The Distributional Consequences of Trade Liberalization:
Consumption Tariff versus Investment Tariff Reduction*

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Abstract

This paper uses numerical simulations to highlight the contrasting effects of consumption and investment tariff reductions on the dynamic adjustments of key measures of aggregate activity and inequality. The consumption tariff has only a weak effect on activity. If implemented instantaneously it leads to a negligible reduction in wealth inequality but a substantial increase in income inequality. If gradual, it causes a more significant decline in wealth inequality but a milder increase in income inequality. A comparable reduction in an investment tariff increases activity significantly. It leads to a significant long-run reduction in wealth inequality if implemented instantaneously, which is moderated if introduced gradually. It is associated with a tradeoff between the short-run and long-run effects on income inequality, reducing it in the very short run, while increasing it slightly over time. The simulations are supplemented with extensive sensitivity analysis, suggesting some sensitivity to key structural parameters.

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1. Introduction

The last three decades have witnessed dramatic trade liberalization and in particular tariff reductions. Between 1984 and 2010 average tariff rates declined from around 22% to around 6%, with an average of around 10% for the entire period.\(^1\) Over a similar period, the comprehensive database developed by Milanovic (2014) indicates that income inequality within countries, as measured by the Gini coefficient has increased steadily from around 0.30 to 0.45. This apparent correlation raises the important question concerning the nature of the relationship between these two variables, an issue that has recently been receiving increased attention in the literature, both from a theoretical standpoint, as well as empirically.

Two alternative mechanisms immediately come to mind. The first, based on the traditional Heckscher-Ohlin trade model, suggests that given differences in the relative endowments of skilled versus unskilled labor between developed and developing economies, a tariff reduction will likely reduce inequality in a developing economy, but increase it in an advanced economy; see Jaumotte, Lall, and Papageorgiou (2013). Alternatively, using a standard one-sector growth model with accumulating physical capital and endogenous labor supply, Rojas-Vallejos and Turnovsky (2015) show that the increase in labor supply following a tariff reduction will tend to increase the return to capital over time, thereby encouraging capital accumulation and increasing income inequality.

On the empirical side, there is a growing literature exploring the relationship between various trade liberalization policies and inequality and yielding a range of results.\(^2\) For example, Savvides (1998) finds that among less developed countries, more open economies experienced increased income inequality during the late 1980s. However, he found that trade policy has had no effect on income inequality in developed countries. Harrison and Hanson (1999) find that trade reform has increased wage inequality for the case of Mexico while Beyer, Rojas, and Vergara (1999) show that for Chile, openness widens the wage gap between unskilled and skilled workers, which in turn raises income inequality. Milanovic and Squire (2005) find that tariff reduction is associated with more wage

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\(^1\) These estimates are based on the data on Trade and Import Barriers data at the World Bank compiled by Francis K.T. Ng consisting of 170 countries over the period 1981-2010.

\(^2\) Much of this literature, particularly as it related to developing countries is reviewed by Goldberg and Pavcnik (2007).
inequality in poorer countries, while the reverse applies in richer countries, contradicting the predictions of the Heckscher-Ohlin model. Bourguignon and Morrison (1990) analyze the relationship between income distribution and foreign trade in developing economies, with one of their findings being that protectionism seems to increase income inequality. Edwards (1997) presents a similar argument. Using data from Deininger and Squire (1997), he shows that the correlation between trade distortions and inequality is positive although not strongly statistically significant. His analysis seems to be robust to different measures of trade openness despite data limitations.

Several recent studies use cross-country panel data sets to determine the causes of rising income inequality. Jaumotte, Lall, and Papageorgiou (2013) find that trade liberalization is associated with lower income inequality, with most of the observed increase being due primarily to the impact of technological change. In contrast, Rojas-Vallejos and Turnovsky (2016) obtain a positive relationship between tariff liberalization and income inequality, the differences likely reflecting the differences in the data sets used. Finally, Lim and Mc Nelis (2014) use a panel to estimate the impact of trade-openness and other controllers on the income Gini coefficient. They find a non-monotonic relationship that depends upon the level of development of the country.

Apart from the alternative channels whereby a specific tariff may be viewed as impacting the economy, another possible explanation for the conflicting empirical results is the diverse nature of tariffs themselves, a factor that may be inadequately reflected in the empirical measures adopted. In this respect, much of the theoretical literature is restricted to tariffs on consumption. While this is important, most open economies – particularly developing economies – also impose substantial tariffs on the imports of investment goods. Indeed, there is a literature comparing the consequences of consumption versus investment tariffs on the dynamics of the aggregate economy; see Brecher and Findlay (1983), Brock and Turnovsky (1993), Osang and Pereira (1996), Osang and Turnovsky

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3 Because of data limitations, Jaumotte et al. (2013) use inequality data based on both income and consumption expenditure surveys. As they note, mixing these two concepts makes a comparison of levels of inequality across countries potentially misleading. The inequality measure employed by Rojas-Vallejos and Turnovsky (2016) uses only income data.

4 Other recent studies include: Stewart and Berry (2000), Dreher and Gaston (2008), and Bergh and Nilsson (2010).

As these papers demonstrate in various ways, investment tariffs have very different consequences for the aggregate economy than do consumption tariffs, an implication that is supported by recent empirical evidence; see Estevadeordal and Taylor (2013). Second, empirical studies typically measure tariff rates by dividing tariff revenues by GDP, thereby confounding the two tariff rates which as just noted impact the economy in sharply contrasting ways.\(^7\)

Since the ongoing trade liberalization has involved a general reduction in tariffs, in this paper we introduce both a consumption tariff and an investment tariff and contrast their respective consequences for wealth and income inequality.\(^8\) As is well known, in order to derive explicit distributional implications of tariff policy one must impose some restrictions on the economy, and we do so by adopting the “representative consumer theory of distribution” [Caselli and Ventura, 2000]. This involves imposing two key assumptions: (i) homogeneity of agents’ utility, and (ii) attributing the underlying source of inequality to the heterogeneity in agents’ initial endowments of their physical and financial assets. Under these conditions, aggregation results pioneered by Gorman (1953) enable the macroeconomic equilibrium and distribution across agents to be determined sequentially.

First, summing over individual agents leads to a macroeconomic equilibrium in which aggregate quantities are determined independently of any distributional measures. Having determined the macroeconomic equilibrium, the corresponding factor returns determine the time paths for agents’ relative wealth, which can then be transformed to yield a measure of wealth inequality. The dynamics of this measure depend upon the relative consumption and labor income along the transitional path and its consequences for the differential savings rates across the distribution of heterogeneous agents. Finally, the distribution of income across agents is determined by the interaction of the evolving wealth distribution with the changing share of income from wealth in total personal income. The fact that the consumption tariff and investment tariff generate sharply contrasting aggregate dynamics, which are in turn the source of the distributional dynamics, underscores the importance of explicitly

\(^6\) Roldos (1991) introduces a common tariff on consumption and investment and hence cannot contrast their effects on the economy.

\(^7\) This is true of the tariff measures adopted by both Jaumotte et al. (2013) and Rojas-Vallejos and Turnovsky (2016).

\(^8\) For example in 1985 China had a tariff on capital of 35.4% and a consumption tariff of 78.7%, which by 2000 were reduced to 13.4% and 24.7% respectively. In this respect there is a literature dealing with the issue of the preferred method from a welfare standpoint of tariff reduction. These include the “concertina method” and the “method of radial reductions” and are briefly summarized by Brock and Turnovsky (1993).
differentiating between the two types of tariffs.

Much of the literature assessing the effects of tariffs on investment makes the polar assumption that all capital goods are imported; see Osang and Pereira (1996), Brock and Turnovsky (1993), Osang and Turnovsky (2000). While this may be convenient, it is unrealistic since comprehensive empirical evidence suggests that on average about 60% of total investment expenditures are on nontraded capital goods; see Bems (2008). To take account of this, we distinguish between traded and nontraded capital, thus enabling us to evaluate how their relative significance and substitutability in production impacts the growth-inequality tradeoff.

The framework we adopt is the standard two sector dependent economy model pioneered by Salter (1959) and Corden (1960). Using this framework, we first derive a number of theoretical implications linking the effects of tariff reduction on both the aggregate economy and its distributional consequences. The sharply contrasting ways the two tariffs directly impinge on the economy accounts for their sharply contrasting effects on both the aggregate and distributional dynamics. It reflects a basic characteristic of the dependent economy model, namely that long-run factor returns, sectoral allocations, and relative prices are determined entirely by production conditions. As a consequence, the consumption tariff by impacting final demand has no long-run effect on these quantities. It determines sectoral outputs solely through its impact on sectoral labor allocation as required to ensure market clearance in the nontraded goods market. In contrast, the investment tariff has a direct impact on the sectoral production structure. In this case we can show that provided the elasticity of substitution between the two capital goods is less than unity, a reduction in the investment tariff will tend to raise the long-run sectoral capital intensities in both sectors, as well as the long-run wage rate.

The evolution of the aggregate economy drives the dynamics of wealth inequality via its impact on two factors during the transition. First, to the extent that discounted aggregate consumption increases (in contrast to completely adjusting on impact), this will tend to increase wealth inequality. This is because wealthy people tend to consume relatively less and save relatively more. Second, to the extent that discounted wage earnings, assumed to be uniform across agents, increase, this has the opposite effect. In addition to incorporating this wealth effect, income inequality is likely to increase due to a reduction in the aggregate consumption expenditure-wealth ratio, and the fact that the rich
save relatively more (consume relatively less) than do the poor.

A key factor influencing the impact of tariffs on distribution concerns the speed with which the tariff reduction is implemented. This is important, since in practice there is substantial variation in the rate at which trade liberalizations have proceeded.\(^9\) We consider two scenarios. In the first, the tariff reduction is completed instantaneously, and we compare this to the alternative case where it is implemented gradually over time. While the time path affects only the transitional path of the aggregate variables, it has not only transitional, but also permanent, consequences for both wealth and income inequality.

Because of the complexity of the model, it is necessary to conduct the dynamic analysis using numerical simulations. In calibrating the model, we set parameters so as to approximate a plausible initial equilibrium structure that will facilitate our understanding of the channels through which tariffs influence the equilibrium, rather than to replicate any specific economic episode. In particular, we assume initial consumption tariff and investment tariff rates of 22% and 11% respectively, with the differential of the two rates being typical of the relative rates.

Starting from that point, we consider a 10 percentage point reduction in each tariff in turn. For the benchmark parameterization we find that in the long run, the reduction in the consumption tariff decreases nontraded output by about 0.8%, while increasing traded output by 1%, resulting in a small increase in GDP of less than 0.2% and an increase in capital stock of 0.34%. Reducing the investment tariff by 10 percentage points is much more stimulating, consistent with the empirical evidence of Estevadeordal and Taylor (2013). The outputs of the nontraded and the traded sector increase by about 1% and 3%, respectively, GDP rises by around 2.5%, while capital stock increases by about 6%.

The aggregate dynamics cause the two tariffs to have sharply contrasting distributional effects. A reduction in the consumption tariff that is completed immediately has a negligible negative effect on wealth inequality, while reducing it by over 1%, if implemented gradually. The long-run reduction in the consumption-wealth ratio dominates the reduction in wealth inequality, so that income

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\(^9\) For example, we have examined the sample of 45 countries developed by Forbes (2000). There we see substantial variation in the speed with which countries reduced their tariffs over the period 1990-2010. Generally we find that advanced economies tended to reduce them at a much slower rate than have developing countries (approximately 6% per annum versus 12%).
inequality is increased by approximately 1.4% if the tariff reduction is immediate, but reduced to around 0.5% if the decrease occurs gradually. In contrast, the immediate reduction in the investment tariff reduces wealth inequality by 1.10%, and by 0.85% if it occurs gradually. Finally, the immediate reduction of the investment tariff has a much weaker effect on income inequality (0.07%) than does the consumption tariff (1.4%). But in contrast to the latter, if implemented gradually has a relatively larger effect on income inequality.

The paper is organized as follows. Section 2 sets out the analytical framework, while Section 3 derives and characterizes the macroeconomic equilibrium. Section 4 characterizes the distributions of wealth, and income, and derives the main analytical results. Section 5 describes the calibration, while Sections 6 and 7 illustrate these results with numerical simulations describing the long-run and transitional effects of the alternative modes of tariff reduction. Section 8 compares the impact of the tariff reductions on the dynamics of wealth and income inequality, while Section 9 summarizes some of the sensitivity analysis. The main observation here is that the impact on both the time path and long run responses of wealth and income inequality to the investment tariff are potentially sensitive to the relative importance of the two capital goods in production and degree of substitutability and complementarity. Section 10 concludes, while technical details are presented in the Appendix.

2. Macroeconomic Model

The basic analytical framework employed is the two-sector dependent small open economy model.10 Households consume a domestically produced tradable good, a domestic nontradable good, and an imported consumption good, the latter subject to a tariff, $\tau_c$. They also accumulate some of the nontraded good as capital, which for expositional convenience and as a reflection of its nontradable characteristic, we designate as “structures”. In addition, households import a second capital good, denoted as “equipment”, which is subject to a tariff, $\tau_e$. They also have access to an international financial market where they can borrow or lend. We shall emphasize the case of international borrowing, so that the economy is a debtor nation, although the case of a lender is perfectly symmetric.

10 The model is a modification of Brock and Turnovsky (1994) to allow for heterogeneity among agents. It differs in one other respect from the earlier model in that agents’ access to the world financial market is subject to frictions.
The only role played by the government is to collect the tariff revenues, which it then redistributes uniformly back to the households. A key focus of the analysis is to highlight the contrasting channels whereby the two tariffs impinge on the macro economy, ultimately manifesting in sharply contrasting consequences for the evolution of wealth and income inequality.

