Long term care policy with nonlinear strategic bequests[*]

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Abstract

We study the design of long term care (LTC) policy when children differ in the degree of altruism, which determines their cost of providing informal care. Parents do not observe their children's altruism, but they can commit to a bequests rule specifying bequests conditional on the level of informal care. Unlike in the traditional strategic bequest model parents cannot extract all the surplus from the transfer for care exchange with their children. Instead, they use a nonlinear bequest rule to screen for the children's degree of altruism. In the laissez-faire, the help provided by less altruistic children is distorted downwards in order to minimize the rent of altruistic ones. Social LTC insurance affects these distortions and the distribution of rents between parents and children. The policy is designed to maximize a weighted sum of parents' and children's utility.

The optimal *uniform* public LTC insurance depends on the attitude towards risk of parents. Under DARA (decreasing absolute risk aversion) preferences, public LTC insurance exacerbates the distortion of informal care. Consequently, the optimal public LTC coverage provides less than full insurance. The opposite is true under IARA (increasing absolute risk aversion) preferences.

A *nonuniform* policy, which conditions LTC benefits on bequests provides full insurance even for the risk of having children with a low degree of altruism. The level of informal care provided by less altruistic children is distorted and the direction of the distortion depends on children's weight in the welfare function. The level of informal care always increases in the children's' welfare weight, irrespective of the parents' degree of risk aversion.

JEL classification: H2, H5.

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1   Introduction

Old age dependency and the need for Long-Term-Care (LTC) it brings about represents a major societal challenge in most developed countries. Due to population ageing the number of dependent elderly with cognitive and physical diseases will increase dramatically during the decades to come; see Cremer *et al.* (2012). Dependency represents a significant financial risk of which only a small part is typically covered by social insurance. Private insurance markets are also thin. Instead, individuals rely on their savings or on informal care provided by family members. Currently the family is the main provider and informal care represents roughly 2/3 of total care (Norton 2002). Informal provision has no direct bearing on public finances, but whether this solidarity is sustainable at its current level is an important question. Sources of concerns are numerous. The drastic change in family values, the increasing number of childless households, the mobility of children, the increasing labor participation of women are as many factors explaining why the number of dependent elderly who cannot count on family solidarity, at least not for the full amount of care they need, is increasing. Furthermore, it is not clear if the important role played by informal care is desirable. Its real cost are often "hidden". In particular, it may indeed impose a significant burden on the caregivers which is both financial and psychological.

In a nutshell, the current situation is inefficient as it leaves some elderly without proper care and often imposes a considerable burden on caregivers. This market failure creates a potential role of public intervention through social LTC provision or insurance. However, the public LTC policy will interact with informal care and most probably involve some crowding out. The latter is often considered as a major problem in the literature but in reality its impact is mixed. It is negative for the parents but positive for caregivers.

Informal care can be motivated by some form of altruism, result from implicit exchange between generations or be "imposed" by social norms. Knowing the foundation

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of informal care is very important to see how family assistance will react to the emergence of private or public scheme of LTC insurance. In this paper we consider a setting where intergenerational exchanges are based on a care vs. bequest (or gift) exchange. Children differ in their level of altruism towards dependent parents which in turn determines their cost of providing informal care. Our framework is inspired by the strategic bequest approach, but it differs from the conventional model in a crucial way because we assume that parent's do not observe their children's' cost of providing informal care.[1] Like in the conventional model we assume that parents can commit to a bequest rule specifying bequests conditional on the level of informal care provided. However, because of the asymmetry of information this does no longer allow them to extract the full surplus generated by the exchange from their children. Even though parents can use nonlinear rules to screen for the children's degree of altruism they will have to leave a positive "rent" to some of the children.

In the *laissez-faire*, the help provided by less altruistic children is distorted downwards in order to minimize the rent enjoyed by the most altruistic ones. Parents are not insured against the dependency risk nor of course can they buy insurance against the risk that their children have a low degree of altruism.

We then introduce social LTC into this setting. It is designed to maximize a weighted sum of parent's an children's utilities. In other words we explicitly account for the wellbeing of caregivers.

In the first part of the paper we consider a uniform social LTC policy. It consists in providing a given level of LTC to *all* dependent individuals; the policy is financed by a uniform lump-sum tax. We show that this policy may affect these distortions and affect the distribution of rents between parents and children. The optimal policy then involves a tradeoff between providing insurance to parents and enhancing the utility of the caregivers. In the absence of informal care, the social LTC insurance should fully

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[1]Bernheim *et al.* (1985)

insure the risk of dependence. With informal care and strategic bequests, we show that the optimal public LTC insurance depends on the attitude towards risk of parents. Under DARA (decreasing absolute risk aversion) preferences, public LTC insurance exacerbates the distortion of informal care. Better insurance coverage makes dependent parents less reliant on care from children with a relatively low level of altruism, so that distorting down informal care is not too costly. Consequently, under DARA, the optimal public LTC coverage provides less than full insurance. The opposite is true under IARA (increasing absolute risk aversion) preferences, in which case the government should provide more than full insurance against the risk of dependence, in order to minimize distortions. The main lesson that emerges from this section is that the design of LTC policies crucially depends on intergenerational interactions, particularly when bequests (or gifts) for care transactions are involved.

In the second part of the paper consider nonlinear policies where bequests are publicly observable and LTC benefits can be conditioned on bequests (or gifts). The LTC policy can then screen for the children's degree of altruism even when the level of informal care is observable only to parents. The underlying problem presents methodological challenges because we have to deal with a "nested" principal-agent problem. While the policy can screen for children's cost of care, this is done only indirectly via the parents. These do not observe their child's degree of altruism either but since they observe informal care, they have superior information.

We show that, while with a uniform policy the results crucially depend on parents' risk aversion, this is no longer true when the policy is restricted by informational considerations only and can be non uniform. In that case the available policy instrument are sufficiently powerful to ensure that parents are always fully insured, even against the risk of having less altruistic children. And since they are fully insured, risk aversion no longer matters. Even more strikingly, the tradeoff between the provision of insurance to parents and the concern for the welfare of the caregivers which drives the results for

a uniform policy is not longer relevant when nonuniform policies are considered.

The paper is structured as follows. In Section 2 we present the model. In Section 3 on uniform LTC policies and characterize the optimal public insurance scheme when parents have full information and under asymmetric information. In Section 4 we analyze non-linear policies. We conclude in Section 5.

