

# Two-Sided Capital Taxes

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## Abstract

I show that in economies with risk sharing frictions and aggregate demand externalities, home capital gains should be taxed at a higher rate than foreign capital gains. The differential tax rate, serving a novel Pigovian role, corrects distorted portfolios and promotes macroeconomic stability. This result provides a stark contrast to standard arguments claiming that capital income from all sources should be taxed at the same rate (e.g. Gordon and Hines (2002)). Moreover, I argue that a constant differential capital tax can complement a generally time-varying capital control policy. Finally, I show that tax changes can take the form of beggar-thy-neighbor policies even when the amount of capital in each country is fixed and a country cannot manipulate its terms of trade.

## Introduction

International capital tax arrangements have varied from pure source country-based taxation to double taxation of foreign gains.<sup>1</sup> Economists have traditionally viewed differential taxation of home and foreign capital gains suboptimal. For example, in their literature review on international taxation, Gordon and Hines (2002) write:

*"The results derived by Diamond and Mirrlees (1971) still imply that production will be efficient under an optimal tax system, as long as there are no relevant restrictions on the types of commodity taxes or factor taxes available. As a result,*

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<sup>1</sup>See for example Gordon and Hines (2002) and Gorter and de Mooij (2001).

*under such a "residence-based tax" on capital, residents should face the same tax rate on their return to savings regardless of the industries or countries in whose financial securities they invest."*

An implicit assumption behind such arguments is that the laissez-faire investment portfolios are efficient.<sup>2</sup> This paper rather argues that generally the equilibrium portfolios are distorted and a differential tax rate can correct for such inefficiencies in allocations.

The analysis is based on two key ingredients. First, risk sharing frictions such as information acquisition costs tilt portfolios toward domestic assets. The existence of such frictions is consistent with the well known equity home bias phenomenon and the lack of international risk sharing (Coeurdacier and Rey, 2013).<sup>3</sup>

The second ingredient is that nominal rigidities give rise to aggregate demand externalities. Here the planner values consumption increases in recessions more than the households and in booms less than the households. This implies positive public benefits from macroeconomic stabilization and risk sharing. Throughout most of this paper, I assume fixed exchange rates which makes dealing with asymmetric shocks using monetary policy difficult. However, the analysis is more generally valid in other contexts with aggregate demand externalities or similar public benefits from consumption smoothing.

The main results of the analysis are as follows. First, assuming no risk sharing frictions and symmetric countries, both home and foreign gains should be taxed at the same rate despite the aggregate demand externalities. While the equilibrium stock prices are generally incorrect, the equilibrium attains the maximal amount of risk sharing and the relative equilibrium stock positions are efficient. This result can be seen as an extension of the uniform taxation result discussed for example by Gordon (1986) and Gordon and Hines (2002). However, in the case of risk sharing frictions, home capital gains should be taxed at a higher rate than foreign capital gains. Here the differential tax rate corrects for the portfolio distortions and promotes macroeconomic stability with positive externalities.

There are also cases in which a higher tax rate for foreign asset generated income can be justified. Namely, this happens when risk sharing frictions are small but domestic assets provide good hedges for productivity shocks in the sectors with rigid prices. However, the model does not automatically

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<sup>2</sup>The analysis of Diamond and Mirrlees (1971) excludes externalities.

<sup>3</sup>See also Sihvonen (2016) for a related analysis

imply such hedging benefits for domestic assets, though they can be justified under some assumptions for the correlation structure between productivity shocks.

In the case of symmetric countries, optimal tax changes typically benefit both countries. However, asymmetries can lead to beggar-thy-neighbor type behavior in capital tax setting. This happens for example in the case when assets in the home country have higher expected return due to higher risk. Then both countries would benefit from reducing their position in the high return, high risk asset. Now assume the home country raises the tax rate on home assets. This results in domestic agents reducing their positions in the home asset and increasing their positions in the foreign asset. If the foreign country leaves taxes unchanged, its residents must make the opposite shifts in asset positions. Ultimately, they are left with a worse portfolio that is more tilted towards high return, high risk assets.

However, such concerns are mitigated when both countries choose taxes optimally. Generally, I show that an equilibrium exists also absent coordination or commitment.

This type of beggar-thy-neighbor policy based on manipulating the composition of households' investment portfolios is different from the terms-of-trade manipulation discussed in the capital control literature (Costinot and Werning, 2014). However, broadly we can see our results as complementing those of the capital control literature. Especially, this paper suggests that differential capital taxation can serve as a type of macroprudential policy. Here I argue that differential taxation can supplement other related policies such as capital controls.

I also consider a number of extensions of the model. First, I show that a differential capital tax serves an important role also in economies in which a uniform capital tax can distort savings decisions. Second, in the baseline model I assume a non-tradable sector with one period rigid prices. Assuming that all goods are tradable or that part of the firms can adjust prices each period can imply quantitatively smaller benefits from macroeconomic stabilization. This may reduce the optimal difference between the capital tax on home and foreign sources. Still, differential capital taxation generally plays a role even in such economies.

**Related Literature** This paper contributes to the vast literature on optimal capital taxation, especially the work on its role in open economies. Moreover, the paper bears implications to the literature on macroprudential policies and capital controls.

Two famous results make the case for a zero capital tax in closed economies. Atkinson and Stiglitz (1976) study optimal commodity and labor income taxation. They assume preferences that are separable in consumption and labor as well as the availability of non-linear taxes on labor. The authors find that the optimal tax arrangement can rely solely on labor taxes: differential commodity taxation results in distortions and is not optimal. In this framework, a capital tax can be interpreted as a differential tax between consuming now or at a later date. The theorem implies that both should be taxed at the same rate, hence a zero capital tax is optimal.

The other famous argument for a zero capital tax is the so called Chamley-Judd result. Judd (1985) considers optimal redistributive taxation in a deterministic model with workers and capitalists. He finds that in the long run the optimal capital tax rate is zero. At the steady-state any tax on capital investment leads to lower capital stock and wages that more than offsets the transfer received by the workers. A similar argument is given by Chamley (1986). Straub and Werning (2015) point out some technical problems with both papers that limit the scope of the Chamley-Judd result.

More closely, this paper relates to the literature on optimal taxation in open economies.<sup>4</sup> Gordon (1986) argues that in a small open economy, corporate tax rate is optimally set to zero. In his deterministic setting, capital investment is infinitely elastic to the home investment return. Any tax levied on corporations results in capital flight to other countries and is entirely borne by the workers. For further analysis on the effects of tax competition, see Razin and Sadka (1991) and Gordon (1992).

Governments might still find it optimal to tax regular savings. Gordon (1986) argues that different tax rates on capital gains from home and foreign sources would result in distortions in optimal portfolios: both should be taxed at the same rate. As mentioned, Gordon and Hines (2002) further note that different tax rates would contradict the classic Diamond and Mirrlees (1971) type arguments that production is efficient under the optimal tax system. This type of uniform taxation generally also implies that capital should be taxed under a residence- rather than source-based system.

Such results tend to implicitly rely on the assumption that investment portfolios are optimal absent distortionary taxation. In my model the equilibrium portfolios are generally inefficient and a differential tax rate can be used to correct for such distortions. Contrary to most of the literature on capital taxation that studies the tax arrangement resulting in the smallest distortions, this paper therefore argues that differential capital taxation

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<sup>4</sup>For a more comprehensive review, see Gordon and Hines (2002)

serves an important role as a Pigovian, i.e. corrective tax.

The idea of capital taxation as correcting for market failures is not entirely new. For example Aiyagari (1995) considers a model with consumption in private and public goods, where the agents face idiosyncratic labor shocks. The model features a type of pecuniary externality: a precautionary savings motive pushes the interest rate inefficiently low. The government optimally taxes capital income to raise the before tax interest rate to equal the rate of time preference. For additional arguments for a positive capital tax see e.g. Golosov et al. (2003) and Conesa et al. (2009).

Perhaps more closely, Naito (1999) argues that source-based capital taxation can be optimal when different types of workers are imperfect substitutes in production. For example a higher tax rate for capital gains from industries employing more skilled workers can shift the tax burden between unskilled and skilled workers and complement a non-linear income tax. However, to my knowledge no paper has suggested a role for differential taxation in correcting for distortions in allocations between home and foreign assets.

Here my paper is closer to the literature on capital controls and macroprudential policies. First, Costinot and Werning (2014) explain how a country can extract rents by using capital controls to manipulate its terms-of-trade. This finding echoes the beggar-thy-neighbor policies discussed in this paper, but is ultimately based on a different mechanism.

Farhi and Werning (2012) explain how a country can use capital controls as a type of stabilization tool to regulate fund inflows and outflows. Their model includes nominal rigidities that gives rise to similar aggregate demand externalities as in this paper. However, in their model the agents have only access to bonds and they focus on optimal policy from the perspective of a single small open economy. Schmitt-Grohe and Uribe (2013) also provide an analysis of optimal capital controls in a small open economy model with nominal rigidities. While the focus of these papers is different, I argue that capital control policies can more broadly complement a differential taxation policy discussed in this paper.

In a related paper, Farhi and Werning (2014) study optimal fiscal transfers in two models with nominal rigidities. The authors find that in some cases the constrained efficient solution can be implemented with portfolio taxes. A more general analysis of optimal macroprudential interventions in models with nominal rigidities is provided by Farhi and Werning (2016). However, it is hard to map the results of these papers into those of the capital taxation literature.

Finally, the analysis relates to the literature on risk sharing frictions and equity home bias. For a discussion of how costly foreign equity ownership

can translate into equity home bias, see Lewis (1999). In related work, Sihvonen (2016) notes that such risk sharing frictions can also be microfounded for example by information costs or heterogenous beliefs. For the equilibrium implications of costly foreign ownership, see for example Bhamra et al. (2014).

## 1 The Model

This section lays down the baseline model used in the analysis. Later I consider multiple extensions of the model introduced here.

Assume there are two countries: home (H) and foreign (F) with sizes  $\omega > 0$  and  $1 - \omega$ . Each country is populated by a unit measure of identical households. Furthermore assume there are two stocks, one for each country. Assume a mixed endowment-production economy in which the tradable good is given by a random endowment, but the non-traded good is produced in each country using labor as the sole input. It is instructive to think of the non-tradable good as a domestic service sector and the tradable good as industrial production. The endowment is distributed as dividends to stockholders and as labor income to residents. Most of the results extend to the case where the stock also represent claims to non-tradable profits.

The home agents can trade the home stock without further costs. However they receive only a fraction  $e^{-f}$  of the returns of the foreign stock. Later I analyze both the case of no frictions ( $f = 0$ ) and a case with frictions ( $f > 0$ ). For more elaborate microfoundations for such risk sharing frictions, see Sihvonen (2016).<sup>5</sup> The next section introduces capital taxes, which also modify the returns received by households.

In the following I will explicitly state the households', firms' and the planner's problem as well as the corresponding equilibrium conditions.

### 1.1 Households

The household preferences are

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<sup>5</sup>This type of simple stock market friction has been considered for example by Lewis (1999). The friction is similar to the iceberg cost model used in the trade literature (Krugman, 1991) in that part of the tradable good is effectively lost due to trade costs. Assuming that part of the cost is rebated back to agents would affect the results quantitatively but not qualitatively. In the related paper I also show that informational signals or biased beliefs which increase the expected return of the home stock can have both similar positive and normative implications than a simple cost that lowers the expected return of the foreign stock.

$$\sum_{t=0}^{\infty} \zeta_{i,t} U_i(c_{i,T,t}, c_{i,NT,t}, N_{i,t}), \quad i = H, F, \quad (1)$$

where  $c_{i,T,t}$  is tradables consumption,  $c_{i,NT,t}$  is non-tradables consumption and  $N_{i,t}$  is labor used in non-tradables production. The preferences are separable in consumption and labor

$$U_i(c_T, c_{NT}, N) = g_i(c_T, c_{NT}) - h_i(N). \quad (2)$$

Here  $g_i$  is a twice differentiable homothetic utility function with the partial derivatives  $g_{i,1} > 0$ ,  $g_{i,1,1} < 0$ ,  $g_{i,2} > 0$  and  $g_{i,2,2} < 0$  and  $h_i$  is a twice differentiable, strictly increasing and convex function  $h'_i > 0$ ,  $h''_i > 0$ . Specifically we later consider CRRA preferences over a CES aggregator. Then  $g(c_{NT}, c_T) = g(C) = \frac{1}{1-\gamma} C^{1-\gamma}$ <sup>6</sup>, where

$$C = \begin{cases} \left( a^{\frac{1}{\phi}} c_T^{\frac{\phi-1}{\phi}} + (1-a)^{\frac{1}{\phi}} c_{NT}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}, & \text{if } \phi \neq 1, \\ c_T^a c_{NT}^{1-a}, & \text{if } \phi = 1. \end{cases} \quad (3)$$

where  $0 < a < 1$ . Later we also use the disutility of labor function  $h(N) = \frac{1}{1+\sigma} N^{1+\sigma}$ .

$\zeta_{i,t}$  is an endogenous discount factor given by  $\zeta_{i,t+1} = \zeta_{i,t} \beta_i(c_{T,i,t})$ , where  $\beta : \mathbb{R}_+ \rightarrow [0, 1)$  is non-increasing. This is used to pin down steady-state wealth levels that are generally indeterminate absent any stationarity inducing device (see Schmitt-Grohe and Uribe (2003) and Rabitsch et al. (2015)). In the case of symmetric countries, the natural approximation point is one with the same wealth level in each country (zero net wealth). To simplify expressions in the case of symmetric countries we therefore set  $\beta(c_{T,i,t}) = \beta$ , where  $\beta$  is constant. In the case of asymmetric countries, we set  $\beta(c_{T,i,t}) = \beta_i c_{T,i,t}^{-\eta_i}$ , where  $\eta_i > 0$ .  $\beta(\cdot)$  could also be a function on an aggregator  $C$ .

