

# Sustainability Narrowness

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## **Abstract**

We study the resilience of a multiplex socio-ecological system (SES) which we structure from the spheres composing the sustainability Venn diagram. The SES network is subject to dynamics of spread of a global reform through the knock-on effect. The model outcomes reveal that high probability of reform completion on an SES layer through nodes previously reformed on other SES layers is necessary and sufficient to obtain positive density of reformed nodes on that layer. Full density can only be reached in the absence of risk of reform abrogation. The opposite case prevents the equilibrium density from reaching a steady state. The numerical simulation results show that the combination of likely probability of reform completion and of proportional influence of all layers yields the maximum magnitude of efficiency of the knock-on effect. We thus provide a formalized argument in favor of giving equal weight to all aspects of sustainable development.

*Keywords:* bioeconomics, socio-ecological systems, multiplex networks, sustainability, resilience

# 1 Introduction

2 The concepts of sustainable development and sustainability have been introduced to re-  
3 duce economic disparities, social exclusion and environmental degradation. No need to  
4 remind that the concept of sustainability came to the fore after the publication of the  
5 well-known Brundtland commission report (Tomlinson, 1987). Among different represen-  
6 tations of sustainability, the Venn diagram (Fig. 1), where the three spheres of economy,  
7 society and environment overlap (Mebratu, 1998), is the most widespread and referred  
8 to.<sup>1</sup> The representation involves the simultaneous pursuit of economic prosperity, social  
9 equity and environmental quality (Elkington, 2002). As can be easily noticed, the area  
10 in which all the objectives coincide is very narrow. For instance, in a study conducted by  
11 Estapé-Dubreuil et al. (2016), the authors do not find a decision-making factor used in  
12 micro-investing that takes all three dimensions of sustainability into account.

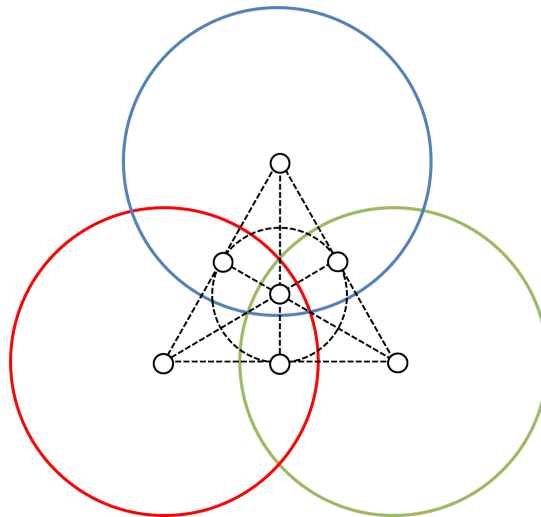


Figure 1: Graphical representation of sustainability in form of a Venn diagram composed of economic (blue), social (red) and environmental (green) areas. The inside network corresponds to the Fano plane.

13 In the outer years, frameworks laying bare the interconnectedness of humans and  
14 their environment, such as socio-ecological systems (SES), have been developed (Ostrom,  
15 2009). They are considered to better describe the dynamics of interactions between human  
16 communities and their environment (Waltner-Toews et al., 2008). Indeed, polycentric

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<sup>1</sup>Despite the alternative representation in form of concentric circles (Mitchell, 2000), where the economic area is embedded in the social area, itself being inside the natural environment, the reasoning behind seems rather distant from the current mentalities.

17 systems are better adapted to social-ecological dynamics because these coupled systems  
18 effectively link scales via diverse information flow capabilities (Ostrom, 2010). In detail,  
19 SES are composed of anthropogenic and natural elements interacting through temporal,  
20 spatial and organizational scales. When SES are represented in form of a network, the  
21 latter is composed of nodes, such as natural components, resource users, civil players,  
22 voters, economic actors or regulatory organizations, and of linkages between those nodes,  
23 like exchanges or transfers of money, energy, information and strategies.

24 The analysis of SES sustainability has been principally conducted through that of re-  
25 silience, provided that collapses result from the lack of resilience (Gonzalès and Parrott,  
26 2012).<sup>2</sup> Therefore, Carpenter et al. (2012) emphasized that public policies for general  
27 resilience must overcome budget limitations, address trade-offs, be acceptable to many  
28 competing interests, and overcome barriers in the structure of existing institutions. Man-  
29 aging for resilience then requires legal framework to be reformed in order to accommodate  
30 the SES dynamic processes (Garmestan and Benson, 2013). As a matter of fact, adaptive  
31 management is unlikely to be effective without reform, and without adaptive manage-  
32 ment, environmental governance is unlikely to succeed (Ruhl, 2005). In continuity of  
33 their insights, we wish to study the possibility of reform on multilayered networks as well  
34 as to measure the efficiency of spread of change in such systems, should their actual state  
35 be reputed to be obsolete or in jeopardy.<sup>3</sup>

36 The exemplification of the foregoing can be done through the recommendation for fiscal  
37 reforms in G20 countries, that would at once benefit economic growth, social inclusion  
38 and environmental outcomes, which entails significant changes in tax structures, increased  
39 emphasis on environmental taxation and a review of environmentally harmful subsidies  
40 (OECD et al., 2012). Another example within reach is relative to the Sendai treaty for  
41 disaster risk reduction (Aitsi-Selmi et al., 2015) as a major agreement of the post-2015  
42 development agenda; the latter is voluntary and non-binding, in which the stakeholders  
43 issued from different backgrounds need to make a concerted effort at reducing the impact  
44 of a natural hazard. But how can a decentralized change of such scope be implemented?  
45 To answer that question, we consider sustainability to be achievable, by means of a reform  
46 initiated by public authorities or the civil society, whenever the states of SES components

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<sup>2</sup>A system is considered to be resilient when its structure adapts to perturbations while continuing to function, be it at the expense of changes (Liu et al., 2007).

<sup>3</sup>The concept of reform assumes the presence of a crisis which could be solved through corrective actions.

