

EXTENSIVE MARGIN EFFECTS OF TAX EVASION WITH MOBILITY BETWEEN THE LEGAL AND HIDDEN SECTORS

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Abstract

This paper incorporates mobility between the legal and black economies into a model of tax evasion with endogenous labor supply, in which underreporting is possible in one sector but is impossible in the other. After applying this framework to various tax evasion scenarios, we have found that the results of the effects along the extensive margin (number of evaders) become more robust and conclusive than those along the intensive margin usually considered by the literature. In particular, it is shown that the following facts reduce the number of evaders: a) Larger and more progressive evasion penalties; b) Higher detection probabilities; c) An increase in the legal sector wage rate; d) A decrease in the moonlighting wage rate; e) Higher costs for creating opportunities to evade; f) Lower opportunities to evade, and g) Greater psychological costs of tax evasion.

When hours of illegal work are also taken into account, policies c), d) and g) continue being valid to reduce total tax evasion, provided that then the sign of the effects along the extensive margin coincides with that of the effects along the intensive margin. The same holds for policies a) and b) in connection with low- and middle-income groups and for policies e) and f) in connection with high-income groups, but not vice-versa given that, in that last case the sign of such effects are contradictory.

Keywords: income taxation; tax evasion; extensive margin responses

JEL classification: C6-D6-D8-H2

1. Introduction

The phenomenon of tax evasion is and always has been common throughout the world, both in developed and underdeveloped countries. However, it was not until the 1970s that the topic emerged as an important area in the theory of taxation, with most theoretical work based on expected utility theory and conducted within a partial equilibrium framework. In the early models of Allingham and Sandmo (1972), Yitzhaki (1974), and others, taxable income is assumed to be exogenously given, and tax reporting behavior is presented as a case of portfolio allocation under uncertainty where individuals can decide whether to report their earnings or not. If they choose not to report them, they face the probability of detection, and if detected, they pay a penalty.¹

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¹ Excellent overviews are found in Slemrod and Yitzhaki (2002) and Sandmo (2012).

Another line of research extended the portfolio model to include labor supply responses and thereby allow for endogenous taxable income. Within this approach (e.g., Andersen, 1977, Baldry, 1979; Pencavel, 1979; Cowell, 1981, 1985), Sandmo (1981) provides an interesting framework on which to build our understanding of the phenomenon of tax evasion. In it, there are two classes of taxpayers, evaders and non-evaders, working in two separate sectors, the regular labor market and the irregular or hidden labor market. Although the evaders may or may not evade taxes, depending on the values of the relevant tax parameters (i.e., the frequency of audits, the penalty rate, and the tax rate), the non-evaders have no opportunity to conceal their taxable earnings. In this way, the income and substitution effects, together with the proportion of evaders and non-evaders, determine the final impact of tax evasion. Sandmo (1981) finds that the optimal audit and penalty rates are complex functions dependent not only on the risk aversion of individuals but also on their labor supply decisions. However, given that his purpose was to incorporate tax evasion into the optimal income tax setting, Sandmo (1981) assumed only two skill levels for the sake of simplicity, one for non-evaders and one for evaders, therefore precluding mobility responses between the two labor markets.

In the present paper, Sandmo's (1981) framework is generalized to a continuum of earning abilities and mobility between the regular and hidden sectors. More specifically, individuals differ in terms of their earnings ability and may have distinct hourly wages whether they work in the informal sector (where the productivity of labor is lower) or in the formal sector. Moreover, thanks to the use of two participation conditions, individuals have the choice of working in either of the two sectors; consequently, the size of each group of taxpayers, evaders and non-evaders, is endogenously determined. In this context, what the participation conditions express is that an individual will leave one of the labor markets if his utility is less than what would be obtained in the other labor market in terms of consumption, leisure, and, possibly, expected tax evasion benefits. It follows that in our setting, individuals must face two simultaneous decisions, whether to work or not and what sector to work in, the regular sector or the hidden sector; the result is therefore a combination of a tax evasion model and an occupational choice model.

Apart from this, we have incorporated into the analysis the economic and moral costs of tax evasion. Among the economic costs are the expenses arising for the creation of evasion opportunities. Among the moral or psychological costs are some non-pecuniary factors, such as the bad conscience deriving from the act of underreporting and the social stigma attached to the discovery of tax fraud (see Allingham and Sandmo, 1972 and Gordon, 1989). The implication of this is that taxpayers may differ not only in terms of their earnings abilities and wage rates (which would potentially be different in the two labor markets according to the different productivities of labor in each sector) but also in terms of their economic costs of evasion, their opportunities to evade taxes, and their moral constraints on doing something illegal.

One of the advantages of our approach is a greater consideration of the role played by the extensive margin of tax evasion than in previous studies on the subject. According to Sandmo (2012), the amount of tax evasion undertaken can be related either to the extent of non-compliance by the individual evader (the intensive margin) or to the number of taxpayers who engage in evasion (the extensive margin). In the literature, most attention has been devoted to the intensive margin dimension, but the decision to

work in the hidden economy must not be presupposed but predicted as a consequence of the model. However, while the traditional view of the extensive margin has been based on the existence of an interior solution in the informal (or self-employed) sector, our emphasis in the present paper is mostly concentrated on the extensive margin effects that result from mobility between the formal and informal sectors. Note in this sense that, besides the direct effects on tax evasion produced by public policies, there are also the indirect effects on tax evasion through occupational changes to jobs with fewer or even zero opportunities to evade. This kind of extensive margin effects is more important than the standard one based on the existence of an interior solution since it decides the size of the hidden economy.

As a second advantage, the model explains up to five different scenarios that shape the diversity of the phenomenon of tax evasion. Thus, while in some of these scenarios moonlighting becomes impossible, in others individuals have more than one source of income, so that they may declare one of these and fail to declare the other. On the other hand, while in some cases the number of hours worked is fixed, in others it is flexible. More specifically, in the first scenario it is assumed that there is a legal labor market coexisting with a hidden labor market in which employees have a flexible labor supply and report their income through a self-declaration scheme according to their perceptions of risk. The second scenario differs from the first in that, as happens in the Allingham-Sandmo model, labor supplies in both markets are exogenous to the problem. The third scenario considers two sectors, one where there is self-employment carried out on a flexible schedule and another where the audit rate equals one because employees' income tax is collected through a pay-as-you-earn (PAYE) scheme operated by the employer. In the fourth scenario, individuals decide between working in a single job on a fixed schedule and working in a job that is compatible with other off-the-books activities, the earnings from which are difficult to tax. The fifth scenario is identical to the fourth, except for the fact that all regular jobs have a fixed work schedule.

As a result of the advantages mentioned, our approach allows a more realistic explanation of why some people engage in tax evasion and some do not. It is well known in this respect that one of the shortcomings of the standard model is that either the whole taxpayer population engages in some tax evasion or everyone reports their true income.² By contrast, it is observed that, except for some situations where non-pecuniary aspects play a critical role in a taxpayer's decision, evading or not evading is just a question of having or not having the opportunity of doing it without being caught. There is also the argument against the standard model that it predicts too much tax evasion in light of the empirical evidence (see for instance Slemrod and Yitzhaki, 2002, p. 1421). To counter these critics, we have seen that, in the scenarios mentioned, while some taxpayers have a 100% probability of being detected, those in the hidden sector are allowed to report part of their true earnings, according to the different opportunities at their disposal. On the same lines, in order to bring our results closer to the observed gap between the regular tax rate and the expected penalty rate when taxpayers are confronted with a favorable gamble, we have incorporated the economic and moral costs of tax evasion into our model. In this way, the apparent inconsistency between theory and observation that results in the standard model has been removed because the

² This is in part a consequence of the fact that most of the tax evasion literature implicitly assumes that the tax system involves self-declaration. However, according to Andreoni et al. (1998), only about 30% of taxpayers evade taxes.

characterization of an interior solution in the hidden sector includes both the expected cost of the evader and the expected disutility attached to being caught evading tax.

Our main findings are summarized in Tables 1 and 3 of section 8. Table 1 relates to the effects from policy changes along the extensive margin while Table 4 depicts the effects of such changes on total tax evasion, therefore including both the extensive and intensive margins. We shall see that, given the unambiguous results that arise concerning the effects on the extensive margin, those on total tax evasion are easy to obtain by just searching for the conditions that ensure the intensive margin effects to present an identical sign. In this way, we conclude that the study of tax evasion makes no sense without considering, as done by the literature, the extensive margin effects for the mobility of taxpayers between the official and the unofficial sectors, provided that they may have an opposite sign to those along the intensive margin.

The remainder of the paper is organized as follows. In [Section 2](#), we set up the model for both the regular and irregular sectors of the economy. [Section 3](#) analyzes the connection between mobility conditions in the two labor markets and the extensive margin of tax evasion. The effects of changes in the different parameters of the model are discussed in [Sections 4, 5, and 6](#). In [Section 7](#), we examine other possible scenarios of tax evasion with restrictions in hours worked and differences in wage rates. The effects on total tax evasion from policy changes are studied in [section 8](#). [Section 9](#) concludes.

2. The model

2.1 Preliminaries

Our model builds on the classic work of Sandmo (1981). The population is represented by a single parameter, σ , which describes ability in efficiency hours. The parameter σ is distributed on $[\underline{\sigma}, \bar{\sigma}] \subseteq \mathbf{R}^+$ according to a density function $f(\sigma) \equiv F'(\sigma) > 0$. While $F(\sigma)$ is considered common knowledge, σ is taken to be private knowledge.

There are two labor markets or sectors: a regular or formal one where there is no evasion and an irregular or informal one where evasion is possible. Individuals working in the regular sector are called “non-evaders” and will be identified by “ n ”; those working in the irregular sector are called “evaders” and will be identified by “ e ”.

In Sandmo (1981), while evaders are assumed to be expected utility maximizers, the non-evaders are not. This makes sense in his model since no mobility is allowed between both labor markets. In what follows, however, instead of presupposing that the distinction between non-evaders and evaders responds to considerations of choice, or to ethical values, we shall assume that all individuals are potential evaders but that they behave as non-evaders when the probability of inspection is 100%. The psychological and moral costs of tax evasion, on the other hand, will be considered in [section 6](#) below.

