio for the US between 2000-2016 (World bank data, International Monetary Fund, Global Financial Stability Report 2016). Capital to asset ratio is a mean that helps financial regulatory determine the minimum level of capital that banks must have. These kind of indexes assure regulators that an intermediary agent can absorb a reasonable amount of loss and complies with statutory Capital requirements.
The present paper develops a dynamic equilibrium model of majority of tax codes and explores the impact of the financial regulation on the economy. There is expansive extant literature in banking and regulation with each study proposing different modeling on relating intermediary liabilities and assets, for example see Allen and Gale (2005), Diamond and Rajan (2005), He and Krishnamurthy (2013), Berger et al. (2016) and Repullo and Suarez (2013) and an empirical work like Adrian and Shin (2010). The present paper’s modeling on intermediary is more in line with He and Krishnamurthy (2013) and Iacoviello (2015). The latter proves that when the intermediary agent is constrained, a loan default can provoke a considerable recession. Intermediary agent is constrained on raising liabilities by a fraction of its assets. In addition adjustment cost on asset prevents bankers dramatically altering the asset.

The present paper assesses the impact of the permanent shock in deduction of the mortgage interest rate, deduction of imputed rental income, property tax and land transfer tax in tax codes and the change in the regulation of the intermediary agent that directly affect financial friction for bankers. The present value of tax revenue change is fixed for all changes to provide the possibility of comparing between policies. The paper shows all the changes have positive effects on present value of GDP change over time. This positive impact is due to the important role of deposits in the model as an alternative for the patient household to transfer its saving to the next period. These results corroborate those of Gervais (2002) in the sense of GDP augmentations and welfare. He uses a dynamic general equilibrium life-cycle economy model to show the impact of the tax deductions used in the US in housing capital and housing service. The financial intermediary agent in his model pools deposits to raise loans for borrow-
ers and provide residential capital. Defining an intermediary in his model simplifies the exposition, it does not face any constraint. In contrast, the intermediary agent in the model presented in this paper faces a lending constraint and it is not a direct player in the demand or supply side of housing. Gervais’ paper presents tax deductions used in this paper’s results in welfare gains in the long run that our result confirm the same thing. In our model, The policies have positive welfare effects for all houseowners. Due to the changes, patient and impatient households consume more and they reduce their labor share in the economy. In addition bankers consume more due to the positive effect on deposit. Renters consume less and raise their labor share. This negatively impacts their welfare.

This paper is informed by different strands of the literature. Taxation (e.g income tax or any housing taxation etc) is one of them. Chatterjee and Eyigungor (2015) discuss tax treatment of housing in crisis and how tax deduction of mortgage interest payments and rental income could affect extensiveness of a crisis. Chambers et al. (2009) apply an asymmetric tax treatment on a general equilibrium framework, where rental price and interests rates are endogenous and house price is fixed. They conclude that reducing mortgage interest deduction has a positive effect on housing service and accordingly on welfare in an individual heterogeneous model. Sommer and Sullivan (2013) use the model in the previous paper to show the impact of the preferential tax treatment of housing on different prices and housing ownership in economy. They argue that reducing deductions results in a drop in house prices, which consequently leads to greater housing ownership. Property taxes are one element of government revenue, they show this action would increase federal income tax revenue, but decrease government revenue. Albouy and Hanson (2014) and Albouy (2009) assess the role of taxation especially income taxes on the place of residence and the quality of life. They show how a deduction in tax code could provide a form of cost-of-living adjustment and change the household optimal decision. Iacoviello and Pavan (2013) analyze the impact of defining housing and mortgage on the volatility of the aggregate economy. They study the volatility of housing investment and GDP, and the procyclicality of debt in an incomplete market. There is a single production and there is a single saving way to transfer to the next period. In the model proposed by the present paper households have more alternatives to transfer savings over time e.g investing in housing, capital and safe asset of the bank, whereas in the model Iacoviello and Pavan’s there is no risk-free asset. They conclude that the more relaxed borrowing constraints, the more decline in output volatility. Gervais and Pandey (2008) study the impact of
repealing the deduction of mortgage interest in federal tax liability. They explore rearranging household balance sheets and which decreasing the amount of interest income taxes collected. In the present paper the variety of changes in tax code are analyzed.

The present paper is closely related to Alpanda and Zubairy (2016). They examine the impact of various tax policies, direct and indirect housing taxes, on macroeconomic variables in a general equilibrium context. Their model consists of housing and goods sectors, housing and capital productions and heterogeneous households in three types, patient(lender and saver), impatient(borrower), and renter(hand to mouth) households. This paper is made significantly different from theirs by the introduction of another type of household which collects the deposit from lenders and lends to borrowers. This intermediary agent is called banker. The existence of bankers comes from the fact that in the real world households have access to bonds, but since they do not have the expertise, it is too costly for them to invest in such assets directly. The ability of raising liabilities by banker is controlled by a restriction which constrains liabilities to a fraction of banker’s asset. The restriction on banker’s lending ability is related to the regulation literature that will be examined presently. Alpanda and Zubairy (2016) assess the welfare effects of each policy and compare them in terms of GDP loss and welfare effects. The mentioned tax policies are increasing the property tax rate, eliminating the mortgage interest deduction, increasing taxation of imputed rental income and eliminating of depreciation allowance for rental income. In the present paper, the first three policies plus the impact of change in land transfer tax (the literature on this tax comes in following paragraph) and the change in banking regulation as one of the most important means to control economy by government or higher regulatory are assessed. In order to maintain comparability between each policy, any change has the same impact on the present value of tax revenue. They find that the elimination of the mortgage interest deduction would be the most effective policy in raising tax revenue and less is lost in output and welfare compared to other policies. In our model, the change in property tax which is a direct change in housing-related taxes has the greater present value GDP gain in comparison with other tax policies and the change in the financial regulation has the larger GDP gain in all mentioned policies.

There are different taxes on housing in US tax codes. The most important one is the property tax on houses households in which people currently reside (Englund, 2003). Another tax on housing is Land transfer tax (LTT) which may apply to households considering changing home. The main studies discuss property tax But little research has been conducted on the impact of LTT on the economy. In this paper this tax and its

effect are briefly discussed. Dachis et al. (2008) and Dachis et al. (2011) study the effect of LTT on Toronto’s housing market. They show that raising LTT by 1.1% could cause a 15% drop in house ownership and subsequently cause a housing price decrease of about 1%. They urge that raising LTT results in welfare loss. Figure 4 shows the rate of moves in the US between 1986-2016 according to US Census Bureau Reports 2016.

![Figure 4: The US mover rate (%)](image)

A discussion about housing generally includes consideration for mortgage and its return. The treatment of Borrowing and collateral constraint in the literature is another focus of this paper. Linneman and Wachter (1989) study the impact of borrowing constraint through micro data on homeownership propensity. Kiyotaki et al. (2011) studies the interaction between borrowing constraints, housing prices, and economic activity. Krishnamurthy (2003) develops a model à la Kiyotaki et al. (1997) and using collateral constraint to present how the degree of amplification with debt changes. Borrowing and collateral constraint are among the most studied topic after crisis. Barakova et al. (2014) examine the housing market boom between 2003 and 2007 and the impact of borrowing constraints on the probability of homeownership. Ebner (2013) shows higher borrowing could potentially be the result of expectations of future house price increases. He says the borrowing constraint positively affects house purchase and influences home equity withdrawals. In the present paper, household borrowers are faced with a collateral constraint. Housing is used as collateral against borrowing, as well as providing utility. Binding the collateral constraint provides a shadow cost by the risk free rate and the cost of capital faced by borrowers. The mortgage market and borrowing constraint is more realistic in this model (Alpanda et al., 2014). The borrowing constraint only affects the new loans. There is some inertia and there is a fraction of borrowers who would like to change mortgage. This realistic model is dissimilar to other models in which the constraint restricts
the entire stock. The price of housing and capital are determined by the housing and capital producers who invest in housing and non-housing goods subject to an adjustment cost (Smets and Wouters, 2007).