47 or nodes can be efficiently updated. Put differently, if the update among the system  
48 constituents spreads sufficiently, the system is considered to be controllable, in the sense  
49 that some agents manage to drive the others,<sup>4</sup> via the network connectivity, toward the  
50 objectives at stake. In consequence, allowing for the overall system controllability, through  
51 the medium of control theory, becomes a necessary condition to ensure sustainability.<sup>5</sup>

52 The Fano plane (Fig. 1) corresponds to the network variant of the Venn diagram. If we  
53 conceive sustainability as achievable through the spread of reform among all constituents,  
54 it would necessitate that all Laplacian eigenvalues – which denote the number of connected  
55 components in a graph – be distinct, which cannot be verified in a triple Steiner system  
56 (Aguilar and Gharesifard, 2015). This implies that the system controllability, for the  
57 purpose of sustainability attainment, is theoretically impossible to achieve. On the other  
58 side, interconnected multiplexes or multilayered networks are a class of dynamic networks  
59 introduced to model real-world complex systems, in which the nodes are connected via  
60 more than one type of links (Mucha et al., 2010; Lee et al., 2015). The particularity of  
61 a multiplex system is the functional coupling between the layers of a certain kind. The  
62 network layers are then constituted of links of different types. The field has become one  
63 of the major contemporary topics in network theory (Lee et al., 2015).

64 In order to go beyond the limits imposed by the Fano plane, we decide to study  
65 resilience through a diffusion of a global reform on a multiplex SES network. The latter  
66 can be envisaged as such since SES express their robustness through the ability to change  
67 over time (Mucha et al., 2010; Gonzalès and Parrott, 2012). That way, we structure the  
68 network from the three spheres forming the Venn diagram, with a functional coupling  
69 between economy, society and environment, which we subject to dynamics of reform  
70 through the knock-on effect, such that the spread of reform on a node comes from the  
71 neighborhood or from the counterparts previously reformed. This approach is motivated  
72 by the lack of dynamic perspectives and of full interrelatedness among the components of  
73 sustainability (Lozano, 2008b).

74 The model outcomes reveal that high probability of reform completion on an SES layer  
75 through nodes previously reformed on other SES layers is necessary and sufficient to obtain  
76 positive density of reformed nodes on that layer. As for full density, it can only be reached

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<sup>4</sup>The leading of agents concerns the practice of negotiations and discussions.

<sup>5</sup>Shastri et al. (2008) use a system theory-based approach, with an optimal control problem formulation, in the interest of deriving time-dependent management strategies.

77 in the absence of risk of reform abrogation. The opposite case prevents the equilibrium  
78 density from reaching a steady state. The numerical simulation results show that the  
79 combination of likely probability of reform completion and of proportional influence of  
80 all layers yields the maximum magnitude of efficiency of the knock-on effect. We thus  
81 provide a formalized argument in favor of giving equal weight to all aspects of sustainable  
82 development. Our clarification also opens an interesting debate on sustainability issues.

83 After this starting section, the dynamic behavior of the multiplex network, studied at  
84 the levels of a layer and of multiple layers, is modeled in Section 2. Section 3 is devoted  
85 to illustrating simulation examples. Section 4 discusses the implications of the theoretical  
86 results.

## 87 2 Model

88 Following the methodology developed by Wei et al. (2016), we consider an interacting  
89 multiplex network, such as the one depicted in Fig. 2, in which the population of agents<sup>6</sup>  
90 is distributed among  $L_n$  layers, where  $n = 1, \dots, 3$ . Each layer contains  $N$  nodes, with  
91  $i = 1, \dots, N$ , with different intra-layer connectivity. Let  $A^n$ , for  $n = 1, \dots, 3$ , be the  
92 adjacency matrix of  $L_n$  with nonnegative elements  $(a_{ij}^n)_{N \times N}$ , for  $i = 1, \dots, N$ . Two nodes  
93 are connected when  $a_{ij}^n = 1$ ; and  $a_{ij}^n = 0$  otherwise. Each node in  $L_n$  is connected to  
94 its counterparts in  $L_{-n}$ , such that there exists a systematic link between the nodes of  
95 different layers.

96 Agents can either be target nodes or reformed nodes, which, in the latter case, have  
97 previously been targets of the reform. In order to get reformed, a target node has to be  
98 sufficiently open for reform. In case a target node from  $L_n$  is connected to at least one  
99 intra-layer reformed node, let  $\beta_n \in [0, 1]$ ,  $n = 1, \dots, 3$ , be the probability of openness for  
100 reform on layer  $n$ . Should the reform be called into question, let  $\mu_n \in [0, 1]$ ,  $n = 1, \dots, 3$ ,  
101 be the risk that a reformed node from  $L_n$  gets abrogated, such that it returns back to its  
102 original state.

103 Given the inter-layer connectivity of the multiplex network, let  $k_n$ , such that  $\sum_{n=1}^3 k_n =$   
104 1, be the parameter of influence emanating from either layer. For example, whenever a

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<sup>6</sup>For illustrative purposes, the population of agents can be interpreted as a set of countries, the actors of which evolve in different SES layers, that are pursuing common reforms, or as a set of stakeholders, representing various spheres of the society, committed to the same purpose.

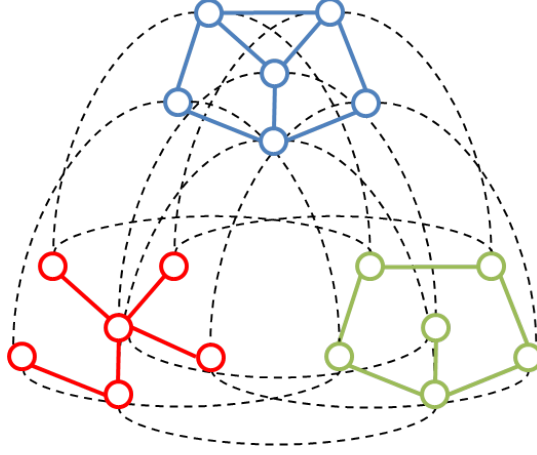


Figure 2: Example of a multiplex network composed of economic (blue), social (red) and environmental (green) layers. Each of them is composed of six connected nodes, that is  $L_n = \{1, 2, 3, 4, 5, 6\}$ , for  $n = 1, \dots, 3$ .