2 Model

Consider a generation of identical parents. When old they face the risk of being dependent with probability π, while they are independent and healthy with probability $(1 - \pi)$. When young they earn a given labor income of $w\overline{T}$ of which they save s. They have preferences over consumption when young, $c \geq 0$, consumption when old and healthy, $d \geq 0$, and consumption, including LTC services, when old and dependent, $m \geq 0$. Their preferences are quasilinear in consumption when young. Risk aversion is introduced through the concavity of second period state dependent utilities. Their expected utility is given by

$$EU = w\overline{T} - s + (1 - \pi) U(s) + \pi E[H(m)], \tag{1}$$

with $m = s + a - \tau(a)$, where $a \geq 0$ is informal care provided by children, while $\tau(a)$ is a transfer (bequest or gift) from parents to children. We assume that this transfer can be conditioned on informal care and assume that parents can commit to this "bequest rule". This is in line with the "strategic bequest" literature. However, our model differs from the traditional literature on exchange based intergenerational transfers in that we assume that parents may not perfectly observe their child's preferences. In other words, we assume that children differ in their degree of altruism which in turn determines their cost of providing care. This heterogeneity is represented by a parameter β which is not publicly observable including to parents. We can think about this parameter as representing the child's degree of altruism, to which the cost of providing care is

inversely related. Assume that β is distributed over $\left\{\underline{\beta}, \overline{\beta}\right\}$. In other words, there are only two levels. The low one $\underline{\beta}$ occurs with probability $\lambda \in]0, 1[$, while the high level $\overline{\beta}$ occurs with probability $(1 - \lambda)$.

Children's cost of providing care a to their parent is given by $v(a, \beta)$, with $v_a > 0$, $v_\beta < 0$, $v_{aa} > 0$, $v_{a\beta} < 0$, where subscripts denote partial derivatives. This cost is increasing and convex in the level of informal care. It *decreases* in β which amounts to saying that $\overline{\beta}$ is the more altruistic child for whom it is less costly to provide care. Furthermore $v_{a\beta} < 0$ implies that the *marginal* cost of informal care also decreases with β.

The children's utility from helping their parents in case of dependence is

$$U_c = c_c - v(a, \beta) \geq 0, \tag{2}$$

where $c_c = \tau(a)$, that is the transfer from their parents.

Children choose a to maximize U_c

$$\tau'(a) = v_a(a, \beta), \tag{3}$$

and the solution to this problem is denoted $a(\beta)$. Observe that a also depends on $\tau(\cdot)$, exactly like labor supply depends on the tax function in a Mirrleesian-type optimal income tax problem.

Anticipating their child's' behavior but ignoring their degree of altruism β, parents choose s and $\tau(a)$ to maximize utility given by

$$EU = w\overline{T} - s + (1 - \pi) U(s) + \pi E_\beta \left[H(s + a(\beta) - \tau(a(\beta)))\right]. \tag{4}$$

To solve this problem we consider the equivalent mechanism design problem where parents parents choose $a(\beta)$ and $\tau(\beta)$ to maximize

$$EU = w\overline{T} - s + (1 - \pi) U(s) + \pi E_\beta \left[H(s + a(\beta) - \tau(\beta)\right] \tag{5}$$

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subject to the relevant participation constraints, as well as the incentive constraints, which come about when the parents do not observe β. These conditions will be described below.

Within this framework, we study the public provision of LTC benefits financed by a lump-sum tax on parents first-period consumption. This policy may be supplemented by a taxation (or subsidization) of the downward intergenerational transfer τ. The policy is determined to maximize social welfare which is given by a weighted sum of parents and children's expected utilities. Parents' weight is normalized to one, while children's utility is weighted by $\gamma \geq 0$.

The timing is as follows. The LTC policy is decided upon in stage 1, before parents and children make their decisions. In stage 2, parents choose their level of savings s and commit to the bequest rule $\tau(a)$. In stage 3, the dependence status of the parents is realized and children choose a according to (3).

While we concentrate on the asymmetric information case, we start by considering the full information benchmark, that is the solution parents can achieve when they observe their child's degree of altruism. We also characterize the optimal LTC policy for this benchmark.

3 Uniform LTC benefit

In this section, we restrict the policy to a uniform g financed by a lump-sum tax. In other words we consider a universal public provision (or subsidization) of LTC. In practice, the government may not be able to observe bequests. Even when the government could observe bequests it may be politically unfeasible to condition g on τ. In Section 4, we consider the case where g can be conditioned on bequests, so that the government to screen for different levels of β. We first charactherize the optimal policy when parents can observe their children's level of altruism. We then turn to the asymmetric information case.

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3.1 Full information benchmark

Since β takes only two values, it is convenient to introduce the following notation

$$\tau(\overline{\beta}) = \overline{\tau} \qquad ; \qquad \tau(\underline{\beta}) = \underline{\tau},$$
$$a(\overline{\beta}) = \overline{a} \qquad ; \qquad a(\underline{\beta}) = \underline{a},$$
$$m(\overline{\beta}) = \overline{m} \qquad ; \qquad \tau(\underline{\beta}) = \underline{m}.$$

Parent's choose s *ex ante,* that is before the state of health and β is realized and both are observed by parents.

With these notation the parent's problem can be written as

$$\max_{\overline{a},\underline{a},\overline{\tau},\underline{\tau},s} \quad w\overline{T} - \pi g - s + (1 - \pi) U(s) + \pi \left[\lambda H(s + \underline{a} - \underline{\tau} + g) + (1 - \lambda) H(s + \overline{a} - \overline{\tau} + g) \right] \tag{6}$$

$$\text{s.t.} \qquad \underline{\tau} - v(\underline{a}, \underline{\beta}) \geq 0, \tag{7}$$

$$\overline{\tau} - v(\overline{a}, \overline{\beta}) \geq 0. \tag{8}$$

Conditions (7) and (8) represent the children's participation constraint. While children take the bequest rule $\tau(a)$ as given, they have the option not to make any exchanges with their parents: no care and no transfer. In this case their utility is normalized to an exogenously given constant which without loss of generality is normalized at zero.