If $i = n, e$ and $\tilde{w}^i > 0$ denotes the wage per efficiency hour in each labor market, the hourly wage of a person who produces σ efficiency units per hour worked comes to be given by $w^i = \tilde{w}^i \cdot \sigma$. It seems natural to expect that the degree of technology will be lower in the irregular sector, so the value of the marginal productivity of labor in the hidden sector will also be lower and therefore $\tilde{w}^n \geq \tilde{w}^e$ or equivalently $w^n \geq w^e$. The

justification is that production in the hidden sector becomes usually less capital-intensive than in the legal sector. Note that this implies that tax evasion introduces a loss of efficiency, as it encourages more skilled workers to migrate to less efficient jobs.

Let L^i stand for reported labor in each labor market. Reported earnings to the tax authority $w^i L^i$ are subject to a non-linear tax schedule:

$$T(w^i L^i) = -a + t \cdot (w^i L^i)^\alpha, \alpha \geq 0 \quad (1)$$

This specification is found in Pencavel (1979). The parameter $a \geq 0$ is a lump sum grant while t and α are the components of the marginal tax rate $\alpha t \cdot (w^i L^i)^{\alpha-1}$. Clearly $T(w^i L^i)$ is progressive, linear, or regressive according to whether $\alpha > 1$, $\alpha = 1$, or $\alpha < 1$.

Let E^i stand for unreported labor in each labor market. Unreported earnings $w^i E^i$ are subject to a non-linear penalty schedule:

$$M(w^i E^i) = \theta \cdot (w^i E^i)^\beta, \beta \geq 0 \quad (2)$$

where θ and β are the parameters that penalize the taxpayer for his under-reporting. Note that β is responsible for the progressivity, linearity, or regressivity of $M(w^i E^i)$ according to whether $\beta > 1$, $\beta = 1$, or $\beta < 1$, respectively.

All individuals, evaders and non-evaders, share the same preference ordering, which is captured by the von Neumann–Morgenstern utility function:

$$EU^i = (1-p)U(C_1^i, L^i + E^i) + pU(C_2^i, L^i + E^i), \quad (i = n, e) \quad (3)$$

The sub-utility function $U(C^i, L^i + E^i)$ is assumed to be twice continuously differentiable and strictly quasi-concave, increasing in consumption C^i and decreasing in hours worked $L^i + E^i$. The parameter p on the other hand denotes the probability of detection of tax evasion, understood as a taxpayer's subjective perception of the frequency of audits.

Concerning the functions C_1^i and C_2^i , they reflect the alternative budget constraints when evasion is either undetected or detected, that is,

$$C_1^i = w^i L^i - t \cdot (w^i L^i)^\alpha + a + w^i E^i \quad (4)$$

$$C_2^i = w^i L^i - t \cdot (w^i L^i)^\alpha + a + w^i E^i - \theta \cdot (w^i E^i)^\beta. \quad (5)$$

2.2 The regular labor market ($i = n$)

In this market, we have $p = 1$ because total earnings are subject to withholding by the employers or equivalently because there is a PAYE scheme such as the one operating in

the United Kingdom and elsewhere. Consequently, $E^n = 0$ and the maximization of (3) subject to (4) and (5) is equivalent to the maximization of $U(C^n, L^n)$ subject to $C^n = w^n L^n - t(w^n L^n)^\alpha + a$.

Let $\{C_\sigma^n, L_\sigma^n\}$ be the optimal bundle for non-evaders with ability σ . From the first-order conditions for utility maximization, we derive the indirect utility function:

$$V^n(t, \alpha, a, w^n) = U(C_\sigma^n, L_\sigma^n). \quad (6)$$

2.3 The irregular labor market ($i = e$)

In this market, $1 > p \geq 0$ and so tax evasion is possible and only detectable at a certain cost. Because we saw that $E^n = 0$, one may simplify the notation by writing E for E^e .

Individuals maximize their expected utility (3) subject to constraints (4) and (5). If $\{C_{1,\sigma}^e, C_{2,\sigma}^e; L_\sigma^e, E_\sigma\}$ depicts the solution for an evader, the Lagrangian will be given by:

$$\begin{aligned} Z_\sigma^e = & EU_\sigma^e - \lambda_{\sigma,1}^e [C_{\sigma,1}^e - w^e L_\sigma^e + t(w^e L_\sigma^e)^\alpha - a - w^e E_\sigma] \\ & - \lambda_{\sigma,2}^e [C_{\sigma,2}^e - w^e L_\sigma^e + t(w^e L_\sigma^e)^\alpha - a - w^e E_\sigma + \theta \cdot (w^e E_\sigma)^\beta], \quad (7) \end{aligned}$$

where $\lambda_{\sigma,1}^e$ and $\lambda_{\sigma,2}^e$ are the Lagrange multipliers. From the first-order conditions for the maximization of Z_σ^e , we get the (expected) indirect utility function:

$$V^e(t, \theta, \alpha, \beta, a, p, w^e) \equiv (1-p)U(C_{\sigma,1}^e, L_\sigma^e + E_\sigma) + pU(C_{\sigma,2}^e, L_\sigma^e + E_\sigma). \quad (8)$$

3. The extensive margin of tax evasion

3.1 Extensive margin without mobility between the two sectors

Here, we are interested in characterizing the extensive margin within the informal sector described in sub-section 2.3. This will help capture how the extensive margin of tax evasion works when there is mobility between the two sectors.

In our model, the case of absence of mobility between the two sectors coincides with Sandmo's (1981) framework, once extended to a continuum of abilities. Because Cowell's (1985) model is equivalent to Sandmo's (1981) framework with the legal sector deleted, the discussion that follows will concern both. Later, we shall take advantage of this fact when we plug Cowell's (1985) analysis into ours.

In both Sandmo (1981) and Cowell (1985), the extensive margin of tax evasion is implied by the condition for an interior solution in the hidden sector (i.e., for $E_\sigma > 0$):

$$\alpha \cdot (w^e L_\sigma^e)^{\alpha-1} > p\beta\theta (w^e E_\sigma)^{\beta-1}. \quad (9)$$

Condition (9) says that for tax evasion to be optimal it is necessary and sufficient that the expected marginal penalty rate is less than the regular marginal tax rate.³ One sees immediately that (9) reduces to the well-known condition $t \geq p \cdot \theta$ for the special case where $\alpha = \beta = 1$. The problem with condition (9) is that it does not enable us to examine how the number of evaders reacts to changes in the different policy parameters.

3.2 Extensive margin with mobility between the two sectors

With mobility, condition (9) alone no longer characterizes the extensive margin of tax evasion, given that it is perfectly compatible with a superior utility level in the legal sector. For this reason, we need some complementary requirements for condition (9).

The utility function (3) is assumed to be part of a more general preference ordering about the utility attainable in each labor market. Such an ordering takes the form:

$$u(\tau, C^n, L^n, C_1^e, C_2^e, L^e, E) = \tau \cdot EU^n + (1 - \tau)EU^e; \quad \tau = \{0, 1\}. \quad (10)$$

Here, τ stands for a binary function that, according to whether $\tau = 1$ or $\tau = 0$, provides the choice to work in the legal sector or in the hidden sector. Since quasi-concavity is not preserved under addition, it will be assumed from now on that $u(\tau, C^n, L^n, C_1^e, C_2^e, L^e, E)$ is a concave function.

An individual taxpayer must solve the following problem.

Setup 1 Find the solution $\{\tau_\sigma, C_\sigma^n, L_\sigma^n, C_{\sigma,1}^e, C_{\sigma,2}^e, L_\sigma^e, E_\sigma\}$ that maximizes $u(\tau, C^n, L^n, C_1^e, C_2^e, L^e, E)$ subject to constraints (4)–(5) and $\tau = \{0, 1\}$.

LEMMA 1 The following conditions are necessary and sufficient for Setup 1:

$$V^n(t, \alpha, a, w^n) - V^e(t, \theta, a, \alpha, \beta, p, w^e) \geq 0, \quad \forall \sigma: \tau_\sigma = 1 \quad (\text{NE})$$

$$V^e(t, \theta, \alpha, \beta, a, p, w^e) - V^n(t, \alpha, a, w^n) \geq 0, \quad \forall \sigma: \tau_\sigma = 0 \quad (\text{EV})$$

Proof: It is sufficient to note that Setup 1 is equivalent to the following program.

Setup 2 Find both the solution for $\{C^n, L^n, C_1^e, C_2^e, L^e, E\}$ and the corner solution for τ that maximize $\tau \cdot EU^n + (1 - \tau)EU^e$ subject to constraints (4)–(5) and $0 \leq \tau \leq 1$.

The reader may check that conditions (NE) and (EV) are both necessary and sufficient provisos for Setup 2, therefore also for Setup 1.

³The derivation of (9) is obtained from the first-order conditions for the maximization of Lagrangian (7), based on the facts that L_σ^e and E_σ are perfect substitutes, so that $U_L(\cdot, L_\sigma^e + E_\sigma) = U_E(\cdot, L_\sigma^e + E_\sigma)$ and $U_C(C_{\sigma,1}^e, L_\sigma^e + E_\sigma) < U_C(C_{\sigma,2}^e, L_\sigma^e + E_\sigma)$.

Conditions (NE) and (EV) describe the optimal share of the shadow sector in GDP. Where $\tau_\sigma = 0$, condition (EV) applies, and the individual is labeled an evader. Therefore, it is through condition (EV), together with (9), that the extensive margin from tax evasion is characterized.

Interestingly enough, an easy way to ensure that condition (9) holds when condition (EV) holds is to assume that $w^n > w^e$, as is shown by the following lemma.⁴

LEMMA 2 *If there is a wage gap in favor of the legal sector, the extensive margin of tax evasion is implied in the choice of the hidden sector by the taxpayer.*

if the wage gap favors the legal sector, any work in the hidden sector implies some evasion

Proof: Just note that $U[w^e L_\sigma^e - t(w^e L_\sigma^e)^\alpha + a, L_\sigma^e] < U[w^n L_\sigma^e - t(w^n L_\sigma^e)^\alpha + a, L_\sigma^e]$. Therefore, evaders choosing $E_\sigma = 0$ could still improve their situation by working L_σ^e hours in the regular sector.

