Accountability in Complex Procurement Tenders

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Abstract

In this paper we address the issue of corruption in terms of favoritism at the design stage of a complex procurement auction. The risk of corruption arises because the community has preferences over the projects it procures but lacks the competence to translate those preferences into an operational technical specification. This task is delegated to a public officer who may collude with one of the firms and let that firm's interest determine the design of the project. We investigate the value of engaging the competing firm in a simple accountability mechanism. We find that significant improvement can be achieved compared with accountability based on random challenge. The level of penalty needed to fully deter corruption is drastically reduced and independent of the complexity of the project. Instead, it depends on the degree of differentiation within the industry. Below the threshold, corruption occurs with some positive probability. Moreover corruption under alert based accountability tends to move the equilibrium specification toward more standard design when the community favors standardization. The investigated mechanism uses minimal information and commitment ability from the part of the community. And since it is played before the official tender, it can be implemented at minimal cost.

1 Introduction

The economic significance of public procurement in Europe is considerable: in 2010 a total of 2 406 billion euros - or around 20% of EU GDP - was spent by governments, the public sector and utility service providers on public works, goods and services.[1] A recent study commissioned by the EC developed a novel methodology to identify the costs of corruption. For practical reasons the study focused on a subset of all public procurement worth 447 billion euros (19% of this total expenditure)[2]. According to their findings the public loss encountered in corrupt projects amounts to 29% of the project

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[1]PWC and Ecory (2013)

[2]It corresponds to the tenders published in the Official Journal and the TED-database in 2010 (Tender Electronic Daily)

value in urban utility construction, 20% in roads and rail and 16% in water & waste. [3] The direct cost of corruption i.e., the cost that can be directly attributed to corrupt decisions in the five sectors under study is estimated in the study to between 1.4 – 2.2 billion euros.[4] The probability of corruption is estimated between 28–43% in waste & water treatment plants and up to 37–53% of (airport) runway construction works. This study provides a forceful confirmation that even in developed economies corruption in procurement remains a major challenge.

As known from numerous studies and analysis (see e.g., TI Integrity Pact in the Water sector) corruption can occur at each of the five stages of the procurement process including: 1. Needs assessment; 2. Preparation of bidding documents including the technical specification; 3. Contractor selection; 4. Contract execution; 5. Final accounting and audit. However, the above mentioned EC study only to a limited extent and only indirectly investigates corruption in the two first stages. Out of 27 red flags used systematically in the study only two can be viewed as somehow related: "preferred supplier indication" and "complaints from non-winning bidders". Yet, private sector actors and specialized anti-corruption organizations as Transparency International are well aware that often when the tender is announced "the winner is already known" partly because the project has been fine-tailored to one of the bidder e.g., as part a bid rigging process (see Kosenok and Lambert-Mogiliansky 2007). Yet, "fine-tailoring" that is corruption at the design stage has received very little attention and very few policy recommendations ever address it - none in the above mentioned EC study. However the associated costs both in terms of social welfare (the project does not respond to social needs) and overcost (the project is designed to maximize the winner's rents) can be very significant even if the rest of the procurement process is clean. To the best of our knowledge only the so-called Integrity Pact, a framework developed by Transparency International to fight corruption in procurement, addresses this corruption risk (we below return to this mechanism). When it comes to the legal basis for sanctioning favoritism in design, the picture is even more extreme. According to e.g., the French legislation Art. 432-14. The act of providing an unjustified advantage is punished by 2 years and 200 000 euros. The only few cases that have been judged in court concern the use of brand name in the technical specification.

[3]Such direct public losses in corrupt/grey cases are typically a result of: - Cost overruns; - Delays of implementation and/or; - Loss of effectiveness (including inferior quality and questionable usefulness).

[4](Road/rail, Waters & waste, Urban utility/construction/ Training/ research and development/

A main reason for the remarkable lack of attention to corruption at the design stage is presumably the particularly difficult nature of the associated probe. Indeed any technical specification of a project tends to favor some firm at the expense of others. This is true even if the design is the social economic efficient one. In order to establish whether or not favoritism has taken place special knowledge is required. In some cases a small detail can be sufficient to seriously reduce competition. An example of a technical specification that clearly excludes competitors is the request for a specific brand. Such a feature secures the win of the firm that owns the brand at a high cost for the public. Prohibiting the use of brand name is by now a standard provision in most Procurement laws. But besides this one gross instance of favoritism, it is very difficult *for an outsider* to detect a technical requirement aimed at unduly favoring a specific firm. Now we can reverse the burden of proof so the procurement official may face an obligation to justify her choices. It is however not practically feasible to provide justification for all aspects of a complex project nor is it feasible to process such information in limited time. Therefore the central problem becomes the selection of which aspect(s) of the technical specification to request justification about. The concept of Transparency International's Integrity Pact (IP) relies on the idea that different stake-holders can provide valuable contributions.[5] So in particular within an IP, civil society and potential bidders are invited to participate to social hearings and request justification at various stages including the design stage. In this paper we provide a formal analysis of a procedure that allows competing firms to participate in a mechanism to hold the procurement agent accountable for the announced technical specification.

More specifically we consider a situation in which two firms compete for a complex project. The public official who manages the procurement auction is willing to take bribes to distort the technical specification of the project to favor a firm. One of the firms has close connections with the public official so they collude whenever favoritism is profitable. The representative citizen has preferences for the project but she lacks the knowledge and the time to translate those into a technical specification. She delegates that task to the public official who has significant discretion by force of his private information. We are interested in assessing the value of engaging the competing firm in a mechanism of

[5] An Integrity Pact is a complex arrangement involving a commitment from the government agency and the firm, an agreement on how to uphold and monitor the commitment with the participation of all stake-holders including civil society.

accountability in order to contain favoritism in project design. More precisely we compare a mechanism of accountability relying on random challenge (RCA) with a one relying on alerts sent by the competing firm, alert based accountability (ABA). The principle of the mechanisms is as follows. Before the submission of offers, one aspect of specification announced by the public official is selected at random (RCA) or by the rival (ABA). The agent is then requested to justify his decision with respect to that aspect. If he fails, the tender is relaunched with a new technical specification and the agent is penalized.

Our first central result (Theorem 1) characterizes the threshold for complete corruption deterrence with alert based accountability. We find that it depends on the industry structure as expressed by the rival's comparative advantage. This contrasts sharply with the threshold that applies in random challenged based accountability which is a function of the complexity of the project (dimensionality of the technical specification) and is therefore generally significantly larger. So for instance, corruption never occurs with ABA (even with zero penalty) if the rival has no comparative advantage independently of the project's complexity while it is systematic with RCA when the penalty is low relative to the complexity. In our second result (Theorem 2) we establish that the threshold is tight i.e., below that threshold any equilibrium is characterized by corruption with positive probability. We find that with RCA, the risk of corruption is most severe when the firm would lose the contract in the absence of corruption. In contrast with ABA, corruption occurs with some positive probability in states where the firm earns positive rents but may yield no net gain in expectation. The general setting does not allow for a more advanced characterization of the patterns of corruption below the threshold. In order to go further, we fix an industry structure in an example which enables illustrating additional features. We show that ABA induces complex patterns of behavior. We have multiple equilibria where the corrupt firm plays a mixed strategy so favoritism occurs with a probability less than one and yields no expected gains. We also find that with ABA corruption is more likely to move the specification toward a more standard technical solution. This is because the corrupt firm responds to the competitor's challenging strategy in a context where by assumption standard specification are more likely.

Our analysis supports the idea that engaging interested parties in accountability can be very beneficial. In our context engaging the competing firm in accountability allows for very significant improvements in terms of corruption containment be it total or partial.

In particular we find ABA increases the efficiency of the much advocated anti-corruption policy that promotes standardization in project design. Finally, from a practical point of view, a critical argument in favor of alert-based accountability is that it does not rely on rather unrealistic assumptions about the citizen's ability to access and process information and her ability to commit on a sophisticated and potentially difficult-to-enforce mechanism. Moreover alerts and justification take place even before firms prepare their offers on the first announced project specification. Therefore alert based accountability does not imply additional cost for honest firms and minimally burdens the procurement process.

The issue of accountability has been addressed in the political science and political economy literature (e.g., Persson et al.,1997) with emphasis on election rules and organizational structure. Our approach shares common features with the literature on optimal monitoring with ex-post verification (cf Townsend, 1979, and Gale and Hellwig, 1985). In contrast with Townsend, for example, we are interested in a mechanism with ex-ante commitment and we do not consider an explicit cost of verification. Moreover we are interested in the participation of competing firms. Our paper shares some features with the literature on inspection games. Indeed a main issue is to investigate how the inspected responds to the inspector's verification strategy. However and in contrast with that literature we are not concerned with statistical analysis and the structure of our inspection problem does not lend itself to the type of approach developed for inspection games. Another strand of literature is relevant to the issue of accountability, persuasion games (cf. Glazer and Rubinstein, 2004, 2006, Forges Koessler 2008). Although our concern is with the monitoring properties of the investigated mechanisms, from an ex-post perspective "alert based accountability" is a specific persuasion game and its analysis is the core of our contribution. From an applied point of view our contribution is to demonstrate how a simple and little costly mechanism may allow stake-holders to hold procurement agents accountable and thereby contain corruption.