The choice variables of each household in country  $i$  are:  $(c_{i,T,t}, c_{i,NT,t}, N_{i,t}, S_{ii,t}, S_{ij,t})$ . Here  $S_{ii,t}$  and  $S_{ij,t}$  represent country  $i$ 's holdings of country  $i$  and  $j$  equity respectively. The household budget constraint at any time period  $t$  is given by

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<sup>6</sup> $g(C) = \log(C)$ , for  $\gamma = 1$ .

$$p_{i,NT,t}c_{i,NT,t} + p_{i,T,t}c_{i,T,t} + p_{S_i,t}S_{ii,t+1} + p_{S_j,t}S_{ij,t+1} = (p_{S_i,t} + p_{i,T,t}d_{i,t})e^{-\tau_{ii}^S}S_{ii,t} + (p_{S_j,t} + p_{j,T,t}d_{j,t})e^{-f}e^{-\tau_{ij}^S}S_{ij,t} + l_{i,t}p_{i,T,t} + W_{i,t}N_{i,NT,t} + \Pi_{i,t} + T_{i,t}, \quad i = H, F, j = -i, \quad (4)$$

Here  $S_{ii,t}$  is country  $i$ 's holdings of its own equity. Moreover,  $W_{i,t}$  is the wage from the non-tradable sector. Furthermore  $\Pi_{i,t}$  represents profits from the non-tradable sector.  $e^{-\tau_{ii}^S}$  is a tax on domestic stock holdings.<sup>7</sup> Moreover,  $T_{i,t}$  represents government transfers to the households.

The intratemporal condition each period is

$$\frac{U_{NT,i,t}}{U_{T,i}} = \frac{p_{NT,t,i}}{p_{T,t,i}} \equiv p_{t,i} \quad i = H, F, \quad (5)$$

Because  $g$  is homothetic, there is a function  $\alpha(p)$  s.t.  $c_{NT} = \alpha(p)c_T$ . Specifically in the CES case we have  $\alpha(p) = \frac{a}{1-a}p^{-\phi}$ . The labor choice FOC is

$$-\frac{U_{N,i,t}}{U_{NT,i,t}} = \frac{W_{t,i}}{p_{NT,t,i}}, \quad i = H, F. \quad (6)$$

The Euler equation, written in terms of the traded good is

$$\mathbb{E} \left[ \frac{U_{T,i,t+1}}{U_{T,i,t}} \frac{p_{T,t,i}}{p_{T,t+1,i}} R_{ij} \right] = 1, \quad i = H, F \quad j = H, F. \quad (7)$$

Here

$$R_{ii,t} = \frac{p_{S_i,t+1} + d_{i,t+1}}{p_{S_i,t}} e^{-\tau_{ii}^S}, \quad i = H, F, \quad (8)$$

$$R_{ij,t} = \frac{p_{S_j,t+1} + d_{j,t+1}}{p_{S_j,t}} e^{-f} e^{-\tau_{ij}^S}, \quad i = H, F, j = -i. \quad (9)$$

## 1.2 Non-Tradables Producers, One Period Rigid Prices

The non-tradable good is produced by a competitive firm, which combines a continuum of varieties  $j \in [0, 1]$  using a CES technology. The non-tradables production is given by

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<sup>7</sup>Given this form for frictions and taxes, the budget constraint assumes positive stock positions. We mainly consider regions where this is true for both home and foreign stock-holdings though the results could be extended to cases where the positions can be negative.



$$Y_{NT,i,t} = \left( \int_0^1 Y_{NT,i,j}^{1-\frac{1}{\vartheta}} dj \right)^{\frac{1}{1-\frac{1}{\vartheta}}}, \quad (10)$$

where  $\vartheta > 1$  is the elasticity of substitution. Each variety  $j$  is produced by a monopolistic entrepreneur using the technology  $Y_{NT,i,t} = A_{NT,i,t}N_{i,t}$ .<sup>8</sup> The demand for variety  $j$  is given by  $c_{NT,i,t} \left( \frac{p_{NT,i,j,t}}{p_{NT,i,t}} \right)^{-\vartheta}$ , where  $p_{NT,i,t} = \left( \int_0^1 p_{NT,i,j,t}^{1-\vartheta} \right)^{\frac{1}{1-\vartheta}}$  is the price of the tradable good. As in Obstfeld and Rogoff (1995), we assume the price of each variety is set one period in advance. The problem of each entrepreneur is

$$\max_{\{p_{i,j,NT,t}(x^{t-1})\}_{t=1}^{\infty}} \mathbb{E}_0 \left[ \sum_{t=1}^{\infty} \Lambda_{i,t} \left( p_{NT,i,t} - \frac{W_{NT,i,t}(1 + \tau_{L,i})}{A_{NT,i,t}} \right) c_{NT,i,t} \left( \frac{p_{NT,i,j,t}}{p_{NT,i,t}} \right)^{-\vartheta} \right] \quad (11)$$

where  $\Lambda_{i,t}$  is a stochastic discount factor and  $s^t$  is a history of state variables.  $\tau_{L,i}$  is government labor subsidy. It is set so that the equilibrium is efficient at symmetric points, offsetting the monopolistic mark-up. This assumption is not necessary but simplifies some of the results. Overall, the price setting condition is not important for our results. When domestic non-tradable firms are fully owned by the domestic households, we have  $\Lambda_{i,t} = \beta^k \frac{U_{i,T,t+i}}{U_{i,T,t}} \frac{p_{i,T,t}}{p_{i,T,t+i}}$ .

**Monetary Policy** Similarly to Farhi and Werning (2014), we assume a monetary authority chooses the tradables price to maximize the sum of welfare in two countries.

$$\max_{\{p_{T,t}(\mathbf{x}^t)\}_{t=0}^{\infty}} \lambda_H V_H + \lambda_F V_F \quad (12)$$

where the consumption levels might in general depend on the tradables price.

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<sup>8</sup>I prove all results with more general production function  $\chi(N)$ ,  $\chi'() > 0$  and  $\chi''() \leq 0$

### 1.3 Alternative Assumptions for Prices

We also consider alternative pricing assumptions.

**Flexible Non-Tradables Price** Now let the monetary authority also choose  $p_{NT,H,t}$  and  $p_{NT,F,t}$  at period  $t$ . In practice such equilibria can be decentralized through the use of labor subsidy.

**Floating Exchange Rate** Now the non-tradables prices are set in advance by the firms. However, due to floating exchange rate, each country can have separate tradables prices.

### 1.4 Government

The government taxes the stock returns received by the households. Capital taxation is residence-based in the sense that gains of home households are taxed only in the home country and vice versa. Specifically, the country  $i$  government taxes the returns of the two stocks at rates  $(\tau_{ii}^S, \tau_{ij}^S)$ . We assume the government needs to satisfy a period by period budget constraint. Each government also uses a labor subsidy, collecting the necessary funds from households.

$$G_{i,t} + T_{i,t} = \tag{13}$$

$$(p_{Sj,t} + p_{j,T,t}d_{j,t})e^{-f}S_{ij,t}(1 - e^{-\tau_{ij}^S}) + (p_{Si,t} + p_{i,T,t}d_{i,t})e^{-f}(1 - e^{-\tau_{ii}^S})S_{ii,t} + W_{i,t}\tau_i^L \tag{14}$$

Here  $G_{i,t}$  represents government expenditures and  $T_{i,t}$  government transfers. In this paper we focus on the corrective role of taxation. Therefore, throughout most of this paper we for simplicity set  $G_{i,t} = 0$ . However, extending the results to the case with  $G_{i,t} > 0$  would be straightforward. Moreover, in some cases we consider an additional tax instrument: a tax on the tradable goods distributed as labor income. This modifies the government budget constraint in a straightforward way.

### 1.5 Equilibrium

The market clearing conditions are given by

**Goods Markets** The market clearing condition for the tradable good is given by

$$\begin{aligned} \omega c_{H,T,t} + (1 - \omega)c_{F,T,t} + \omega S_{HF} \left( d_{F,t} + \frac{p_{SH,t}}{p_{T,t}} \right) (1 - e^{-f}) + \\ (1 - \omega) S_{FH} \left( d_{H,t} + \frac{p_{SH,t}}{p_{T,t}} \right) (1 - e^{-f}) = y_{H,t} + y_{F,t}. \end{aligned} \quad (15)$$

Where  $y_{H,t}$  and  $y_{F,t}$  are the aggregate home and foreign tradables endowments. For the non-tradable good, we have

$$c_{NT,i,t} = A_{NT,i,t} N_{i,t} \quad i = H, F \quad (16)$$

**Labor Markets**

$$N_{i,j,t} = N_{i,j,t}^{demand} \quad j = H, F \quad (17)$$

**Stock Market** We normalize the stock supply to 1. Then the market clearing condition for equity becomes

$$\omega S_{ii,t} + (1 - \omega) S_{ij,t} = 1, \quad i = H, F, \quad j = -i, \quad (18)$$

Uncertainty is represented by the variables  $\mathbf{x}_t = (d_{H,t}, d_{F,t}, l_{H,t}, l_{F,t}, A_{NT,H,t}, A_{NT,F,t})$ . We first set  $A_{NT,t,H} = 1$ ,  $A_{NT,t,F} = 1$ , though consider the effects of such productivity shocks later. We assume  $\mathbf{x}_t$  follows

$$\log(\mathbf{x}_t) = \log(\bar{\mathbf{x}}) + \Delta(\log(\mathbf{x}_{t-1}) - \log(\bar{\mathbf{x}})) + \varepsilon_t \quad (19)$$

To guarantee stationarity we assume the roots of  $|I - \xi \Delta| = 0$  are greater than 1 in modulus.  $\varepsilon_t$  is assumed independently and identically distributed. For the approximations it is convenient to assume, as in Devereux and Sutherland (2010), that each component of  $\varepsilon_t$  is symmetrically distributed on the line  $[-\epsilon, \epsilon]$ .

The state variables can also be defined as  $Z_t = \{\mathbf{x}_t, S_{HH,t}, S_{HF,t}, P_{NT,H,t}, P_{NT,F,t}\}$ , giving the equilibrium a Markov structure. A competitive equilibrium is a sequence of goods prices  $\{\mathbf{p}(\mathbf{x}^t)\}_{t=0}^{\infty}$ , stock prices  $\{\mathbf{p}_S(\mathbf{x}^t)\}_{t=0}^{\infty}$ , consumption decisions  $\{\mathbf{c}(\mathbf{x}^t)\}_{t=0}^{\infty}$ , stock positions  $\{\mathbf{S}(\mathbf{x}^t)\}_{t=0}^{\infty}$ , labor supply decisions  $\{\mathbf{N}(\mathbf{x}^t)\}_{t=0}^{\infty}$  and government policies  $\{\tau^L(\mathbf{x}^t), \tau^S(\mathbf{x}^t)\}_{t=0}^{\infty}, \{\mathbf{T}(\mathbf{x}^t)\}_{t=0}^{\infty}$  such that

- Given prices, the consumption decisions, stock positions and labor supply decisions solve each household's problem characterized by intratemporal conditions and Euler equations.

- Given prices and allocation, the non-tradables prices solve each firm's problem.
- Given prices and allocation, the tradables prices solve the monetary authority's problem.
- The government budget constraints are satisfied.
- All markets clear.

## 1.6 Decentralized Tax Setting: Two Groups of Countries

The tax setting problem with two countries is in general complicated. This is because the countries internalize the price impacts of the tax changes. While such effects are unlikely to change the solutions materially, we here choose to consider a modified simpler problem. Instead of two countries, we consider a case of two groups of countries a home group and a foreign group. Each group consists of a continuum of ex ante and ex post identical countries. This also simplifies the analysis relative to the planner problem because the countries can take the tradables and stock prices as given.<sup>9</sup>

We assume the groups have in place the following tax treaty: capital gains from countries within the same group are treated the same as home capital gains. Assuming no arbitrage, the price of each country's stock within the same group must be priced the same. Therefore there is no utility loss from such an agreement. Similarly, there will be no benefits from treating the capital gains from two different countries in the foreign group differently.

Therefore the problem of each country is reduced to deciding the tax rate applied to gains from the home group and foreign group. More formally, we assume that before the birth of households at  $t = 0$ , the governments choose their tax policy at  $t = -1$ .

Because each country is small, its choices have no effect on the equilibrium tradables and stock prices. Therefore the problem of each government is to choose a state contingent path for capital taxes given the equilibrium price processes. However, throughout most of this paper we consider approximations of the equilibrium conditions. It will turn out that the optimal differential tax rates are approximately constant.

Formally, we solve the optimal tax policies in the following way. First, we solve the following planner problem for each country  $k$

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<sup>9</sup>The sizes of the groups correspond to those of the countries introduced before  $\omega$  and  $1 - \omega$

$$\max_{\{\mathbf{S}(x^t), \mathbf{p}_{NT,t}(x^{t-1})\}_{t=0}^{\infty}} V_k. \quad (20)$$

Note that here we allow the planner to also alter the non-tradables price. In practice this can be attained through the use of labor subsidy. Here,  $V_H$  is the value function in the following competitive equilibrium, where the agents take the chosen quantities as given. Second, we find the tax rates that support the efficient solution in a decentralized equilibrium.

Given the two groups, the household and firm problems remain the same. Assuming each country in a given group is weighted equally, the problem of the monetary authority becomes

$$\max_{\{\mathbf{p}_{T,t}(x^t)\}_{t=0}^{\infty}} \lambda_H \int_0^1 V_{H,k} dk + \lambda_F \int_0^1 V_{F,k} dk \quad (21)$$

However, assuming symmetric choices among the countries in a group,  $V_{H,k} = V_H$  for some  $V_H$  for all  $k$  and similarly for the foreign group. Then we can drop the integrals and the monetary authority problem is essentially as before. Given symmetric choices, also the market clearing conditions are similar to before.

## 2 Analysis

### 2.1 A Simple Benchmark: A Uniform Taxation Result

I start the analysis with a simple benchmark by rederiving the uniform taxation result of Gordon (1986) using a two period model. While the later sections consider second order approximations, the analysis in this section is exact. Heuristically, the idea of the proposition is that a differential tax distorts allocations between different stocks. Originally, he considers a deterministic model, here I extend the result to a case with uncertainty. As explained in the previous chapter we consider the case of two groups of countries introduced before. This assumption is similar to Gordon (1986) who also assumed that the countries are small.