In what follows, we shall see that an important advantage of using conditions (NE) and (EV) is that it enables exploring how the formal and informal sectors are affected by changes in the different parameters of the model. If, in addition, it is assumed that $w^n > w^e$, we shall also know how such changes affect the extensive margin of tax evasion and therefore the number of evaders.

4. Comparative statics: enforcement parameters

Some of the questions one would wish to ask are fairly obvious, for example: ‘Will increasing the penalty rates or the probability of detection reduce evasion activity? Others are more subtle: ‘Will an increase in the progressivity of the regular tax system encourage evasion?’ or: ‘Will evasion activity be affected by movement in relative wage rates?’ In this section, we obtain extensive margin responses to changes in the marginal penalty rate and the probability of detection in both cases when the penalty is imposed on the amount of undeclared earnings and on the amount of evaded taxes.

Call $S_\sigma^n \equiv V^n(t, a, w^n) - V^e(t, \theta, a, p, w^e)$ and $S_\sigma^e \equiv V^e(t, \theta, a, p, w^e) - V^n(t, a, w^n)$ the informational rent of an σ non-evader and an σ evader, respectively, understood as the excess of their indirect utility over their reservation utility. From (6) and (8) we get:

$$\frac{\partial S_\sigma^e}{\partial \theta} = -\lambda_{\sigma,2}^e \cdot (w^e E_\sigma)^{\beta} < 0 \quad (11)$$

⁴Sandmo (2012, p. 13) refers to the situation where taxpayers in the illegal sector opt for the corner solution $E_\sigma = 0$ by saying that, unlike the non-evaders who are honest by necessity (their probability of detection equals one), the evaders, if honest, are honest by choice.

$$\frac{\partial S_{\sigma}^e}{\partial \beta} = -\lambda_{\sigma,2}^e \theta \cdot (w^e E_{\sigma})^{\beta} \log(w^e E_{\sigma}) < 0 \quad \text{if } w^e E_{\sigma} > 1 \quad (12)$$

$$\frac{\partial S_{\sigma}^e}{\partial p} = -U(C_{\sigma,1}^e, L_{\sigma}^e + E_{\sigma}) + U(C_{\sigma,2}^e, L_{\sigma}^e + E_{\sigma}) < 0. \quad (13)$$

for all σ such that $\tau_{\sigma} = 0$. Conditions (11)–(13) allow us to *An increase in θ , β , and p generates a migration flow to the legal sector of those evaders indifferent about working in either of the two sectors.*

These results extend the literature on tax evasion with endogenous labor supply to include the extensive margin responses to variations in deterrence when the economy includes both a legal sector and a hidden sector. It suggests that governments should enforce those activities where the number of taxpayers indifferent about evading or not is significant, as it is here where public policies become more effective in the fight against tax evasion. Most notably, it is not only the magnitude of the audit and penalty scheme, measured by p and θ , but also its degree of progressivity, measured by β , that matters in reducing the dimension of the hidden economy.

We also find that, since both the probability of detection and the penalty system work to deter the growth of the shadow economy, the government may impose the combination of p and θ (and/or β) that is more appealing to taxpayers' degree of risk aversion. It seems reasonable to think in this sense that taxpayers with a high degree of risk aversion will prefer a higher p together with a lower b and θ (and/or β) because, despite the increase in the probability of getting caught, the consequences become less severe.

A question that sometimes arises is whether a large penalty with a small probability of detection would be a more powerful tax evasion deterrent than a high probability of detection with a small penalty. In our case, the question should be reformulated in the sense of whether a large penalty with a small probability of detection is a more powerful deterrent to work in the shadow economy than a high probability of detection with a small penalty. To answer the question, consider the following relationship between the penalty and the probability of detection:

$$p\theta = \kappa \text{ (constant)}. \quad (14)$$

This implies that the expected penalty rate for one dollar remains constant after raising the penalty rate, so the audit rate must be adequately reduced. Differentiating (14):

$$\frac{dp}{d\theta} = -\frac{p}{\theta}. \quad (16)$$

Using equation (16), we obtain, after approximating using a Taylor expansion the term $U(C_{\sigma,1}^e, L_{\sigma}^e + E^e) - U(C_{\sigma,2}^e, L_{\sigma}^e + E_{\sigma})$, also considering the fact that $C_{\sigma,1}^e - C_{\sigma,2}^e = \theta \cdot (w^e E_{\sigma})^{\beta}$:

$$\left. \frac{\partial S_{\sigma}^e}{\partial \theta} \right|_{p\theta = \kappa} = \frac{1}{2} U_{CC}(C_{\sigma,2}^e, L_{\sigma}^e + E_{\sigma}) \theta \cdot (w^e E_{\sigma})^{2\beta} < 0. \quad (17)$$

Clearly, the sign follows from risk aversion. Condition (17) leads to Proposition 2. As a result, we find that, *If the penalty rate is increased but the efforts to detect tax evaders are adjusted so as to keep the expected penalty rate for tax evasion unaltered, there will be a decrease in the number of risk-averse evaders.*

A similar result was previously derived by Christiansen (1980) and Koskela (1983) for the portfolio model of Allingham and Sandmo (1972) and by Andersen (1977) for the extended model with labor supply responses. In their analysis, however, instead of considering the extent of tax evasion in terms of the number of evaders, their focus is on the extent of non-compliance by the individual evader and therefore on the intensive margin of tax evasion. Apart from this, Andersen (1977) proceeded by assuming that the utility function is additively separable in income and hours of work. Proposition 2 not only confirms the result for the extensive margin of tax evasion but does so without imposing any special structure on the utility function.

Now, assume that, as in Yitzhaki (1974), the penalty is a function of the amount of taxes the individual tried to evade, so that instead of function (2) we have:

$$M(w^j E^i) = \theta \cdot [t \cdot (w^j L^i + w^j E^i)^{\alpha} - t \cdot (w^j L^i)^{\alpha}]^{\beta}. \quad (18)$$

The Lagrangian (7) then turns into:

$$\begin{aligned} Z_{\sigma}^e = & EU_{\sigma}^e - \lambda_{\sigma,1}^e [C_{\sigma,1}^e - w^e L_{\sigma}^e + t(w^e L_{\sigma}^e)^{\alpha} - a - w^e E_{\sigma}] \\ & - \lambda_{\sigma,2}^e \{ C_{\sigma,2}^e - w^e (L_{\sigma}^e + E_{\sigma}) + t \cdot (w^e L_{\sigma}^e)^{\alpha} - a + \theta \cdot [t \cdot (w^e L_{\sigma}^e + w^e E_{\sigma})^{\alpha} - t \cdot (w^e L_{\sigma}^e)^{\alpha}]^{\beta} \} \end{aligned} \quad (19)$$

, where $\lambda_{\sigma,1}^e$ and $\lambda_{\sigma,2}^e$ denote the marginal utility of income of a σ -evader in states 1 and 2. In consequence, the equivalent conditions to (11) and (12) and (17) are now:

$$\frac{\partial S_{\sigma}^e}{\partial \theta} = -\lambda_{\sigma,2}^e \cdot [t \cdot (w^e L_{\sigma}^e + w^e E_{\sigma})^{\alpha} - t \cdot (w^e L_{\sigma}^e)^{\alpha}]^{\beta} < 0 \quad (20)$$

$$\begin{aligned} \frac{\partial S_{\sigma}^e}{\partial \beta} = & \\ & - \lambda_{\sigma,2}^e \cdot [t \cdot (w^e L_{\sigma}^e + w^e E_{\sigma})^{\alpha} - t \cdot (w^e L_{\sigma}^e)^{\alpha}]^{\beta} \log [t \cdot (w^e L_{\sigma}^e + w^e E_{\sigma})^{\alpha} - t \cdot (w^e L_{\sigma}^e)^{\alpha}] < 0 \end{aligned} \quad (21)$$

provided that $t \cdot (w^e L_{\sigma}^e + w^e E_{\sigma})^{\alpha} - t \cdot (w^e L_{\sigma}^e)^{\alpha} \geq 1$. Concerning condition (17), the equivalent condition can be now written as:

$$\left. \frac{\partial S_{\sigma}^e}{\partial \theta} \right|_{p\theta = \kappa} = \frac{1}{2} U_{CC}(C_{\sigma,2}^e, L_{\sigma}^e + E_{\sigma}) \theta \cdot [t \cdot (w^e L_{\sigma}^e + w^e E_{\sigma})^{\alpha} - t \cdot (w^e E_{\sigma})^{\alpha}]^{2\beta} < 0. (22)$$

Because inequalities (13) and (14) are not affected by the change in the penalty function, Proposition 3 follows as a direct consequence of conditions (21) and (22). *Propositions 1 and 2 continue to hold as they stand if the penalty is a function of the tax evaded rather than a function of the unreported earnings.*

is based on the theoretical insight that the probability of detection (the frequency of audits) and the penalty for evasion are policy substitutes. If one wishes to achieve a given degree of deterrence, this may be achieved by high probabilities and low penalties or by low probabilities and high penalties. The concern for low costs of tax administration leads one to favour the second alternative. However, such a policy might lead to the horizontal equity argument of unacceptable high penalties for a few for violations committed by many. A counterargument might be that one could then just set penalties so high that nobody would evade taxes. But for penalties to be socially acceptable, they probably must be set so that in the eyes of the general public, they “fit the crime”.

Just to mention the obvious, the robustness of the results in Propositions 1 and 2 is reinforced by the fact that the effectiveness of policy instruments against tax evasion is confirmed under the two penalty systems (2) and (18).