Under full information, the parent can extract all the surplus, and both participation constraints will be binding.[2] Substituting for $\underline{\tau}$ and $\overline{\tau}$ from (7) and (8) set equal to zero the parent's problem can then be rewritten as

$$\max_{\overline{a},\underline{a},s} \quad P^f = w\overline{T} - \pi g - s + (1 - \pi) U(s) + \pi \left[\lambda H(s + \underline{a} - v(\underline{a}, \underline{\beta}) + g) \right.$$
$$\left. + (1 - \lambda) H(s + \overline{a} - v(\overline{a}, \overline{\beta}) + g) \right]$$

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[2] When a participation constraint is not binding parents can increase the corresponding a and/or decrease τ, thereby increasing their expected utility.

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The first-order conditions (FOC) with respect to the remaining choice variable are given by

$$\frac{\partial P^f}{\partial \overline{a}} = (1 - \lambda)H'(\overline{m})[1 - v_a(\overline{a}, \overline{\beta})] = 0, \tag{9}$$

$$\frac{\partial P^f}{\partial \underline{a}} = \lambda H'(\underline{m})[1 - v_a(\underline{a}, \underline{\beta})] = 0, \tag{10}$$

$$\frac{\partial P^f}{\partial s} = -1 + (1 - \pi)U'(s) + \pi[\lambda H'(\underline{m}) + (1 - \lambda)H'(\overline{m})] = 0. \tag{11}$$

The first two conditions imply

$$1 = v_a(\overline{a}, \overline{\beta}) = v_a(\underline{a}, \underline{\beta}), \tag{12}$$

which is quite intuitive. Since parents have to pay exactly the cost (to their child) of the informal care they receive, they equalize marginal costs to marginal benefits, which are equal to one. Note surprisingly, this implies $\overline{a} > \underline{a}$ and $\overline{m} > \underline{m}$: more altruistic children provide more informal care and their parents enjoy a larger amount of total care, m, in case of dependency. To decentralize this solution parents must then choose τ so that $\tau'(\overline{a}) = \tau'(\underline{a}) = 1$.

Observe that neither $\overline{a}$ nor $\underline{a}$ depend on g. Consequently, a uniform g can never achieve full insurance for β; the parent with a more altruistic child will always be better off.

Within this framework, we study the public provision of a uniform LTC benefit g financed by a lump-sum tax on parents first period consumption πg. Since the parents have full information, the children's utility will be zero no matter what. Consequently the weight of children in social welfare γ is of no relevance at this point and we can just as well neglect this term in the welfare function. The government's problem is then given by

$$\max_{g} \quad G^f = w\overline{T} - \pi g - s + (1 - \pi)U(s) + \pi\left[\lambda H(s + \underline{a} - v(\underline{a}, \underline{\beta}) + g)\right.$$
$$\left. + (1 - \lambda)H(s + \overline{a} - v(\overline{a}, \overline{\beta}) + g)\right], \tag{13}$$

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where s, $\underline{a}$ and $\overline{a}$ are given by the solution to the parent's problem given g.

Using the envelope theorem (for the induced effect on s) and recalling the the levels of a do not depend on g we have[3]

$$\frac{\partial G^f}{\partial g} = -\pi + \pi[\lambda H'(\underline{m}) + (1 - \lambda)H'(\overline{m})] = 0. \tag{14}$$

Not surprisingly this condition equalizes marginal costs and benefits of g. Using the parent's FOC for s implies

$$U'(s) = [\lambda H'(\underline{m}) + (1 - \lambda)H'(\overline{m})] = E_\beta[H'(m)] = 1. \tag{15}$$

This condition states that the three possible uses of (first-period) income, direct consumption c, deferred consumption s and LTC insurance, g must have the same expected utility.

Observe that with *ex ante* identical individuals the distinction between private and public insurance is of no relevance. We have implicitly assumed that there is no private insurance market for LTC. If, on the other hand a fair private insurance market were available, individuals would spontaneously buy the socially optimal level of insurance. In that case no government intervention is required.

3.2 Asymmetric information

Parents' problem

Except for the policy design, the previous section has presented a rather standard strategic bequest model. Parents have all the bargaining power and have full information about their child's degree of altruism. We now turn to the more interesting case where parents do not know their child's type. We then have to add two incentive constraints to the parents' problem. The objective function does not change and the participation constraints continue of course to apply. The parents' problem can then be stated as

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[3]We assume that the second order condition holds.

follows

$$\max_{\overline{a},\underline{a},\overline{\tau},\underline{\tau},s} \quad P^{as} = w\overline{T} - \pi g - s + (1 - \pi)U(s) + \pi[\lambda H(s + \underline{a} - \underline{\tau} + g)$$

$$+ (1 - \lambda)H(s + \overline{a} - \overline{\tau} + g)] \tag{16}$$

$$\text{s.t.} \quad \underline{\tau} - v(\underline{a}, \underline{\beta}) \geq 0, \tag{17}$$

$$\overline{\tau} - v(\overline{a}, \overline{\beta}) \geq 0, \tag{18}$$

$$\overline{\tau} - v(\overline{a}, \overline{\beta}) \geq \underline{\tau} - v(\underline{a}, \overline{\beta}), \tag{19}$$

$$\underline{\tau} - v(\underline{a}, \underline{\beta}) \geq \overline{\tau} - v(\overline{a}, \underline{\beta}). \tag{20}$$

This is a rather standard mechanism design problem and one can easily show that one obtains the "usual" pattern of binding incentive and participation constraints. To be precise, the participation constraint of the high cost (low altruism) type child will be binding so that

$$\underline{\tau} = v(\underline{a}, \underline{\beta}). \tag{21}$$

Further, the incentive constraint of $\overline{\beta}$, the low cost, high altruism type, (19), is binding. Using (21) this condition can be written as

$$\overline{\tau} = v(\overline{a}, \overline{\beta}) + [v(\underline{a}, \underline{\beta}) - v(\underline{a}, \overline{\beta})], \tag{22}$$

where the term in brackets on the RHS represents the "rent" of $\overline{\beta}$. In words, because more altruistic children can always mimick the less altruistic ones, at a lower cost, they have to receive a transfer which exceeds the cost of the care they provide. This is particularly interesting from our perspective because it implies that the utility of the caregivers is no longer exogenously given. Some of them now receive a transfers which puts them above their reservation utility level and this transfer may at least potentially depend on the LTC policy.