2.1 Firms

Domestic production takes place in two sectors – tradable and nontradable – by a single aggregate representative firm. Output of the tradable good, \( Y_T \), (taken to be the numeraire) is produced using structures \( (S_T) \), equipment \( (E_T) \), and labor \( (L_T) \) by means of a linearly homogeneous neoclassical production function, which we may write in intensive form as:

\[
Y_T = F(S_T, E_T, L_T) \equiv f(s_T, e_T)L_T, \quad \text{where} \quad s_T \equiv \frac{S_T}{L_T}, \quad e_T \equiv \frac{E_T}{L_T}.
\]  

(1a)

Analogously, the nontraded good \( Y_N \) is produced using structures \( (S_N) \), equipment \( (E_N) \), and labor \( (L_N) \) by means of a second linearly homogeneous production function:

\[
Y_N = H(S_N, E_N, L_N) \equiv h(s_N, e_N)L_N, \quad \text{where} \quad s_N \equiv \frac{S_N}{L_N}, \quad e_N \equiv \frac{E_N}{L_N}.
\]  

(1b)

Structures, equipment, and labor can move freely and costlessly between sectors, subject to the sectoral allocation constraints

\[
S_T + S_N = S \quad (2a)
\]

\[
E_T + E_N = E \quad (2b)
\]

\[
L_T + L_N = 1 \quad (2c)
\]

where \( S \) and \( E \) denote the aggregate capital stocks accumulated by households and \( L = 1 \) are the short-run supplies available to firms for allocation across the two sectors. The relative price, \( p \), of nontraded output in terms of traded output serves as a proxy for the real exchange rate so that the total capital stock, \( K \), in the economy measured in terms of the traded good is \( K = pS + E \). The representative firm allocates its productive inputs to maximize profits, so that the returns to the two types of capital,
$r_s, r_e,$ and the wage rate, $w$, all expressed in terms of the numeraire satisfy the conventional static efficiency conditions, expressed in intensive per capita units

\[ r_s \equiv f_s(s_T, e_T) = ph_s(s_N, e_N) \]  
\[ r_e \equiv f_e(s_T, e_T) = ph_e(s_N, e_N) \]  
\[ w \equiv f(s_T, e_T) - s_T f_s(s_T, e_T) - e_T f_e(s_T, e_T) = p \left[ h(s_N, e_N) - s_N h_s(s_N, e_N) - e_N h_e(s_N, e_N) \right] \]

In addition, the sectoral allocation equations (2a)-(2c) expressed in terms of intensive units become:

\[ S = s_T L_T + s_N (1 - L_T) \]  
\[ E = e_T L_T + e_N (1 - L_T) \]

### 2.2 Consumers

The economy is populated by a mass 1 of infinitely-lived individuals, indexed by $j$, who are identical in all respects except for their initial endowments of structures, $S_{j,0}$, equipment, $E_{j,0}$ and their initial debt position, $Z_{j,0}$. In this respect we should note that while there are many sources of heterogeneity, initial endowments are arguably among the most significant.\(^\text{11}\) Since we are interested in distribution and inequality we shall focus on individual $j$’s relative holdings of capital and bonds, $s_j(t) = S_j(t)/S(t)$, $e_j(t) = E_j(t)/E(t)$, $z_j(t) = Z_j(t)/Z(t)$, where $Z(t)$ denotes the economy-wide average stock of debt. Initial relative endowments, $s_{j,0}, e_{j,0}, z_{j,0}$ have mean 1 and relative standard deviations, $\sigma_{s,0}, \sigma_{e,0}, \sigma_{z,0}$ across agents.\(^\text{12}\) Each agent is also endowed with one unit of time that he can allocate to labor in the traded sector, $L_{j,T}$, or in the nontraded sector, $L_{j,N}$, implying $L_{j,T}(t) + L_{j,N}(t) = 1$. With a continuum of agents, the economy-wide supply of labor in each sector is $L_z = \int_0^1 L_{j,z} \, dz \quad (z = T, N)$ and other aggregates are defined analogously, with the economy-wide labor supply fixed inelastically at unity.

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\(^\text{11}\) Compelling evidence supporting this view is provided by the well known studies by Piketty (2011) and Stiglitz (2012).

\(^\text{12}\) These initial endowments can be perfectly arbitrary and therefore consistent with any required non-negativity constraints. They may also be correlated across agents. As will become apparent in the course of the analysis, the form of the distribution of the initial endowments will be reflected in the evolving distributions of wealth and income.
Each individual $j$ has lifetime utility that depends upon the following isoelastic function of the domestically produced tradable good, $C_{j,T}$, the domestically produced nontradable good, $C_{j,N}$, and the imported consumption good, $C_{j,F}$, that is subject to a tariff, $\tau_c$. All households have identical preferences described by:

$$
U_j = \frac{1}{\gamma} \int_0^\infty \left[ (C_{j,T})^\theta (C_{j,N})^{1-\theta} (C_{j,F})^\eta \right] e^{-\beta t} dt
$$

(5)

$$
0 \leq \theta \leq 1, \ 0 < \eta, -\infty < \gamma \leq 1, \ \theta \gamma < 1, \ \eta \gamma < 1
$$

where $1/(1-\gamma)$ is the household’s intertemporal elasticity of substitution, $\theta$ measures the relative importance of the domestic traded vs. nontraded consumption, $\eta$, parameterizes the relative importance of the imported consumption good, and $\beta$ is the subjective discount rate. The remaining restrictions in (5) ensure concavity of the utility function in the three consumption goods.

Households own both capital stocks, which depreciate at the constant rates, $\delta_S$ and $\delta_E$, respectively. Thus, each household’s investment expenditures on structures, $I_{j,S}$, and equipment, $I_{j,E}$, are governed by the conventional accumulation equations

$$
\dot{S}_j = I_{j,S} - \delta_S S_j
$$

(6a)

$$
\dot{E}_j = I_{j,E} - \delta_E E_j
$$

(6b)

We assume that the agent chooses his rates of consumption, $C_{j,T}(t), C_{j,N}(t), C_{j,F}(t)$, and rates of accumulation of structures, $S_j(t)$, equipment, $E_j(t)$, and international debt, $Z_j(t)$, so as to maximize intertemporal utility, (5), subject to his capital accumulation equations, (6a) and (6b), and flow budget constraint. Expressed in terms of units of domestic tradable output as numeraire, this is

$$
\dot{Z}_j(t) = C_{j,T} + pC_{j,N} + (1 + \tau_c)C_{j,F} + pI_{j,S} + (1 + \tau_c)I_{j,E} - w - r_s S_j - r_e E_j - T_j + i \left( \frac{Z}{pS} \right) Z_j
$$

(7)

where we normalize the (international) price of the imported consumption good and imported investment good to equal the unitary price of the tradable consumption good.

Equation (7) asserts that the agent’s expenditures comprise his consumption of the domestic
A tradable good, his consumption and investment expenditures on the nontradable good, the imported consumption good and equipment, both inclusive of their respective tariffs, together with the interest owing on his holdings of debt. His earnings include wages, income from his holdings of nontraded and traded capital, and lump-sum transfers received from the government. To the extent that expenditures exceed income the agent accumulates debt and vice versa.

The budget constraint is written from the standpoint of a borrower, although $Z_j < 0$ corresponds to a lender. Whether the equilibrium turns out to be one in which the agent is a debtor or creditor depends upon the relative magnitudes of the rate of time preference and the given world rate of interest, $i^*$. In either case, a key element of the model is that while the economy has access to the international capital market, it faces a friction in the form of increasing borrowing costs expressed by the relationship

$$i \left( \frac{Z}{pS} \right) = i^* + \omega \left( \frac{Z}{pS} \right).$$

This equation asserts that the financial friction facing the economy is in the form of a borrowing premium $\omega(.)$ over the fixed world interest rate. The premium is a positive convex function of its debt relative to its level of development as parameterized by its stock of domestically produced (nontraded) capital. There are several alternative ways to formulate this borrowing constraint, but the qualitative implications are essentially identical.\(^{13}\) The shape of the function reflects the degree of openness of the economy with respect to the financial market, and the fact that the debt is normalized by the market value of nontraded capital means that larger, wealthier, economies are less constrained by the financial friction implicit in (8). While the individual household takes the borrowing costs as given, the equilibrium cost of capital is determined by their collective actions.

Performing the optimization yields the following first order optimality conditions:

\(^{13}\) This formulation of borrowing costs dates back to Bardhan (1967) who expressed it as an increasing function of debt alone. A commonly adopted alternative is to specify the borrowing premium in terms of the ratio of debt to output, as a measure of the country’s debt servicing ability. Another alternative in the present context is to express it in terms of aggregate capital inclusive of equipment. However, these alternatives lead to analytical complications without adding insight, since they are driven by structures as the fundamental state variable. Hence we adopt the simpler formulation, which in any event may serve as the best index of the country’s level of development, and best proxy the lending risks associated with that economy.
\[ \theta \left( C_{j,T} \right)^{\eta-1} \left( C_{j,N} \right)^{\eta(1-\theta)} \left( C_{j,F} \right)^{\eta} = \lambda_j \]  

(9a)

\[ (1-\theta) \left( C_{j,T} \right)^{\eta} \left( C_{j,N} \right)^{\eta(1-\theta)-1} \left( C_{j,F} \right)^{\eta} = p\lambda_j \]  

(9b)

\[ \eta \left( C_{j,T} \right)^{\eta} \left( C_{j,N} \right)^{\eta(1-\theta)} \left( C_{j,F} \right)^{\eta-1} = (1 + \tau_c)\lambda_j \]  

(9c)

\[ i \left( \frac{Z}{pS} \right) = \beta - \frac{\dot{\lambda}_j}{\lambda_j} \]  

(9d)

\[ \frac{r_c}{p} - \delta_s + \frac{\dot{p}}{p} = \beta - \frac{\dot{\lambda}_j}{\lambda_j} \]  

(9e)

\[ \frac{r_c}{(1 + \tau_c)} - \delta_e + \frac{\dot{\tau}_c}{(1 + \tau_c)} = \beta - \frac{\dot{\lambda}_j}{\lambda_j} \]  

(9f)

where \( \lambda_j \) is agent \( j \)'s marginal utility of wealth, which for a debtor is the marginal utility of reducing debt. Equations (9a)-(9c) are standard static efficiency conditions, equating the marginal benefits of the three consumption goods to their respective marginal costs, while (9d) - (9f) are conventional arbitrage conditions equating the rates of return on investments and borrowing to the rate of return on consumption. One point of interest is that while the return to investing in equipment declines with the current tariff, it increases with its rate of change \( \dot{\tau}_c \). This is because a higher tariff next period increases the incentive to invest in the current period. This is particularly relevant, since programs to reduce tariffs invariably take place over extended periods of time, in which case the changing tariff rates become important. Finally, the following transversality conditions hold

\[ \lim_{t \to \infty} \lambda_j pS e^{-\rho t} = 0; \lim_{t \to \infty} \lambda_j E e^{-\rho t} = 0; \lim_{t \to \infty} \lambda_j Z e^{-\rho t} = 0; \]  

(9g)

Defining agent \( j \)'s total consumption expenditure, in terms of the traded good, inclusive of the consumption tariff, \( \tau_c \), by \( C_j \equiv C_{j,T} + pC_{j,N} + (1 + \tau_c)C_{j,F} \), we may express the agent’s consumption expenditures as

\[ C_{j,T}(t) = \left( \frac{\theta}{1+\eta} \right) C_j(t) \]  

(10a)
\[ p C_{j,n}(t) = \left( \frac{1-\theta}{1+\eta} \right) C_j(t) \]  
(10b)

\[ (1+\tau_c)C_{j,F}(t) = \left( \frac{\eta}{1+\eta} \right) C_j(t) \]  
(10c)

Thus each individual consumes the three consumption goods in the same proportion.

As noted, programs of trade liberalization, and specifically tariff reductions typically involve extensive negotiations and therefore are likely to be implemented gradually over an extended period of time. To allow for this we assume that tariffs are adjusted gradually from the initial rates, \( \tau_{c,0}, \tau_{c,0} \), to their post liberalization rates, \( \tilde{\tau}_c, \tilde{\tau}_c \), in accordance with the known path

\[ \tau_z(t) = \tilde{\tau}_z + (\tau_{z,0} - \tilde{\tau}_z)e^{\nu z t}, \quad z = c, e \]  
(11)

The parameter, \( \nu_z > 0 \), thus specifies the speed with which the tariff change occurs, and hence defines the time path it follows. The conventional assumption, where the tariff is fully adjusted instantaneously, is obtained by letting \( \nu_z \to \infty \) in (11). But the more general specification introduced in (11) is important. This is because, as the numerical simulations will demonstrate, there is a sharp contrast between how \( \nu_z \) affects the adjustment of aggregate quantities and their distributions across agents. As one would expect, the time path of tariffs affects the transitional path of the aggregate economy and not the aggregate steady state. But in contrast, it influences both the time paths and the steady-state levels of both wealth and income inequality, thereby having permanent distributional consequences.

Taking the time derivatives of (10a) - (10c), and combining with (9d) - (9f), we obtain

\[ \frac{\dot{C}_{j,T}}{C_{j,T}} = \frac{\dot{C}_j}{C_j} = \frac{1}{1-\gamma(1+\eta)} \left[ (1-\theta)\gamma \left( \frac{r_c}{p} - \delta \right) + \left[ 1-\gamma(1-\theta) \right] i \left( \frac{Z}{pS} \right) - \beta - \eta \gamma \left( \frac{\tilde{\tau}_c}{1+\tilde{\tau}_c} \right) \right] \]  
(12a)

\[ \frac{\dot{C}_{j,F}}{C_{j,F}} = \frac{\dot{C}_j}{C_j} = \frac{1}{1-\gamma(1+\eta)} \left[ (1-\theta)\gamma \left( \frac{r_c}{p} - \delta \right) + \left[ 1-\gamma(1-\theta) \right] i \left( \frac{Z}{pS} \right) - \beta - \eta \gamma \left( \frac{\tilde{\tau}_c}{1+\tilde{\tau}_c} \right) \right] \]  
(12b)

The assumption that the change in the tariff rate occurs at a constant proportionate rate, and is completed only asymptotically, is made purely for analytical convenience. It is straightforward to generalize (11) to the case where the new level of the tariff is reached in finite time, \( T \). The analysis could also be modified to allow for the increase in the tariff to follow a more general time path, and the same general qualitative conclusions would emerge.

