Compensation of third party victims, and liability sharing rules in oligopolistic markets

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Abstract

Accidents causing environmental damages and/or harm to several (third party) victims may result from the joint action of competing firms, in such a way that it may be too costly or impossible for Courts to disentangle the specific contribution of each firm (cf noise pollution by Orly airport 1988, Cass. 2e civ, No 86-12.543; abestos litigation in USA 1994, Becker v. Baron Bros). Courts in many jurisdictions have the opportunity in such contexts to conclude for firms liability *in solidum*, and to allocate the burden of damages between offenders thanks to alternative apportionment rules. In this paper, we analyze the impacts of such liability sharing arrangements (*per capita* vs market share rule) on output and care decisions in an oligopoly. For a symmetric oligopoly, we find that compared to the *per capita* rule, the market share rule leads to a lower output level but also to lower care expenditures at equilibrium. However as the net effect on the expected harm to victims is ambiguous, it is not clear that the market share rule is dominating the *per capita* rule. Moreover, we also show that no sharing arrangement induce the optimal levels of output and care expenditures. For an asymmetric oligopoly, we find that equilibrium market shares between low cost firms and high cost firms are more dispersed under the *per capita* rule than under the market share rule. This suggests the existence of strong ties between competition law and liability law, some liability regimes for joint offenders cases developing more anticompetitive effects than others. To the least, our analysis shows that the per capita rule (market share rule) provides (preserves) competitive advantages to the most (least) efficient firms. We also find that high costs firms produce more and invest more in care under the *per capita* rule than under the market share rule; in contrast, the comparison is undertermined for low costs firms, and thus for the industry.

Keywords: Strict liability; liability apportionment; market share liability; environmental liability; imperfect competition.

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1 Introduction

Very often, firms pertaining to an industry are located on the same area, close to their markets or to some common facilities (airlines companies and airports; chemical activities or refineries and ports and so on). The issue is that the baseline output or economic activity of these firms may also produce externalities (nuisances, or accidents) to third party victims. In such contexts, hard uncertainty may prevail regarding the specific firm being at the origin of the externality, or even it is not feasible to disentangle the influence of each firm on the harm borne by victims. In this paper, we analyze the output and care decisions of firms competing *à la Cournot*, in these kinds of contexts, i.e. when they manufacture a good that may jointly harm some victims (different from their customers). Our main purpose is to compare two different rules of liability apportionment which are generally considered by Courts to share damages between joint offenders when hard uncertainty exists, namely the per capita rule on the one hand, and on the other, the market share rule (also called, market share liability).

Market share liability traditionnally applies when a plaintiff can establish that a product (or one of its components) caused her an injury but cannot identify among manufacturers the actual tortfeasor(s). In such cases, courts hold manufacturers collectively liable and each one of them compensate a portion of the damages proportional to their market share at the time of the injury. First used by the Californian Supreme Court in Sindell vs Abbott Laboratories, this legal doctrine has been mostly applied by US courts in toxic torts (torts in which an exposure to a chemical is at the origin o the injury) such as abestos[1]or MTBE [2], a gasoline additive. In Europe, some courts have also used the market share liability in this way. Following the recent *Distilbène litigation* [3], one have acknowledged a revival of the debate in France in tort cases with several potential tortfeasors. The debate mainly focused on the same issues as it was observed previously in USA (see Dillbary 2011). Some French scholars (Molfessis 2015, Quézel-Ambrunaz 2010) have argued in favor of traditional solutions adopted for damages apportionment (leading to an equal allocation of the damage) compared to the market share rule from the perspective of consistency to admitted theories of causation, and adequacy to jurisprudence. Others have motivated the latter rule on the grounds that market shares may be a proxy for the likelihood of individual liability, in contexts of joint liability characterized by hard uncertainty and ambiguous causation (Ferey and G'sell 2013, G'sell 2010).

But this debate has also considered the potential extensions of the market share liability, both in scope (domains of law) and space (see Thieffry 2013). There exists a potential for shifting from the traditional rules of apportionment (including dividing the damages to victims equally between offenders) to market share liability in the area of industrial activities with potential environmental or health harms

[1] *Becker v. Baron Bros*, 649 A.2d 613 (N.J. 1994).
[2] *In re Methyl Tertiary Butyl Ether*, 175 F. Supp. 2d 593 (S.D.N.Y. 2001).
[3] See TGI Nanterre 10 april 2014 n 12/12349 and n 12/13064. Interestingly, both cases concerns the diethylstilbestrol (DES) , a product delivered to pregnant women and which caused years later injuries to the children exposed *in utero*.

(see also Hamilton and Sunding 2000). In France, civil courts had already used activity level to apportion damages among firms.[4] At the European level, the environnemental liability established by the Directive 2004/35/CE also allow this solution. The directive let to the member states law the determination of the liability apportionment in case of multi party causation. Interestingly, the french law, at the article L. 162-18 of the environmental code establishes that when an environmental harm has multiple causation, then the damage must be divided among operators in proportion of their participation to the harm.[5]

Although Courts have used the traditional solutions (including dividing the damages to victims equally between offenders) up to now, they may be able to use market share as a proxy for the participation to the harm caused by a particular industry when it is impossible to disentangle the influence of each individual offender on the total harm to victims. Indeed, a large range of industrial activities, from the chemical to the oil sectors, have the power to provoke joint harm to several victims as third parties, or more broadly speaking to environment. From this perspective, one also have to acknowledge that in raising the issue of liability and liability sharing, it turns out to be difficult to ignore the market structure where firms operate, and the imperfection of competition. Indeed, a crucial aspect regarding the extension of liability sharing arrangements to industrial activities relates to their impacts on the strategic interactions between firms, and the intensity of competition. This is the central issues of our paper.

The first literature to which our paper is connected is about joint and several liability in contexts with multiple tortfeasors (see Kornhauser and Revesz (2000) for a review). Focusing on the incentives to take care, Landes and Posner (1980), Shavell (1987) and Kornhauser and Revesz (1989) have first considered the standard question of the comparative advantages of strict liability vs negligence. Later on, Miceli and Segerson (1991) turn to the issue of firms entry, but do not explicitly consider strategic interactions between firms.

Also clearly related to our work is the literature about product liability and imperfect competition. The seminal papers by Spence (1977), Polinsky (1980) and Polinsky and Rogerson (1983) who first discussed the interplay between market power and standard liability rules have been extended to different market set up in the recent period (Baniak and Grajzl 2016, Baumann and Friehe 2015; Baumann, Friehe and Rasch 2015; Chen and Hua 2015; Daughety and Reinganum 2014). Contrary to the issues discussed here, those works are not considering the situation of third parties as victims, (i.e. victims without any contractual arrangement or market relationship with the industry), nor the context of joint liability, or the comparison of alternative liability sharing arrangements. Also worthy to note is the paper by Hamilton and Sunding (2000) who discuss the issue of firms entry in an asymmetric (quantity) oligopoly, as a response to an increase in their liability; nevertheless, apart of considering the case of product liability,

[4] See for instance Cass. 2e civ., 20 juill. 1988, n 86-12.543. In the case, airplanes' noise pollution caused several injuries to the neighboring houses of an airport. The french suprem court, *Cour de Cassation*, held airlines companies collectively liable and each of them had to compensate a portion of damages proportional to their activity level.

[5] Article L. 162-18 du Code de l'environnement: Lorsqu'un dommage à l'environnement a plusieurs causes, le coût des mesures de prévention ou de réaration est réparti par l'autorité visée au 2 de l'article L. 165-2 entre les exploitants, à concurrence de la participation de leur activité au dommage ou à la menace imminente de dommage.

they do not provide the characterization of the optimum, nor the comparative analysis between different sharing arrangements.

Finally, it is also worth quoting the paper by Nussim and Tabbach (2009) who challenge the foundations of the standard model of unilateral accident and care, for the reason that it treats care decisions as a non-durable input. They argue instead that very often care expenditures have a durable nature, that is precautionary measures "may be effective or endure for all activity level, and certainly need not to be taken per unit of activity". As a typical example of durable precaution, one may think to investment in specific infrastructures corresponding to a (large) fixed cost, independent from the level of output.

Regarding the debates discussed above (liability sharing rules and internal consistency of law, or as proxy of probabilities), we take a agnostic view, and rather focus on a related point although neglected, which is the impact of liability sharing on market under imperfect competition, and the implications for the society/economy. Important is to remind that we consider here the potential of extension outside of product liability, but cases such as environmental law, competition law etc. The questions we ask, in cases absent of any (knowledge of the) possibility to disentangle multiple offenders' responsibility, are: which liability sharing arrangement (no liability, equal share, market share) can be considered as the best outcome ? Does it allow to reach an efficient outcome?

To do so, we use the basic framework of an oligopoly *à la Cournot*, analyzing the simultaneous choice of care and output by competing firms producing an homogenous good. The expected harm to third parties who are not consumers of the good, is related to the market supply of good and to the aggregate expenditures in precaution. Firms operate in the industry under a rule of strict liability, augmented of a rule of damages apportionment. On the one hand, the harm can be equally shared among the firms (*per capita* rule), which is one of most common rule used by the Courts. On the other hand, the harm can shared among firms in proportion of their market share (market share liability). We believe that two other features of our set up are also specific to the problematic of environmental liability: first, we assume that the level of activity in the industry affects the expected harm to victims in a cumulative way, i.e. the expected harm per unit of output increases with the total industry output at a more-than-proportional rate.[6]; second, we assume that precaution is "durable" in the sense of Nussim and Tabbach (2009).

