OPTIMAL HUMAN CAPITAL BEQUEATHING

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Abstract. When parents endow their offspring with human capital and the effectiveness with which they do so depends on their own, the decentralized allocation of resources through markets cannot deliver, under laissez-faire, the benevolent planner's outcome maximizing the representative agent's welfare. Specifically, the market level of human capital is too low. Most importantly, this happens to be the case *even when parents internalize perfectly in their utility the value of their investment for their children*. The problem is not therefore one of an externality not internalized, but rather the impossibility of replicating in a decentralized way, under laissez-faire, the kind of intergenerational coordination that a benevolent planner can achieve. The planner's allocation can nonetheless be decentralized through the market subsidizing labor income at the expense of a lump-sum tax on saving returns.

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1. Introduction

Households choose, among other things, the human capital they endow their offspring with for altruistic reasons. This takes typically the form of education expenditures that determine their offspring human capital. The effectiveness of households' education efforts arguably depends also on the parents' own human capital, in such a way that —all other things equal— any given amount invested translates into a higher human capital for the offspring when compounded with a high human capital of the parents. Be as it may, one would expect at any rate that when parents internalize the correct value that their own human capital and the education effort they make entail for their children's well-being, they would be able to choose the right amount, from a social viewpoint. Interestingly enough, as it is shown below, this is not the case. It turns out that even when the parents assess correctly the value for their children of the education effort they make for them —as well as how their own education compounds with it— and choose in a decentralized way their educational investment efforts through the market, the latter cannot deliver (under laissez-faire) the right allocation. By "*the right allocation*" I mean the allocation that a planner would choose in their stead in order to maximize the well being of the representative agent of the economy. This will be shown to be the case below for stationary allocations.

To truly grasp the extent to which this should be surprising, it should be noted that when I refer to a situation in which "*parents assess correctly the value for their children of their education effort and how their own education compounds with it*" I mean exactly that: parents add to their utility from consumption the *actual* value function $V(e^t, h^t)$ —weighted by a parameter $\beta \in (0, 1)$ measuring their degree of altruism— of their children's optimization as a function of the parents' choice of education for their children e^t, and the parents' own human capital h^t. This modeling choice is not only the only one consistent with the parents' rationality —since they know that their children's decision problem is the same as theirs and, knowing their own value function, they therefore know their children's, from which they could only depart in their objective at the price of being irrational— but is also made in order to give all the chances to the decentralized allocation of resources to deliver the best outcome for the representative agent. And still, it falls short of doing so. Why is it so?

From the analysis below it follows that —since the parents internalize correctly the impact of their educational choice on their offspring through the value function $V(e^t, h^t)$— it is not a missing externality in their optimization which lies at the heart of the result. What drives the result is the inescapable fact that, while parents can take into account how their choices impact their children's, they nonetheless cannot choose for them. This is a constraint from which a benevolent planner, by definition, is freed: he or she chooses for everyone, and therefore can do better than what households can do in a decentralized way.

In some sense, the main lesson to be drawn from this result is that internalizing all kinds of externalities needs not always be enough: there are limits to what can be done in a decentralized way... *under laissez-faire*. Indeed, if one is willing to intervene, the planner's allocation *can* be decentralized. This requires a policy that steers households choices to it through incentives though. Specifically, since the market delivers too few efficient units of labor, labor income needs to be subsidized —while funding this by means of a non-distortionary lump-sum tax on the savings returns— in order to give parents the right incentive to invest more in their children's education.

2. Model

Consider and economy of identical 2-period lived overlapping generations of households consuming both when young and old, out of their labor income when young and the returns to their savings when old. Population grows by a positive factor n every period, and households can invest when young in their offspring education some nonnegative amount.

The representative household born at t is endowed when young with h^t effective units of labor —identified to the household's human capital— resulting from the *per* children educational investment made at $t-1$ by its parent household e^{t-1} and the parent household's own human capital h^{t-1} through a human capital production function $H(e^{t-1}, h^{t-1})$. Effective units of labor are remunerated at a wage rate w_t at period t.