For simplicity assume there are two periods  $t = 0$  and  $t = 1$  and that consumption happens only at period  $t = 1$ . Assume that each home government needs to finance a consumption  $G_H$  at time  $t = 1$  and the foreign government a consumption  $G_F$  at  $t = 1$ . Each government relies on capital taxation to raise the revenues necessary. Normalizing the price of the tradable good to 1, each government budget constraint now becomes:

$$G_i + T_i = d_{j,t} S_{ijk} \tau_{ijk}^S + d_i \tau_{iik}^S S_{iik,t}, i = H, F, j = -i, k \in [0, 1], \quad (22)$$

where  $\tau_{iik}^S$  and  $\tau_{ijk}^S$  are the capital tax rates of home and foreign income and  $S_{ijk,t}$  and  $S_{iik,t}$  are the amounts of foreign and home stocks held by households. Moreover,  $d_{i,t}$  and  $d_{j,t}$  are dividends from home and foreign stocks and  $T_{i,t,k}$  is government transfers. Further assume that initially each country owns only home stocks. Dropping the  $k$  subscripts for simplicity, each period  $t = 0$  budget constraint becomes

$$S_{ii} p_{S,i} + S_{ij} p_{S,j} = p_{S,i}, \quad i = H, F, j = -i \quad (23)$$

where  $S_{ii}$  is country  $i$ 's holdings of country  $i$ 's equity and  $p_{S,i}$  is country  $i$ 's equity price. Here we normalize the supply of each stock to one. For simplicity abstract away from non-tradables consumption. The time  $t = 1$  budget constraint is then given by:

$$c_{T,i} = d_i(1 - \tau_{ii}^S) S_{ii} + d_j(1 - \tau_{ij}^S) S_{ij} + T_i, \quad i = H, F, j = -i. \quad (24)$$

Plugging in the government budget constraints, the household budget constraints become

$$c_{T,i} = d_i S_{ii} + d_j S_{ij} - G_i, \quad i = H, F, j = -i. \quad (25)$$

Each government decides its taxation strategy for period  $t = 1$  at  $t = 0$ . Note that the tax revenue is generally time-varying but the government expenditure is fixed. Therefore we assume that taxes are set so that even in the worst case, the revenue is sufficient to finance the government consumption. The residual capital tax revenue is distributed back to agents via transfers  $T_i$ . Given equilibrium stock positions  $(S_{iik}, S_{ijk})$ , the taxation strategy  $(\tau_{ii}^S, \tau_{ij}^S)$  is viable if

$$G_i \leq \inf d_j \times S_{ij} \tau_{ij}^S + \inf d_i \times S_{ii} \tau_{ii}^S, \quad i = H, F, j = -i. \quad (26)$$

Moreover, viability requires that the taxes rates are below 100 per cent. We now consider the capital tax choices in each country. The following proposition summarizes the uniform taxation result.

**Proposition 1** (Uniform Taxation Result in the Simple Benchmark Model). *Consider the simple two period model (with no externalities) introduced in this section. Assume there is a viable taxation strategy with equal tax rates  $(\tau^S, \tau^S)$ . Then this taxation strategy is optimal.*

*Proof.* Consider one small home country. Examine the problem of a planner who, in addition to deciding the tax rates, can choose the equilibrium quantities in the country. Its problem can be written

$$\max_{c_{T,H}, S_{HH}, S_{HF}, \tau_{HH}, \tau_{HF}} \mathbb{E}[U(c_{T,H})] \quad (27)$$

subject to (one can show that the first two constraints bind with equality)

$$S_{HH}p_{SH} + S_{HF}p_{SF} = p_{SH} \quad (28)$$

$$c_{T,H} = d_H(1 - \tau_{HH}^S)S_{HH} + d_F(1 - \tau_{HF}^S)S_{HF} + T_H \quad (29)$$

$$G_H \leq \inf d_F \times S_{HF} \tau_{HF}^S + \inf d_H \times S_{HH} \tau_{HH}^S, \quad (30)$$

As above the period  $t$  budget constraint becomes

$$c_{T,H} = d_H S_{HH} + d_F S_{HF} - G_H \quad (31)$$

Assume that the viability constraint does not bind. Then the optimal portfolio choice is characterized by the Euler equation

$$\mathbb{E}[U_C(r_H - r_F)] = 0. \quad (32)$$

Moreover the planner is indifferent with respect to different viable tax rates.

Now consider decentralizing this planner solution. Consider equal tax rates  $(\tau^S, \tau^S)$ . Now the Euler equation in decentralized equilibrium is

$$\mathbb{E}\left[U_C\left(r_H(1 - \tau^S) - r_F(1 - \tau^S)\right)\right] = 0 \quad (33)$$

But the tax rates cancel out. That is the Euler equation becomes

$$\mathbb{E}[U_C(r_H - r_F)] = 0. \quad (34)$$

But if these tax rates are viable, they implement the planner solution.  $\square$

## 2.2 Flexible Prices or Floating Exchange Rate

I next consider the main model introduced before. From now on I for simplicity assume there is no government spending to be financed and focus solely on the optimal difference between home and foreign capital taxes. Therefore here capital taxes serve only a corrective, Pigovian, role. The structure of this and the following sections is the following. I first consider an equilibrium with no capital taxes. I then solve for the planner solution. Finally, I discuss how the planner solution can be implemented with the correct capital taxes.

First consider the case of flexible prices or a floating exchange rate. Perfect adjustment of relative prices implies that the marginal utilities of consumption corresponding to the equilibrium and planner solutions are the same. This implies that the equilibrium stock positions are efficient. The following proposition formalizes this result.

**Proposition 2** (Main Model, The Cases with No Aggregate Demand Externalities). *Consider the main model, but assume the non-tradables prices are flexible or the exchange rate is floating. Now prices can be set so that the marginal utilities from the perspective of the household and country (planner) are the same. There is no need for corrective taxation.*

*Proof.* I leave subscripts out for simplicity. The marginal utility from the perspective of a country with respect to tradables consumption is

$$\hat{V}_C(c_T, p) = \alpha(p)U_{NT} + U_T + U_N \frac{\alpha(p)}{A} = U_T(1 + \alpha(p)p) + U_N \frac{\alpha(p)}{A} = U_T(1 + p\alpha(p)\tau) \quad (35)$$

where

$$\tau = 1 + \frac{1}{A} \frac{U_N}{U_{NT}}, \quad (36)$$

is the labor wedge. With flexible prices the planner can set  $p_{NT} = AW$ , which corresponds to the flexible price equilibrium with competitive firms. On the other hand the intertemporal condition is given by  $\frac{U_N}{U_{NT}} = -\frac{W}{p_{NT}}$ . Plugging in we obtain  $\tau = 0$  and hence  $\hat{V}_C = U_T$ , i.e. the marginal utilities from the perspective of the country and household are now the same and there are no demand externalities.



Only the relative prices in each country matter. With flexible exchange rates the country can have separate tradables prices. Now for any non-tradables prices there is a tradables price that attains  $\tau = 0$ . Let us set the tradables price at that level in each country. This can be supported when the exchange rate takes the value of the ratio of tradables prices. Therefore again  $\hat{V}_C = U_T$ . Therefore the equilibrium solution must be efficient from the perspective of a small country. □

Despite such a theorem, settings with flexible exchange rates might also feature aggregate demand externalities. For example Karabarounis (2014) finds that a measure of the labor wedge  $\tau$  is non-zero and procyclical also in the US. On the other hand the procyclicality of  $\tau$  is a key driving force of our later results concerning the efficiency of stock positions. There are therefore good reasons to believe that our results are more generally valid, though the frictions causing such behavior are beyond the scope of this paper.

### 2.3 Symmetric Countries, Aggregate Demand Externalities Induced by Nominal Rigidities

I now proceed to analyse the case with nominal rigidities, first focusing on the case of symmetric countries. For tractability I consider approximations of the equilibrium and planner solutions similar to Devereux and Sutherland (2010), Tille and van Wincoop (2010) and Coeurdacier and Gourinchas (2013). Denote log deviations by tildes, relative values (Home -Foreign) by hats and approximation points by bars. To simplify algebra the proofs of this and the next subsection follow Tille and van Wincoop (2010) and assume that the fees and taxes are 2nd order terms. To obtain similar expressions we could alternatively approximate both the fees and taxes around 0.<sup>10</sup>

To avoid notational clutter, we leave the  $k$  subscripts, denoting a given country in a group, out from the formulas.

We focus on comparing a laissez-faire equilibrium with an equilibrium where all countries choose taxes optimally. This latter solution is called the planner solution. In the next section we consider the effects of asymmetric tax changes. A second order approximation of the relative home and foreign Euler equations yield the following lemma.

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<sup>10</sup>To obtain the result that  $\mathbb{E}_t[\hat{r}_{t+1}] = 0$  in a first order approximation we could assume that there is a small group of investors who do not face the stock market friction.

**Lemma 1.** *Assume one -period rigid non-tradables prices (and flexible or fixed tradables price). A second order approximation of the relative Euler equations corresponding to the laissez-faire equilibrium and an equilibrium where each country chooses positions optimally (planner solution) is given by*

$$2f - \gamma \text{cov}_t(\hat{c}_{t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0, \quad (37)$$

$$2f - \psi \text{cov}_t(\hat{c}_{t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0, \quad (38)$$

where  $\psi > \gamma$ .

For proof see appendix. Note that here  $\gamma$  is the relative risk aversion coefficient of the households. The planner solution takes a similar form with  $\gamma$  replaced by  $\psi > \gamma$ . Here the planner solution coincides with an equilibrium with more risk averse agents. The nominal rigidities imply positive public benefits from macroeconomic stabilization. The lemma shows that this can be seen as an increase in the efficient level of risk aversion.

Evaluating the covariance term requires first order solutions for  $\hat{c}_{t+1}$  and  $\hat{r}_{t+1}$ . Providing a solution to these variables is not necessary for most of our results. However, for clarity the following lemma gives an expression for the first order solution

**Lemma 2.** *A first order approximation of the equilibrium system has the form*

$$\hat{r}_{t+1} = \frac{R_2}{1 - R_1 S p_S} \epsilon_{t+1} + \mathcal{O}(\epsilon^2), \quad \hat{c}_{t+1} = \left[ \frac{D_1 S p_S}{1 - R_1 S p_S} R_2 + D_2 \right] \epsilon_{t+1} + D_3 Z_{t+1} + \mathcal{O}(\epsilon^2), \quad (39)$$

where  $S$  is the zero order value of stock position and  $p_S$  the corresponding zero order price. Moreover,  $R_1$  and  $D_1$  are scalars and  $D_2$  and  $R_2$  are vectors.<sup>11</sup>

The proof is similar to that in Devereux and Sutherland (2010). The approximation is independent of whether we consider the equilibrium or planner problem. Note that in a first order approximation  $\mathbb{E}_t[\hat{r}_{t+1}] = 0$ , so that  $\hat{r}_{t+1}$  is a zero mean i.i.d. variable. The stock positions corresponding to the second order approximation of the relative Euler equation are constant. These stock position are the so called zero order position discussed in Devereux and Sutherland (2010) and Tille and van Wincoop (2010). We can define the zero order stock positions solving the equilibrium and planner solutions as follows.

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<sup>11</sup>The stock positions are excluded from the state variables  $Z_t$ . A natural approximation point for variables other than stock positions is the deterministic steady-state.

**Definition** The zero order equilibrium stock position  $S^{eq}$ , is the position that solves the 2nd order approximation of the equilibrium Euler equations (given the linear approximation of other parts of the model) around an approximation point. Similarly the zero order stock position corresponding to the "planner" problem  $S^{plan}$  with rigid prices is the position that solves the 2nd order approximation of the relative Euler equation from the perspective of each country.

The following lemma helps understand the solution. It shows that the equilibrium zero order stock position is the position that maximizes a quadratic approximation of the agent's utility. Similarly the zero order stock position chosen by the planner is the position that maximizes a quadratic approximation of the planner's objective. Therefore we can think of the zero order stock positions as solving a type of mean-variance problem. Here the planner who internalizes the benefits of macroeconomic stabilization penalizes consumption variance more than the agents.

**Lemma 3** (A lemma that is not used but provided for clarity). *i) Given a symmetric solution and fixed prices, we can approximate the utility functions corresponding to equilibrium and planner solutions so that*

$$V^{eq}(S) = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \tilde{c}_{T,H,t}(2) + \tilde{c}_{T,F,t}(2) - \frac{1}{2} \gamma (Var(\tilde{c}_{H,t}) + Var(\tilde{c}_{F,t})) \right] + \mathcal{O}(\epsilon^3), \quad (40)$$

where

$$\mathbb{E}_0[\tilde{c}_{T,H,t}(2) + \tilde{c}_{T,F,t}(2)] = -2(1-S)p_S \tilde{f} \quad (41)$$

and

$$V^{plan}(S) = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \tilde{c}_{T,H,t}(2) + \tilde{c}_{T,F,t}(2) - \frac{1}{2} \psi (Var(\tilde{c}_{H,t}) + Var(\tilde{c}_{F,t})) \right] + \mathcal{O}(\epsilon^3), \quad (42)$$

where

$$\mathbb{E}_0[\tilde{c}_{T,H,t}(2) + \tilde{c}_{T,F,t}(2)] = -2(1-S)p_S \tilde{f} \quad (43)$$

ii)  $S^{eq}$  maximizes  $V^{eq}(S)$  subject to a first order approximation of the budget constraints.  $S^{plan}$  maximizes  $V^{plan}(S)$  subject to a first order approximation of the budget constraints.

For proof: see the appendix. Note that this result is provided for clarity, but is not used when proving any of the other results in this paper.

I next state the main result of this section.