In this set-up, two important results emerge. The first one is that, regarding the objective of safety (preserving victims well being), no liability sharing regime strictly dominates the other one. The second result is that no sharing arrangement has the power to mimic the optimal levels of output and care expenditures. In details, assuming constant marginal costs of production we show that the equilibrium for a symmetric oligopoly output level of the industry is larger under a per capita rule than under a market share apportionment – which makes victims worse off *ex post*, since the harm in case of accident is higher. In contrast, and related to this first effect on the output, the equilibrium level of care expenditures is

[6]See also Daughety and Reinganum (2014) for a discussion in the context of product liability in the domain of medecine, food safety, but also pollution.

lower under the market share liability than under the equal sharing arrangement – which deteriorates the situation *ex ante* of the victims. In some sense, the analysis of the determinants and comparison of optimal decisions to equilibrium levels of output and care display some uncomfortable, and clearly deceptive results: generally speaking, there is no necessity for the optimal output (care expenditures) to be smaller (larger) than what emerges market discipline and the incentives create by liability in the context of imperfect competition. In turn, for an asymmetric oligopoly, we find that equilibrium market shares between low cost firms and high cost firms are more dispersed under the *per capita* rule than under the market share rule. We also find that high costs firms produce more and invest more in care under the *per capita* rule than under the market share rule; in contrast, the comparison is undertermined for low costs firms, and thus for the industry. Finally, we perform some robustness checks for our analysis, relaxing the constant marginal cost assumption. We show that only minor differences occur under the alternative assumption of increasing marginal costs of production, indeed the our major conclusions still hold.

Section 2 introduces the model. Section 3 analyzes the equilibrium with Cournot competition under two different liability sharing arrangements, the *per capita* vs market sharing rule. Section 4 compares the two equilibrium outcomes with liability sharing to the no liability regime, the social optimum, and to the private monopoly solution. Section 5 proposes some robustness checks, based on an alternative costs structure for firms. Section 6 concludes.

2 The model

The situation we are focusing on is one where the good produced provides some benefits to society (to consumers of the good), but accidental events may occur during the production process and victims in case of accident have no contractual nor market relationship with firms; in particular, victims and consumers are not the same persons. Moreover, we are considering a case of multiple liability but in a sense uncertain, since it is not possible to disentangle the influence of each individual firm on the aggregate harm to victims.[7]

To this end, we introduce a very simple model of imperfect competition, where firms may harm some victims/the environment and thus invest in precautionary measures to reduce the cost of liability. We consider the market for an homogenous product, where $N > 2$ firms compete *à la Cournot*. Both consumers and firms are risk neutral. The quantity of goods produced by firm i is denoted $q_i (i = 1, 2, .., N)$, and $Q = \sum_{i=1}^{N} q_i$ represents the aggregate output of industry. The market demand is given by $P(Q) = a - bQ$, $(a > 0, b > 0)$, consumers being not harmed by the product.

We assume that the expected harm $H(X, Q)$ has the form $H(X, Q) = Q^2 h(X)$, with $X = \sum_{i=1}^{N} x_i$

[7] Firms are located at the same place and experience an accident at the same moment; or the damage is diffuse, being the outcome of several minor failures in the production process which are not observable by outside parties, but having large and cumulative effects above some threshold on victims or the environment in the long run etc.

and x_i representing the level of care of firm $i = 1, 2, .., N$; moreover, we assume that for any $X > 0$, $h'(X) < 0$, and $h''(X) > 0$. This specific assumption captures the fact that the expected harm is related to the industry output ($\frac{\partial H}{\partial Q} = 2Qh(X) > 0$), without any possibility to disentangle the influence of each firm ($\frac{\partial H}{\partial q_i} = \frac{\partial H}{\partial Q}$); moreover the effect is cumulative ($\frac{\partial^2 H}{\partial Q^2} = 2h(X) > 0$) meaning that the higher the level of aggregate activity, the higher the marginal (expected) harm. Similarly, firms' individual precautionary measures are supposed to have the same (negative) impact on the expected damage ($\frac{\partial H}{\partial x_i} = \frac{\partial H}{\partial X} = Q^2 h'(X) < 0$), but returns to scale in the care activity are decreasing ($\frac{\partial^2 H}{\partial X^2} = Q^2 h''(X) > 0$). Finally, remark that this specific functional form may be understood as a case where the probability of accident is captured by $h(X)$, assuming $h(X) < 1$, and the damage in case an accident occurs is scaled by the (square of) output Q^2. Although this is not the unique interpretation, we will adhere to it throughout the paper.

Let us turn to the productive costs of firms. Since we want to capture the situation where care is considered as a specific input in the process of production, with a durable nature, we will assume that the total cost of production of firm i is $C_i(q_i, x_i) = k_i(q_i) + c_i(x_i)$, where $c_i(x_i)$ is the cost of care, and is $k_i(q_i)$ the production cost associated with the other inputs. The durable nature of care being captured by the fact that the cost of care as a specific input of production does not depend on firm's activity level (since it is independent from the cost of the other productive inputs used by a firm to produce the good). For the sake of simplicity, we will assume that $k_i(q_i) = k_i q_i$, $k_i > 0$, and $c_i(x_i)$ satisfies $c_i' > 0$ and $c_i'' > 0$. Moreover, regarding the characteristics of the technology available to firms, we will consider that the N firms are split in two groups, such that firms in a group are homogenous, but firms pertaining to group 1 have total and marginal costs of production and care which are lower compared with firms in group 2; formally, there exist $n_1 > 1$ firms characterized by k_1, c_1, and $n_2 > 1$ firms($n_1 + n_2 = N$) characterized by $k_2 > k_1, c_2^{(i)}(x) > c_1^{(i)}(x)$ for $i = 0, 1$ $\forall x > 0$. In order that equilibrium exists, we obviously assume that $a > k_2$.

3 Oligopoly equilibrium with liability sharing

We remind that we consider here a situation where the individual influence of each firm on the aggregate expected harm $H(X, Q)$ cannot be disentangled. For that reason, we consider that Courts set for the liability of firms *in solidum*, and decide that damages to victims are shared between all firms. Let us denote as $L_i(X, Q)$ the amount of compensation accruing to firm $i = 1, 2, ..., N$; we will consider alternative arrangements in the next paragraphs, according to which, firm's i liability is calculated such that $L_i(X, Q) = s_i \times H(X, Q)$ where s_i is firm's i liability share ($i = 1, 2, ..., N$).

3.1 Care and output under *per capita* apportionment

Let us assume first that strict liability is augmented with a damage rule consisting in an equal share of the damage between the firms: they have to compensate victims with an equal share in the expected harm ($s_i = \frac{1}{N}$, $\forall i = 1, ..., N$), such that the liability accruing to firm i is $L_i^{pc}(X,Q) = \frac{1}{N}H(X,Q) = \left(\frac{1}{N}\right)Q^2h(X)$.

In this case, a firm in group i chooses q_i, x_i a level of output and a level of care in order to maximize its profit:

$$\begin{aligned} \pi_i(q_i, x_i) &= P(Q)q_i - C_i(q_i, x_i) - L_i^{pc}(X,Q) \\ &= (a - b(q_i + Q_{-i}))q_i - k_iq_i - c_i(x_i) - \left(\frac{1}{N}\right)(q_i + Q_{-i})^2 h(x_i + X_{-i}) \end{aligned} \tag{1}$$

where we denote $Q_{-i} = \sum_{s \neq i} q_s$, $Q = q_i + Q_{-i}$, $X_{-i} = \sum_{s \neq i} x_s$, and $X = x_i + X_{-i}$. The first-order conditions for firm i, $\forall i = 1, 2$, require that q_i, x_i satisfy respectively:[8]

$$a - 2bq_i - bQ_{-i} = k_i + \frac{2}{N}(q_i + Q_{-i})h(X) \tag{2}$$

$$-h'(X).\frac{1}{N}Q^2 = c_i'(x_i) \tag{3}$$

Condition (2) means that the individual supply is set at a level where the marginal market proceeds are equal to the full marginal cost, including the marginal cost associated with productive input expenditures, plus the marginal cost associated with liability (share in the expected harm). We observe that firms do not fully bear the total harm to victims (RHS in (2), the marginal increase in the excepted harm). Due to the per capita apportionment, they support only $1/N$ of it. From condition (3), we deduce that the the level of care is such that the marginal cost of care expenditures equals the marginal benefit associated with the decrease in liability (expected harm). The same remark applies: each firm invests in care activity according to their private marginal benefit (LHS in (3)) reflecting the decrease in expected liability, which is only $1/N$ of (victims) full marginal benefits. We return to this in more details in the next section, however these simple observations suggest that strict liability with equal damage sharing rule introduces distortions both on the output and care levels.

Turning now to the equilibrium analysis, one remarks that conditions (2)-(3) are identical for any firm pertaining to a given group, since the expression of the marginal market proceeds is identical for any firm in group i, i.e. $a - 2bq_i - bQ_{-i} = a - b(1 + n_i)q_i - bn_jq_j$ (j representing the other group). As a result, all firms in group i will produce an aggregate supply Q_i ($= n_iq_i$) and will invest in the same level

[8] We assume that at equilibrium, the individual output level in each group does not bind $q^{\max}$ the capacity constraint of the firm, i.e. $q_i < q_i^{\max}$, $\forall i$.