Households at t can save when young by both lending k^t to firms at a rental rate r_{t+1} to be paid next period, and holding real balances M^t/p_t of monetary savings M^t,[1] where p_t is the price level of consumption at t. The representative household born at t makes then saving and consumption choices, c_0^t and c_1^t when young and old

[1]The possibility of monetary savings is necessary for the market decentralization of the planner's steady state not to be degenerate. Indeed, in an overlapping generations economy with population growth the planner aims at (1) a level of savings equating the representative household's inter-

respectively, that provide a utility $u(c_0^t, c_1^t)$ to which it adds the the utility obtained by each of its n children, weighted by an altruism factor $\beta \in (0,1)$.

3. Household's optimal choice

Given the human capital it is endowed with as a result of the interaction of education received and its parents' human capital e^{t-1}, h^{t-1} respectively, the period t representative household aims at maximizing —with respect to its consumption and saving choices (in capital and money) c_0^t, c_1^t, k^t, M^t and educational effort e^t (and under the first and second period budget constraints and the human capital formation technology)— its overall utility comprising the utility it derives from its consumption profile $u(c_0^t, c_1^t)$, plus the maximum overall utility $V(e^t, h^t)$ of each of its n children, weighted by an altruism factor β,[2] that is to say

$$
\begin{aligned}
V(e^{t-1}, h^{t-1}) = \max_{c_0^t, c_1^t, k^t, M^t, e^t, h^t} u(c_0^t, c_1^t) + n\beta V(e^t, h^t) \\
c_0^t + k^t + \frac{M^t}{p_t} + ne^t \leq w_t h^t \\
c_1^t \leq r_{t+1} k^t + \frac{M^t}{p_{t+1}} \\
h^t \leq H(e^{t-1}, h^{t-1})
\end{aligned}
$$

for given consumption and factor prices $p_t, p_{t+1}, w_t, r_{t+1}$, parent choices e^{t-1}, h^{t-1}, and $n < \frac{1}{\beta}$.[3]

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temporal marginal rate of substitution with the return to savings *and* (2) maximizing the net output —which requires a level of capital equating its marginal productivity to the population growth factor. The amount of total savings necessary to achieve (1) is typically different from the amount of capital savings necessary for (2). At a market equilibrium, the presence of an asset other than capital allows to fill the gap between desired savings and net output maximizing capital. In the absence of such a means of saving other than lending capital to firms, the planner's optimality conditions will be met by a competitive equilibrium only in degenerate cases.

[2] Although it is obvious from the first period budget constraint that at the solution the third constraint is always binding, the recursive way in which human capital is formed requires h^t to be included in t's problem *as if* it was a variable of choice, which is actually none, since it is determined by e^{t-1} and h^{t-1}, that is to say by $e^{t-1}, e^{t-2}, e^{t-3}, \ldots$

[3] The extent to which households *can* be altruistic is linked to the population growth factor. Should the altruism factor β exceed the reciprocal of the population growth factor $\frac{1}{n}$, there would not

The first order conditions necessarily characterizing the household choice are

$$\begin{pmatrix} u_0(c_0^t, c_1^t) \\ u_1(c_0^t, c_1^t) \\ 0 \\ 0 \\ n\beta V_e(e^t, h^t) \\ n\beta V_h(e^t, h^t) \end{pmatrix} = \lambda_0^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ \frac{1}{p_t} \\ n \\ -w_t \end{pmatrix} + \lambda_1^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{1}{p_{t+1}} \\ 0 \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

along with the constraints binding, where —from the envelope theorem—[4]

$$V_e(e^t, h^t) = \mu^{t+1} H_e(e^t, h^t)$$
$$V_h(e^t, h^t) = \mu^{t+1} H_h(e^t, h^t)$$

that is to say

$$\frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} = \frac{p_t}{p_{t+1}} = r_{t+1}$$
$$u_0(c_0^t, c_1^t) = \beta \mu^{t+1} H_e(e^t, h^t)$$
$$\mu^t - w_t u_0(c_0^t, c_1^t) = n\beta \mu^{t+1} H_h(e^t, h^t)$$

so that the period t representative household's **optimal choice** $c_0^t, c_1^t, k^t, M^t, e^t$ and

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exist a value function V allowing to define the representative household's problem —specifically, the right-hand side would fail to be a contraction of the space containing V, and the existence of the (fixed point) V allowing for the representative household's problem to be well-defined is not guaranteed. Alternatively, when fertility is endogenous, the altruism discount factor can be assumed to decrease fast enough in the population growth factor —see Becker, Murphy and Tamura (1994).