The reason for this is the homogeneity of the utility function (5) which causes individuals to maintain fixed relative consumption over time. This introduces a “zero root” into the dynamics of the distributional measures, as a result of which their equilibrium values become path dependent; see Atolia, Chatterjee, and Turnovsky (2012) where this issue is discussed in detail in the context of a Ramsey model.

\[ 12 \]
\[
\dot{C}_{j,N} = \frac{1}{1 - \gamma(1 + \eta)} \left[ 1 - \gamma(\theta + \eta) \right] \left( \frac{r_s}{p} - \delta_s \right) + \gamma(\theta + \eta) i \left( \frac{Z}{pS} \right) - \beta - \eta \gamma \left( \frac{\dot{\tau}_c}{1 + \tau_c} \right) \]  
\tag{12b}
\]

\[
\dot{C}_{j,F} = \frac{1}{1 - \gamma(1 + \eta)} \left[ (1 - \theta) \gamma \left( \frac{r_s}{p} - \delta_s \right) + [1 - \gamma(1 - \theta)] i \left( \frac{Z}{pS} \right) - \beta - (1 - \gamma) \left( \frac{\dot{\tau}_c}{1 + \tau_c} \right) \right] \]  
\tag{12c}
\]

With the right hand side of equations (12) being common to all agents, these equations imply that each individual, \( j \), will choose the same growth rate for the three consumption goods as well as for their respective total consumption, \( C_j \). Because of the linearity of the individual optimality conditions, (10), aggregation over individuals is straightforward. Thus, summing these equations over all agents, we can express the equilibrium aggregate economy-wide consumption levels, \( C_T(t), C_N(t), C_F(t) \), respectively in terms of the total consumption expenditure, \( C \equiv C_T + pC_N + (1 + \tau_c)C_F \), namely

\[
C_T(t) = \left( \frac{\theta}{1 + \eta} \right) C(t) \]  
\tag{13a}
\]

\[
pC_N(t) = \left( \frac{1 - \theta}{1 + \eta} \right) C(t) \]  
\tag{13b}
\]

\[
(1 + \tau_c)C_F(t) = \left( \frac{\eta}{1 + \eta} \right) C(t) \]  
\tag{13c}
\]

which together with (10) and (12) implies

\[
\frac{\dot{C}_{j,T}}{C_{j,T}} = \frac{\dot{C}_T}{C_T} = \frac{\dot{C}_j}{C_j}; \quad \frac{\dot{C}_{j,N}}{C_{j,N}} = \frac{\dot{C}_N}{C_N}; \quad \frac{\dot{C}_{j,F}}{C_{j,F}} = \frac{\dot{C}_F}{C_F} \quad \text{for all } j \]  
\tag{14}
\]

In particular, we may write \( C_j = \varphi_j C \), where \( \int_0^1 \varphi_j d\gamma = 1 \), and \( \varphi_j \), which defines agent \( j \)'s relative consumption, is constant over time for each \( j \), and is yet to be determined; see eq. 29 below.

\section*{2.3 The government}

To isolate the impact of the tariff reduction, the domestic government is assigned a very minor role, simply levying the tariffs on the imported consumption good and equipment and then rebating
the revenues to consumers as lump-sum transfers.\textsuperscript{16} It issues no debt, nor conducts any other expenditures, maintaining a balanced budget in accordance with

\[ T(t) = \tau_c(t)C_F(t) + \tau_c(t)I_E(t) \tag{15} \]

To avoid ad hoc distributional effects we assume further that the tariff revenues are rebated uniformly across the agents so that \( T_j(t) = T(t) \) for each \( j \).

3. Macroeconomic equilibrium

First, combining equations (12a), (13a) with (14) we may express the equilibrium dynamics of aggregate consumption in the form

\[
\frac{\dot{C}}{C} = \frac{1}{1 - \gamma(1 + \eta)} \left[ (1 - \theta)\gamma \left( \frac{r_c}{p} - \delta_s \right) + [1 - \gamma(1 - \theta)]i \left( \frac{Z}{pS} \right) - \beta - \eta\gamma \left( \frac{\dot{\tau}_c}{1 + \tau_c} \right) \right]
\tag{16}
\]

Second, equilibrium in the domestic nontraded goods market clearance implies

\[ Y_N = h(s_N, e_N)(1 - L_T) = C_N + I_s \tag{17} \]

where \( I_s = \dot{S} + \delta_S S \) is the total gross investment in structures and is obtained by aggregating (6a) across domestic consumers. Substituting for (13b), this can be expressed in the form of the nontraded capital accumulation equation

\[ \dot{S} = h(s_N, e_N)(1 - L_T) - \left( \frac{1 - \theta}{1 + \eta} \right) \frac{C}{p} - \delta_S S \tag{17'} \]

Next, aggregating over the individual budget constraints (7), noting the linear homogeneity of the production functions, using the sectoral allocation conditions, (2), the optimality conditions, (3), and the government budget constraint, (15), yields the current account relationship

\textsuperscript{16} Another assumption that would also isolate the role of the tariff would be to assume that the tariff revenues are allocated to government expenditure which has no impact on private behavior. Alternatively, if we were to assume that the tariff revenues are spent on some activity that enhances private productivity say, we would have the difficulty of disentangling the effect of the tariff from that the productive effect of the expenditure.
\[ \dot{Z}(t) = C_T + C_F + I_E - f(s_T, e_T) L_T + i \left( \frac{Z}{pS} \right) Z \]  

(18)

where \( I_E = \dot{E} + \delta E \) is the total gross investment in equipment. Equation (18) asserts that the aggregate rate of accumulation of debt equals the trade deficit plus the interest owing on the country’s net holdings of foreign debt. Substituting for \( C_T, C_F \) and \( I_E \) this can be expressed as

\[ \dot{Z}(t) = \left( \frac{\theta}{1 + \eta} + \frac{\eta}{(1 + \eta)(1 + \tau_e)} \right) C + \dot{E} + \delta E - f(s_T, e_T) L_T + i \left( \frac{Z}{pS} \right) Z \]  

(18’)

Finally, the dynamics of the relative price is obtained by combining the optimality conditions pertaining to debt and structures, given by (9d) and (9e), respectively, namely

\[ \frac{\dot{p}(t)}{p(t)} = i \left( \frac{Z}{pS} \right) - \left( \frac{r_s}{p} - \delta_s \right) \]  

(19)

Solving equation (19) forward yields

\[ p(t) = \int_{t}^{\infty} r_s(u)e^{-\int_{\tau}^{t}[i(t)+\delta_t]d\tau} du \]  

(19’)

highlighting how \( p(t) \) serves a dual role, as an asset price reflecting the discounted value of future returns to nontraded capital, as well as the relative price of two consumption goods.

Equations (16), (17’), (18’) and (19) describe the basic macroeconomic dynamics. However, the accumulation of debt, (18’), also increases with \( \dot{E} \), which turns out to be dependent upon that of \( Z(t), S(t), p(t) \). To see this, and to incorporate this dependence into the equilibrium dynamics, we need to reconsider the short-run equilibrium.

Recalling (3a)-(3c), and combining with (9d) and (9f), while using the time derivative of (11) we obtain

\[ f_s(s_T, e_T) = ph_s(s_N, e_N) \]  

(20a)

\[ f_e(s_T, e_T) = ph_e(s_N, e_N) \]  

(20b)

\[ f(s_T, e_T) - s_T f_s(s_T, e_T) - e_T f_e(s_T, e_T) = p \left[ h(s_N, e_N) - s_N h_s(s_N, e_N) - e_N h_e(s_N, e_N) \right] \]  

(20c)
\[ f_e(s_T, \tau_T) = \left[ i \left( \frac{Z}{pS} \right) + \delta_E \right] (1 + \tau_e) + \nu_e(\tau_e - \tilde{\tau}_e) \]  
(20d)

These four equations determine the short-run sectoral allocation ratios\(^{17}\)

\[ s_r = s_r(p, Z/S, \tau_e, \tilde{\tau}_e); \quad e_r = e_r(p, Z/S, \tau_e, \tilde{\tau}_e) \]  
(21a)

\[ s_N = s_N(p, Z/S, \tau_e, \tilde{\tau}_e); \quad e_N = e_N(p, Z/S, \tau_e, \tilde{\tau}_e) \]  
(21b)

From these equations we see that the short-run effect of the consumption tariff on the sectoral factor allocation occurs solely through its impact on the relative price. The nature of this response depends upon whether the tariff change is completed instantaneously, or occurs gradually. In addition to an analogous price effect, the tariff on investment, by raising the price of equipment directly, impacts the profit maximizing sectoral allocation of productive inputs. The effects of structures and debt on the sectoral allocation occur gradually over time.

Substituting (21) into (4a) we may solve for \( L_T \)

\[ L_T = \frac{S - s_N(p, Z/S, \tau_e, \tilde{\tau}_e)}{s_r(p, Z/S, \tau_e, \tilde{\tau}_e) - s_N(p, Z/S, \tau_e, \tilde{\tau}_e)} \equiv L_T(p, Z, S, \tau_e, \tilde{\tau}_e) \]  
(22a)

thus yielding the short-run sectoral labor allocations in terms of the dynamically evolving variables, \( p, Z, S \). Combining the terms in (21a), (21b), and (22a) with (4b) we see that the market clearing condition for equipment can be expressed as

\[ E = e_r(p, Z/S, \tau_e, \tilde{\tau}_e)L_T(p, Z, S, \tau_e, \tilde{\tau}_e) + e_N(p, Z/S, \tau_e, \tilde{\tau}_e)(1 - L_T(p, Z, S, \tau_e, \tilde{\tau}_e)) \equiv E(p, Z, S, \tau_e, \tilde{\tau}_e) \]  
(22b)

Equation (22b) merits two observations. First, the short-run responses in the sectoral factor allocations arising from discrete changes in the relative price and possibly the investment tariff itself, generate instantaneous adjustments in total equipment. With no trade impediments these can be imported instantaneously. Second, taking the time derivative of (22b) yields

\(^{17}\) The short-run responses can be easily determined from (20). They depend critically upon the sectoral capital intensities. The impact effects are similar to those obtained by Brock and Turnovsky (1994) and details are available in an expanded version of the paper.
\[
\dot{E}(t) = E_p \dot{p}(t) + E_\tau \dot{Z}(t) + E_S \dot{S}(t) + E_{\tau_e} \dot{\tau}_e(t)
\]

which highlights the dependence of \( \dot{E} \) in \((18')\). Substituting this expression into \((18')\) yields

\[
(1 - E_\tau) \dot{Z}(t) = \left( \frac{\theta}{1 + \eta} + \frac{\eta}{(1 + \eta)(1 + \tau_r)} \right) C + \delta_k E - f(s_r, e_r) L_r + i \left( \frac{Z}{pS} \right) Z
\]

\[+ E_p \dot{p}(t) + E_S \dot{S}(t) + E_{\tau_e} \dot{\tau}_e(t)\]  

\((18'')\)

where \( \dot{p}, \dot{S}, \dot{\tau}_e \) are obtained from \((19), (17')\) and \((11)\), respectively.

This completes the description of the structural model, the detailed solution to which is summarized in the Appendix. At any instant of time the expressions \((21a), (21b), \) and \((22a)\) yield the short-run solutions for the sectoral allocations \( s_r, s_N, e_r, e_N, L_r \) in terms of the dynamically evolving variables, \( \dot{S}, \dot{Z}, \dot{C}, \dot{p} \) given by \((17'), (18''), (16), \) and \((19)\), together with \((11)\) for the case where tariffs are adjusted gradually. One key observation is that the aggregate equilibrium is independent of any distributional considerations. This is a consequence of the homogeneity assumptions and a reflection of the “representative consumer theory of distribution” that our approach embodies.

In Section 7 below we shall analyze the local dynamics following a decrease in the tariff rates, by linearizing the dynamic equations about their steady state. The formal structure of this system is set out in the Appendix, where the unique stable adjustment path is characterized. For the numerical simulations conducted, including the extensive sensitivity analysis, we find that the system exhibits saddlepoint behavior in the neighborhood of the steady state.\(^{18}\) Given the specified trajectory for tariffs, that may or may not evolve sluggishly depending upon \( \nu_z \), the stable transitional path is a two-dimensional stable manifold, along which both capital and foreign bonds evolve gradually, while the relative price and employment may respond instantaneously to new information as it comes available.

### 3.1 Steady-state equilibrium

The steady-state equilibrium (denoted by tildes) is attained when \( \dot{S} = \dot{Z} = \dot{C} = \dot{p} = \dot{E} = \dot{\tau}_e = \dot{\tau}_c = 0 \). As is characteristic of the standard two sector-two good dependent economy model,

\(^{18}\) By examining the characteristic equation of the dynamic system one can derive formal conditions for there to be two positive and two negative eigenvalues. However, this is not only very tedious, and ultimately not very illuminating, and we find it much more useful to rely on our simulation results to establish the plausibility of this desired root configuration.
steady state is determined in two stages. In the first, the sectoral capital labor ratios and relative price are determined. Having derived these, aggregate market clearing conditions determine the equilibrium levels. Thus, combining (20a)-(20d), together with (9d) and (9f), yields:

\[(\tilde{s}_T, \tilde{e}_T) = \tilde{p} h_s(\tilde{s}_N, \tilde{e}_N)\] (23a)

\[(\tilde{s}_L, \tilde{e}_L) = \tilde{p} h_e(\tilde{s}_N, \tilde{e}_N)\] (23b)

\[f(\tilde{s}_T, \tilde{e}_T) - \tilde{s}_T f_s(\tilde{s}_T, \tilde{e}_T) - \tilde{e}_T f_e(\tilde{s}_T, \tilde{e}_T) = \tilde{p} \left[ h(\tilde{s}_N, \tilde{e}_N) - \tilde{s}_N h_s(\tilde{s}_N, \tilde{e}_N) - \tilde{e}_N h_e(\tilde{s}_N, \tilde{e}_N) \right] \] (23c)

\[f_s(\tilde{s}_T, \tilde{e}_T) = \left[ \beta + \delta_F \right] (1 + \tilde{e}_s)\] (23d)

\[h_s(\tilde{s}_N, \tilde{e}_N) = \beta + \delta_S\] (23e)

These five equations determine the long-run equilibrium values of \(\tilde{s}_T, \tilde{s}_N, \tilde{e}_T, \tilde{e}_N\), together with \(\tilde{p}\), independently of demand conditions, including the consumption tariff, \(\tau_c\). As a further consequence, the long-run wage rate and return to capital are unaffected, while sectoral outputs, \(\tilde{Y}_T, \tilde{Y}_N\), move in proportion to sectoral labor movements, \(\tilde{L}_T, \tilde{L}_N\); see equations (1a, 1b). In contrast, by directly affecting the rate of return on investment in equipment, (23d), the investment tariff impacts all these quantities. In the case that the sectoral production functions \(f(s_T, e_T), h(s_N, e_N)\) are of the CES form with a common elasticity of substitution less than unity \((\sigma < 1)\), so that structures and equipment are complements in production, it is straightforward to show that a decrease in the investment tariff increases all four sectoral capital-labor ratios. The effect on the relative price, however, depends upon the relative sectoral intensities of the two capital goods. These responses are critical causes of the contrasting distributional effects of the two tariffs, which we summarize in the following proposition:

**Proposition 1:** (i) A reduction in the consumption tariff has no long-run effect on the sectoral capital intensities, \(\tilde{s}_T, \tilde{s}_N, \tilde{e}_T, \tilde{e}_N\), the wage rate, \(\tilde{w}\), or the returns to structures or equipment, \(\tilde{r}_s, \tilde{r}_e\). Sectoral output responses are strictly proportional to sectoral labor reallocation.