of care x_i, their equilibrium values being described according to the conditions:

$$a - k_i = \left(b + \frac{2}{N}h(X) + \frac{b}{n_i}\right) Q_i + \left(b + \frac{2}{N}h(X)\right) Q_j, \forall i, j = 1, 2 \tag{4}$$

$$-h'(X).\frac{1}{N}Q^2 = c_i'(x_i), \forall i = 1, 2 \tag{5}$$

Solving (4), we verify that at equilibrium the aggregate and individual supply in group i, Q_i^{pc}, q_i^{pc} $\forall i = 1, 2$, and the aggregate output of the industry Q^{pc} are given by:

$$Q_1^{pc} = \frac{n_1}{1+N}\left(\frac{(a-k_1) + n_2(k_2-k_1)\left(1 + \frac{2}{bN}h(X^{pc})\right)}{b + \frac{2}{1+N}h(X^{pc})}\right) = n_1.q_1^{pc}$$

$$Q_2^{pc} = \frac{n_2}{1+N}\left(\frac{(a-k_2) - n_1(k_2-k_1)\left(1 + \frac{2}{bN}h(X^{pc})\right)}{b + \frac{2}{1+N}h(X^{pc})}\right) = n_2.q_2^{pc}$$

$$Q^{pc} = \frac{N}{1+N}\left(\frac{a - \frac{n_1}{N}k_1 - \frac{n_2}{N}k_2}{b + \frac{2}{1+N}h(X^{pc})}\right) = Q_1^{pc} + Q_2^{pc}$$

where X^{pc} is the aggregate expenditures in care. We can also assess the equilibrium individual market share of firms in each group:

$$\frac{q_1^{pc}}{Q^{pc}} = \frac{Q_1^{pc}}{n_1.Q^{pc}} = \frac{1}{N}\frac{(a-k_1) + n_2(k_2-k_1)\left(1 + \frac{2}{bN}h(X^{pc})\right)}{a - \frac{n_1}{N}k_1 - \frac{n_2}{N}k_2}$$

$$\frac{q_2^{pc}}{Q^{pc}} = \frac{Q_2^{pc}}{n_2.Q^{pc}} = \frac{1}{N}\frac{(a-k_2) - n_1(k_2-k_1)\left(1 + \frac{2}{bN}h(X^{pc})\right)}{a - \frac{n_1}{N}k_1 - \frac{n_2}{N}k_2}$$

Now, using (5), at equilibrium the individual level of care x_i^{pc} for $i = 1, 2$ is implicitly defined as a best response to the aggregate care and industry output level by:

$$-h'(X^{pc}).\frac{1}{N}\left(Q^{pc}\right)^2 = c_i'(x_i^{pc}), \forall i = 1, 2$$

One can observe that:

Proposition 1. *i)* $\frac{q_1^{pc}}{Q^{pc}} > \frac{q_2^{pc}}{Q^{pc}}$. *ii) In group 1 (respectively in group 2), the individual market share increases (decreases) with the joint probability of accident,* $h(X^{pc})$. *iii)* $x_1^{pc} > x_2^{pc}$.

Proof: i) Consider first that $k_2 > k_1 \Rightarrow \frac{(a-k_1)+\frac{n_2}{b}(k_2-k_1)\left(b+\frac{2}{N}h(X^{pc})\right)}{a-\frac{n_1}{N}k_1-\frac{n_2}{N}k_2} > 1$, and thus we obtain: $\frac{q_1^{pc}}{Q^{pc}} > \frac{1}{N}$. By the same token, $k_2 > k_1 \Rightarrow \frac{(a-k_2)-\frac{n_1}{b}(k_2-k_1)\left(b+\frac{2}{N}h(X^{pc})\right)}{a-\frac{n_1}{N}k_1-\frac{n_2}{N}k_2} < 1$, and thus we obtain: $\frac{q_2^{pc}}{Q^{pc}} < \frac{1}{N}$. ii) Straightforward. iii) Straightforward, given that the marginal benefit of care is the same in both groups (and decreases with x), but the marginal costs of care (that increases with x) satisfy $c_2'(x) > c_1'(x)$ for any $x > 0$. ■

Remark that all the basic results in comparative statics for an oligopoly *à la* Cournot still hold (i.e. influence of k_2, k_1, N). Moreover, it is also easy to verify that, compared to a no liability rule, the liability cost for firms has a negative impact on quantities Q_i^{pc}, q_i^{pc} $\forall i = 1, 2$, and Q^{pc}. However, result ii) in proposition 1 is more specifically worth to notice: it means that, still comparing to the no liability rule, firms pertaining to group 1 (respectively, group 2) obtain a higher (smaller) equilibrium market share, under strict liability (with equal sharing).

3.2 Care and output under market share apportionment

Let us assume now that strict liability is augmented with a damage rule based on the individual market share of each firm: firms have to compensate only a share of the expected harm determined in proportion to their market share ($s_i = \frac{q_i}{Q}$, $i = 1, .., N$), such that the liability accruing to firm i is $L_i^{ms}(X, Q) = \frac{q_i}{Q} H(X, Q) = q_i Q h(X)$.

A firm in group $i = 1, 2$ chooses now q_i, x_i a level of output and care which maximize the profit :

$$\begin{aligned} \pi_i(q_i, x_i) &= P(Q)q_i - C_i(q_i, x_i) - L_i^{ms}(X, Q) \\ &= (a - b(q_i + Q_{-i}))q_i - k_i q_i - c_i(x_i) - q_i(q_i + Q_{-i})h(x_i + X_{-i}) \end{aligned} \tag{6}$$

where we use the same notations as before. The first-order conditions for firm i require that q_i, x_i satisfy $\forall i = 1, 2$:[9]

$$a - 2bq_i - bQ_{-i} = k_i + (2q_i + Q_{-i})h(X) \tag{7}$$

$$-h'(X).q_i Q = c_i'(x_i) \tag{8}$$

Once more, (7)-(8) have the same interpretation compared to (2)-(3). However, two characteristic features are noticeable here. Comparing conditions (7) and (2), it is obvious that once more firms do not fully bear the social cost of their market activity (RHS in (2)), and in thus the marginal cost due to their individual liability is smaller (RHS in (7)) than at optimum. Finally, comparing equation (8) and (3), we conclude that in setting of their care expenditures, firms only consider their private marginal benefit (LHS in (8)) reflecting the decrease in their expected liability, which is smaller than at optimum (LHS in (3)). Hence, the same conclusion applies here: strict liability with a damage sharing rule based on market share introduces distortions both on the output and care levels.

Turning again to the equilibrium analysis, one also remarks that conditions (7)-(8) are identical for

[9] Once more, we assume that at equilibrium, the individual output level in each group does not bind $q^{\max}$ the capacity constraint of the firm, i.e. $q_i < q_i^{\max}$, $\forall i$.

any firm pertaining to a given group, since the expression of the marginal market proceeds is identical for any firm in group i, i.e. $a - 2bq_i - bQ_{-i} = a - b(1+n_i)q_i - bn_jq_j$ (j representing the other group). As a result, all firms in group i will produce an aggregate supply Q_i $(= n_iq_i)$ and will invest in the same level of care x_i, their equilibrium values being described according to the conditions:

$$a - k_i = \left(\frac{1+n_i}{n_i}\right)(b + h(X))\,Q_i + (b + h(X))\,Q_j,\ \forall i, j = 1, 2 \tag{9}$$

$$-h'(X).\frac{Q_i}{n_i}Q = c_i'(x_i),\ \forall i = 1, 2 \tag{10}$$

Solving (8), we verify that at equilibrium the aggregate and individual supply in group i, Q_i^{ms}, q_i^{ms} $\forall i = 1, 2$, and the aggregate supply of the industry Q^{ms}, are given by:

$$\begin{aligned} Q_1^{ms} &= \frac{n_1}{1+N}\left(\frac{(a-k_1) + n_2\,(k_2 - k_1)}{b + h(X^{ms})}\right) = n_1q_1^{ms} \\ Q_2^{ms} &= \frac{n_2}{1+N}\left(\frac{(a-k_2) + n_1\,(k_1 - k_2)}{b + h(X^{ms})}\right) = n_2q_2^{ms} \\ Q^{ms} &= \frac{N}{1+N}\left(\frac{a - \frac{n_1}{N}k_1 - \frac{n_2}{N}k_2}{b + h(X^{ms})}\right) = Q_1^{ms} + Q_2^{ms} \end{aligned}$$

where X^{ms} is the aggregate expenditures in care. We can also assess the equilibrium individual market share of firms in each group:

$$\begin{aligned} \frac{q_1^{ms}}{Q^{ms}} &= \frac{Q_1^{ms}}{n_1.Q^{ms}} = \frac{1}{N}\frac{(a-k_1) + n_2\,(k_2 - k_1)}{a - \frac{n_1}{N}k_1 - \frac{n_2}{N}k_2} \\ \frac{q_2^{ms}}{Q^{ms}} &= \frac{Q_2^{ms}}{n_2.Q^{ms}} = \frac{1}{N}\frac{(a-k_2) - n_1\,(k_2 - k_1)}{a - \frac{n_1}{N}k_1 - \frac{n_2}{N}k_2} \end{aligned}$$

Now, using (8), the equilibrium level of care x_i^{ms} for $i = 1, 2$ is implicitly defined as a best response to (decreasing in) the aggregate care level by (using that $\frac{Q_i}{n_iQ} = \frac{q_i}{Q}$):

$$-h'(X^{ms}).\frac{q_i^{ms}}{Q^{ms}}\,(Q^{ms})^2 = c_i'(x_i^{ms}), \forall i = 1, 2$$

One can observe that:

Proposition 2. *i)* $\frac{q_1^{ms}}{Q^{ms}} > \frac{q_2^{ms}}{Q^{ms}}$. *ii) The individual market share in both groups is independent from the joint probability of accident,* $h(X^{ms})$*. iii)* $x_1^{ms} > x_2^{ms}$.