105 reform comes from the counterparts in other layers, the weighted probability that a target  
 106 node from  $L_n$  is open for reform issued by other layers amounts to  $(1 - k_n)\beta_n$ .

107 Finally, consider  $p_{n,i}(t) \in [0, 1]$  to be the probability that node  $i$  from  $L_n$  gets reformed  
 108 at time  $t$ , such that its complement corresponds to the probability that  $i$  remains a target  
 109 node.<sup>7</sup>

## 110 2.1 Intra-layer connectivity

### 111 2.1.1 Dynamics

112 Before moving forward to multiplex networks, let us start with a single layer in order to  
 113 study the intra-layer connectivity. The evolution of  $p$  for node  $i$  from  $L_n$  is formalized in  
 114 the form of a dynamical equation

$$\begin{aligned}
 p_{n,i}(t+1) &= (1 - p_{n,i}(t))(1 - q_{n,i}(t)) + p_{n,i}(t)(1 - \mu_n) \\
 &= 1 - q_{n,i}(t) - p_{n,i}(t) + p_{n,i}(t)q_{n,i}(t) + p_{n,i}(t) - p_{n,i}(t)\mu_n \\
 &= 1 + p_{n,i}(t)(q_{n,i}(t) - \mu_n) - q_{n,i}(t)
 \end{aligned} \tag{1}$$

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<sup>7</sup>The network model puts emphasis on the fact that agents do not necessarily engage in a binding cooperative game, but instead follow (allow themselves to be influenced by) their neighborhood or (by) their counterparts evolving on other layers. In point of fact, in dynamic game theory, the spread of a strategy, or that of a practice, takes place after the individual comparison between alternative payoffs, where high-payoff strategies propagate in the population of players.

115 for  $n = 1 \setminus \{2, 3\}$  and  $i = 1, \dots, N$ , where  $q_{n,i}(t) = \prod_{j=1}^N (1 - \beta_n a_{ij}^n p_{n,j}(t))$  represents  
 116 the probability that node  $i$ , despite being open for reform, does not get reformed by  
 117 neighbor  $j$ .

118 The discrete dynamics corresponds to the sum of the composed probability that open  
 119 node  $i$  is reformed by at least one neighbor and of the composed probability that reformed  
 120 node  $i$  is not subject to abrogation. Rewriting the equation enables us to highlight the  
 121 fact that the level of  $p_{n,i}$  at time step  $t + 1$  depends on the gap between the probabilities  
 122 of reform failure and abrogation at time  $t$ .

123 According to the technique used in Dragicevic (2015), solving the dynamical equation  
 124 reduces to solving the nonhomogeneous recurrence relation  $p_{n,i}(t)$ , where  $c_1(q_{n,i}(t) - \mu_n)$   
 125 is the associated homogeneous recurrence relation with a solution of  $c_1(q_{n,i}(t) - \mu_n)^{t-1}$ .  
 126 The nonhomogeneous part yields  $c_2 = \frac{(q_{n,i}(t)-1)(1-(q_{n,i}(t)-\mu_n)^t)}{(q_{n,i}(t)-\mu_n-1)}$  from which we obtain the  
 127 stationarity expression of

$$p_{n,i}^*(t) = \frac{(q_{n,i}(t) - 1)(1 - (q_{n,i}(t) - \mu_n)^t)}{q_{n,i}(t) - \mu_n - 1} \left[ \frac{(q_{n,i}(t) - \mu_n)^t}{(q_{n,i}(t) - \mu_n)^t - (q_{n,i}(t) - \mu_n)^{t-1}} \right] \quad (2)$$

128 for  $n = 1 \setminus \{2, 3\}$  and  $i = 1, \dots, N$ . After considering the above, three cases may be  
 129 observed.

130 The first case corresponds to  $p_{n,i}^*(t) = 0 \Leftrightarrow q_{n,i}(t) = \mu_n$ . The probability that node  
 131  $i \in L_n$  gets reformed at time  $t$  by any neighbor from the layer is null if the probability  
 132 that a target node does not get reformed is equal to the probability that a reformed node  
 133 becomes abrogated.

134 The second case corresponds to  $p_{n,i}^*(t) > 0 \Leftrightarrow q_{n,i}(t) < \mu_n$ . The probability that node  
 135  $i \in L_n$  gets reformed at time  $t$  by any neighbor from the layer is positive if the probability  
 136 that a target node does not get reformed is less than the probability that a reformed node  
 137 becomes abrogated.

138 The third case corresponds to  $p_{n,i}^*(t) = 1 \Leftrightarrow \mu_n = 0$ . The certainty that node  $i \in L_n$   
 139 gets reformed at time  $t$  by any neighbor from the layer occurs if the probability that a  
 140 reformed node becomes abrogated is equal to zero. The following proposition ensues.

141 **Proposition 1** *In a network exclusively dependent on intra-layer connectivity, although*  
 142 *the risk of abrogation might annul a reform project conducted on that layer, a higher level*

143 *than that of reform failure is necessary and sufficient to achieve the possibility of reform;*  
 144 *the certainty of reform implies the absence of risk of abrogation.*

145 The necessity is straightforward from the expression of  $p_{n,i}^*(t)$ . The sufficiency comes  
 146 from the construction of probability  $q_{n,i}(t)$ , which is itself dependent on  $p_{n,i}(t)$ .

### 147 **2.1.2 Density**

148 The probability dynamics previously obtained enable us to study the density of reformed  
 149 agents in a network exclusively dependent on intra-connectivity.

$$\begin{aligned} \rho_i^*(t) &= \frac{1}{N} \sum_{j=1}^N p_{n,i}^*(t) \\ &= \frac{1}{N} \sum_{j=1}^N \frac{(q_{n,i}(t) - 1)(1 - (q_{n,i}(t) - \mu_n)^t)}{q_{n,i}(t) - \mu_n - 1} \left[ \frac{(q_{n,i}(t) - \mu_n)^t}{(q_{n,i}(t) - \mu_n)^t - (q_{n,i}(t) - \mu_n)^{t-1}} \right] \end{aligned} \quad (3)$$

150 for  $n = 1 \setminus \{2, 3\}$  and  $i = 1, \dots, N$ , where  $q_{n,i}(t) = \prod_{j=1}^N (1 - \beta_n a_{ij}^n p_{n,j}^*(t))$  is the prob-  
 151 ability that node  $i$ , despite being open for reform, does not get reformed by neighbor  
 152  $j$ .