Substituting for the transfers from (21) and (22) into the objective function, the

parent's problem can then be rewritten as

$$\max_{\overline{a},\underline{a},s} \quad P^{as} = w\overline{T} - \pi g - s + (1-\pi)U(s) + \pi\left[\lambda H(s + \underline{a} - v(\underline{a},\underline{\beta}) + g)\right. \tag{23}$$

$$\left. + (1-\lambda)H(s + \overline{a} - v(\overline{a},\overline{\beta}) - v(\underline{a},\underline{\beta}) + v(\underline{a},\overline{\beta}) + g)\right]. \tag{24}$$

The first-order conditions are given by

$$\frac{\partial P^{as}}{\partial \overline{a}} = (1-\lambda)H'(\overline{m})[1 - v_a(\overline{a},\overline{\beta})] = 0, \tag{25}$$

$$\frac{\partial P^{as}}{\partial \underline{a}} = \pi\left\{\lambda H'(\underline{m})[1 - v_a(\underline{a},\underline{\beta})] - (1-\lambda)H'(\overline{m})[v_a(\underline{a},\underline{\beta}) - v_a(\underline{a},\overline{\beta})]\right\} = 0, \tag{26}$$

$$\frac{\partial P^{as}}{\partial s} = -1 + (1-\pi)U'(s) + \pi[\lambda H'(\underline{m}) + (1-\lambda)H'(\overline{m})] = 0. \tag{27}$$

From equation (25) we obtain $1 = v_a(\overline{a},\overline{\beta})$, which is the full information condition for $\overline{\beta}$, the low cost type; see equation (12). This is the traditional no distortion at the top result that, given the quasi linearity, not just applies to the *rule*, but also to the actual level of care, $\overline{a}$, which is the same as in the full information solution. Consequently, it continues to be independent of g.

Turning to $\underline{a}$, rearranging (26) yields

$$[1 - v_a(\underline{a},\underline{\beta})] = \frac{(1-\lambda)H'(\overline{m})}{\lambda H'(\underline{m})}[v_a(\underline{a},\underline{\beta}) - v_a(\underline{a},\overline{\beta})] > 0, \tag{28}$$

where we have used $v_{a\beta} < 0$, which implies $\Delta v_a = v_a(\underline{a},\underline{\beta}) - v_a(\underline{a},\overline{\beta}) > 0$. Consequently $v_a(\underline{a},\underline{\beta}) < 1$ and we have a downward distortion for $\underline{a}$. Intuitively, $\Delta v_a > 0$ accounts for the fact that the rents of the low cost type increase with $\underline{a}$. The downward distortion allows parents to mitigate these rents. Equation (28) also implies that $\underline{a}$ depends on g, as well as the bequests left to both types, according to (21) and (22).

Optimal LTC policy when parents do not observe β

We continue to consider a linear policy consisting of a uniform g financed by a lump-sum tax. The previous subsection has shown that since the parents no longer have full information, some of the children's, namely the more altruistic will now how a positive

utility level. Moreover, their utility is affected by the LTC policy via its impact on the parents' optimization problem. Consequently, the weight of children in social welfare γ will now be relevant. When this weight is positive, LTC policy will now strike a balance between providing insurance coverage to parents and the concern for the wellbeing of the caregivers.

The government's problem is then given by

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\begin{aligned}
\max_g \quad G^{as} =& w\overline{T} - \pi g - s + (1-\pi)U(s) + \pi \left[\lambda H(s + \underline{a} - v(\underline{a}, \underline{\beta}) + g) \right. \\
& + (1-\lambda)H(s + \overline{a} - v(\overline{a}, \overline{\beta}) - [v(\underline{a}, \underline{\beta}) - v(\underline{a}, \overline{\beta})] + g)\big] \\
& + \gamma\pi(1-\lambda)[v(\underline{a}, \underline{\beta}) - v(\underline{a}, \overline{\beta})], \quad (29)
\end{aligned}
$$

where $\overline{a}, \underline{a}$ and s are determined by the solution to the parents' problem. Observe that parents' utility is expressed as the solution to the reformulated problem (24) which is an unconstrained optimization because the relevant IC and participation constraints have been substituted into the objective function.

Using the envelope theorem according to which we can neglect the derivatives of parents utility wrt. a's and s, the FOC is given by[4]

$$
\frac{\partial G^{as}}{\partial g} = -\pi + \pi[\lambda H'(\underline{m}) + (1-\lambda)H'(\overline{m})] + \pi(1-\lambda)\gamma[v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \overline{\beta})]\frac{\partial \underline{a},}{\partial g} = 0, \quad (30)
$$

Recall that the term $\Delta v_a = v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \overline{\beta})$ is positive. Observe that the children's' utility does not directly depend on s and that $\overline{a}$ does not depend on g, nor does the high costs child's utility which is always equal to the exogenous participation level (which we have normalized at zero). Consequently the only behavioral response on behalf of the parents that is relevant in (30) is $\partial \underline{a},/\partial g$ which measures how g affects the level of care provided by the more altruistic child. The sign of this expression will in turn determine how the level of care affects the caregivers utility.

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[4]We assume that the second order condition holds.

Consider first the case where caregivers utility is not included in social welfare, that is $\gamma = 0$. Then, using the parent's FOC, expression (30) can be written as

$$U'(s) = [\lambda H'(\underline{m}) + (1 - \lambda)H'(\overline{m})] = 1.$$

This is the same *rule* as under full information which was given by (15). In both case we have $U'(s) = 1$ so that the *level* of s is also the same. However, the levels of m will differ from the full information solution which in turn implies that the *level* of g will also in general be different, even though the rule is the same.

We use superscripts f and as to refer to the solutions of the full information and asymmetric information problems, specified by (13) and (29) respectively. When $g = g^f$ we have $\lambda H'(\underline{m}) + (1 - \lambda)H'(\overline{m}) > 1$ because then $\underline{m}^{as} < \underline{m}^f$ (because $\underline{a}$ is distorted downward) and $\overline{m}^{as} < \overline{m}^f$ (because the low cost children now receive a positive rent); consequently we have $g^{as} > g^f$: the optimal level of LTC benefits from the parent's perspective is larger under asymmetric information than under full information. Intuitively, g is increased to compensate in part for the downward distortion (in a) that parents create to mitigate children's rents.

Let us now turn to the case where $\gamma > 0$, which includes the utilitarian solution with $\gamma = 1$. Then g is no longer solely determined to provide insurance to parents. The optimal LTC policy now also accounts for the impact of g on informal care and thus on children's utility (rents). Roughly speaking, when $\partial \underline{a}/\partial g > 0$ one can expect that the effect described for $\gamma = 0$ is reinforced by the effect of g on children's rents. Since rents increase in $\underline{a}$, increasing g increases rents. In this case we have $g^{as} > g^f$. However, when $\partial \underline{a}/\partial g < 0$, the two effects go in opposite directions. Either way this discussion shows that as soon as $\gamma > 0$ the results will crucially depend on the sign of $\partial \underline{a}/\partial g$.