5. Comparative statics: changes in other parameters of the model

5.1 Extensive margin effects of changes in marginal tax rates

The sign of $\partial S_{\sigma}^n / \partial t = -\lambda_{\sigma}^n \cdot (w^n L^n)^{\alpha} + (\lambda_{\sigma,1}^e + \lambda_{\sigma,2}^e) \cdot (w^e L^e)^{\alpha}$ is clearly undefined without imposing structure on the utility function. And the same happens with the signs of $\partial S_{\sigma}^e / \partial t$, $\partial S_{\sigma}^n / \partial \alpha$, $\partial S_{\sigma}^e / \partial \alpha$, $\partial S_{\sigma}^n / \partial a$ and $\partial S_{\sigma}^e / \partial a$. Moreover, the indeterminacy continues being present even if the penalty is a function of the amount of tax evaded, given that $\partial S_{\sigma}^n / \partial t = -\lambda_{\sigma}^n w^n L_{\sigma}^n + (\lambda_{\sigma,1}^e + \lambda_{\sigma,2}^e) w^e L_{\sigma}^e + \lambda_{\sigma,2}^e \theta w^e E_{\sigma}$.

Since the effects on the intensive margin from changes in t , α or a exhibit the same degree of ambiguity (see Baldry, 1979, Pencavel, 1979, and Cowell, 1981), our analysis leads to the conclusion that the parameters governing the tax rate cannot be safely used to modify the taxpayers' reporting decisions. Note that this may be viewed as an argument against the extended opinion that evasion justifies a reduction in marginal tax rates.

5.2 Effects of an increase in the wage gap in favor of the legal sector

It can easily be verified that $\forall \sigma$ such that $\tau_{\sigma} = 0$:

$$\frac{\partial S_{\sigma}^e}{\partial w^n} = -\lambda_{\sigma}^n L_{\sigma}^n [1 - \alpha t (w^n L_{\sigma}^n)^{\alpha-1}] < 0 \quad (23)$$

, where λ_σ^n denotes the marginal utility of income of a non-evader. If we substitute w^n for w^e , the sign of this partial derivative with respect to w^e becomes less clear, as the reader may easily check. *Consequently, any increase in non-evaders' wage rate will tend to reduce the size of the hidden sector. The effect of an equivalent increase in the evaders' wage rate remains undetermined.*

Recall that in our model, an increase in w^i is equivalent to an increase in $\tilde{\omega}^i$. The idea behind the proposition is that an increase in the productivity of labor in the legal sector produced by either a technological improvement or by education would offset the potential advantages of tax evasion in the hidden sector. It is also worth noting that the result is associated with a gain in efficiency, as the increase in the ratio of marginal productivities in each labor market $\tilde{\omega}^n/\tilde{\omega}^e$ reduces the distortion introduced by tax evasion by encouraging skilled workers to migrate to more efficient jobs. In section 7 we shall see that the result in Proposition 4 is more general since it also extends to the intensive margin effects of tax evasion and is in accordance with the empirical evidence.

Summing up the comparative statics analysis of our model in the preceding and current sections, we note that, although it does not yield any clear-cut results on the effects of changes in the tax rate, unambiguous results can be derived for the two instruments which are of particular interest for policy purposes, i.e. the penalty rate and the probability of detection. The penalty rate is a parameter over which the government exercises direct control, while the probability of detection would be assumed to be controlled indirectly through the amount and efficiency of resources spent on detecting tax evasion. The model implies that these two policy tools are substitutes for each other. While the expected tax yield would fall with a decrease of p , the loss of tax revenue could be compensated by an increase of θ .

6. Economic and psychological costs of tax evasion

It should be stressed that despite the fact that the extensions presented in this section could have been introduced from the start without modifying the results in Propositions 1–4, it seemed convenient for expositional reasons to do it separately. We shall see that while the economic costs of tax evasion enter the budget conditions for the taxpayer, the psychological costs affect his expected utility function.

6.1 Costs for concealing income and creating opportunities to evade

While taxpayers differ in their opportunities to evade, these opportunities are inversely related to the potential evasion costs that people have to bear. The more opportunities a taxpayer has, the lower will be his evasion costs.

In our case, a simple way to incorporate the evasion costs is to write the budget conditions for the taxpayer when evasion is either undetected or detected as:

$$C_1^e = w^e L^e - t \cdot (w^e L^e)^\alpha + a + w^e E - c(E, \zeta_\sigma) \quad (24)$$

$$C_2^e = w^e L^e - t \cdot (w^e L^e)^\alpha + a + w^e E - \theta \cdot (w^e E)^\beta - c(E, \zeta_\sigma). \quad (25)$$

Here ζ_σ denotes the effective evasion opportunities. We assume ζ_σ to be exogenous and increasing.

We also assume that the cost function is convex in E , so that $c_{EE}(E, \zeta) \geq 0$ and in addition satisfies $c_E(E, \zeta) > 0$, $c_\zeta(E, \zeta) < 0$, $c_{E\zeta}(E, \zeta) \leq 0$, $c(0, \zeta) = c(E, \infty) = 0$, and $c(E, 0) = \infty$. Note that all taxpayers with $\zeta_\sigma = 0$ must be non-evaders because, for them, the probability of inspection turns out to be 100%. In this way, the separation of the population into evaders and non-evaders can now be explained not only in terms of the wage differences or the audit probability in each sector but also in terms of the cost of evasion, which is a function of the cost of the acquisition of information, the opportunities for underreporting, and the amount of unreported work.

Regarding the condition for an interior solution in the hidden sector, condition (9) now becomes:

$$w^e \alpha (w^e L_\sigma^e)^{\alpha-1} \geq p w^e \beta \theta (w^e E_\sigma)^{\beta-1} + p \cdot c_E(E_\sigma, \zeta_\sigma) \quad (26)$$

Clearly, the costs of further evasion make the condition for positive underreporting to be optimal more restrictive in the sense of a larger gap between the marginal tax rate and the marginal expected penalty. Therefore, the resulting expected gain is no longer enough for the individual to engage in tax evasion, unless such a gap also covers the expected marginal cost $p \cdot c_E(E_\sigma, \zeta_\sigma)$. In this way, the predictions of the model are made more consistent with the empirical observation that despite the existence of low audit probabilities and modest penalties, tax compliance is high in modern tax systems.

It is also worth observing that while the indirect utility function for a non-evader continues in this case being $V^n(t, \alpha, a, w^n) = U(C_\sigma^n, L_\sigma^n)$, for an evader it comes to be:

$$V^e = V^e(t, \theta, \alpha, \beta, a, p, c, w^e) \quad (27)$$

Therefore, we find that $\forall \sigma$ such that $\tau_\sigma = 0$:

$$\frac{\partial E_\sigma^e}{\partial c} = -(\lambda_{\sigma,1}^e + \lambda_{\sigma,2}^e) < 0 \quad (28)$$

But, alternatively to (27) we may formulate the indirect utility function as:

$$V^e = V^e(t, \theta, \alpha, \beta, a, p, \zeta_\sigma, w^e) \quad (29)$$

In this case, condition (28) turns out to be:

$$\frac{\partial E_\sigma^e}{\partial \zeta_\sigma} = -(\lambda_{\sigma,1}^e + \lambda_{\sigma,2}^e) \cdot c_\zeta > 0. \quad (30)$$

so that the number of tax evaders diminishes with the administrative cost derived from tax evasion.

A reduction in evasion opportunities will tend to reduce the size of the informal sector. Proposition 5 is policy-relevant as it suggests that an alternative to raising enforcement levels is to increase the fixed costs and reduce the opportunities for tax evasion.

6.2 Psychological costs of tax evasion

In addition to the associated income loss, there are non-pecuniary factors affecting the taxpayer's utility in his decision on whether or not to evade taxes. Among them, Gordon (1989) and Sandmo (2012) propose as extensions of the portfolio model with exogenous income the bad conscience provoked by tax evasion for the taxpayer. In this subsection, the extension is made in connection with the model of tax evasion with labor supply responses.

In our case, a simple way to incorporate this question is to write expected utility as:

$$EU^e = (1-p)U(C_1^e, L^e + E) + pU(C_2^e, L^e + E) - \delta(E). \quad (29)$$

where $\delta(E) \geq 0$ denotes the disutility due to the bad conscience associated with the act of underreporting. It makes sense to assume $\delta(0) = 0$, $\delta'(E) > 0$, and $\delta''(E) > 0$.

Concerning the characterization of the extensive margin, condition (9) now becomes:

$$w^e \alpha (w^n L_\sigma^e)^{\alpha-1} \geq p w^e \beta \theta (w^e E_\sigma)^{\beta-1} + p \delta'(E_\sigma). \quad (30)$$

One sees immediately that, as in the case of the economic costs of tax evasion, the disutility of evasion has the effect of making the condition for positive underreporting to be optimal more restrictive, in the sense of a larger gap between the marginal tax rate and the marginal expected penalty. The difference is now given by the expected marginal disutility of underreporting $p \delta'(E_\sigma)$.

Proceeding similarly as for condition (28), we obtain $\forall \sigma$ such that $\tau_\sigma = 0$:

$$\frac{\partial S_\sigma^e}{\partial \delta} = -1 < 0. \quad (31)$$

It turns out that the sign of the effects on the number of evaders of a rise in the disutility attached to tax evading is definitively negative. *An increase in the disutility attached to either the act of tax evasion will tend to reduce the hidden sector.*

This result reinforces the opinion that the fear of feelings of guilt has a great effect on the decision to become a tax evader. Moreover, as in the case of , the result helps to circumvent in part the critics concerning the prediction by the standard model of too much tax evasion by reducing the need for an exaggerated estimate by the taxpayers of the probability of detection that prevents them from exploiting opportunities that are apparently profitable. From a policy perspective, the result suggests that fiscal

authorities should devote resources and efforts to implement consciousness-raising campaigns against tax evaders.

7. Other scenarios of tax evasion

One of the criticisms that could be made of our analysis in the preceding sections is that, as in most theoretical studies on tax evasion, it presupposes that there are unrestricted opportunities open to the individual to vary his participation in both labor markets, so L^n, L^e , and E operate as variable functions. On the other hand, we have limited the attention to the case where there is a single wage rate for all kinds of earnings, either on-the-books or off-the-books, developed in the hidden sector. Moreover, working in a particular job was presupposed to be incompatible with other work activities. As a result of these limitations, of the four possible situations described in the Introduction, all the model predictions contained in Propositions 1–6 are restricted to the first scenario and other possibilities such as the phenomenon of moonlighting become ignored.