2 The Setting

Projects. A community, identified with a representative citizen (she), has decided to realize a complex project, e.g. a major construction work aimed at fulfilling specific social economic objectives. The citizen has no prior knowledge nor expertise to formulate the technical specification corresponding to those objectives. She cannot design the project, choose among various versions or even identify the various relevant aspects of the project. The complexity of the project is captured by its multi-dimensionality: a project corresponds to a n-dimensional vector of specifications, $\mathbf{q} = (q_1, ..., q_n) \in \{0, 1\}^n$.

The citizen delegates to a public officer (a civil servant, he) the task of translating the community's decision into the technical specifications of the project and of running a procurement auction to allocate the project to a private contractor. The public officer has specific and technical knowledge that enables him to fulfill these tasks. In particular, he has private information about $\overline{\mathbf{q}}$, the project the citizen would opt for if she were able to figure out the various technical aspects and the costs and benefits associated to them. The public officer uses his knowledge of $\overline{\mathbf{q}}$ to choose the final project $\widehat{\mathbf{q}}$, possibly different from $\overline{\mathbf{q}}$, to be auctioned off in the procurement auction.

The nature of the community's needs is supposed to be the outcome of a random process: the random variables $\bar{q}_i$, for $i = 1, .., n$, are assumed to be identically independently distributed with $\bar{q}_i = 1$ with probability ε and 0 with probability $1 - \varepsilon$. We interpret $\bar{q}_i = 0$ as a standard specification and $\bar{q}_i = 1$ as a sophisticated specification; standard specifications are assumed to be more likely than sophisticated ones: $\varepsilon < 1/2$.[6]

Private contractors. Several firms compete to win the procurement auction and become the unique contractor in charge of realizing the project $\widehat{\mathbf{q}}$. Each firm is characterized by its technological know-how, its experience, its formal intellectual property, the talent and ability of its employees; we capture these characteristics through a (multi-dimensional) type $\theta \in \{0, 1\}^n$. We assume that the cost of realizing a project with specifications $\mathbf{q}$ for a firm of type θ is given by:[7]

$$C(\mathbf{q}, \theta) = c \cdot \#\{i; q_i = 1, \theta_i = 0\},$$

that is, we assume that the only cost a firm incurs is when it produces a sophisticated specification while it does not have the technology for it.

We consider two firms, firm α and firm β characterized respectively by θ^α and θ^β

[6] It is a common practice to favor standardized solutions rather than sophisticated ones in public procurement. It is justified by a concern to promote competition to keep down public expenditure. It is also included in many procurement guidelines with the motivation that it helps contain favoritism.

[7] We let $\#A$ denote the cardinal of set A.

and the profile of firms' types is called the *industry structure*. The multi-dimensionality assumption implies that, if neither $\theta^\alpha \geq \theta^\beta$ nor $\theta^\alpha \leq \theta^\beta$, each firm has a comparative advantage and, depending upon the announced project, either firm may win the procurement auction (absent favoritism).

The procurement process. The process includes two stages. First, the public officer produces and announces a technical description $\widehat{\mathbf{q}}$ of the project to be procured. Then, the project $\widehat{\mathbf{q}}$ is allocated to one of the private contractors through a first-price procurement auction under complete information. If $C(\widehat{\mathbf{q}}, \theta^i) < C(\widehat{\mathbf{q}}, \theta^j)$ (respectively $=$) for $(i, j) = (\alpha, \beta)$ or (β, α), i wins the auction (respectively with probability 1/2), the price paid by the community corresponds to the cost of the losing firm, i.e. to $C(\widehat{\mathbf{q}}, \theta^j)$, and the winner's profit is equal to $C(\widehat{\mathbf{q}}, \theta^j) - C(\widehat{\mathbf{q}}, \theta^i)$.

Corruption. We focus on corruption at the design stage i.e., the risk of favoritism in the choice of the technical specification of the project. We assume that only one predetermined firm, firm α, has close connections with the public officer and that the public officer is corruptible i.e., willing to distort the project to favor firm α in exchange of a bribe. Finally, we are not concerned with the allocation of rents between the public officer and firm α i.e., by the magnitude of the bribe. Firm α and the public officer get together before the public announcement of the tender, share the information about the socially preferable project $\overline{\mathbf{q}}$ and jointly decide upon the project technical specification $\widehat{\mathbf{q}}$ to be tendered so as to maximize their aggregate expected surplus (under full information). We discuss these assumptions in the concluding section.

Accountability. The public officer is paid a fixed salary, normalized to 0, that cannot be made contingent on the specifications of the final project nor based on a revelation mechanism, because of the inability of the citizen to describe ex ante what a project is. But the public officer is accountable towards the community: he has to justify his decisions if they are challenged and he has to suffer punishment in case of eventual misconduct. We assume that the public officer is capable of justifying an honest decision with respect to any dimension. So the failure to provide a justification is taken as a proof of misconduct. Let L denote the monetary value of the punishment; it captures e.g. the officer's disutility of demotion or of being fired. Moreover, in the case of proven misconduct, the procurement auction based on the (distorted) project specification is canceled, a new tender is launched and firm α is excluded from it.[8]

Accountability is constrained by the fact that giving account and justifying all dimensions of a project is prohitively costly for complex projects. To capture this feature, we assume that only one specification of the project can be challenged and justified at finite cost, normalized to zero. So, the public officer is challenged on one specification i of the announced project $\widehat{\mathbf{q}}$ and he has to justify it by providing adequate evidence that $\bar{q}_i = \widehat{q}_i$; if the announced project differs from the corresponding specification in the socially preferable project, i.e. if $\bar{q}_i \neq \widehat{q}_i$, there is proof of his misconduct while if $\bar{q}_i = \widehat{q}_i$, no such proof is established even though he may have distorted other specifications in the final project.

Absent any guidance, the citizen cannot distinguish among the various characteristics of a project nor describe them, so that he can only challenge one randomly drawn specification of the project, what we call *accountability based on random challenge.*

We investigate another procedure based on the possibility of letting rival firms choose the dimension of the project to be challenged. The idea is that firms in the industry share the knowledge of the industry structure that is, they know each others' types and, in particular, firm α's opponents know which vector of specifications benefits firm α and therefore could be the outcome of corruption. Then, after the final project $\widehat{\mathbf{q}}$ has been made public, firm α's opponents can provide guidance as to which characteristic of the project raises more suspicion than others with regards to the likelihood of corruption by firm α. Our *alert-based accountability* procedure therefore relies on firm β (the sole opponent here) recommending one specification of the project to be challenged after $\widehat{\mathbf{q}}$ is announced but before the procurement auction takes place.

Summary. At this point, it is worth summarizing the information and game structure.

1. The public officer learns $\overline{\mathbf{q}}$.

2. The citizen commits publicly to an accountability procedure.

3. The public officer and firm α jointly design the project $\widehat{\mathbf{q}}$ that is announced.

4. The announced accountability procedure is implemented.

[8] We do not specify the new tender, we only need to assume that the initial rivals of firm α, here firm β, expect positive profits from the new tender procedure if firm α is excluded.

5. If no proof of misconduct is found, the tender proceeds with the announced project specification $\widehat{\mathbf{q}}$. If misconduct is proven, a new tender is organized without firm α and the public officer is punished with a penalty L.

3 Simple accountability procedures

This section introduces some technical notation and characterizes two benchmark cases namely, the no accountability and accountability based on random challenge.

Fix an industry structure $(\theta^\alpha, \theta^\beta)$. Given a project $\mathbf{q}$, firm α's profit in the auction is equal to $\sup\{\pi(\mathbf{q}), 0\}$ with $\pi(\mathbf{q}) \equiv C(\mathbf{q}, \theta^\beta) - C(\mathbf{q}, \theta^\alpha)$. It is immediate to see that $\max_{\mathbf{q}\in Q} \pi(\mathbf{q}) = \pi(\theta^\alpha)$.[9] If $\pi(\bar{\mathbf{q}})$ is maximal, i.e. equal to $\pi(\theta^\alpha)$, firm α has nothing to gain from engaging in corruption. Moreover, given the risk of punishment, there is no point in engaging in corruption that leads to zero profits. So, when corruption occurs changing $\bar{\mathbf{q}}$ into $\widehat{\mathbf{q}}$, it must necessarily be the case that: $\pi(\bar{\mathbf{q}}) < \pi(\theta^\alpha)$ and $0 < \pi(\widehat{\mathbf{q}})$. Consequently, if firm α has no comparative advantage, i.e. if $\pi(\theta^\alpha) \leq 0$, there cannot be any corruption in equilibrium. So, from now on, we assume that the industry structure is such that $\pi(\theta^\alpha) > 0$.