Proof: i) First, remark that $k_2 > k_1 \Rightarrow \frac{(a-k_1)+n_2(k_2-k_1)}{a-\frac{n_1}{N}k_1-\frac{n_2}{N}k_2} > 1$, and thus we obtain: $\frac{q_1^{ms}}{Q^{ms}} > \frac{1}{N}$; ii) and by the same token, $k_2 > k_1 \Rightarrow \frac{(a-k_2)-n_1(k_2-k_1)}{a-\frac{n_1}{N}k_1-\frac{n_2}{N}k_2} < 1$, and thus we obtain: $\frac{q_2^{ms}}{Q^{ms}} < \frac{1}{N}$. ii) Straightforward. iii) Straightforward, given that $\frac{q_1^{ms}}{Q^{ms}} > \frac{q_2^{ms}}{Q^{ms}}$ (the marginal benefit of care $-h'(X^{ms}).\frac{q_i^{ms}}{Q^{ms}}\,(Q^{ms})^2$ is higher for group 1 than for group 2) and that $c_2'(x) > c_1'(x)$ for any $x > 0$ (the marginal cost of care is higher for group 2 than for group 1). ■

Remark once more that all the basic results in comparative statics for an oligopoly *à la* Cournot still hold (i.e. influence of k_2, k_1, N). Moreover, it is also straightforward that, compared to a no liability rule, the liability cost for firms has a negative impact on quantities Q_i^{ms}, q_i^{ms} $\forall i = 1, 2$, and Q^{ms}. However, result ii) in proposition 2 is also worth to notice, since it means that the equilibrium market shares are not affected by strict liability augmented with the market share rule, i.e. the equilibrium market shares under strict liability and no liability are equal in each group.

4 Comparison of apportionment rules

To begin with, we will start with the analysis of a symmetric oligopoly, assuming that there exists only firms of type 1 (the lowest cost, both in terms of production and in terms of care activity). Then we will return to the asymmetric case.

4.1 Preliminary: symmetric oligopoly

Assuming that all firms pertain to the same group (let say, group 1), under a *per capita* rule, the first order conditions are written at equilibrium (using $q_i = q$ and $x_i = x$ $\forall i = 1, ..., N$) as:

$$
\begin{aligned}
a - b(1+N)q &= k_1 + 2qh(X) \quad (11)\\
-h'(X).Nq^2 &= c_1'(x) \quad (12)
\end{aligned}
$$

In turn, under strict liability and the market share rule, the first-order conditions now are written (also using $q_i = q$ and $x_i = x$ $\forall i = 1, ..., N$) as:

$$
\begin{aligned}
a - b(1+N)q &= k_1 + (1+N)\,qh(X) \quad (13)\\
-h'(X).Nq^2 &= c_1'(x) \quad (14)
\end{aligned}
$$

Comparing (11) and (13), we observe that they have the same LHS (marginal market proceeds, which decrease in q); but their RHS (corresponding to the marginal cost of liability) is different and satisfy all else equal (for a given X) $2qh(X) < (1+N)qh(X)$ since $N > 2$. As a result, it comes that for a given value of aggregate care expenditures, X, we obtain that $q^{pc}(X) > q^{ms}(X)$ (and thus $Q^{pc}(X) > Q^{ms}(X)$). However, we have to take into account the feedback effect of care activity levels (indeed, X is not the same but is specific to each liability regimes, as we now explain). For that purpose, let us turn now to conditions (12) and (14); we observe that they have a similar expression. However, given that $q^{pc} > q^{ms}$, it comes that $x^{pc} > x^{ms}$ (and thus $X^{pc} > X^{ms}$). To complete the argument, let us return to the determination of the output levels, and assess the feedback effect of care activity on equilibrium outputs. Note that since $X^{pc} > X^{ms}$, we obtain $h(X^{pc}) < h(X^{ms})$: this means the cost of liability passed to each firm with the adjustment of precautionary measures is smaller under the first regime (equal sharing) than under the second (market share), implying a smaller operating cost for firms, and thus reinforcing the differential effect on the level of market output initially analyzed.

Finally, it is straightforward that in the symmetric case, the equilibrium market share of firms only depends on the total number of firms, whatever the rule of liability apportionment. Solving (11), it is easy to verify that the equilibrium aggregate and individual output levels under the *per capita* rule are given by $Q^{pc} = \frac{N}{1+N} \frac{a-k_1}{b+\frac{2}{1+N}h(X^{pc})} = Nq^{pc}$ where X^{pc} is the aggregate expenditures in care; this yields $\frac{q^{pc}}{Q^{pc}} = \frac{1}{N}$. Now, solving (13), the aggregate and individual levels of output under the market share rule are given by: $Q^{ms} = \frac{N}{1+N} \frac{a-k_1}{b+h(X^{ms})} = Nq^{ms}$, with X^{ms} the aggregate level of care; this yields $\frac{q^{ms}}{Q^{ms}} = \frac{1}{N}$.

We summarize the results in the next proposition:

Proposition 3. *In a symmetric oligopoly, when liability is allotted between firms according to their market share, both the aggregate market supply and the aggregate expenditures in care are smaller at equilibrium than when liability is equally shared among firms ($Q^{ms} < Q^{pc}$; $X^{ms} < X^{pc}$; $h(X^{ms}) > h(X^{pc})$). In contrast, the equilibrium expected damage may be larger as well as smaller ($(Q^{ms})^2 .h(X^{ms}) \gtrless (Q^{pc})^2 .h(X^{pc})$). Nevertheless, firms have the same market share at equilibrium under both rules: $\frac{1}{N}$.*

It is important to remark that under both liability sharing rules, firms obtain exactly the same market share at equilibrium. Thus, the impact of liability sharing is mainly driven by the market adjustment: liability sharing implies a contraction in the individual level of output (and thus the contraction of the market supply allows an increase in the equilibrium price), this one in turn providing firms with incentives to reduce their expenditures in precaution. As previously discussed, this market adjustment is more important under the liability regime based on market share liability rule, than under the equal share rule. In a sense, this means that the consumers of the good are more affected (in terms of surplus/utility loss) under the market share liability rule, than under the equal sharing arrangement. In contrast, the effect on victims in terms of expected damage is ambiguous, meaning that whether the market share is better or worse than the equal share liability regime in controlling risky activities, is still an open issue.

Indeed, from the *ex post* point of view, victims are better off under the market share liability rule, than under the equal sharing arrangement: in cases where an accident occurs, the effective damage to victims is smaller under the market share liability rule, than under the equal sharing arrangement. However, as a result of weaker incentives to invest in care provided by the market share liability rule, compared to the equal sharing arrangement, the probability of accident is also larger with the first rule. Hence, the *ex ante* point of view does not allow to conclude.

4.2 Asymmetric oligopoly

We now return to the asymmetric case, and compare the per capita rule of apportionment with the market share rule of apportionment. A first result we will use later one can be directly established:

Proposition 4. *Under the per capita rule, equilibrium market shares between firms with high and low costs are more dispersed than under the market share rule:* $\frac{q_1^{pc}}{Q^{pc}} > \frac{q_1^{ms}}{Q^{ms}} > \frac{1}{N} > \frac{q_2^{ms}}{Q^{ms}} > \frac{q_2^{pc}}{Q^{pc}}$.

Proof: This results from our previous observations (and from a direct comparison between the different market shares expressions) that under the market share rule, firms in each group have the same equilibrium market share than under no liability, while, under the equal share arrangement, firms in group 1 (group 2) have a higher (respectively, smaller) equilibrium market share than under no liability. ■

Let us now compare the individual level of outputs obtained under each rule; according to conditions (2) and (7), q_i^{pc}, q_i^{ms} $\forall i = 1, 2$ respectively satisfy:

$$a - 2bq_i - bQ_{-i} = k_i + \frac{2}{N}\left(q_i + Q_{-i}\right)h(X)$$
$$a - 2bq_i - bQ_{-i} = k_i + \left(2q_i + Q_{-i}\right)h(X)$$

Remark that these two conditions have the same LHS (marginal market proceeds, which decrease in q_i); but the RHS (corresponding to the marginal cost of liability, which increases in q_i) in (2) is larger than the RHS in (7) (for a given X) $\frac{2}{N}\left(q_i + Q_{-i}\right)h(X) < \left(2q_i + Q_{-i}\right)h(X)$ given that $N > 2$. As a result, it comes that for an exogenous value of aggregate care expenditures, X, the individual output level reached under the *per capita* rule is larger than under the market share rule: $q_i^{pc}(X) > q_i^{ms}(X)$ $\forall i = 1, 2$ – in turn this also implies that the industry supply is also larger under the *per capita* rule than under the market share rule, for a X exogenously given:[10] $Q^{pc}(X) > Q^{ms}(X)$. The intuition of the result is as follows. As we know from first order conditions, a firm chooses its level of output such that the marginal market proceeds equal the increase of the individual expected liability (victims' expected compensation accruing to her). However, under the *per capita* this latter increases proportionally to the market output, whereas, under the market share apportionment it increases more than proportionally to the market output. Since the marginal benefit is the same in both situations, then firms always produce more under a per capita rule than under a market share rule of apportionment.

But once more to establish a complete comparison regarding the equilibrium levels (e.g. for X endogenously given), we have to take into account the feedback effect of care activity levels (indeed, X

[10] Obviously, this can be verified by direct comparison since using the equilibrium values, we have: $Q^{pc}(X) = \frac{N}{1+N}\left(\frac{a - \frac{n_1}{N}k_1 - \frac{n_2}{N}k_2}{b + \frac{2}{1+N}h(X)}\right) < Q^{ms}(X) = \frac{N}{1+N}\left(\frac{a - \frac{n_1}{N}k_1 - \frac{n_2}{N}k_2}{b + h(X)}\right)$.

is not the same but is specific to each liability regimes, as we now explain). For that purpose, let us turn now to conditions (11) and (16), which can be written respectively as:

$$
\begin{aligned}
-h'(X).\frac{1}{N}Q^2 &= c_i'(x_i), \forall i = 1, 2 \\
-h'(X).\frac{q_i}{Q}Q^2 &= c_i'(x_i), \forall i = 1, 2
\end{aligned}
$$

Thus in contrast to what holds for the determination of market outputs, they have the same RHS (the marginal cost of care, which increases in x_i). However the difference in both LHS (corresponding to the individual marginal benefit of liability, which decreases in x_i) reflects the influence of the market equilibrium, which may run in different directions, depending on the group of a firm:

i) focusing on firms in group 2, we have shown that $\frac{q_2^{ms}}{Q^{ms}} < \frac{1}{N}$ which, combined with $Q^{pc}(X) > Q^{ms}(X)$, implies $\frac{1}{N}\left(Q^{pc}(X)\right)^2 > \frac{q_2^{ms}}{Q^{ms}}\left(Q^{ms}(X)\right)^2$ and thus $x_2^{pc} > x_2^{ms}$; this also implies that the aggregate expenditure in care for group 2 is larger under the equal share rule than under the market share: $X_2^{pc} > X_2^{ms}$;

ii) in contrast, for firms in group 1, we have shown that $\frac{q_2^{ms}}{Q^{ms}} > \frac{1}{N}$ which goes in the opposite direction to the influence of $Q^{pc}(X) > Q^{ms}(X)$, implying that $\frac{1}{N}\left(Q^{pc}(X)\right)^2 \lesseqgtr \frac{q_1^{ms}}{Q^{ms}}\left(Q^{ms}(X)\right)^2$ and thus $x_1^{pc} \lesseqgtr x_1^{ms}$ and $X_1^{pc} \lesseqgtr X_1^{ms}$.

