Funding Global Environmental Public Goods through Multilateral Financial Mechanisms

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Abstract

This paper examines multilateral financial mechanisms (MFMs) that address global environmental public good problems. MFMs provide grants to promote sustainable development, jointly producing (global) public benefits and (domestic) private benefits. I model MFMs and show how to implement efficient outcomes through burden sharing. This is the first paper, to my knowledge, to design an efficient mechanism for contexts with joint production. Comparative static analysis reveals how grant disbursements affect public good provision. Counterintuitively, increasing grant funding to a country can decrease its contributions to environmental quality. Moreover, grant reallocations can have varied welfare consequences: they can be redistributive, Pareto improving, or immiserating. I demonstrate how welfare outcomes, both in terms of efficiency and equity, are shaped by the distribution of grants across countries and by substitute and complement relationships between goods.

1 Introduction

Global environmental problems like climate change, biodiversity loss, and deforestation all pose significant threats to humans and ecosystems alike. Yet despite the immense costs of environmental degradation, collective action problems have hindered international mitigation efforts. In response to these challenges, multilateral financial mechanisms (MFMs) have become increasingly prevalent as a means for providing global environmental public goods. The Global Environment Facility, Climate Investment Funds, and the emerging Green Climate Fund are just a few examples of such institutions.

MFMs play an important role by coordinating financing efforts for international public goods. In particular, they specify a burden sharing arrangement that outlines funding obligations for each member country. The funds are then distributed among member countries as grants to finance environmental projects. These grants help launch projects that jointly produce local (private) and global (public) benefits in line with the goal of sustainable development. For example, an MFM may fund green energy projects in China, which can enhance local environmental quality and create jobs in China while also helping meet global climate goals.

In this paper, I construct and analyze of model of MFMs to examine how their design affects incentives, environmental quality, and welfare. I show how a burden sharing system can be designed to yield a Pareto optimal equilibrium, and I also show how the allocation of grants affects equilibrium provision and welfare. As a result of joint production, efficiency and distribution are intertwined, and an MFM's decisions will have important consequences in both dimensions. I demonstrate how shifting grants across countries will, in some cases, simply benefit recipient countries at the expense of countries that lose grants. This result is intuitive and indicates that grant disbursements can be useful for addressing distributional concerns. However, depending on countries' preferences and the original distribution of grants, it is also possible for such a redistribution to cause Pareto improvements or even immiseration, whereby all countries are made worse off. As such, MFMs should take care when choosing disbursement amounts, as attempts to meet distributional goals can potentially cause inefficiencies.

International public good provision has been a topic of intensive study. However, prior analysis has focused primarily on treaties (e.g., Carraro & Siniscalco (1993); Barrett (1994); Hoel & Schneider (1997); Kolstad (2007)), examining participation and compliance in coalitions that provide public goods. Often, the utility or payoff function is modeled in a parsimonious manner with linearly separable benefits and costs (e.g., Nordhaus (2015); Barrett (2013); Harstad (2012)). Even those that allow for more flexible tradeoffs do not account for local co-benefits from public good production (e.g., Gerber & Wichardt (2009); Eyckmans (1999, 1997)). This paper considers a richer setting with more general preferences. Moreover, this model captures projects that jointly produce local and global benefits, a distinguishing feature of MFM sustainable development efforts.

On the latter point, this work builds on and extends an active literature on impure public goods, which are goods that jointly produce private and public benefits (Cornes & Sandler, 1984; Kotchen, 2005; Chan & Kotchen, 2014). Unlike prior literature, which focuses on analysis of suboptimal Nash equilibria, the present work demonstrates how international institutions can implement efficient outcomes through a burden sharing mechanism. This mechanism draws insights from prior research on efficient public goods provision, such as Lindahl (1958), Mas-Colell & Silvestre (1989), and Buchholz *et al.* (2011), but it applies to a novel context with impure public goods that have international spillovers.

Overall, this work highlights an important element of international public good provision that is typically ignored: that efforts to improve the global environment have local co-benefits. Thus, not only does it matter *how much* of a public good is provided, but *where* the public good is produced also has significant implications for both efficiency and equity. This work is directly applicable to the policy making process, as it shows how multilateral financial mechanisms can improve environmental quality while addressing the dual goals of efficiency and equity simultaneously.

2 Model

There are $i = 1, ..., n$ member countries in the MFM. Each country's preferences are represented by a strictly increasing and strictly quasiconcave utility function $u^i(x^i, l^i, G)$, where x^i represents i's domestic consumption, l^i captures local co-benefits from projects sited in country i, and G is global environmental quality. u is indexed by i, allowing for countries to have heterogeneous preferences.