We now turn to the study of this sign which requires a closer look at the comparative statics of the parent's problem under asymmetric information. The following lemma is established in Appendix A.1.

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Lemma 1 *When the parent's utility in case of dependence $H(m)$ exhibits IARA we have $\partial\underline{a}/\partial g > 0$; when $H(m)$ exhibits DARA we have $\partial\underline{a}/\partial g < 0$.*

This lemma shows that the way the level of care provided by the high cost (low altruism child) is affected by g via the parent's problem depends on the parents' attitude towards risk. Intuitively these results can be understood as follows. With DARA, as g increases, parents become *less* risk averse. Then, distorting downwards becomes less costly for parents and they prefer a lower level of $\underline{a}$. This is because reducing m in the bad state of nature becomes less costly. The case with IARA is exactly symmetrical. Note that empirically DARA appears to receive more support; see Friend and Blume (1975).

Using equation (30) and Lemma 1 we can study the effect of γ on $g^{as}(\gamma)$. For instance, we can compare the utilitarian level ($\gamma = 1$), $g^{as}(1)$ with $g^{as}(0)$, the level achieved when children are not accounted for in SWF. With DARA we know from Lemma 1 that $\underline{a}$ decreases as g increases which in turn implies that the utility of the more altruistic child decreases; recall that $\Delta v_a > 0$. Consequently equation (30) evaluated at $g^{as}(0)$ is negative so that $g^{as}(1) < g^{as}(0)$. Under IARA these effects are reversed and we obtain $g^{as}(1) > g^{as}(0)$. This result also goes through for intermediate levels of γ. Totally differentiating (30) yields

$$\frac{\partial g}{\partial \gamma} = -\frac{\pi(1-\lambda)[v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \overline{\beta})]\frac{\partial \underline{a},}{\partial g}}{SOC} \tag{31}$$

which has the same sign as $\partial\underline{a}/\partial g > 0$. Accordingly, under DARA g decreases as γ increases while it increases in γ under IARA.

Using the FOC for the parents (27), equation (30) implies that, whenever $\gamma > 0$

$$U'(s) < 1 < E(H'(m)) \qquad \text{if} \qquad \partial\underline{a}/\partial g < 0,$$

while we have

$$U'(s) > 1 > E(H'(m)) \qquad \text{if} \qquad \partial\underline{a}/\partial g > 0.$$

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Intuitively, the utility of dependent parents is distorted down with respect to the full information case under DARA. In this case, providing full insurance against the risk of dependence would push parents to cut the utility of altruistic children (by distorting down a̲). Then, from the social planner perspective, there is a trade-off between insurance and children's utility, leading to less than full insurance. Accordingly, parents have an incentive to save more and this increases their consumption if healthy. Under IARA these effects are reversed.

The results obtained in this section are summarized in the following proposition.

Proposition 1 *Consider the case where children's altruism is not observable including to parents and where policy is restricted to a LTC benefit g financed by a lump sum tax. Informal care is observable only by parents. We have*

(i) The risk of having children with a low degree of altruism is not fully insured.

(ii) If children's utility has no weight in social welfare, the optimal LTC insurance scheme implies full insurance against dependence. This is achieved by a uniform benefit that is larger than in the full information case.

(iii)If the weight of children in social welfare is strictly positive, two cases may arise.

(a) When the parent's utility in case of dependence H(m) exhibits IARA a the optimal LTC insurance scheme implies more than full insurance against dependence; the optimal insurance benefit g is higher than under full information. Furthermore, the transfer increases with the weight of children in social welfare.

(b) When H(m) exhibits DARA, the optimal LTC insurance scheme implies less than full insurance against dependence. The transfer decreases with the weight of children in social welfare.

4   Nonlinear policies

We now consider nonlinear policies where bequests are publicly observable and g can be conditioned on τ. The LTC policy can then screen for β. We continue to assume

that a is observable only to parents. The underlying problem presents methodological challenges because we have to deal with a "nested" principal-agent problem.[5] While the policy can screen for β this is done only indirectly via the parents. These do not observe their child's β either but since they observe informal care, they have superior information.

We proceed exactly like in the previous section. We start with the full information solution, then consider the case where only parents have full information. Finally we turn to the interesting case where neither the parents nor the government can observe the children's type β. Parents observe a but the government does not. We will assume throughout the section that the government cannot make any *direct* transfer to children.

4.1   Full information solution

In this section, we assume that both parents and the government have full information. The government sets $\overline{g}, \underline{g}, \overline{\tau}, \underline{\tau}$, anticipating the choices of the parents. Parents set a such that $\tau - v(a, \beta) = 0$; we can thus define $a^f(\tau, \beta)$ as the solution to this equation. We have

$$\frac{\partial a^f}{\partial \tau} = \frac{1}{v_a}$$

$$\frac{\partial a^f}{\partial \beta} = -\frac{v_\beta}{v_a} > 0$$

Parents also choose s *ex ante* to maximize utility

The government now maximizes

$$\max_{\overline{g}, \underline{g}, \overline{\tau}, \underline{\tau}} \quad G^{ff} = w\overline{T} - \pi(\lambda \underline{g} + (1-\lambda)\overline{g}) - s + (1-\pi) U(s)$$

$$+ \pi \left[\lambda H(s + a^f(\underline{\tau}, \underline{\beta}) - \underline{\tau} + \underline{g}) + (1-\lambda)H(s + a^f(\overline{\tau}, \overline{\beta}) - \overline{\tau} + \overline{g}) \right]$$

---

[5] The problem considered by Guesnerie and Laffont (1978) has a similar structure. They nonlinear taxation of a monopolist who itself used nonlinear pricing.

