# **Taxation and Aggregate Price Stickiness**

## ZHIYONG AN

Division of Enterprise Risk Management Fannie Mae Email: anzy2008@gmail.com

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## Abstract

We study the impact of taxation on aggregate price stickiness in New Keynesian economics. We show that taxation increases aggregate price stickiness. Moreover, we show that the magnitude of the impact is first-order. Our results imply that taxes act as automatic destabilizers on the supply side, which is in sharp contrast with the traditional role of automatic stabilizers played by taxes on the demand side.

Key words: Automatic Stabilizers; New Keynesian Economics; Price Stickiness; Taxation

JEL classification: E; H

#### I. INTRODUCTION

Although price stickiness is central to Keynesian models, in most such models it has no solid microeconomic foundation. Thus construction of microeconomic foundations for price stickiness is a priority for New Keynesian economists. In order to meet this challenge, New Keynesian economists have put forward two parallel ideas, namely, small menu costs by Mankiw (1985) and near-rationality by Akerlof and Yellen (1985).<sup>1</sup>

The papers in this literature share three common features. First, they assume that all the firms are homogeneous in the sense that they have the same price-adjustment barrier.<sup>2</sup> Second, they show that a second-order "small" price-adjustment barrier for an individual firm to adjust its price can cause changes in money supply to have first-order "large" effect on real economic variables. Finally, the parameter  $\beta$ , which is the fraction of the firms that keep their prices unchanged following a money supply shock, is exogenous. In the initial equilibrium of their models, each firm sets its price to maximize profit. Then, they introduce a money supply shock  $\varepsilon$  into their models.<sup>3</sup> Following the money supply shock, they assume that  $\beta$  fraction of the firms change their price to maximize profit. They either assume a general  $\beta$  between zero and one (Akerlof and Yellen, 1985) or assume  $\beta$  is equal to one (Mankiw, 1985; Blanchard and Kiyotaki, 1987; and Ball and Romer, 1989, 1990, and 1991) in their models. In a word,  $\beta$  is exogenous in their models.

<sup>&</sup>lt;sup>1</sup> The following studies in this literature (e.g., Blanchard and Kiyotaki, 1987; and Ball and Romer, 1989, 1990, and 1991), in general, expand on the near-rationality model by Akerlof and Yellen (1985). The key difference is that they derive their results from basic optimization assumptions so that explicit welfare calculations are allowed.

 $<sup>^{2}</sup>$  By the definition of small menu costs and near-rationality, it is obvious that they are equivalent routes to the same place. Therefore, for the convenience of exposition, we give them a unified terminology in this paper, i.e., price-adjustment barrier.

<sup>&</sup>lt;sup>3</sup> More specifically,  $\mathcal{E}$  is the fraction change in the money supply.

An (2009) first endogenizes  $\beta$  in the near-rationality model by Akerlof and Yellen (1985) by assuming that the firms are heterogeneous in the sense that they have different priceadjustment barriers. Because  $\beta$  can be considered as an indicator of aggregate price stickiness by definition, he further characterizes the behavior of aggregate price stickiness by studying the properties of the endogenized  $\beta$ . He obtains three key results: (1)  $\lim_{\epsilon \to 0} \beta(\varepsilon) = 1$ ; (2)  $\frac{d\beta}{d\varepsilon}\Big|_{\varepsilon=0} = 0$ ; and (3) the possibility of multiple equilibrium values of  $\beta$ . The first two results imply that prices are not only sticky, but price stickiness is very significant for small money supply shocks in a well-defined sense. The last result is due to the so-called "strategic complementarity" (Cooper and John, 1988). This result is important because it implies the possibility of coordination failures are not completely competing paradigms to explain economic fluctuations, but can be compatible with each other.

In this paper, we study the impact of taxation on aggregate price stickiness in New Keynesian economics. We achieve this by introducing corporate profit taxation into An (2009). We obtain two key results. First, we show that corporate profit taxation increases aggregate price stickiness. Moreover, we show that the magnitude of the impact is first-order.