[4] The derivative of the value $V(e^t, h^t)$ of the Lagrangian of the problem faced by generations $t+1$

$$u(c_0^{t+1}, c_1^{t+1}) + n\beta V(e^{t+1}, h^{t+1}) - \lambda_0^{t+1}\left(c_0^{t+1} + k^{t+1} + \frac{M^{t+1}}{p_{t+1}} + ne^{t+1} - w_{t+1}h^{t+1}\right)$$
$$- \lambda_1^{t+1}\left(c_1^{t+1} - r_{t+2}k^{t+1} - \frac{M^{t+1}}{p_{t+2}}\right) - \mu^{t+1}\left(h^{t+1} - H(e^t, h^t)\right)$$

with respect to e^t, i.e. $V_e(e^t, h^t)$, is indeed $\mu^{t+1} H_e(e^t, h^t)$, and similarly for $V_h(e^t, h^t) = \mu^{t+1} \cdot H_h(e^t, h^t)$.

human capital endowment h^t are necessarily characterized by

$$\frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} = \frac{p_t}{p_{t+1}} = r_{t+1}$$

$$\frac{1}{H_e(e^t, h^t)} \frac{1}{\beta} \frac{u_0(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} = w_{t+1} + n \frac{H_h(e^{t+1}, h^{t+1})}{H_e(e^{t+1}, h^{t+1})}$$

$$c_0^t + k^t + \frac{M^t}{p_t} + ne^t = w_t h^t$$

$$c_1^t = r_{t+1} k^t + \frac{M^t}{p_{t+1}}$$

$$h^t = H(e^{t-1}, h^{t-1})$$

4. Market equilibria

At a market equilibrium, capital and labor are remunerated by their marginal productivities, so that

$$w_t = F_L(\frac{k^{t-1}}{n}, h^t)$$

$$r_{t+1} = F_K(\frac{k^t}{n}, h^{t+1})$$

and the allocation is feasible if

$$c_0^t + \frac{c_1^{t-1}}{n} + k^t + ne^t = F(\frac{k^{t-1}}{n}, h^t)$$

which requires

$$\frac{M^t}{p_t} = \frac{1}{n} \frac{M^{t-1}}{p_t}$$

that is to say

$$\frac{M^t}{M^{t+1}} = n$$

The set of conditions defining a market equilibrium allocation therefore follows.

Definition. *A* **market equilibrium** *is any allocation* $c_0^t, c_1^t, k^t, M^t, e^t, h^t$ *and* *prices p_t such that, for all t,*

$$\begin{aligned}
\frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} &= \frac{p_t}{p_{t+1}} = F_K(\frac{k^t}{n}, h^{t+1}) \\
\frac{1}{H_e(e^t, h^t)} \frac{1}{\beta} \frac{u_0(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} &= F_L(\frac{k^t}{n}, h^{t+1}) + n \frac{H_h(e^{t+1}, h^{t+1})}{H_e(e^{t+1}, h^{t+1})} \\
c_0^t + k^t + \frac{M^t}{p_t} + ne^t &= F_L(\frac{k^{t-1}}{n}, h^t) h^t \\
c_1^t &= F_K(\frac{k^t}{n}, h^{t+1}) k^t + \frac{M^t}{p_{t+1}} \\
h^t &= H(e^{t-1}, h^{t-1}) \\
n &= \frac{M^t}{M^{t+1}}
\end{aligned}$$

A specific instance of market equilibrium allocation is a stationary one that treats all households equally, as characterized in the following definition.