In this section, we explore among other situations that in which the individual may opt for working either at a single job incompatible with other activities or at another job that can be combined with a second off-the-books job. In this context, it is quite likely that the only possibility of variation in work hours is on the off-the-books job, which implies that we have to allow for the possibility that the labor supplies L^n and L^e may be fixed. Moreover, in order to look at the problem in a more realistic way, we assume that while the non-evaders continue to be paid w^n , we allow for the possibility that the evaders may have two different wage rates: w^e when working at the on-the-books activity and \hat{w}^e when working at the off-the-books activity. In this last case, the interpretation would be that, instead of a choice between having or not the possibility of reporting all or part of the true earnings, the choice is between working or not in a job that has access to additional employment in the black market economy.

7.1 Other variants of our model

For easy reference, let us call A the regular sector job, B the on-the-books irregular sector job, and C the off-the-books irregular sector job. In all of the scenarios that follow we shall have a problem of occupational choice where the individual chooses between working in job A, where taxable earnings $w^n L^n$ are withheld through a PAYE scheme operated by the employer, and one of these alternative possibilities: either job B or both job B and job C. In the first of such possibilities, taxable earnings from job B are subjected to a self-declaration scheme, so that the reported part to the tax authority is denoted by $w^e L^e$ and the unreported part by $w^e E$. In the second of such possibilities, the earnings $w^e L^e$ from job B are totally reported while those $\hat{w}^e E$ from job C are totally unreported. We start with the description of the scenario analyzed in the preceding sections, to continue with other four scenarios that stem as variants of our basic model.

Scenario 0: L^n and $L^e + E$ flexible; $w^e = \hat{w}^e$

In this case job C is again absent and the interpretation would be that there is a legal labor market with a 100 percent probability of detection coexisting with a hidden labor market in which employees have a flexible labor supply and report their income through

a self-declaration scheme according to their perceptions of risk. An example could be that of a freelancer who may opt for either working exclusively for one company which always withholds tax payments or working for many different companies which may not always withhold tax payments. Consequently, taxable earnings from job B may be reported ($w^e E = 0$), underreported ($w^e L^e > 0$ and $w^e E > 0$), or totally unreported ($w^e L^e = 0$) to the tax authorities. The policy implications from changes in the parameters of the model for this scenario were already studied in sections 2, 3, 4, 5 and 6 above.

Scenario 1: L^n and $L^e + E$ fixed; $w^e = \hat{w}^e$

The situation just differs from scenario 0 in that, as happens in the Allingham-Sandmo model, labor supplies in both markets are exogenous to the problem. Probably, the major interest of scenario 1 is that, together with scenario 0, defines the polar situations from which derive the more realistic and interesting scenarios below.

Scenario 2: L^n fixed and $L^e + E$ flexible; $w^e = \hat{w}^e$

The interpretation would be that the individual chooses between two legitimate possibilities, one of which involves paid-employment and the other self-employment. An example would be that of someone working for a manufacturer who may opt for leaving the job to set up as a freelancer. Since scenario 2 is an intermediate variant between scenario 0 and scenario 1, everything said concerning them also holds here with the logical qualifications imposed by the nature of the problem studied.

Scenario 3: L^n fixed and $L^e + E$ flexible; $w^e \neq \hat{w}^e$

This scenario corresponds to the case of a wage-earner whose choice lies between working at two jobs, both subject to withholding or PAYE, job A that involves a fixed hours contract and job B that involves a variable hours contract. Further, while A is incompatible with other work activities, B enables supplementing the income by moonlighting at job C with a different wage rate. An example would be that of the manufacturer worker of scenario 2 but with the possibility when freelancing of doing some of-the-books activity.

Probably, a more realistic interpretation would be that the individual decides between working as a wage-earner in fixed hours job A, which is subject to PAYE and is incompatible with any other work activity, and working as a self-employed in a free hours job B, which is subject to SD and is compatible with a moonlighting activity. Moreover, such a moonlighting activity is paid at a lower wage rate \hat{w}^e .

Note that, unlike what happens when $w^e = \hat{w}^e$, conditions (NE) and (EV) do not define the extensive margin of tax evasion. Their role reduces now to deciding in which sector to work, either in job A or in the two different jobs B and C. Now, substituting $w^e E^e$ for $\hat{w}^e E^e$ in budget conditions (4) and (5), we check that the extensive margin of tax evasion is now characterized by the inequality:

$$w^e - w^e \alpha \cdot (w^e L_\sigma^e)^{\alpha-1} \leq \hat{w}^e - p \hat{w}^e \beta \theta (\hat{w}^e E_\sigma^e)^{\beta-1}. \quad (35)$$

In this way, condition (36) indicates that the after-tax wage in the on-the-books market should be less than the expected after tax wage in the off-the-books-market. For the special case where $\alpha = \beta = 1$, (36) reduces to condition (12) in Sandmo (2012).

Scenario 4: L^n and L^e fixed; Eflexible; $w^e \neq \hat{w}^e$

The present scenario describes the most frequent situation since it is quite likely that the only possibility of changes in work hours is on the off-the-books job. The individual often has to decide, when looking for work, between two wage-earning jobs A and B that involve a fixed hours contract, one that is incompatible with other activities and the other that allows working at a second moonlighting job C.

Note also that there is here the possibility that $L^e = 0$, which implies complete specialization in the shadow economy. This is the case of the so-called ‘ghosts’, as those workers who operate in the underground economy, not declaring any form of earnings to the tax authorities at all. Cowell (1985, p. 26) refers to this possibility in the following terms: *‘In fact the way in which the tax administration operates may provide an inducement to do this rather than ‘mix’ legal and illegal activity at relatively low levels of h/H [i.e. $L^e/(L^e + E)$]. After all, if the tax authorities have no record of you whatsoever you may have a significantly better chance of successful evasion than if you do a few legitimate hours a week and thus automatically appear on some official file.’* In our context, the individual decides between two extreme situations: whether to report his total earnings (job A) or whether to report none of his earnings (job C).

7.2 Main results

Let \overline{C}_σ^n , $\overline{C}_{\sigma,1}^e$, $\overline{C}_{\sigma,2}^e$, $\overline{C}_{\sigma,1}^e$ and $\overline{C}_{\sigma,2}^e$ denote the consumption bundles C_σ^n , $C_{\sigma,1}^e$ and $C_{\sigma,2}^e$ for fixed labor supplies $L_\sigma^n = \overline{L}_\sigma^n$, $L_\sigma^e = \overline{L}_\sigma^e$ and $L_\sigma^e + E = \overline{L}_\sigma^e + E$ respectively. Then, replace $V^n(t, \alpha, a, w^n)$ by $U(\overline{C}_\sigma^n, \overline{L}_\sigma^n)$ and $V^e(t, \theta, \alpha, \beta, a, p, w^e)$ by $(1-p)U(\overline{C}_{\sigma,1}^e, \overline{L}_\sigma^e + E_\sigma) + pU(\overline{C}_{\sigma,2}^e, \overline{L}_\sigma^e + E_\sigma)$ in case of scenario 1, $V^n(t, \alpha, a, w^n)$ by $U(\overline{C}_\sigma^n, \overline{L}_\sigma^n)$ in case of scenarios 2 and 3, and $V^e(t, \theta, \alpha, \beta, a, p, w^e)$ by $(1-p)U(\overline{C}_{\sigma,1}^e, \overline{L}_\sigma^e + E_\sigma) + pU(\overline{C}_{\sigma,2}^e, \overline{L}_\sigma^e + E_\sigma)$ in case of scenario 4,

\overline{L}^n denote the hours worked by non-evaders and $\overline{L}^e + E$ those worked by evaders. Then the analysis is identical to that in sub-section 7.2, with the only difference being that now we have the additional constraints $L_\sigma^n = \overline{L}_\sigma^n$ and $L_\sigma^e + E = \overline{L}_\sigma^e + E$. Therefore, instead of λ_σ^n , we will have $U_C(\overline{C}_\sigma^n, \overline{L}^n)$ wherein $\overline{C}_\sigma^n = w^n \overline{L}^n - t(w^n \overline{L}^n)^\alpha + a$ in (23), and instead of Z_σ^e , we will have $Z_\sigma^e + \xi_\sigma^e \cdot [\overline{L}_\sigma^e + E_\sigma - (L_\sigma^e + E_\sigma)]$ where ξ_σ^e denotes the standard Lagrange multiplier in (7) and (19). This means that all of the equations derived in sections 5 and 6 hold point by point and, consequently, also the same results with the logical qualifications imposed by the nature of the problem studied.

The reader may check that if $V^n(t, \alpha, a, w^n)$ is replaced by $U(\overline{C}_\sigma^n, \overline{L}_\sigma^n)$. (6)

$$\begin{aligned} V^e(t, \theta, \alpha, \beta, a, p, w^e) &\equiv (1-p)U(\overline{C}_{\sigma,1}^e, \overline{L}_\sigma^e + E_\sigma) + pU(\overline{C}_{\sigma,2}^e, \overline{L}_\sigma^e + E_\sigma) \\ V^e(t, \theta, \alpha, \beta, a, p, w^e) &\equiv (1-p)U(\overline{C}_{\sigma,1}^e, \overline{L}_\sigma^e + E_\sigma) + pU(\overline{C}_{\sigma,2}^e, \overline{L}_\sigma^e + E_\sigma) \end{aligned} \quad (8)$$

In scenarios 3 and 4, again we have $U_C(\overline{C}_\sigma^n, \overline{L}_\sigma^n)$ in place of λ_σ^n in condition (23). On the other hand, while the indirect utility function for a non-evader $U(\overline{C}_\sigma^n, \overline{L}_\sigma^n)$ does not change, for an evader it turns out to be:

$$V^e = V^e(t, \theta, \alpha, \beta, a, p, w^e, \hat{w}^e) \quad (36)$$

This means that we must replace $w^e E^e$ by $\hat{w}^e E^e$ in (11), (12), (20), (21), and (22). But beyond these changes, the analysis above continues to hold. Consequently, all the results in Propositions 1–6 also apply to the present scenario of tax evasion, with the natural qualifications imposed by the nature of the problem studied. The only analytical difference is that, fortunately, the indeterminacy observed in sub-section 5.1 concerning the effects of changes in w^e when $w^e = \hat{w}^e$ vanishes in this case and we may obtain $\forall \sigma$ such that $\tau_\sigma = 0$:

$$\frac{\partial S_\sigma^n}{\partial w^e} = -(\lambda_{\sigma,1}^e + \lambda_{\sigma,2}^e) L_\sigma^e \cdot [1 - \alpha \cdot (w^e L_\sigma^e)^{\alpha-1}] < 0 \quad (37)$$