This paper has been organized as follows. Section 2 presents the model that is composed of defining four types of household, firms and government. The optimal conditions and market clearing are also explained in section 2. Section 3 presents the calibration of the baseline model. Section 4 analyses the deterministic model in the case of permanent shocks to tax policies and the regulation on banker. The changes are compared in terms of the present value of tax revenue change, the present value of GDP change and welfare effect on their transition paths between before and after shock steady states. Section 5 offers a conclusion on the findings of this paper.

2 Model

The model is a closed-economy model with heterogeneous households. There are four types of households in the economy who live infinitely. The first type, patient households, buy the houses for personal use and rental purposes. In addition, patient households own capital in the economy which is lent to the firms. Firms use the lent capital and labor from all households to produce goods for consumption, investment and government expenditure. For this type of household, there is a possibility to invest through the deposit in intermediary agent which is here referred to as banker. The key consideration of this paper is to assess the potential of bankers to change the behavior of the economy. Bankers are agents that pool deposits from patient households as liabilities and lend them to impatient households as mortgages and to government as loans to cover lump sum transfers and expenditure. The banker consumes and is limited to a borrowing constraint that defines the applied leverage ratio in the economy set by the government or higher regulatory.

The second type of households is impatient household. Impatient households buy houses using the loan from the banker (i.e mortgage). Collateral constraint restricts this household’s borrowing to the value of their housing. Housing provides utility benefit for households except bankers and impatient households use it as collateral for borrowing from banker. The third type of households, renters, work for the firms and rent houses from patient households. The role of government in this economy is mainly to transfer taxes as lump sum to all households. There are taxes on income revenue (on wage, deposit and capital interest) and property taxes. The latter is divided to a tax on current house
and a tax on the value of the newly bought houses (land transfer tax). Government also borrows from bankers to pay the transfer and its expenditure. Based on the US data there is a possibility of deduction of property tax for owners and deduction of taxation on imputed rental income for patient households. Housing in this economy has a dynamic supply as well as all other variables.

2.1 Household

There are 4 types of households, patient(saver), impatient (borrower), renter and banker. There is a unit measure of infinitely lived household of each type\(^3\) in the economy. In the paper superscribe \(P, I, R, B\) stand for Patient, Impatient and Renter households and banker, respectively. All types of households benefit from consumption, housing and leisure time in the utility function.

2.1.1 Patient households

Patient household utility function is,

\[
E_t \sum_{\tau=t}^{\infty} \beta_T^{\tau-t} \left\{ \log c^P_T + \varphi_h \log h^P_{T-1} - \varphi_l \frac{(l^P_T)^{1+\iota}}{1+\iota} \right\}
\]

where \(t\) presents time, \(\beta_P < 1\) is the discount factor that is greater than discount factor of other households. \(\varphi_h\) and \(\varphi_l\) are the coefficient that present the relative importance of housing and leisure in the utility function and \(\iota\) is the inverse of the Frisch-elasticity of labor supply. The patient households’ budget constraint is

\[
(1 + \tau_c) c^P_t + p^h_t I^{Ph}_t + p^k_t I^k_t + d_t \leq
w^P_t + p^R_t h^R_{t-1} + (1 + r_t) d_{t-1} + r^k_t k_{t-1} + \Gamma^P_t - \tau_w[w^P_t + (p^R_t - \delta_h)(h^R_{t-1} + I^R_t)]
- \tau_p p^h_t (h^P_{t-1} + h^R_{t-1}) - \tau_d d_{t-1} - \tau_k (r^k_t - \delta_k) k_{t-1} - \tau_p p^h_t (h^P_{t-1} + h^R_{t-1})
- \tau_l \zeta p^h_t (h^P_t + h^R_t) - AC^P_t
\]

where patient households’ housing and capital investment, \(I^{Ph}, I^k\), respectively are

\[
I^{Ph}_t = [h^P_t - (1 - \delta_h)h^P_{t-1}] + [h^R_t - (1 - \delta_h)h^R_{t-1}]
\]

\[
I^k_t = k_t - (1 - \delta_k)k_{t-1}
\]

\(^3\) The same arrangement as Iacoviello (2005)
Patient households consume non housing consumption \( c^P \), invest in residential and rental purpose houses, \( h^P \) and \( h^R \) respectively, and receive the rent \( p^R \) on each unit of rental housing from renter household. They invest and transfer their savings through the periods by investing deposit \( d \) to the intermediary agent. Patient households are owners of capital. Each period firms borrow the capital from patient households and after producing goods return the undepreciated part to the household. Households buy houses with relative housing price \( p^h \). They accumulate capital from capital producer to lend to firms with relative capital price \( p^k \). Patient households work for firms and receive \( w^P \), as wage. \( l^P \) is the share of patient in the population, \( \Gamma^P \) is transfer for patient from the government and \( r \) is the interest rate on the deposit. There are different types of taxes in the economy. \( \tau_w \) is the tax on income composed by wage and rental income, \( \tau_p \) is property tax on current residential houses, \( \tau_l \) is land transfer tax on new houses and \( \zeta \) is the fraction of the population who changes houses. There is a depreciation rate \( \delta_h \) on houses and \( \delta_k \) on capital. \( I_r \) is a variable that has a value between 0 and 1, depending on the policy of the government. It is the parameter which presents the deduction of taxation on imputed rental income for patient households. The last term in the budget constraint is the adjustment cost \( AC^P \), which ensure a smooth change between houses.\(^4\)

The FOC with respect to residential houses is

\[
(1 + \tau_l \zeta + \frac{\psi_h h^P}{h^R} (h^P_t - h^P_{t-1}))p^h_t = \\
\beta_P E_t [\frac{\varphi^P_{t+1}}{\lambda^P_t} (1 - \delta_h - \tau_p (1 - \tau_w))p^h_{t+1} - I_r \tau_w (p^R_{t+1} - \delta_h) + \frac{\psi_h h^P}{h^R} p^h_{t+1} (h^P_{t+1} - h^P_t)]
\]

(2.5)

The optimal condition with respect to rental houses is

\[
(1 + \tau_l \zeta + \frac{\psi_h h^R}{h^R} (h^R_t - h^R_{t-1}))p^h_t = \\
\beta_P E_t [\frac{\lambda^P_{t+1}}{\lambda^P_t} ((1 - \delta_h - \tau_p (1 - \tau_w))p^h_{t+1} + (1 - \tau_w)p^R_{t+1} + \tau_w \delta_h + \frac{\psi_h h^R}{h^R} h^R_{t+1} (h^R_{t+1} - h^R_t))]
\]

(2.6)

where where \( \lambda^P \) is the Lagrangian multiplier of the budget constraint. The optimal conditions respect to residential and rental purpose houses are so that the marginal utility of

\(^4\)Adjustment cost is \( AC^P_t = \frac{\psi_w}{2^n} p_t^h (h^P_t - h^P_{t-1})^2 - \frac{\psi_w}{2^n} p_t^R (h^R_t - h^R_{t-1})^2 \).
consumption is equal to the marginal cost of consumption. The FOC equates the marginal cost of buying one unit of residential/rental purpose house at time \( t \) with discounted utility gain and the house value after tax and depreciating. The FOCs with respect to deposit and capital respectively are

\[
1 = \beta P \mathbb{E}_t \left[ \frac{\lambda^P_{t+1}}{\lambda^P_t} (1 + (1 - \tau_d) r_{t+1}) \right] \tag{2.7}
\]

\[
p^k_t = \beta P \mathbb{E}_t \left[ \frac{\lambda^P_{t+1}}{\lambda^P_t} ((1 - \delta_k) p^k_{t+1} + (1 - \tau_k) r^k_{t+1} + \tau_k \delta_k) \right] \tag{2.8}
\]

The FOC respect to deposit/capital equates the cost the household pays to deposit/capital one more unit with the discounted gain next period. And finally optimal condition for share of labor equates the marginal loss of leisure time with the gain in the case of working in the firm\(^5\).