**Theorem 1** (Higher home tax rate, Symmetric Countries). *Assume one period fixed non-tradables prices (and flexible or fixed tradables price). The planner solution can be implemented using differential capital taxation. The optimal differential tax rate is constant in a 2nd order approximation. It is given by  $\tau^S = f(1 - \frac{\gamma}{\psi})$ .*

*Proof.* Including the differential tax rate the Euler equations corresponding to the equilibrium and planner solutions become

$$2f - 2\tau^S - \gamma cov_t(\hat{c}_{t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (44)$$

$$2f - \psi cov_t(\hat{c}_{t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0, \quad (45)$$

where I assumed  $cov_t(p_{T,t+1}, \hat{r}_{t+1}) = 0$ , i.e. symmetric treatment by the monetary authority (natural with symmetric countries). Solving  $cov_t(\hat{c}_{t+1}, \hat{r}_{t+1})$  from the first condition and plugging in to the latter we obtain

$$2f - \frac{\psi}{\gamma}(2f - 2\tau^S) = 0. \quad (46)$$

From which we can solve

$$\tau^S = f(1 - \frac{\gamma}{\psi}), \quad (47)$$

All the other conditions are the same. Because the tax revenues are distributed back to the agents, the budget constraints remain unchanged.

It is easy to show that this tax rate is also efficient from the perspective of each small country.

□

Note that this result can be understood in two ways. First,  $\tau^S = f(1 - \frac{\gamma}{\psi})$  implements the efficient zero order stock positions. Second, it is also the

optimal differential tax rate given any 2nd order approximation of Euler equations.

Note that with flexible prices  $\psi = \gamma$  and therefore  $\tau_S = 0$ . This is the Gordon uniform taxation result discussed in the previous section. Absent externalities, differential taxation merely distorts investment portfolios and is not optimal. Therefore the above theorem can be seen as a generalization of the Gordon result.

The theorem also extends the scope of the uniform taxation result. That is we also have  $\tau_S = 0$  when  $f = 0$ . With symmetric countries and no risk sharing frictions, uniform taxation is optimal despite nominal rigidities. To understand this result note that using the above lemma, absent frictions the equilibrium and planner problems then become one of minimizing consumption variance. Despite the fact that the planner's effective risk aversion is higher, there is no trade-off between fees and risk sharing. Because the equilibrium solution minimizes the consumption variance in both countries given the budget constraints, a planner cannot achieve more risk sharing.

To obtain a difference between the planner and equilibrium solutions given symmetric countries, we need both aggregate demand externalities induced by nominal rigidities ( $\psi > \gamma$ ) and risk sharing frictions ( $f > 0$ ). Now there is a trade-off between risk sharing benefits and costs of foreign equity ownership. The planner values risk sharing more and hence chooses a higher foreign stock position.

Note that we assumed that all countries engage in optimal taxation policies, which preserves the symmetry of the problem. However assuming symmetric treatment by the monetary authority, one can show that the above formula for the optimal tax rate holds for each country even when some countries abstain from optimal tax changes. In the next section we analyze cases in which only some of the countries choose taxes optimally.

The table below summarizes the results of this chapter.

Table 1: Optimal Differential Capital Taxation in the Symmetric Case

Risk Sharing Frictions	Agg. Demand Externalities (Nom. Rigidities)	
	<b>No</b>	<b>Yes</b>
<b>No</b>	Tax same	Tax same
<b>Yes</b>	Tax same	Tax home higher

## 2.4 Asymmetric Countries, Aggregate Demand Externalities Induced by Nominal Rigidities

We now consider the case of asymmetric countries. With symmetric countries all parties typically benefit from similar deviations from equilibrium values. Absent differential taxation the equilibrium features excessive home bias in both countries. If the home countries increase the tax rate on home gains, households increase their holdings of foreign equity and reduce their holdings of home equity. The foreign households must take the opposite positions even if the foreign countries leave tax rates unchanged. But this implies that the foreign country ends up reducing home bias and is hence also better off.

In this section we see that this does not necessarily hold in the case of asymmetric countries. We again consider the case of two groups of countries, home and foreign. At  $t = -1$  each country chooses the capital tax rate pertaining to income from the home and foreign groups.

We first proceed with some simplifying results.

**Lemma 4.** *Given any equilibrium the second order term  $\mathbb{E}_t[\hat{r}_{t+1}] + \frac{1}{2}\mathbb{E}_t[\tilde{r}_{H,t+1}^2 - \tilde{r}_{F,t+1}^2]$  is not time varying.*

*Proof.* A 2nd order approximation of the home Euler equation gives.

$$\mathbb{E}_t[\hat{r}_{t+1}] + f + \mathbb{E}_t[\tilde{r}_{H,t+1}^2 - \tilde{r}_{F,t+1}^2] = \gamma^* cov_t(\tilde{c}_{H,t+1}, \hat{r}_{t+1}) + \gamma_p cov_t(\tilde{p}_{T,t+1}, \hat{r}_{t+1}) \quad (48)$$

Where  $\gamma^* = \gamma - \eta$  is the risk aversion coefficient modified by the endogenous discount factor. Evaluating the two covariance terms requires first order approximations of  $\tilde{c}_{H,t}$ ,  $\tilde{p}_{T,t}$  and  $\hat{r}_t$ . In a first order approximation  $\hat{r}_{t+1}$  is of the form  $\hat{r}_{t+1} = \hat{R}_1 \varepsilon_{t+1}$ . Because  $\varepsilon_t$  are identically distributed, they are homoskedastic. Now the two covariance term on the right must be constant. Therefore the term on the left is constant. This result does not change when adding a constant tax. □

The following definition helps simplify the formulas.

**Definition** Let the drift (instantaneous growth rate) of asset  $i$  be  $\mu_i = \mathbb{E}_t[\tilde{r}_{t+1,i}] + \frac{1}{2}\mathbb{E}_t[\tilde{r}_{t+1,i}^2]$

Given the previous lemma  $\mu_H - \mu_F$  is constant, though each term separately is not necessarily constant. For now note that the relative Euler equations from the perspective of the households and countries are given by:

$$\mu_H - \mu_F + f - \gamma_H^* \text{cov}_t(\tilde{c}_{t+1,H}, \hat{r}_{t+1}) - \gamma_{H,p} \text{cov}_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (49)$$

$$\mu_H - \mu_F - f - \gamma_F^* \text{cov}_t(\tilde{c}_{t+1,F}, \hat{r}_{t+1}) - \gamma_{F,p} \text{cov}_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (50)$$

$$\mu_H - \mu_F + f - \psi_H^* \text{cov}_t(\tilde{c}_{t+1,H}, \hat{r}_{t+1}) - \psi_{H,p} \text{cov}_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (51)$$

$$\mu_H - \mu_F - f - \psi_F^* \text{cov}_t(\tilde{c}_{t+1,F}, \hat{r}_{t+1}) - \psi_{F,p} \text{cov}_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (52)$$

Here

$$\gamma_p = -p_T \frac{\frac{\partial U_T/p_T}{\partial p_T}}{U_T/p_T} \Big|_{(\bar{c}, \bar{p})} > 0, \quad (53)$$

and

$$\psi_p = -p_T \frac{\frac{\partial V_T/p_T}{\partial p_T}}{V_T/p_T} > 0. \quad (54)$$

represent price hedging effects. Moreover,

$$\gamma_i^* = \gamma_i - \eta_i, \quad \psi_i^* = \psi_i - \eta_i \quad (55)$$

represent risk aversion terms modified by the endogenous discount factor. Note that because  $\gamma_i < \psi_i$ , also  $\gamma_i^* < \psi_i^*$ . Moreover, for typical parameter values  $\psi_F > \gamma_F$ . We now proceed to analyse cases in which only one party changes tax rates.

**Proposition 3.** *Assume all prices are fixed (alternatively  $\text{cov}(\tilde{p}, \hat{r}) = 0$  so that the monetary authority treats each country symmetrically) and there are no risk-sharing frictions. Let there be no differential capital taxation in place. Then each country would benefit from reducing the position in the high return (drift) assets and increasing the position in the low return (drift) assets.*

*Proof.* Now the conditions become

$$\mu_H - \mu_F - \gamma_H^* \text{cov}_t(\tilde{c}_{t+1,H,k}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (56)$$

$$\mu_H - \mu_F - \gamma_F^* \text{cov}_t(\tilde{c}_{t+1,F,k}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (57)$$

We can solve from the first equation

$$\text{cov}_t(\tilde{c}_{t+1,H,k}, \hat{r}_{t+1}) = \frac{\mu_H - \mu_F}{\gamma_H^*} \quad (58)$$

Assume that at equilibrium values  $\mu_F > \mu_H$ . Now the country benefits from switching investment in to the low return asset when

$$\mu_H - \mu_F - \psi_H^* \text{cov}_t(\tilde{c}_{t+1,H,k}, \hat{r}_{t+1}) > 0 \quad (59)$$

From this we can solve

$$\frac{\mu_H - \mu_F}{\psi_H^*} - \text{cov}_t(\tilde{c}_{t+1,H,k}, \hat{r}_{t+1}) > 0 \quad (60)$$

Plugging in the equilibrium condition we obtain

$$(\mu_H - \mu_F) \left[ \frac{1}{\psi_H^*} - \frac{1}{\gamma_H^*} \right] > 0 \quad (61)$$

Which is true because  $\psi_H^* > \gamma_H^*$  and we assumed  $\mu_F > \mu_H$ . Symmetrically, it can be shown that the foreign country would benefit from making the same changes. □

Because each home and foreign country would prefer to change positions in the same way, there is no room for simple swaps between home and foreign assets that would increase welfare in both countries. A home country would benefit from increasing the tax on the higher return asset. However, this would be at the cost of a foreign country which would benefit from countering by increasing the tax rate of the same asset.

This conclusion does typically not change much once we allow for flexible tradables prices. To see this, allowing for flexible tradables prices we have

$$\mu_H - \mu_F - \gamma_H^* \text{cov}_t(\tilde{c}_{t+1,H,k}, \hat{r}_{t+1}) - \gamma_{H,p} \text{cov}_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) = 0 \quad (62)$$

From this we can solve,



$$\text{cov}(\tilde{c}_{t+1,H,k}, \hat{r}_{t+1}) = (\mu_H - \mu_F) \frac{1}{\gamma_H^*} - \frac{\gamma_{H,p}}{\gamma_H} \text{cov}_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) = 0 \quad (63)$$

Now a home country benefits from reducing its position in the high return domestic asset and increasing its position in the lower return foreign asset when

$$\text{cov}(\tilde{c}_{t+1,H,k}, \hat{r}_{t+1}) < (\mu_H - \mu_F) \frac{1}{\psi_H^*} - \frac{\psi_{H,p}}{\psi_H} \text{cov}_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) = 0 \quad (64)$$

Plugging in the equilibrium condition we obtain

$$(\mu_H - \mu_F) \left( \frac{1}{\psi_H^*} - \frac{1}{\gamma_H^*} \right) + \left( \frac{\gamma_{H,p}}{\gamma_H^*} - \frac{\psi_{H,p}}{\psi_H^*} \right) \text{cov}_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) > 0 \quad (65)$$

Here there are additional benefits from the transaction when  $\text{cov}_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) > 0$  and  $\frac{\gamma_{H,p}}{\gamma_H^*} > \frac{\psi_{H,p}}{\psi_H^*}$ . This condition is true for standard parameter values. But the analogous condition for the foreign country is

$$(\mu_H - \mu_F) \left( \frac{1}{\psi_F^*} - \frac{1}{\gamma_F^*} \right) + \left( \frac{\gamma_{F,p}}{\gamma_F^*} - \frac{\psi_{F,p}}{\psi_F^*} \right) \text{cov}_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) > 0 \quad (66)$$

But typically also for the foreign country  $\frac{\gamma_{F,p}}{\gamma_F^*} > \frac{\psi_{F,p}}{\psi_F^*}$ . Therefore also this mechanism tends to favor reducing positions in the same stock in all countries which would violate the market clearing condition. Again, we have not found a situation where simple home-foreign equity swaps would increase welfare in all countries.

However, risk sharing frictions can still make it possible for countries in both groups to benefit from a home-foreign equity swap. This happens when the after fee return of the home stock is higher than the after fee return of the foreign stock, namely when  $|\mu_H - \mu_F| < f$ .

Now we can summarize our findings in the following proposition.

**Proposition 4.** *Ignore price effects and assume that at equilibrium values we have  $|\mu_H - \mu_F| < f$ . Let there be no differential capital taxation in use. Then both home and foreign countries can benefit from simple home-foreign equity swaps.*

This proposition is modified only slightly when we include price effects. Then the friction needs to be both higher than the expected return difference and the price covariance term in all countries. However, assuming  $|\mu_H - \mu_F| > f$  and ignoring price effects, both home and foreign countries would benefit from reducing their position in the same asset and therefore benefit from increasing the tax rate of the high return assets. However, such policies would necessarily be at the cost of the other countries. This is summarized in the following proposition

**Proposition 5** (Beggars-thy-neighbor Policies). *Ignore price effects and assume that at equilibrium values we have  $|\mu_H - \mu_F| > f$ . Let there be no differential capital taxation in use. Then a home country could benefit at the cost of foreign countries by increasing investment in the low return asset and reducing investment in the high return asset. This can be implemented by increasing the relative tax rate of the high return asset.*

This result follows easily from the above discussion.<sup>12</sup> The right type of home-foreign asset shift can be implemented with the correct tax changes.

The figures below illustrate the above results. In both figures we set price effects to zero, though they could be modified to incorporate them. We could also allow the friction to be different in each country.

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<sup>12</sup>Note that given our assumptions, the steady-state wealth levels are determined solely by the endogenous discount factors; we need  $\frac{\beta_H(\bar{c}_H)}{\beta_F(\bar{c}_F)} = 1$ . Therefore the tax changes affect only the relative asset positions. However, the idea of the proposition works more generally. In practice the foreign country is induced to changes in positions though price changes that widen the gap between equilibrium and efficient choices.