For a given socially preferable project $\bar{\mathbf{q}}$, corruption by firm α may take the form of upgrades of some specifications i, so that $\bar{q}_i = 0$ and $\hat{q}_i = 1$, or/and downgrades of some other specifications i, so that $\bar{q}_i = 1$ and $\hat{q}_i = 0$. Given the risk of being caught, firm α engages in corruption to upgrade specification i only if this is strictly profitable i.e., only if $\theta_i^\alpha = 1$ and $\theta_i^\beta = 0$. For any project $\mathbf{q}$, let

$$S_u(\mathbf{q}) \equiv \left\{j; q_j = \theta_j^\alpha = 1, \theta_j^\beta = 0\right\}$$

denote the set of characteristics that could be the outcome of an upgrade in the profile $\mathbf{q}$, and $s_u(\mathbf{q})$ its cardinal. The maximal number of characteristics that could be the outcome of upgrades is: $s_u^{max} \equiv \max_{\mathbf{q}} s_u(\mathbf{q}) = s_u(\theta^\alpha)$. Similarly, firm α would engage in a downgrade of specification i only if $\theta_j^\alpha = 0$ and $\theta_j^\beta = 1$, and for any project $\mathbf{q}$, let

$$S_d(\mathbf{q}) \equiv \left\{j; q_j = \theta_j^\alpha = 0, \theta_j^\beta = 1\right\}$$

[9]Note that any $\mathbf{q}$ that differs from θ^α on any characteristics i such that $\theta_i^\alpha = \theta_i^\beta$ also reaches this maximum.

denote the set of characteristics that could be the outcome of a downgrade in the profile $\mathbf{q}$, with cardinal $s_d(\mathbf{q})$, and $s_d^{max} \equiv \max_{\mathbf{q}} s_d(\mathbf{q}) = s_d(\theta^\alpha)$.

So, the set of characteristics that raise suspicion ex post in a project $\widehat{\mathbf{q}}$ is given by $S(\widehat{\mathbf{q}}) \equiv S_u(\widehat{\mathbf{q}}) \cup S_d(\widehat{\mathbf{q}})$, with cardinal $s(\widehat{\mathbf{q}}) \equiv s_u(\widehat{\mathbf{q}}) + s_d(\widehat{\mathbf{q}})$. Straightforward algebra yields the following result:

Lemma 1 *: For any* $\mathbf{q}$*, the following holds:*[10] $\pi(\mathbf{q}) = c(s(\mathbf{q}) - s_d^{max})$.

A given accountability procedure generates a probability of punishment and cancellation $P(\widehat{\mathbf{q}}, \bar{\mathbf{q}})$ when the socially preferable project is $\bar{\mathbf{q}}$ and the announced project is $\widehat{\mathbf{q}}$, with $P(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \mathbf{0}$ by definition. Firm α and the public officer design the final project $\widehat{\mathbf{q}}$ that maximizes their expected joint surplus taking into account this probability of being caught:

$$\pi(\widehat{\mathbf{q}})(1 - P(\widehat{\mathbf{q}}, \bar{\mathbf{q}})) - P(\widehat{\mathbf{q}}, \bar{\mathbf{q}})L. \tag{1}$$

In the no-accountability situation, corruption is never detected nor punished, $P(\widehat{\mathbf{q}}, \bar{\mathbf{q}}) = 0$, and in equilibrium the procurement auction bears on a project that best fits firm α's capacities, i.e. a project that maximizes $\pi(\widehat{\mathbf{q}})$ within the set of possible distortions. This immediately leads to the following Lemma:

Lemma 2 *(No accountability): In the absence of accountability procedure, there is corruption with probability* 1 *for all* $\bar{\mathbf{q}}$ *such that* $\pi(\bar{\mathbf{q}}) < \pi(\theta^\alpha)$*, all specifications* i *such that* $\bar{q}_i = \theta_i^\beta \neq \theta_i^\alpha$ *are distorted and the final project is always such that* $\pi(\widehat{\mathbf{q}}) = \pi(\theta^\alpha)$.

The lemma simply confirms that, absent accountability, corruption arises whenever it generates a strict gain in the procurement auction for firm α.

Let us now investigate accountability based on random challenge, i.e. the procedure that is based on the challenge of a specification randomly drawn within $\{1, 2, ..., n\}$ with probability $\frac{1}{n}$. Then, we obtain the following proposition.[11]

Proposition 1 *(Accountability based on random challenge): Under accountability based on random challenge, if* $L > (n-1)c$*, there is no distortion in equilibrium, whatever the socially preferable specification* $\bar{\mathbf{q}}$*, while if* $L < (n-1)c$*, there exists some* $\bar{\mathbf{q}}$ *that is distorted with probability 1 in equilibrium; moreover, in this last case, suppose* $\bar{\mathbf{q}}$ *and* $\bar{\mathbf{q}}'$ *are such that* $S(\bar{\mathbf{q}}) \subset S(\bar{\mathbf{q}}')$ *strictly, then:*

[10] Note that $\pi(\theta^\alpha) = cs_u^{max}$.

[11] Proofs are relegated in the Appendix.

- *if* $\pi(\bar{\mathbf{q}}') > \pi(\bar{\mathbf{q}}) \geq 0$ *and there is corruption at* $\bar{\mathbf{q}}'$*, then there is also corruption at* $\bar{\mathbf{q}}$ *in equilibrium*

- *and if* $\pi(\bar{\mathbf{q}}) < \pi(\bar{\mathbf{q}}') \leq 0$ *and there is corruption at* $\bar{\mathbf{q}}$*, then there is also corruption at* $\bar{\mathbf{q}}'$ *in equilibrium.*

This proposition shows first that when $L > (n-1)c$ there is no corruption at all in equilibrium whatever the socially preferable project. Conversely, there always states of nature, i.e. values of socially preferable project $\bar{\mathbf{q}}$, for which corruption occurs with probability 1 when $L < (n-1)c$. So, the threshold $L \lesseqgtr (n-1)c$ that determines whether there is corruption or not, depending on the industry structure, is tight. The key message here is obviously not that corruption does not occur for large punishments and occurs for low punishments, this is rather immediate; it is rather that the critical level of punishment is potentially large, of the same order as the dimensionality of projects.

The second and less intuitive message from the proposition is that, when $L < (n-1)c$, corruption is more prone to occur in situations in which firm α does not hold too strong an advantage in supplying the socially preferable project so that it would tie with firm β in the procurement auction without corruption. The intuition for this result runs as follows. Corruption induces a potential gain but it also induces a risk of losing the punishment plus the profit that would have been obtained without engaging in corruption. This no-corruption profit is measured by $\sup\{\pi(\bar{\mathbf{q}}); 0\}$. Therefore, whenever firm α holds a net technological advantage over its rival in supplying the socially preferable project, i.e. whenever $\pi(\bar{\mathbf{q}}) \geq 0$, the higher this technological advantage, the larger the loss from being caught in the corruption stage and therefore the less likely corruption. So, when L falls slightly below $(n-1)c$, corruption appears first for some states of nature such that $\pi(\bar{\mathbf{q}}) \leq 0$.

What if, absent corruption, firm α would lose the auction, i.e. if $\pi(\bar{\mathbf{q}}) < 0$? Engaging in corruption from such a state of nature so as to induce a strictly profitable profile $\widehat{\mathbf{q}}$ requires more distortions than at $\bar{\mathbf{q}}'$ such that $S(\bar{\mathbf{q}}) \subset S(\bar{\mathbf{q}}') \subset S(\widehat{\mathbf{q}})$ and $\pi(\bar{\mathbf{q}}') = 0$, hence a higher probability of getting caught, while the default option, i.e. no corruption yields the same null expected profit. Therefore, if corruption is prevented for any $\bar{\mathbf{q}}'$ such that $\pi(\bar{\mathbf{q}}') =\geq 0$, then it should also be prevented for any $\bar{\mathbf{q}}$ such that $\pi(\bar{\mathbf{q}}) < 0$ and conversely, if corruption occurs for some $\bar{\mathbf{q}}$ such that $\pi(\bar{\mathbf{q}}) < 0$, it should also occur for some $\bar{\mathbf{q}}'$ such that $\pi(\bar{\mathbf{q}}') = 0$.

So, the states of nature in which firm α is the most likely to engage in corruption correspond to situations in which firm α has no net advantage over firm β and firm α would tie with firm β in the procurement auction. In other words, corruption is most difficult to fight when it involves distorting the project so as to make firm α win the procurement auction instead of tying, rather than when the issue is to increase its (already) winner's profit.

4 Alert-based accountability

Given an industry structure $(\theta^\alpha, \theta^\beta)$ that is common knowledge among the two firms, the alert-based procedure determines a Bayesian game to be played between them. Firm α privately knows the state of nature characterized by $\bar{\mathbf{q}}$ and its strategy maps any socially preferable project, $\bar{\mathbf{q}}$ into an announced project $\widehat{\mathbf{q}}$; it can either engage in corruption by upgrading some specification and downgrading some others, or refrain from engaging in corruption. Firm β observes the announced project $\widehat{\mathbf{q}}$, forms posterior beliefs about $\bar{\mathbf{q}}$ given $\widehat{\mathbf{q}}$, and its strategy maps any announced project into a specification i to be flagged, i.e. which the public officer should justify.

It is intuitive that whenever corruption takes place, firm β has to play mixed strategies. If it were to select always one deterministic specification for any announced project $\widehat{\mathbf{q}}$, firm α would never engage in corruption by distorting this specification and firm β would be worse off than under a uniform random alert strategy, with which it would always have a positive probability of cancelling the tender.