The FOCs are

$$\frac{\partial G^{ff}}{\partial \overline{g}} = -\pi(1-\lambda) + \pi(1-\lambda)H'(\overline{m}) = 0, \tag{32}$$

$$\frac{\partial G^{ff}}{\partial \underline{g}} = -\pi\lambda + \pi\lambda H'(\underline{m}) = 0, \tag{33}$$

$$\frac{\partial G^{ff}}{\partial \overline{\tau}} = \pi(1-\lambda)H'(\overline{m})\left[\frac{\partial a^f(\overline{\tau}, \overline{\beta})}{\partial \tau} - 1\right] = 0, \tag{34}$$

$$\frac{\partial G^{ff}}{\partial \underline{\tau}} = \pi\lambda H'(\underline{m})\left[\frac{\partial a^f(\underline{\tau}, \underline{\beta})}{\partial \tau} - 1\right] = 0, \tag{35}$$

and parents' choose s so that

$$\frac{\partial G^{ff}}{\partial s} = -1 + (1-\pi)U'(s) + \pi[\lambda H'(\underline{m}) + (1-\lambda)H'(\overline{m})] = 0. \tag{36}$$

Combining these equations yields

$$H'(\overline{m}) = H'(\underline{m}) = U(s) = 1$$

and

$$v_a(\overline{a}, \overline{\beta}) = v_a(\underline{a}, \underline{\beta}) = 1$$

These expressions have a simple interpretation. With full information, a nonuniform LTC policy can provide full insurance not only against the risk of dependence, but also against the risk of having children with a low degree of altruism (and a high cost of care). Informal care a is set at the efficient levels for each child.

4.2 Only parents have full information

Since the government no longer observes β, we have to add the relevant *IC* constraints. The problem is then given by

$$
\max_{\overline{g},\underline{g},\overline{\tau},\underline{\tau}} \quad G^{af} = w\overline{T} - \pi(\lambda\underline{g} + (1-\lambda)\overline{g}) - s + (1-\pi)U(s)
$$
$$
+ \pi\left[\lambda H(s + a^f(\underline{\tau},\underline{\beta}) - \underline{\tau} + \underline{g}) + (1-\lambda)H(s + a^f(\overline{\tau},\overline{\beta}) - \overline{\tau} + \overline{g})\right]
$$
$$
(37)
$$

$$
\text{s.t.} \qquad a^f(\underline{\tau},\underline{\beta}) - \underline{\tau} + \underline{g} \geq a^f(\overline{\tau},\underline{\beta}) - \overline{\tau} + \overline{g} \tag{38}
$$

$$
a^f(\overline{\tau},\overline{\beta}) - \overline{\tau} + \overline{g} \geq a^f(\underline{\tau},\overline{\beta}) - \underline{\tau} + \underline{g} \tag{39}
$$

We shall assume that (39), the constraint from the low cost type to the high cost type is binding. This constraint is effectively violated at the full information solution characterized in Subsection 4.1. To see this recall that this solution implies

$$
a^f(\overline{\tau},\overline{\beta}) - \overline{\tau} + \overline{g} = a^f(\underline{\tau},\underline{\beta}) - \underline{\tau} + \underline{g} < a^f(\underline{\tau},\overline{\beta}) - \underline{\tau} + \underline{g},
$$

so that

$$
\overline{g} < \underline{g} + \overline{\tau} - \underline{\tau} + a^f(\underline{\tau},\overline{\beta}) - a^f(\overline{\tau},\overline{\beta}),
$$

which violates condition (39). Substituting the incentive constraint into G^{af} the government's problem can then be rewritten as

$$
\max_{\underline{g},\overline{\tau},\underline{\tau}} \quad G^{af} = w\overline{T} - \pi\left(\underline{g} + (1-\lambda)\left(\overline{\tau} - \underline{\tau} + a^f(\underline{\tau},\overline{\beta}) - a^f(\overline{\tau},\overline{\beta})\right)\right) - s + (1-\pi)U(s)
$$
$$
+ \pi\lambda H(s + a^f(\underline{\tau},\underline{\beta}) - \underline{\tau} + \underline{g}) + \pi(1-\lambda)H(s + a^f(\underline{\tau},\overline{\beta}) - \underline{\tau} + \underline{g}). \tag{40}
$$

The FOCs are given by

$$
\frac{\partial G^{af}}{\partial \underline{g}} = -\pi + \pi\left[\lambda H'(\underline{m}) + (1-\lambda)H'(\overline{m})\right] = 0, \tag{41}
$$

$$
\frac{\partial G^{af}}{\partial \overline{\tau}} = \pi(1-\lambda)\left[\frac{\partial a^f(\overline{\tau},\overline{\beta})}{\partial \tau} - 1\right] = 0, \tag{42}
$$

$$
\frac{\partial G^{af}}{\partial \underline{\tau}} = \pi\lambda H'(\underline{m})\left[\frac{\partial a^f(\underline{\tau},\underline{\beta})}{\partial \tau} - 1\right] + \pi(1-\lambda)\left(H'(\overline{m}) - 1\right)\left[\frac{\partial a^f(\underline{\tau},\overline{\beta})}{\partial \tau} - 1\right] = 0. \tag{43}
$$

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Recall that $a^f(\underline{\tau}, \overline{\beta}) > a^f(\underline{\tau}, \underline{\beta})$, which implies $\overline{m} = s + a^f(\underline{\tau}, \overline{\beta}) - \underline{\tau} + \underline{g} > \underline{m} = s + a^f(\underline{\tau}, \underline{\beta}) - \underline{\tau} + \underline{g}$. Hence, condition (41) implies that $H'(\overline{m}) < 1$. Furthermore using our assumption on a^f along with (35) which defines $\underline{\tau}^{ff}$ we have

$$\frac{\partial a^f(\underline{\tau}^{ff}, \overline{\beta})}{\partial \tau} > \frac{\partial a^f(\underline{\tau}^{ff}, \underline{\beta})}{\partial \tau} = 1,$$

so that at $\underline{\tau}^{ff}$ the first term on the RHS of (43) is zero while the second term is negative, which in turn implies $\underline{\tau}^{af} < \underline{\tau}^{ff}$ (from the SOC). This is not surprising. In order to relax the IC constraint, the optimal policy distorts $\underline{\tau}$ downwards which leads to a downward distortion on $\underline{a}$. Conversely, $\overline{\tau}$ and $\overline{a}$ are not distorted; condition (42) is identical to its full information counterpart (34).

Combining (41) with (36), the first-order condition for parents' saving , yields

$$U'(s) = \left[\lambda H'(\underline{m}) + (1 - \lambda)H'(\overline{m})\right] = 1$$

As in the case with uniform transfers, the optimal LTC policy implies full insurance against dependence but under asymmetric information, it is not possible to provide insurance against the risk of having children with a low degree of altruism.