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19 See Turnovsky (1997) for a detailed characterization of the dynamic structure of the standard dependent economy model.
(ii) If $\sigma \leq 1$, a reduction in the investment tariff increases all sectoral capital intensities, $\tilde{s}_r, \tilde{s}_N, \tilde{e}_r, \tilde{e}_N$, raising $\tilde{w}$. It reduces the return on equipment, $\tilde{r}_e$, while its effect on structures, $\tilde{r}_s$, reflects that of $\tilde{p}$.

The remaining steady-state conditions are:

\begin{align*}
\tilde{S} &= \tilde{s}_s \tilde{L}_r + \tilde{s}_N (1 - \tilde{L}_r) \\
\tilde{E} &= \tilde{e}_r \tilde{L}_r + \tilde{e}_N (1 - \tilde{L}_r) \\
\tilde{h}(\tilde{s}_N, \tilde{e}_N) (1 - \tilde{L}_r) - \left( \frac{1 - \theta}{1 + \eta} \right) \tilde{C} - \delta_s \tilde{S} &= 0 \\
\tilde{f} \left( \tilde{s}_r, \tilde{e}_r \right) \tilde{L}_r - \beta \tilde{Z} &= \left( \frac{\theta}{1 + \eta} + \frac{\eta}{(1 + \eta)(1 + \tilde{r}_e)} \right) \tilde{C} + \delta_e \tilde{E} \\
\tilde{i} \left( \frac{\tilde{Z}}{\tilde{p} \tilde{S}} \right) &= \beta
\end{align*}

Having obtained $\tilde{s}_r, \tilde{s}_N, \tilde{e}_r, \tilde{e}_N$, and $\tilde{p}$, equations (24a)-(24e) determine the equilibrium sectoral labor allocation, $\tilde{L}_r$, the equilibrium capital stocks, $\tilde{S}, \tilde{E}$, the stock of debt, $\tilde{Z}$ and aggregate consumption, $\tilde{C}$. The following points with regard to the steady state (24) merit comment. First, given $\tilde{p}$, the debt to structures capital ratio adjusts to drive the equilibrium borrowing rate to the given rate of time preference. Second, the investment tariff, $\tilde{r}_e$, exerts its entire impact via the production decisions in (24). Third, the role of the consumption tariff, $\tilde{r}_c$, is to ensure that the total demand for imports (consumption plus investment) is consistent with the country’s resources generated by its production, adjusted for its debt-serving commitments. Finally, we note that from (23) and (24) we can infer all other equilibrium quantities such as the sectoral outputs in (1) and the various components of consumption in (13).

4. **Wealth and income inequality**

We now analyze the consequences of tariff liberalization for the evolution of wealth and income inequality.
4.1 Wealth inequality

To abstract from any direct, but arbitrary, discretionary distributional effects arising from lump-sum transfers, we assume that tariff revenues are rebated uniformly across the agents, namely $T_j(t) = T(t)$, for all $j$. For convenience, we shall price imported equipment inclusive of the tariff, so that the gross wealth of household $j$, measured in terms of traded output is

$$V_j = pS_j + (1 + \tau_p)E_j - Z_j$$

(25)

where we assume that $V_j > 0$ so that the agent has net positive wealth and is therefore solvent. Taking the time derivative of (25), using the individual’s budget constraint, (7), the arbitrage condition (9e), and the distributional assumption $T_j(t) = T(t)$, the rate of wealth accumulation for agent $j$ is given by

$$\dot{V}_j(t) = i \left( \frac{Z}{pS} \right) V_j(t) + w(t) + T - C_j$$

(26)

and aggregating over all agents $j$ yields

$$\dot{V}(t) = i \left( \frac{Z}{pS} \right) V(t) + w(t) + T - C$$

(26')

Next, we define individual $j$’s share of aggregate wealth to be $v_j = V_j/V$. Taking the time derivative of $v_j$ and combining with (26) and (26'), together with $C_j = \varphi_j C$, we obtain

$$\dot{v}_j = \frac{1}{\beta} \left[ (C(t) - w(t) - T(t))(v_j - 1) + (1 - \varphi_j)C(t) \right]$$

(27)

Equation (27) indicates how the evolution of an individual agent’s relative wealth depends upon the evolution of aggregate gross consumption expenditure, the real wage rate, as well as his own specific endowments as reflected in $v_j$ and $\varphi_j$.

Before solving for $v_j(t)$ we consider some of the steady-state relationships between consumption and wealth. First, considering (26’) at steady state and using (24e), we see that

$$\bar{C} = \beta \bar{V} + \bar{w} + \bar{T}$$

(28)
so that aggregate steady-state consumption equals the income from wealth, plus wage income, plus the tariff revenue. Next, considering (26) at steady state and subtracting (28) yields:

$$\tilde{C}_j - \bar{C} = \beta \bar{V} (\bar{v}_j - 1) = \beta (\bar{V} - \bar{V})$$  \hspace{1cm} (29)

From (29) we see that if agent $j$’s wealth places him above the average, his long-run marginal propensity to consume (inclusive of the tariff) out of his above-average component of his wealth equals $\beta$. Moreover, since (28) implies $\bar{C} > \beta \bar{V}$, it follows that the average long-run propensity to consume out of wealth exceeds $\beta$, implying that wealthier agents save proportionately more and consume proportionately less.

From (29) we obtain $\varphi_j - 1 = (\beta \bar{V} / \bar{C})(\bar{v}_j - 1)$, enabling us to write (27) as:

$$\dot{v}_j = \frac{1}{\bar{V}} \left\{ [C(t) - w(T) - T(t)](v_j - 1) + (\beta \bar{V} / \bar{C})(1 - \bar{v}_j)C(t) \right\}$$  \hspace{1cm} (30)

To analyze the evolution of relative wealth, we linearize (30) around the steady state $[\bar{C}, \bar{w}, \bar{V}, \bar{T}]$. Omitting details, the linearized equation becomes (see Appendix A.2):

$$\dot{v}_j = \beta (v_j - \bar{v}_j) + (\bar{v}_j - 1) \frac{(\bar{w} + \bar{T})}{\bar{V}} \left\{ \frac{C}{\bar{C}} - \frac{(w + T)}{(w + T)} \right\}$$  \hspace{1cm} (31)

The key observation about (31) is that the coefficient of $v_j(t) > 0$. Thus in order for the long-run distribution of wealth to be non-degenerate, each agent’s relative wealth must remain bounded. To achieve this requires that the solution for $v_j(t)$ is given by the forward-looking solution:

$$v_j(t) - 1 = (\bar{v}_j - 1) \left\{ 1 + (\bar{w} + \bar{T}) \int_0^t \left[ \frac{w(u) + T(u)}{\bar{w} + \bar{T}} - \frac{C(u)}{\bar{C}} \right] e^{-\beta(u-t)} du \right\}$$  \hspace{1cm} (32)

Setting $t = 0$ in (32), determines the steady-state relative wealth, $\bar{v}_j$ in terms of the relative wealth at time 0, namely

---

20 Since $\beta \bar{V} / \bar{C} < 1$ we can immediately infer that consumption inequality, which remains constant over time, is less than long-run wealth inequality.

21 The procedure we are following is developed in greater detail in Turnovsky and García-Peñalosa (2008).

22 Otherwise $v_i \to \pm \infty$, depending upon the agent’s initial endowment.
\[(\tilde{v}_j - 1) = (v_j(0) - 1) \left\{ 1 + \frac{(\tilde{w} + \tilde{T})}{\tilde{v}} \right\} \int_0^\infty \left[ \frac{w(u) + T(u)}{\tilde{w} + \tilde{T}} - \frac{C(u)}{C} \right] e^{-\beta u} du \right\}^{-1} \quad (32')\]

and letting \( t \to \infty \) in (32) we see that \( \lim_{t \to \infty} v_j(t) = \tilde{v}_j \).

In general, the initial jumps in \( p(0) \), and \( C(0) \), following a structural change, including an investment tariff decrease, will cause an initial jump in \( v_j(0) \) from its previous stationary level. For the simulations we perform this turns out to be extremely small, and it will be exactly zero if initially all agents hold the same portfolio shares, as we shall henceforth assume, in which case \( v_j(0) = v_{j,0} \). 23

Because of the linearity of (32) and (32') across agents, these equations, which describe a specific individual’s relative asset position, can be directly transformed into a corresponding relationship describing the relative distribution of wealth across agents, as measured by its coefficient of variation, which therefore serves as a convenient measure of wealth inequality. For notational convenience let

\[ \chi(t) = \left\{ 1 + \frac{(\tilde{w} + \tilde{T})}{\tilde{v}} \right\} \int_0^\infty \left[ \frac{w(u) + T(u)}{\tilde{w} + \tilde{T}} - \frac{C(u)}{C} \right] e^{-\beta (u-t)} du \] \quad (33a)

in which case

\[ \sigma_v(t) = \chi(t) \hat{\sigma}_v \] \quad (33b)

\[ \sigma_{v,0} = \chi(0) \hat{\sigma}_v \] \quad (33c)

and hence

\[ \sigma_v(t) = \frac{\chi(t)}{\chi(0)} \sigma_{v,0} \] \quad (33d)

Thus given \( \sigma_{v,0} \), (33a-d) determine the entire time path of \( \sigma_v(t) \).

Written in this way we can identify the elements driving the dynamics of relative wealth, and therefore its distribution across agents, which occur as the economy traverses its transitional path. The first is the time path of the discounted present value of labor income plus tariff revenues, which we refer to as “gross labor income”. With wage income and tariff revenues being uniformly distributed across agents, the more rapidly these approach their new steady-state values following a tariff change,

\[ 23 \text{ That is we assume } S_{i,j} / S_{i,j,0} = E_{i,j} / E_{i,j,0} = Z_{i,j} / Z_{i,j,0} \text{ for each } i, j. \]
the less the accumulated wealth differences along the transitional path and the smaller the effect on wealth inequality. This is compared to the discounted present value of consumption along the transitional path. Since wealthier people consumer relative less and save relatively more, thereby accumulating wealth at a faster rate, more rapid growth in aggregate consumption is associated with increasing wealth inequality. We may summarize this in:

**Proposition 2:** To the extent that gross labor income is increasing (decreasing) during the transition it will lead to a permanent decrease (increase) in wealth inequality. To the extent that aggregate consumption is increasing (decreasing) it will lead to a permanent increase (decrease) in wealth inequality. Both responses depend upon the speed with which they are occurring.

To actually compute \( v_j(t) - 1 \) and \( \sigma_j(t) \) along the transitional path we substitute the solutions for \( w(t) - \tilde{w} \), \( T(t) - \tilde{T} \) as described in the Appendix A.1.

### 4.2 Income inequality

There are several natural measures of income inequality that one can consider. We focus on disposable income, taken to include net income earned on wealth, labor income, plus the transfers received from the tariff revenues. Using the arbitrage conditions (9d) – (9f), agent \( j \)’s income is

\[
Q_j(t) = i(t)V_j(t) + w(t) + T(t)
\]

with aggregate income being:

\[
Q(t) = i(t)V(t) + w(t) + T(t)
\]

so that the agent’s relative income, \( q_j(t) = Q_j(t)/Q(t) \), is

---

24 Specifically we define net income from wealth expressed in terms of the traded good as numeraire to be \((r_t + \delta \bar{p}_t)S_j + (r_c + \bar{r}_c - \delta_c(1 + r_c)E - iZ_j\) , which nets out depreciation (appropriately priced) and adjusts for capital gains. Substituting (9d) to (9f) this reduces to \( i(t)V(t) \). An alternative measure would be gross income inequality \( Q_j(t) = i(t)V_j(t) + w(t) \). With tariff revenues being a small percentage of GDP the difference between that and our chosen income measure is minor.
\[ q_j(t) - 1 = \frac{i(t)V(t)}{i(t)V(t) + w(t) + T(t)} (v_j(t) - 1) \]  

(35)

Again, because of the linearity of (35) we can express the relationship between relative income and relative wealth in terms of the corresponding standard deviations of their respective distributions, \( \sigma_q \) and \( \sigma_v \) namely

\[ \sigma_q(t) = \frac{i(t)V(t)}{i(t)V(t) + w(t) + T(t)} \sigma_v(t) \equiv \zeta(t) \sigma_v(t) \]  

(36)

where \( \zeta(t) \equiv \frac{i(t)V(t)}{[i(t)V(t) + w(t) + T(t)]} \) denotes the share of income from wealth in total disposable income. Hence, at any instant of time inequality can be decomposed into two elements. The first is the dynamics of wealth inequality, \( \sigma_v(t) \); the second is the dynamics of factor returns as they impact the share of income from net wealth, \( \zeta(t) \).