Consequently, we consider mixed (behavioral) strategies: let $\rho_i(\widehat{\mathbf{q}})$, for any $i \in S(\hat{\mathbf{q}})$, denote the probability that firm β flags specification i when the final project is $\widehat{\mathbf{q}}$, with $\rho_i(\hat{\mathbf{q}}) \geq 0$ and $\sum_{i \in S(\hat{\mathbf{q}})} \rho_i(\hat{\mathbf{q}}) = 1$ and let $p_i(\bar{\mathbf{q}})$, for any $i \in S(\theta^\alpha) \backslash S(\bar{\mathbf{q}})$, denote the probability that firm α chooses to engage in corruption and distort characteristic i, with $p_i(\bar{\mathbf{q}}) \geq 0$ so that $1 - \sum_{i \in S(\theta^\alpha) \backslash S(\bar{\mathbf{q}})} p_i(\bar{\mathbf{q}}) \geq 0$ is equal to the probability that firm α does not engage in corruption at $\bar{\mathbf{q}}$.

As can be expected, corruption is deterred under alert-based accountability whenever the penalty for being caught is large enough. The more interesting question is to characterize what "large enough" means and how it compares with the corresponding threshold under accountability based on random challenge.

Theorem 1 *(Complete deterrence): Assume* $L > s_d^{max}c$*; the equilibrium outcome under alert-based accountability is unique and corresponds to the no-corruption outcome whatever the socially preferable project* $\bar{\mathbf{q}}$.

The theorem provides a threshold on the magnitude of penalties that deters corruption in any state of nature: more precisely, for any penalty such that $L > s_d^{max}c$, there does not exist another equilibrium outcome than the no corruption outcome.

The threshold depends on the industry structure $(\theta^\alpha, \theta^\beta)$ through the number of specifications on which firm β has a competitive advantage over firm α. Note also that $s_d^{max}c = C(\theta^\beta, \theta^\alpha)$; that is, the corruption deterrence threshold corresponds to a value of the punishment that is equal to firm α's cost in realizing the project that best fits with firm β's technological capacities.[12] The smaller the number of specifications for which firm β has a competitive advantage over firm α, the easier it is to prevent corruption under alert-based accountability.

The extreme case corresponds to the situation in which firm α unambiguously dominates firm β is terms of technological advantages, so that $s_d^{max} = 0$; in this case, we have the following remarkable corollary.

Corollary 1 *(No corruption by a superior firm): When firm* α *is more technologically efficient that firm* β*, i.e. when* $s_d^{max} = 0$*, firm* α *never engages in corruption under alert-based accountability.*

To understand this result, consider a socially preferable project is characterized by $s_u(\bar{\mathbf{q}}) \geq 0$. Firm α would earn $\pi(\bar{\mathbf{q}}) = s_u(\bar{\mathbf{q}})c$ if it does not try to manipulate the final project. If instead it distorts (upgrades) $k \geq 1$ specifications so as to increase its rival's cost by kc, it would generate a probability $\frac{k}{s_u(\bar{\mathbf{q}})+k}$ of being caught and would thus earn an expected payoff equal to:

$$(s_u(\bar{\mathbf{q}}) + k)c \times \{1 - \frac{k}{s_u(\bar{\mathbf{q}}) + k}\} - L \times \frac{k}{s_u(\bar{\mathbf{q}}) + k} = s_u(\bar{\mathbf{q}})c - \frac{kL}{s_u(\bar{\mathbf{q}}) + k},$$

which is smaller than the no-corruption payoff for any$L$ and k. In other words, there is room for corruption under alert-based accountability only when firm α can dilute firm β's suspicion over both types of manipulations, i.e. upgrades and downgrades,

[12]This corresponds to the generalization of this theorem to more general symmetric cost functions.

i.e. both inducing sophisticated specifications that are favorable to firm α and avoiding sophisticated specifications that are favorable to its rival. When dilution of distortions among possible downgrades is impossible, the gains from distortion are wiped out by the increase in the probability of being caught.

The second remark concerns the comparison with the random challenge procedure. The critical level of punishment that deters entirely corruption under alert-based accountability is lower than the corresponding level of punishment under the random challenge procedure: given that $s_u^{max} > 0$ by assumption and $s_u^{max} + s_d^{max} \leq n$, it follows that $s_d^{max} c \leq (n-1)c$. It is potentially much lower when there is little differentiation in the industry as captured by proportion of specifications over which firm β has a technological advantage over firm α. When this proportion s_d^{max}/n is small, firm α has limited possibilities of dilution of the upgrades it can induce and corruption is therefore more easily deterred. A crucial determinant of the efficiency of alert-based accountability is therefore related to the extent of differentiation in the industry as captured by the strength of the competitor compared to the complexity of projects involved.. The stronger the competitor the more difficult it is to fully deter favoritism in project design.

The better performance of the alert-based accountability procedure comes obviously from the ability of firm β to exploit ex-post information about the set of specifications that raise suspicion in the announced project. Challenges can be concentrated on the subset of specifications that raise suspicion *ex post*, i.e. on $S(\hat{\mathbf{q}})$. Indeed, if the verification procedure could be ex ante specified conditional on the announced project $\hat{\mathbf{q}}$, a (uniformly distributed) random challenge procedure over $S(\hat{\mathbf{q}})$ conditional on $\hat{\mathbf{q}}$ being observed would deter corruption in all states of nature. Note that the proof of the theorem precisely exhibits an equilibrium that attain the no-corruption outcome such that $\rho_i(\hat{\mathbf{q}}) = \frac{1}{s(\hat{\mathbf{q}})}$ for $i \in S(\hat{\mathbf{q}})$; this is precisely what an ex-ante specified random challenge procedure restricted on $S(\hat{\mathbf{q}})$ would generate. So, alert-based accountability is equivalent to the possibility of committing ex ante on a contingent uniform verification procedure on $S(\hat{\mathbf{q}})$ *as far as* we are concerned with completely wiping out the occurrence of corruption. The next result and the example section that follows show however that alert-based accountability performs differently from this contingent random verification procedure when $L < s_d^{max} c$. The critical argument in favor of alert-based accountability is that it does not rely on rather unrealistic assumptions about the citizen's ability to access and process information and her ability to commit on a sophisticated and potentially difficult-to-enforce mechanism conditional on the announced final project.

Assume now that the punishment is not large so that $L < s_d^{max} c$. We show that corruption necessarily takes place with positive probability so that the threshold $L \lesseqgtr s_d^{max} c$ that has been characterized in the previous theorem is actually "tight". Moreover, we can characterize states of nature in which corruption occurs.

Theorem 2 *(Corruption onto firm α's prefered project): When $L < s_d^{max} c$, any announced project $\hat{\mathbf{q}}$ such that $\pi(\hat{\mathbf{q}}) = \pi(\theta^\alpha) = s_u^{max} c$ is the outcome of corruption with positive probability in any equilibrium under alert-based accountability.*

When penalties are not sufficient to completely wipe out corruption, Theorem **??** tells us that when the set of suspects is maximal $s_u^{max} + s_d^{max}$ so that $\pi(\hat{\mathbf{q}}) = \pi(\theta^\alpha)$, there is necessarily corruption with positive probability in any equilibrium. Therefore, Theorem 2 confirms that the threshold $s_d^{max} c$ is indeed tight in the alert-based accountability procedure: when L is larger, there is no corruption in any equilibrium, while when L is smaller, there is some corruption in any equilibrium.

Compared to the case of accountability based on random challenges, Theorem **??** also points toward a modification in the form of equilibrium strategies when there is corruption. Under alert-based accountability, as soon as $L < s_d^{max} c$, when the final project that is announced yields maximal profit to firm α, there is positive probability that there has been active corruption.[13] Therefore, corruption becomes an issue under alert-based accountability not so as to enable firm α to win the tender, as opposed to a tie, but rather so as to induce a perfect fit of the final project with firm α technological abilities.

Given the complexity of the game induced by laert-based accountability in general, we are not able to characterize more precise properties of equilibrium strategies in the general setting when $L < s_d^{max} c$. So, in the next section, we develop an example with a specific industry structure and we characterize several types of corruption equilibria in this simplified setting.

[13]Of course, there is a positive prabability that it corresponds the socially preferable project, in which case firm α need not engage in corruption at all.

5 An illustrative example

The specific setting. Let us normalize $c = 1$ and assume that the project under consideration has many dimensions $n > 3$ but the firms differ in their technological ability only with respect to 3 specifications, say $i = 1, 2, 3$; so, when describing a project, we will only write down the first 3 specifications. We shall focus on the following industrial structure: $\theta^\alpha = (1, 1, 0, ...)$ and $\theta^\beta = (0, 0, 1, ...)$, so that firm α's highest possible gain from winning the project is $\pi(\theta^\alpha) = 2$ which obtains when its specification mirrors firm α's technology (at least on the 3 first dimensions).

No accountability. In the absence of any accountability, firm α engages in corruption with the public officer whenever $\bar{\mathbf{q}}$ is such that $\pi(\bar{\mathbf{q}}) < 2$ and the outcome of corruption will be the profile that best fits firm α's technology, i.e. the public officials announces some $\hat{\mathbf{q}}$ =$(1, 1, 0, .., \bar{q}_i, ..)$ so that the coalition (firm and public agent) gains $\pi(\hat{\mathbf{q}}) = 2$. Only the three first specification are ever distorted and therefore we refer to them as the set of suspects.

Note that for n large relative to 3, unconstrained corruption results in a variety of project designs being announced, designs that do not correspond to firm $\alpha's$ technological profile on the $n - 3$ last dimensions. Therefore, in a context where not all public officers are corruptible, it is relatively easy to convince the citizen - who lacks the information necessary to identify the set of suspicious specifications - that the procedure is clean by pointing out the many specifications that do not favor firm α i.e., do not respond to its technological profile.