### 2.1.2 Impatient households

Impatient households have a utility function in the same fashion as Patient ones,

\[
E_t \sum_{\tau=t}^{\infty} \beta^{t-\tau} \{ \log c^I_{\tau} + \varphi_h \log h^I_{\tau-1} - \varphi_l \frac{(l^I_{\tau})^{1+\iota}}{1+\iota} \} \tag{2.9}
\]

in order to make borrowing and lending easier for these agents, the impatient discount factor is assumed to be less than that of patient households, \( \beta_I < \beta_P \). Their budget constraint is,

\[
(1 + \tau_c) c^I_t + p^h_I I^{ih}_t + (1 + \tau^b_I) M_{t-1} \leq w^I_t I^l_t + M_t + \Gamma^I_t - \tau w[w^I_t I^l_t - I_m r^b_t M_{t-1} + I_r (p^R_t - \delta_h) h^I_{t-1} - \tau_p \delta^b h^I_{t-1}]
- \tau_p p^h_I h^I_{t-1} - \tau_l \zeta p^h_I h^I_t - AC^I_t \tag{2.10}
\]

Impatient households consume \( c^I \) and buy residential purpose house \( h^I \) and invest in housing

\[
I^{ih} = h^I_t - (1 - \delta_h) h^I_{t-1} \tag{2.11}
\]

\(^5\)Appendix 6.1
Impatient households get loans $M$ as mortgages from bankers to buy houses. They work for firms and get $w^I$ as the wage of impatient. $I^I$ is the share of impatient labor in the population, $T^I$ is the transfer for impatient from the government and $r^b$ is the interest rate on the mortgage. $0 \leq I_m \leq 1$ is a variable that measures how much interest payments on borrowing are deductible. When it is equal to one the interest payment is fully deductible. The tax code is the same as patient household, also there is an adjustment cost on changing houses$^6$.

Collateral constraint is$^7$,

$$M_t \leq \rho_m M_{t-1} + (1 - \rho_m) \theta p^b_t h^I_t$$

(2.12)

where $\theta$ is the fraction of assets that can be collateralized for borrowing, and $\rho_m$ determines the persistence in the borrowing constraint. This constraint reflects the reality of mortgage borrowing. It prevents a huge change in the mortgage from one period to another alongside constraints the mortgage to the collateral housing. In the real world in any period only a fraction of households change mortgages so the borrowing constraint only affects the new loans. This realistic model is in contrast to other models in which the constraint restricts the entire stock.

The first order conditions with respect to impatient houses is

$$(1 + \tau_I - \frac{\lambda^m_I}{\lambda^I_t}(1 - \rho_m)\theta + \frac{\psi}{h} (h^I_t - h^I_{t-1}))p^b =$$

$$\beta_I E_t[\frac{\varphi}{h} \lambda^I_{t+1} h^I_t + \lambda^I_{t+1} ((1 - \delta - \tau_p (1 - \tau_w))p^b_{t+1} - I_t r_w (p^b_{t+1} - \delta_h) + \frac{\psi}{h} p^b_{t+1} (h^I_{t+1} - h^I_t))]$$

(2.13)

where $\lambda^I$ is the Lagrangian multiplier of the budget constraint and $\lambda^m$ is the Lagrangian multiplier of the collateral constraint. The optimal condition with respect to mortgage,

$$1 - \frac{\lambda^m_I}{\lambda^I_t} = \beta_I E_t[\frac{\lambda^I_{t+1}}{\lambda^I_t} (1 + (1 - I_m \tau_w) r^b_{t+1} - \frac{\lambda^m_{t+1}}{\lambda^I_t} \rho_m)]$$

(2.14)

The optimal condition respect to impatient household’s houses is so that it equates the cost of raising one unit of impatient house to the discounted gain from the house after tax plus the utility gain from loosening the collateral constraint by raising one unit of impatient

$^6 AC^I_t = \frac{\psi}{2\delta} p^b_t (h^I_t - h^I_{t-1})^2$

$^7$ in the same form of Justiniano et al. (2015)
house. The optimal condition respect to mortgage equates the discounted marginal cost of raising one unit of mortgage plus utility loss from tightening the collateral constraint to the marginal gain plus the discounted utility gain from loosening the collateral constraint next period.

2.1.3 Renter households

Renter households’ utility function is,

\[
E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \{ \log e^{R}_{\tau} + \varphi_h \log h^{R}_{\tau-1} - \varphi_t \frac{(I^{R})^{1+\tau}}{1+\tau} \}
\]  

(2.15)

the discount factor of renters is the same as impatient one. They benefit from consumption, renting houses and leisure time. Their budget constraint is

\[
(1 + \tau_c) c^{R}_{t} + p^{R}_{t} h^{R}_{t-1} \leq (1 - \tau_{wr}) w^{R}_{t} l^{R}_{t} + \Gamma^{R}_{t}
\]  

(2.16)

Renter household consumes \( c^{R} \). Renter households rent rents houses from patient households. They work and have the wage \( w^{R} \) and pays a different wage tax \( \tau_{wr} \) (rather than other type of households) as income tax on renters’ income. \( I^{R} \) is the share of renter labor in the population and \( \Gamma^{R} \) is the transfer for renters from the government. They do not have the ability to borrow or invest and they spend what they earn. They are referred to as *hand to mouth*.

The first order conditions with respect to rental housing,

\[
p^{R}_{t} = \frac{\varphi_h}{\lambda^{R} h^{R}_{t-1}}
\]  

(2.17)

where where \( \lambda^{R} \) is the Lagrangian multiplier of the budget constraint.

2.1.4 Bankers

The representative banker is an agent in this economy. Banker consumes and pools the deposit of patient households as liabilities and loans to the government and impatient households as the banker ’s asset \( a \). Asset \( a \) is composed of \( b^{\theta} \), the loan to the government and \( M \), the mortgage to impatient households,

\[
a_{t} = b^{\theta}_{t} + M_{t}
\]  

(2.18)
Banker benefits consumption in every period. Banker’s utility function and budget constraint are,

$$\max E_t \sum_{\tau = t}^{\infty} \beta^{\tau-t} \log c_\tau$$

$$(1 + \tau_c) c_t^B + (1 + r_t) d_{t-1} + a_t + \frac{\psi_b (a_t - a_{t-1})^2}{\bar{a}} = d_t + (1 + r^b_t) a_{t-1}$$ (2.19)

Every period, banker consumes after returning the last period deposit and its interest to patient households and raising loans as mortgages to impatient and government loans. $r^b$ is the interest rate on loans. It is the same interest rate that impatient households pay on mortgages. In the same time banker receives new deposits and the return on last period loans. Bankers are constrained to issue liabilities by the amount of assets in its portfolio$^8$,

$$d_t \leq \phi a_t$$ (2.20)