4.3   Asymmetric information

Consider now the case where neither the parents nor the government can observe children's types. The government proposes a menu $\left((\overline{\tau}, \overline{g}), (\underline{\tau}, \underline{g})\right)$. The only choice left to parents is then to fix the level of a associated with each option. As long as $\overline{\tau} > \underline{\tau}$, parents set these levels of informal care, such that the participation constraint of $\underline{\beta}$ and the incentive constraint of $\overline{\beta}$ are satisfied. Formally, the levels of a are then defined by

$$\underline{\tau} = v\left(\underline{a}^{as}, \underline{\beta}\right)$$

and

$$\overline{\tau} = v\left(\overline{a}^{as}, \overline{\beta}\right) + v\left(\underline{a}^{as}, \beta\right) - v\left(\underline{a}^{as}, \overline{\beta}\right).$$

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The optimal (nonuniform) LTC policy is then determined by solving the following problem

$$\max_{\overline{g},\underline{g},\overline{\tau},\underline{\tau}} \quad G^{aa} = w\overline{T} - \pi(\lambda\underline{g} + (1-\lambda)\overline{g}) - s + (1-\pi)U(s)$$

$$+ \pi\left[\lambda H(s + \underline{a}^{as} - \underline{\tau} + \underline{g}) + (1-\lambda)H(s + \overline{a}^{as} - \overline{\tau} + \overline{g})\right] \quad (44)$$

$$\lambda H(s + \underline{a}^{as} - \underline{\tau} + \underline{g}) + (1-\lambda)H(s + \overline{a}^{as} - \overline{\tau} + \overline{g}) \geq H(s + \underline{a}^{as} - \underline{\tau} + \underline{g}) \quad (45)$$

$$\lambda H(s + \underline{a}^{as} - \underline{\tau} + \underline{g}) + (1-\lambda)H(s + \overline{a}^{as} - \overline{\tau} + \overline{g}) \geq H(s + a^{as}(\overline{\tau}, \underline{\beta}) - \overline{\tau} + \overline{g}) \quad (46)$$

$$\underline{\tau} = v\left(\underline{a}^{as}, \underline{\beta}\right) \quad (47)$$

$$\overline{\tau} = v\left(\overline{a}^{as}, \overline{\beta}\right) + v\left(\underline{a}^{as}, \underline{\beta}\right) - v\left(\underline{a}^{as}, \overline{\beta}\right) \quad (48)$$

$$\overline{\tau} < \underline{\tau} \quad (49)$$

The IC constraints of the parents are now given by equation (45) and (46). Since parents have no private information, the social planner has just has to ensure that they will not prefer a policy schedule over the other. In other words (45) implies that parents should implement a menu $((\overline{\tau}, \overline{a}^{as}), (\underline{\tau}, \underline{a}^{as}))$, rather than preferring a pooling contract $(\underline{\tau}, \underline{a}^{as})$. The condition holds as long as $\overline{a}^{as} - \overline{\tau} + \overline{g} \geq \underline{a}^{as} - \underline{\tau} + \underline{g}$. Condition (46) implies that parents do not prefer a pooling contract $(\underline{\tau}, a^{as}(\overline{\tau}, \underline{\beta}))$.

To solve the government problem, we will first assume that (45) and (46) hold, and that $\overline{\tau} > \underline{\tau}$. We will verify *ex post* that the solution to the unconstrained problem fulfils these constraints. Substituting (47) and (48) in the objective function, the problem can be rewritten as

$$\max_{\overline{g},\underline{g},\overline{a},\underline{a}} \quad G^{aa} = w\overline{T} - \pi(\lambda\underline{g} + (1-\lambda)\overline{g}) - s + (1-\pi)U(s)$$

$$+ \pi\lambda H(s + \underline{a} - v\left(\underline{a}, \underline{\beta}\right) + \underline{g})$$

$$+ \pi(1-\lambda)H(s + \overline{a} - v\left(\overline{a}, \overline{\beta}\right) - v\left(\underline{a}, \underline{\beta}\right) + v\left(\underline{a}, \overline{\beta}\right) + \overline{g}) \quad (50)$$

The FOCs are given by

$$\frac{\partial G^{aa}}{\partial \overline{g}} = -\pi(1-\lambda) + \pi(1-\lambda)H'(\overline{m}) = 0, \tag{51}$$

$$\frac{\partial G^{aa}}{\partial \underline{g}} = -\pi\lambda + \pi\lambda H'(\underline{m}) = 0, \tag{52}$$

$$\frac{\partial G^{aa}}{\partial \overline{a}} = \pi(1-\lambda)H'(\overline{m})\left[1 - v_a(\overline{a}, \overline{\beta})\right] = 0, \tag{53}$$

$$\frac{\partial G^{aa}}{\partial \underline{a}} = \pi\lambda H'(\underline{m})\left[1 - v_a(\underline{a}, \underline{\beta})\right] - \pi(1-\lambda)\left[H'(\overline{m}) - \gamma\right]\left[v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \overline{\beta})\right] = 0. \tag{54}$$

Conditions (51) and (52), combined with the parents' FOC with respect to savings imply that

$$U'(s) = H'(\underline{m}) = H'(\overline{m}) = 1$$

This shows that under asymmetric information between parents and children, the optimal non uniform LTC insurance scheme provides full insurance not only against the risk of dependence, but also against uncertain altruism. This is in stark contrast with the result obtained with a uniform policy where full insurance could not be achieved.

Informal care is set at its first best level for the more altruistic children, as it is shown in (53). Conversely, the optimal level of informal care provided by less altruistic children is distorted. Combining with (52), (54) can be rewritten as

$$\pi\lambda H'(\underline{m})\left[1 - v_a(\underline{a}, \underline{\beta})\right] - \pi(1-\lambda)(1-\gamma)\left[v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \overline{\beta})\right] = 0.$$

Since v_a decreases in β, the direction of the distortion depends on the sign of $(1-\gamma)$. If children have a lower weight than parents in the social welfare function ($\gamma < 1$), then $\underline{a}$ is distorted down. However, this distortion will be smaller than the one that parents would implement in the absence of a LTC policy. If parents and children have the same weight, ($\gamma = 1$), there will be no distortion of the informal care. Finally, if children have a higher weight than parents, there will be an upward distortion of informal care. The

level of informal care $\underline{a}$ always increases in γ irrespective of the degree of risk aversion of the parents.[6]

Proposition 2 *Consider the case where children's altruism is not observable including to parents and where LTC benefits g can be conditioned on the transfer τ paid by parents to children in exchange for informal care. Informal care is observable only to parents. We have*

(i) The risk of having children with a low degree of altruism is fully insured.