As a result, the feedback effect of care expenditures on equilibrium outputs is in all ambiguous. On the one hand, since $x_2^{pc} > x_2^{ms}$, the aggregate expenditure in care for group 2 is larger under the equal share rule than under the market share: $X_2^{pc} > X_2^{ms}$; on the other hand, group 1 may invest in care less under the equal share rule than under the market share: $X_1^{pc} \lesseqgtr X_1^{ms}$. However, since the cost of liability for both kinds of firms is passed through the joint probability of accident $h(X)$, it is not possible to conclude.

It can be expected that as firms of group 2 are numerous and over represented in the industry, then there will exist a tendency for that at equilibrium, aggregate output and care expenditures are larger under the per capita rule than under the market share rule: $Q^{pc} > Q^{ms}$ and $X^{pc} > X^{ms}$. In contrast, the net effect is still ambiguous when in contrast when firms of group 1 are numerous and over represented in the industry.

4.3 A no liability regime

Before proceeding to the comparison the social optimum, or alternative market structures, it is worth to briefly consider the case with no liability. For this purpose, it is straightforward that considering the equilibrium values found for example under the market share rule, and assuming that $h(X) = 0$, we obtain the equilibrium values under the no liability regime for the aggregate and individual supply in group i, Q_i^{nl}, q_i^{nl} $\forall i = 1, 2$, and the aggregate supply of the industry Q^{nl}, are given by:

$$\begin{aligned}
Q_1^{nl} &= \frac{n_1}{(1+N)\,b}\left((a-k_1)+n_2\,(k_2-k_1)\right) = n_1 q_1^{nl} \\
Q_2^{nl} &= \frac{n_2}{(1+N)\,b}\left((a-k_2)+n_1\,(k_1-k_2)\right) = n_2 q_2^{nl} \\
Q^{nl} &= \frac{N}{(1+N)\,b}\left(a-\frac{n_1}{N}k_1-\frac{n_2}{N}k_2\right)
\end{aligned}$$

where X^{ms} is the aggregate expenditures in care. We can also assess the equilibrium individual market share of firms in each group:

$$\begin{aligned}
\frac{q_1^{nl}}{Q^{nl}} &= \frac{1}{N}\frac{(a-k_1)+n_2\,(k_2-k_1)}{a-\frac{n_1}{N}k_1-\frac{n_2}{N}k_2} \\
\frac{q_2^{nl}}{Q^{nl}} &= \frac{1}{N}\frac{(a-k_2)-n_1\,(k_2-k_1)}{a-\frac{n_1}{N}k_1-\frac{n_2}{N}k_2}
\end{aligned}$$

whereas, the equilibrium level of care is obviously $x_i^{nl} = 0$ for $i = 1, 2$.

The next proposition collects two main implications of the no liability regime:

Proposition 5. *Under the no liability regime: i) equilibrium market shares in both groups are equal to those obtained under the market share rule, meaning that they are less dispersed than under the per capita rule:* $\frac{q_1^{pc}}{Q^{pc}} > \frac{q_1^{ms}}{Q^{ms}} = \frac{q_1^{nl}}{Q^{nl}} > \frac{1}{N} > \frac{q_2^{nl}}{Q^{nl}} = \frac{q_2^{ms}}{Q^{ms}} > \frac{q_2^{pc}}{Q^{pc}}$*; ii) in both groups, firms output level is larger than under any of both regimes of liability sharing:* $Q_i^{nl} > \max\left(Q_i^{pc}, Q_i^{ms}\right)$, $\forall i = 1, 2$.

Proof. i) is obtained by direct comparison. ii) The comparison is also straightforward and left to the reader. ■

Since Courts have the opportunity to release firms of any liability in case where they cannot disentangle their individual responsibility in causing the damage to victims, the proposition means that their customers are better off under the no liability regime, whereas the victims are clearly worse off (ex ante as well as ex post), since both the expected damages and the post accident damages are larger than under any liability regime.

5 Imperfect competition and social welfare considerations

5.1 Social welfare maximization

We determine the socially optimal level of care and output, associated with the maximization of social welfare, which is defined as the sum of consumers' total utility $\int_0^Q P(z)dz$, Q being the total quantity they consume, minus the total production costs of the output (including the cost of care) augmented of the expected harm. A benevolent planner can directly use the fact that decisions regarding care activity and output are separated from a technological point of view, since care is durable (i.e. does not interfere with the use of other productive inputs). Hence, given that firms produce a homogenous good (firms outputs are perfect substitutes for consumers), and that all firms pertaining two a group are identical in terms of marginal cost of production and care, the social planner will simply choose a quota of output for each group, respectively Q_1, Q_2, and allocate an equal share of this quota to each firm in a group: $q_1 = \frac{Q_1}{n_1}$ and $q_2 = \frac{Q_2}{n_2}$. The same argument explains that the social planner will set an aggregate level of care expenditures in each group with the same level of care expenditures for each firm in a group, respectively x_1, x_2. As a result, the social welfare function can be finally written as a function of (Q_1, Q_2, x_1, x_2):

$$\begin{aligned} SW(Q_1, Q_2, x_1, x_2) &= \int_0^Q P(z)dz - n_1 C_1\left(\frac{Q_1}{n_1}, x_1\right) - n_2 C_2\left(\frac{Q_2}{n_2}, x_2\right) - H(Q, X) \qquad (15) \\ &= a\left(Q_1 + Q_2\right) - \frac{b}{2}\left(Q_1 + Q_2\right)^2 - k_1 Q_1 - k_2 Q_2 \\ &\quad - n_1 c_1(x_1) - n_2 c_2(x_2) - \left(Q_1 + Q_2\right)^2 h(n_1 x_1 + n_2 x_2) \end{aligned}$$

with $Q = Q_1 + Q_2$. Let us consider the derivatives of (15) with respect to Q_i, x_i, $\forall i = 1, 2$:

$$\begin{aligned} \frac{\partial SW}{\partial Q_i} &= a - bQ - k_i - 2Qh(X) \\ \frac{\partial SW}{\partial x_i} &= -h'(X).Q^2 - c_i'(x_i) \end{aligned}$$

Inspection of the first line above suggests that we cannot have simultaneously $\frac{\partial SW}{\partial Q_1} = 0$ and $\frac{\partial SW}{\partial Q_2} = 0$ given that $k_1 < k_2$, when $x_i > 0$ is set according to $\frac{\partial SW}{\partial x_i} = 0$ for $i = 1, 2$. Hence, the optimal solution will depend on the total capacity of production in group 1 which has the lowest cost structures. Let us denote this capacity as $\bar{Q}_1$ (corresponding to the individual capacity $\bar{q}_1 = \frac{\bar{Q}_1}{n_1}$), it comes that:

a) either the output level and care level denoted as (Q^w, x^w), which are the solution to:

$$a - bQ^w = k_1 + 2Q^w h(X^w) \qquad (16)$$

$$-h'(X^w).\left(Q^w\right)^2 = c_1'\left(x^w\right) \qquad (17)$$

with $Q^w = n_1 q^w$ and $X^w = n_1 x^w$, are such that $Q^w < \bar{Q}_1$ – and in this case, it is socially optimal to set $q_2 = 0 = x_2$, and let only firms in group 1 to produce and invest in care. As a result, (Q^w, x^w) described by (16)-(17) gives the optimal solution. Condition (16) means that, the social optimum is such that the output level in group 1 must be pushed to the point where the marginal market proceeds (equal for each group) are equal to total marginal costs (including the marginal cost associated with productive inputs, and the one associated with liability). From condition (17), we deduce that the social optimum requires, for firms in group 1, that the marginal cost of care expenditures equals the marginal benefit associated with the decrease in liability (expected harm).

Using equations (16), the optimal output levels (both aggregate and individual) and the equilibrium market share of firms when the constraint on the capacity of production in group 1 does not bind may be written as:

$$\begin{aligned} Q^w &= \frac{a-k_1}{b+2h(X^w)} = n_1 q^w \\ \frac{q^w}{Q^w} &= \frac{1}{n_1} \end{aligned}$$

where $X^w = n_1 x^w$ is set according to (17).