The traditional Keynesian economics focuses entirely on the demand side and reaches the conclusion that taxes serve as automatic stabilizers on the demand side (e.g., Auerbach and Feenberg, 2000; and Auerbach, 2009). In sharp contrast, we focus solely on the supply side and reach the conclusion that taxes contribute to aggregate price stickiness, which implies that taxes act as automatic destabilizers on the supply side. Therefore, the net impact of taxes on economic

fluctuations is theoretically ambiguous because in reality, it is obviously a mixture of the supply side effect stressed in this paper and the traditional demand side effect.

Our work might be related with Kleven and Kreiner (2003), but with three key differences. First, they still assume that all the firms are homogeneous in the sense that they have the same price-adjustment barrier, while we assume that the firms are heterogeneous in the sense that they have different price-adjustment barriers. Second, they still assume that  $\beta$  is exogenous,<sup>4</sup> while we have endogenized  $\beta$  by following An (2009). Finally, they concentrate on the impact of taxation on an individual firm's profit loss, while we directly focus on the impact of taxation on aggregate price stickiness (i.e.,  $\beta$ ). Overall, we thus view our work as an advancement of their contribution.

The remainder of the paper is organized as follows. Section II presents the model. Section III illustrates our work using an example. Finally, Section IV briefly concludes the paper.

#### II. MODEL

#### II.1. A Review of Akerlof and Yellen (1985) and An (2009)

Akerlof and Yellen (1985) assume a monopolistically competitive economy with a fixed number of identical firms in their model. Each firm's sales depend on the level of real aggregate demand and the firm's own price relative to the aggregate price level.

In the initial equilibrium, each firm sets its price to maximize profit, under the assumption that a change in its own price has no effect on the prices charged by rivals or on the aggregate price level. That is, each firm is assumed to be a Bertrand maximizer.

<sup>&</sup>lt;sup>4</sup> More specifically, they assume that  $\beta$  is equal to one.

Then, Akerlof and Yellen introduce a money supply shock into their model. More specifically, they assume that the money supply changes by a fraction  $\varepsilon$ . Following the money supply shock, they assume a fraction  $\beta$  of the firms do not change their prices, while the remaining  $(1 - \beta)$  of the firms change their prices to maximize profit.

If a firm does not change its price following the money supply shock, it will incur a profit loss that is a function of both  $\varepsilon$  and  $\beta$ . We denote this loss function as  $L(\varepsilon, \beta)$ . Akerlof and Yellen show that  $L(\varepsilon, \beta)$  is only second-order with respect to  $\varepsilon$ , i.e.,  $\frac{\partial L(\varepsilon, \beta)}{\partial \varepsilon}\Big|_{\varepsilon=0} = 0$ . In this sense, if an individual firm does not change its price following the money supply shock, its

behavior is suboptimal, but still near-rational.

In order to endogenize  $\beta$ , An (2009) keeps all the assumptions made by Akerlof and Yellen, except making one change. That is, he assumes that the firms are no longer homogeneous, but heterogeneous in the sense that they have different price-adjustment barriers. In other words, he assumes that there is a distribution of price-adjustment barriers among the firms; and moreover, the distribution is common knowledge among them.

In more detail, he assumes that each firm has a positive price-adjustment barrier  $c_i > 0$ , where *i* is the firm index. The price-adjustment barriers for all the firms ( $\{c_i\}$ ) follow a certain distribution, which is common knowledge among all the firms. For notational convenience, let's use *F* to denote the cumulative distribution function (CDF) of the price-adjustment barriers. He assumes that *F* is first-order differentiable and strictly increasing. Because  $c_i > 0$  for each firm *i*, he has F(0) = 0. Because *F* is first-order differentiable and strictly increasing, he has F' > 0and F'(0, ...) > 0, where F' is the first-order derivative of *F*. After analyzing the pricing decision process of each firm following a money supply shock and its aggregate outcome, he reaches the equilibrium equation as follows:

$$1 - \beta = F(L(\varepsilon, \beta)).$$
<sup>(1)</sup>

Equation (1) gives us the equilibrium value of  $\beta$ . By Equation (1), he further shows that:

(1) 
$$\lim_{\varepsilon \to 0} \beta(\varepsilon) = 1$$
; (2)  $\frac{d\beta}{d\varepsilon}\Big|_{\varepsilon=0} = 0$ ; and (3) the possibility of multiple equilibrium values of  $\beta$ .