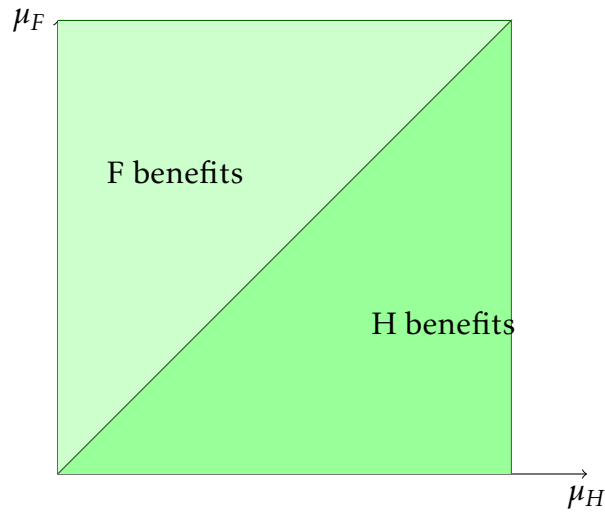


Figure 1: Home tax rate of home assets increased,  $f = 0$

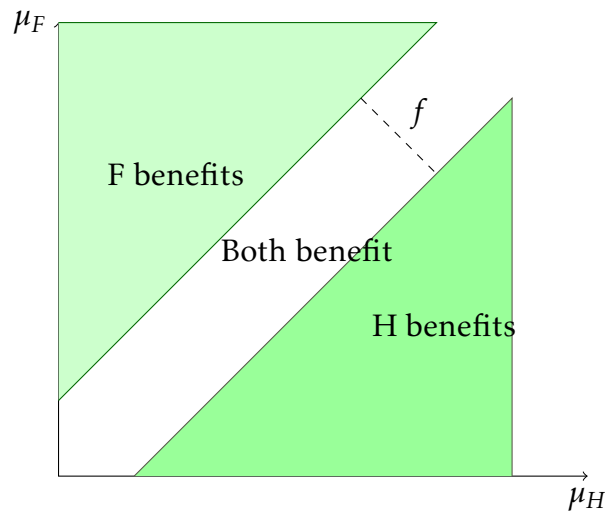


Figure 2: Home tax rate of home assets increased,  $f > 0$

Given the above discussion we might be worried that an equilibrium does not exist when all countries can engage in tax changes. Now we show that an equilibrium exists, assuming that all countries choose taxes optimally.

**Proposition 6.** *Consider a game where the home and foreign countries choose tax rates for home and foreign capital income at  $t = -1$ . A (Nash) equilibrium exists. The relative tax rates are constant in a 2nd order approximation.*

*Proof.* The proof is constructive. First consider the exact conditions. The tax rates in each country are set so that the Euler equations hold from the perspective of the countries. The generally state dependent equilibrium tax rates  $(\tau_{HH,t}^S, \tau_{HF}^S(Z_t), \tau_{FH,t}^S(Z_t), \tau_{FF,t}^S(Z_t))$  are such that for all  $Z_t$

$$\mathbb{E}_t \left[ \frac{U_{T,i,t+1}}{U_{T,i,t}} \frac{p_{T,t}}{p_{T,t+1}} R_i e^{-\tau_{ii}^S} \middle| Z_t \right] = 1, \quad i = H, F. \quad (67)$$

$$\mathbb{E}_t \left[ \frac{U_{T,i,t+1}}{U_{T,i,t}} \frac{p_{T,t}}{p_{T,t+1}} R_{-i} e^{-f} e^{-\tau_{i,-i}^S} \middle| Z_t \right] = 1, \quad i = H, F \quad (68)$$

$$\mathbb{E}_t \left[ \frac{V_{T,i,t+1}}{V_{T,i,t}} \frac{p_{T,t}}{p_{T,t+1}} R_i \middle| Z_t \right] = 1, \quad i = H, F \quad (69)$$

$$\mathbb{E}_t \left[ \frac{V_{T,i,t+1}}{V_{T,i,t}} \frac{p_{T,t}}{p_{T,t+1}} R_{-i} e^{-f} \middle| Z_t \right] = 1, \quad i = H, F \quad (70)$$

That is the tax rates are such that the Euler equations hold from the perspective of each country. If this were not the case for some country, it could benefit from tax changes. If for example,  $\mathbb{E}_t \left[ \frac{U_{T,H,t+1}}{U_{T,H,t}} R_H e^{-\tau_{HH}^S} \middle| Z_t \right] > 1$ , the country could benefit from tax changes inducing the agents consume less today and invest the proceeds to the home stock.

In a 2nd order approximation, the differential tax rate used by the home country is characterized by

$$\mu_H - \mu_F + f - \psi_H^* cov_t(\tilde{c}_{t+1,H}, \hat{r}_{t+1}) - \psi_{H,p} cov_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (71)$$

$$\mu_H - \mu_F + f - \tau_H^S - \gamma_H^* cov_t(\tilde{c}_{t+1,H}, \hat{r}_{t+1}) - \gamma_{H,p} cov_t(\tilde{p}_{t+1,T}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (72)$$

The covariance terms in the first equation are constant and hence  $\mu_H - \mu_F$  is constant. Therefore  $\tau_H^S$  in the latter equation is also constant.  $\square$

**Numerical Example** I illustrate the above results with a simple example. For now let there be only two periods. First consider a model with symmetric countries. Further assume that the households face a positive stock market friction and that each country initially holds equal amounts of each stock.

Table 2A gives the utilities, stock positions and prices in an equilibrium without taxes. Here the equilibrium is naturally symmetric.

Table 2B gives the same statistics in case the home countries choose taxes optimally but foreign countries abstain from capital taxation. Now the home countries increase the relative tax rate of capital income from home sources. This induces smaller domestic asset positions in both countries. The price of the home asset declines. Due to increased risk sharing both countries are better off, though the utility of home households increases more.

Finally, table 2C shows the equilibrium quantities when all countries choose taxes optimally. The symmetry of the equilibrium is now restored. Risk sharing increases further. The utility of the foreign households increases and that of home households decreases relative to the case when only home countries tax. However, both countries are better off compared to the case with no taxation.

Table 2A: Symmetric Countries, No Taxation

	<b>Positions</b>		<b>Prices</b>		<b>Utilities</b>	
$S_{HH}$	0.75	$p_{SH}$	1	$V_H$	0.807	
$S_{HF}$	0.25	$p_{SF}$	1	$V_F$	0.807	

Table 2B: Symmetric Countries, Only Home Countries Tax

	<b>Positions</b>		<b>Prices</b>		<b>Utilities</b>	
$S_{HH}$	0.633	$p_{SH}$	0.909	$V_H$	0.817	
$S_{HF}$	0.379	$p_{SF}$	1	$V_F$	0.812	

Table 2C: Symmetric Countries, All Countries Tax

	<b>Positions</b>		<b>Prices</b>		<b>Utilities</b>	
$S_{HH}$	0.595	$p_{SH}$	1	$V_H$	0.815	
$S_{HF}$	0.405	$p_{SF}$	1	$V_F$	0.815	

Now consider the effect of asymmetries. Assume the model is still symmetric with one exception: the home stock has twice the dividend variance of the foreign stock. For simplicity, assume the households receive no labor income and that there are no stock market frictions. Table 3A shows the positions, prices and utilities absent any capital taxes. Because the problems

of home and foreign households are symmetric, they invest equal amounts in each stock. However, the riskier home stock trades at lower price.

Table 3B shows the situation when home countries choose capital taxes optimally. Now due to increased tax rate on the home asset, home households increase their relative positions in the foreign asset. The price of the high risk foreign asset increases, which induces the foreign households to make the opposite changes. The utility of the home households is increased at the cost of foreign households: the tax change constitutes a beggar-thy-neighbor policy.

Finally table 3C shows the equilibrium when all countries choose taxes optimally. Now again all households invest equal amounts in the stocks. Because the allocation corresponds to that with no capital taxes, the utilities are again at the old level. However, in order for the households to be willing to hold the riskier stock with higher tax rate, its price is now lower.

Table 3A: Asymmetric Assets, No Taxation

Positions		Prices		Utilities	
$S_{HH}$	0.5	$p_{SH}$	0.919	$V_H$	0.941
$S_{HF}$	0.5	$p_{SF}$	1	$V_F$	0.941

Table 3B: Asymmetric Assets, Only Home Countries Tax

Positions		Prices		Utilities	
$S_{HH}$	0.409	$p_{SH}$	0.878	$V_H$	0.943
$S_{HF}$	0.580	$p_{SF}$	1	$V_F$	0.935

Table 3C: Asymmetric Assets, All Countries Tax

Positions		Prices		Utilities	
$S_{HH}$	0.5	$p_{SH}$	0.754	$V_H$	0.941
$S_{HF}$	0.5	$p_{SF}$	1	$V_F$	0.941

## 2.5 Capital Taxation and Capital Controls

In this paper I have argued that a differential capital tax can be used to enhance macroeconomic stability. But how does differential capital taxation

relate to other types of stabilization tools such as capital controls?

This question is particularly important because in many papers the capital control effectively takes the form of capital tax or subsidy. For example in Costinot and Werning (2014), the planner effectively modifies market interest rates using dynamic taxation. Moreover, in Farhi and Werning (2012) capital controls are used to correct inefficiencies caused by similar nominal rigidities as in this paper.

Costinot and Werning (2014) argue that a country can use a capital control policy to manipulate its term of trade analogously to a monopolist extracting rents from buyers. A similar argument is offered by Farhi and Werning (2012). This mechanism is absent from our setting where the countries can only trade a single tradable good. Moreover, in the appendix I show that the presence of two tradable goods does not qualitatively alter the results concerning the optimal differential taxation of home and foreign capital gains.<sup>13</sup>

Above I showed that in asymmetric settings tax changes can take the form of beggar-thy-neighbor policies. Here a country is able to use taxation to increase its holdings of "low-risk" assets with positive externalities. The residents in foreign countries with passive taxation policies must subsequently decrease their holdings of the same assets, and are hence left with worse portfolios. This mechanism is clearly separate from the terms-of-trade manipulation discussed in the capital control literature. Moreover, it is different from the competition issues discussed in the taxation literature (e.g. Razin and Sadka (1991)).

But could the planner not use a dynamic capital control policy as a type of stabilization policy as in Farhi and Werning (2012)? Generally the answer is yes, but this does not affect our results for the optimal differential taxation between income from home and foreign sources.

In this paper I have focused on studying the efficiency of equilibrium in a 2nd order approximation. For the purposes of this section, it is important to make a distinction between two approaches: the efficiency of what I called zero order stock positions and efficiency in a generic 2nd approximation. While these approaches yield equivalent results for the optimal differential tax, they have different implications for the optimal level of capital taxes.

As explained in Devereux and Sutherland (2011), the zero order stock positions can be seen as positions pertaining to a type of deterministic steady-state obtained as the limit of the economy when the size of uncertainty

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<sup>13</sup>The terms-of-trade manipulation channel is absent from this setting because there is a continuum of each type of country.

approaches zero. In such a steady-state, there is clearly no role for dynamic capital control policies. Moreover, the zero order positions are determined through a relative Euler equation. Therefore only the difference between tax rates of home and foreign assets is relevant.

However, generally the level of capital taxes matters even in a 2nd order approximation. To understand the determination of the optimal level of capital taxation, note that the exact optimality condition for the optimal home capital gains rate is such that the planner's version of the Euler equation holds. Consider the Euler equations for the home asset in each country. For simplicity assume that prices are fully fixed, including flexible tradables price modifies the expression only slightly. Applying a 2nd order approximation and deducting the two conditions gives the result summarized by the following proposition

**Proposition 7.** *Assume fully fixed prices. Given a 2nd order approximation, the optimal difference between the home and foreign tax rate of the home asset is*

$$\tau_{t,FH}^S - \tau_{t,HH}^S = f\left[\frac{\gamma}{\psi} - 1\right] - \gamma \left[ \frac{\gamma_{pru}}{2\gamma} - \frac{\psi_{pru}}{2\psi} \right] \mathbb{E}_t[\tilde{c}_{H,t+1}^2 - \tilde{c}_{F,t+1}^2] + \quad (73)$$

$$\gamma \left[ \frac{\gamma_{pru}}{2\gamma} + \gamma - \frac{\psi_{pru}}{2\psi} - \psi \right] [\tilde{c}_{H,t}^2 - \tilde{c}_{F,t}^2] + \mathcal{O}(\epsilon^3), \quad (74)$$

where

$$\gamma_{pru}(\bar{p}, \bar{c}) = c_T^2 \frac{\partial^2 U_T}{\partial c_T^2} \Big|_{(\bar{p}, \bar{c})}, \quad (75)$$

and the same coefficient corresponding to the coefficient from the perspective of the country is

$$\psi_{pru}(\bar{p}, \bar{c}) = c_T^2 \frac{\partial^2 V_T}{\partial c_T^2} \Big|_{(\bar{p}, \bar{c})}. \quad (76)$$

For proof: see appendix. Evaluating the expression requires first order solution for  $\tilde{c}_{H,t}$  and  $\tilde{c}_{F,t}$ .

This equation specifies the optimal difference between capital taxes of the home asset in the home and foreign country as a function of the first order terms for consumption. A similar expression would be obtained for



any traded asset such as a bond. Note that the optimal difference is generally time-varying. Moreover, this equation specifies the optimal difference between capital taxes for a given asset, yet the overall level of capital taxes is still indeterminate.

An increase in the relative home capital tax of an asset induces the home agents to reduce their positions in the asset. One way to see this is that the home agents borrow funds from the foreign agents. This intuition would be especially germane if the asset were a bond.

The above expression helps us map the results to those of the capital control literature. Here we can see the variation in  $\tau_{t,FH}^S - \tau_{t,HH}^S$  as a dynamic capital control strategy used to adjust portfolio flows. However, as we have shown the planner also employs differential asset taxation to correct for the portfolio composition of households. Moreover, we showed that given a 2nd order approximation the optimal differential capital tax is constant. Therefore we can see the constant differential tax as complementing capital control strategies discussed in the literature.

### 3 Extensions and Modifications

In this section we consider extensions and modifications of the results in the previous section. Special attention is given to two additional considerations. First, we discuss extending the results to a case with stochastic productivity shocks in the non-tradable sector. Second we discuss the role of differential capital taxation in economies with elastic aggregate savings. The appendix also provides alternatives to the model with a non-tradable sector and one period rigid prices.