Each country has income $w^i > 0$ that it spends on domestic consumption x^i and contributions c^i to the MFM's trust fund. The contributions are aggregated in the trust fund $C = \sum_{i=1}^n c^i$ and allotted to countries according to the grant disbursement system $\mathbf{s} = (s^1, ..., s^n)$, where s^i represents the proportion of the trust fund given to country i. Hence, i will receive s^iC to invest in sustainable development projects.

Projects y are priced at p, so that $py^i = s^iC$. Furthermore, $\sum_{i=1}^n s^i = 1$, so that all contributions to the trust fund are ultimately disbursed as project grants. These projects yield local benefits l as well as global benefits G. For example, l could represent local air quality improvements or local economic stimulus generated by a renewable energy project, while G could capture global climate change mitigation. The inclusion of l in the model implies that the production of public goods G can also have co-benefits l within producer countries, an expansion on the typical public good provision framework.[1]

To simplify analysis, I make several normalizations without loss of generality. First, I normalize the price of projects to $p = 1$. Second, I choose units of local co-benefits so that each project in country i generates one unit of local amenity l^i, and I choose units of G so that each project creates one unit of the global public good G. I assume throughout that all projects around the world are equally effective in providing G.[2] Given the aforementioned normalizations, the following identities will hold: $G = C$ and $l^i = s^i C = s^i G$.

In practice, MFMs make decisions in two stages. First, grant disbursements $\mathbf{s}$ are determined. Then, countries make their contribution decisions in the second stage according to a burden sharing rule.[3] I begin by modeling the second stage, taking $\mathbf{s}$ as exogenous. I will then describe how changes in the grant disbursement system in the first stage affect subsequent contributions.

Burden sharing

The MFM announces a burden sharing rule $\tau = (\tau^1, ..., \tau^n)$ and invites countries to convene and decide upon a target public good provision level $\tilde{G}$. τ specifies the proportion of $\tilde{G}$ to be funded by each country, and countries take this sharing rule as given. According to the agreement, country i is obliged to contribute $\tilde{c}^i = \tau^i \tilde{G}$, which will be supplemented by $\tilde{C}_{-i} = (1 - \tau^i)\tilde{G}$ from others, implying a matching rate of $\frac{1-\tau^i}{\tau^i}$ for i's contribution.

In practice, MFMs instate a *pro-rata provision* to ensure compliance with the burden sharing rule. This is common practice among MFMs like the GEF (GEF, 2013). The provision specifies that if a country falls short of its funding obligation $\tau^i \tilde{G}$, all other countries are permitted to reduce their contributions by the same proportion, thus maintaining the relative contributions specified by the burden sharing rule.[4,5] This provision plays a critical role in shaping incentives. In terms of the model, it implies that i will believe that its contributions c^i will be matched by others' contributions of $C_{-i} = \frac{1-\tau^i}{\tau^i} c^i$, not only when it gives the prescribed level of $\tilde{c}^i = \tau^i \tilde{G}$, but for any value $0 \leq c^i \leq \tilde{c}^i$ that it chooses.[6]

Given the sharing rule τ and grant disbursements $\mathbf{s}$, country i's private maximization problem is

$$\max_{x^i, c^i} u^i(x^i, l^i, G) \text{ subject to}$$

$$w^i \geq x^i + c^i; \quad G = c^i + C_{-i}; \quad C_{-i} = \frac{1 - \tau^i}{\tau^i} c^i; \quad \text{and} \quad l^i = s^i G.$$

[1]The present setup implies that countries only obtain l through contributions to the MFM's trust fund. This could arise under the plausible assumption that the MFM provides a cheaper means for producing l than direct investments. It is possible to generalize so that countries can invest in l independently of the MFM, but this would complicate analysis without providing additional insight. Chan & Kotchen (2014) show that the availability of additional impure public goods does not materially change the comparative statics of the model.

[2]This is a simplifying assumption the helps focus attention on the results of interest. Without this assumption, projects would tend to be sited in countries with the lowest costs of providing G for cost-effectiveness; even so, the primary insights from the paper will continue to hold.

[3]For example, the GEF pre-specifies grant disbursements through its System of Transparent Allocation of Resources (STAR) in order to give countries "predictability of funding and flexibility of programming" (GEF, 2010). Then countries contribute to the GEF's trust fund based on a burden sharing arrangement.

[4]This punishment is credible. Suppose some k deviates from its burden share (i.e., $c^k \leq \tilde{c^k} = \tau^k \tilde{G}$). Then any country $j \neq k$ can request a partial refund that reduces its contribution by the same proportion. That is, the allowable refund is such that j's final contribution satisfies $\frac{c^j}{c^k} \geq \frac{\tilde{c}^j}{\tilde{c}^k} \Leftrightarrow c^j \geq \frac{\tau^j}{\tau^k} c^k$. Given the opportunity, each country has the incentive to accept its refund because $MRS(x_i, \tilde{G}) < 1$, as long as $\tilde{G}$ exceeds the level of public good provision in the uncoordinated Nash equilibrium. Thus, $c^j = \frac{\tau^j}{\tau^k} c^k$ for all j after refunds, and relative contributions specified by the burden sharing rule will be maintained.