Up to now, we assumed an exogenous number of firms (in group 1, n_1). However, it is worth to remark that (16)-(17) do not depend on the size of group 1. This means that from the social point of view, it is irrelevant which firm effectively produce and invest in care. As a consequence, if fixed costs are associated with producing the good and/or investing in care, then the optimal number of firms in group 1 that might be active is equal to $n_1^w = 1$ (e.g. in order to avoid the duplication of fixed costs, and thus to allow maximizing social welfare).

b) or $Q^w > n_1\bar{q}_1$ and thus it is socially optimal to let both kinds of firms producing and investing in care. In this case, the total output of the industry Q^w is allocated to the two groups as follows: :

$$\begin{aligned} Q_1^w &= \bar{Q}_1 \\ Q_2^w &= Q^w - \bar{Q}_1 \\ Q^w &= \frac{a-k_1}{b+2h(X^w)} \end{aligned}$$

where X^w denotes the total value of care expenditures. On the one hand, the total supply in group 1 corresponds to its capacity, each active firm in group 1 producing an output equal to $\bar{q}_1 = \frac{\bar{Q}_1}{n_1}$; on the other hand, each active firm in group 2 produces an output equal to $q_2^w = \frac{\hat{Q}^w - \bar{Q}_1}{n_2}$ (assuming $Q_2^w < n_2\bar{q}_2$). The individual market share of a firm in group 1 and group 2 is respectively:

$$\begin{aligned} \frac{q_1^w}{Q^w} &= \frac{\bar{Q}_1}{n_1 Q^w} = \frac{\bar{q}_1}{Q^w} \\ \frac{q_2^w}{Q^w} &= \frac{1}{n_2}\left(1 - \frac{\bar{Q}_1}{Q^w}\right) \end{aligned}$$

In turn, given that $c_1'(x) < c_2'(x)\ \forall x > 0$, it is socially worth to let only firms in group 1 investing in care (i.e. the optimal care level in group 2 is $x_2^w = 0$), such that the optimal amount of care expenditure in group 1, $x_1^w > 0$, satisfies:

$$-h'(X^w).\left(Q^w\right)^2 = c_1'\left(x_1^w\right)$$

with $X^w = n_1.x_1^w$. If fixed costs are associated with producing the good and/or investing in care, then n_2^w the optimal number of firms in group 2 that might be active solves $n_2^w \bar{q}_2 = Q^w - \bar{Q}_1$ (e.g. in order to avoid the duplication of fixed costs, and thus to allow maximizing social welfare).

5.2 Inefficient liability sharing rules in the symmetric oligopoly

To begin with, let us consider the symmetric case. For the purpose of illustrating the driving forces running the comparison between the optimal solution and the equilibrium, let us write conditions (15) and (16) as (assuming that the exogenous number of firms pertaining to group 1 is N, with $Q = Nq$):

$$\begin{aligned} a - bNq &= k_1 + 2Nqh(X) \\ -h'(X).\left(Nq\right)^2 &= c_1'(x) \end{aligned}$$

The first line shows that compared with what occurs under Cournot competition (whatever the liability sharing rule), the optimal level of output is the result of two opposite effects: on the one hand, a higher marginal benefit (LHS in the first equation) in terms of market proceeds; on the other hand, a higher marginal cost of liability (respectively RHS of the first equation, for a given X). The net effect is thus ambiguous, all else equal, and the optimal individual output (and thus the optimal market supply) may be smaller, as well as larger than under any regime of liability sharing.

The second line shows now that, when both the accident externality (full expected damage) and the market externality are jointly internalized, the marginal benefit (LHS) associated with care expenditures is, all else equal (specifically for a given output level Q or q), larger than under any regime of liability sharing - the marginal cost of precautionary measures being the same; thus, all else equal (for a given q) this suggests the tendency for optimal care expenditures to be larger than the equilibrium one[11]. However, the final size of the marginal benefit of care depends on the output level, meaning that given the ambiguity in the comparison between q^w, q^{pc} or q^{ms}, then the comparison between optimal care expenditures and equilibrium ones is also ambiguous.

The results are summarized in the next proposition (for an exogenous number of firms in group 1):

Proposition 6. *Whatever the damage sharing arrangement adopted (equal share vs market share), strict*

[11] This tendancy is exhacerbed (mitigated) when the optimal output is larger (smaller) than at the Cournot equilibrium.

liability augmented with a damage sharing rule leads to an inefficient outcome, both in terms of output and care expenditures, in a symmetric oligopoly with durable care. However, both the direction and size of the distortions is undetermined: the equilibrium levels of output and care expenditures may be too high, as well as too low compared the optimal ones.

Simple calculations may help in understanding the different forces driving the total effect. Since the market adjustment process is all in this framework, we can calculate for example:

$$\begin{aligned}
Q^w - Q^{pc} &= \frac{a-k_1}{b+2h(X^w)} - \frac{N}{1+N}\frac{a-k_1}{b+\frac{2}{1+N}h(X^{pc})} \\
&= \left(\frac{a-k_1}{1+N}\right)\frac{b-2N\left(h(X^w)-\frac{1}{N}h(X^{pc})\right)}{(b+2h(X^w))\left(b+\frac{2}{1+N}h(X^{pc})\right)} \\
Q^w - Q^{ms} &= \frac{a-k_1}{b+2h(X^w)} - \frac{N}{1+N}\frac{a-k_1}{b+h(X^{ms})} \\
&= \left(\frac{a-k_1}{1+N}\right)\frac{b-2N\left(h(X^w)-\frac{1+N}{2N}h(X^{ms})\right)}{(b+2h(X^w))\,(b+h(X^{ms}))}
\end{aligned}$$

This suggests that, in a case where the market demand is weakly (highly) sensitive to price – in the sense that b is large (respectively, small) – there is a pressure for the optimal output to be set at a higher (lower) level compared to the Cournot level, whatever the sharing arrangement prevailing. However, the feedback influence of the difference in care expenditures and probability of accident, may work in different directions. Remark specifically that it is not necessary that $h(X^w) > h(X^{ms})$ to have an opposite tendency for the optimal output to be smaller than at equilibrium: if $h(X^w)$ is not too small compared to $h(X^{ms})$, then $h(X^w) - \frac{1+N}{2N}h(X^{ms}) > 0$ and $h(X^w) - \frac{1}{N}h(X^{pc}) > 0$ may hold.

5.3 Inefficient liability sharing rules in the asymmetric oligopoly

Finally, we consider here the issue of the distortions introduced by strict liability under the different liability sharing arrangements considered here.

It is obvious that two cases deserve consideration, depending on whether $Q^w < \bar{Q}_1$ or $Q^w > \bar{Q}_1$.

a) Let us consider first that the optimal solution is such that $Q^w < n_1\bar{q}_1$; the general conclusions are straightforward:

Proposition 7. *Whatever the damage sharing arrangement adopted (equal share vs market share) under Cournot competition and durable care, strict liability augmented with a damage sharing rule leads to an inefficient outcome regarding the allocation of output and care efforts: firms in group 1 (group 2) produce and invest in care not enough (respectively, too much) compared to what is socially efficient. However, at the industry level, both the direction and size of the distortions is undetermined: the equilibrium levels of output and care expenditures may be too high, as well as too low compared the optimal ones. On the other* *hand, when fixed cost exist, the number of firms in group 1 and as well as group 2 is excessive, compared to the optimal number.*

In this scenario where $Q^w < n_1\bar{q}_1$, the result is immediate since we have shown that it is socially wasteful to let firms in group 2 produce and invest in care; only firms in group 1 should be allowed to produce and undertake care investment. Moreover, as fixed costs exist (associated with production and/or care activity), the number of active firms might be equal to 1.

b) Considering now the alternative scenario where the optimal solution verify $Q^w > n_1\bar{q}_1$, we arrive only at small variations compared to the former results:

Proposition 8. *Whatever the damage sharing arrangement adopted (equal share vs market share) under Cournot competition and durable care, strict liability augmented with a damage sharing rule leads to an inefficient outcome regarding the allocation of output and care efforts: firms in group 1 produce not enough compared to what is socially efficient, whereas their care expenditures may be excessive or insufficient; in turn, firms in group 2 may produce too much or not enough compared to what is socially efficient, while their care expenditures are excessive. However, at the industry level, both the direction and size of the distortions is undetermined: the equilibrium levels of output and care expenditures may be too high, as well as too low compared the optimal ones.*

In this scenario with a binding capacity of production, it is worth reminding that for the purpose of maximizing social welfare, firms in group 1 should be entitled to produce up to their full capacity, which is not biding under Cournot competition; while the role of firms in group 2 is only residual, affording the difference $Q^w - n_1\bar{q}_1$, and might be not allowed to invest in care.

5.4 Private monopoly

Finally, we turn to the issue of market monopolization (Polinsky 1980, Polinsky and Rogerson 1983, Daughety and Reinganum 2014).

Assume that total production and care expenditures are set under the conditions of a private monopoly. The monopoly will also directly use the characteristic features of its technology, and simply choose a quota of output for each group, respectively Q_1, Q_2 allocating an equal share of this quota to each firm in a group, $q_1 = \frac{Q_1}{n_1}$ and $q_2 = \frac{Q_2}{n_2}$; and it will set the same level of care expenditures for each firm in a group, respectively x_1, x_2. As a result, the monopoly profits can be written as:

$$\begin{aligned}\Pi_m(Q_1, Q_2, x_1, x_2) &= P(Q)Q - n_1 C_1\left(\frac{Q_1}{n_1}, x_1\right) - n_2 C_2\left(\frac{Q_2}{n_2}, x_2\right) - H(Q, X) \qquad (18)\\ &= (a - b(Q_1 + Q_2))(Q_1 + Q_2) - k_1 Q_1 - k_2 Q_2 \\ &\quad - n_1 c_1(x_1) - n_2 c_2(x_2) - (Q_1 + Q_2)^2 h(n_1 x_1 + n_2 x_2)\end{aligned}$$

with $Q = Q_1 + Q_2$.

We will first analyze the equilibrium of the monopoly, and then compare to the optimal solution and the to the oligopoly equilibrium.