153 We observe that  $\rho_i^*(t) = 0 \Leftrightarrow q_{n,i}(t) = \{1, \mu_n\}$ . This implies that the density of  
 154 reformed nodes is equal to zero in case of certainty that node  $i$  has not been reformed by  
 155 any neighbor from  $L_n$  at time  $t$ , or when its probability of not being reformed equals that  
 156 of being abrogated. In greater depth, these properties give the following.

$$\begin{aligned} q_{n,i}(t) = 1 &\Leftrightarrow \prod_{j=1}^N (1 - \beta_n a_{ij}^n p_{n,j}^*(t)) = 1 \\ &\Leftrightarrow (1 - \beta_n a_{ij}^n p_{n,j}^*(t))^N = 1 \\ &\Leftrightarrow \beta_n = \frac{1}{a_{ij}^n p_{n,j}^*(t)} = 0 \end{aligned} \quad (4)$$

157 for  $n = 1 \setminus \{2, 3\}$  and  $i = 1, \dots, N$ . While  $a_{ij}^n p_{n,j}^*(t) = 1/\beta_n$  represents the eigenvalue  
 158 of the adjacency matrix  $A$ , the reversed expression, that is  $\beta_n = 1/a_{ij}^n p_{n,j}^*(t)$ , corresponds  
 159 to the spillover threshold for the policy engaged in  $L_n$ . In this case, it amounts to zero,  
 160 which implies that the policy knock-on effect shall be sterile. The second equality yields



$$\begin{aligned}
q_{n,i}(t) = \mu_n &\Leftrightarrow \prod_{j=1}^N (1 - \beta_n a_{ij}^n p_{n,j}^*(t)) = \mu_n \\
&\Leftrightarrow (1 - \beta_n a_{ij}^n p_{n,j}^*(t))^N = \mu_n \\
&\Leftrightarrow \beta_n = \frac{1 - \mu_n^{\frac{1}{N}}}{a_{ij}^n p_{n,j}^*(t)} \tag{5}
\end{aligned}$$

161 for  $n = 1 \setminus \{2, 3\}$  and  $i = 1, \dots, N$ , where the spillover threshold amounts to  $\frac{1 - \mu_n^{1/N}}{a_{ij}^n p_{n,j}^*(t)}$ .  
162 We have  $\lim_{p_{n,j}^*(t) \rightarrow 0} \frac{1 - \mu_n^{1/N}}{a_{ij}^n p_{n,j}^*(t)} = \infty$  and  $\lim_{p_{n,j}^*(t) \rightarrow 1} \frac{1 - \mu_n^{1/N}}{a_{ij}^n p_{n,j}^*(t)} = 1 - \mu_n^{1/N}$ . In the first case,  
163 as the probability of reforming node  $i$  by neighbor  $j$  at time  $t$  goes to zero, the spillover  
164 threshold goes to the unattainable level of infinity, which yields a zero density of reformed  
165 nodes. In the second case, as the probability of reforming node  $i$  by neighbor  $j$  at time  $t$   
166 tends to one, the spillover threshold amounts to  $1 - \mu_n^{1/N}$ , that is zero for large values of  
167  $N$ . We thus fall on the same property in both cases.

168 Likewise, we observe that  $\rho_i^*(t) > 0 \Leftrightarrow q_{n,i}(t) < \mu_n$ . The result implies that the  
169 density of reformed nodes is strictly positive when the probability that node  $i$  has not  
170 been reformed by any neighbor at time  $t$  is less than that of being abrogated. In detail,  
171 we have the following

$$\begin{aligned}
q_{n,i}(t) < \mu_n &\Leftrightarrow \prod_{j=1}^N (1 - \beta_n a_{ij}^n p_{n,j}^*(t)) < \mu_n \\
&\Leftrightarrow (1 - \beta_n a_{ij}^n p_{n,j}^*(t))^N < \mu_n \\
&\Leftrightarrow \frac{1 - \mu_n^{\frac{1}{N}}}{a_{ij}^n p_{n,j}^*(t)} < \beta_n < \frac{1}{a_{ij}^n p_{n,j}^*(t)} \tag{6}
\end{aligned}$$

172 for  $n = 1 \setminus \{2, 3\}$  and  $i = 1, \dots, N$ . We have  $\lim_{p_{n,j}^*(t) \rightarrow 0} \left\{ \frac{1 - \mu_n^{1/N}}{a_{ij}^n p_{n,j}^*(t)}, \frac{1}{a_{ij}^n p_{n,j}^*(t)} \right\} = (\infty, \infty)$   
173 and  $\lim_{p_{n,j}^*(t) \rightarrow 1} \left\{ \frac{1 - \mu_n^{1/N}}{a_{ij}^n p_{n,j}^*(t)}, \frac{1}{a_{ij}^n p_{n,j}^*(t)} \right\} = (1 - \mu_n^{1/N}, 1)$ . When the probability of reforming  
174 node  $i$  by neighbor  $j$  at time  $t$  goes to zero, the interval in which stands the spillover  
175 threshold tends to the unrealistic level of infinity. On the contrary, as the probability of  
176 reforming node  $i$  by neighbor  $j$  at time  $t$  approaches certainty, the spillover threshold lies  
177 within zero and one for large values of  $N$ . Therefore, a substantial high probability of  
178 reform completion enables to reach positive density of reformed nodes.

179 Finally,  $\rho_i^*(t) = 1 \Leftrightarrow \mu_n = 0$ . In consequence, the absence of risk of reform abrogation  
 180 enables to strike full density of reformed nodes.