Parameter $\phi$ is the leverage ratio that presents the ratio of the liabilities over the asset of the banker. Constraint (2.20) is set by government or higher regulatory. It is what we can see in the financial regulation literature and Basel core principles (Supervision, 2006). Bankers face an adjustment cost in their budget constraint that is external to the agent and guarantees a smooth change in assets. The optimality condition for deposit and loans are

$$1 = \frac{\lambda^\phi_t}{\lambda^B_t} + \beta_B E_t \frac{\lambda^B_{t+1}}{\lambda^B_t} (1 + r_{t+1})$$ (2.21)

$$1 + \psi_b \frac{a_t - a_{t-1}}{\bar{a}} = \frac{\lambda^\phi_t}{\lambda^B_t} \phi + \beta_B E_t \frac{\lambda^B_{t+1}}{\lambda^B_t} (1 + r_{t+1}^b + \psi_b \frac{a_{t+1} - a_t}{\bar{a}})$$ (2.22)

where $\lambda^B_t, \lambda^\phi_t$ are the Lagrangian multiplier of the constraints. In this structure loans are less liquid than deposits. The optimal condition with respect to deposit equates the today payoff of raising one unit of deposit with the discounted expected cost of that. One unit more deposit means one unit more payoff but it tightens the constraint and reduces the utility much as the Lagrangian multiplier of the constraint normalized by budget constraint’s Lagrangian multiplier($\lambda^\phi/\lambda^B$). On the other hand optimal condition respect to the asset equates the discounted tomorrow’s payoff to the today’s cost of raising one unit of assets(loans). Raising one unit of loan means consuming one unit less today

$^8$see Iacoviello (2015)
but having one unit discounted return more tomorrow. The greater the loan, the looser
the constraint and the more utility (much as the Lagrangian multiplier of the constraint
normalized by budget constraint’s Lagrangian multiplier times the leverage ratio ($\phi$$\lambda^e/\lambda^B$).
Banks are indifferent between pooling deposit and raising loans so it results in the positive
budget constraint’s Lagrangian multiplier since the banker’s discount factor is less than
the patient household’s. Because of the equation on the adjusted returns on deposits and
loans, since the leverage ratio is less than one, deposits are more liquid than loans.

### 2.2 Firms

A perfectly competitive non-housing good market is characterized by constant returns to
scale. Identical firms of measure one are producing a homogeneous final good according
to the Cobb-Douglas technology. Profit maximization determines factor prices and they
are equal to their marginal products. All households except bankers work for the firm but
they have different labor elasticity. The firm rent the capital from patient households to
produce consumption goods.

\[
Y^f_t = A_t k_{t-1}^\alpha (l_t^P)^{\iota_P} (l_t^I)^{\iota_I} (l_t^R)^{\iota_R} \quad (2.23)
\]

\[
\Pi^f_t = Y^f_t - w_t^P l_t^P - w_t^I l_t^I - w_t^R l_t^R - r_t^k k_{t-1} \quad (2.24)
\]

Market factors resulted by the first order condition respect to capital and labor, respectively,
are

\[
\frac{\alpha Y^f_t}{k_{t-1}} = r_t^k \quad (2.25)
\]

\[
(1 - \alpha)\frac{Y^f_t}{l_i^i} = w_i^i, \quad i = P, I, R \quad (2.26)
\]

### 2.3 Capital and housing producer

In the economy there are perfectly competitive capital and housing producers who produce
capital and housing used by households subject to an adjustment cost. The capital and
housing investment by producers are

\[
[1 - \frac{\psi_x}{2}(\frac{i_t^x}{i_{t-1}^x})^2]i_t^x = x_t - (1 - \delta_x)x_{t-1} \quad x = k, h
\]
where \( h_t = h^P_t + h^I_t + h^R_t \) is total housing. A producer maximizes her benefit as

\[
E_t \sum_{\tau=t}^{\infty} \beta^\tau \lambda^P_t \left[ p^\tau_r(x_\tau - (1 - \delta_w)x_{\tau-1}) - \delta_w^\tau \right] \quad x = k, h
\]

subject to equation (2.27).^9

\[ (2.28) \]

2.4 Government

Government gets all the taxes on income revenue, property and land transfer and redistribute them as lump-sum to agents except to financial agent. Total tax \( T_t \) is composed of

\[
T_t = \tau_c C_t + \tau_w[w^P_t l^P_t + (p^R_t - \delta_h)(h^R_{t-1} + I_t h^P_{t-1}) - \tau_k(h^P_{t-1} + h^R_{t-1})] + \tau_d r_t d_{t-1}
+ \tau_p(h^P_{t-1} + h^R_{t-1}) + \tau_i (h^P_t + h^R_t) + \tau_k (r^k_t - \delta_k) k_{t-1}
+ \tau_w[w^I_t l^I_t - I^m_t r^m_t M_{t-1} + I_r (p^R_t - \delta_h) h^I_{t-1} - \tau_p h^I_{t-1}] + \tau_p h^I_{t-1} + \tau_i h^I_t
+ \tau_{wr} w_t r^R_t
\]

where \( C_t = c^P_t + c^I_t + c^R_t + c^B_t \) is total households’ consumption. Total tax is composed of taxes on consumption, patient income tax, impatient income tax and renter income tax(line 4). The government spends the total tax plus the borrowing from the banker to cover the payment as transfers to all households except bankers (relative to their type), its loan to bankers and to spend for its expenditure. Hence

\[
b^g_t + T_t = (1 + r^b_t) b^g_{t-1} + g_t + \Gamma^P_t + \Gamma^I_t + \Gamma^R_t
\]

where

\[
\Gamma_i = \varphi_i Y^f_t - \rho_b b^g_{t-1}, \quad i = I, P, R.
\]

Equ. (2.31) describes transfers to each household type. \( Y^f \) is non-housing good production from firms. \( \varphi_i \) are level parameters specific to the type of household that show how much the government helps any type. \( \rho_b \) determines the response of transfers to government debt. This coefficient is set to avoid ponzi game by the government so transfers are adjust to the government debt.

---

^9For details see Appendix 6.1
2.5 Market clearing

Non-housing good market clearing condition equates firm’s output to total consumption, total housing investment, capital investment and government expenditure,

\[ Y_t^f = C_t + i_t^h + i_t^k + g_t \] (2.32)

where \( i_t^h \) is the housing investment of producers, \( i_t^k \) is the capital investment of producers and \( h_t \) is total housing. In this paper GDP\(^{10} \) is introduced by

\[ Y_t = (1 + \tau_c)C_t + p^R h_{t-1} + i_t^h + i_t^k + g_t \] (2.33)

that is firm’s output plus consumption tax and housing provides consumption services.

An equilibrium is a set of prices \((p^h, p^k, p^R, r, r^h, r^k)\) and allocations \((c^p, c^l, c^r, c^b, h^P, h^I, h^R)\) so that maximize the household utility functions subject to all constraints, market factors and market clearings.

3 Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Target</th>
<th>Model</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_P, \beta_I, \beta_B )</td>
<td>0.9916, 0.9852, 0.957</td>
<td>( r = 0.04, r_b = 0.0538 )</td>
<td>( r = 0.04, r_b = 0.0538 )</td>
<td>BIR</td>
</tr>
<tr>
<td>( \varphi_h )</td>
<td>0.247</td>
<td>( h/Y = 5.2 )</td>
<td>( h/Y = 5.2 )</td>
<td>FOF</td>
</tr>
<tr>
<td>( \varphi_l )</td>
<td>0.81</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_h, \delta_k )</td>
<td>0.0096, 0.0141</td>
<td>( k/Y = 6 )</td>
<td>( k/Y = 6 )</td>
<td>NIPA</td>
</tr>
<tr>
<td>( \psi_P, \psi_I, \psi_R )</td>
<td>0.040, 0.036, 0.030</td>
<td>( tr/Y = 0.08 )</td>
<td>( tr/Y = 0.08 )</td>
<td>NIPA</td>
</tr>
<tr>
<td>( \tau_{w}, \tau_{wr} )</td>
<td>0.31, 0.21</td>
<td>( T/Y = 0.27 )</td>
<td>( T/Y = 0.27 )</td>
<td></td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.7</td>
<td>( M/h_I = 0.7 )</td>
<td></td>
<td>AHS</td>
</tr>
<tr>
<td>( \iota_P, \iota_I, \iota_R )</td>
<td>0.22, 0.54, 0.24</td>
<td>( h_P/h = 0.37 ), ( h_I/h = 0.43 )</td>
<td>( h_P/h = 0.37 ), ( h_I/h = 0.43 )</td>
<td>FOF</td>
</tr>
<tr>
<td>( \alpha, A )</td>
<td>0.2047, 1.805</td>
<td>( h_P/h = 0.37, h_R/h = 0.2 )</td>
<td>( h_I/h = 0.43, h_R/h = 0.2 )</td>
<td>optimal conditions, ( r_k, k )</td>
</tr>
</tbody>
</table>