Assuming that the economy starts out in an initial steady state, (36) reduces to

\[ \frac{\bar{\sigma}_q}{\bar{\sigma}_v} = \left( \frac{\beta \bar{V}}{\beta \bar{V} + \bar{w} + \bar{T}} \right) \bar{\sigma}_v \]  

(37)

and dividing (36) by (37) we derive the following expression for income inequality relative to the initial long-run inequality

\[ \frac{\sigma_q(t)}{\bar{\sigma}_q} = \zeta(t) \left[ \frac{\beta \bar{V} + \bar{w} + \bar{T}}{\beta \bar{V}} \right] \frac{\sigma_v(t)}{\bar{\sigma}_v} \]  

(38)

In steady state (38) reduces to:

\[ \frac{\bar{\sigma}_q}{\bar{\sigma}_v} = \left( \frac{\bar{C}/\bar{V}}{\bar{C}/\bar{V}} \right) \bar{\sigma}_v \]  

(38’)

so that long-run income inequality varies positively with long-run changes in wealth inequality and inversely with changes in the gross consumption-wealth ratio. The latter response reflects the fact wealthier people consume relatively less and save relatively more. Thus we may state:

**Proposition 3:** To the extent that a tariff reduction reduces the gross consumption-
wealth ratio it will increase long-run income inequality. To the extent that it decreases (increases) wealth inequality it will decrease (further increase) income inequality.

Thus, the overall effect of a tariff change will reflect both these effects. As our numerical simulations suggest, for plausible calibrations a reduction in either tariff will likely reduce wealth inequality. In addition both tariffs tend to reduce the $C/V$ ratio. In general, the decline in response to a reduction in $\tau_c$ is sufficiently large to dominate the wealth inequality effect causing long-run income inequality to rise. However, the decline following the reduction in $\tau_e$ is somewhat smaller, reducing the increase in income inequality; indeed in some cases it is actually dominated by the wealth inequality effect so that income inequality actually declines.

5. Numerical Analysis

Because of the complexity of the model, to analyze the dynamics of the aggregate economy as well as the dynamics of wealth and income inequality requires numerical simulations. To study the local dynamics of the economy following a reduction in tariffs, we employ the linearized system (A.4). The simulations are based on the constant elasticity utility function, (5), while the sectoral production functions are specified as follows:

$$ Y_T = A_T \left[ \alpha_T (S_T)^{\rho} + (1 - \alpha_T) (E_T)^{\rho} \right] \frac{\omega_T}{\rho} (L_T)^{1-\rho} \equiv A_T \left[ \alpha_T (s_T)^{\rho} + (1 - \alpha_T) (e_T)^{\rho} \right] \frac{\omega_T}{\rho} L_T $$

$$ Y_N = A_N \left[ \alpha_N (S_N)^{\rho} + (1 - \alpha_N) (E_N)^{\rho} \right] \frac{\omega_N}{\rho} (L_N)^{1-\rho} \equiv A_N \left[ \alpha_N (s_N)^{\rho} + (1 - \alpha_N) (e_N)^{\rho} \right] \frac{\omega_N}{\rho} L_N $$

The production functions for both sectors are specified to be of CES form between the two capital goods, which together combine with labor in Cobb-Douglas form. The elasticity of substitution between the two capital goods is assumed to be the same in the two sectors, and we make the plausible assumption that structures and equipment are complements in production and assume $\sigma = 1/(1+\rho) \leq 1$. The coefficients $\alpha_T, \alpha_N$ characterize the relative intensity of nontraded to traded capital in the two production functions, while $1 - \omega_T, 1 - \omega_N$ parameterize the degree of labor intensity. In addition, the borrowing (or lending) constraint is specified by
\[ i = i^* + \zeta \left( e^{\frac{z}{pS}} - 1 \right) \]

which reflects convex increasing borrowing costs.

### 5.1 Calibration

The parameters used to calibrate the benchmark economy are summarized in Table 1.A, with the equilibrium values of key macro quantities reported in Table 1.B. These are taken to be typical of an emerging trade-dependent open economy.

Turning first to preferences, \( \gamma = -1.5 \) implies an intertemporal elasticity of substitution equal to 0.4, well within the range of empirical estimates; see Guvenen (2006). The exponents on preferences, \( \theta = 0.4, \eta = 0.2 \) imply that 50% of total consumption expenditures are on tradables, consistent with evidence of Lombardo and Ravenna (2012).\(^{25}\) The rate of time preference of 5% in conjunction with the world interest rate of 3.5%, together with the parameterization of the borrowing premium \( (a = 0.06, \xi = 1) \) implies that the economy is a debtor with an equilibrium debt-GDP ratio of around 0.43. In practice, the net position of countries in the international financial market varies substantially and our equilibrium figure is close to the average of major emerging economies like Brazil, Indonesia, Turkey, and Mexico.\(^{26}\)

Information on the production structure of two sector models is sparse, although detailed information is provided by Bems (2008). Using a UN sample exceeding 110 countries over the period 1960-2004, he finds that an average of around 60% of aggregate investment expenditures are devoted to nontradables, an allocation that he finds is extremely stable over time. Assuming depreciation rates of structures and equipment \( \delta_s = 0.075 \) and \( \delta_e = 0.10 \), respectively, in steady-state the constraint \( \tilde{p}\tilde{I}_s = 0.6(\tilde{p}\tilde{I}_s + \tilde{I}_e) \) translates to \( \tilde{p}\tilde{S}/\tilde{E} = 2.0 \), a ratio that our steady state closely matches (2.05).

Since the nontraded sector includes services that are more labor intensive, we set \( \omega_r = 0.44, \omega_n = 0.28 \), implying corresponding sectoral elasticities of labor productivity of 0.56 and 0.72, respectively. Given the rest of the calibration this yields an equilibrium labor share of output of

\(^{25}\) Using the definition of tradable consumption based on the share in CPI expenditure, Lombardo and Ravenna (2012) find the mean tradable consumption share in a sample of 25 industrial and emerging economies to be around 55% or 49%, depending upon the threshold used to define tradable.

\(^{26}\) World Bank Development Indicators.
around 0.65. Further constraints are obtained by applying the equilibrium sectoral allocation conditions (20). As a benchmark we assume that the two sectors are equally intensive in the two capital goods, so that \( \alpha_T = \alpha_N = \alpha \). In this case (23a)-(23c) imply

\[
\frac{\tilde{s}_T}{\tilde{s}_N} = \frac{\tilde{e}_T}{\tilde{e}_N} = \left(1 - \frac{\omega_N}{\omega_T}\right) \frac{\omega_T}{\omega_N} = 2.02
\]

which, together with the capital market clearing conditions (24a), (24b), implies

\[
\frac{\tilde{p}\tilde{s}_T}{\tilde{e}_T} = \frac{\tilde{p}\tilde{s}_N}{\tilde{e}_N} = \frac{\tilde{p}\tilde{S}}{E} = 2.05
\]

Conditions (39a,b) are independent of \( \alpha \), the choice of which is constrained by (23d).27 This is most transparent for the Cobb-Douglas production function, when this relationship simplifies to:

\[
A_T \alpha_T (1 - \alpha) \left( \frac{\tilde{s}_T}{\tilde{e}_T} \right)^{\omega_T} \left( \frac{\tilde{e}_T}{\tilde{e}_N} \right)^{\omega_N - 1} = (\beta + \delta_E)(1 + \tau_e) = 0.1665
\]

The fact that expenditure on structures dominates that on equipment, raises the productivity of traded capital, yielding a return that is inconsistent with the external return (0.1665) if \( \alpha \) is too small. Accordingly, we set \( \alpha = 0.68 \) as a benchmark. In addition, since structures and equipment are presumably complements in production we set \( \sigma = 2/3 \) as the benchmark, although we allow it to vary between 0.2 and 1.25.

The implied aggregate capital-output ratio is 2.57. The share of GDP produced in the traded sector 0.47 is close to the average of the set of economies reported by Morshed and Turnovsky (2004). With around 41% of labor employed in that sector this implies the ratio of labor productivity in the traded sector to that in the nontraded sector to be about 1.29. A recent study employing a panel of 56 countries by Mano and Castillo (2015) shows how this productivity ratio varies across regions and industries. Our estimate of 1.29 is remarkably close to their all sample benchmark ratio of 1.26.28

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27 We may also observe that precisely the same constraints are implied by (20a-20c), together with (24a, 24b). Thus the sectoral capital proportionalities described by (39a, 39b) hold throughout the entire transitional paths following either tariff reduction.

28 The equilibrium consumption-wealth ratio implies that the ratio of income inequality to that of wealth inequality, as measured by the relative coefficient of variation is around 0.14. While the Gini coefficient on wealth is uniformly significantly larger than that of income, this ratio overstates the difference. The main reason for this is because much of income inequality is due to wage inequality, which we abstract from here. To incorporate this element would require us to
The base tariff on consumption is set at 22%, while the investment tariff is set at 11%, which is close to the average of low and middle income countries for around 1990; see World Bank (2015). Together they generate a tariff revenue of around 3.2% of GDP, which is very close to the average of emerging economies for the period 1995-2000.29

While we do not attempt to calibrate to a specific economy or episode, we view this calibrated equilibrium as providing a plausible benchmark designed to facilitate our understanding of the mechanisms in play as the economy evolves over time in response to a tariff reduction, the specification of which is as follows. Starting from the initial benchmark, $\tau_c = 0.22, \tau_e = 0.11$, in each case we specify a a 10 percentage point reduction in two alternative ways.30 The first assumes the reduction is completed instantaneously. The second specifies the reduction to occur gradually, at the constant rate of 10% per year (cf. footnote 9). In the latter case, the reduction is completed only asymptotically, although it is straightforward to impose a finite time horizon. The key point is that the moment the tariff policy is announced, its future levels along the transitional path become fully anticipated and begin to influence behavior.

6. Tariff reductions: long-run aggregate effects

We begin by comparing the long-run effects a 10 percentage point reduction in the consumption tariff $\tau_c$ from 22% to 12% with a similar reduction in the investment tariff $\tau_e$ from 11% to 1%. We shall focus on the benchmark parameterization $\alpha_{\tau} = \alpha_N = 0.68$ presented in Table 2. As already noted, the long-run effects on the aggregate economy are identical, whether the tariff is eliminated instantly or gradually over time.

6.1 Reduction in consumption tariff

The reduction in $\tau_c$ leaves the long-run relative price, $\tilde{p}$, sectoral capital intensities introduce differential labor skills, complicating further what is already quite a complex model. Despite this caveat, we feel that the measure provides reasonable guidance to the relative impacts of the changes in tariff policy on both wealth and income inequality.

29 See OECD (2015, Graph 9.3)
30 The combination of these tariff cuts reduces the share of tariff revenues to GDP from 3.2% to around 1.4%, which is the average of the developing countries over the period 2008-2012; see OECD (2015, Graph 9.3)
(\(\tilde{\tau}_T, \tilde{\tau}_N, \tilde{e}_T, \tilde{e}_N\)), and factor returns unchanged, consistent with (23). The reduced consumption tariff lowers the domestic price of the imported consumption good, encouraging more trade, shifting resources from the nontraded to the traded sector. Thus \(L_T\) increases by around 0.5 percentage points (1.21%) with a corresponding reduction in \(L_N\). The constancy of the sectoral capital intensities implies that the two capital goods both move in the same proportions. With \(s_T > s_N, e_T > e_N\), (24a) and (24b) imply that in long-run equilibrium the total stocks of both capital goods increase slightly, with the increase in \(\tilde{E}\) slightly exceeding that of \(\tilde{S}\). This results in an overall increase in the total capital stock, \(\tilde{K}\), of around 0.34%. In addition, with the sectoral capital-labor ratios, \(s_T\) etc., remaining unchanged, the reallocation of resources to the traded sector means that \(\tilde{Y}_T\) increases while \(\tilde{Y}_N\) declines. Moreover since \(s_T/s_N = e_T/e_N > 1\) the increase in traded output exceeds the decline in nontraded output. The resulting marginal increase in GDP is less than that of \(\tilde{K}\), and the overall capital-output ratio rises. The reduction in nontraded output, coupled with the resources necessary to replace the depreciated nontraded capital, leaves less of the nontraded output available for consumption. The proportionality of the three consumption components [see (13)] thus requires overall consumption, \(\tilde{C}\), to decline. With the long-run borrowing cost tied to \(\beta\), the long-run increase in \(\tilde{S}\) is matched by a proportional long-run increase in foreign debt, \(\tilde{Z}\). This offsets the increase in capital and overall gross wealth, \(\tilde{V}\), increases by 0.36%. Finally, the decline in \(\tilde{C}\), coupled with the increase in \(\tilde{V}\) implies a reduction in the equilibrium \(\tilde{C}/\tilde{V}\) of 1.54%.

### 6.2 Reduction in investment tariff

With \(\alpha_T = \alpha_N\), reducing the tariff on investment, \(\tau_e\), by 10 percentage points raises both sectoral traded capital intensities \((e_T, e_N)\) by around 10.3% and with the two capital goods being complements the nontraded capital intensities \((s_T, s_N)\) also rises proportionately, but by a lesser amount of around 2.95%. With labor supply fixed the reduction in \(\tau_e\) raises the relative abundance of capital, and with \(\omega_T > \omega_N\) raises the relative productivity of labor in the traded sector. Thus in order for the return to labor to be equalized across sectors, the relative price of nontraded output increases by around 0.90%. In addition, the increased trade shifts resources to the traded sector, and while this is met primarily by an increase in capital, it also involves a modest reallocation of labor. As a result
of this, with sectoral capital intensities in the traded sector exceeding those in the nontraded sector [see (39a)], the overall increases in the two capital goods exceed the sectoral increases \((d\hat{S} = 3.14\%, d\hat{E} = 10.5\%)\), respectively. A further consequence of the increase in capital is that despite the reallocation of labor, both traded and nontraded output increase, although the overall increase is less than that of capital, so that \(K/Y\) increases. In contrast to the reduction in \(\tau_c\), the increase in nontraded output, is sufficient, given the need to replace the depreciated nontraded capital for consumption to increase. With the long-run borrowing cost tied to \(\beta\), the long-run increase in the value of nontraded capital \(\hat{p}\hat{S}\) is matched by a proportional long-run increase in foreign debt, \(\hat{Z}\). This partially offsets the increase in capital and overall gross wealth, \(\hat{V}\), increases by 2.59\%. This increase in \(\hat{V}\) exceeds the increase in \(\hat{C}\), yielding a modest reduction in the equilibrium \(\hat{C}/\hat{V}\) of 1.13\%.

