

Nudge in networks*

Benjamin Ouvrard[†] Anne Stenger[‡]

March 3, 2017

Abstract

This paper presents a model of voluntary contributions for a local public good, with individuals in a fixed network (complete, circle, line and star), based on the model of Bramoullé and Kranton (2007). We first characterize the equilibrium conditions in the absence of external incentives. We then consider the introduction of an informational nudge (announcement of the socially optimal contribution), both under complete and incomplete information regarding individuals' positions in the network. We show that, regardless of the regulator's level of information, an informational nudge may induce higher levels of aggregate contributions in circle and complete networks, and reduces strategic uncertainty, as long as individuals' sensitivity to the nudge (or their interest in the public good that is provided) is high enough. However, in star and line networks, the level of information available to the regulator matters since a nudge may not necessarily increase the level of aggregate contributions or reduce strategic uncertainty. Our main conclusion is therefore that a nudge policy should target specific individuals in specific networks. Moreover, we consider a "second best" nudge for line networks under incomplete information because the socially efficient profile of contributions may be complex to implement in such a situation.

Keywords : nudge; network; local public goods; information disclosure.

JEL Codes : C72, D83, H41.

*We would particularly like to thank Gisèle Umbhauer, Yann Bramoullé and Jocelyn Donze for their helpful comments. We are also grateful to the participants of the BIOECON conference (Cambridge, 2016) and the French-German Workshop on Experimental Economics (Konstanz, 2016).

[†]BETA, CNRS, bouvrard@unistra.fr

[‡]INRA, LEF, BETA, anne.stenger@nancy.inra.fr

1 Introduction

As economic agents, we are always part of a group, regardless of the group status: researchers working in a lab with colleagues working on similar topics, workers who are members of unions, households living in cities (or villages) with neighbors, individuals connected through social networks (Facebook, Twitter, LinkedIn, etc.), among others. In other words, in most of their activities, individuals are more or less connected to each other through social links and in this way form a network (structured or not). Because individuals are linked, their decisions may depend on what the others do. For instance, teenagers may be influenced by the tastes of their peers who are in their social networks. Researchers may discover new papers to read thanks to colleagues. Thus, information is diffused through networks. Taking networks into consideration when looking at the provision of local public goods seems relevant since these goods are shared (Tiebout 1956, Berglas and Piner 1981, Scotchmer 2002).

In this paper we consider a model of voluntary contributions to a local public good through networks. During the two last decades, a growing number of researchers have considered the influence of networks on individuals' decisions (Allouch 2015, Bloch and Zenginobuz 2007, Bramoullé and Kranton 2007, Bramoullé *et al.* 2014, Sanditov and Arora 2016). Although the model is general, we have some applications to environmental settings in mind (provision of environmental services, protection of local biodiversity, air quality, soil quality, etc.). Notwithstanding, this model could also be applied to more general situations (roads, national defense, etc.).

Our choice to consider networks is motivated by the results of different studies that have shown that the fact of belonging to a network may motivate participation in green activities. Using a panel of 765 individuals in Belfast, Ireland, Kurz *et al.* (2007) provide evidence that an individual's participation in recycling activities is influenced by his or her own neighbors' participation. McCallum *et al.* (2007) obtained similar results in their study of six communities in New Zealand. They show that citizens' participation in green activities is influenced by the community. Finally, Videras *et al.* (2012) estimate the determinant of pro-environmental behavior using a panel of 452 individuals. In addition to the level of education and salary, the ecological profile of the individual's community explains those behaviors.

We have adapted the model proposed by Bramoullé and Kranton (2007). In their paper, the authors focus on the study of Nash equilibria to describe the commonly observed behavior of an individual. In our paper, contrary to these authors, we are concerned with incentives to increase the level of contribution to the local public good. In particular, we consider non-monetary incentives, namely nudges (Thaler and Sunstein 2009). Nudges are simple actions to induce individuals to act in a given direction, at no cost (or at low cost), and without restricting their set of actions. The nudge we consider is the disclosure of the socially optimal contribution. Our modelization is close to that proposed by Harding and Hsiaw (2014), Farhi and Gabaix (2015) and Ouyard and Spaeter (2016). In the latter, the authors compare the efficiency of a tax with that of a nudge to increase individual contributions to environmental quality. They show that the nudge scenario may be as efficient as a tax scenario, with agents who are highly sensitive to environmental matters.

Up until now, research on nudges has essentially focused on their effects on the behavior of individuals in the fields of energy conservation (Allcott 2011, Ayres *et al.* 2013, Costa and Kahn 2013, Ferraro and Price 2013), organ donation (Johnson and Goldstein 2003), and savings (Madrian and Shea 2001). These studies highlight encouraging results

concerning the use and efficiency of nudges. Allcott (2011) studied the impact of normative messages to decrease household electricity consumption. Home Energy Reports were sent to similar households and included the mean energy consumption, emoticons¹ and advice. An average decrease of electricity consumption of 2% was observed. Interestingly, it would have been necessary to increase electricity prices between 11% and 20% in the short run to obtain the same result. Similar results were obtained by Ayres *et al.* (2013), Costa and Kahn (2013) and Ferraro and Price (2013).