And if β and θ are not too high, so that the penalty does not exceed the irregular income:

$$\frac{\partial S_\sigma^n}{\partial \hat{w}^e} = -\lambda_{\sigma,1}^e E_\sigma - \lambda_{\sigma,2}^e E_\sigma \cdot [1 - \beta \theta (\hat{w}^e E_\sigma)^{\beta-1}] < 0 \quad (38)$$

The analysis (scenario 4) again turns out to be identical to that in sub-section 7.2, with the only difference being that, apart from $L_\sigma^n = \overline{L}_\sigma^n$, now we have the additional constraint $L_\sigma^e = \overline{L}_\sigma^e$. Therefore, instead of Z_σ^e , we will have $Z_\sigma^e + \xi_\sigma^e \cdot (L_\sigma^e - \overline{L}_\sigma^e)$ in (7) and (19), where ξ_σ^e denotes the standard Lagrange multiplier. This means that all of the equations derived in sections 5 and 6 hold point by point and, consequently, also the results in Propositions 1–6, together with conditions (37) and (38), with the natural qualifications imposed by the nature of the problem studied.

The following table summarizes most of the comparative statics results derived in this paper.

Table 1

Effects on the number of evaders of an increase in the various model parameters.

Different scenarios of tax evasion

	0	1	2	3	4
a	????				
t, α	????				
θ	↓↓↓↓↓				
β	↓↓↓↓↓				
p	↓↓↓↓↓				
$\theta (p\theta = \kappa)$	↓↓↓↓↓				
w^n	↓↓↓↓↓				
w^e	???	↑↑			
\hat{w}^e	———	↑↑			
c	↓↓↓↓↓				
ζ	↑↑↑↑↑				
δ	↓↓↓↓↓				

Table 1 describes the direction in which E_σ moves, given an increase in one of the model's parameters. Let us then turn to Note that scenario 1 in Table 1 coincide with the one depicted in section 4, where L^n , L^e and are variable, and $w^e = \hat{w}^e$. Regarding scenarios 2, 3 and 4 of tax evasion, they coincide with those depicted in subsection 7.1, 7.2 and 7.3 respectively.

If policymakers are only interested in reducing the inefficiency produced by evasion, what this table indicates is that raising or lowering most of the policy parameters will definitively reduce the distortion in the distribution of workers between labor markets.

Clearly, all the results in the preceding sections continue to hold with the natural qualifications imposed by the nature of the problem studied when L^n and L^e may be either fixed or variable. The only possible change arises if the evader taxpayer operates by taking a second job off the books with a different wage rate, as then Proposition 4 must be extended to allow for changes in the various wage rates in the terms depicted by conditions (37) and (38) above.

A first implication of this proposition is that, as in the case without an hour restriction, with an hour restriction the effectiveness of compliance tools θ , β , b , and p is more clear-cut as regards their impact along the extensive margin than along the intensive margin. As a second implication, Proposition 7 means that, in the absence of the associated ethical limits and economic costs, the social planner could raise at will the policy parameters (i.e. $p, \theta, \beta, w^n, \hat{w}^e, \zeta, \delta$ and s) and force all workers into the legal

market where the intensive margin effects of tax evasion are inexistent. All this leads to concluding that those individuals who are indifferent about working in either of the two labor markets because the (EV) and (NE) constraints are binding play a crucial role in reducing the number of tax evaders. This is because they allow making a cost-benefit study, in terms of resources and penalties versus reductions of the hidden sector of the economy, to determine the optimal changes that should be made in the policy parameters. Consequently, research on identifying and understanding this group of taxpayers seems highly desirable, given that it is in the context of indifferent taxpayers where reductions of the hidden economy can be made with minimal penalty changes and cost of resources.

The results in Table 1 suggest that the planner should change indefinitely the parameters of the model and thus force all workers into the legal market. The question is however how is that we do not observe this in reality. The reply is threefold. First, there are administrative costs and legal limits underlying any change of the parameters governing the model (i.e. detection probabilities, evasion penalties, opportunities for tax evasion, moral costs of evasion, etc.). Secondly, beyond reducing tax evasion, governments usually have other objectives to pursue such as the redistribution of income or the stabilization of the economy. And third, reducing the number of evaders does not necessarily mean reducing tax evasion, as we shall observe in what follows.

8 Total tax evasion: extensive effects and intensive effects

The preceding section shows what happens to the relative size of the legal and hidden sectors when changing the policy parameters. It does not, however, make clear predictions about the total amount of evasion, which presumably is the primary object of interest. To achieve this goal, we must first check what happens with the effects on hours of illegal work from changes in public policies.

8.1 Effects along the intensive margin of tax evasion

In this sub-section, the attention is focused on the effects from policy parameters that, in jobs B and C, arise for the five scenarios considered in the last section.

Already in their early work, Allingham and Sandmo (1972, pp. 338–339) recognized that, after extending the model to take account of labor supply decisions, they had not been able to come up with any interesting and reasonably simple results. Precisely, due to the indeterminacies that arise, Cowell (1985) proposed imposing some structure on the problem, for the different four scenarios considered above in absence of job A. In particular, for most of the cases studied, he assumed functional separability on the functions L_σ^e and E_σ in the sense of Drèze and Modigliani (1972), or equivalently $\partial^2(U_L/U_C)/\partial C^2 = 0$.⁵ Under this condition and some additional assumptions, he proved

⁵ This requirement involves the utility specification $U(C^i, L^i + E^i) \equiv M(g(C^i) + h(C^i) \cdot (L^i + E^i))$, where $M' > 0$ and $h > 0$. This specification allows separating the following two problems that must be solved by the individual simultaneously, i.e. ‘How much leisure time shall I sacrifice?’; and ‘How shall I allocate my working time amongst “on the books” and “off the books” activities?’ (see Cowell 1985, p. 20). , unlike what happens in the case of an additively separable in consumption and leisure case utility

that higher penalty rates decrease the hours of informal work, higher audit rates decrease the hours of informal work, and higher tax rates increase the hours of informal work. Nevertheless, for the first two results Cowell (1985) further assumed that there is both decreasing absolute risk aversion and increasing relative risk aversion and for the third result that there is a backward-bending vertical labor supply.

Using Cowell's findings, in the Appendix it is shown that the behavior of the model for the various scenarios coincides with that depicted in Table 2.

Table 2
Effects on total hours of illegal work from a rise in the various parameters of the model.

Scenarios of tax evasion

0 1 2 3 4

a $\uparrow^{1,6}$ $\uparrow^{2,3}$ $\uparrow^{1,6}$ $\uparrow^{1,6}$ \downarrow^4

θ \downarrow^1 \downarrow \downarrow^1 \downarrow^1 $\uparrow\downarrow^7$

β \downarrow^1 \downarrow \downarrow^1 \downarrow^1 $\uparrow\downarrow^7$

p \downarrow^1 \downarrow \downarrow^1 \downarrow^1 $\uparrow\downarrow^7$

w^n 0 0 0 0 0

δ 0 0 0 0 0

c $\downarrow^{1,6}$ $\downarrow^{2,3}$ $\downarrow^{1,6}$ $\downarrow^{1,6}$ $\uparrow\downarrow^5$

Notes

1. Functional separability
2. Decreasing absolute risk aversion
3. $\delta(E_\sigma)$ is constant
4. Leisure is normal
5. \uparrow if leisure is superior, \downarrow if leisure is inferior
6. Consumption is normal
7. \uparrow if labor supply is forward rising, \downarrow if backward bending.

This table describes the direction in which E_σ moves, given an increase in one of the model's parameters. Where this depends on further assumptions about individual behavior this is indicated in the notes at the foot of the table.

function, one may distinguish between an individual's risk aversion and the shape of his labor supply function.

Concerning the wage rate in the official sector w^n , we immediately find that, since job A is incompatible with any other work activity, E_σ is independent of w^n ; note in this sense that first order conditions for individual maximization prove that E_σ is only affected by w^e and \hat{w}^e under the five possible situations. And something similar can be said concerning δ as long as its derivative does not vary after the parameter change (see the Appendix).

For the rest of parameters, Table 2 takes advantage of the fact that, in the absence of job A, scenarios 1, 2, 3 and 4 coincide with Cowell's (1985) variants of his basic model, namely:

1 Allingham-Sandmo	$L^e + E$ fixed
2 SD (self-declaration)	$L^e + E$ flexible, functional separability, $w^e = \hat{w}^e$
3 PAYE	as in 2 but $w^e \neq \hat{w}^e$
4 PAYE	L^e fixed, E flexible

8.2 Extensive plus intensive margin effects of tax evasion

If we compare Table 1 with Table 2, we will see that the difference lies, not only in the emphasis is put on either the extensive or the intensive margin effects, but also in the fact that the various variants of Cowell's (1985) model do not consider the existence of labor market A.

In what follows, we shall combine our findings, relative to parametric changes along the extensive margin of tax evasion, with our and Cowell's (1985) results in Tables 2 and 3, relative to parametric changes along the intensive margin. This will be done after noting that the four variants of Cowell's (1985) model correspond to scenarios 1, 2, 3 and 4 in the absence of labor market A.

Thus, if we accept the assumption in Cowell (1985) of functional separability, an increase in θ or p will always reduce tax evasion in terms of both the number of evaders and the hours worked in the shadow economy. Therefore, if we accept his assumption of functional separability, an increase in θ or p will not only reduce self-employment but also tax evasion both in terms of number of evaders and unreported earnings.