5.4.1 Equilibrium analysis

Let us consider the derivatives of (4) with respect to Q_i, x_i, $\forall i = 1, 2$:

$$\begin{aligned} \frac{\partial \Pi_m}{\partial Q_i} &= a - 2bQ - k_i - 2Qh(X) \\ \frac{\partial \Pi_m}{\partial x_i} &= -h'(X).Q^2 - c_i'(x_i) \end{aligned}$$

The first derivative suggests once more that we cannot have simultaneously $\frac{\partial \Pi_m}{\partial Q_1} = 0$ and $\frac{\partial \Pi_m}{\partial Q_2} = 0$ given that $k_1 < k_2$ when $\frac{\partial \Pi_m}{\partial x_i} = 0$ for $i = 1, 2$. Hence, the monopoly equilibrium will also depend on the total capacity of production in group 1 which has the lowest cost structures. Still denoting this capacity as $\bar{Q}_1$ (and the individual capacity $\bar{q}_1 = \frac{\bar{Q}_1}{n_1}$), it comes that:

a) either the output level and care level denoted as (Q^m, x^m), given by the conditions:

$$a - 2bQ^m = k_1 + 2Q^m h(X^m) \tag{19}$$

$$-h'(X^m).\left(Q^m\right)^2 = c_1'\left(x^m\right) \tag{20}$$

with $Q^m = n_1 q^m$ and $X^m = n_1 x^m$, satisfying $Q^m < \bar{Q}_1$ – and then it is profit maximizing for the monopoly to set $Q_2 = 0 = x_2$, and let firms in group 1 only to produce and invest in care. As a result, the equilibrium for the monopoly is described by (Q^m, x^m). Conditions (19)-(20) have the same general interpretation as (16)-(17) to which they closely look like. Condition (19) means that the monopoly will increase the output level in group 1 up to the point where the marginal market proceeds are equal to the total marginal cost of production (including the marginal cost associated with productive inputs, and the one associated with liability). From conditions (20), we deduce that the equilibrium in monopoly requires, for firms in group 1, that the marginal cost of care expenditures equals the marginal benefit associated with the decrease in liability (expected harm).

Solving (19), the equilibrium is associated with a level of output and care expenditures given by:

$$Q^m = \frac{1}{2}\frac{a - k_1}{b + h(X^m)} = n_1 q_1^m$$

where $X^m = n_1 x^m$ solve (20). Finally, the individual market share at equilibrium of a firm is simply equal to $\frac{q_1^m}{Q^m} = \frac{1}{n_1}$.

b) or $Q^m > \bar{Q}_1$, and it is rational for the monopoly that both kinds of firms produce and invest in care. In this case, the total output of the monopoly is set according to the level: $Q^m = \frac{1}{2}\frac{a-k_1}{b+2(\hat{X}^m)}$ where $\hat{X}^m$ denotes the total value of care expenditures. On the one hand, the total supply in group 1 corresponds to its capacity, each active firm in group 1 producing an output equal to $\bar{q}_1 = \frac{\bar{Q}_1}{n_1}$; on the other hand, each active firm in group 2 produces an output equal to $q_2^m = \frac{\hat{Q}^m - \bar{Q}_1}{n_2}$ (assuming $Q_2^m < n_2\bar{q}_2$). The individual market share of a firm in group 1 and group 2 is respectively:

$$\begin{aligned}\frac{q_1^m}{Q^m} &= \frac{\bar{Q}_1}{n_1 Q^m} = \frac{\bar{q}_1}{Q^m}\\ \frac{q_2^m}{Q^m} &= \frac{1}{n_2}\left(1 - \frac{\bar{Q}_1}{Q^m}\right)\end{aligned}$$

In turn, given that $c_1'(x) < c_2'(x)\ \forall x > 0$, it is worth for the monopoly to let only firms in group 1 investing in care (i.e. the equilibrium care level in group 2 is $x_2^m = 0$), such that the equilibrium amount of care expenditure in group 1, $x_1^m > 0$, satisfies:

$$-h'(X^m).\left(Q^m\right)^2 = c_1'\left(x_1^m\right)$$

with $X^m = n_1.x_1^m$. If fixed costs are associated with producing the good and/or investing in care, then n_2^w the number of firms in group 2 that might be active solves $n_2^m\bar{q}_2 = Q^m - \bar{Q}_1$ (e.g. in order to avoid the duplication of fixed costs, and thus to allow maximizing social welfare).

5.4.2 Comparisons

We will ignore here the issue of the capacity constraint, in order to simplify. This amounts to assume that the industry is composed of identical firms.

The direct comparison between the monopoly equilibrium and the optimal solution in this scenario where the capacity constraint in group 1 does not bind shows that the efficient incentives to invest in care are preserved under a private monopoly, since (20) is identical to (17) - meaning that for any given level of output, the monopoly will choose an optimal response in term of care. However, the marginal market proceeds (LHS in (19)) of a monopoly are smaller than at optimum (LHS in (16)), meaning that for a given level of care, the monopoly will supply a lower level of output. In all, the conclusion is that under a private monopoly, the equilibrium is associated with a lower level of output and care expenditures, compared to the optimal levels: $Q^m = \frac{1}{2}\frac{a-k_1}{b+h(X^m)} < Q^w = \frac{a-k_1}{b+h(X^w)}$, $X^m < X^w$.

In turn, comparing the monopoly to the symmetric oligopoly (whatever the apportionment rule) suggest that ambiguity prevails; let us write (16)-(17) as follows:

$$
\begin{aligned}
a - 2bNq &= k_1 + 2Nqh(X) \\
-h'(X^m).\,(Nq)^2 &= c_1'(x)
\end{aligned}
$$

It comes that the monopoly faces both marginal market proceeds and a full marginal cost of production (including liability burden) higher than under a symmetric oligopoly; this implies that, for any given level of care activity, the comparison between the output levels is ambiguous (the monopoly may produce less or more than a symmetric oligopoly). In turn, for any given output level, the monopoly faces a marginal benefit associated with care expenditures larger than a symmetric oligopoly – indeed, the monopoly faces efficient incentives, as remarked before – implying that the monopoly invest more in care expenditures than the oligopoly, for any exogenous output level. In all, the comparison between the equilibrium characteristics of the monopoly and the oligopoly is not conclusive.

6 Robustness

In this section, we assess the impact of a different costs structure on our main findings. Let us assume that in group $i = 1, 2$, the full production cost, including care expenditures, is given by $C_i(q_i, x_i) = \frac{1}{2}k_i q_i^2 + c_i(x_i)$, still assuming that $k_2 > k_1$. We briefy review the consequences for the oligopoly market under *per capita*, market share, and no liability rules, and compare to the optimal solution.

6.1 The *per capita* rule

In this case, a firm in group i chooses q_i, x_i a level of output and a level of care in order to maximize its profit:

$$
\pi_i(q_i, x_i) = (a - b(q_i + Q_{-i}))q_i - \frac{1}{2}k_i q_i^2 - c_i(x_i) - \left(\frac{1}{N}\right)(q_i + Q_{-i})^2 h(x_i + X_{-i})
$$

The first-order conditions for firm i, $\forall i = 1, 2$, require that q_i, x_i satisfy respectively:

$$
\begin{aligned}
a - 2bq_i - bQ_{-i} &= k_i q_i + \frac{2}{N}(q_i + Q_{-i})\,h(X) && (21) \\
-h'(X).\frac{1}{N}Q^2 &= c_i'(x_i) && (22)
\end{aligned}
$$

It is immediate that the difference between conditions (21) and (2) lies their RHS, the marginal cost of production being now proportional to q_i. Note however, that all else equal, this does change the influence of the cost structure on individual decisions (all else equal, $k_2 q + \frac{2}{N}Qh(X) > k_1 q + \frac{2}{N}Qh(X) \Rightarrow$

$q_2(X) < q_1(X)$). In contrast, conditions (22) and (3) are identical. Turning now to the equilibrium analysis, we verify that the aggregate and individual supply in group i, Q_i^{pc}, q_i^{pc} $\forall i = 1,2$, and the aggregate output of the industry Q^{pc} are given by:

$$\begin{aligned} Q_1^{pc} &= \frac{a.n_1}{N}\left(\frac{b+k_2}{\Delta}\right) = n_1.q_1^{pc} \\ Q_2^{pc} &= \frac{a.n_2}{N}\left(\frac{b+k_1}{\Delta}\right) = n_2.q_2^{pc} \\ Q^{pc} &= a\left(\frac{b+\frac{n_2}{N}k_1+\frac{n_1}{N}k_2}{\Delta}\right) \end{aligned}$$

where $\Delta = \frac{1}{N}(b+k_2)(b+k_1) + \left(b+\frac{2}{N}h(X^{pc})\right)\left(b+\frac{n_1}{N}k_2+\frac{n_2}{N}k_1\right)$ and X^{pc} is the aggregate expenditures in care. We can also assess the equilibrium individual market share of firms in each group:

$$\begin{aligned} \frac{q_1^{pc}}{Q^{pc}} &= \frac{1}{N}\frac{b+k_2}{b+\frac{n_2}{N}k_1+\frac{n_1}{N}k_2} \\ \frac{q_2^{pc}}{Q^{pc}} &= \frac{1}{N}\frac{b+k_1}{b+\frac{n_2}{N}k_1+\frac{n_1}{N}k_2} \end{aligned}$$

Remark that now, they do not depend on the probability of accident $h(X^{pc})$. Moreover, we verify that the inequalities $\frac{q_1^{pc}}{Q^{pc}} > \frac{1}{N} > \frac{q_2^{pc}}{Q^{pc}}$ still hold since $k_2 > k_1$. Finally, we also obtain $x_1^{pc} > x_2^{pc}$.

6.2 The market share rule

A firm in group $i = 1,2$ chooses now q_i, x_i a level of output and care which maximize the profit :

$$\pi_i(q_i, x_i) = (a - b(q_i + Q_{-i}))q_i - \frac{1}{2}k_i q_i^2 - c_i(x_i) - q_i(q_i + Q_{-i})h(x_i + X_{-i})$$

The first-order conditions for firm i require that q_i, x_i satisfy $\forall i = 1,2$:

$$a - 2bq_i - bQ_{-i} = k_i q_i + (2q_i + Q_{-i})h(X) \tag{23}$$

$$-h'(X).q_i Q = c_i'(x_i) \tag{24}$$

Comparing conditions (23) and (7), it is obvious that once more the RHS is changed with the same impact between firms in group 1 and in group 2 (all else equal, $k_2 q + (q+Q)h(X) > k_1 q + (q+Q)h(X) \Rightarrow q_2(X) < q_1(X)$). In contrast, (24) and (8) are identical.