In another paper, Banerjee *et al.* (2014) studied the effect of providing information about the behavior of other individuals under the implementation of an agglomeration bonus scheme. They considered landowners arranged in a circle, each landowner having two direct neighbors. In the first treatment, subjects received information about the behavior of their two closest neighbors. In the second treatment, subjects received information about the four closest neighbors, i.e., they also received information about their direct neighbors' neighbors. Banerjee *et al.* (2014) showed that subjects receiving more information on other subjects were more likely to coordinate their actions on the Pareto optimal outcome. In particular, in the first treatment, the share of subjects choosing the Pareto optimal outcome decreased from 63% in the first period to 4% after 30 periods. In the second treatment, the share of subjects choosing the Pareto optimal outcome decreased from 73% in the first period to 18% after 30 periods.

To the best of our knowledge, no modelization of the implementation of a nudge in networks has been proposed. In this paper, we propose to fill this gap and to study how a nudge may induce higher levels of contributions for a local public good such as environmental quality. From a public policy point of view, the interest of nudging is to propose a low cost form of regulation. In this paper, we first propose a theoretical model in which we consider individuals arranged in a network (complete, circle, star or line).² The objective is then to study the equilibria under the implementation of a nudge. As in Ouyard and Spaeter (2016), the nudge we consider is the announcement of the socially optimal contribution.

As in Bramoullé and Kranton (2007), we expect that the individual's position explains her level of contribution. Indeed, there is a spatial heterogeneity due to the number of direct neighbors (especially in star and line networks). This spatial heterogeneity may require, at the social optimum, some individuals to contribute more than others. Thus, following the example of Sunstein (2013), we should vary the content of the nudge to take this heterogeneity into account. However, this is possible only in the case of complete information from the regulator point of view. Under incomplete information, such refinement is not possible and the strategy must be adapted.

The rest of the paper is organized as follows. Section 2 takes a look at the models of voluntary contributions in networks and the modelizations of the reaction to a nudge. Section 3 describes the basic model. The implementation of a nudge under complete information is discussed in Section 4. We relax the assumption of complete information in Section 5. In Section 6, we discuss the results we obtained. Lastly, a conclusion is proposed in Section 7.

¹If the energy consumption of the household was below (above) the mean, a smiley (sad emoticon) was used.

²The complete network is a special case in which each individual is linked to everyone else.

2 Related literature

Our paper can first be related to others that propose a theoretical model of agents in networks. We can distinguish between models of voluntary contributions with and without spillovers. We first characterize models without spillovers and then concentrate on models with spillovers.

Bramoullé and Kranton (2007) and Bramoullé *et al.* (2014) propose a model in which individuals are part of a network and can contribute to a local public good. They adopt a positive approach³ and study the existence of Nash equilibria. They show that the network's structure shapes the Nash equilibria.⁴ Different networks are considered (circle, star, complete, etc.) corresponding to different situations (neighborhood, individuals connected by the Internet and social networks, hierarchical relationships, etc.). These networks especially differ in the number of direct neighbors (the closest neighbors an individual interacts with). More recently, Allouch (2015) proposed a model similar to Bramoullé and Kranton (2007) and Bramoullé *et al.* (2014), except that agents have a budget constraint. The author shows that under the assumption of normality of both the private and the public goods, a unique Nash equilibrium exists.

The second strand of literature includes the study of spillovers. In the model proposed by Bloch and Zenginobuz (2007), individuals belong to jurisdictions, and the hypothesis is that individuals from other jurisdictions may benefit from the local public good provided by a given jurisdiction. When the spillovers are symmetric between jurisdictions, the authors show that a unique Nash equilibrium exists. However, in the case of asymmetric spillovers, the existence of a unique Nash equilibrium is made possible only if these spillovers are sufficiently low. Lastly, Sanditov and Arora (2016) propose a model in which economic agents have a *ego-utility* (which comes from the consumption of public and private goods) and a *social utility* (corresponding to the sum of utility spillovers from their neighbors). The authors identify two opposite effects. Indeed, agents have an interest in contributing to the public good since they receive a social benefit (primary effect). However, when agents consume the private good, their gain in utility also benefits the others through the spillovers (secondary effect). The equilibrium level is higher than the one when considering purely selfish individuals (who do not consider the effect of spillovers), but is still lower than the socially optimal one.