On the other hand, concerning the wage rate in the official sector w^n , since job A is incompatible with any other work activity, E_σ is independent of w^n and only the extensive margin effects of tax evasion count; note in this sense that first order conditions for individual maximization prove that E_σ is only affected by w^e and \hat{w}^e under the possible four scenarios. Therefore, total tax evasion decreases with any increase in w^n without the need to assume any of the conditions in Cowell (1985).

Also worth recalling that scenario 4 includes the important case of an individual who specializes entirely in illegal activity (i.e. $L^n = L^e = 0$).

Table 3 represents the intersection between Table 1 and Table 2. It provides sufficient conditions for the extensive margin effects for tax evasion in Table 1 to become also total effects for tax evasion.

Table 3

Effects on total tax evasion (number of hours of illegal work plus number of evaders) from a rise in the various parameters of the model

Different scenarios of tax evasion

	0	1	2	3	4
θ	↓ ¹	↓	↓ ¹	↓ ¹	↓ ⁶
β	↓ ¹	↓	↓ ¹	↓ ¹	↓ ⁶
p	↓ ¹	↓	↓ ¹	↓ ¹	↓ ⁶
w^n	↓	↓	↓	↓	↓
δ	↓	↓	↓	↓	↓
c	↓ ^{1,5}	↓ ^{2,3}	↓ ^{1,5}	↓ ^{1,5}	↓ ⁴
ζ	↑ ^{1,5}	↓ ^{2,3}	↑ ^{1,5}	↑ ^{1,5}	↓ ⁴

Notes:

1. Functional separability
2. Decreasing absolute risk aversion
3. $\delta(E_\sigma)$ is constant
4. Leisure is inferior
5. Consumption is normal
6. Forward-rising labor supply

Table 3 provides sufficient conditions for the extensive margin effects on tax evasion in Table 1 to become total tax evasion effects. Clearly, the results in the first two rows are important since they do not rely on any special assumption on the objects of the model. The reason lies in that E_σ , being independent of w^n and δ , only the extensive margin effects of tax evasion count. This finding is in accordance with some empirical evidence pointing out that the share of evaded taxes over GDP decreases with the stage of economic development, which to a great extent depends on productivity. In this respect, Crane and Nourzad (1986) have documented that tax evasion in the United States over the period 1947–1981 has fallen in relative terms when income has grown. A similar pattern was found by Schneider et al. (2011), who demonstrated that the relative size of the hidden economy over the period 1999–2007 has decreased for 162 countries, whereas the non-weighted average of GDP per capita has increased for the same countries and over the same period. Along the same lines, great differences in the ability to generate tax revenue among developing and developed countries have been shown by Gordon and Li (2009), among others.

Also relevant is the result concerning the effectiveness of the psychological cost derived from the act of underreporting, since it also applies to all scenarios without the need of any further assumption. It does seem that sentiments of shame and guilt reduce to a greater extent than expected the perceived benefits from cheating since their effects are projected both along the extensive and the intensive margins for the five scenarios under study. As a matter of fact, the result is a consequence of Sandmo (2012, p. 11): *“It is worth emphasizing, however, that neither alternative leads to any change in the comparative statics predictions of the model for the case of an interior solution; the difference that they make relates solely to the explanation of the extensive margin of tax compliance behavior.”* Although he refers only to the extensive margin due to the existence of an interior solution, we have checked that that the extensive margin due to mobility between formal and informal sectors also proves to crucially count too.

Concerning the various scenarios, the most outstanding ones are scenario 2 and scenario 4, given that neither of them assumes the existence of functional separability.

The only problem is that changes in w^n and δ are not immediate but involve long term policies which require time and efforts to get results. By contrast, θ and p can be changed more easily but their effects are less clear-cut. Thus,

The results concerning scenario 2 are interesting since they imply that the Allingham and Sandmo conclusions concerning the penalty and audit rates also hold in a more general environment. In it there is a third labor market, the tax and penalty schedules are nonlinear and both the economic and psychological costs of tax evasion are also present in the analysis.

But the most policy relevant findings relate to scenario 4. This is important since this is the situation that can be viewed as the more frequently found in practice. Here the audit and penalty system against tax evasion, same as the progressivity of the penalty function. It is a bit surprising that the effectiveness of important tax instruments such as the audit and penalty rates, same as the progressivity of the penalty function, heavily depend on not only the particular tax evasion scenario considered but also the income level of the group of individuals to which they are applied.

Note that the result on ζ is a direct consequence of the result on c .

Comparing $c(E, \zeta_\sigma)$ and ζ with θ , β and p the empirical evidence concerning labor supply indicates that a higher wage may result in a smaller number of working hours (for a comprehensive review, see Killingsworth, 1983; for a brief review, see Ehrenberg and Smith, 1991).

An interesting implication of this formulation is that it leads to a less optimistic view of the effectiveness of using penalty taxation as deterrence to tax evasion. In the new version of the model it will still be true that an increase in the penalty rate leads to less evasion. But because evasion decreases, the “conscience tax” $B'(E)$ also goes down, and this diminishes the effect of the penalty tax. In other words, the stronger extrinsic incentive to truthful reporting reduces the intrinsic incentive to behave honestly.

When it comes to the inferior sub-table of Table 3, the purpose is to impose structure on the model so as to find unambiguous results for those parameters that, in the different scenarios, do not offer. The implication of this is that, under the assumptions in Cowell (1985), most of his results continue being valid when, as a replacement for both his on-the-books and off-the-books activities, the taxpayer may engage in a third formal activity incompatible with any other additional job. . Either functional separability or a forward-rising labor supply is sufficient for an increase in θ and/ or p to imply a reduction of total tax evasion. However, the applicability is restricted to the case $b = 0$; $\alpha = \beta = 1$; $c(E, \zeta_\sigma) = 0$; $\delta = 0$; $s = 1$, so that the analysis ignores the psychological and compliance costs of tax evasion, the possibility of a fixed fine and the consideration of nonlinear tax and penalty schedules.

Of course, the more modest findings are related to the effects from \hat{w}^e provided that it involves more stringent assumptions and only applies to a limited number of scenarios. In any case, it suggests that improving technology and education in the moonlighting job boosts tax evasion.

9. Concluding comments

This paper addresses an understudied topic in the literature: the extensive margin effects of tax evasion that stem from changes in the different policy parameters. To do it, we have developed a model on tax evasion with labor supply responses that solves some of the shortcomings in the standard deterrence framework of tax compliance. The analysis has been focused on four tax evasion scenarios that capture many of the situations found in the real world. We observed that if the absence of ambiguity is to be considered a good thing, our findings reinforce the theory of tax evasion with endogenous labor supply. Not only do they confirm the results of the effects along the intensive margin that arise under several assumptions from changes in the penalty and audit systems but they even become more robust and conclusive. Moreover, they apply to most of the typical tax evasion scenarios that are found in the real world. Our findings shed light, not only on the customary effects from changes in the penalty and audit rates, but also on other aspects that also affect the decision to evade taxes, such as the relative wage rates in each labor market, the economic costs of tax evasion, and the bad conscience associated with the act of underreporting. In particular, it is shown that the following facts induce migration to the legal sector:

- a) Higher wage rates in the legal sector;
- b) Higher psychological costs of tax evasion;
- c) Higher costs for creating opportunities to evade;
- d) Lower opportunities to evade;
- e) Higher evasion penalties;
- f) More progressive evasion penalties;
- g) Higher detection probabilities;
- h) Higher evasion penalties combined with lower detection probabilities.

In contrast, for most of the above scenarios the signs of the partial derivatives of the audit and penalty system become unclear even in the absence of income effects. These indeterminacies have been stressed by Baldry (1979), Pencavel (1979), and Cowell (1981) by showing that when gross income is endogenous to the model, the simple

conclusions in Allingham and Sandmo (1972) and Yitzhaki (1974) regarding the effects of changes in the tax rate, probability of audit and penalty rate are unlikely to be robust. The conclusion that arises is that the effectiveness of compliance tools and the rest of policy parameters, such as the wage rates and the psychological and economic costs of tax evasion, is much more clear-cut as regards their impact along the extensive margin than along the intensive margin. This implies in turn that the whole exercise done in the above sections is worthwhile, given the certitude that it adds to our knowledge about the effects from changes in the different parameters of the model, when the emphasis is put on the number of evaders instead of on the hours of informal work.

The only limits to the application of tax policies a) to h) are, apart from the costs in terms of the resources needed for their application (i.e. administration costs, education to improve productivity, campaigns to favor the payment of taxes, etc.), there is the fact that penalties should be socially acceptable, in the sense of “fitting the crime” in the eyes of taxpayers (note that, in absence of such limits, the social planner could raise at will the policy parameters and force all workers into the legal market). Because of this, an interesting situation arises when there are intervals such that the participation conditions (NE) and (EV) are satisfied as equality, therefore indicating the existence of a large mass of individuals who are indifferent about working in either of the two sectors. Under such circumstances, drastic reductions of the number of evaders may be achieved with just a moderate increase of the penalty, the cost of inspections or any other of the other parameters of the model. The conclusion is that it should be explored the importance of those groups of indifferent individuals to decide if it is worth or not to implement changes in any of the available policy instruments.

Since conditions a) to h) do not impose any special requirements or assumptions on the variables of the model, the requirements or assumptions for a reduction in the hours of illegal work (intensive margin) suffice for a reduction in total tax evasion. In this sense, we have provided conditions for policies a) to h) to also ensure reductions in the number of hours worked by the individual evader (intensive margin), therefore becoming also conditions for total tax evasion. When this is the case, long term policies c), d) and g) seem to be the most effective to combat tax evasion. This is because, not only they apply without the need of further assumptions, but also because they are relevant for all of the five scenarios considered. Points c), d) and g) are long term policies in the sense that they require long term expending on education in order to improve productivity and produce changes in ethical values and attitudes towards taxation and illegal work.

Concerning the other tax policies, it is a bit surprising that the effectiveness of important tax instruments such as the audit and penalty rates, same as the progressivity of the penalty function, heavily depend on different aspects such as the particular tax evasion scenario considered, the utility function chosen and the income level of the group of individuals to which they are applied. Their effectiveness depends while a) and b) are much more appropriate for middle and low income groups, e) and f) are more effective to prevent tax evasion by high-income groups. This means that, while the traditional instruments such as the probability of detection or the penalty rate prove to be more effective for combating tax evasion undertaken by middle and low-income individuals, for high-income individuals the more suitable policies are those focused on increasing the administrative costs of illegal work, or reducing the opportunities for tax evasion.