#### 3.1 Productivity Shocks in the Non-Tradable Sector

Now assume  $(A_{NT,H}, A_{NT,F})$  are stochastic. For simplicity still consider the case of symmetric countries. The equilibrium condition is

$$2f - \gamma cov_t(\hat{c}_{t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (77)$$

The planner condition is

$$2f - \psi cov_t(\hat{c}_{t+1}, \hat{r}_{t+1}) + \psi_A cov_t(\hat{A}_{t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (78)$$

where

$$\psi_A = A \frac{\frac{\partial U_T(1+\alpha(p)p\tau)/p_T}{\partial A}}{U_T(1+\alpha(p)p\tau)/p_T} > 0 \quad (79)$$

The expression for  $\psi_A$  is given in the appendix. Now the planner benefits from increasing home ownership and reducing foreign ownership when

$$2f \underbrace{\left[1 - \frac{\psi}{\gamma}\right]}_{\leq 0} + \psi_A \text{cov}_t(\hat{A}_{t+1}, \hat{r}_{t+1}) > 0 \quad (80)$$

If the expression is positive absent taxes, the countries could implement such an equilibrium by taxing capital gains from foreign sources at a higher rate. We obtain the following proposition.

**Proposition 8.** *Home equity returns should be taxed at lower rate than foreign returns when domestic equity gives a good hedge for productivity shocks in the sector with rigid prices and risk sharing frictions are small (so that the above equation holds).*

However, home stock returns are generally not positively correlated with productivity shocks in the non-tradable sector. This is because non-tradable profits are generally not increasing in the productivity of non-tradable firms. To see this in a simple way, note that non-tradables profits are positive when

$$\tau > \frac{\tau_L}{1 + \tau_L} \quad (81)$$

and negative otherwise. But the labor wedge  $\tau$  is decreasing in  $A_{NT}$ .

### 3.2 Capital Taxation with Elastic Aggregate Savings

In the above environment households can only save through holding stocks that represent claims to an endowment. Effectively there is no way to store consumption goods into the next period. This means that in the aggregate the households cannot be saving the wrong amount, though in general the asset portfolios could be distorted. This property would not be materially affected if we replaced the tradables endowment with tradables goods produced using a labor input and a fixed capital input. Similarly, we could allow for capital investment by firms financed through retained earnings.

Given the classic arguments raised against capital taxation (e.g. Atkinson and Stiglitz (1976)), one might worry that our results do not hold in a setting

where capital taxes can distort aggregate savings decisions. In this section I therefore consider a simple model where capital taxation generally alters the total amount saved. We see that even in such an environment, it is optimal to use differential capital taxation to correct for distortions in investment portfolios. Assume there are two time periods  $t = 0$  and  $t = 1$  and consider a model with two symmetric countries. Assume that at  $t = 0$  there is only consumption of tradable goods. Therefore the households' preferences are given by

$$u(c_{T,i,0}) + \mathbb{E}[U(c_{T,i,1}, c_{NT,i,1}, N)], \quad i = H, F, \quad (82)$$

where  $U$  has the same form as before. At  $t = 0$  all agents hold an endowment  $y_0$  of tradable goods. They have access to home and foreign investment technologies. The home technology delivers  $A_H s_{H,H}$  units of the tradable good at  $t = 1$  for  $s_{H,H}$  units saved by home households at  $t = 0$  and the foreign technology  $A_F s_{H,F} e^{-f}$  goods for  $s_{H,F}$  units saved. The returns for the foreign household are defined symmetrically. Both  $A_H$  and  $A_F$  are stochastic and have the same statistical properties, plus some correlation structure.

In addition to savings, the agents receive tradable goods at  $t = 1$  through labor income  $l_i$  that is an exogenously given stochastic variable. Now the period  $t = 0$  budget constraint for tradable goods is given by

$$c_{T,i,0} + s_{i,H} + s_{i,F} = y_0, \quad i = H, F. \quad (83)$$

Moreover, the period  $t = 1$  budget constraint is

$$c_{T,i,1} = l_i + s_{i,H} A_H + s_{i,F} A_F e^{-f}, \quad i = H, F. \quad (84)$$

Absent externalities, i.e. when prices at  $t = 1$  are fully flexible, taxing savings can now distort aggregate savings decisions. A uniform capital tax  $e^{-\tau}$  lowers the returns received from savings that are then given by  $A_H s_{i,H} e^{-\tau}$  and  $A_H s_{i,H} e^{-f-\tau}$ . This typically reduces the amounts saved at  $t = 0$  and affects the period  $t = 1$  income. On the other hand taxing the exogenously given labor income is fully non-distortionary. Therefore, assuming the labor income is higher than government expenditure, raising the full amount needed through a tax on labor is optimal.

As before, we next introduce demand externalities into the model. That is, we assume that the non-tradables prices are set one period in advance. The problems of the firm and monetary authority are as before. The period  $t = 1$  marginal utility from the perspectives of a household and a country take the old form. The following proposition summarizes the properties of optimal tax rate.

**Proposition 9.** *In the model with elastic aggregate savings, the planner solution can be implemented with capital taxes. The differential tax rate is as before, i.e. home equity is taxed at  $f(1 - \frac{\gamma}{\psi})$  higher rate than foreign equity.*

The proof is similar as before. The rates are such that the planner versions of the Euler equations hold.<sup>14</sup> This shows that our results are not driven by the assumption that the tradables good is given by an endowment.

### 3.3 Frictions and Taxes: Different Forms

For tractability, the baseline model assumes that the friction takes the form of a holding cost. Then the optimal tax that offsets this friction is a tax on the value of holdings. However, we could also assume as in e.g. Bhamra et al. (2014) that the friction is proportional to the dividend, not position value. Then the optimal tax is also a dividend tax. As before we obtain tractable expressions, either by assuming the friction and tax are second order terms as in Tille and van Wincoop (2010) or by approximating each around 0. Given symmetric countries, the optimal tax that offsets a friction proportional to dividend is  $\frac{\bar{d}}{p_S} f(1 - \frac{\gamma}{\psi})$ . Here  $\bar{d}$  and  $p_S$  are the zero order dividend and stock price respectively. Each is independent of the frictions and taxes.

We could also consider a capital tax on realized gains, that is on dividends and capital gains when the stock is sold. However given a 2nd order approximation, the equilibrium stock position is constant. Therefore, given our approximation such a tax is equivalent to a dividend tax. Overall, we would not expect the form of capital tax to be important. Any tax that shifts the stock positions in the right direction should be beneficial.

### 3.4 Residence- vs. Source-Based Taxation and Tax Competition

Above I assumed that capital taxation is carried out only by the country of residence. Could an efficient solution be implemented under a purely source-based taxation system?

Generally, the answer is clearly no. To see this, consider the case of symmetric countries. Assume the efficient solution requires that home capital gains are taxed at higher rate than foreign gains. Let the capital tax

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<sup>14</sup>In a general setting this requires that a capital tax may be negative, i.e. take the form of a subsidy.

rates in the two countries be  $\tau_H^{S^*}$  and  $\tau_F^{S^*}$ . That is now the home households pay  $\tau_F^{S^*}$  from their foreign investments and vice versa. Now optimal taxation from the perspective of the home country requires  $\tau_F^{S^*} > \tau_H^{S^*}$ . On the other hand efficient taxation from the perspective of the foreign country requires  $\tau_H^{S^*} > \tau_F^{S^*}$ . Both cannot be true.

Therefore, implementing the efficient solution generally requires residence-based taxation. However, the optimal tax system can be implementable using a combination of source- and residence-based tax systems. To see this consider again the previous symmetric country setting. Assume both countries employ a source-based capital tax of  $\tau_s^S$ . In addition each country administers a residence-based tax of  $\tau_r^S$  on home assets. Now the tax rate on domestic capital gains in both countries is  $\tau_r^S$  higher than the tax rate on foreign gains. Assume  $\tau_r^S = f(1 - \frac{\gamma}{\psi})$ , which corresponds to the optimal differential tax derived before. Now the equilibrium attains the efficient solution assuming that  $\tau_s^S$  is chosen optimally.

Source-based capital taxation could give rise to tax competition. Assume that the taxes are set to yield an efficient allocation. Now a home country might want to raise additional revenues by increasing the tax rate of foreign holders of the home stock. This, however, is not a problem given our assumption of two groups of countries. If a country increased the tax rate on foreign investors, they would shift into equity issued in other countries within the same group. This mechanism is similar to that discussed by Razin and Sadka (1991), who argue that tax competition should lead to residence- rather than source-based taxation. However, the result might not hold if there are no perfect substitutes to the equity issued by the country.

Tax competition can be a more serious concern when capital is free to migrate between countries. When corporations directly or indirectly have to bear part of the capital tax burden, a concern with increasing capital taxes is that businesses might move to countries with lower taxes. Absent uncertainty and coordination, Razin and Sadka (1991) argue that perfect tax competition will lead to zero capital taxes in each country. While this result requires a deterministic setting, tax competition might still more generally impede the implementation of efficient tax systems. For example assume efficiency requires that home capital gains are taxed higher than foreign gains. However, a country might be reluctant to raise the tax on home gains if worried about capital migration. Overall, in very general settings we would still expect a role for coordination in implementing efficient capital tax arrangements.

## 4 Conclusions

I have provided an analysis of optimal capital taxation in economies with aggregate demand externalities and risk sharing frictions. According to standard arguments, gains from home and foreign sources should be taxed at the same rate. This paper suggests the contrary. I argue that the differential tax rate on home and foreign gains serves an important role in correcting for distortions in equilibrium portfolios and supporting macroeconomic stability.

Throughout this paper I assumed that the equilibrium portfolios are privately optimal. Here the financial friction is best interpreted as a cost representing for example information collection efforts. In a companion paper (Sihvonen, 2016), I posit that the fee can also be interpreted as representing heterogenous beliefs. This interpretation does not necessarily change the results of this paper. Namely, I show that when the welfare criterion is non-paternalistic, a belief difference is essentially equivalent to a holding cost. However, when the friction is behavioral and the welfare criterion is paternalistic, the planner is also concerned about correcting inefficiencies in private choices. This generally also yields an interesting additional channel through which differential capital taxation can increase welfare.

The analysis suggests other venues for further research. For example I microfound the positive public benefits from consumption smoothing using nominal rigidities. It would be interesting to try to obtain similar positive externalities through alternative mechanisms. Channels to consider could include pecuniary externalities due to incomplete markets as in Brunnermeier and Sannikov (2014) or the interaction between financing constraints and prices as in Bianchi (2011).

## 5 Appendix A: Additional Extensions

In this first part of the appendix I provide some alternatives to the setting with a non-tradable sector and one period rigid prices. Such settings may feature quantitatively smaller benefits from macroeconomic stabilization and hence smaller differential tax rate. However, we will see that differential taxation plays a role even in such economies.

## 5.1 Two Tradable Goods

It is quite easy to extend the analysis to concern multiple tradable goods. For simplicity consider again the symmetric model. Assume there are two input goods, one for each country, used to produce the aggregate tradable good. Let the home stock represent claims on the endowment of the home good and the foreign stock a claim on the endowment of the foreign good. The aggregate tradable good is then given by

$$C_{T,H} = \left( a_I^{\frac{1}{\phi_I}} c_{I,H}^{\frac{\phi_I-1}{\phi_I}} + (1-a_I)^{\frac{1}{\phi_I}} c_{I,F}^{\frac{\phi_I-1}{\phi_I}} \right)^{\frac{\phi_I}{\phi_I-1}}, \quad (85)$$

where  $a_I$  measures bias towards the home input good. Because of home bias in the goods market, the price indices in the two countries can be different. The intratemporal conditions imply that the home price for the aggregate tradable good is given by

$$p_{T,H} = \left( a_I p_{I,H}^{1-\phi_I} + (1-a_I) p_{I,F}^{1-\phi_I} \right)^{\frac{1}{1-\phi_I}} \quad (86)$$

For simplicity assume there are no frictions. At equilibrium values we have

$$-\gamma_H cov_t(\tilde{c}_{t+1,H}, \hat{r}_{t+1}) - \gamma_{H,p} cov_t(\tilde{p}_{t+1,H}, \hat{r}_{t+1}) = 0 \quad (87)$$

On the other hand, the planner benefits from the transaction when

$$-\psi_H cov_t(\tilde{c}_{t+1,H}, \hat{r}_{t+1}) - \psi_{H,p} cov_t(\tilde{p}_{t+1,H}, \hat{r}_{t+1}) > 0 \quad (88)$$

Plugging in the equilibrium condition and rearranging, we obtain

$$\left[ \frac{\gamma_{p,H}}{\gamma} - \frac{\psi_{p,H}}{\psi_H} \right] cov_t(\tilde{p}_{t+1,H}, \hat{r}_{t+1}) > 0 \quad (89)$$

The expression in brackets is typically positive. Then the home country benefits from increasing the position in the foreign asset and reducing the position in the home asset when  $cov_t(\tilde{p}_{t+1,H}, \hat{r}_{t+1}) < 0$ . Now because the tradables price index is different in the foreign country, we can also have  $cov_t(\tilde{p}_{t+1,F}, -\hat{r}_{t+1}) > 0$  so that the foreign country benefits from taking the opposite positions. This type of change can be implemented by increasing

the tax rate of the home asset in both countries. Similarly an opposite covariance pattern suggest that the foreign asset should be taxed at a higher rate in both countries. Empirically, the correlation between prices and equity returns is small (Van Wincoop and Warnock (2010), Coeurdacier and Gourinchas (2013)), indicating that the price channel is less important for capital taxation.

## 5.2 Home and Foreign Good

Now assume, similarly to for example Gali and Monacelli (2005) that there are only two goods produced using labor in each country. Both goods are tradable and each country has a preference towards the consumption of the home good. Let the only shocks be those to the productivity in each country. Assume the model is symmetric and the prices of each good are fixed one period in advance. The stocks represent claims to the firm profits in each country.