A comparison of the responses in rows 2 and 3 illustrates the contrasting impacts of the two alternative forms of tariff reduction. In particular, a 10 percentage point reduction in \(\tau_c\) increases the total capital stock, \(K\), by 0.36\% and an increase in GDP of 0.16\%. In contrast, a 10 percentage point reduction in \(\tau_e\) increases the capital stock by 6.17\% and an increase in GDP of around 2.58\%. The strong growth and output effects of an investment tariff reduction is in general agreement to the recent simulation and empirical evidence provided by Estevadeordal and Taylor (2013).\(^\text{31}\)

### 7. Tariff reductions: Transitional paths

We now compare the transitional dynamics associated with the two forms of tariff reduction. These are illustrated in Figs. 1 and 2, respectively, where the sharp contrasts in the dynamic responses are clearly evident. These differences reflect the fact that \(\tau_c\), by influencing the relative price of the imported consumption good, impacts the economy via final demand, while \(\tau_e\), by raising the cost of imported capital, influences production. In both cases we compare the adjustment paths where the tariff reduction is completed instantaneously, with the alternative scenario where it occurs gradually.

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\(^{31}\) Estevadeordal and Taylor’s simulations show that a 10% reduction in the investment tariff raises long-run GDP by 4.60\% if all capital is traded and by 1.48\% if it is mostly nontraded. Assuming a 40-60\% split, as in our analysis, this suggests an overall increase in GDP of around 2.72\%, close to our estimate of 2.58\%. Their empirical analysis suggests that a 10% reduction in the overall tariff will raise the annual growth rate by between 0.3\% and 0.4\%, which over the long run (50 years say) would raise the capital stock by between 16\% and 22\%. Since their tariff is also on intermediate goods, and accounts for around 75\% of the growth effect, this suggests that the 10% reduction in the investment tariff will raise the long-run capital stock by at least 5\%, to which our estimate of 6.2\% is quite close.
at the rate of 10% per period.

7.1 Reduction in consumption tariff

The dynamics of the aggregate quantities where the 10 percentage point reduction in the consumption tariff is completed instantaneously are illustrated by the dashed lines in the panels of Fig. 1. On impact, the reduction of $\tau_c$ reduces total consumption expenditure, $C(0)$, from 0.980 to 0.967; [Fig 1.(1b)]. With imports cheaper, the demand for nontraded consumption declines, causing the relative price, $p(0)$, to drop albeit very slightly by around 0.01% [Fig. 1.(1a)]. The decline in tariffs stimulates trade, moving resources from the nontraded to the traded sector. To maintain equilibrium in the factor markets, the sectoral capital-labor ratios must satisfy the constraints, dictated by (39a), namely, $s_r = 2.02s_N$, $e_r = 2.02e_N$, so that $d\hat{s}_r = d\hat{s}_N, d\hat{e}_r = d\hat{e}_N$. This constraint, together with (i) the fact that $S$ is fixed in the short run, (ii) total labor supply is inelastic, and (iii) the traded sector is more intensive in both capital goods $(s_r > s_N, e_r > e_N)$ implies that the reallocation of resources to the traded sector is accomplished primarily by a reallocation of labor from the nontraded to the traded sector $(dL_r > 0, dL_N < 0)$; see Fig. 1.(2a) where $L_r(0)$ increases from 0.414 to 0.416. With labor moving to the traded sector, where it is relatively less productive $(\omega_r > \omega_N)$ wages immediately decline slightly; see Fig. 1.(2b). This movement of labor is accompanied by a reallocation of nontraded capital such that its ratio to labor in both sector declines. In addition, with equipment and structures being complements in production, $e_r, e_N$ both decline as well [Figs. 1.(3a)-(4b)]. As a result of this factor reallocation initial traded output increases by around 0.31%, while nontraded output declines by around 0.26% [Figs. 1.(5a, 5b)].

These initial responses generate subsequent dynamics. Thus the reduction in $C/p$ exceeds the decrease in $Y_N$ requiring that $S$ starts to accumulate in order for the nontraded goods market to clear. Likewise, the net effect of the increase in imported consumption stems from the decline in $\tau_c$ and the increase in imported capital dominates the increase in $Y_T$ so that debt starts to accumulate. Third, the initial increase in borrowing costs and the return to nontraded capital causes the growth rate of consumption to rise, while with the increase in the former exceeding that of the latter $p$ starts to rise. As $Z$ and $S$ start to increase these generate more dynamics propelling the economy to its new steady
state. In some cases, such as \( C \) and \( p \) the adjustments are very minor. In other cases, they are more significant and involve subsequent reversals. For example, after approximately 5 periods of increase, debt will begin to decline. This reflects the fact that in the short run with \( S \) sluggish, the increase in traded output is met primarily by increasing imported capital together with the migration of labor. As \( S \) is accumulated albeit slightly, producers substitute away from machinery, the imports of which decline, reducing the level of debt. Clearance in the structural capital market along the transition implies\( \hat{S} = \hat{s}_r + (s_r - s_n)(\hat{L}_r/S) \). With the sectoral capital-labor ratios adjusting in proportion at a rate faster than that of overall structures, and with \( s_r > s_n \) it follows that following its initial jump to the traded sector, labor must be gradually reallocated back toward the nontraded sector.

The dynamics in the case where tariffs are reduced gradually at the rate of 10% per period are illustrated by the solid lines. In all cases, the aggregates converge to the same steady state, although the transitional paths diverge dramatically. This reflects the fact that the tariff reduction embodies two factors: (i) an implementation effect, which operates when the tariff reduction is actually introduced, and (ii) a wealth effect, which comes into effect the instant the policy is announced. In the case of immediate complete tariff reduction both effects come into effect simultaneously, while with a gradual adjustment, the implementation effect only builds up gradually over time.

Thus, the knowledge that in the long run the consumption tariff will be reduced raises agents’ wealth. However, since in the short run tariffs remain unchanged, they channel their initial additional purchasing power toward the nontraded good. This raises the price of the nontraded good, causing resources to be reallocated to the nontraded sector, the output of which rises leading to an initial accumulation of nontraded capital. Over time, as the tariffs are gradually reduced, resources are gradually reallocated back to the traded sector, as the implementation effect gains strength.

Whether the reduction in the consumption tariff is completed instantaneously, or implemented gradually, the magnitudes of the dynamic adjustments are small. This reflects the fact that \( \tau_c \) has no long-run impact on the economy’s production structure, as described by sectoral capital intensities and relative price. The bulk of the adjustment occurs through the reallocation of labor across the sectors and the reduction in aggregate consumption (inclusive of the tariff).
7.2 Reduction in investment tariff

The dynamic adjustments following a 10 percentage point reduction in the investment tariff from 11% to 1% are illustrated in Fig. 2, with the instantaneous and gradual tariff adjustments again being denoted by the dashed and solid lines respectively. A striking contrast with the consumption tariff is that, because \( \tau_e \) directly impacts the production structure, it causes more substantial quantitative adjustments, both in the short run and over time.

On impact, the 10 point reduction in \( \tau_e \) will reduce the price of equipment, causing an immediate increase in imported capital of around 8.6% [Fig. 2.(7a)]. With structures and equipment being complementary in production, this immediately raises the demand for structures, which are produced in the nontraded sector and therefore take time to acquire. To produce the necessary nontraded output, labor is immediately deployed to the nontraded sector, causing an immediate reduction in \( L_T \) from 0.414 to 0.385 (increase in \( L_N \) from 0.586 to 0.615) [Fig. 2.(5a,5b)]. The increase in imported capital coupled with a reallocation of the existing structures across the two sectors causes \( e_T, e_N \) to both increase by 10%, while \( s_T, s_N \) both rise by somewhat more than 2% [Figs. 2.(3a-4b)]. Thus, the net immediate effect of the tariff reduction is a paradoxical one: output of the nontraded sector increases by 6.4%, while that of the traded sector declines by 4.8% [see Figs. 2.(5a,5b)].

The reduction in the tariff on investment requires the economy to balance two conflicting objectives. First, must produce the structures necessary to maintain productive efficiency, given the complementarity of the two investment goods in production. Second, it needs to take advantage of the trade liberalization resulting from the reduced tariff. To achieve this, the economy immediately deploys 3% of the total labor force to the nontraded sector, thus accumulating the nontraded output as capital at a rapid rate. It then immediately begins to reallocate the labor back to the traded sector [see Figs. 2.(2a, 6a)], in effect making the deployment of labor very temporary. Following this strategy, after just one period, \( Y_T \) has been restored to its initial equilibrium level and it continues to increase beyond that, while \( Y_N \) declines correspondingly [Fig. 2.(5b)]. This rapid initial reduction in traded output is reflected in a corresponding rapid initial accumulation of foreign debt, which is also reversed after a couple of periods, when the productive resources have been moved to the traded sector and
exports have been increased [Fig. 2.(8b)].

At the same time, the reallocation of capital across sectors causes an initial rise in the gross return to structures, \( r_s \), from its initial equilibrium of 13.48% to around 13.64% [Fig. 2.(6b)], which in turn leads to an initial increase in the relative price, \( p(0) \), of approximately 0.8%, consistent with (19’) and illustrated in Fig.2.(1a)]. The higher price of nontraded goods leads to a reduction in their consumption, so that the net effect is to increase net total consumption \( C(0) \) by approximately 0.5%, from 0.989 to 0.984, as illustrated in Fig 2.(1b).

From Fig. 2 it is evident that the rapid adjustment characteristic of the initial stages is driven by the need to increase the stock of structures (which takes time) in response to the increase in equipment (available instantaneously through trade). Much of the adjustment is therefore completed during the first 2-3 periods immediately following the reduction in tariffs, after which the evolution proceeds at a much more leisurely pace.\(^{32}\)

The dynamic adjustments following a gradual reduction in \( \tau_e \) contrast sharply with those we have just been discussing. Again the contrast arises due to the distinction between the “implementation effect”, which arises when the tariff reduction begins to take effect, and the “wealth effect”, which occurs when information about the impending tariff reduction is revealed. In this case, since the tariff reduction proceeds gradually it impacts the economy by slightly reducing the required rate of return on investment in equipment, \( f_e^r \), (via \( \dot{r}_e \) in (20d)) from around 0.1665 to 0.1617. This induces firms to reallocate equipment from the nontraded to the traded sector, and given their complementarity, likewise for structures. The net effect is a slight reduction in \( f_s \), leading to a reduction in the relative price of nontraded goods, \( p \) in accordance with (19’), which in turn leads to an immediately decline in \( V \). Given the initial stock of structures, \( S \), and the relative sectoral intensities, \( s_r > s_N \), the mild increase in \( s_T \), dominates the mild drop in \( s_N \) leading to an slight initial reallocation of labor to the nontraded sector [see (22a)]. This in turn together with the corresponding reallocation of equipment to the traded sector necessitates an immediate reduction in \( E \). With \( E \) actually declining on impact, there is no need to immediately raise \( S \), (as when \( \tau_e \) declines instantaneously). As a result, the

\(^{32}\) Formally this is the result of the large negative eigenvalue \( \mu_i = -0.976 \) which dominates the dynamics at the beginning of the transition.
allocation of capital toward the traded sector suffices to immediately raise traded output. Over time, as \( \tau_e \) gradually declines the forces associated with the implementation effect gradually increase and the aggregate economy evolves to the new equilibrium.

Comparing Figs. 1 and 2 highlights two sharply contrasting aspects of the transitional dynamics. The first is that between the immediate versus the gradual tariff reduction. But in addition, in either of these cases the dynamic adjustments generated by the two tariffs are also in sharp contrast, this being a reflection of their differential impact the economy.

8. Dynamics of Wealth and Income Inequality

As discussed in Section 4, the dynamics of wealth inequality are driven by the discounted sum of expected future consumption relative to that of labor income inclusive of tariff rebates, (gross labor income); see (32). Knowing the time path for wealth inequality, the dynamics of income inequality then depends upon the evolution of the share of income from wealth relative to that of personal income; see (36). To facilitate our understanding of the link between the aggregate dynamics and the distributional implications, it is helpful to consider Figs. 1.(1b), 2.(1b) (aggregate consumption), Figs. 3.(1a,3a), (gross labor income), and Figs. 3.(1b, 3b) (capital income share of wealth).

8.1 Reduction in consumption tariff

Comparing Fig. 1.(1b) with Fig. 3.(1a) with we see that when the reduction in the consumption tariff, \( c^\tau \), is completed instantaneously, the initial declines in \( C \) and \( w+T \) are such that aggregate consumption approaches its steady state at a slightly slower rate than does gross labor income. Since this is so uniformly along the transitional path, and since gross labor income is equally distributed across agents, wealth inequality falls, albeit very slightly. As illustrated in Fig 3.(2a) long-run wealth inequality is reduced by 0.13%. In addition, the discrete 10 percentage point reduction in \( \tau_e \) by causing an immediate decline in \( Q \) causes \( iV/Q \) to increase by around 1.27%, which over time increases further to around 1.5%. Adjusting for the slight decline in wealth inequality, this translates to a long-run increase in income inequality of around 1.42%; see Fig. 3.(2b).
In contrast, if $\tau_c$ is reduced only gradually, $C$ adjusts at a slower rate relative to $(w+T)$ and there is more transitional time for relative income to adjust and wealth inequality declines by 1.02%. The increase in $iV/Q$ also proceeds gradually and coupled with the larger long-run decline in wealth inequality nets out to a smaller increase in income inequality of around 0.51%.