Random challenge based accountability. With random challenge based accountability, Proposition **??** shows that if $L > n-1$, there cannot be any corruption in equilibrium, while if $L < n - 1$, there is a strict expected gain in engaging in corruption for firm α in at least one state of nature $\bar{\mathbf{q}}$ and the largest risk of corruption is found in states $\bar{\mathbf{q}}$ such that $\pi(\bar{\mathbf{q}}) = 0$. More precisely, the following claim holds:

Claim 1 *: For L such that $n - 1 > L > \sup\{n - 2; \frac{2n}{3} - 2; \frac{n}{2} - 1\}$ corruption occurs in equilibrium for states of nature $\bar{\mathbf{q}}$ such that $\pi(\bar{\mathbf{q}}) = 0$ but not for $\bar{\mathbf{q}}$ such that $\pi(\bar{\mathbf{q}}) \in \{-1, 1\}$*

Proof. If $\bar{\mathbf{q}}$ is such that $\pi(\bar{\mathbf{q}}) = 1$, i.e. for $(1, 0, 0, ...)$ or $(0, 1, 0, ...)$ or $(1, 1, 1, ...)$, engaging in corruption to induce $\widehat{\mathbf{q}} = (1, 1, 0, ...)$ involves one distortion and is strictly profitable (resp. dominated by no corruption) iff: $2(1-\frac{1}{n})-\frac{L}{n}>1$ (resp. <1)$\Leftrightarrow L<n-2$ (resp. $L>n-2$).

If $\bar{\mathbf{q}}$ is such that $\pi(\bar{\mathbf{q}})=0$, i.e. for $(0,0,0,...)$ or $(0,1,1,...)$ or $(1,0,1,...)$, engaging in corruption to induce $\widehat{\mathbf{q}}=(1,1,0,...)$ involves two distortions and is strictly profitable (resp. dominated by no corruption) iff: $2(1-\frac{2}{n})-\frac{2L}{n}>0$ (resp. <0)$\Leftrightarrow L<n-2$ (resp. $L>n-2$). Engaging in corruption to induce $\widehat{\mathbf{q}}$ such that $\pi(\widehat{\mathbf{q}})=1$ involves only one distortion and is strictly profitable (resp. dominated by no corruption) iff: $1(1-\frac{1}{n})-\frac{L}{n}>0$ (resp. <0)$\Leftrightarrow L<n-1$ (resp. $L>n-1$).

If $\bar{\mathbf{q}}$ is such that $\pi(\bar{\mathbf{q}})=-1$, i.e. for $(0,0,1,...)$, engaging in corruption to induce $\hat{\mathbf{q}}=(1,1,0,...)$ involves three distortions and is strictly profitable (resp. dominated by no corruption) iff: $2(1-\frac{3}{n})-\frac{3L}{n}>0$ (resp. <0)$\Leftrightarrow L<\frac{2n}{3}-2$ (resp. $L>\frac{2n}{3}-2$). Engaging in corruption to induce $\widehat{\mathbf{q}}$ such that $\pi(\widehat{\mathbf{q}})=1$ involves two distortions and is strictly profitable (resp. dominated by no corruption) iff: $1(1-\frac{2}{n})-\frac{2L}{n}>0$ (resp. <0)$\Leftrightarrow L<\frac{n}{2}-1$ (resp. $L>\frac{n}{2}-1$).[14] The statement in the claim follows. ■

When $\pi(\bar{q})=0$, firm α has an incentive to distort one specification to its advantage so that $\pi(\widehat{\mathbf{q}})=1$. This shows that with random challenges the most severe risk of corruption arises when firm α would tie with its rival in the procurement auction and thus earn zero profit. Corruption, in that interval of values for L, amounts to securing a win while incurring the smallest risk of detection. This means that with accountability based on random challenge the most serious risk of favoritism arises when the allocation outcome, i.e. which firm wins, is at stake rather than when favoritism brings about larger profit.

Alert based accountability. Consider now alert based accountability. First, note that firm β will concentrate its alert strategy and send red flags only on specifications that might be distorted, i.e. on possible suspects $i\in\{1,2,3\}$. Given the symmetric structure of the model, we will w.l.o.g. focus on an alert strategy for firm β that is symmetric *within* the set of suspect upgrades, i.e. between $i=1$ and $i=2$ when there are both suspects, and that is symmetric among all non-suspect specifications. As a consequence, a strategy for firm β is fully described by $\rho\left(\widehat{\mathbf{q}}\right)$, the probability for flagging the set of suspect upgrades $\{1,2\}$, the complementary probability applies to the only suspect downgrade $\{3\}$. More specifically:[15]

[14]Note that for $n=6$, $\frac{2n}{3}-2=\frac{n}{2}-1$ and then for any $n>6$, $\frac{2n}{3}-2>\frac{n}{2}-1$.

[15]A subscript on ρ is used to refer to the total number of suspect upgrades in $\widehat{\mathbf{q}}$

- if $\widehat{\mathbf{q}} = (1, 1, 0, ...)$, flag $i = 1$ with probability $\frac{\rho_2}{2}$, $i = 2$ with probability $\frac{\rho_2}{2}$ and $i = 3$ with probability $1 - \rho_2$, for some $\rho_2 \in (0, 1)$;

- if $\widehat{\mathbf{q}} = (1, 0, 0, ...)$ or $(0, 1, 0, ...)$, flag the specification in $\{1, 2\}$ that is such that $\widehat{\mathbf{q}}_i = 1$ with probability ρ_1 and $i = 3$ with probability $1 - \rho_1$;

- if $\widehat{\mathbf{q}} = (1, 1, 1, ...)$, flag specifications 1 and 2 with equal probability 1/2;

- if there is only one suspect specification, flag it with probability 1;

- for any other $\widehat{\mathbf{q}}$, flag specifications with equal probability 1/3.

As an illustration of Theorem **??** and **??**, whenever $L > 1$ there is no corruption in equilibrium, because here $s_d^{max} = 1$ (and $s_u^{max} = 2$) while whenever $L < 1$, there is corruption in any equilibrium. The critical value $L^* = 1$ for corruption deterrence is tight: it is smallest penalty that ensures a corruption-free environment. This threshold is much lower than in the case of accountability based on random challenge, in particular when the dimensionality of projects is high.

We now go a bit further than the general analysis of the previous sections by showing first that when $L < 1$, corruption exhibits an interesting pattern.

Claim 2 *When $L < 1$, the following constitutes an equilibrium of the alert-based accountability game.*

- *Firm α engages in corruption as follows:*

  - *Induce $\widehat{\mathbf{q}} = (1, 1, 0, ...)$ with probability 1 if $\bar{\mathbf{q}} = (1, 1, 1, ...)$ by forcing the choice of a standardized specification $\hat{q}_3 = 0$ for $i = 3$;*

  - *Induce $\widehat{\mathbf{q}} = (1, 1, 0, ...)$ with probability 1 if $\bar{\mathbf{q}} = (1, 0, 1, ...)$ (or $(0, 1, 1, ...)$) by forcing the choice of a standardized specification $\hat{q}_3 = 0$ on $i = 3$ and of a sophisticated specification $\hat{q}_i = 1$ for $i = 2$ (or $i = 1$);*

  - *Induce $\widehat{\mathbf{q}} = (1, 1, 0, ...)$ with probability $k = \frac{\varepsilon^2}{(1-\varepsilon)^2}$ (and refrain from corruption with the complement probability) if $\bar{\mathbf{q}} = (1, 0, 0, ...)$ (or $(0, 1, 0, ...)$) by randomizing the choice of a sophisticated specification $\hat{q}_i = 1$ for $i = 2$ (or $i = 1$);*

– *Induce* $\widehat{\mathbf{q}} = (1, 1, 0, ...)$ *with probability* $k = \frac{\varepsilon^2}{(1-\varepsilon)^2}$ *(and refrain from corruption with the complement probability) if* $\bar{\mathbf{q}} = (0, 0, 0, ...)$ *by randomizing the choice of joint sophisticated specifications* $\hat{q}_1 = \hat{q}_2 = 1$*;*

– *Refrain from corruption in other states of nature.*

- *Firm* β *follows an alert strategy with* $\rho_1 = \frac{1}{1+L}$ *and* $\rho_2 = \frac{2}{2+L}$ *(beliefs are specified in the proof below).*

Proof. The exhaustive analysis of all cases for firm α's strategy is tedious and uninformative. So, we just provide the analysis of one case. As an example, consider state $\bar{\mathbf{q}}$ =$(0, 0, 0, ...)$. Firm α can choose maximal corruption to induce $\widehat{\mathbf{q}} = (1, 1, 0, ...)$, which yields an expected profit equal to $2(1 - \rho_2) - \rho_2 L = 0$ given firm β's candidate strategy It can choose to refrain from corruption which yields an expected profit equal to 0. There is another possible choice that corresponds to (partial) corruption: it consists in inducing $\widehat{\mathbf{q}} = (1, 0, 0, ...)$ (or equivalently in inducing $\widehat{\mathbf{q}} = (0, 1, 0, ...)$), which yields an expected profit equal to $1(1 - \rho_1) - \rho_1 L = 0$. Therefore, firm α's strategy is sequentially rational at $\bar{\mathbf{q}}$ =$(0, 0, 0, ...)$. The analysis of all other cases follows similar steps.