We assume CRRA preferences over a CES aggregator  $g(c_{NT}, c_T) = g(C) = \frac{1}{1-\gamma} C^{1-\gamma}$ , where

$$C_H = \begin{cases} \left( a^{\frac{1}{\phi}} c_{HH}^{\frac{\phi-1}{\phi}} + (1-a)^{\frac{1}{\phi}} c_{HF}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}, & \text{if } \phi \neq 1 \\ c_{HH}^a c_{HF}^{1-a} & \text{if } \phi = 1 \end{cases} \quad (90)$$

Moreover, we assume  $a > \frac{1}{2}$  so that there is home bias in preferences. Assume prices are set so that at the symmetric approximation point, the labor wedge is zero so that the marginal utility from the perspective of the country and households coincide. Similarly to before, equilibrium zero order stock positions solve

$$2f - \gamma \text{cov}_t(\tilde{c}_H^H - \tilde{c}_F^F, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (91)$$

Next solve an expression for the planner solution. An approximation of the home Euler equation gives

$$f - \psi^{**} \text{cov}_t(\tilde{c}_{H,t+1}^H, \hat{r}_{t+1}) - \kappa \text{cov}_t(\tilde{c}_{F,t+1}^H, \hat{r}_{t+1}) + \psi_A^{**} \text{cov}_t(\tilde{A}_{H,t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (92)$$

The expressions for  $\psi^{**}$  and  $\kappa$  are given in the appendix. Similarly for the foreign country



$$-f - \psi^{**} \text{cov}_t(\tilde{c}_{F,t+1}^F, \hat{r}_{t+1}) - \kappa \text{cov}_t(\tilde{c}_{H,t+1}^F, \hat{r}_{t+1}) + \psi_A^{**} \text{cov}_t(\tilde{A}_{F,t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (93)$$

Deduct the conditions to yield:

$$2f - \psi^{**} \text{cov}_t(\tilde{c}_H^H - \tilde{c}_F^F, \hat{r}_{t+1}) - \kappa \text{cov}_t(\tilde{c}_F^H - \tilde{c}_H^F, \hat{r}_{t+1}) + \psi_A \text{cov}_t(\hat{A}_{t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (94)$$

To understand the connection between  $\tilde{c}_H^H - \tilde{c}_F^F$  and  $\tilde{c}_F^H - \tilde{c}_H^F$ , note that due to fixed prices we have exactly

$$\tilde{c}_H^H - \tilde{c}_F^F = \hat{C} \quad (95)$$

$$\tilde{c}_F^H - \tilde{c}_H^F = -\hat{C} \quad (96)$$

Plugging in we can write the equilibrium and planner conditions as

$$2f - \gamma \text{cov}_t(\hat{C}_{t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (97)$$

$$2f - \psi^{***} \text{cov}_t(\hat{C}_{t+1}, \hat{r}_{t+1}) + \psi_A \text{cov}_t(\hat{A}_{t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (98)$$

Where we defined  $\psi^{***} \equiv \psi^{**} - \kappa$ . The following lemma helps to map the discussion into previous results.

**Lemma 5.** *The following relationship holds:  $\gamma < \psi^{***} < \psi^{**}$ .*

For proof see appendix B.

The equation above is similar to that derived before in the case of productivity shocks. Because  $\psi^{***} > \gamma$ , again when the friction is large, the planner would benefit from increasing foreign stock position and reducing home stock positions. However, because  $\psi^{***} < \psi^{**}$  this mechanism is weaker.<sup>15</sup> As before higher returns for the home country imply increased demand for the home good by home households. However, this effect is smaller because at the same time lower returns by foreign households imply reduced demand for the home good by foreign households.

<sup>15</sup>Note, however, that the comparison is between  $\psi^{***}$  and  $\psi^{**}$  not between  $\psi^{***}$  and  $\psi$ . However, we could choose the parameters so that  $\psi^{**} = \psi$ .

### 5.3 Calvo Price Setting

The monetary policy problem with tradable and non-tradable goods and staggered price setting à la Calvo (1983) is in general complicated and beyond the scope of this paper. However, here we note that partial price adjustment can potentially alleviate the nominal rigidity problem.

Assume that a measure  $1 - \theta$  of firms can change their prices, while  $\theta$  of firms hold prices constant. The problem of each entrepreneur is

$$\max_{\{p_{i,j,NT,t}(x^t)\}} \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \Lambda_{i,t+k} \theta^k (p_{NT,j,t} - MC_{j,t+k}^n) Y_{t+k,j} \right] \quad (99)$$

Here

$$Y_{t,j} = c_{NT,i,t} \left( \frac{p_{NT,i,j,t}}{p_{NT,i,t}} \right)^{-\vartheta} \quad (100)$$

$$MC_{t,j} = \frac{W_{NT,i,t}(1 + \tau_{L,i})}{A_{NT,i,t}} \quad (101)$$

The FOC is

$$\mathbb{E}_t \left[ \sum_{k=0}^{\infty} \Lambda_{i,t+k} \theta^k \left( p_{NT,j,t} - \frac{\vartheta}{\vartheta - 1} MC_{j,t+k}^n \right) Y_{t+k,j} \right] = 0 \quad (102)$$

Next we show how partial price adjustment can alleviate the nominal rigidity problem. We can see this result as follows. The amount of variety  $i$  consumed is  $\left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} c_{NT}$ . For simplicity, we assume the monetary authority can choose the non-tradables prices that can be adjusted. Consider a strategy where the authority chooses the non-tradables prices so that the corresponding labor wedge is zero. In appendix B I show that then the risk aversion coefficient from the perspective of the country has the form

$$\psi^C = (1 - \theta)\gamma + \theta\psi^{C*} > \gamma \quad (103)$$

Here  $\psi^{C*} > \gamma$  is the risk aversion coefficient seen before modified for price dispersion. Here the price adjustment tends to tilt the risk aversion

coefficient downwards to  $\gamma$  and in this sense alleviate the nominal rigidity problem. However, depending on the equilibrium price distribution,  $\psi^{C^*}$  may be higher than  $\psi$ .

## 6 Appendix B: Proofs

### 6.1 Proof of Lemma 1

A second order approximation of the relative home and foreign Euler equations yields

$$\mathbb{E}_t \left[ \hat{r}_{t+1} + \frac{1}{2} [\tilde{r}_{t+1,F}^2 - \tilde{r}_{t+1,H}^2] - \gamma(\bar{p}, \bar{c}) \Delta \hat{c}_{t+1,T,H} \hat{r}_{t+1} - \gamma_p(\bar{p}, \bar{c}) \tilde{p}_{t,T} \hat{r}_{t+1} \right] + \mathcal{O}(\epsilon^3) = 0 \quad (104)$$

$$\mathbb{E}_t \left[ \hat{r}_{t+1} + \frac{1}{2} [\tilde{r}_{t+1,F}^2 - \tilde{r}_{t+1,H}^2] - \gamma(\bar{p}, \bar{c}) \Delta \hat{c}_{t+1,T,F} \hat{r}_{t+1} - \gamma_p(\bar{p}, \bar{c}) \tilde{p}_{t+1,T} \hat{r}_{t+1} \right] + \mathcal{O}(\epsilon^3) = 0 \quad (105)$$

where each term is 2nd order and

$$\gamma(\bar{p}, \bar{c}) = -c_T \frac{\frac{\partial U_T/p_T}{\partial c_T}}{U_T/p_T} \Big|_{(\bar{p}, \bar{c})} \quad (106)$$

and

$$\gamma_p(\bar{p}, \bar{c}) = -p_T \frac{\frac{\partial U_T/p_T}{\partial p_T}}{U_T/p_T} \Big|_{(\bar{p}, \bar{c})}. \quad (107)$$

Deducting the conditions gives

$$2f - \gamma \mathbb{E}_t[\hat{c}_{t+1} \hat{r}_{t+1}] + \mathcal{O}(\epsilon^3) = 0 \quad (108)$$

Lemma 2 shows that the conditional mean of  $\hat{r}_{t+1}$  is always zero. Therefore we have

$$2f - \gamma \text{cov}_t(\hat{c}_{t+1}, \hat{r}_{t+1}) + \mathcal{O}(\epsilon^3) = 0 \quad (109)$$

For generality assume a production function  $\chi(N)$ ,  $\chi'(\cdot) > 0$ ,  $\chi''(\cdot) \leq 0$ . Then the marginal utility in the country becomes

$$\hat{V}_c(c_T, p) = U_T(1 + \alpha(p)p) + U_N \frac{\partial \chi^{-1}}{\partial c_T} \alpha(p) \quad (110)$$

The planner's FOC w.r.t.  $S_{HH}$  gives

$$\mathbb{E}_0 \left[ \frac{\partial V_H(\mathbf{S}, p)}{\partial S_{HH}} - \frac{\partial V_F(\mathbf{S}, p)}{\partial S_{FH}} \right] = 0 \quad (111)$$

$$\Leftrightarrow \mathbb{E}_0 \left[ \frac{\partial \hat{V}_H(c_{H,T}, p)}{\partial c_{H,T}} d_H - \frac{\partial \hat{V}_F(c_{F,T}, p)}{\partial c_{F,T}} d_H e^{-f} \right] = 0 \quad (112)$$

Now a second order approximation gives

$$\text{Cov}(-\psi(\bar{p}, \bar{c}) \Delta \hat{c}_T, \tilde{d}) = -\tilde{f} \quad (113)$$

where

$$\begin{aligned} \psi(\bar{p}, \bar{c}) &= \bar{c}_T(1 + \alpha(p)p) \times \\ &= \frac{(\bar{\alpha} g_{2,1}(\bar{\alpha} \bar{c}_T, \bar{c}_T) + g_{2,2}(\bar{\alpha} \bar{c}_T, \bar{c}_T)) - \frac{h''(\chi^{-1}(\bar{\alpha} \bar{c}_T)) \left( \frac{\partial \chi^{-1}}{\partial c} \bar{\alpha} \right)^2 + h'(\chi^{-1}(\bar{\alpha} \bar{c}_T)) \frac{\partial^2 \chi^{-1}}{\partial c^2} \bar{\alpha}^2}{1 + \bar{\alpha} \bar{p}}}{g_2(\bar{\alpha} \bar{c}_T, \bar{c}_T)(1 + \bar{\alpha} \bar{p}) - h'(\chi^{-1} \bar{\alpha} \bar{c}_T) \frac{\partial \chi^{-1}}{\partial c_T} \bar{\alpha}} \end{aligned} \quad (114)$$

Now the stock position can be solved similarly to above.  $\square$

To see that  $\psi > \gamma$  notice:

$$\begin{aligned} \psi(\bar{p}, \bar{c}) &= \bar{c}_T \times \\ &= \frac{(\bar{\alpha} g_{2,1}(\bar{\alpha} \bar{c}_T, \bar{c}_T) + g_{2,2}(\bar{\alpha} \bar{c}_T, \bar{c}_T)) - \frac{h''(\chi^{-1}(\bar{\alpha} \bar{c}_T)) \left( \frac{\partial \chi^{-1}}{\partial c} \bar{\alpha} \right)^2 + h'(\chi^{-1}(\bar{\alpha} \bar{c}_T)) \frac{\partial^2 \chi^{-1}}{\partial c^2} \bar{\alpha}^2}{1 + \bar{\alpha} \bar{p}}}{g_2(\bar{\alpha} \bar{c}_T, \bar{c}_T) - \frac{h'(\chi^{-1}(\bar{\alpha} \bar{c}_T)) \frac{\partial \chi^{-1}}{\partial c_T} \bar{\alpha}}{1 + \bar{\alpha} \bar{p}}} \\ &> - \frac{\alpha g_{21}(\bar{\alpha} \bar{c}_T, \bar{c}_T) + g_{22}(\bar{\alpha} \bar{c}_T, \bar{c}_T)}{g_2(\bar{\alpha} \bar{c}_T, \bar{c}_T)} \bar{c}_T = \gamma(\bar{p}, \bar{c}) \end{aligned} \quad (115)$$

where we used  $h'(\cdot) > 0$  and  $h''(\cdot) > 0$ ,  $\frac{\partial \chi^{-1}}{\partial c} > 0$  and  $\frac{\partial^2 \chi^{-1}}{\partial c^2} \geq 0$  and the inequality assumes the denominator is positive.  $\square$

Assuming CRRA preferences over a CES aggregator, we have  $U_T = \left(\frac{p_T}{P}\right)^{1-\gamma\phi} a^{\frac{1}{\phi}} c_T^{-\gamma}$ .  $\gamma(\bar{p}, \bar{c}) = \gamma$ . Then the planner's risk aversion is globally higher. Assuming a linear production function we have

$$\psi(\bar{p}, \bar{c}) = \frac{\gamma + \left(\frac{p_T}{P}\right)^{\gamma\phi-1} a^{-\frac{1}{\phi}} h''\left(\frac{\bar{\alpha}\bar{c}_T}{A}\right) \left(\frac{\bar{\alpha}}{A}\right)^2 \bar{c}^{1+\gamma} (1 + \bar{\alpha}\bar{p})^{-1}}{1 - \left(\frac{p_T}{P}\right)^{\gamma\phi-1} a^{-\frac{1}{\phi}} h'\left(\frac{\bar{\alpha}\bar{c}_T}{A}\right) \frac{\bar{\alpha}}{A} / (1 + \bar{\alpha}\bar{p})} > \gamma \quad (116)$$

## 6.2 Proof of lemma 3

I consider a symmetric equilibrium. That is I assume stock positions are of the form  $(S, 1 - S, 1 - S, S)$  and compute utilities for different values of  $S$ . We assume prices are entirely fixed, allowing for a flexible tradables prices modifies the expressions only slightly. I first approximate  $V_{H,t}^{eq} = \mathbb{E}[g(c_{H,T,t}, \alpha(p)c_{H,T,t})]$ , where  $p$  is the relative price, around mean values  $\bar{c}$ ,  $\bar{p}$  that also correspond to the deterministic steady-state.

$$V_{H,t}^{eq} = \mathbb{E}[\bar{V} + U_c(\bar{c}, \alpha(\bar{p})\bar{c})\bar{c}\tilde{c}_{H,t} + \frac{1}{2}U_{cc}(\bar{c}, \alpha(\bar{p}))\bar{c}^2\tilde{c}_{H,t}^2] + \mathcal{O}(\epsilon^3) \quad (117)$$