[5]Gerber & Wichardt (2009) also employ a deposit-refund mechanism to encourage public good provision. Although the specific mechanism differs, the deposit-refund system is useful in both cases for deterring deviation from a prescribed goal.

[6]$c^i > \tilde{c}^i$ will not be individually rational.

Combining the first three constraints and the normalizations above, this problem can be reframed as a choice of x^i, l^i, and G:

$$\max_{x^i,l^i,G} u^i(x^i, l^i, G) \text{ subject to}$$
$$w^i \geq x^i + \tau^i G \quad \text{and} \quad l^i = s^i G.$$

The first order condition is $MRS^i(x^i, G) \equiv \frac{s^i u_l(\cdot)+u_G(\cdot)}{u_x(\cdot)} = \tau^i$, where subscripts denote derivatives. Taking burden shares as given, each country will have a most-preferred level of global environmental quality captured by its demand function $G^i_\tau(\tau^i, w^i, s^i)$. That is, country i's demand depends only on its own burden share, income, and grant disbursement. For any given set of burden shares τ, countries may differ in their demand for global environmental quality G. However, burden shares can be adjusted to achieve agreement on the desired G.[7]

Definition 1. A burden sharing equilibrium is a set of burden shares $(\tau^{1*}, ..., \tau^{n*})$ and a corresponding allocation $(x^{1*}, ..., x^{n*}, l^{1*}, ..., l^{n*}, G^*)$ for which

a) $G^* = G^1_\tau(\tau^1, w^1, s^1) = ... = G^n_\tau(\tau^n, w^n, s^n)$;

b) $x^{i*} = w^i - \tau^{i*} G^*$ and $l^{i*} = s^i G^*$ for all i; and

c) $\sum_{i=1}^n \tau^{i*} = 1$.

In burden sharing equilibrium, there is consensus regarding the desired level of global environmental quality G^* (condition a). This ensures that all countries have the incentive to contribute their specified shares. Moreover, countries' budget and production constraints are met (condition b), and the public good is fully funded (condition c). The burden sharing equilibrium is akin to a Lindahl equilibrium in a context with joint production.

Assumption: (Monotonicity) Preferences are such that $\frac{\partial G^i_\tau}{\partial \tau^i} < 0$. That is, global environmental quality is an ordinary good that follows the law of demand.

Proposition 1. *For a given* $\mathbf{s}$*, there exists a unique burden sharing equilibrium. This burden sharing equilibrium can be implemented through the sharing rule* $\tau^* = (\tau^{1*}, ..., \tau^{n*})$*, and the resultant allocation* $(x^{1*}, ..., x^{n*}, l^{1*}, ..., l^{n*}, G^*)$ *is Pareto optimal.*

A proof is provided in the Appendix. Notably, this allocation arises naturally from countries' self-regarding actions when the pro-rata provision is in place and the appropriate burden sharing rule is announced. Hence, the optimal allocation (for a given $\mathbf{s}$) can be implemented by the MFM using the sharing rule $\tau^* = (\tau^{1*}, ..., \tau^{n*})$. Because $\tau^i < 1$ for all i, global public good provision G^* will necessarily be larger under burden sharing than in Nash equilibrium, thus solving the underprovision problem.

I will refer to τ^* as the $\mathbf{s}$-optimal sharing rule, as it is efficient conditional on $\mathbf{s}$. This sharing rule can be derived as a function of the primitives of the model (income, grant disbursements, and utilities), as shown in the proof of this proposition. Given the generality of this derivation, MFM can use this in wide-ranging contexts as an algorithm for assigning burden shares τ^*, thereby achieving consensus on the optimal G^*.

[7] Alternatively, countries could directly negotiate over τ so that there will be consensus on G.

Disbursements

So far, I have focused on optimal burden sharing for a given set of grant disbursements $\mathbf{s}$. In practice, MFMs often choose grant disbursements to fulfill distributional or equity criteria (GEF, 2010). As I show below, disbursements can help meet these goals, but they can also have important efficiency implications.

First, consider how a change in grant disbursement s^i affects a single country's demand for global environmental quality G^i_τ.

Lemma 1. *A country's demand for environmental quality G^i_τ can increase or decrease in its grant disbursement s^i.*

Demand for G does not necessarily increase in s^i. Rather, complement and substitute relationships shape the demand response. The general proof of this lemma requires only a slight modification to the virtual price method of Cornes & Sandler (1984, 1994), so a full exposition is excluded for the sake of brevity. Instead, let us focus on a particular case to illustrate this point most clearly.