Definition. *A* **market steady state** *is, therefore, a profile c_0, c_1, k, m, e, h such that*

$$\begin{aligned}
\frac{u_0(c_0, c_1)}{u_1(c_0, c_1)} &= n = F_K(\frac{k}{n}, h) \\
\frac{\frac{1}{\beta} - nH_h(e, h)}{H_e(e, h)} &= F_L(\frac{k}{n}, h) \\
c_0 + k + m + ne &= F_L(\frac{k}{n}, h) h \\
c_1 &= F_K(\frac{k}{n}, h) k + nm \\
h &= H(e, h)
\end{aligned}$$

We will now compare the allocations that the market can deliver at a steady state, with the kind of steady state that a planner would choose, as characterized in the next section.

5. Planner's steady state

A benevolent planner would only support a feasible steady state profile of consumptions c_0, c_1 that maximizes the representative household's overall steady state utility, which comprises the utility the household derives from its own consumption profile $u(c_0, c_1)$ plus the overall steady state utility of each of its children —of which it has n and are weighted by the altruism factor β— comprising each the utility it derives from its own consumption profile $u(c_0, c_1)$, plus the overall steady state utility of each of its children... that is to say, the planner maximizes

$$u(c_0, c_1) + n\beta\Big(u(c_0, c_1) + n\beta\big(u(c_0, c_1) + n\beta(u(c_0, c_1) + \dots)\big)\Big)$$
$$= \frac{1}{1 - n\beta} u(c_0, c_1)$$

given that $n < \frac{1}{\beta}$, or what amounts to the same thing, the planner aims at maximizing $u(c_0, c_1)$, since the first factor in the right-hand side of the equation above amounts to a mere scaling factor.

Therefore, the planner's steady state is a profile c_0, c_1, k, e, h solution to[5]

$$\max_{c_0, c_1, k, e, h} u(c_0, c_1)$$
$$c_0 + \frac{c_1}{n} + k + ne \leq F(\frac{k}{n}, h)$$
$$h \leq H(e, h)$$

It is thus worth noting that the planner's ability to choose for all households a stationary allocation makes the households' altruism to appear to be seemingly irrelevant in the planner's problem. Indeed, taking it into account does not make

[5] As a matter of fact, for the planner's objective households' altruism towards their offspring is —even taken into account— inessential. Indeed, the planner aims at maximizing the utility of the representative agent under a *stationary allocation* of resources. Specifically, the overall utility of a representative agent comprises the utility he derives from his own consumptions $u(c_0, c_1)$ plus the discounted (by β) value of the overall utility of his n children, which in turn comprises the utility they derive on their own consumptions $u(c_0, c_1)$ and their discounted value of the overall utilities of their offspring... that is to say

$$u(c_0, c_1) + n\beta u(c_0, c_1) + n^2\beta^2 u(c_0, c_1) + \dots$$

so that —under the assumption $n < \frac{1}{\beta}$— the planner aims at maximizing $\frac{1}{1-n\beta} u(c_0, c_1)$ or, equivalently, $u(c_0, c_1)$.

the planner consider an objective distinct from the one that would correspond in the case of selfish households, as opposed to altruistic ones. The reason is that in being able to choose consumption profiles for all generations and focusing on stationary ones makes the altruism effect boil down to a scaling factor in the planner's objective, with no impact on the optimal allocation. Alternatively, it can be understood as follows: even when the planner looks after the representative agent's consumptions only *directly*, it realizes —through the feasibility constraint— that the education effort e enters in the determination of the contemporaneous (because if the stationarity) efficient units of labor necessary for output and, hence, consumption, which leads it to *indirectly* provide for h through e, behaving this way altruistically *de facto* even if in the selfish interest of the representative agent.

The planner's steady state solution to the problem above is, therefore, a profile c_0, c_1, k, e, h necessarily satisfying

$$\begin{pmatrix} u_0(c_0, c_1) \\ u_1(c_0, c_1) \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ \frac{1}{n} \\ 1 - F_K(\frac{k}{n}, h)\frac{1}{n} \\ n \\ -F_L(e, h) \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 0 \\ 0 \\ -H_e(e, h) \\ 1 - H_h(e, h) \end{pmatrix}$$

along with the planner's constraints binding, which provides the following necessary characterization of a planner's steady state.