#### **II.2. Introduce Taxation into An (2009)**

Now, we introduce corporate profit taxation into An (2009). For notational convenience, let's use t to denote the corporate profit tax rate.

Because the profit loss of an individual firm resulting from keeping its price unchanged following a money supply change also depends on t, we augment the profit loss function from  $L(\varepsilon,\beta)$  to  $L(\varepsilon,\beta,t)$ . In addition, because corporate profit taxation implies that the government

shares the profit loss, we have 
$$\frac{\partial L(\varepsilon, \beta, t)}{\partial t} < 0$$
.

Thus, corresponding to Equation (1), we could write the equilibrium equation for our extended model as follows:

$$1 - \beta = F(L(\varepsilon, \beta, t)).$$
<sup>(2)</sup>

Equation (2) gives us the equilibrium value of  $\beta$  in our extended model.

By Equation (2), it is straightforward to verify that the three properties of  $\beta$  obtained by An (2009) still hold in this extended model.

As  $\frac{\partial L(\varepsilon, \beta, t)}{\partial t} < 0$ , F' > 0, and  $F'(0_+) > 0$ , it is straightforward to prove that

 $\frac{\partial \beta(\varepsilon,t)}{\partial t} > 0$  by Equation (2), which has two important implications. First, corporate profit taxation increases aggregate prices stickiness. Moreover, the magnitude of the impact of corporate profit taxation on aggregate price stickiness is first-order.

The above two new results suggest that taxes act as automatic destabilizers. In sharp contrast, the traditional Keynesian economics show that taxes act as automatic stabilizers (e.g., Auerbach and Feenberg, 2000; and Auerbach, 2009). How to reconcile the difference? The difference is due to that we focus solely on the supply side, while the traditional Keynesian economics concentrates entirely on the demand side. In reality, the net effect of taxes on economic fluctuations is obviously a mixture of both the supply side effect emphasized by this paper and the traditional demand side effect, and is thus theoretically ambiguous.

## III. EXAMPLE

In this section, we use an example to illustrate our model. This example is adapted by introducing corporate profit taxation into the one in An (2009).

Let's consider a monopolist with a constant cost curve and a linear demand curve. Suppose the constant cost is 0 and demand is  $q = m - p + \overline{p}$ , where *m* is the money supply, *p* is the price of the product of the individual firm, and  $\overline{p}$  is the aggregate price level. In the initial equilibrium, each firm is setting its price to maximize its own profit, taking the aggregate price level as given. Each individual firm's own price has negligible effect on the aggregate price level. Suppose the corporate profit tax rate is *t*. Now, we introduce a money supply shock. Following the money supply shock, the new demand curve is  $q = m(1 + \varepsilon) - p + \overline{p}$ . We assume the price-adjustment barriers follow a uniform distribution u[0, A], which is common knowledge among all the firms.

Following the money supply shock, if a fraction  $\beta$  of the firms keep their price unchanged and if the monopolist decides to keep his original optimal price unchanged as well rather than charge the new optimal price, he will lose:

$$L = (1 - t)m^2(1 - x)^2,$$

where  $x = \frac{p_m}{m}$  and  $p_m$  is the new optimal price, *L* is the loss function, and *x* satisfies Equation (3):

$$(1+\varepsilon) - 2x + x^{(1-\beta)} = 0.$$
(3)

The derivation of the loss function follows the same procedures as those in An (2009). The details of how to derive the loss function are available in Appendix 1.

Therefore, if  $\beta = 1$ , then  $x = 1 + \frac{\varepsilon}{2}$  by Equation (3) and we get the minimum loss  $L_{\min} = 0.25(1-t)m^2\varepsilon^2$ . If  $\beta = 0$ , then  $x = 1 + \varepsilon$  by Equation (3) and we get the maximum loss  $L_{\max} = (1-t)m^2\varepsilon^2$ . Thus, if  $\beta \in [0, 1]$ , then  $x = 1 + k(\beta)\varepsilon$ , and we get the general loss function

$$L(\varepsilon,\beta,t) = (1-t)m^{2}(1-x)^{2} = (1-t)m^{2}k(\beta)^{2}\varepsilon^{2},$$
(4)