Consider countries with constant elasticity of substitution (CES) preferences with the general form: $u^i(x^i, l^i, G) = \left(\alpha_i (x^i)^\rho + \beta_i (l^i)^\rho + \gamma_i G^\rho\right)^{1/\rho}$, where $\rho \leq 1$. Here, $\frac{\partial G^i_\tau}{\partial s^i} > 0$ if $0 < \rho \leq 1$, i.e., if x^i, l^i, and G are substitutes (a full derivation is given in the Appendix). However, $\frac{\partial G^i_\tau}{\partial s^i} < 0$ when the goods are complements ($\rho < 0$). Hence, a country's response to changes in grants s^i depends critically on the relationship between private and public goods. The intuition is as follows: as s^i increases, l^i becomes easier to obtain (cheaper). If domestic consumption x^i is a complement (substitute) for local co-benefits l^i, then i will demand more (less) x^i. As such, i will allocate more of its budget to x^i at the expense of contributions to the trust fund, resulting in lower global environmental quality.[8]

It follows that *equilibrium* provision may also increase or decrease when grant disbursements are altered, as the equilibrium response is the composition of individual responses. That is, $\frac{dG^*}{ds^i}$ can be positive or negative, and the direction of change will depend on the magnitudes of (i) changes in grant disbursements and (ii) the demand elasticity with respect to grants across countries (which in turn depends on complement/substitute relationships as described before). In equilibrium, the optimal burden shares adjust as $\mathbf{s}$ changes, and the magnitude and direction of these changes will also depend on these two factors.

Welfare

The preceding analysis sets the stage for welfare analysis, which is critical to assessing the desirability of any grant disbursement system. Importantly, the choice of grant disbursements has significant implications for welfare, both in terms of efficiency and distribution.

Recall that the $\mathbf{s}$-optimal burden sharing rule will yield an optimal allocation of goods, *conditional on* $\mathbf{s}$. Interestingly, the burden sharing equilibrium associated with one value of $\mathbf{s}$ may be Pareto dominated by the burden sharing equilibrium associated with another value of $\mathbf{s}$; therefore, the choice of grants affects efficiency, and some vectors of disbursements may be Pareto inferior.

For parsimony, consider a simple world with two countries $i = 1, 2$ that are assigned grant disbursements s^1 and s^2.[9] I can derive optimal burden shares $\tau^{i*}(G^*, w^i, s^i)$, which are based on the preferences of each country as well as incomes and grant disbursements. In the burden sharing equilibrium, the following conditions will

[8] For more general preferences, the sign of $\frac{\partial G^i_\tau}{\partial s^i}$ is ambiguous because changes in s^i affect both virtual prices for l and G; the net effect will depend on the relative strength of own-price and cross-price effects. CES preferences offer an informative boundary case that sharply identifies the role of substitute/complement relationships.

[9] Including more countries complicates exposition without materially affecting the conclusions.

hold for $i = 1, 2$:

$$\begin{aligned} \tau^{1*} + \tau^{2*} &= 1 \\ G^{1*} = G^{2*} &= G^* \\ s^1 + s^2 &= 1 \\ x^{i*} + \tau^{i*} G^* &= w^i \\ s^i G^* &= l^{i*} \end{aligned}$$

Consider an adjustment in grant disbursements and the resultant effect on each country's utility. Taking the total derivative of the utility function, I obtain

$$du^i = u_x^i dx^* + u_l^i dl^* + u_G^i dG^*.$$

Proposition 2. *A redistribution of grants from one country to another can increase utility for both countries (Pareto improvement), decrease utility for both countries (immiseration), or increase utility for one country while decreasing utility for the other (inter-country utility tradeoff).*

This result arises because the change in grant disbursements $\mathbf{s}$ has two effects in equilibrium: it changes the quantity of local co-benefits l^i enjoyed by a country, and it also changes the burden share τ^{i*} assigned to that country based on the $\mathbf{s}$-optimal burden sharing rule. As such, the shift in grants has the following effects (see Appendix for derivation):

$$\frac{du^1}{ds^1} = \underbrace{-u_x^1 G^* \frac{d\tau^{1*}}{ds^1}}_{\text{cost of larger burden share}} + \underbrace{u_l^1 G^*}_{\text{benefit of additional grants}} \tag{1}$$

$$\frac{du^2}{ds^1} = \underbrace{-u_x^2 G^* \frac{d\tau^{2*}}{ds^1}}_{\text{benefit of reduced burden share}} - \underbrace{u_l^2 G^*}_{\text{cost of lost grants}} . \tag{2}$$