Let us now analyze firm β's flagging strategy. Observing $\widehat{\mathbf{q}} = (1, 1, 0, ...)$ and given firm α's strategy, firm β conjectures that specification $i = 1$ (or specification $i = 2$) has been distorted with probability equal to:

$$\Pr\{\bar{\mathbf{q}} = (0, 1, 0, ...)\}.k + \Pr\{\bar{\mathbf{q}} = (0, 0, 0, ...)\}.k + \Pr\{\bar{\mathbf{q}} = (0, 1, 1, ...)\}.1,$$

and that specification $i = 3$ has been distorted with probability equal to:

$$\Pr\{\bar{\mathbf{q}} = (1, 1, 1, ...)\}.1 + \Pr\{\bar{\mathbf{q}} = (0, 1, 1, ...)\}.1 + \Pr\{\bar{\mathbf{q}} = (1, 0, 1, ...)\}.1.$$

Given the independence of the realization of specifications in the socially preferable project, the value of k in the Claim makes these probabilities equal. This means that all three specifications $\{1, 2, 3\}$ are equally suspect for $\widehat{\mathbf{q}} = (1, 1, 0, ...)$ and that firm β's flagging strategy is sequentially rational at $\widehat{\mathbf{q}} = (1, 1, 0, ...)$. As no other profile $\widehat{\mathbf{q}}$ can be the outcome of corruption given firm α's candidate strategy, firm β is willing to randomize in any way after any other observation. This completes the characterization of the equilibrium in this example. ∎

Compared to the random challenge procedure, the alert-based accountability procedure does not only lower the critical penalty that ensures a corruption-free environment. It also changes the nature of corruption whenever it occurs, i.e. when L is small. Under alert-based accountability, when L is slightly below the critical level, corruption may take place in various states of nature, some of them such that firm α already enjoys a strong advantage compared to firm β. Moreover, in such states of nature, corruption may take place with a probability strictly smaller than 1. In equilibrium the extent of favoritism reflects the asymmetry in the occurence of $\overline{q}_i = 1$ and $\overline{q}_i = 0$. The smaller ε the smaller the extent of equilibrium corruption in those states.

Suppose that the projects are of high dimensionality, n is large and assume that $L < 1$. Under accountability based on random challenge, the proof of the first claim shows that in all states of nature, corruption occurs with probability 1 resulting in a project $\widehat{\mathbf{q}} = (1, 1, 0, ...)$, so that this project will always be observed ex post. Under alert-based accountability, the proposed equilibrium shows that corruption does not occur with probability 1 for all states of nature in equilibrium. In particular, in states of nature such that $\bar{q}_3 = 0$ that require firm α to upgrade specification $i = 1$ or $i = 2$ from $\bar{q}_i = 0$ to $\hat{q}_i = 1$, or both. The reason is that observing some $\hat{q}_i = 1$, for $i = 1$ or 2, raises suspicion much more strongly than observing $\hat{q}_3 = 0$ for firm β as sophisticated specifications are less likely than standard ones in the socially preferable project. Therefore, such $\hat{q}_i = 1$ is flagged more often than $\hat{q}_3 = 0$, to the point that firm α is indifferent between inducing these upgrades or not. Of course, by the same token, firm α always downgrades specification $i = 3$ with probability 1 whenever this is relevant. This remarkable finding shows that under alert-based accountability, downgrades are more likely to occur than upgrades. This reflects the general property of social preferences: standardized specifications are more likely than sophisticated ones in the socially preferable projects.

We thus find that with ABA the much advocated anti-corruption policy that emphasizes standardization $\varepsilon < 1/2$ receives new power: i. corruption occurs with a probability less than 1 and yields no net gain in expectation, ii. corruption tends to further favor standardization.

The previous equilibrium is by no means unique. Indeed, the game played under alert-based accountability is complex and multiple equilibria exist in general. To illustrate, it is possible to prove that there are other equilibria with the same alert strategy as above for firm β and a similar corruption strategy for firm α except that:[16]

- in states of nature $\bar{\mathbf{q}} = (1, 0, 0, ...)$ and $(0, 1, 0, ...)$, it induces a change from $\bar{q}_i = 0$ for $i = 1$ or 2 to $\hat{q}_i = 1$ with probability r;
- and in state of nature $\bar{\mathbf{q}} = (0, 0, 0, ...)$, it induces a change from $\bar{q}_i = 0$ to $\hat{q}_i = 1$ for both specifications $i = 1$ and $i = 2$ with probability s such that

$$kr + s = k^2(k + 1).$$

In such an equilibrium, it is still the case that only profile $\hat{\mathbf{q}} = (1, 1, 0, ...)$ can be the outcome of corruption. But, contrary to what Theorem **??** or the above equilibria might suggest, it is not always the case that corruption, when it occurrs, leads to maximal profit for α, i.e. to $\widehat{\mathbf{q}} = (1, 1, 0, ...)$. There also exist *partial corruption* equilibria, where corruption does not lead to maximal profit, as appears in the following claim.

Claim 3 *The alert strategy for firm β characterized in the previous claim and the following class of corruption strategegs for firm α constitute equilibria under alert-based accountability:*

- *Induce $\widehat{\mathbf{q}} = (1, 1, 0, ...)$ with probability 1 if $\bar{\mathbf{q}} = (1, 1, 1, ...)$ by forcing the choice of a standardized specification $\hat{q}_3 = 0$ for $i = 3$;*
- *Induce $\widehat{\mathbf{q}} = (1, 0, 0, ...)$ with probability $v \in [0, 1]$ and $\widehat{\mathbf{q}} = (1, 1, 0, ...)$ with probability $1 - v$ if $\bar{\mathbf{q}} = (1, 0, 1, ...)$ by forcing the choice of a standardized specification $\hat{q}_3 = 0$ for $i = 3$ and randomizing the choice of a sophisticated specification $\hat{q}_2 = 0$ for $i = 2$; similarly for $\bar{\mathbf{q}} = (0, 1, 1, ...)$;*
- *Induce $\widehat{\mathbf{q}} = (1, 1, 0, ...)$ with probability $r \in [0, 1]$ if $\bar{\mathbf{q}} = (1, 0, 0, ...)$ or $(0, 1, 0, ...)$ by randomizing the choice of a sophisticated specification $\hat{q}_i = 1$ on $i = 2$ or $i = 1$;*
- *Induce $\widehat{\mathbf{q}} = (1, 1, 0, ...)$ with probability $s \geq 0$, or $\widehat{\mathbf{q}} = (1, 0, 0, ...)$ with probability $t \geq 0$, or $\widehat{\mathbf{q}} = (0, 1, 0, ...)$ with probability t, or refrain from corruption with probability $1 - s - 2t \geq 0$ if $\bar{\mathbf{q}} = (0, 0, 0, ...)$ by randomizing the choice of a sophisticated specification $\hat{q}_i = 1$ on $i = 2$ and/or $i = 1$.*

[16] The proof is a special case of the next claim below, hence omitted

Proof. It is a simple matter a computation to show that the following conditions, on top of the conditions ensuring that probabilities are within $[0,1]$, characterizes equilibria of the game under alert-based accountability:

$$\begin{aligned} t &= k^2 v \\ s + kr &= k^2[k + (1-v)] \end{aligned}$$

where the first condition guarantees that $i = 1$ and $i = 3$ raise equal suspicion at $\widehat{\mathbf{q}} = (1, 0, 0, ...)$ (with a similar condition at $\widehat{\mathbf{q}} = (0, 1, 0, ...)$) and the second condition guarantees that all three specifications raise equal suspicion at $\widehat{\mathbf{q}} = (1, 1, 0, ...)$.[17] ■

In this more general class of equilibria, corruption does not necessarily mean maximal corruption, i.e. corruption so as to induce $\widehat{\mathbf{q}} = (1, 1, 0, ...)$. Corruption may also develop so as to induce $\widehat{\mathbf{q}} = (1, 0, 0, ...)$ or $\widehat{\mathbf{q}} = (0, 1, 0, ...)$ with positive probability, i.e. as partial corruption. This is another way by which alert-based accountability helps curb corruption problems: it may reduce the scope of corruption.

6 Concluding remarks

The model we have investigated has many specific features. Some of them are modeling choices made to ensure tractability, e.g. the binary structure of specifications (standard/sophisticated), the additive cost functions, the definition of types, the first-price auction format; these, we think, are not critical. Others correspond to central and relevant features of the situation we are interested in, as already discussed in the Introduction:

- citizens cannot easily design complex projects or mechanisms ex ante, they are unable to assess the relevance of the specifications of a project and they have limited resources in checking them out ex post;
- public officers have some technical knowledge and expertise about the community's needs. They work under administrative rules with limited monetary incentives so that they are not incentivized to use their knowledge and expertise in the socially preferable manner;

[17] The equilibrium that we discussed extensively above is such that $s = r = k^2$ and $t = v = 0$. The class of equilibria with similar properties corresponds to $t = v = 0$ and $kr + s = k^2(k+1)$.

- through their use of the technology and their market interactions, firms know each others much better than public authorities do.