8.2 Reduction in investment tariff

Reducing the investment tariff instantaneously, causes both $C$ and $(w+T)$ to immediately increase on impact. Again, the initial response moves gross labor income closer to its new steady state than it does consumption, so that for similar reasons to those above, wealth inequality declines steadily, eventually falling by 1.11%; see Fig. 3. (4a). On impact, $iV/Q$ declines sharply, due almost entirely to the immediate rise in the wage rate and therefore labor income and causing a very temporary decrease in income inequality of 1.28%. Thereafter, $iV/Q$ rises rapidly as structures are accumulated to match the additional equipment, with $iV/Q$ essentially reaching its long run increase of around 1.13% after a couple of periods, with inequality increasing by almost 1%. Thereafter, the gradual decline in income inequality reflects the decline in wealth inequality and in the long run income inequality by just 0.07%.

If $\tau_c$ is reduced only gradually, wealth inequality rises initially before gradually declining. This reflects the fact that the smaller increase in $w$ [see Fig. 2.(2b)], coupled with the sluggishness of the tariff revenues means that the initial increase in $w+T$ is much smaller so that it actually declines relative to its long-run responses during the early stages of the transition. With this adverse short-run response of labor income, his leads to an initial increase in wealth inequality, which eventually is reversed over time. The relative share of income from wealth, $iV/Q$ adjusts more gradually and when considered in conjunction with the non-monotonic adjustment of wealth inequality, leads to a non-monotonic adjustment in income inequality.

8.3 Tariffs and Tradeoffs

The responses in the benchmark economy highlight the sharp contrasts between the two tariffs and particularly the tradeoffs they involve between capital accumulation (growth) and inequality.
Reducing the consumption tariff, $\tau_c$ by 10 percentage points is characterized by the following:

(i) A weak effect on economic growth as reflected by a small increase in the capital stock.
(ii) A small reduction in wealth inequality if implemented instantaneously, a larger reduction if introduced gradually.
(iii) A substantial and rapid increase in income inequality if implemented instantaneously, a more gradual and milder increase if introduced gradually.

Reducing the tariff on investment, $\tau_e$ by 10 percentage points has the following effects:

(i) A powerful effect on capital accumulation
(ii) A significant gradual reduction in wealth inequality if implemented instantaneously, which is reduced if introduced gradually and even associated with an increase in wealth inequality in the short run.
(iii) A tradeoff between the short-run and long-run effects on income inequality, reducing it in the very short run, while increasing it over time. The tradeoff is augmented when the tariff reduction is implemented gradually.

9. Sensitivity Analysis

Table 3 reports the sensitivity of the long-run responses of capital and the inequality measures to the reductions in the consumption tariff and the investment tariff, respectively. We focus on three critical parameters: (i) an increase in the relative importance of the imported consumption good, $\eta$, from 0.2 to 0.4; (ii) a decrease in the relative importance of nontraded capital in production, $\alpha$, from 0.68 to 0.38; varying the elasticity of substitution of the two capital goods in production between $\sigma = 0.2$ (strong complements) and $\sigma = 1.25$ (substitutes).

9.1 Sensitivity to $\eta$

The qualitative responses to reductions in the two tariffs remain essentially as for the benchmark case discussed in Sections 6-8. Unsurprisingly, the larger $\eta$ increasing the relative importance of the imported consumption directly, strengthens the effects of the reduction in $\tau_c$. 

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although the relative responses to immediate versus gradual tariff reductions remain unchanged. The effects of the reduction in $\tau_c$, by impacting production, are barely affected by a structural change involving consumption tastes, $\eta$.

9.2 Sensitivity to $\alpha, \sigma$

As noted in Section 5 our choice of $\alpha_T = \alpha_N = \alpha = 0.68$ is dictated by the compelling empirical evidence suggesting that approximately 60% of aggregate investment is devoted to nontradables, with equal sectoral intensity of the two capital goods across the two sectors serving as a natural benchmark. Our results are robust to moderate variations in these sectoral intensities ($\alpha_T = 0.73, \alpha_N = 0.63$) and the reversal ($\alpha_N = 0.73, \alpha_T = 0.63$). Despite the evidence supporting the relative constancy of the composition of investment expenditures, there is inevitably some variation both over time and across nations. Thus, as an alternative we assume that production in both sectors is relatively equipment-intensive and set $\alpha = 0.38$. This implies an expenditure ratio $pS/E \approx 0.4$, approximating that of a number of heavily trade-dependent economies, such as Hong Kong, Guatemala, Nicaragua, Slovakia, Tanzania; see Bems (2008). While identifying nontraded capital with structures and traded capital with equipment, and treating them as complements in production, as we have been doing, is natural, interpreting them more broadly as representing general baskets of capital goods, in which case they may be substitutes, is also plausible.

From the second panel of Table 3 we see that the impact of the reduction in the consumption tariff, $\tau_c$, on the growth and distributional measures is quite insensitive to the substantial variations in the production characteristics. Again, this is unsurprising since the long-run equilibrium production structure and factor returns are independent of the consumption tariff.

In contrast, the growth and distributional consequences of the reduction in the investment tariff vary considerably with variations in the production elasticities, $\alpha, \sigma$. The range of responses of the inequality measures are clearly illustrated in Fig. 4. The diversity of the dynamic paths of wealth inequality reflects the sensitivity of $\chi(t)/\chi(0)$ to variations in the production elasticities $\alpha, \sigma$. In addition to this, the evolution of income inequality depends upon the impact of $\alpha, \sigma$ on the relative share of capital income, $\zeta(t)$. 

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In most cases, wealth inequality declines, as in the benchmark case. However, wealth inequality will increase when the two capital goods are strong complements ($\sigma = 0.2$) and the tariff is reduced only gradually. This is because on impact gross labor income exceeds its new steady-state level, so that during the subsequent transition it declines, impacting poor individuals more, and thus leading to an increase in wealth inequality. Whether income inequality rises or falls and the extent to which this depends upon the speed of tariff reduction implementation is highly sensitive to the two productive elasticities. In all cases we find that $\bar{C}/\bar{V}$ falls, as in the benchmark case, thus, exerting an increasing effect on income inequality. Whether long-run income inequality rises or falls depends upon whether this term dominates the likely decline in wealth inequality and this is in turn sensitive to the structural production characteristics. In particular, whether a gradual reduction in the investment tariff is less harmful to income inequality than an immediate reduction depends critically upon the substitutability of the two capital goods.

10. Conclusions

In this paper we have investigated the relationship between trade liberalization, in the form of tariff reduction, and the increase in inequality that has characterized most countries in recent years. Most of the literature examining this issue has been at the aggregate level, but the fact that consumption tariffs and investment tariffs operate in dramatically different ways and that a substantial fraction of capital is nontraded requires that the issue be addressed in a disaggregated model. Accordingly, we have done so employing a two-sector dependent economy model augmented to include both traded and nontraded capital.

Our main conclusions, based on the benchmark calibration, suggest the following tradeoffs between the two tariffs and their respective impacts on activity and distribution.

(i) Reducing the consumption tariff has only a weak effect on activity, most of which is due to the sectoral reallocation of labor. It leads to a negligible reduction in wealth inequality if implemented instantaneously, and a larger reduction if introduced gradually. This contributes
to a substantial and rapid increase in income inequality if implemented instantaneously, but a
more gradual and milder increase if introduced gradually.

(ii) A comparable reduction in the tariff on investment has the following contrasting effects. It
increases output and the aggregate capital stock significantly. It leads to a significant long-run
reduction in wealth inequality if implemented instantaneously, which is moderated if
introduced gradually and even associated with an increase in wealth inequality in the short run.
It is associated with a tradeoff between the short-run and long-run effects on income inequality,
reducing it in the very short run, while increasing it over time. The tradeoff is augmented when
the tariff reduction is implemented gradually.

These numerical simulations suggest one important policy conclusion. Suppose a policymaker
wishes to reduce tariffs with minimal impact on income inequality. These results suggest that this
objective can be achieved by removing the tariff on investment quickly, and the consumption tariff
more gradually. However, this policy conclusion is sensitive to key structural parameters. Thus, if an
economy is highly dependent upon imported capital, both tariffs should be reduced gradually.
Table 1

A. The Benchmark Economy

<table>
<thead>
<tr>
<th>Preference parameters:</th>
<th>$\gamma = -1.5, \theta = 0.40, \eta = 0.20, \beta = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production parameters:</td>
<td>$\alpha_r = 0.68, (0.38); \alpha_n = 0.68, (0.38); \omega_r = 0.44, \omega_n = 0.28; \sigma = 0.67(0.2,1.25)$</td>
</tr>
<tr>
<td>Productivity parameters:</td>
<td>$A_T = 1.0, A_N = 1.0$</td>
</tr>
<tr>
<td>Depreciation rate:</td>
<td>$\delta_s = 0.075, \delta_e = 0.10$</td>
</tr>
<tr>
<td>World interest rate:</td>
<td>$i^* = 0.035$</td>
</tr>
<tr>
<td>Premium on borrowing:</td>
<td>$a = 0.06$</td>
</tr>
<tr>
<td>Weight on the premium:</td>
<td>$\xi = 1$</td>
</tr>
<tr>
<td>Tariffs:</td>
<td>$\tau_c = 0.22, \tau_e = 0.11$</td>
</tr>
</tbody>
</table>

B. Key Equilibrium Quantities

<table>
<thead>
<tr>
<th>Capital-Output ratio:</th>
<th>2.57</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption to Wealth:</td>
<td>0.35</td>
</tr>
<tr>
<td>Debt-GDP:</td>
<td>0.43</td>
</tr>
<tr>
<td>Tariff revenues/GDP</td>
<td>3.2%</td>
</tr>
<tr>
<td>$\tilde{p}\tilde{S}/\tilde{E}$</td>
<td>2.05</td>
</tr>
<tr>
<td>Labor productivity traded/nontraded</td>
<td>1.28</td>
</tr>
<tr>
<td>Labor share of income:</td>
<td>0.65</td>
</tr>
<tr>
<td>Traded share of GDP</td>
<td>0.47</td>
</tr>
<tr>
<td>Labor employed in traded sector, $L_T$</td>
<td>0.41</td>
</tr>
<tr>
<td>Wealth inequality</td>
<td>1</td>
</tr>
<tr>
<td>Income inequality</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Table 2
Key steady-state equilibrium ratios and steady-state responses to tariff reduction

Benchmark case ($\alpha_T = \alpha_N = 0.68; \sigma = 2/3$)

A. Aggregate quantities

<table>
<thead>
<tr>
<th></th>
<th>YN</th>
<th>YT</th>
<th>Y</th>
<th>S</th>
<th>E</th>
<th>p</th>
<th>K</th>
<th>K/Y</th>
<th>LT</th>
<th>Z/Y</th>
<th>C</th>
<th>V</th>
<th>C/V</th>
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</thead>
<tbody>
<tr>
<td>Initial Eq</td>
<td>0.604</td>
<td>0.592</td>
<td>1.242</td>
<td>1.991</td>
<td>1.046</td>
<td>1.078</td>
<td>3.193</td>
<td>2.571</td>
<td>0.414</td>
<td>0.428</td>
<td>0.980</td>
<td>2.775</td>
<td>0.353</td>
</tr>
<tr>
<td>Reduce $\tau_c$ by 0.10% Change</td>
<td>0.599 (0.83)</td>
<td>0.598 (1.01)</td>
<td>1.244 (0.16)</td>
<td>1.998 (0.35)</td>
<td>1.050 (0.38)</td>
<td>1.078 0</td>
<td>3.204 (0.34)</td>
<td>2.576 (0.19)</td>
<td>0.419 (1.21)</td>
<td>0.429 (0.29)</td>
<td>0.968 (-1.22)</td>
<td>2.785 (0.36)</td>
<td>0.348 (-1.54)</td>
</tr>
<tr>
<td>Reduce $\tau_e$ by 0.10% Change</td>
<td>0.610 (1.06)</td>
<td>0.610 (3.04)</td>
<td>1.274 (2.58)</td>
<td>2.053 (3.11)</td>
<td>1.156 (10.5)</td>
<td>1.088 (0.93)</td>
<td>3.390 (6.17)</td>
<td>2.661 (3.50)</td>
<td>0.417 (1.64)</td>
<td>0.435 (1.33)</td>
<td>0.993 (2.59)</td>
<td>0.349 (-1.13)</td>
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</tr>
</tbody>
</table>

B. Distributional measures

<table>
<thead>
<tr>
<th></th>
<th>$\tilde{\sigma}_v$</th>
<th>$\sigma_v(0)$</th>
<th>$\tilde{\sigma}_q$</th>
<th>$\sigma_q(0)$</th>
<th>$\tilde{\sigma}_y$</th>
<th>$\sigma_y(0)$</th>
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<tr>
<td></td>
<td>Discrete change</td>
<td>Gradual change</td>
<td>Discrete change</td>
<td>Gradual change</td>
<td>Discrete change</td>
<td>Gradual change</td>
</tr>
<tr>
<td>Initial Eq</td>
<td>$\tau_c = 0.22; \tau_e = 0.11$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.1416</td>
<td>0.1416</td>
<td>0.1416</td>
</tr>
<tr>
<td>Reduce $\tau_c$ by 0.10% Change</td>
<td>0.9988 (-0.12)</td>
<td>0.9898 (-1.02)</td>
<td>0.1434 (1.27)</td>
<td>0.1436 (1.42)</td>
<td>0.1416 (-0.01)</td>
<td>0.1423 (0.51)</td>
</tr>
<tr>
<td>Reduce $\tau_e$ by 0.10% Change</td>
<td>0.9890 (-1.10)</td>
<td>0.9915 (-0.85)</td>
<td>0.1398 (-1.28)</td>
<td>0.1417 (0.07)</td>
<td>0.1382 (-2.40)</td>
<td>0.1420 (0.32)</td>
</tr>
</tbody>
</table>
### Table 3: Sensitivity analysis