In light of these results on total tax evasion and the fact that many of the general policy recommendation would be to use that

In principle, one would have expected a more complex model to yield fewer unambiguous results than in the two-class framework.

Despite the fact that increasing the number of parameters usually increases the degree of ambiguity increases, our results on total tax evasion show that this is no longer the case provided that the extensive margin of tax evasion derived from mobility between sectors are taken into account. The exception, however, has been the marginal tax rate since, in this context, the responses to potential policy changes are not clear-cut, and this happens either if the penalty is a function of the amount of income unreported or a function of the amount of tax evaded, either if the tax-schedule is linear or nonlinear, and either if the assumptions in Cowell (1981, 1985) are considered or not; recall in this author showed that if there is functional separability, labor supply is backward bending, absolute risk aversion is decreasing and relative risk aversion is increasing, then an increase in the marginal tax rate increases the proportion of declared income.⁶ Consequently, our results on the extensive margin of tax evasion derived from mobility between sectors provide an additional argument against the extended opinion that higher tax rates encourage tax evasion, even without relying, as in Sandmo (1981), on welfare considerations. These would say that “*...the ambiguity is due to the fact that the black labour market is less distorted than the regular one, and a tax increase that pushes labour from the more distorted to the less distorted sector may represent an efficiency gain to the economy.*”

It therefore seems that, same as concluded in Sandmo (1981), the role of the marginal tax rate should be concentrated on the design of optimal redistribution policies rather than on the fight against tax evasion. Fortunately, the present model has the advantage that it may be easily transformed into an optimal taxation setting with tax evasion decisions, given that the participation constraints (NE) and (EV) are especially suitable for this purpose.⁷

Sandmo (1985): “*Is the existence of tax evasion an argument for a lower marginal tax rate? We have seen that the optimal tax analysis does not offer any clear conclusion on this point, and my own inclination is to say that, at least as long as tax evasion is not an overwhelming social problem, the choice of the marginal tax rate should be governed by the more standard efficiency and equity concerns. The penalty and audit rate are instruments better targeted on the decision to evade taxes.*”

In any case, our major conclusion would be that it makes no sense the study of tax evasion without considering, as traditionally done by the literature, the extensive margin effects for the mobility of taxpayers between the official and the unofficial sectors. This is because the effects from changes in the policy parameters may have opposite sense along the intensive and extensive margins. This is what happens, not only in connection with the marginal tax rate, but also with the enforcement parameters when applied to high-income classes and the administrative costs of tax evasion when applied to middle-

⁶ In fact, our analysis on mobility between sectors even invalidates the result in Yitzhaki (1974) with exogenous labor supply.

⁷ In a paper under preparation, an extension of the present model is used to analyze the implications of underreporting for optimal redistribution through labor income taxation.

and low-income classes. Moreover, depending on the specific tax evasion scenario the sign of effects along the extensive margin will coincide or differ with those on the intensive margin under the same circumstances and for the same parameter changes.

To end with, it would be interesting to explore to what extent the results in this paper remain valid when, apart from wage income, capital income is subject to taxation. The underlying framework could also be generalized to consider dynamic incidence factors, a broader range of government activities, and the impacts of an open economy. Another extension could be to allow for differences among taxpayers in terms of their attitudes to risk, along the lines followed by Boadway et al. (1991) or Pestieau and Posen (1991).

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APPENDIX

It should be mentioned that, although Cowell (1985) restricted the attention to linear tax and penalty schemes, by following his analysis the results in Table 3 can be easily shown to also apply by operating with nonlinear tax and penalty schemes. And something similar can be said concerning the parameters regulating the economic costs of tax evasion since, despite they were ignored in Cowell's model, their effects in Table 3 on irregular labor supply directly arise after noting that changes in c are equivalent in terms of E_σ to changes in $-a$.

Now, consider scenario 2. Making $x_2 = t\alpha w^e (w^e L_\sigma^e)^{\alpha-1} - c_E$ and $y_2 = t\alpha w^e (w^e L_\sigma^e)^{\alpha-1} - \theta\beta w^e (w^e E_\sigma)^{\beta-1} - c_E$, the first order conditions become:

$$(1-p)U_C^1 \cdot x_2 + pU_C^2 \cdot y_2 - \delta'(E_\sigma) = 0 \quad (39)$$

where the superscripts denote evaluation of the expression in the two states of nature 1 and 2. Totally differentiating conditions (39), we obtain, since $x_2 > 0$ and $y_2 < 0$:

$$\frac{\partial E_\sigma}{\partial c} = \frac{(1-p)U_C^1 x_2}{G_2} \cdot \left(\frac{U_{CC}^1}{U_C^1} - \frac{U_{CC}^2}{U_C^2} \right) < 0 \text{ if there is decreasing absolute risk aversion and } \delta(E_\sigma) \text{ is constant.}$$

$$\frac{\partial E_\sigma}{\partial c_E} = \frac{(1-p)U_C^1 + pU_C^2}{G_2} < 0$$

$$\frac{\partial E_\sigma}{\partial \delta} = 0$$

It is worth emphasizing, however, that neither alternative leads to any change in the comparative statics predictions of the model for the case of an interior solution; the difference that they make relates solely to the explanation of the extensive margin of tax compliance behavior.

$$\frac{\partial E_\sigma}{\partial \theta} = \frac{p\beta\hat{w}^e (w^e E_\sigma)^{\beta-1} U_C^2 x_2 + p(w^e E_\sigma)^\beta U_{CC}^2 y_2}{G_2} < 0$$

$$\frac{\partial E_\sigma}{\partial \beta} = p\theta\hat{w}^e (\hat{w}^e E_\sigma)^\beta \frac{(\hat{w}^e E_\sigma)^{-1} (1 + \beta z_2) U_C^2 + U_{CC}^2 y_2 z}{G_2} < 0$$

$$\frac{\partial E_\sigma}{\partial p} = \frac{U_C^1 x_2 - U_C^2 y_2}{G_2} < 0$$

where $EU_C = (1-p)U_C^1 + pU_C^2$, $z = \log(\hat{w}^e E_\sigma)$ and $G_2 < 0$ by the second-order condition for utility maximization in scenario 2.

Now, consider scenario 4. Making $x_4 = \hat{w}^e - c_E$ and $y_4 = \hat{w}^e - \theta\beta\hat{w}^e (\hat{w}^e E_\sigma)^{\beta-1} - c_E$, the first order conditions are:

$$(1-p)U_E^1 + pU_E^2 + (1-p)U_C^1 \cdot x_4 + pU_C^2 \cdot y_4 - \delta'(E_\sigma) = 0 \quad (37)$$

After totally differentiating equation (38), we obtain:

$$\frac{\partial E_\sigma}{\partial c} = \frac{(1-p)(U_{CC}^1 x_4 + U_{EC}^1) + p(U_{CC}^2 y_4 + U_{EC}^2)}{G_4} \quad (38)$$

$$\frac{\partial E_\sigma}{\partial c_E} = \frac{(1-p)U_C^1 + pU_C^2}{G_4} < 0$$

$$\frac{\partial E_\sigma}{\partial \zeta_\sigma} = \frac{EU_C \cdot c_{E\zeta} + [(1-p)(U_{CC}^1 x_4 + U_{EC}^1) + p(U_{CC}^2 y_4 + U_{EC}^2)]c_\zeta}{G_4} \quad (39)$$

$$\frac{\partial E_\sigma}{\partial \delta'} = \frac{1}{G_4} < 0 \quad (40)$$

$$\frac{\partial E_\sigma}{\partial \theta} = p(\hat{w}^e E_\sigma)^\beta \frac{E_\sigma^{-1} U_C^2 + U_{CC}^2 y_4 + U_{EC}^2}{G_4} \quad (41)$$

$$\frac{\partial E_\sigma}{\partial \beta} = p\theta\hat{w}^e (\hat{w}^e E_\sigma)^\beta \frac{(\hat{w}^e E_\sigma)^{-1} \cdot (1 + \beta z_2) U_C^2 + (U_{CC}^2 y_4 + U_{EC}^2) \cdot z}{G_4} \quad (42)$$

$$\frac{\partial E_\sigma}{\partial p} = \frac{(U_C^1 x_4 + U_E^1) - (U_C^2 y_4 + U_E^2)}{G_4} \quad (43)$$

where G_4 stands for the second-order condition for utility maximization in scenario 4. Concerning (38), an increase in the opportunities to evade will increase illegal work hours if leisure is an inferior good, i.e. *Also the price of leisure is the wage rate foregone. Will an increase in the wage rate cause you to work more or less etc. - At low incomes leisure is normal (the price goes up you consume less, by working more), at high incomes leisure is inferior (the price goes up you consume more because the increase in wage rate increases your income allowing you to relax). At lower wage rates you will work more if offered an increase in the wage rate. The marginal benefit of working has gone up, so you work more so that again $MB=MC$. The substitution effect dominates. At some point your income will rise to a point where you will choose to work less if offered a pay rise. This is because a pay rise allows you afford more leisure time. You can maintain a similar living standard whilst having more time to relax. Also at higher incomes the quality of your leisure time is much greater (you can afford speed boats etcetc). In this region an increase in the wage rate causes MC to increase by more than MB (the extra money means you would rather enjoy spending it in your leisure time, than earning a bit more). Because MC has gone up by more than MB you work less. Now the income effect dominates.*

For showing the negative sign in (38) and (40), we shall follow Cowell's (1981) reasoning: "*..both (39) and (40) will be negative/positive as the labor supply curve is forward rising/backward bending. In the case of (17) the right-hand side is proportional to the labor-supply effect of a change in the marginal tax rate in the conventional labor supply model under certainty. In the case of (18), the right-hand side will be positive or negative according as $U, x - U$, is an increasing/decreasing function of x . However, the differential of this with respect to x is seen to be $U_r + [U, x-U,] h$, which for $x = wr$ is the numerator in (17).*"¹⁶ So once again the argument follows.

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