Clearly, the net effect on a country's welfare will depend on the relative benefits and costs of the grant redistribution in equilibrium. We can shed further light on this tradeoff by rearranging Expressions 1 and 2. An increase in s^1 and accompanying decrease in s^2 will improve utility for both countries if

$$\frac{u_l^1}{u_x^1} > \frac{d\tau^{1*}}{ds^1} \tag{3}$$

$$\frac{u_l^2}{u_x^2} < -\frac{d\tau^{2*}}{ds^1}. \tag{4}$$

Thus, if the value of additional environmental projects outweighs the cost of taking on a larger burden share, then a country will benefit from receiving more grants. Similarly, for a country that loses grants, utility will increase if its burden share decreases enough to outweigh the loss of environmental projects.

There are two primary features that shape these welfare outcomes: complement/substitute relationships between private and public consumption and the initial distribution of grants.

Complement and substitute relationships

Welfare outcomes will depend upon whether countries treat private and public consumption as complements or substitutes. The following two corollaries assume CES preferences to illustrate this point.

Corollary 1. *If l and G are complements, then a reallotment of grants will increase utility for the recipient country at the expense of the country that loses grants. Pareto improvements and immiseration are not possible.*

If l and G are complements, $\frac{d\tau^{i*}}{ds^i} < 0$, so the equilibrium burden share τ^{i*} is decreasing in s^i. Therefore, a country that receives additional grants will also be assigned a *lower* burden share in equilibrium, both of which enhance the country's utility. On the other hand, its counterpart will be unambiguously worse off; that country will lose grants and also be assigned a higher burden share. In essence, this change in grant allocation redistributes welfare from one country to the other.

Corollary 2. *If l and G are substitutes, then a reallotment of grants can have a wide range of welfare outcomes, as in Proposition 2.*

If l and G are substitutes, $\frac{d\tau^{i*}}{ds^i} > 0$, and a redistribution of grants will have countervailing effects on each country's utility. The country that gains grants will also be faced with a larger burden share, while the country that loses grants will benefit from a lower burden share. In such a setting, Pareto improvements and immiseration are both possible, as is a simple redistribution of welfare. The net welfare consequences will depend on the relative magnitudes of these effects and the initial distribution of grants, as discussed below.

These corollaries have important ramifications, as countries at different stages of development will face distinct tradeoffs between private consumption and environmental quality. Whereas very poor countries may substitute between environmental quality and economic growth, wealthier ones may see the two as complementary. The reasoning behind this mirrors the logic of the environmental Kuznets curve (Dinda, 2004). For example, clean air and clean water may become increasingly important to a growing middle class that seeks outdoor recreation activities, and these environmental amenities can complement other domestic investments like education and transportation infrastructure. For the poorest countries, on the other hand, it is likely that domestic expenditures (e.g., education, infrastructure, public health programs) substitute for environmental quality due to the necessity of such projects (Shibayama & Fraser, 2014).

Distribution of grants

The distribution of grants will also influence welfare, both in terms of efficiency and equity. The following example helps elucidate this point, and it also provides further clarity on the tradeoffs between grant reallocations and changes in burden shares.

Example 1. Suppose the two countries have identical preferences defined by $u^i(x^i, l^i, G) = x^i + \sqrt{l^i} + \sqrt{G}$. Assuming an interior solution, I can compute optimal burden shares and equilibrium environmental quality:

$$\tau^{i*} = \frac{\sqrt{s^i} + 1}{\sqrt{s^1} + \sqrt{s^2} + 2}$$

$$G^* = \left(\frac{\sqrt{s^1} + \sqrt{s^2} + 2}{2}\right)^2.$$

u^1 increases if

$$2\sqrt{s^1}\sqrt{1 - s^1} + 1 - 2s^1 + 3\sqrt{1 - s^1} - \sqrt{s^1} > 0, \tag{5}$$

which holds if $0 < s^1 < b$, where $b \approx 0.875$. Likewise, u^2 increases if

$$\sqrt{1-s^1} + 1 - 2\sqrt{s^1}\sqrt{1-s^1} - 3\sqrt{s^1} - s^1 > 0, \tag{6}$$

which holds if $0 < s^1 < a$, where $a \approx 0.125$.

Corollary 3. *If the allotment of grants is sufficiently skewed, then redistributing grants from a grant-rich country to a grant-poor country will yield Pareto improvements. Redistributing grants from a grant-poor country to a grant-rich country will cause immiseration.*

Intuitively, a country with very few grants will have scarce local projects, which makes the marginal benefits of s^i high. Redistributing grants to increase this country's disbursement would greatly enhance its utility, outweighing the fact that it will have a larger burden share in equilibrium. Meanwhile, a country with a large wealth of grants will also benefit from the redistribution; although it suffers a loss of local projects and associated co-benefits, this loss is more than compensated by the fact that its counterpart will take on a larger burden share. Therefore, grant reallocations can potentially benefit all parties.