Table 1: Free parameters, targets are defined annually

Table 1 presents the value of the parameters which are chosen to get the targets annually in the data. \( \beta_P \) is set to 0.9916 to target the safe asset interest rate on deposit equal to 4\% annually. \( \beta_I \) is set to 0.9852 to have a Lagrange multiplier on household loans that is equivalent to a 200 basis point spread on the risk-free rateas (Alpanda and

\(^{10}\)that is consistent with NIPA’s data
\( \beta_B \), banker’s discount factor is set to get \( r_b = 0.0538 \) considering 30-Year Fixed Rate Mortgage Average in the US from Banking Information and Regulation data (BIR, Board of Governors of the Federal Reserve System). With these calibrations, Total Consumption over GDP is set to \( C/Y = 51\% \) and \( c^P/C = 33\%, c^I/C = 43\%, c^R/C = 22\%, c^B/C = 2\% \). The coefficient of leisure time in the utility function \( \varphi_l \) is calibrated to insure the labor supply of each household is 30\%. The coefficient of importance of housing in the utility function \( \varphi_h \) is chosen to get housing value over GDP equal to 5.2 as the Flow of Funds Accounts (FOF; Federal Reserve Board). \( \delta_h \) is set to 0.0096 to get housing investment over GDP by patient household equal to 5\% and \( \delta_k \) equal to 0.0141 to get capital over GDP equal to 6 based on the the National Income and Product Accounts (NIPA, Bureau of Economic Analysis). Based on the same data, \( \vartheta_P, \vartheta_I, \vartheta_R \) are set to 0.04, 0.036, 0.030, respectively, to target total transfer over GDP, \( tr/Y = 0.08 \). \( \tau_w, \tau_{wr} \) are calibrated to 0.31, 0.21 to get income tax, \( T/Y = 0.27 \) as Zubairy (2014).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \iota )</td>
<td>1</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>( \rho_m )</td>
<td>0.85</td>
<td>Alpanda and Zubairy (2016)</td>
</tr>
<tr>
<td>( \tau_k, \tau_c, \tau_p, \tau_d )</td>
<td>0.4, 0.05, 0.14/4, 0.15</td>
<td>Zubairy (2014)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.9</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>( \psi_b )</td>
<td>0.25</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>( \psi_k, \psi_h, \psi_a )</td>
<td>8, 30, 0.1</td>
<td>Alpanda and Zubairy (2016)</td>
</tr>
<tr>
<td>( \rho_b )</td>
<td>0.001</td>
<td>Avoiding indeterminacy</td>
</tr>
<tr>
<td>( \tau_r )</td>
<td>0.0125</td>
<td>NCSL</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>0.1483/4</td>
<td>US Census Bureau</td>
</tr>
</tbody>
</table>

Table 2: Deep Parameters

Based on the 2011 American Housing Survey (AHS; Census Bureau) data \( \theta \) is calibrated to 0.7 to target \( M/h_I = 0.7 \) in the steady state. Following the same data and the 2001 Residential Finance Survey (RFS; Census Bureau), \( \iota_P, \iota_I, \iota_R \) are set to 0.22, 0.54, 0.24 respectively to target \( h_P/h = 0.37, h_I/h = 0.43, h_R/h = 0.20 \). Government expenditure is calibrated to 18\% based on NIPA, and to have government loan\( (b_g) \) over GDP 30\%. \( \rho_b \), the elasticity of transfers to government debt, is calibrate to 0.001 to avoid indeterminacy (Leeper et al., 2010). \( \tau_l \) is calibrated to 0.0125 that is the average land transfer tax in US based on data from National Conference of State Legislatures (NCSL). On the other hand, this tax is not applied on all households. Only the mover should pay land transfer tax. Based on the US Census Bureau Reports the average rate of mover between 1986-2016 is \( \zeta = 14.83\%/4 \) per quarter. \( \alpha, A \) are set to 0.2047, 1.805 based on the optimal conditions and the relation between \( r_k, k \) and to insure \( k/Y = 6 \). Frisch elasticity is set
to one from Smets and Wouters (2007). In order to get high persistence in the borrowing constraint, $\rho_m$ is set to 0.85 as Alpanda and Zubairy (2016). The leverage ratio $\phi$, and the coefficient of banker’s lending adjustment cost $\psi_b$ are set to 0.9, 0.25, respectively, from Iacoviello (2015). Coefficient of capital and housing producer adjustment cost $\psi_k, \psi_h$ are calibrated to 8, 30 and the coefficient of housing investment adjustment cost $\psi_a = 0.1$ following Christiano et al. (2005) and Alpanda and Zubairy (2016). Capital, consumption, property and deposit taxes($\tau_k, \tau_c, \tau_p, \tau_d$) are set to 0.4, 0.05, 0.14, 0.15 respectively based on the US tax codes (Zubairy, 2014).

Table 3 presents the steady state value of all variables over GDP for the baseline model.

<table>
<thead>
<tr>
<th>Variable/GDP</th>
<th>symbol/Y</th>
<th>Steady State/Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>$c_P, c_I, c_R, c_B$</td>
<td>0.17, 0.22, 0.11, 0.01</td>
</tr>
<tr>
<td>Housing</td>
<td>$h_P, h_I, h_R$</td>
<td>2, 2.2, 1</td>
</tr>
<tr>
<td>Tax</td>
<td>$T$</td>
<td>0.27</td>
</tr>
<tr>
<td>Bankers’ asset</td>
<td>$a$</td>
<td>1.87</td>
</tr>
<tr>
<td>Mortgage</td>
<td>$M$</td>
<td>1.56</td>
</tr>
<tr>
<td>Government loan</td>
<td>$b_g$</td>
<td>0.30</td>
</tr>
<tr>
<td>Deposit</td>
<td>$d$</td>
<td>1.68</td>
</tr>
<tr>
<td>non-housing output</td>
<td>$Y_f$</td>
<td>0.82</td>
</tr>
<tr>
<td>wages</td>
<td>$w_P, w_I, w_R$</td>
<td>0.54, 0.97, 0.43</td>
</tr>
<tr>
<td>Government Exp.</td>
<td>$g$</td>
<td>0.18</td>
</tr>
<tr>
<td>Transfers</td>
<td>$tr_P, tr_I, tr_R$</td>
<td>0.03, 0.03, 0.02</td>
</tr>
<tr>
<td>Investments</td>
<td>$i^k, i^h$</td>
<td>0.08, 0.05</td>
</tr>
</tbody>
</table>

Table 3: Steady state of the benchmark model

4 Effects of Permanent shocks

The aim of this section is to assess the role of bankers in deterministic economy when there is a permanent shock in the housing taxes and deduction conditions. The paper analyses the effect of the tightening or loosening the economy by changing the leverage ratio of the intermediary agent. There are permanent shocks in period one in the system of equations for deduction of mortgage interest, $I_m$, for deduction of imputed rental income, $I_r$, for property tax, $\tau_p$, land transfer tax, $\tau_l$ and change in the financial regulation-leverage ratio, $\phi$. The only difference between $\tau_p$ and $\tau_l$ is the former impacts on the current housing where household lives and the latter impacts the houses it would like to buy in
this period. However the nature of change in the regulation is different than changes in the tax code. It is interesting to examine how government or higher regulatory could raise tax revenue and effect economy indirectly. This model is a *Perfect foresight model* and there is a perfect foresight path which describes transition between the base line steady state and the new steady state for the variables on 1000 periods. In period zero agents are not aware of these permanent shocks at period one. All the results in the tables are based on the value of the variables on their transition path between the initial and terminal steady state. The exogenous variables $I_m, I_r, \tau_p, \tau_l, \phi$ in Table 4 are changed to get the present value of tax revenue gain equal to 50% to preserve the comparability. The present value of tax revenue change is calculated by,

$$PV_T = \frac{1}{T_0} \sum_{t=0}^{\infty} \beta^t (T_t - T_0)$$

(4.1)

where $\beta$ is a weighted average of discount factors of agents in the economy and it is equal to $\frac{\beta_p}{\beta_c} \beta^P + \frac{\beta_l}{\beta_c} \beta^I + \frac{\beta_b}{\beta_c} \beta^B$. $T_0$ is the total tax steady state at period zero without any shock. The present value of GDP change in the table are also calculated in an analogous fashion. Table 4 presents the change in the present value of GDP change based on perfect foresight solution.