181 **Proposition 2** *In a network exclusively dependent on intra-layer connectivity, high prob-*  
 182 *ability of reform completion is necessary and sufficient to obtain positive density of re-*  
 183 *formed nodes on that layer; full density can only be reached in the absence of risk of reform*  
 184 *abrogation.*

## 185 2.2 Intra- and inter-layer connectivities

186 The consideration of an interacting multiplex network makes the density both dependent  
 187 on intra- and inter-layer connectivities. We have

$$\begin{aligned} \rho_i^*(t) &= \frac{1}{N} \sum_{j=1}^N p_{n,i}^*(t) \\ &= \frac{1}{N} \sum_{j=1}^N \frac{(q_{n,i}(t) - 1)(1 - (q_{n,i}(t) - \mu_n)^t)}{q_{n,i}(t) - \mu_n - 1} \left[ \frac{(q_{n,i}(t) - \mu_n)^t}{(q_{n,i}(t) - \mu_n)^t - (q_{n,i}(t) - \mu_n)^{t-1}} \right] \end{aligned} \quad (7)$$

188 for  $n = 1, \dots, 3$  and  $i = 1, \dots, N$ , where  $q_{n,i}(t) = \prod_{j=1}^N (1 - k_{-n} \beta_n a_{ij}^n p_{n,j}^*(t))$  is the  
 189 probability that node  $i$ , despite being open for reform, does not get reformed by neighbor  
 190  $j$  either through intra- or inter-layer connectivity. This time,  $q_{n,i}(t)$  is also dependent on  
 191  $k_{-n}$ , be it the influence coming from the reformed counterparts in other layers.

192 Once again, we observe that  $\rho_i^*(t) = 0 \Leftrightarrow q_{n,i}(t) = \{1, \mu_n\}$ . More specifically,  $k_{-n} \beta_n =$   
 193  $(1 - k_n) \beta_n = \frac{1}{a_{ij}^n p_{n,j}^*(t)} = 0$ . Despite the influence of nodes from both the neighborhood of  
 194 node  $i$  and from layers  $L_{-n}$  through  $k_{-n}$ , the policy knock-on effect will be vain. When  
 195  $q_{n,i}(t) = \mu_n$ , the spillover threshold amounts to  $\beta_n = \frac{1 - \mu_n^{1/N}}{k_{-n} a_{ij}^n p_{n,j}^*(t)} = \frac{1 - \mu_n^{1/N}}{(1 - k_n) a_{ij}^n p_{n,j}^*(t)}$ . We  
 196 have  $\lim_{k_n \rightarrow 0} \frac{1 - \mu_n^{1/N}}{(1 - k_n) a_{ij}^n p_{n,j}^*(t)} = \frac{1 - \mu_n^{1/N}}{a_{ij}^n p_{n,j}^*(t)}$ . By that, when the combined influence from layers  
 197  $L_{-n}$  is high enough, their policy knock-on effect will depend on the probability that node  
 198  $i$  gets reformed by node  $j$  via  $k_{-n}$  at time  $t$ . As  $p_{n,j}^*(t) \rightarrow 1$ , the spillover threshold  
 199 is zero for large values of  $N$ . In parallel, we have  $\lim_{k_n \rightarrow 1} \frac{1 - \mu_n^{1/N}}{(1 - k_n) a_{ij}^n p_{n,j}^*(t)} = \infty$ , be it  
 200 another unattainable threshold level. In both cases, zero density of reformed nodes will  
 201 be achieved.

202 Again, we observe that  $\rho_i^*(t) > 0 \Leftrightarrow q_{n,i}(t) < \mu_n$ . This comes down to  $\frac{1 - \mu_n^{1/N}}{(1 - k_n) a_{ij}^n p_{n,j}^*(t)} <$   
 203  $\beta_n < \frac{1}{(1 - k_n) a_{ij}^n p_{n,j}^*(t)}$  or  $\beta_n \in (0, 1)$ , when  $p_{n,j}^*(t) \rightarrow 1$ , for large values of  $N$ . When the

204 likelihood of reforming node  $i$  by node  $j$  through  $k_{-n}$  at time  $t$  is close to certainty, the  
205 spillover threshold lies within zero and one for large values of  $N$ . Thereby,  $\rho_i^*(t) > 0$  can  
206 be obtained through high probability of achieving reform in other layers.

207 As for  $\rho_i^*(t) = 1 \Leftrightarrow \mu_n = 0$ , reaching full density of reformed nodes implies a risk of  
208 reform abrogation equal to zero.

209 **Proposition 3** *In an interacting multiplex network both dependent on intra and inter-*  
210 *layer connectivities, high probability of reform completion on a layer through nodes re-*  
211 *formed on other layers is necessary and sufficient to obtain positive density of reformed*  
212 *nodes on that layer; full density can only be reached in the absence of risk of reform*  
213 *abrogation.*

214 Let us now analyze the stability of equilibrium density by considering  $\rho_i^*(t)$  as a Lya-  
215 punov function candidate. The latter is then assumed to be a rate function (Mesquita  
216 and Hespanha, 2010). The time derivative is equal to