Definition. *That is to say, the* **planner's steady state** *is a profile* c_0, c_1, k, e, h *such that*

$$\begin{aligned} \frac{u_0(c_0, c_1)}{u_1(c_0, c_1)} &= n = F_K(\frac{k}{n}, h) \\ \frac{n - nH_h(e, h)}{H_e(e, h)} &= F_L(\frac{k}{n}, h) \\ c_0 + \frac{c_1}{n} + k + ne &= F(\frac{k}{n}, h) \\ h &= H(e, h) \end{aligned}$$

6. Market vs Planner steady states

From the market and planner steady state definitions above, it follows that they differ in that they are characterized by

$$\frac{\frac{1}{\beta} - nH_h(\bar{e}, \bar{h})}{H_e(\bar{e}, \bar{h})} = F_L(\frac{\bar{k}}{n}, \bar{h})$$
$$\frac{n - nH_h(e^*, h^*)}{H_e(e^*, h^*)} = F_L(\frac{k^*}{n}, h^*)$$

respectively or, equivalently,

$$\frac{1}{\beta} = F_L(\frac{\bar{k}}{n}, \bar{h})H_e(\bar{e}, \bar{h}) + nH_h(\bar{e}, \bar{h})$$
$$n = F_L(\frac{k^*}{n}, h^*)H_e(e^*, h^*) + nH_h(e^*, h^*)$$

Note that in the case of parent-to-children one-to-one transfers —*i.e.* when $h^t = e^{t-1}$ so that $H_e = 1$ and $H_h = 0$— it turns out that

$$F_L(\frac{k^*}{n}, h^*) = n < \frac{1}{\beta} = F_L(\frac{\bar{k}}{n}, \bar{h})$$

so that the market steady state delivers a too high wage rate —because of too little efficient units of labor being supplied, as it will be seen below.

while $F_K(\frac{\bar{k}}{n}, \bar{h}) = n = F_K(\frac{k^*}{n}, h^*)$, so that

$$F(\frac{\bar{k}}{n}, \bar{h}) = \bar{k} + F_L(\frac{\bar{k}}{n}, \bar{h})\bar{h}$$

$$F(\frac{k^*}{n}, h^*) = k^* + F_L(\frac{k^*}{n}, h^*)h^*$$

3bis. Household's optimal choice under policy

Consider now a policy consisting of (1) taxing/subsidizing[6] household t's labor income at a rate τ_t, while (2) transferring/taxing a lump-sum T_{t+1} when old. The representative household faces then the problem

[6]Depending on the sign of the rate.

$$
\begin{aligned}
V(e^{t-1}h^{t-1}) = \max_{c_0^t, c_1^t, k^t, M^t, e^t, h^t} & \; u(c_0^t, c_1^t) + n\beta V(e^t, h^t) \\
& c_0^t + k^t + \frac{M^t}{p_t} + ne^t \leq (1+\tau_t) w_t h^t \\
& \qquad c_1^t \leq r_{t+1} k^t + \frac{M^t}{p_{t+1}} + T_{t+1} \\
& \qquad h^t \leq H(e^{t-1}, h^{t-1})
\end{aligned}
$$

for given policies τ_t, T_t, consumption and factor prices $p_t, p_{t+1}, w_t, r_{t+1}$, parent choices e^{t-1}, h^{t-1}, and $n < \frac{1}{\beta}$.

The first order conditions necessarily characterizing the household choice are

$$
\begin{pmatrix} u_0(c_0^t, c_1^t) \\ u_1(c_0^t, c_1^t) \\ 0 \\ 0 \\ n\beta V_e(e^t, h^t) \\ n\beta V_h(e^t, h^t) \end{pmatrix} = \lambda_0^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ \frac{1}{p_t} \\ n \\ -(1+\tau_t) w_t \end{pmatrix} + \lambda_1^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{1}{p_{t+1}} \\ 0 \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}
$$

along with the constraints binding, where —from the envelope theorem—

$$
\begin{aligned}
V_e(e^t, h^t) &= \mu^{t+1} H_e(e^t, h^t) \\
V_h(e^t, h^t) &= \mu^{t+1} H_h(e^t, h^t)
\end{aligned}
$$