Turning again to the equilibrium analysis, we verify that the aggregate and individual supply in group

i, Q_i^{pc}, q_i^{pc} $\forall i = 1,2$, and the aggregate supply of the industry Q^{ms}, are given by:

$$\begin{aligned} Q_1^{ms} &= \frac{a.n_1}{1+N}\left(\frac{b+h(X^{ms})+k_2}{\Omega}\right) = n_1 q_1^{ms} \\ Q_2^{ms} &= \frac{a.n_2}{1+N}\left(\frac{b+h(X^{ms})+k_1}{\Omega}\right) = n_2 q_2^{ms} \\ Q^{ms} &= \frac{a.N}{1+N}\left(\frac{b+h(X^{ms})+\frac{n_1}{N}k_2+\frac{n_2}{N}k_1}{\Omega}\right) \end{aligned}$$

where $\Omega = (b+h(X^{ms}))\left[(b+h(X^{ms}))+\frac{1+n_1}{1+N}k_2+\frac{1+n_2}{1+N}k_1\right]+\frac{k_1.k_2}{1+N}$ and X^{ms} is the aggregate expenditures in care. We can also assess the equilibrium individual market share of firms in each group:

$$\begin{aligned} \frac{q_1^{ms}}{Q^{ms}} &= \frac{1}{N}\frac{b+h(X^{ms})+k_2}{b+h(X^{ms})+\frac{n_1}{N}k_2+\frac{n_2}{N}k_1} \\ \frac{q_2^{ms}}{Q^{ms}} &= \frac{1}{N}\frac{b+h(X^{ms})+k_1}{b+h(X^{ms})+\frac{n_1}{N}k_2+\frac{n_2}{N}k_1} \end{aligned}$$

Remark that now, they do depend on the probability of accident $h(X^{ms})$, such that $\frac{q_1^{ms}}{Q^{ms}}$ $\left(\frac{q_2^{ms}}{Q^{ms}}\right)$ is decreasing (increasing) in $h(X^{ms})$. On the other hand, we verify that the inequalities $\frac{q_1^{ms}}{Q^{ms}} > \frac{1}{N} > \frac{q_2^{ms}}{Q^{ms}}$ still hold since $k_2 > k_1$. Finally, we still obtain that $x_1^{ms} > x_2^{ms}$.

6.3 The no liability regime

Once more, it is straightforward that considering the equilibrium values found for example under the market share rule, and assuming that $h(X) = 0$, we obtain the equilibrium values under the no liability regime for the aggregate and individual supply in group i, Q_i^{nl}, q_i^{nl} $\forall i = 1,2$, and the aggregate supply of the industry Q^{nl}, are given by:

$$\begin{aligned} Q_1^{nl} &= \frac{a.n_1}{N}\left(\frac{b+k_2}{\Delta'}\right) = n_1.q_1^{nl} \\ Q_2^{nl} &= \frac{a.n_2}{N}\left(\frac{b+k_1}{\Delta'}\right) = n_2.q_2^{nl} \\ Q^{nl} &= a\left(\frac{b+\frac{n_2}{N}k_1+\frac{n_1}{N}k_2}{\Delta'}\right) \end{aligned}$$

where $\Delta' = \frac{1}{N}(b+k_2)(b+k_1)+b\left(b+\frac{n_1}{N}k_2+\frac{n_2}{N}k_1\right)$. We can also assess the equilibrium individual market share of firms in each group:

$$\begin{aligned}\frac{q_1^{nl}}{Q^{nl}} &= \frac{1}{N}\frac{b+k_2}{b+\frac{n_2}{N}k_1+\frac{n_1}{N}k_2}\\ \frac{q_2^{nl}}{Q^{nl}} &= \frac{1}{N}\frac{b+k_1}{b+\frac{n_2}{N}k_1+\frac{n_1}{N}k_2}\end{aligned}$$

The next proposition collects the two main implications that result from the comparison of equlibria under the three regimes, *per capita* rule, market share rule, and no liability:

Proposition 9. *i) Equilibrium market shares in each group are equal under the per capita rule and the no liability regime, and more dispersed than under the market share rule:* $\frac{q_1^{pc}}{Q^{pc}} = \frac{q_1^{nl}}{Q^{nl}} > \frac{q_1^{ms}}{Q^{ms}} > \frac{1}{N} > \frac{q_2^{ms}}{Q^{ms}} > \frac{q_2^{nl}}{Q^{nl}} = \frac{q_2^{pc}}{Q^{pc}}$*; ii) for both groups of firms, output levels under no liability are larger than under any of both regimes of liability sharing:* $Q_i^{nl} > \max\left(Q_i^{pc}, Q_i^{ms}\right)$, $\forall i = 1, 2$.

Proof. i) is obtained by direct comparison. ii) The comparison is also straightforward and left to the reader. ■

Indeed, the main change is related to the i) part of the proposition.

6.4 Social welfare maximization

The social welfare function, as a function of (Q_1, Q_2, x_1, x_2), can now be written:

$$\begin{aligned}SW(Q_1,Q_2,x_1,x_2) &= a\left(Q_1+Q_2\right)-\frac{b}{2}\left(Q_1+Q_2\right)^2-\frac{1}{2n_1}k_1Q_1^2-\frac{1}{2n_2}k_2Q_2^2 \\ &\quad -n_1c_1(x_1)-n_2c_2(x_2)-\left(Q_1+Q_2\right)^2h(n_1x_1+n_2x_2)\end{aligned} \quad (25)$$

with $Q = Q_1 + Q_2$. The first order conditions are now :

$$a - bQ = \frac{k_i}{n_i}Q_i + 2Qh(X),\ \forall i = 1, 2 \quad (26)$$

$$-h'(X).Q^2 = c_i'\left(x_i\right),\ \forall i = 1, 2 \quad (27)$$

Again, compared to (16)-(17), the main noticeable change is related to the term $\frac{k_i}{n_i}Q_i$ in (26), since (27) is similar to (17). All else equal, any former comment also extends to this new cost structure.

Solving (26), we obtain the aggregate and individual supply in group i, Q_i^w, q_i^w $\forall i = 1, 2$, and the aggregate supply of the industry Q^w, are given by:

$$
\begin{aligned}
Q_1^w &= a\left(\frac{b+2h(X^w)+\frac{k_2}{n_2}}{\nabla}\right) = n_1.q_1^w \\
Q_2^w &= a\left(\frac{b+2h(X^w)+\frac{k_1}{n_1}}{\nabla}\right) = n_2.q_2^w \\
Q^w &= 2a\left(\frac{b+2h(X^w)+\frac{k_1}{2n_1}+\frac{k_2}{2n_2}}{\nabla}\right)
\end{aligned}
$$

where $\nabla = (b+2h(X^w))\left(\frac{k_1}{n_1}+\frac{k_2}{n_2}\right)+\frac{k_1k_2}{n_1n_2}$. We can also assess the equilibrium individual market share of firms in each group:

$$
\begin{aligned}
\frac{q_1^w}{Q^w} &= \frac{1}{2n_1}\frac{b+2h(X^w)+\frac{k_2}{n_2}}{(b+2h(X^w))+\frac{k_1}{2n_1}+\frac{k_2}{2n_2}} \\
\frac{q_2^w}{Q^w} &= \frac{1}{2n_2}\frac{b+2h(X^w)+\frac{k_1}{n_1}}{(b+2h(X^w))+\frac{k_1}{2n_1}+\frac{k_2}{2n_2}}
\end{aligned}
$$

As a result, it is easy to verify that this modification arising at the level of the cost structure does not change our main conclusions, specifically, propositions 6 and 7 are still valid.

7 Conclusion

The paper shows that for an oligopoly the activity of which may endanger third parties victims and/or may harm the environment, it is easy to compare the consequences for the consumers of the good associated with liability under different regimes of damage sharing; still, things are less clear for the victims, despite a no liability regime always represents the worse situation for the victims. As a main conclusion, it is not clear that the market share rule is dominating the *per capita* rule in contexts where there is a hard uncertainty that prevents from disentangling the influence of each firm on the occurrence of accidents and harm borne by victims. Under strict liability with damage sharing, firms have an incentive to reduce the output level, because of the additional cost due to liability load, the contraction in output being larger under the market share rule than under the equal sharing arrangement. In turn, as the output decreases, firms' liability exposure is reduced, justifying to cut individual expenditures in care. As a result, regarding the issue of the control of risk through tort law and liability regime, there is a trade-off between a smaller damage in case of accident (market share rule) or a smaller probability of accident (equal share arrangement). Turning to the issue of efficiency, the analysis is quite deceptive. Compared to the discipline imposed by liability under imperfect competition, the optimal level of output is influenced by contradictory forces (both the marginal benefit and the marginal cost associated with production increase), which cast some doubts on the general structure of incentives exerted on care expenditures.

Also worth of interest is the result along which in a asymmetric oligopoly, alternative regimes of liability are not equivalent in terms of competitive/anti competitive effects; specifically we have found for different assumptions on production costs that the *per capita* rule leads to more dispersion in equilibrium market shares between high and low costs firms. In a sense, the *per capita* rule gives an additional competitive advantage to the most efficient firms (having the best costs structure), whereas the market share rule is more defensive for high costs firms, less efficient. A neglected aspect in the arguments of pros and cons the market share apportionment solution, comes from the fact that these market shares are not exogenous, but they reflect the structure of incentives designed by tort law and liability rules, given the characteristic features of the competitive environment. Hence, assessing the influence of alternative competitive environments in order to better understand the ties between competition law and liability law is also at the top of our agenda of research.

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