$$\begin{aligned}
\rho_i'^*(t) &= \frac{(q_{n,i}(t) - 1)(1 - (q_{n,i}(t) - \mu_n)^t)}{q_{n,i}(t) - \mu_n - 1} \left( \frac{q_{n,i}(t) - \mu_n}{(q_{n,i}(t) - \mu_n)^t - (q_{n,i}(t) - \mu_n)^{t-1}} \right)^{N-1} \quad (8) \\
&\times \left[ \frac{q'_{n,i}(t)}{(q_{n,i}(t) - \mu_n)^t - (q_{n,i}(t) - \mu_n)^{t-1}} \right] \\
&- \left( \frac{q_{n,i}(t) - \mu_n}{(q_{n,i}(t) - \mu_n)^t - (q_{n,i}(t) - \mu_n)^{t-1}} \right)^{N-1} \\
&\times \left[ \frac{(q_{n,i}(t) - \mu_n) (q_{n,i}(t) - \mu_n)^t \left( \frac{tq'_{n,i}(t)}{q_{n,i}(t) - \mu_n} + \ln(q_{n,i}(t) - \mu_n) \right)}{\left( (q_{n,i}(t) - \mu_n)^t - (q_{n,i}(t) - \mu_n)^{t-1} \right)^2} \right] \\
&+ \left( \frac{q_{n,i}(t) - \mu_n}{(q_{n,i}(t) - \mu_n)^t - (q_{n,i}(t) - \mu_n)^{t-1}} \right)^{N-1} \\
&\times \left[ \frac{(q_{n,i}(t) - \mu_n) (q_{n,i}(t) - \mu_n)^{t-1} \left( \frac{(t-1)q'_{n,i}(t)}{q_{n,i}(t) - \mu_n} + \ln(q_{n,i}(t) - \mu_n) \right)}{\left( (q_{n,i}(t) - \mu_n)^t - (q_{n,i}(t) - \mu_n)^{t-1} \right)^2} \right] \\
&+ \frac{(q_{n,i}(t) - \mu_n)^t}{(q_{n,i}(t) - \mu_n)^t - (q_{n,i}(t) - \mu_n)^{t-1}} \left( \frac{(q_{n,i}(t) - 1)(1 - (q_{n,i}(t) - \mu_n)^t)}{q_{n,i}(t) - \mu_n - 1} \right)^{N-1} \\
&\times \left[ \frac{(1 - (q_{n,i}(t) - \mu_n)^t)q'_{n,i}(t)}{q_{n,i}(t) - \mu_n - 1} - \frac{(q_{n,i}(t) - 1)((1 - (q_{n,i}(t) - \mu_n)^t)q'_{n,i}(t))}{(q_{n,i}(t) - \mu_n - 1)^2} \right] \\
&- \left( \frac{(q_{n,i}(t) - 1)(1 - (q_{n,i}(t) - \mu_n)^t)}{q_{n,i}(t) - \mu_n - 1} \right)^{N-1} \\
&\times \left[ \frac{(q_{n,i}(t) - 1)(q_{n,i}(t) - \mu_n)^t \left( \frac{tq'_{n,i}(t)}{q_{n,i}(t) - \mu_n} \right) + \ln(q_{n,i}(t) - \mu_n)}{q_{n,i}(t) - \mu_n - 1} \right] \\
&\geq 0
\end{aligned}$$

217 We know, by definition of  $q_{n,i}(t)$ , that its derivative resumes to that of  $-p'_{n,j}{}^*(t) < 0$ .  
218 As a consequence, whenever  $q_{n,i}(t) < \mu_n \leq 1$ , which corresponds to the criterion for  
219 obtaining positive density of reformed nodes,  $\rho_i'^*(t) > 0$ , such that the equilibrium density  
220 is unstable in the sense of Lyapunov. It implies that the reform spread on layers can be  
221 withdrawn in time. The result is in accordance with our previous results, for positive  
222 density also depends on the tradeoff between the risks of failing to reform and that of  
223 abrogating the reform. In fact, according to the model outcomes, high probability of  
224 abrogation signifies that the reform has been previously adopted by a number of nodes.  
225 Albeit, what triggers the reform diffusion also prevents it from attaining stationarity.

226 **Proposition 4** *In an interacting multiplex network both dependent on intra- and inter-*  
227 *layer connectivities, high risk of reform abrogation prevents the equilibrium density of*

228 reformed nodes from reaching a steady state.

### 229 3 Simulations

230 Based on the properties and conditions previously obtained, the aim of this section is to  
 231 illustrate, through simulations, the levels of spillover thresholds as well as the potential  
 232 measures of diffusion.

#### 233 3.1 Intra-layer connectivity

234 Fig. 3 illustrates the spillover threshold values above which the policy knock-on effect is  
 235 operational. We observe a sequence of decreasing convex curves with a corner equilibrium,  
 236 at  $\{p_{n,j}^*(t), \mu_n\} = (0, 1)$ , from which arise the belt-shaped areas, that delimit the levels of  
 237  $\beta_n$ , colored in shades of blue. It verifies the property of  $\mu_n = q_{n,i}(t)$ . The same can be  
 238 noticed for  $p_{n,i}^*(t) = 1$ , where  $\mu_n = 0$ .

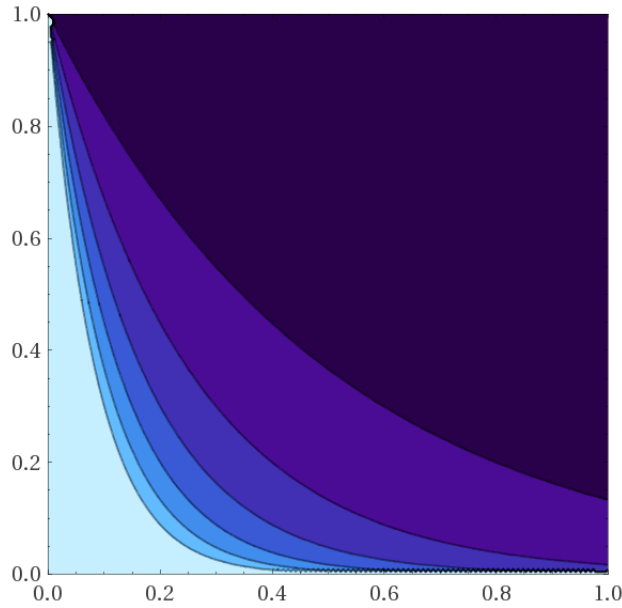


Figure 3: Levels of spillover thresholds  $\beta_n$  in a single layer. The  $x$ -axis corresponds to the probability ( $p_{n,j}^*(t)$ ) that node  $i$  from  $L_n$  is reformed by neighbor  $j$  at time  $t$ . The  $y$ -axis denotes the probability ( $\mu_n$ ) that a reformed node from  $L_n$  gets abrogated. While the light blue area corresponds to higher values of spillover threshold, that is  $\lim_{p_{n,j}^*(t) \rightarrow 0, \mu_n \rightarrow 0} \beta_n = 0.23$ , dark blue areas match with levels of spillover threshold of  $\lim_{p_{n,j}^*(t) \rightarrow 1, \mu_n \rightarrow 1} \beta_n = 0.00^+$ .