<table>
<thead>
<tr>
<th>Policy</th>
<th>Parameter</th>
<th>from→to</th>
<th>$PV_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>deduction of mortgage interest</td>
<td>$I_m$</td>
<td>1 → 0.875</td>
<td>-0.0590</td>
</tr>
<tr>
<td>deduction of imputed rental income</td>
<td>$I_r$</td>
<td>0 → 0.040</td>
<td>-0.0843</td>
</tr>
<tr>
<td>property tax</td>
<td>$\tau_p$</td>
<td>0.014 → 0.015</td>
<td>0.0061</td>
</tr>
<tr>
<td>land transfer tax</td>
<td>$\tau_l$</td>
<td>0.0125 → 0.0137</td>
<td>0.0060</td>
</tr>
<tr>
<td>financial regulation</td>
<td>$\phi$</td>
<td>0.9 → 0.934</td>
<td>0.0239</td>
</tr>
</tbody>
</table>

Table 4: $PV_Y$ from the change in the exogenous variables to gain $PV_T = 0.5$

Table 5 presents the percentage change of variables from the baseline steady state to the new steady state after related shock in Table 4.

<table>
<thead>
<tr>
<th>Policy change on</th>
<th>$Y$</th>
<th>$Y^f$</th>
<th>$C$</th>
<th>$h$</th>
<th>$T$</th>
<th>$tr$</th>
<th>$M$</th>
<th>$d$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_m$</td>
<td>-0.064</td>
<td>0.014</td>
<td>0.073</td>
<td>-0.52</td>
<td>0.33</td>
<td>-0.28</td>
<td>-1.40</td>
<td>3.41</td>
<td>0.014</td>
</tr>
<tr>
<td>$I_r$</td>
<td>-0.099</td>
<td>0.008</td>
<td>0.085</td>
<td>-0.73</td>
<td>0.36</td>
<td>-0.31</td>
<td>-0.95</td>
<td>4.2</td>
<td>0.008</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>0.031</td>
<td>0.007</td>
<td>0.083</td>
<td>-0.72</td>
<td>0.35</td>
<td>-0.30</td>
<td>-0.73</td>
<td>4.23</td>
<td>0.007</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>0.030</td>
<td>0.007</td>
<td>0.083</td>
<td>-0.73</td>
<td>0.35</td>
<td>-0.30</td>
<td>-0.74</td>
<td>4.2</td>
<td>0.007</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.079</td>
<td>-0.100</td>
<td>-0.26</td>
<td>1.14</td>
<td>0.27</td>
<td>-0.54</td>
<td>2.14</td>
<td>12.7</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

Table 5: % change in the SS of the variables after permanent shock
Figure 5: % Dev. from SS, Permanent shocks on $I_m, I_r, \tau_p, \tau_l, \phi$

Figure 5 plots the response to change on all fiscal policy and in the banking regulation through the leverage ratio. The paths in the figure present the transition path from the baseline steady state to the new steady state after related shock that lasts for ever. Table 6 presents the percentage change of consumption and housing of all types of households between the initial steady state and the new steady state.

<table>
<thead>
<tr>
<th>Policy change on</th>
<th>$c^P$</th>
<th>$c^I$</th>
<th>$c^R$</th>
<th>$c^B$</th>
<th>$h^P$</th>
<th>$h^I$</th>
<th>$h^R$</th>
<th>$p^R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_m$</td>
<td>0.20</td>
<td>-0.09</td>
<td>-0.04</td>
<td>3.41</td>
<td>0.20</td>
<td>-1.40</td>
<td>-0.04</td>
<td>0</td>
</tr>
<tr>
<td>$I_r$</td>
<td>0.17</td>
<td>-0.06</td>
<td>-0.05</td>
<td>4.23</td>
<td>-0.83</td>
<td>-0.95</td>
<td>-0.05</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>0.15</td>
<td>-0.05</td>
<td>-0.05</td>
<td>4.23</td>
<td>-0.61</td>
<td>-0.73</td>
<td>-0.93</td>
<td>0.89</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>0.15</td>
<td>-0.05</td>
<td>-0.05</td>
<td>4.21</td>
<td>-0.62</td>
<td>-0.74</td>
<td>-0.94</td>
<td>0.89</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.72</td>
<td>-0.02</td>
<td>-0.18</td>
<td>-28.6</td>
<td>0.72</td>
<td>2.14</td>
<td>-0.18</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6: % change in the SS of Consumption and Housing

Figure 6 presents the responses to permanent shock on housing taxes and the leverage ratio on consumption and housing of any type of household.
The following section presents the results in Table 4-7 and Figures 5-6. The results are discussed in terms of transition path and the impact on GDP present value followed by welfare.

4.1 Results

4.1.1 Deduction of mortgage interest

By reducing deduction of mortgage interest $I_m$ from one to 87%, the marginal cost of having an additional unit of mortgage increases. This change directly targets the impatient household’s decision on mortgage. Table 4 in line one presents the changes of variables by 13% reducing in $I_m$. The present value of the tax revenue gain rises by 50% as imposed by the experiment. This change results in a negative present value of GDP presented in the first row of Table 4. This effect can be explained in light of the fact that the change makes a rise in capital, deposit and consumption and reduces total housing.

Increasing the marginal cost of raising mortgage reduces the mortgage taken by impatient households directly. The less mortgage, the less impatient consumption and the less impatient house demands because of the collateral constraint. Tightening the collateral constraint through the drop in impatient house demands decreases the shadow price of the collateral constraint. A drop in the shadow price directly raises the borrowing
interest rate\textsuperscript{11}. Since the leverage ratio is constant, a raise in borrowing interest rate is followed by a raise in the safe asset interest rate. It is verifiable by checking banker’s optimal conditions which were explained in the banker section. The higher the interest rate on the safe asset, the higher the deposit. On the other hand, an initial drop in impatient house demands increases the housing price and motivates patient households to raise residential and rental houses investment. The latter causes a drop in rental housing price which raises renters’ consumption because of the income effect. Some time after the shock, the interest rate decreases and reverts to its steady state which results in less deposit, patient consumption and patient housing. In addition the rental price goes toward the new steady state and rental housing drops and is replaced by patient houses which provides more utility. The drop in the rental price motivate patient households to invest in capital and intermediary safe asset. Raising capital increases the non-housing output which results in more income wage due to equation (2.26). The major difference between non-housing output and GDP is $pR^h$. So on the contrary with non-housing output, GDP drops because of the drop in the rental price. Higher return next period results in a drop in the marginal rate of substitution ($\lambda^P$) and a raise in patient consumption. It is a result of the Euler equation. In addition, both the income effect and the high investment in safe assets and capital make an increase in patients’ consumption. Since $\varphi_h$ (the coefficient which presents the importance of housing the utility function) is fixed, lower marginal rate of substitution motivates patient households to have more residential houses. Lower marginal rate of substitution increases the discounted utility gain and the house value after tax and depreciating in the steady state whereas the marginal cost of buying one unit of residential purpose house at time $t$ is fixed in the steady state. This increase is caused by the optimality condition which equates the marginal cost of buying one unit of residential purpose house at time $t$ with discounted utility gain and the house value after tax and depreciating. The more deposit, the more asset for banker and the more government loan (mortgage declines because of an increase in the raising mortgage cost due to the smaller deduction rate). The increase in non-housing output increases the transfers. On the other hand some time after the shock, non-housing output moves to the new steady state and government loans rise which causes a drop in transfers. Less transfer causes a decline in both impatient’s and renters’ consumption.