(ii) Informal care is set at its first best level for the more altruistic children.

(iii) The level of informal care provided by less altruistic children is distorted and the direction of the distortion depends on children's weight in the welfare function. It has the same sign as $(\gamma - 1)$ so that a downward (upward) distortion occurs when the weight of the children is lower (higher) than one.

(iv) The level of informal care $\underline{a}$ always increases in γ irrespective of the parents' degree of risk aversion.

Note that while with a uniform policy the results crucially depend on parents' risk aversion this is no longer true when the policy is restricted by informational considerations only, and can be non uniform. In that case the available policy instruments are sufficiently powerful to ensure that parents are always fully insured, even against the risk of having less altruistic children. And since they are fully insured, risk aversion no longer matters. Even more strikingly, the tradeoff between the provision of insurance to parents and the concern for the welfare of the caregivers which drives the results for a uniform policy is no longer relevant under nonuniform policies.

---

[6]It remains to be checked that the solution characterized above satisfy the parents' IC constraints. Observe that the solution to the government problem implies $\underline{m} = \overline{m}$. Then, conditions (45) and (46) are satisfied. Furthermore, as long as $\gamma \leq 1$, $\underline{a} < \overline{a}$, which implies that $\underline{\tau} < \overline{\tau}$.

5   Conclusion

We study the design of long term care (LTC) policy when informal care from children to dependent parents is due to a bequest motive. Parents do not observe their children's altruism, but they can commit to a bequests rule specifying bequests conditional on the level of informal care.

We show that social LTC insurance affects these distortions and the distribution of rents between parents and children. The social welfare function is a weighted sum of parents' and children's utility.

The optimal *uniform* public LTC insurance depends on the attitude towards risk of parents. Under DARA (decreasing absolute risk aversion) preferences, public LTC insurance exacerbates the distortion of informal care. Consequently, the optimal public LTC coverage provides less than full insurance. The opposite is true under IARA (increasing absolute risk aversion) preferences.

A *nonuniform* policy, which conditions LTC benefits on bequests provides full insurance even for the risk of having children with a low degree of altruism. The level of informal care provided by less altruistic children is distorted and the direction of the distortion depends on children's weight in the welfare function. The level of informal care always increases in the children's' welfare weight, irrespective of the parents' degree of risk aversion.

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Appendix

A.1 Proof of Lemma 1

Since $\bar{a}$ is independent off g, we can use the FOCs (26) and (27) to study the comparative statics of the solution.

Using subscripts to denote partial derivatives. Define

$$H = \begin{bmatrix} P_{\underline{a}\underline{a}}^{as} & P_{s\underline{a}}^{as} \\ P_{\underline{a}s}^{as} & P_{ss}^{as} \end{bmatrix},$$

and

$$D = \begin{bmatrix} -P_{\underline{a}g}^{as} \\ -P_{sg}^{as} \end{bmatrix},$$

where

$$P_{s\underline{a}}^{as} = P_{\underline{a}s}^{as} = \pi[\lambda H''(\underline{m})(1 - v(\underline{a},\underline{\beta}) - (1-\lambda)H''(\overline{m})(v_a(\underline{a},\underline{\beta}) - v_a(\underline{a},\overline{\beta}))] = \pi A, \quad \text{(A.1)}$$

$$P_{ss}^{as} = (1-\pi)U''(s) + \pi[\lambda H''(\underline{m}) + (1-\lambda)H''(\overline{m})], \quad \text{(A.2)}$$

$$P_{\underline{a}g}^{as} = \pi\left\{\lambda H''(\underline{m})[1 - v_a(\underline{a},\underline{\beta})] - (1-\lambda)H''(\overline{m})[v_a(\underline{a},\underline{\beta}) - v_a(\underline{a},\overline{\beta})]\right\} = \pi A, \quad \text{(A.3)}$$

$$P_{sg}^{as} = \pi[\lambda H''(\underline{m}) + (1-\lambda)H''(\overline{m})], \quad \text{(A.4)}$$

and where

$$A = \lambda H''(\underline{m})[1 - v_a(\underline{a},\underline{\beta})] - (1-\lambda)H''(\overline{m})[v_a(\underline{a},\underline{\beta}) - v_a(\underline{a},\overline{\beta})] \quad \text{(A.5)}$$

Using Cramer's rule we obtain

$$\frac{\partial\underline{a}}{\partial g} = \frac{\begin{vmatrix} -P_{\underline{a}g}^{as} & P_{s\underline{a}}^{as} \\ -P_{sg}^{as} & P_{ss}^{as} \end{vmatrix}}{|H|},$$

where $|H| > 0$ from the SOC.

Substituting from (A.1)–(A.4), evaluating the determinant and simplifying successively yields

$$\text{sgn}\left(\frac{\partial\underline{a}}{\partial g}\right) = \text{sgn}\left(\begin{vmatrix} -A\pi & A\pi \\ \pi[\lambda H''(\underline{m}) + (1-\lambda)H''(\overline{m})] & (1-\pi)U''(s) + \pi[\lambda H''(\underline{m}) + (1-\lambda)H''(\overline{m})] \end{vmatrix}\right)$$

$$= \text{sgn}\left(-\pi A[(1-\pi)u''(s)]\right) = \text{sgn}(A)$$

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To sum up we have to study the sign of A defined by (A.5). Substituting from (28) and rearranging yields

$$A = (1 - \lambda)\Delta v_A \left[H''(\underline{m})\frac{H'(\overline{m})}{H'(\underline{m})} - H''(\overline{m}) \right].$$

Because $\Delta v_A > 0$, this expression has the same sign as the term in brackets on the RHS.

Consequently we have

$$A > 0 \qquad \Longleftrightarrow \qquad \frac{H''(\underline{m})}{H'(\underline{m})} > \frac{H''(\overline{m})}{H'(\overline{m})}$$

$$\Longleftrightarrow \qquad -\frac{H''(\underline{m})}{H'(\underline{m})} < -\frac{H''(\overline{m})}{H'(\overline{m})}.$$

Since $\underline{m} < \overline{m}$ this is true under IARA (Increasing Absolute Risk Aversion), while DARA (Decreasing Absolute Risk Aversion) yields $A < 0$.