The most interesting discussion therefore bears on assumptions that are both restrictive and debatable. Arguably, our model of the corruption stage is quite special. First, competition may unravel to the corruption stage and there could be some competitive corruption game among firms to become the bribing firm. Although the point is valid, we think it is appropriate to work out the case of monopoly in corruption as a first step. Second, it may be more relevant to suppose that negotiations between the public officer and the bribing firm take place under bilateral asymmetric information, the firm not knowing the socially preferable set of specifications and the public officer not knowing the firm's technological advantages. This information structure plus the fact that these negotiations must be secret and that monetary transfers may be largely hindered should lead to an inefficient outcome for the two parties. Our choice of a game that leads to an efficient agreement for the colluding parties aims at giving the best chances to favoritism, thereby providing a "conservative" picture of what can be achieved in terms of corruption deterrence by the alert-based accountability procedure. Third, the challenge technology and the costs attached to it ultimately imply that the problem boils down to the choice of one characteristics to verify and there are no *false positive*. The possibility of verifying a larger, but fixed-size sample of characteristics is not critical and the model could be extended to an imperfect detection technology.

Given these reservations, our results strongly underline the value of engaging competing firms in accountability in the context of a complex procurement auction. Because the competitor can easily be incentivized and has better information than the citizens, he can more efficiently target the request for justification than the citizen. As a consequence full corruption deterrence becomes feasible at a level of penalty which is much lower and unrelated to the complexity of the project. Instead it depends on the industry profile. When firms are close competitors so they differ over a small number of dimensions full deterrence is achieved at minimal cost. And if the competitor has no comparative advantage, corruption is deterred by the mere prospect of losing the auction if detected, no penalty is needed.

The analysis also shows that when corruption does occur alert based accountability induces a pattern quite different from that induced by accountability based on random challenge. First while the risk of corruption is most severe when the allocation of the project is at stake with RCA, with ABA there is a risk even when the firm already has a solid advantage but corruption often occurs with a probability less a than one and yields no expected gain. Moreover favorism tends to favor standardization and does not always lead to maximal profit for the winning firm. The specific patterns of corruption under ABA are closely related to the asymmetry between standard and sophisticated specifications. Therefore we find that ABA increases the efficiency of the much advocated anti-corruption policy that calls for standardization in technical specification.

Finally, from a practical point of view, a critical argument in favor of alert-based accountability is that it does not rely on rather unrealistic assumptions about the citizen's ability to access and process information and her ability to commit to a sophisticated and potentially difficult-to-enforce mechanism (cf. remark 1). Moreover alerts and verification take place even before firms prepare their offers on the first announced project specification. Therefore alert based accountability does not imply additional cost for honest firms and minimally disturbs the procurement process.

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Appendix

Proof of Proposition ??

If $\bar{\mathbf{q}}$ is distorted into $\widehat{\mathbf{q}}$, there exists $S = S(\widehat{\mathbf{q}})\backslash S(\bar{\mathbf{q}})$, S non empty with cardinal $s > 0$, that corresponds to the set of all specifications that are distorted. The expected value of distorting $\bar{\mathbf{q}}$ into $\widehat{\mathbf{q}}$ is thus equal to: $\pi(\widehat{\mathbf{q}})\left[1 - \frac{s}{n}\right] - \frac{s}{n}L$. Using the fact that $\pi(\widehat{\mathbf{q}}) = \pi(\bar{\mathbf{q}}) + (s_u(\widehat{\mathbf{q}}) - s_u(\bar{\mathbf{q}}))c$, this expected value can be written as:

$$\pi(\bar{\mathbf{q}}) + s.c - \frac{s}{n}\left[\pi(\bar{\mathbf{q}}) + s.c + L\right]. \tag{2}$$

Consider first $\bar{\mathbf{q}}$ such that $\pi(\bar{\mathbf{q}}) \geq 0$. If for any $S \subseteq S(\theta^{\alpha})\backslash S(\bar{\mathbf{q}})$ non-empty with cardinal $s > 0$, this expression is strictly smaller than $\pi(\bar{\mathbf{q}})$, that is if $\pi(\bar{\mathbf{q}}) + L > (n-s)c$, then $\bar{\mathbf{q}}$ is cannot be distorted in equilibrium into $\widehat{\mathbf{q}}$.

So, if $\pi(\bar{\mathbf{q}}) + L > (n-1)c$, there exists no set S of potential distortions that could be implemented through corruption in equilibrium and $\bar{\mathbf{q}}$ cannot be distorted at all.

Finally, if $L > (n-1)c$, then for any $\bar{\mathbf{q}}$ such that $\pi(\bar{\mathbf{q}}) \geq 0$, it follows that $\pi(\bar{\mathbf{q}}) + L > (n-1)c$. One concludes that if $L > (n-1)c$, there cannot be any corruption starting from any profile $\bar{\mathbf{q}}$ such that $\pi(\bar{\mathbf{q}}) \geq 0$.

Assuming $L > (n-1)c$, suppose that there exists $\bar{\mathbf{q}}$ with $\pi(\bar{\mathbf{q}}) < 0$ that is distorted into some $\widehat{\mathbf{q}}$ with positive probability in equilibrium, i.e. suppose that:

$$\pi(\widehat{\mathbf{q}}) - \frac{(s(\widehat{\mathbf{q}}) - s(\bar{\mathbf{q}}))}{s(\widehat{\mathbf{q}})}[\pi(\widehat{\mathbf{q}}) + L] \geq 0.$$

Then, there exists $\bar{\mathbf{q}}'$ such that $S(\bar{\mathbf{q}}) \subset S(\bar{\mathbf{q}}') \subset S(\widehat{\mathbf{q}})$ all strictly, i.e. $s(\bar{\mathbf{q}}) < s(\bar{\mathbf{q}}') < s(\widehat{\mathbf{q}})$, and $\pi(\bar{\mathbf{q}}') = 0$. So,

$$\pi(\widehat{\mathbf{q}}) - \frac{(s(\widehat{\mathbf{q}}) - s(\bar{\mathbf{q}}'))}{s(\widehat{\mathbf{q}})}[\pi(\widehat{\mathbf{q}}) + L] > \pi(\widehat{\mathbf{q}}) - \frac{(s(\widehat{\mathbf{q}}) - s(\bar{\mathbf{q}}))}{s(\widehat{\mathbf{q}})}[\pi(\widehat{\mathbf{q}}) + L] \geq 0. \tag{3}$$

This means that corruption takes place starting from $\bar{\mathbf{q}}'$, which contradicts the first part of the proof above. To sum up, if $L > (n-1)c$ there is no corruption at all in equilibrium of the random challenge procedure.

Suppose now that $L < (n-1)c$ and consider a socially preferable project $\bar{\mathbf{q}}$ such that $\pi(\bar{\mathbf{q}}) = 0$. Using (??), it comes that at such a profile, firm α is strictly better off making one distortion within $S(\theta^\alpha)\backslash S(\bar{\mathbf{q}})$ than refraining from engaging in corruption. Therefore, there is necessarily corruption in equilibrium at such socially preferable project.

The first inequality in (**??**) leads immediately to the last part of the proposition.

Proof of Theorem ??

Sketch of the proof (to be re-written)

1. Suppose there is corruption from $\bar{\mathbf{q}}$ to $\widehat{\mathbf{q}}$ with $\pi(\bar{\mathbf{q}}) < 0$, i.e.:

$$\pi(\hat{\mathbf{q}}) - \left(\frac{s(\hat{\mathbf{q}}) - s(\bar{\mathbf{q}})}{s(\hat{\mathbf{q}})}\right)(\pi(\hat{\mathbf{q}}) + L) \geq 0.$$

Then there exists $\bar{\mathbf{q}}'$ such that $S(\bar{\mathbf{q}}) \subset S(\bar{\mathbf{q}}') \subset S(\hat{\mathbf{q}})$ all strictly, i.e. $s(\bar{\mathbf{q}}) < s(\bar{\mathbf{q}}') < s(\hat{\mathbf{q}})$, and $\pi(\bar{\mathbf{q}}') = 0$, such that

$$\pi(\hat{\mathbf{q}}) - \left(\frac{s(\hat{\mathbf{q}}) - s(\bar{\mathbf{q}}')}{s(\hat{\mathbf{q}})}\right)(\pi(\hat{\mathbf{q}}) + L) > \pi(\hat{\mathbf{q}}) - \left(\frac{s(\hat{\mathbf{q}}) - s(\bar{\mathbf{q}}')}{s(\hat{\mathbf{q}})}\right)(\pi(\hat{\mathbf{q}}) + L) \geq 0 = \pi(\bar{\mathbf{q}}'),$$

that is, there is corruption at $\bar{\mathbf{q}}'$. So, one can basically restrict attention to $\bar{\mathbf{q}}$ such that $\pi(\bar{\mathbf{q}}) \geq 0$.

2. **Existence of a no-corruption equilibrium**. Consider the following alert strategy for firm β: for any $\hat{\mathbf{q}} \in Q^+$, flag any specification $i \in S(\hat{\mathbf{q}})$ with probability $\frac{1}{s(\hat{\mathbf{q}})}$. We claim that this alert strategy induces firm α never to engage in corruption, whatever the state of nature. Note that if this holds, then it is easy to construct beliefs so that this alert strategy is sequentially rational firm β at any $\hat{\mathbf{q}} \in Q^+$ since there is no corruption on the equilibrium path.