\textsuperscript{11}Shadow price or utility gain is calculated from the Lagrangian multiplier of collateral constraint, $\lambda_m$. See the optimality conditions for impatient households with respect to impatient houses and mortgage equation (2.13)-(2.14)
4.1.2 Deduction of imputed rental income

Deduction of imputed rental income targets both patient and impatient households. Increasing the costs of owning houses by 4% makes a gain of 50% in the present value of tax revenue and the loss of GDP (about 8%) which is about 2.5% less than the loss of the deduction of mortgage interest.

Impatient households are more sensitive to changes in comparison to patient ones as housing is the only way to transfer their wealth is housing whereas patient households have a variety of alternatives to choose from, for example, housing investment deposit and capital. Raising $I_r$ firstly affects impatient housing with a drop. Hence similar to the change in the deduction of mortgage interest, patient households have incentive to buy more houses for residential or renting purposes. This, in turn, reduces the rental price. A drop in the rental price increases renter consumption due to the income effect. Some time after the shock, the rental price moves to the new steady state and rental housing drops off. With the change in deduction of imputed rental income, patient households prefer to invest more in safe assets and capital. Since the collateral constraint is tighter, due to the drop in impatient housing, the mortgage declines. Raising deposit and declining mortgage means that the government is capable of having more loan that decreases the transfer. In addition an increase in capital generates increased output and wage.

4.1.3 Property and Land transfer tax

These two taxes effect the economy in the same way, as evidenced in the tables and figures. The former applies to current houses while the latter only effects movers. Property tax is raised from 0.014 to 0.015 (7%) and land transfer tax increases from 0.0125 to 0.0137 (10%). These two changes work in the same manner while the former is applied on all homeowners and the latter is applied only on the fraction of population which moves and changes house. However only a fraction of the population pays LTT, almost the same change in LTT and property tax provides same effects and this shows how important this tax could be to the economy and how government or social planner can choose between these two ways of direct taxing. These changes make an increase equal to 50% gain in the present value of tax revenue changes alongside about 0.6% gain in the present value of GDP changes. The present value of GDP for these changes, on the contrary with deductions, is positive. As housing value in the steady state has a large share equal to 5.2 times of GDP, any direct intervention, in comparison with the indirect one, could make a big impact. The GDP gain in direct changes against the loss in indirect changes
could be explained by the role of housing. According to Table 6, in the steady state, only direct changes effect the rental price (positively). This augmentation in the price makes a positive effect on GDP and causes a higher GDP for direct changes rather than indirect ones.

Increasing housing taxes reduces total housing demand in the long run. When the shock occurs, impatient households decrease their house demands. This not only causes a drop in mortgages but also a drop in the housing price. Patient households benefit the low price to raise their houses. The transition paths are largely comparable to other changes. The only difference is the positive effect on the rental price in the long term that is due to the drop in the rental housing investment. Because of the initial drop in rental price patient household prefers to convey their investment through the safe asset and capital that makes a positive income effect with more output. Since collateral constraint limits mortgage to the collateralized housing for impatient household, a drop in housing reduces the mortgage and increase the government loans.

4.1.4 Change in Financial Regulation

The government or higher regulatory can control the economy by changing the regulation of banker. The leverage ratio controls the banker liabilities in respect to its assets. Raising the leverage ratio means loosening the economy, this means more deposit, mortgage and government loan. In the benchmark model the leverage ratio is set to 0.9. To gain 50% tax revenue more than benchmark, the leverage ratio is raised by 3.7%. This change increases the present value of GDP by 2%. Adding two optimal conditions of banker,\((2.21)-(2.22)\), around the steady state\(^{12}\),

\[
\lambda_t^\phi (1 - \phi) \simeq \beta_B E_t \lambda_t^B (r_{t+1}^b - r_{t+1})
\]

this equation shows the spread between the return on banker assets and the return on deposits becomes smaller whenever the parameter of the financial regulation, i.e leverage ratio, gets higher\(^{13}\). In other words, in the steady state a raise in the leverage ratio decreases the return on assets. When asset constraint \((2.20)\) become looser because, for example because the leverage ratio is raised, the return on the assets gets smaller to make the banker indifference between loans and deposits. The explanation comes from the liquidity of loans and deposit. Deposits are more liquid than loans. In the

\(^{12}\)the adjustment cost is zero in the steady state

\(^{13}\)Note around the steady state for the small change on the leverage ratio \(\lambda^\phi\) and \(\lambda^B\) do not change.
equilibrium, raising one unit of deposit requires raising one over \( \phi \) unit of loans (that is greater than one). A decline on the return on loans, \( r^b \), motivates the government to raise borrowing. On the other hand, the raise in the leverage ratio increases deposits which negatively effects the interest rate on safe assets. This negative impact makes patient households more motivated to raise investments on housing. The more investment on housing and safe assets, the more assets next period and more patient consumption. The raise in patient house demands increases the house price which results in a drop in impatient houses. Consequently the drop in impatient houses tightens the collateral constraint and decreases mortgages. The mechanism of variable changes is similar to other policy changes in previous sections. In addition, from the banker budget constraint, increasing the leverage ratio negatively effects its consumption. However the share of banker consumption in total consumption is minuscule, due to the large negative effect on this consumption, total consumption declines alongside. Some time after shock, due to the decline in the borrowing interest rate, mortgages raise. The direct effect of raising mortgage is increasing impatient housing investments and consumption.

4.2 Welfare effects

One of the means by which one can compare the tax policies and the financial regulation is their effects on welfare. To analyze the impact of each change on welfare of agents, the change in their respective lifetime welfare in terms of annual consumption equivalents that is equal to \( \Lambda_i, i = P, I, R, B \) and is calculated in following equation,

\[
\sum_{t=0}^{\infty} \beta^t U((1 + \Lambda_i) c^0_i, h^0_i, l^0_i) = \sum_{t=0}^{\infty} \beta^t U(c^t_i, h^t_i, l^t_i) \tag{4.3}
\]

Where \( c^0_i, h^0_i, l^0_i \) are consumption, housing and labor of each agent\(^{14}\) in the initial steady state and \( c^t_i, h^t_i, l^t_i \) are the related variables at any period of the transition path plotted in Figures 5-6 till achieving the new steady state after shocks. Positivity of \( \Lambda \) means the change leads agents to be better off. Table 7 presents welfare effects of each change for all agents in the economy and the weighted average of welfare effects, \( \Lambda_A^{15} \).

Exploring the table, the positive effect of increasing taxes, directly and indirectly, and of loosening the economy(by leverage rate) on patient and impatient household is determined. Renter household as the most sensitive agent(it works and consumes-hand to mouth) is always worst off. Increasing taxes and loosening the economy will reduce

\(^{14}\)For the banker it is assumed that housing and labor are equal to zero.