where  $k(\beta) \in [0.5, 1]$ ;  $\frac{dk(\beta)}{d\beta} < 0$ , i.e.,  $k(\beta)$  is strictly decreasing in  $\beta$ ; and for each  $\beta \in [0, 1]$ ,

there is a unique x that satisfies Equation (3). The uniqueness of x for each  $\beta$  can also be shown graphically by drawing the intersection of function  $f(x) = 2x - (1 + \varepsilon)$  and function  $g(x) = x^{(1-\beta)}$ . By Equation (4), it is straightforward to check that  $\frac{\partial L(\varepsilon, \beta, t)}{\partial t} < 0$ .

Because  $1 - \beta = F(L)$  and the price-adjustment barriers follow the uniform distribution u[0, A], i.e.,  $F(y) = \frac{y}{A}$  for  $\forall y \in [0, A]$ ,  $\beta$  must satisfy:

$$\beta = 1 - \frac{(1-t)m^2k(\beta)^2\varepsilon^2}{A}.$$
(5)

By Equation (5), it is straightforward to prove that  $\frac{\partial \beta(\varepsilon, t)}{\partial t} > 0$ .

## **IV.** CONCLUSION

In this paper, we study the impact of taxation on aggregate price stickiness in New Keynesian economics. We show that taxation contributes to aggregate price stickiness. Moreover, we show that the magnitude of the impact is first-order. Our results imply that taxes act as automatic destabilizers on the supply side, which is in sharp contrast with the idea of traditional Keynesian economics that taxes act as automatic stabilizers on the demand side. Taken together, we argue that the net effect of taxes on economic fluctuations is theoretically ambiguous in reality.

## **APPENDIX 1.**

In the initial equilibrium, each firm is taking the aggregate price as given and setting its price to maximize its own profit. Essentially, they are solving the following maximization problem:  $\max_{\{p\}} (1-t) p(m-p+\overline{p})$ . The first order condition for this optimization problem is:

$$m - 2p + \overline{p} = 0. \tag{A1}$$

As each firm is charging the same price, we have

$$p = p \,. \tag{A2}$$

By Equations (A1) and (A2), we have p = p = m.

Now, we introduce a money supply shock, i.e., money supply increases from m to  $m(1+\varepsilon)$ . Following this change in money supply, we assume  $\beta$  fraction of the firms keep their original optimal price unchanged, i.e., their price is still m. However, the remaining  $(1-\beta)$  fraction of the firms change their price and charge the new optimal price. Essentially, they take the new aggregate price as given and solve the following maximization problem:  $\max_{\{p_m\}} (1-t) p_m (m(1+\varepsilon) - p_m + \overline{p}_{new}).$  The first order condition for this optimization problem is:

$$m(1+\varepsilon) - 2p_m + \overline{p}_{new} = 0.$$
(A3)

As  $\beta$  fraction of the firms charge *m* and the remaining  $(1-\beta)$  fraction of the firms charge  $p_m$ , by the definition of aggregate price we have:

$$\overline{p}_{new} = m^{\beta} p_m^{1-\beta}. \tag{A4}$$

Plug Equation (A4) into Equation (A3), we have:

$$m(1+\varepsilon) - 2p_m + m^\beta p_m^{1-\beta} = 0.$$
(A5)

If we define  $x = \frac{p_m}{m}$ , we can rewrite Equation (A5) as:

$$(1+\varepsilon) - 2x + x^{(1-\beta)} = 0.$$
 (A6)

Now, we can write down the profit loss function

$$L(\varepsilon,\beta,t) = (1-t)p_m \left[m(1+\varepsilon) - p_m + \overline{p}_{new}\right] - (1-t)m \left[m(1+\varepsilon) - m + \overline{p}_{new}\right].$$
(A7)

Plug Equation (A4) into Equation (A7), and take advantage of Equation (A5), we have:

$$L(\varepsilon,\beta,t) = (1-t)(m-p_m)^2.$$
(A8)

As 
$$x = \frac{p_m}{m}$$
, we can rewrite Equation (A8) as  $L(\varepsilon, \beta, t) = (1-t)m^2(1-x)^2$ , where x

satisfies Equation (A6).

Thus we have derived the profit loss function in the example.

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