239 The substitutability between the probability of reform and that of abrogation is less  
 240 pronounced for low values of  $p_{n,i}^*(t)$  and  $\mu_n$ . This can be explained by the fact that when

241 the probability of reforming a node is low, the possibility to abrogate that reform is low  
 242 as well, such that the two parameters evolve in a complementary way. As the eventuality  
 243 of abrogation increases, the probability of reform decreases, so that both parameters turn  
 244 substitutable.

245 For high values of  $p_{n,j}^*(t)$ ,  $\beta_n$  is invariably around zero. A spillover threshold close to  
 246 but different from zero implies that the reform can easily spread through the intra-layer  
 247 connectivity. As both  $p_{n,i}^*(t)$  and  $\mu_n$  tend to zero, the spillover threshold increases, and  
 248 the spread by means of intra-layer connectivity becomes less reachable as well.

249 **Result 1** *In a network exclusively dependent on intra-layer connectivity, likely probability*  
 250 *of reform completion irrespective of the probability of reform abrogation is necessary and*  
 251 *sufficient to initiate the knock-on effect.*

252 The first result implies that a decentralized spread of reform on a layer can be con-  
 253 ducted through a few nodes only. In consequence, in absence of a central authority which  
 254 would otherwise impose a vast reform through binding policies, a non-binding directive  
 255 could be implemented by means of the spillover effect.

### 256 3.2 Intra- and inter-layer connectivities

257 Let us now take a closer look at the combined influence from  $k_{-n}$  on the spillover threshold.

258 Fig. 4 also depicts the values of spillover threshold above which the policy knock-on  
 259 effect is operational. We observe increasing concave curves, delimiting a series of belt-  
 260 shaped areas colored in shades of blue, with a gradual transition from complementarity  
 261 to substitutability. The proportional distribution of knock-on effects coming from layers  
 262  $L_{-n}$ , where  $(1 - k_n) \simeq 2/3$ , matches with corner values of  $\{p_{n,j}^*(t), k_n\} = (0^+, 0^+) \cup (1, 1)$ .

263 One interesting result is that  $\beta_n$  only exists for  $k_n \leq 1/3$  when  $p_{n,j}^*(t) \rightarrow 0$ . Thereby,  
 264 whenever the influence from  $L_{-n}$  is less than  $2/3$ , at the levels of probabilities of reform  
 265 – be it through the inter-layer connectivity – close to zero, the knock-on effect fails to  
 266 function. For  $k_n \rightarrow 1$ ,  $\beta_n$  only exists for  $p_{n,j}^*(t) \geq 2/3$ . In this case, the knock-on effect  
 267 will not take place either.

268 For all other configurations, the spread of reform should be operational, with a maxi-  
 269 mum magnitude of efficiency for  $p_{n,j}^*(t) > 1/2$  and  $k_n \leq 1/3$ .

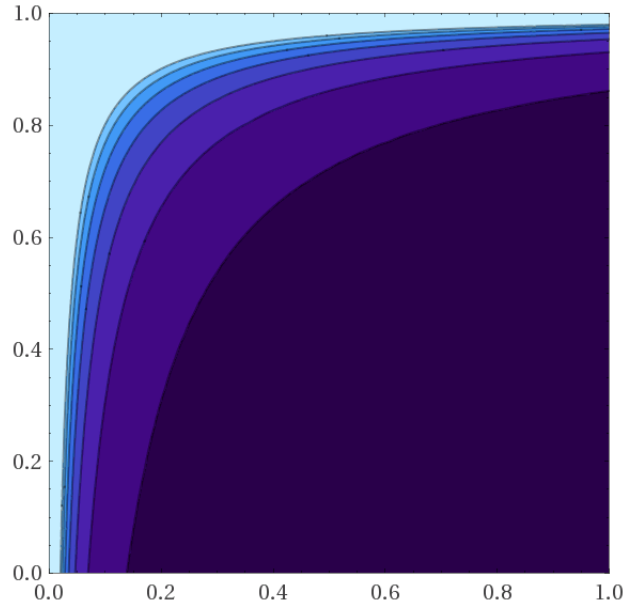


Figure 4: Levels of spillover thresholds  $\beta_n$  in a multiplex network. The  $x$ -axis corresponds to the probability ( $p_{n,j}^*(t)$ ) that node  $i$  from  $L_n$  is reformed by neighbor  $j$  through layers  $L_{-n}$ . The  $y$ -axis denotes the magnitude of influence ( $k_n$ ) from  $L_n$ . While the light blue area corresponds to high levels of spillover threshold, that is  $\lim_{p_{n,j}^*(t) \rightarrow 0, k_n \rightarrow 1} \beta_n = 0.99$ , dark blue areas correspond to low but positive levels of spillover threshold, that is  $\lim_{p_{n,j}^*(t) \rightarrow 1, k_n \rightarrow 0} \beta_n = 0.00^+$ .

270 **Result 2** *In an interacting multiplex network both dependent on intra- and inter-layer*  
 271 *connectivities, the combination of likely probability of reform completion and of propor-*  
 272 *tional influence of all layers yields the maximum magnitude of efficiency of the knock-on*  
 273 *effect.*

274 The second result suggests that a decentralized spread of reform can also be conducted  
 275 through a few nodes only. Nevertheless, unlike the previous case, a non-binding directive,  
 276 which would this time be addressed to the entire multiplex network, could only be imple-  
 277 mented – through the spillover effect – by virtue of a proportional consideration of the  
 278 counterparts from other layers.