Consider a state of nature $\bar{\mathbf{q}} \in \bar{Q}$ at which firm α earns $\pi(\bar{\mathbf{q}}) \geq 0$ without corruption. Consider the possibility of engaging in corruption so as to induce $\hat{\mathbf{q}} \in Q^+$ with positive probability. The expected profit of this move is equal to:

$$\pi(\hat{\mathbf{q}}) - \left(\frac{s(\hat{\mathbf{q}}) - s(\bar{\mathbf{q}})}{s(\hat{\mathbf{q}})}\right)(\pi(\hat{\mathbf{q}}) + L).$$

Note that $\pi(\hat{\mathbf{q}}) = \pi(\bar{\mathbf{q}}) + (s(\hat{\mathbf{q}}) - s(\bar{\mathbf{q}}))\, c$. So, the expected profit of engaging in corruption can be written:

$$\pi(\bar{\mathbf{q}}) - \left(\frac{s(\hat{\mathbf{q}}) - s(\bar{\mathbf{q}})}{s(\hat{\mathbf{q}})}\right)(\pi(\hat{\mathbf{q}}) + L - s(\hat{\mathbf{q}})c) \tag{4}$$

that is: $\pi(\bar{\mathbf{q}}) - \left(\frac{s(\hat{\mathbf{q}})-s(\bar{\mathbf{q}})}{s(\hat{\mathbf{q}})}\right)(L - s_d^{max}c)$. Then, if $L > s_d^{max}c$, engaging in corruption to $\hat{\mathbf{q}}$ is dominated by no corruption at $\bar{\mathbf{q}}$ when $\pi(\bar{\mathbf{q}}) \geq 0$.

3. **Non-existence of an outcome with corruption**. Suppose there is one. Take $\widehat{\mathbf{q}}$ that is outcome of corruption in this equilibrium and order the ρ_i according to $i = 1, ...s(\widehat{\mathbf{q}})$ at this profile from smallest to highest (with possible ties) and let ρ denote the corresponding column $s(\widehat{\mathbf{q}})-$dimensional vector. There are socially preferable projects $\bar{\mathbf{q}}_{\mathbf{t}}$, for $t = 1, ...T(\widehat{\mathbf{q}})$ for which there is corruption with positive probability in equilibrium leading to $\widehat{\mathbf{q}}$.

For any t, one has:

$$s(\widehat{\mathbf{q}})c - \{\sum_{i \in S(\widehat{\mathbf{q}}) \backslash S(\bar{\mathbf{q}}_{\mathbf{t}})} \rho_i\}(\pi(\widehat{\mathbf{q}}) + L) \geq s(\bar{\mathbf{q}}_{\mathbf{t}})c$$

that is,

$$\sum_{i \in S(\widehat{\mathbf{q}}) \backslash S(\bar{\mathbf{q}}_{\mathbf{t}})} \rho_i \leq \frac{c(s(\widehat{\mathbf{q}}) - s(\bar{\mathbf{q}}_{\mathbf{t}}))}{\pi(\widehat{\mathbf{q}}) + L}. \tag{5}$$

Moreover, for any specification $i \in S(\widehat{\mathbf{q}})$ such that $\rho_i > 0$, there exists some socially preferable project that indexed by t such that $i \in S(\widehat{\mathbf{q}}) \backslash S(\bar{\mathbf{q}}_{\mathbf{t}})$, that is i is one of the distorted specifications in the corruption outcome starting from $\bar{\mathbf{q}}_{\mathbf{t}}$.

Let $\epsilon_{\mathbf{t}}$ denote the row $s(\widehat{\mathbf{q}})-$dimensional vector consisting in 0 or 1 such that $\epsilon_{\mathbf{t}i} = 1$ if and only if $i \in S(\widehat{\mathbf{q}}) \backslash S(\bar{\mathbf{q}}_{\mathbf{t}})$. (**??**) can be written as:

$$\epsilon_{\mathbf{t}}.\rho \leq \frac{c}{\pi(\widehat{\mathbf{q}}) + L}\epsilon_{\mathbf{t}}.\mathbf{1}$$

with $\mathbf{1}$ denoting the $s(\widehat{\mathbf{q}})-$dimensional column vector consisting of 1 at each line.

Notice that (**??**) can be written as an upper bound on an average value of the ρ_i over the set of distoritions:

$$\frac{1}{(s(\widehat{\mathbf{q}}) - s(\bar{\mathbf{q}}_{\mathbf{t}}))} \sum_{i \in S(\widehat{\mathbf{q}}) \backslash S(\bar{\mathbf{q}}_{\mathbf{t}})} \rho_i \leq \frac{c}{\pi(\widehat{\mathbf{q}}) + L}.$$

So, given that the ρ_i are ordered, the above inequality implies that for any $j \in S(\widehat{\mathbf{q}}) \backslash S(\bar{\mathbf{q}}_{\mathbf{t}})$,

$$\frac{1}{\#\{i \leq j, i \in S(\widehat{\mathbf{q}}) \backslash S(\bar{\mathbf{q}}_{\mathbf{t}})\}} \sum_{i \in S(\widehat{\mathbf{q}}) \backslash S(\bar{\mathbf{q}}_{\mathbf{t}}), i \leq j} \rho_i \leq \frac{c}{\pi(\widehat{\mathbf{q}}) + L}$$

also holds. Let $\epsilon_{\mathbf{t}}^{\mathbf{j}}$ for $j \in S(\widehat{\mathbf{q}})\backslash S(\bar{\mathbf{q}}_{\mathbf{t}})$, denote the truncation of $\epsilon_{\mathbf{t}}$ up to the j-th term, and 0 for larger entries. The last inequality writes as:

$$\epsilon_{\mathbf{t}}^{\mathbf{j}}.\rho \leq \frac{c}{\pi(\widehat{\mathbf{q}})+L}\epsilon_{\mathbf{t}}^{\mathbf{j}}.\mathbf{1}.$$

With this construction, starting from the profiles of $\epsilon_{\mathbf{t}}$ for $t = 1, ...$, it is then possible for any $i \in S(\widehat{\mathbf{q}})$ to construct a row vector $\epsilon^{\mathbf{i}}$ consisting of 1 and 0, such that $\epsilon_{\mathbf{j}}^{\mathbf{i}} = 0$ for $j > i$ and such that

$$\epsilon^{\mathbf{i}}.\rho \leq \frac{c}{\pi(\widehat{\mathbf{q}})+L}\epsilon^{\mathbf{i}}.\mathbf{1}.$$

or, using the matrix $E = (e_{ij})_{ij} = (\epsilon_{\mathbf{j}}^{\mathbf{i}})_{ij}$,

$$E.\rho \leq \frac{c}{\pi(\widehat{\mathbf{q}})+L}E.\mathbf{1}.$$

The matrix E is low-triangular with 1 on its diagonal; hence it is invertible and its determinant is equal to $s(\widehat{\mathbf{q}})$. Let us define the following row vector $m = s(\widehat{\mathbf{q}}).\mathbf{1}^t.E^{-1}$. Note first that all its entries are natural numbers (or 0). Then, left-multiplying both sides (**??**) by m, one gets:

$$m.E.\rho = s(\widehat{\mathbf{q}}).\mathbf{1}^t.\rho = s(\widehat{\mathbf{q}}) \leq \frac{c}{\pi(\widehat{\mathbf{q}})+L}m.E.\mathbf{1} = \frac{c}{\pi(\widehat{\mathbf{q}})+L}s(\widehat{\mathbf{q}})\mathbf{1}^t.\mathbf{1} = \frac{c}{\pi(\widehat{\mathbf{q}})+L}s(\widehat{\mathbf{q}})^2$$

From this, it follows that $L \leq s_d^{max}c$, a contradiction.

Proof of Theorem 2

Consider $\widehat{\mathbf{q}} \in \hat{Q}$ such that $\pi(\widehat{\mathbf{q}}) = \pi(\theta^\alpha)$, so that $\pi(\widehat{\mathbf{q}}) = s_u(\widehat{\mathbf{q}})c = s_u^{max}c$ and $s_d(\widehat{\mathbf{q}}) = s_d^{max}$.

For any $i \in S(\widehat{\mathbf{q}})$, consider the socially preferable project $\bar{\mathbf{q}}^i$ that coincides with the final project $\widehat{\mathbf{q}}$ except for specification i for which it is less favorable to firm α, i.e. such that $\bar{\mathbf{q}}_j^i = \widehat{\mathbf{q}}_j$ for any $j \neq i$ and $\bar{\mathbf{q}}_i^i \neq \widehat{\mathbf{q}}_i$. At $\bar{\mathbf{q}}^i$, firm α has only the choice between not engaging in corruption or distorting specification i.

For $\widehat{\mathbf{q}}$ not to be the outcome of any distortion due to corruption in equilibrium, it must be that for any $i \in S(\widehat{\mathbf{q}})$, $p_i(\bar{\mathbf{q}}^i) = 0$. $p_i(\bar{\mathbf{q}}^i) = 0$ implies that firm α does not engage at all in corruption at $\bar{\mathbf{q}}^i$ and therefore that $\rho_i(\widehat{\mathbf{q}}) \geq \frac{c}{s_u^{max}c+L}$. Summing up over $i \in S(\widehat{\mathbf{q}})$, it comes:

$$\frac{\left[s_u^{max} + s_d^{max}\right] c}{s_u^{max} c + L} \leq 1,$$

which is not compatible with $L < s_d^{max} c$. So, in any equilibrium, there must be corruption leading to any such project $\hat{\mathbf{q}}$.