For the example above, consider $0 < s^1 < a$, where country 2 has the majority of grants and country 1 has relatively few. Here, a transfer of grants from country 2 to country 1 will yield a Pareto improvement. Country 1 will have to take on a larger contribution burden in equilibrium (cost of larger burden share). However, this is outweighed by the value of additional grants and co-benefits that it receives (benefit of additional grants), and in net, u^1 will increase according to Expression 5. Meanwhile, country 2 loses grant funding (cost of lost grants), but this is more than offset by the fact that its burden share will be lowered as country 1 begins to contribute more (benefit of reduced burden share). This yields a higher utility u^2, as well (Expression 6). Thus, in the presence of large disparities, equalizing grant transfers can improve welfare for both.

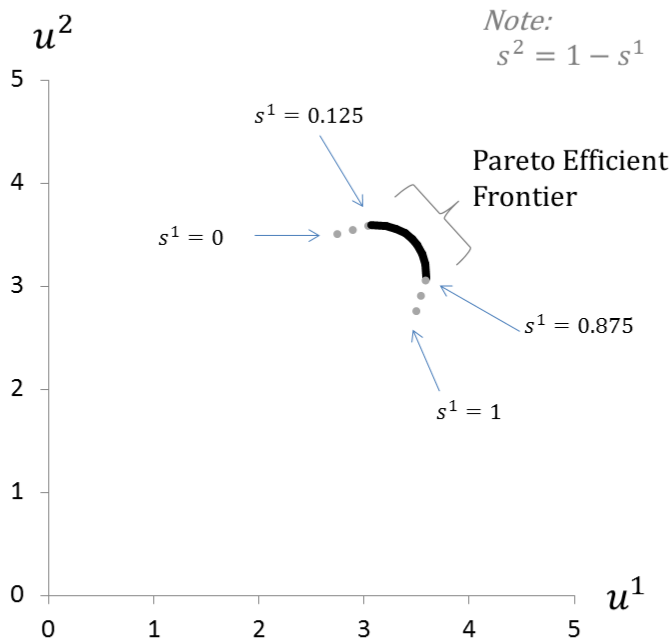
The converse occurs when $s^1 > b$. In this case, a transfer of grants from country 2 to country 1 magnifies the disparities across countries, leading to lower utilities for both. Lastly, if the grant disbursements are such that $a < s^1 < b$, such a grant transfer will increase country 1's utility while decreasing country 2's. That is, when countries' receipts from the trust fund are comparable, adjustments to grant allocations simply redistribute welfare across countries.

Corollary 4. *There exist a multitude of Pareto optimal equilibria that can be implemented by varying grant disbursements.*

Figure 1 illustrates the utilities that arise in burden sharing equilibrium for different values of $\mathbf{s}$ if countries have incomes $w^1 = w^2 = 2$. Highly unequal disbursements (i.e., when $0 < s^1 < a$ or $s^1 > b$) will implement Pareto-dominated equilibria. However, there also exists a wide range of grant disbursements that will yield allocations along a Pareto frontier ($a < s^1 < b$).

Thus, when disbursements are in the range $a < s^1 < b$, grants can be reallocated to achieve different distributional outcomes without sacrificing efficiency, a fact that can be useful for application. In a typical pure public good setting without joint production, there is a unique Pareto optimal allocation that can be achieved through Lindahl pricing; it is well known that the allocation can only be changed through lump sum transfers (see for example Cornes & Sandler (1996)). Here, even without direct transfers of income, there is a wider array of possibilities. As a virtue of this feature, the MFM is empowered to select among equilibria based on its social welfare function. By adjusting grant disbursements within this range, the MFM can influence the final allocation of goods and satisfy distributional goals without distortionary effects.

Figure 1: Equilibrium utilities from burden sharing under varying grant allotments. The bold curve is the Pareto frontier. The dotted lines indicate Pareto-dominated allocations.



3 Discussion

This paper has developed and analyzed a novel model of multilateral financial mechanisms. Unlike prior work on global public goods, the analysis here considers a scenario with joint production of local (private) and global (public) benefits, a critical feature of sustainable development efforts and MFM projects.

I show how optimal outcomes can be achieved through burden sharing, and I analyze the comparative statics of the model to reveal how grant disbursements affect environmental quality and welfare. In practice, MFMs adjust grant disbursements to address distributional or equity concerns. While such adjustments can affect distributional outcomes, I show that they also have important consequences for efficiency, a critical point that is missed by standard analyses that ignore the joint production of public and private goods inherent in sustainable development efforts.