## 279 4 Discussion

280 Many definitions of sustainable development have been proposed, most of which have  
 281 been previously collected by Kirby et al. (1995). According to Lozano (2008b), these  
 282 definitions can be classified in one of the following categories: (1) conventional economic  
 283 perspective; (2) non-environmental degradation perspective; (3) integrational perspective,

284 i.e. encompassing the economic, environmental, and social aspects; (4) inter-generational  
285 perspective; and (5) holistic perspective. Sustainability seen from the economic perspec-  
286 tive is considered to confuse sustainability with economic viability, i.e. sustained growth  
287 and self-sufficiency (Lozano, 2008a), which howbeit should not be marginalized either.

288 The holistic perspective combines the integrational and inter-generational perspectives  
289 (Lozano, 2007) with the search for two dynamic and simultaneous equilibria: the first  
290 between the three pillars of sustainability; the second of continuum in a temporal manner.  
291 However, time planning, as a consideration of the future effects of today's actions and  
292 inactions as paramount, has often been relegated to a secondary role (Seghezzo, 2009). By  
293 modeling sustainability through multiplex networks, we implicitly address sustainability  
294 in a holistic manner, in that we attempt to take into account its different aspects, without  
295 omitting to subject them to time dynamics. From a broader perspective, our results  
296 should be viewed as a proof that multiplex networks can be put to good use to apprehend  
297 the topics relative to the sustainability of SES. To a lesser degree, our framework also  
298 succeeded in measuring the magnitude of spillover effects, which have previously been  
299 tested in Cherry et al. (2003). In order to validate or invalidate our theoretic statements,  
300 additional experimental works could be undertaken. In all cases, the model outcomes  
301 open an interesting debate on sustainability issues.

302 First, notwithstanding the risks of reform failure and abrogation, we do confirm the  
303 theoretical possibility to lead socio-ecological systems toward reforms that are considered  
304 as indispensable. We thus manage to exceed the limits imposed by the topology of the  
305 Fano plane. Second, achieving a worthwhile objective by reforming a multilayered ar-  
306 chitecture ought to be seen as transient, because the population of agents following the  
307 reform path is found to be non-stationary. Hence, monitoring and evaluating the reform  
308 process seem as important as setting it off on a path. Third, we do confirm the narrowness  
309 of the sustainability space, such as one depicted in the well-known Venn diagram. In other  
310 words, in presence of high likelihood of advancing an amendment, the sole proportional  
311 influence of layers constituting SES yields the maximum magnitude of efficiency of the  
312 knock-on effect. However, considering all aspects of sustainable development as of equal  
313 importance does not seem to be of clear evidence yet.

314 Indeed, good reforms offer critical insights on conflict between the various spheres  
315 of economy, society and ecology (Brennan, 2008). For example, Estapé-Dubreuil et al.



316 (2016) show that the criteria used in investment decision-making only depend on two out  
317 of three dimensions of sustainability. It implies that advancing two objectives requires  
318 sacrificing the third one. Timely, we can mention the topic of full employment, which is  
319 considered to be an obligatory macroeconomic objective to achieve sustainable develop-  
320 ment. Yet, full employment and ecological sustainability objectives seem to be in large  
321 conflict (Lawn, 2006). This is probably why, on the occasion of the 21st Conference of  
322 the Parties to the United Nations Framework Convention on Climate Change (COP21), a  
323 call for a deep change in mentalities has been made. Besides, by discriminating the roles  
324 to play by the three pillars underlying the SES setting, one may achieve sustainability,  
325 but at a cost of greater efforts, because the knock-on effect shall be moderately efficient.  
326 Provided that, in addition to the cost of monitoring the overall process of reform imple-  
327 mentation, the sacrificed objective would need to be rehabilitated in the long-run, this  
328 type of strategy can be reasonably evaluated as economically unsound.

329 Despite its apparent abstractness, this work can be used easily to measure the impact  
330 of constraints under which the triple dividend effect (Tanner et al., 2015), while investing  
331 in disaster resilience, would take off. Carpenter et al. (2012) speak about general resilience  
332 as of the capacity of SES to transform in response to unfamiliar, unexpected and extreme  
333 shocks such as natural hazards. Even if a disaster does not occur, investing in resilience  
334 should provide evidence for three types of co-benefits, which are social protection by  
335 saving lives, economic growth by engaging in long-term investments and environmental  
336 benefits by avoiding environmental degradation. Nonetheless, building resilience at this  
337 scale requires to design and implement the right incentives. The last authors enumerate  
338 a list of conditions that enable the achievement of general resilience. Those include  
339 diversity, modularity, openness, reserves, feedbacks, nestedness, monitoring, leadership,  
340 and trust. Not only do our results support the indispensability of these qualitative criteria,  
341 but also provide a formalized cadre for conducting a quantitative analysis of resilience,  
342 from a perspective of interactions in multilayered networks, which is among the pressing  
343 challenges when it comes to incorporating reforms in complex systems, for the concept is  
344 hard to translate into measurable variables.<sup>8</sup>

345 To conclude, let us dwell on the price-regulating mechanisms and the environmental

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<sup>8</sup>This work could also be associated to what Sneddon et al. (2006) term deliberative democracy in a post-Brundtland world, in that it is built on decentralized decision-making and equal treatment of spheres composing sustainable development. In that case, the model enables to measure its efficiency.

346 pricing reforms. If we replace reforms by market-price fluctuations toward optimal prices  
347 for sustainable development (Pearce, 1988), in which prices observed on markets fully in-  
348 corporate social costs and environmental externalities, a reform failure becomes the status  
349 quo on price levels as a result of improper price updates. The same goes with reform abro-  
350 gation, which can then be interpreted as an impediment to market corrections inclusive of  
351 non-economic impacts. Should this be the case, the results of the model indicate that the  
352 pricing – without ever reaching stationarity in the long run – would benefit from equally  
353 considering market supply and demand along with the environmental repercussions of  
354 production and consumption, not forgetting the aspect of social cohesion with respect to  
355 the access to goods and services produced within the society. This statement seems to  
356 mirror that of Kahn (2015), who recalls the imperfect tradeoffs between economy, envi-  
357 ronment and equity. To a certain extent, it also ties up with the idea of making greater  
358 use of full-cost accounting (Richards, 1997) and that of shadow pricing (van Soest et al.,  
359 2006).

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