Approximating the fee around 0 or assuming it is 2nd order,  $\bar{V}$  is independent of stock positions. Due to symmetry, we obtain for the sums of two utilities

$$V_{H,t}^{eq} + V_{F,t}^{eq} = \mathbb{E}\left[U_c(\bar{c}, \alpha(\bar{p}))\bar{c}(\tilde{c}_{H,t} + \tilde{c}_{F,t}) + \frac{1}{2}U_{cc}(\bar{c}, \alpha(\bar{p}))\bar{c}^2(\tilde{c}_{H,t}^2 + \tilde{c}_{F,t}^2)\right] + t.i.p. + \mathcal{O}(\epsilon^3) \quad (118)$$

Moreover, note that

$$\mathbb{E}[\tilde{c}_H + \tilde{c}_F] = -2(1 - S)p_S\tilde{f} \quad (119)$$

Note that  $U_c(\bar{c}, \bar{c}\bar{\alpha}) = g_1(\bar{c}, \bar{c}\bar{\alpha}) + g_2(\bar{c}, \bar{c}\bar{\alpha})\bar{\alpha}$ . However, by the intratemporal condition  $g_2 = g_1\bar{p}$ . Therefore  $U_c(\bar{c}, \bar{c}\bar{\alpha}) = g_1(\bar{c}, \bar{c}\bar{\alpha})(1 + \bar{\alpha}\bar{p})$ . Moreover,  $U_{cc}(\bar{c}, \bar{c}\bar{\alpha}) = g_{11}(\bar{c}, \bar{c}\bar{\alpha}) + g_{12}(\bar{c}, \bar{c}\bar{\alpha})\bar{\alpha} + g_{21}(\bar{c}, \bar{c}\bar{\alpha})\bar{\alpha} + g_{22}(\bar{c}, \bar{c}\bar{\alpha})\bar{\alpha}^2$ . Derivating the intratemporal condition one more time gives  $g_{21} = g_{11}\bar{p}$ . Moreover,  $g_{22} = g_{12}\bar{p}$ . Therefore,  $U_{cc}(\bar{c}, \bar{c}\bar{\alpha}) = (g_{11}(\bar{c}, \bar{c}\bar{\alpha}) + g_{12}(\bar{c}, \bar{c}\bar{\alpha})\bar{\alpha})(1 + \bar{\alpha}\bar{p})$ . Now by dividing by  $U_c(\bar{c}, \alpha(\bar{p}))\bar{c}$  and redefining the value function we obtain:

$$V_{H,t}^{eq} + V_{F,t}^{eq} = -2(1-S)p_S \tilde{f} - \frac{1}{2}\gamma[Var(\tilde{c}_{H,t}) + Var(\tilde{c}_{F,t})] + t.i.p. + \mathcal{O}(\epsilon^3) \quad (120)$$

Therefore we obtain

$$\begin{aligned} V(S) &= \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t [g(c_{H,T,t}(S), c_{F,NT,t}(S)) + g(c_{H,T,t}(S), c_{F,NT,t}(S))] \right] = \\ &= \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left[ -2(1-S)p_S \tilde{f} - \frac{1}{2}\gamma(Var(\tilde{c}_{H,t}) + Var(\tilde{c}_{F,t})) \right] \right] + t.i.p. + \mathcal{O}(\epsilon^3) \quad (121) \end{aligned}$$

The first order approximations of the budget constraints is used to evaluate the variance terms. The approximation for the value function of the planner is obtained using similar arguments.

ii) Follows easily by taking FOC.

### 6.3 Proof of Lemma 5

$\psi^{***} > \psi^{**}$  because  $\kappa > 0$ . To see that  $\psi^{***} > \gamma$ , note

$$\begin{aligned} \psi^{***} &= \psi^{**} - \kappa = -\frac{U_{TT}(1 + \bar{\alpha}\bar{p}) + U_{NN}\frac{1}{\bar{A}^2}\bar{c}_H^H + \frac{U_{NN}\frac{1}{\bar{A}^2}\bar{c}_F^H}{U_T}}{U_T} \\ &= \gamma(1 + \bar{\alpha}\bar{p}) - \frac{U_{NN}}{U_T\bar{A}^2}(\bar{c}_H^H - \bar{c}_F^H) > (1 + \bar{\alpha}\bar{p})\gamma > \gamma \end{aligned} \quad (122)$$

where we used  $U_{NN} < 0$ ,  $\bar{c}_H^H > \bar{c}_F^H$  (home bias in consumption) and  $\bar{\alpha}\bar{p} > 0$ .

### 6.4 Proof of Proposition 8

A 2nd order approximation of the relative Euler equation for the home asset yields

$$\tau_{t,FH}^S + f - \tau_{t,HH}^S - \gamma\mathbb{E}_t[\hat{c}_{t+1} - \hat{c}_t] - \gamma\mathbb{E}_t[\hat{c}_{t+1} - \hat{c}_t, \hat{r}_{t+1}] \quad (123)$$

$$+ \frac{\gamma_{pru}}{2}\mathbb{E}_t[\tilde{c}_{H,t+1}^2 - \tilde{c}_{F,t+1}^2] - \left[ \frac{\gamma_{pru}}{2} + \gamma^2 \right] [\tilde{c}_{H,t}^2 - \tilde{c}_{F,t}^2] + \mathcal{O}(\epsilon^3) = 0. \quad (124)$$

Here

$$\gamma_{pru}(\bar{p}, \bar{c}) = c_T^2 \frac{\partial^2 U_T}{\partial c_T^2} \Big|_{(\bar{p}, \bar{c})}. \quad (125)$$

$\gamma_{pru}$  can be seen as a measure of prudence, the convexity of marginal utility. The equation corresponding to the planner solution is given by

$$f - \psi \mathbb{E}_t[\hat{c}_{t+1} - \hat{c}_t] - \psi \mathbb{E}_t[\hat{c}_{t+1} - \hat{c}_t, \hat{r}_{t+1}] \quad (126)$$

$$+ \frac{\psi_{pru}}{2} \mathbb{E}_t[\tilde{c}_{H,t+1}^2 - \tilde{c}_{F,t+1}^2] - \left[ \frac{\psi_{pru}}{2} + \psi^2 \right] [\tilde{c}_{H,t}^2 - \tilde{c}_{F,t}^2] + \mathcal{O}(\epsilon^3) = 0. \quad (127)$$

Here

$$\psi_{pru}(\bar{p}, \bar{c}) = c_T^2 \frac{\partial^2 V_T}{\partial c_T^2} \Big|_{(\bar{p}, \bar{c})}. \quad (128)$$

Combining the equations, we obtain

$$\left[ \tau_{t,FH}^S - \tau_{t,HH}^S \right] \frac{1}{\gamma} = f \left[ \frac{1}{\psi} - \frac{1}{\gamma} \right] - \left[ \frac{\gamma_{pru}}{2\gamma} - \frac{\psi_{pru}}{2\psi} \right] \mathbb{E}_t[\tilde{c}_{H,t+1}^2 - \tilde{c}_{F,t+1}^2] + \quad (129)$$

$$\left[ \frac{\gamma_{pru}}{2\gamma} + \gamma - \frac{\psi_{pru}}{2\psi} - \psi \right] [\tilde{c}_{H,t}^2 - \tilde{c}_{F,t}^2] + \mathcal{O}(\epsilon^3) \quad (130)$$

or

$$\tau_{t,FH}^S - \tau_{t,HH}^S = f \left[ \frac{\gamma}{\psi} - 1 \right] - \gamma \left[ \frac{\gamma_{pru}}{2\gamma} - \frac{\psi_{pru}}{2\psi} \right] \mathbb{E}_t[\tilde{c}_{H,t+1}^2 - \tilde{c}_{F,t+1}^2] + \quad (131)$$

$$\gamma \left[ \frac{\gamma_{pru}}{2\gamma} + \gamma - \frac{\psi_{pru}}{2\psi} - \psi \right] [\tilde{c}_{H,t}^2 - \tilde{c}_{F,t}^2] + \mathcal{O}(\epsilon^3). \quad (132)$$

## 6.5 Calvo Price Setting

The marginal utility from the perspective of the country is:

$$g_2(\alpha(p)c_T, c_T)(1 + \alpha(p)p) - \int_0^1 h' \left( \frac{1}{A} \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \alpha(p)c_T \right) \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \frac{\alpha(p)}{A} di =$$

$$\int_0^1 g_2(\alpha(p)c_T, c_T)(1 + \alpha(p)p) - h' \left( \frac{1}{A} \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \alpha(p)c_T \right) \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \frac{\alpha(p)}{A} di \quad (133)$$

The fraction  $1 - \theta$  of firms can change their price. These prices are set to correspond to a zero labor wedge. Let us denote these firms by interval  $[0, 1 - \theta]$ . Now we get

$$\begin{aligned}
& \int_0^1 g_2(\alpha(p)c_T, c_T)(1 + \alpha(p)p) - h' \left( \frac{1}{A} \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \alpha(p)c_T \right) \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \frac{\alpha(p)}{A} di \\
&= \int_0^{1-\theta} g_2(\alpha(p)c_T, c_T)(1 + \alpha(p)p) - h' \left( \frac{1}{A} \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \alpha(p)c_T \right) \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \frac{\alpha(p)}{A} di + \\
& \int_{1-\theta}^1 g_2(\alpha(p)c_T, c_T)(1 + \alpha(p)p) - h' \left( \frac{1}{A} \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \alpha(p)c_T \right) \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \frac{\alpha(p)}{A} di \\
& \quad = (1 - \theta)g_2(\bar{\alpha}\bar{c}_T, \bar{c}_T) + \\
& \int_{(1-\theta)}^1 g_2(\alpha(p)c_T, c_T)(1 + \alpha(p)p) - h' \left( \frac{1}{A} \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \alpha(p)c_T \right) \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \frac{\alpha(p)}{A} di
\end{aligned} \tag{134}$$

Therefore the risk aversion coefficient has the form

$$\psi^C = (1 - \theta)\gamma + \theta\psi^{C*} \tag{135}$$

where

$$\begin{aligned}
\psi^{C*} &= -\frac{1}{1 - \theta} \frac{1}{g_2(\bar{\alpha}\bar{c}_T, \bar{c}_T)} \times \\
\frac{\partial}{\partial c_T} \int_{1-\theta}^1 g_2(\bar{\alpha}\bar{c}_T, \bar{c}_T)(1 + \alpha(p)p) - h' \left( \frac{1}{A} \left( \frac{p_{NT}^i}{p_{NT}} \right)^{-\vartheta} \alpha(p)c_T \right) \frac{p_{NT}^i}{p_{NT}} \frac{\alpha(p)}{A} di \Big|_{(\bar{c}, \{\bar{p}_i\}_{1-\theta}^1)} &> \gamma
\end{aligned} \tag{136}$$

## 6.6 Expressions for $\psi_A$ , $\psi_A^{**}$ , $\gamma_p$ , $\psi_p$ , $\psi^{**}$ , $\kappa$ , $\gamma_{pru}$ and $\psi_{pru}$

For simplicity assume a linear production function and zero labor wedge at the approximation point. Moreover assume the power type form posited for  $h$ . Now

$$\psi_A = \frac{-U_{NN} \frac{\bar{\alpha}^2}{\bar{A}^2} - U_N \frac{\bar{\alpha}}{\bar{A}}}{U_T} > 0. \tag{137}$$



$\psi_A^{**}$  takes the same form but  $U_{NN}$  and  $U_N$  are evaluated at  $\frac{\bar{c}_H^H + \bar{c}_F^H}{A}$ . Moreover,

$$\gamma_p = (\gamma\phi - 1) \left[ 1 - a \left( \frac{\bar{p}_T}{\bar{P}} \right)^{1-\phi} \right] + 1 \quad (138)$$

and

$$\psi_p = (\gamma\phi - 1) \left[ 1 + a \left( \frac{\bar{p}_T}{\bar{P}} \right)^{1-\phi} \right] [1 + \bar{p}\bar{\alpha}] + \bar{\alpha}\bar{p}(1 - \phi) - \frac{1}{\bar{A}} \frac{U_N}{U_T} (1 + \sigma) \bar{\alpha}\phi - 1 \quad (139)$$

and

$$\psi^{**} = - \frac{U_{TT}(1 + \bar{\alpha}\bar{p}) + U_{NN} \frac{1}{\bar{A}^2} \bar{c}_H^H}{U_T} \bar{c}_H^H, \quad (140)$$

where  $U_{NN}$  is evaluated at  $\frac{\bar{c}_H^H + \bar{c}_F^H}{A}$ . Moreover,

$$\kappa = - \frac{U_{NN} \frac{1}{\bar{A}^2} \bar{c}_F^H}{U_T} \bar{c}_F^H. \quad (141)$$

Finally, the prudence terms are given by

$$\gamma_{pru} = \frac{U_{TTT}^*}{U_T} \bar{c}_T^2, \quad (142)$$

where

$$U_{TTT}^* = \bar{\alpha}^2 g_{211}(\bar{\alpha}\bar{c}_T, \bar{c}_T) + \bar{\alpha} g_{212}(\bar{\alpha}\bar{c}_T, \bar{c}_T) + \bar{\alpha} g_{221}(\bar{\alpha}\bar{c}_T, \bar{c}_T) + g_{222}(\bar{\alpha}\bar{c}_T, \bar{c}_T). \quad (143)$$

and

$$\psi_{pru} = \frac{U_{TTT}^* - h''' \left( \frac{\bar{\alpha}\bar{c}_T}{\bar{A}} \right) \frac{\bar{\alpha}^3}{\bar{A}^3}}{g_2(\bar{\alpha}\bar{c}_T, \bar{c}_T) - h' \left( \frac{\bar{\alpha}\bar{c}_T}{\bar{A}} \right) \frac{\bar{\alpha}}{\bar{A}}} \bar{c}_T^2 \quad (144)$$

where we for simplicity assumed a linear production function.

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