that is to say

$$
\begin{aligned}
\frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} = \frac{p_t}{p_{t+1}} &= r_{t+1} \\
u_0(c_0^t, c_1^t) &= \beta \mu^{t+1} H_e(e^t, h^t) \\
\mu^t - (1+\tau_t) w_t u_0(c_0^t, c_1^t) &= n\beta \mu^{t+1} H_h(e^t, h^t)
\end{aligned}
$$

so that, necessarily, the household optimal choice is characterized by

$$\frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} = \frac{p_t}{p_{t+1}} = r_{t+1}$$

$$\frac{1}{H_e(e^t, h^t)} \frac{1}{\beta} \frac{u_0(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} = (1 + \tau_{t+1}) w_{t+1} + n \frac{H_h(e^{t+1}, h^{t+1})}{H_e(e^{t+1}, h^{t+1})}$$

$$c_0^t + k^t + \frac{M^t}{p_t} + ne^t = (1 + \tau_t) w_t h^t$$

$$c_1^t = r_{t+1} k^t + \frac{M^t}{p_{t+1}} + T_{t+1}$$

$$h^t = H(e^{t-1}, h^{t-1})$$

4bis. Market equilibria under policy

At a market equilibrium, capital an labor are remunerated by their marginal productivities so that

$$w_t = F_L(\frac{k^{t-1}}{n}, h^t)$$

$$r_{t+1} = F_K(\frac{k^t}{n}, h^{t+1})$$

and the allocation is feasible if

$$c_0^t + \frac{c_1^{t-1}}{n} + k^t + ne^t = F(\frac{k^{t-1}}{n}, h^t)$$

which requires

$$\frac{M^t}{p_t} = \frac{1}{n} \frac{M^{t-1}}{p_t}$$

that is to say

$$\frac{M^t}{M^{t+1}} = n$$

and

$$0 = \tau_t w_t h^t + \frac{1}{n} T_t$$

A market equilibrium is therefore any collection of sequences for $c_0^t, c_1^t, k^t, M^t, e^t, h^t$ and p_t such that, for all t,

$$\frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} = \frac{p_t}{p_{t+1}} = F_K(\frac{k^t}{n}, h^{t+1})$$

$$\frac{1}{H_e(e^t, h^t)} \frac{1}{\beta} \frac{u_0(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} = (1+\tau_t) F_L(\frac{k^t}{n}, h^{t+1}) + n \frac{H_h(e^{t+1}, h^{t+1})}{H_e(e^{t+1}, h^{t+1})}$$

$$c_0^t + k^t + \frac{M^t}{p_t} + e^t = (1+\tau_t) F_L(\frac{k^{t-1}}{n}, h^t) h^t$$

$$c_1^t = F_K(\frac{k^t}{n}, h^{t+1}) k^t + \frac{M^t}{p_{t+1}} + T_{t+1}$$

$$h^t = H(e^{t-1}, h^{t-1})$$

$$n = \frac{M^t}{M^{t+1}}$$

$$0 = \tau_t F_L(\frac{k^{t-1}}{n}, h^t) h^t + \frac{1}{n} T_t$$

A **market steady state** is a profile c_0, c_1, k, m, e, h such that —under policy τ, T—

$$\frac{u_0(c_0, c_1)}{u_1(c_0, c_1)} = n = F_K(\frac{k}{n}, h)$$

$$\frac{\frac{1}{\beta} - n H_h(e, h)}{H_e(e, h)} = (1+\tau) F_L(\frac{k}{n}, h)$$

$$c_0 + k + m + ne = (1+\tau) F_L(\frac{k}{n}, h) h$$

$$c_1 = F_K(\frac{k}{n}, h) k + nm + T$$

$$h = H(e, h)$$

$$0 = \tau F_L(\frac{k}{n}, h) h + \frac{1}{n} T$$

Proposition 1. *The policy τ, T decentralizes the planner's steady state iff labor income is subsidized at a rate*

$$\tau = \frac{1}{H_e(e, h) F_L(\frac{k}{n}, h)} \left(\frac{1}{\beta} - n \right) > 0$$

through a second-period lump-sum tax

$$T = -\frac{n}{H_e(e,h)}\left(\frac{1}{\beta} - n\right)h < 0$$

References

TBW ...