\(^{15}\)\( \Lambda_A = \frac{c^P}{c} \Lambda_P + \frac{c^I}{c} \Lambda_I + \frac{c^R}{c} \Lambda_R + \frac{c^B}{c} \Lambda_B \)
banker welfare that the latter has a considerable impact. The percentages in the table present what percent of the first period consumption of agent is required to make agents indifferent between applying the change and having the benchmark value.

Change in deduction of mortgage interest has 1.75% and 1.03% welfare gain for patient and impatient households, respectively. That means they are better off and instantly to make patient household indifferent between the tax code with fully deduction of mortgage interest and the tax code with partially deduction, there needs to be a per period compensation of 1.75% of its first period consumption. On the contrary, the renter and banker are worst off by $-2.71\%$ and $-1.94\%$. The welfare changes come from the income effect and the changes in consumption. In a model with bankers, patient households have the ability to transfer savings to the next period through their safe asset. Due to the deterministic behavior of the model, patient households increase the consumption by investing more capital and safe asset. In addition, in the new steady state patient and impatient labors in the economy drop while the wage increases as we can see in figure 7. This has a positive effect on the utility function. The renter wage also increases but it is not enough so renters provide more labor rather than their old steady state. Consequently welfare is effected negatively. However the changes in the tax codes increase the new steady state of bankers’ consumption, the drop in consumption at early periods of simulation (due to the drop in deposits) makes the welfare loss. As viewpoint of general welfare, the weighted average of welfare effect shows 0.38% better off.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Lambda_P(%)$</th>
<th>$\Lambda_I(%)$</th>
<th>$\Lambda_R(%)$</th>
<th>$\Lambda_B(%)$</th>
<th>$\Lambda_A(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_m$</td>
<td>1.75</td>
<td>1.03</td>
<td>-2.71</td>
<td>-1.94</td>
<td>0.38</td>
</tr>
<tr>
<td>$I_r$</td>
<td>1.29</td>
<td>0.87</td>
<td>-2.18</td>
<td>-0.25</td>
<td>0.31</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>1.33</td>
<td>0.67</td>
<td>-2.12</td>
<td>-0.26</td>
<td>0.25</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>1.33</td>
<td>0.67</td>
<td>-2.12</td>
<td>-0.27</td>
<td>0.25</td>
</tr>
<tr>
<td>$\phi$</td>
<td>2.54</td>
<td>0.95</td>
<td>-2.38</td>
<td>-33.0</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 7: welfare effects
From the second line of table 7 one can see that by decreasing deduction of imputed rental income has negative implications for renters and bankers hence they are worst off. Renters provide more labor and the income effect is not enough to provide more consumption for them. To make renters indifferent between the tax code with fully deduction of imputed rental income and the tax code with partially deduction, they need a per period compensation of $-2.18\%$ of their first period consumption. A same required per period compensation for patient, impatient and intermediary are $1.29\%$, $0.87\%$, $-0.25\%$ respectively. In comparison with the last case, patient and impatient households are not as well off, renters and bankers are even less so and the weighted average welfare declines to $31\%$.

The signs of welfare change in the case of changing in property and land transfer taxes are the same as other previous changes. Patients, impatient are better off and renters and bankers are worse off due to the income effect and share of labors from each type of household in the new steady state. For patients and bankers this change in the tax code has a welfare effect in the middle of 2 other changes ($1.33\%$, and $-0.26\%$ respectively). This change has a minimum welfare gain for impatient household ($0.67\%$). From the view point of general welfare, the weighted average welfare of changes in property and transfer taxes is the lowest ($25\%$) welfare gain among the changes in tax codes.

There is a huge drop in bankers’ consumption when the leverage ratio increased that has a tremendous negative welfare effect on intermediary agent. Renters are forced to increase their share of labor and consume less. This has a negative effect on their
welfare(−2.38%) which is the largest welfare loss in all changes. Due to an increase in deposit and mortgage, patient and impatient household consume more and they are positively impacted by a raising leverage ratio. The patient household’s welfare adjustment from changing the leverage ratio is 2.54% which is almost twice the same change from other policies and it comes from the big change in deposits. The weighted average welfare for the change in the banker’s leverage ratio is the lowest one among all policies(0.2%).

5 Conclusion

This paper studies a general equilibrium model with heterogeneous agents and discusses the impact of various housing taxes alongside the change in the regulation of financial intermediary are discussed. The model is closed to the real economy where households lack expertise in investing directly in bonds and instead deposit their saving in an expert intermediary agent. The model presents financial friction in the form of collateral constraints for borrower households and lending constraint for intermediary agents. The latter directly effect the demand side of borrowing constraint and provides an effective way to control the economy for governments or higher regulatory. This paper finds that direct change policies(e.g. property tax) discussed above result in GDP gains whereas indirect tax changes(e.g. change in deductions) result in GDP losses. This could be explained in light of the fact that intermediary agent gives a new channel to lender households to transfer savings to next periods.

The findings of this paper show the change in deduction of imputed rental income has the lowest present value of GDP loss among other alternatives. However the change in the financial regulation has the greatest gain among all polices, the change in the property tax which is a direct change in housing-related taxes has the greatest gain among changes in the tax codes. Indirect changes lead the economy to a lower GDP (rather than the initial steady state) in the long term, though direct and financial regulation changes lead to a higher one.

All changing tax policies have negative impacts on total housing whereas the change in the financial regulation augments housing. This augmentation is largely due to looser lending constraint and more mortgage. This results in more housing for borrowers in the new steady state.

On the other hand, welfare effects of the changes can offer another way to compare among the policies. The changes have different impacts on households. Generally, any augmentation in tax revenue has negative welfare effects on renters (as the most sensitive type
of households who are hand to mouth) as well as bankers. Welfare effects on lenders, borrowers are positive. The impact of changes in the regulation on welfare of lender compared with others is large. This difference comes from the direct effect of the financial regulation on deposits. The comparison between the policies through welfare effects depends on the target type of household. For example deduction of mortgage interest rate has highest positive effect on patient households among other tax policies whereas it also has the worst effect on renters. Changing the regulation has a tremendous positive effects on patient and impatient agents. It is the greatest impact among all policies in terms of welfare gain. Analyzing the policies from the view point of general welfare, deduction of mortgage interest rate is the best change and the regulation change is the worst one due to the greatest and lowest welfare gain.

6 Appendix

6.1 FOCs

\[
\varphi_l(l^i_t) = \lambda_i(1 - \tau_w)w^i_t \quad i = P, I \\
\varphi(l^R_t) = \lambda_t^R(1 - \tau_w)w^R_t
\]

\[
p^x - \psi_p p^x \left( \frac{i^x_t}{i^x_{t-1}} - 1 \right) \frac{i^x_t}{i^x_{t-1}} - \psi_p \left( \frac{i^x_t}{i^x_{t-1}} - 1 \right)^2 p^x_t \\
+ \beta \mathbb{E}\left[ \frac{\lambda_{t+1}}{\lambda_t^P} \psi_p p^x_{t+1} \left( \frac{i_{t+1}^x}{i^x_t} - 1 \right) \left( \frac{i_{t+1}^x}{i^x_t} \right)^2 \right] = 1 \quad x = k, h
\]

6.2 Prices in steady state

Table 8 presents the steady state value of the prices per quarter for the baseline model.

<table>
<thead>
<tr>
<th>Price</th>
<th>symbol</th>
<th>Steady State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rental Price</td>
<td>(p^R)</td>
<td>0.0281</td>
</tr>
<tr>
<td>Interest rate on safe asset</td>
<td>(r)</td>
<td>0.0100</td>
</tr>
<tr>
<td>Interest rate on capital</td>
<td>(r^k)</td>
<td>0.0282</td>
</tr>
<tr>
<td>Interest rate on mortgage</td>
<td>(r^b)</td>
<td>0.0135</td>
</tr>
</tbody>
</table>

Table 8: Steady state of the prices per quarter
References