In general, welfare outcomes will depend upon both the distribution of grants as well as the preferences of countries, and the preceding analysis reveals different facets of these tradeoffs. Understanding these relationships is critical to assessing the performance of an MFM. These results are particularly germane to assessing the GEF's design, and historical adjustments to grant disbursements can be considered in this light. For example, the fourth replenishment of the GEF assigned over 38% of grants to Brazil, Russia, India, China, and South Africa (BRICS) (GEF, 2010). However, in the sixth replenishment, grants to the BRICS comprised a much smaller percentage of overall funding at 25% (GEF, 2014). Instead, there have been widespread increases in grants to countries in Africa, the Caribbean, Eastern Europe, Central Asia, and Pacific Islands. This redistribution of course benefits recipient countries. However, it may also help BRICS countries, like India and South Africa, that experience a corresponding decrease in burden shares. Thus, such changes in grant disbursements need not raise objections from countries that lose grants; those countries can in fact gain from such a redistribution if the MFM is attentive to the tradeoffs discussed in this paper.

The model presented here has several limitations that open avenues for further research. In the model, the MFM chooses grant disbursements, and countries in turn take these as given when making contribution decisions; however, in actuality, the MFM's decision-making is not entirely autonomous, so countries may strategically influence grant disbursements. Participation in MFMs is also voluntary, a consideration that I have abstracted away from in this analysis to allow a sharper focus on joint production. As with other similar cost sharing mechanisms (Lindahl, 1958; Mas-Colell & Silvestre, 1989), information asymmetry and preference revelation remain challenges for achieving optimality in practice. Future work can examine these issues in a context with jointly produced private and public benefits to provide a more complete picture of MFMs.

It should also be noted that MFMs like the Global Environment Facility, Climate Investment Funds, and Green Climate Fund have raised only modest amounts to date; the size of these funds is well below the sums necessary to solve global environmental crises like climate change. Even so, MFMs provide a useful framework for public good provision. MFMs can facilitate cooperation between countries and create a foundation for subsequent treaty-making. Moreover, they highlight the local benefits of sustainable development projects and their accompanying welfare implications, which are ignored in standard models of global public good provision. From a practical perspective, MFMs are also attractive because they contract upon contributions to a group fund, which are easily observable; this contrasts with treaties, where agreements are based on emissions or abatement levels, which are more difficult to observe and enforce. Thus, the preceding analysis shows how MFMs can be designed to address environmental challenges, and it also offers more general lessons, both positive and normative, for global public good provision.

4 Appendix

4.1 Burden sharing equilibrium

Buchholz *et al.* (2008) explain how an analogous method to the replacement function approach (Cornes *et al.*, 1999; Cornes & Hartley, 2007) can be used to establish Lindahl equilibrium, and I adapt their methodology here for a setting with joint production. By assumption, G is monotonic and decreasing in τ, so I can invert $G^i_\tau(\tau^i, w^i, s^i)$ to obtain $\tau^i = \phi^i(G, w^i, s^i)$, where ϕ^i represents the marginal willingness to pay for G. Assuming that $\sum_{i=1}^n \phi^i(0, w^i, s^i) \geq 1$,[10] it follows from the intermediate value theorem that there exists a unique G^* such that

$$\sum_{i=1}^{n} \phi^i(G^*, w^i, s^i) = 1. \tag{7}$$

From this, I can solve for G^*, τ^*, $\mathbf{x}^*$ and $\mathbf{l}^*$, where $\tau^{i*} = \phi^i(G^*, w^i, s^i)$, $x^{i*} = w^i - \tau^{i*}G^*$, and $l^{i*} = s^i G^*$ for all i.

By definition of $G^i_\tau(\tau^i, w^i, s^i)$, $MRS^i(w^i - \tau^i G^i_\tau, G^i_\tau) = \tau^i$. In burden sharing equilibrium, $G^* = G^i_\tau$ and $x^{i*} = w^i - \tau^{i*}G^*$, so individual rationality is satisfied according to $MRS^i(x^{i*}, G^*) = \tau^{i*}$. Moreover, $\sum_{i=1}^n \tau^{i*} = 1$ holds, so it follows that the Samuelson condition is satisfied ($\sum_{i=1}^n MRS^i(x^{i*}, G^*) = 1$), and total contributions cover the cost of the public good. Therefore, the burden sharing equilibrium is $\mathbf{s}$-optimal, i.e., it yields a Pareto optimal allocation given the first stage choice of $\mathbf{s}$.