#### A. Variations in $\eta$

<table>
<thead>
<tr>
<th></th>
<th>$K$</th>
<th>$\tilde{\sigma}_v$</th>
<th>$\sigma_q(0)$</th>
<th>$\tilde{\sigma}_q$</th>
<th>$\sigma_q(0)$</th>
<th>$\tilde{\sigma}_q$</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Discrete change</td>
<td>Gradual change</td>
<td>Discrete change</td>
<td>Gradual change</td>
<td>Discrete change</td>
</tr>
<tr>
<td><strong>Reduce $\tau_e$ by 0.10</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta = 0.2$</td>
<td>0.34</td>
<td>-0.12</td>
<td>-0.91</td>
<td>1.27</td>
<td>1.42</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\eta = 0.4$</td>
<td>0.51</td>
<td>-0.17</td>
<td>-1.48</td>
<td>2.22</td>
<td>2.46</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Reduce $\tau_e$ by 0.10</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta = 0.2$</td>
<td>6.17</td>
<td>-1.10</td>
<td>-0.85</td>
<td>-1.28</td>
<td>0.07</td>
<td>-2.40</td>
</tr>
<tr>
<td>$\eta = 0.4$</td>
<td>6.15</td>
<td>-1.04</td>
<td>-0.75</td>
<td>-1.23</td>
<td>0.14</td>
<td>-2.36</td>
</tr>
</tbody>
</table>

#### B. Variations in $\alpha, \sigma$

<table>
<thead>
<tr>
<th></th>
<th>$K$</th>
<th>$\tilde{\sigma}_v$</th>
<th>$\sigma_q(0)$</th>
<th>$\tilde{\sigma}_q$</th>
<th>$\sigma_q(0)$</th>
<th>$\tilde{\sigma}_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Discrete change</td>
<td>Gradual change</td>
<td>Discrete change</td>
<td>Gradual change</td>
<td>Discrete change</td>
</tr>
<tr>
<td><strong>Reduce $\tau_e$ by 0.10</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha = 0.68, \sigma = 0.2$</td>
<td>0.34</td>
<td>-0.07</td>
<td>-0.99</td>
<td>1.24</td>
<td>1.48</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\alpha = 0.68, \sigma = 2/3$</td>
<td>0.34</td>
<td>-0.12</td>
<td>-0.91</td>
<td>1.27</td>
<td>1.42</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\alpha = 0.68, \sigma = 1.25$</td>
<td>0.34</td>
<td>-0.15</td>
<td>-0.87</td>
<td>1.27</td>
<td>1.40</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\alpha = 0.38, \sigma = 0.2$</td>
<td>0.34</td>
<td>-0.13</td>
<td>-0.95</td>
<td>1.25</td>
<td>1.41</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\alpha = 0.38, \sigma = 2/3$</td>
<td>0.34</td>
<td>-0.09</td>
<td>-0.91</td>
<td>1.29</td>
<td>1.46</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\alpha = 0.38, \sigma = 1.25$</td>
<td>0.34</td>
<td>-0.07</td>
<td>-0.86</td>
<td>1.33</td>
<td>1.48</td>
<td>-0.01</td>
</tr>
<tr>
<td><strong>Reduce $\tau_e$ by 0.10</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\alpha = 0.68, \sigma = 0.2$</td>
<td>8.26</td>
<td>-1.50</td>
<td>1.71</td>
<td>-3.75</td>
<td>-0.19</td>
<td>-2.20</td>
</tr>
<tr>
<td>$\alpha = 0.68, \sigma = 2/3$</td>
<td>6.17</td>
<td>-1.10</td>
<td>-0.85</td>
<td>-1.28</td>
<td>0.07</td>
<td>-2.40</td>
</tr>
<tr>
<td>$\alpha = 0.68, \sigma = 1.25$</td>
<td>4.08</td>
<td>-0.23</td>
<td>-2.86</td>
<td>0.47</td>
<td>0.79</td>
<td>-4.41</td>
</tr>
<tr>
<td>$\alpha = 0.38, \sigma = 0.2$</td>
<td>9.47</td>
<td>-3.34</td>
<td>-5.30</td>
<td>-4.37</td>
<td>-1.83</td>
<td>-1.26</td>
</tr>
<tr>
<td>$\alpha = 0.38, \sigma = 2/3$</td>
<td>10.0</td>
<td>-1.45</td>
<td>-3.23</td>
<td>-1.75</td>
<td>0.45</td>
<td>-5.45</td>
</tr>
<tr>
<td>$\alpha = 0.38, \sigma = 1.25$</td>
<td>10.8</td>
<td>-0.30</td>
<td>-3.37</td>
<td>1.24</td>
<td>2.08</td>
<td>-8.37</td>
</tr>
</tbody>
</table>
Fig. 1: Dynamics of consumption tariff reduction
(5a) Traded Output

(5b) Non-traded Output

(6a) Structures

(6b) Returns to structures

(7a) Equipment

(7b) Returns to Equipment

(8a) Tariff adjusted wealth

(8b) Debt
Fig. 2: Dynamics of investment tariff reduction
Wealth inequality
(i) Wealth inequality
(ii) Income inequality

A. Reduction in Consumption Tariff

B. Reduction in Investment tariff

--- : Initial equilibrium
- - - : Instantaneous Reduction

Fig. 3 Effects of tariff reduction on inequality
A. Technology structures intensive; very low substitution: $\alpha = 0.68, \sigma = 0.2$

B. Technology structures intensive; very high substitution: $\alpha = 0.68, \sigma = 1.25$

C. Technology machine intensive; very low substitution: $\alpha = 0.38, \sigma = 0.2$

D. Technology machine intensive; very high substitution: $\alpha = 0.38, \sigma = 1.25$

--- : Initial equilibrium  
: Gradual Reduction  
: Instantaneous Reduction

**Fig. 4 Reduction in tariff on investment: alternative scenarios**
Appendix

A.1 Dynamics of the Aggregate Economy

The aggregate dynamic system is specified by:

\[ \dot{S} = h(s_N, e_N)(1 - L_T) - \left( \frac{1 - \theta}{1 + \eta} \right) C - \delta_S S \quad (A.1a) \]

\[ (1 - E_Z) \dot{Z} = \left( \frac{\theta}{1 + \eta} + \frac{\eta}{(1 + \eta)(1 + \tau_e)} \right) C + \delta_E E - f(s_T, e_T) L_T + i \left( \frac{Z}{pS} \right) Z + E_p \dot{p} + E_S \dot{S} + E_{\tau} \dot{\tau} \quad (A.1b) \]

\[ \dot{p} = p \left[ \left( \frac{Z}{pS} \right) - \left( h(s_N, e_N) - \delta_S \right) \right] \quad (A.1c) \]

\[ \dot{C} = \frac{C}{1 - \gamma(1 + \eta)} \left[ (1 - \theta) \gamma \left( h(s_N, e_N) - \delta_S \right) + [1 - \gamma(1 - \theta)] i \left( \frac{Z}{pS} \right) - \beta - \eta \gamma \left( \frac{\dot{\tau}}{1 + \tau_e} \right) \right] \quad (A.1d) \]

\[ \dot{\tau} = -v_e (\tau_e - \tilde{\tau}_e) \quad (A.1e) \]

\[ \dot{\tau} = -v_e (\tau_e - \tilde{\tau}_e) \quad (A.1f) \]

which we can rewrite as

\[
\begin{pmatrix}
\dot{S} \\
\dot{Z} \\
\dot{p} \\
\dot{C} \\
\dot{\tau}
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
1 - E_Z & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & \frac{C}{1 - \gamma(1 + \eta)} & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
h(s_N, e_N)(1 - L_T) - \left( \frac{1 - \theta}{1 + \eta} \right) C - \delta_S S \\
\left( \frac{\theta}{1 + \eta} + \frac{\eta}{(1 + \eta)(1 + \tau_e)} \right) C + \delta_E E - f(s_T, e_T) L_T + i \left( \frac{Z}{pS} \right) Z \\
p \left[ \left( \frac{Z}{pS} \right) - \left( h(s_N, e_N) - \delta_S \right) \right] \\
\left[ (1 - \theta) \gamma \left( h(s_N, e_N) - \delta_S \right) + [1 - \gamma(1 - \theta)] i \left( \frac{Z}{pS} \right) - \beta \right] \\
-\nu_e (\tau_e - \tilde{\tau}_e) -v_e (\tau_e - \tilde{\tau}_e)
\end{pmatrix}
\]

Letting \( x \equiv (S, Z, p, C, \tau_e, \tau_e) \) and denoting the coefficient matrix by \( B(x) \), the macrodynamic system can be written more compactly in the form

\[ \dot{x} = B(x)M(x) \quad (A.2) \]

To analyze the local dynamics we linearize around steady state, which occurs at \( M(\tilde{x}) = 0 \) and reduces to (24). Thus, the local dynamics are expressed by
This yields a system of the form

$$\begin{pmatrix} \dot{S} \\ \dot{Z} \\ \dot{p} \\ \dot{C} \\ \dot{\tau}_e \end{pmatrix} = B(x) \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} & 0 & m_{16} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} & m_{26} \\ m_{31} & m_{32} & m_{33} & 0 & 0 & m_{36} \\ m_{41} & m_{42} & m_{43} & 0 & 0 & m_{46} \\ 0 & 0 & 0 & 0 & 0 & -\nu_e \\ 0 & 0 & 0 & 0 & 0 & -\nu_e \end{pmatrix} \begin{pmatrix} S - \tilde{S} \\ Z - \tilde{Z} \\ p - \tilde{p} \\ C - \tilde{C} \\ \tau_e - \tilde{\tau}_e \end{pmatrix}$$

(A.4)

where the elements $m_{ij}$ denote the appropriate partial derivatives, evaluated at steady state and incorporate the short-run responses. Thus, for example,

$$m_{16} \equiv \frac{\partial \delta}{\partial \tau_e} = -h(\tilde{s}_N, \bar{e}_N) \frac{\partial \tilde{L}_T}{\partial \tau_e} \bigg|_{x=x} + (1 - \tilde{L}_T) \frac{\partial h(\tilde{s}_N, \bar{e}_N)}{\partial \tau_e} \bigg|_{x=x}$$

where the partial derivatives are obtained from (20a)-(20d) and (22a), (22b). For further convenience, we may write the linearized system (A.4) as $\dot{x}(t) = D(x(t) - \tilde{x})$.

The linearized dynamic system decomposes into two subsystems. The first consists of the endogenous dynamics of $S, Z, p, C$ and includes two sluggish variables, $S, Z$, together with two jump variables $p, C$. Assuming that this is characterized by two stable eigenvalues ($\mu_1, \mu_2 < 0$) this will yield a unique stable transitional path for given tariff rates. In practice, to determine the root structure of this system is impractical, however, extensive numerical simulations confirm this to be the case over virtually all plausible parameter sets.

To the extent that this augmented by gradual adjustments in tariffs, specified by (11), further sluggishness is imposed on the system. Thus, in general the solution to the linearized system (A.4) is of the form:

$$x(t) - \tilde{x} = A_1 \nu_1 e^{\mu_1 t} + A_2 \nu_2 e^{\mu_2 t} + A_3 \nu_3 e^{-\nu_1 t} + A_4 \nu_4 e^{-\nu_2 t}$$

(A.5)
The vectors $v_1, v_2, v_3, v_4$ correspond to the eigenvectors associated with the eigenvalues $\mu_1, \mu_2, -\nu_1, -\nu_2$, respectively. The arbitrary constants $A_i, i=1...4$ are obtained from initial conditions on the sluggish variables $S, Z, \tau_c$, and $\tau_e$. We normalize the eigenvectors, so that their first component is 1. Thus, they can be written as $v_j = (1, \kappa_{2j}, \kappa_{3j}, \kappa_{4j}, \kappa_{5j}, \kappa_{6j})$, for $j = 1,2$. Note that because of the exogeneity of $\tau_c, \tau_e$, $\kappa_{5j} = \kappa_{6j} = 0$. Similarly, we normalize $v_3, v_4$ such that their component associated with the respective tariff is set equal to one, so that $v_3 = (\pi_{11}, \pi_{21}, \pi_{31}, \pi_{41}, 1, 0)$ and $v_4 = (\pi_{12}, \pi_{22}, \pi_{32}, \pi_{42}, 0, 1)$.

We determine the arbitrary constants by evaluating (A.5) at time zero, yielding:

\[
A_1 + A_2 + A_3\pi_{11} + A_4\pi_{12} = (S_0 - \tilde{S})
\]

\[
A_1\kappa_{21} + A_2\kappa_{22} + A_3\pi_{21} + A_4\pi_{22} = (Z_0 - \tilde{Z})
\]

\[
A_3 = (\tau_{c,0} - \tilde{\tau}_c)
\]

\[
A_4 = (\tau_{e,0} - \tilde{\tau}_e)
\]

The solutions for the dynamics of $S(t), Z(t), p(t), C(t)$ are then obtained by substituting for the constants and eigenvectors into (A.5).

These solutions constitute the core aggregate dynamics. Knowing these, the time paths for the remaining aggregate then follow. Specifically, the dynamic adjustments of sectoral intensities can then be computed from (21a), (21b), and the labor allocation and accumulation of equipment from (22a), (22b). The dynamics of sectoral outputs are obtained from (1a), (1b) and $r_s(t), r_e(t), w(t)$ from (4c). The consumption components follow from (13), while equilibrium tariff revenues, necessary to compute equilibrium wealth inequality, then follow from (15) and are given by

\[
T(t) = \tau_c(t)(1 + \tau_c)^{-1}\eta(1 + \eta)^{-1}C(t) + \tau_e(t)[\tilde{E}(t) + \delta_t E(t)].
\]

### A.2 Solution for Wealth Inequality

We begin with (27):

\[
\dot{v}_j = \frac{1}{V} \left\{ [C(t) - w(t) - T(t)](v_j - 1) + (\beta \tilde{V} / \tilde{C})(1 - \tilde{v}_j)C(t) \right\}
\]

(A.6)

Linearizing this equation around steady state we obtain
which using (28) yields

\[ v_i(t) = \beta(v_i - \bar{v}_i) + (\bar{v}_i - 1) \frac{(v_i - \bar{v}_i)}{v_i} + \frac{T}{C - (w + T)} \]

Solving this equation and imposing the transversality condition we obtain

\[ \frac{dv}{dt} = \frac{1}{\nu} \left( C - w - T \right) (v_i - \bar{v}_i) + \left( C - (w + T) \right) \left( v_i - 1 \right) - \beta(v_i - 1) \left( \frac{v_i}{C - w} \right) \]
References