4.2 Constant elasticity of substitution preferences

Suppose all agents have CES preferences $u^i(x^i, l^i, G) = \left(\alpha_i (x^i)^\rho + \beta_i (s^i G)^\rho + \gamma_i G^\rho\right)^{1/\rho}$ with $\rho \leq 1$. The first order condition defines $\tau^i(G, w^i, s^i)$ implicitly as $\tau^i = \frac{\beta_i (s^i)^\rho + \gamma_i}{\alpha_i}\left(\frac{w^i}{G} - \tau^i\right)^{1-\rho}$ for all i. In equilibrium, $\sum \tau^i(G^*, w^i, s^i) = 1$, so I have $n+1$ equations to solve for $n+1$ unknowns $(\tau^{1*}, .., \tau^{n*}, G^*)$. I can differentiate to obtain:

$$\begin{aligned}
\frac{\partial \tau^{i*}}{\partial s^i} &= \rho \frac{\beta_i (s^i)^{\rho-1}}{\alpha_i}\left(\frac{w^i}{G^*} - \tau^{i*}\right)^{1-\rho} - \frac{\partial \tau^i}{\partial s^i}\frac{\beta_i (s^i)^\rho + \gamma_i}{\alpha_i}(1-\rho)\left(\frac{w^i}{G^*} - \tau^{i*}\right)^{-\rho} \\
&= \rho \cdot \frac{\frac{\beta_i (s^i)^{\rho-1}}{\alpha_i}\left(\frac{w^i}{G^*} - \tau^{i*}\right)^{1-\rho}}{1 + \frac{\beta_i (s^i)^\rho + \gamma_i}{\alpha_i}(1-\rho)\left(\frac{w^i}{G^*} - \tau^{i*}\right)^{-\rho}}
\end{aligned}$$

Note that the term expressed as a fraction is positive, so $\frac{\partial \tau^i}{\partial s^i}$ has the same sign as ρ. Therefore, the response of τ^{i*} to s^i depends critically on whether goods are complements or substitutes, as captured by the sign of ρ. τ^{i*} increases in s^i if the goods are substitutes $(0 < \rho \leq 1)$ and decreases in s^i if the goods are complements $(\rho < 0)$. As $\rho \to 0$, this approaches a Cobb-Douglas utility function in the limit, yielding $\frac{\partial \tau^{i*}}{\partial s^i} \to 0$.

[10]When $\sum_{i=1}^n \phi^i(0, w^i, s^i) < 1$, there is a degenerate case in which equilibrium public good provision is trivially 0, as the marginal cost of provision exceeds the marginal willingness to pay.

4.3 Two-country example for utility changes arising from changes in grant disbursements

In equilibrium, the following conditions must be met:

$$\begin{aligned} \tau^{1*} + \tau^{2*} &= 1 \\ s^1 + s^2 &= 1 \\ x^i + \tau^i G^* &= w^i \\ s^i G^* &= l^i \end{aligned}$$

From these conditions, I can derive expressions for utility changes. From the budget and technology constraints I have

$$\begin{aligned} dx^i &= dw^i - (\tau^i dG^* + G^* d\tau^i) \\ dl^i &= s^i dG^* + G^* ds^i \end{aligned}$$

Substituting these into the expression for the derivative of utility $du^i = u_x^i dx + u_l^i dl + u_G^i dG^*$, I obtain

$$\begin{aligned} du^i &= u_x^i(dw^i - (\tau^i dG^* + G^* d\tau^i)) + u_l^i(s^i dG^* + G^* ds^i) + u_G^i dG^* \\ &= (-\tau^i u_x + s^i u_l^i + u_G^i)dG^* + u_x^i dw^i - u_x^i G^* d\tau^i + u_l^i G^* ds^i \\ &= u_x^i dw^i - u_x^i G^* d\tau^i + u_l^i G^* ds^i. \end{aligned}$$

Note that $(-\tau^i u_x + s^i u_l^i + u_G^i)dG^* + u_x^i dw^i - u_x^i G^* d\tau^i + u_l^i G^* ds^i = 0$ by the country's first order condition.

From $s^1 + s^2 = 1$, I have $ds^2 = -ds^1$. Therefore, the total derivatives of utility with respect to s^1 are

$$\begin{aligned} \frac{du^1}{ds^1} &= -u_x^1 G^* \frac{d\tau^1}{ds^1} + u_l^1 G^* \\ \frac{du^2}{ds^1} &= -u_x^2 G^* \frac{d\tau^2}{ds^1} - u_l^2 G^*. \end{aligned}$$

Therefore, a change in grant disbursements that increases s^1 (and decreases s^2) will be Pareto improving if both

$$\begin{aligned} \frac{u_l^1}{u_x^1} &> \frac{d\tau^1}{ds^1} \\ \frac{u_l^2}{u_x^2} &< -\frac{d\tau^2}{ds^1}. \end{aligned}$$

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