Due to the modelization of the reaction to an informational nudge, our paper may also be linked to the implementation of a nudge. In particular, we follow a modelization close to the one proposed by Ouyard and Spaeter (2016). In a model of voluntary contributions to improve environmental quality, the authors compare the efficiency of a tax with the one of a nudge (based on the disclosure of the socially optimal contribution) to increase individuals' contributions. They argue that the marginal reaction to the nudge depends on environmental sensitivity: the higher the sensitivity to the environment is, the higher the marginal reaction to the nudge will be. Contrary to their modelization, we consider the possibility that individuals contribute more than what is announced. Moreover, our approach is more general.

³They are not interested in what individuals *should* contribute, but in what individuals *actually* contribute.

⁴Different Nash equilibria are possible ranging from *distributed* (each individual contributes) to *specialized* (some contribute at the level of the Nash equilibrium; others do not contribute at all), and *hybrid* (between the two previous kinds of equilibria).

Other authors have proposed a modelization to the reaction to a nudge. Farhi and Gabaix (2015) proposed a model in which nudges (disclosure of some information, use of anchors, etc.) are implemented to change the agents' consumption of some goods (e.g., cigarettes). More precisely, the implementation of a nudge makes agents consider another budget constraint instead of the "true" one (they consider a *perceived* budget constraint). Agents are not affected in the same way and, similarly to Ouvrard and Spaeter (2016), the reaction to the nudge depends on an intrinsic characteristic: individual *nudgeability*, which corresponds to the parameter controlling the reaction to the nudge.

Lastly, Harding and Hsiaw (2014) proposed a model in which, contrary to Farhi and Gabaix (2015) and Ouvrard and Spaeter (2016), it is not a regulator who provides the nudges, but individuals themselves through goal-setting (e.g., to reduce energy consumption). Depending on the agent's level of consumption, they model the reaction to the nudge as a disutility (utility) if she consumes more (less) than her objective.

In the next section, we adopt the model proposed by Bramoullé and Kranton (2007), which presents the advantage of more easily studying the equilibria in different structures.

3 Basic model

We now discuss the model we are considering here, which is based on Bramoullé and Kranton (2007). First, we consider private optima. We then study to the study of social optima. As emphasized in the rest of the paper, individuals' positions in the network, particularly in the star and line networks, are essential to characterize their level of contributions.

3.1 Private optima

Let us consider an economy with N individuals. These individuals can voluntarily contribute to a local public good. Let us denote the individual i 's contribution to the local public good as a_i , $a_i \in \mathbb{R}^+$. In this setting, a_i can be expressed in terms of money, time spent on effort, etc. Individuals incur a monetary cost from their contribution to the local public good. For the sake of simplicity, let us assume that the marginal cost of contribution, c , is constant.

Individuals are members of a fixed network. Let us denote the link existing between individuals i and j as $g_{ij} = 1$, and $g_{ij} = 0$ otherwise.⁵ If $g_{ij} = 1$, then individual j benefits from individual i 's contributions. In our setting, the links are not directed so that $g_{ij} = g_{ji}$.⁶ By convention, we also set $g_{ii} = 1$, meaning that individual i benefits from her own contribution. As in Bramoullé and Kranton (2007), individuals' contributions are substitutes for their direct neighbors' contributions. Moreover, let us denote the set of individual i 's direct neighbors as N_i , where $N_i = \{j \in N \setminus i : g_{ij} = 1\}$. Finally, the number of individual i 's direct neighbors is given by the cardinal of this set, $k_i = |N_i|$.

⁵In the context of a neighborhood, $g_{ij} = 1$ if individual j is the direct neighbor of individual i . In the context of social networks (Facebook, Twitter, LinkedIn, etc.), $g_{ij} = 1$ if individual j is a "friend" of individual i .

⁶López-Pintado (2013) proposed a model of voluntary contributions to a public good in networks with directed links.

Individuals benefit from the voluntary contributions according to the benefit function $f(A)$, where $A = \sum_{i=1}^{N_i} a_i$, $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$. Individual i 's total utility is thus given by:

$$U_i(a_i) = f\left(a_i + \sum_{j \in N_i} a_j\right) - ca_i \quad (1)$$

Individuals simultaneously choose their level of contribution. The first-order condition is given by:

$$f'(A^P) - c = 0 \quad (2)$$

where A^P is the Nash equilibrium level of contribution.⁷ Thus, each individual contributes so that the marginal benefit of the local public good equates its marginal cost of provision. In particular, let us denote the level of contribution of individual i 's k_i neighbors as A_{-i} . Considering the level of contribution of her k_i , individual i will contribute if $A_{-i} < A^P$. In that case, she will contribute $a_i = A^P - A_{-i}$. Otherwise, she will not contribute: $a_i = 0$.

Note that a condition to ensure an interior solution is that $f'(0) > c$. In that case, the first-order condition is such that $f'(0) - c > 0$: the individual has an interest in increasing her level of contribution.

In this paper, we consider four different networks: complete, circle, line and star networks. In complete and circle networks, everyone has the same number of neighbors and is observed by the same number of individuals. In line networks, two individuals (each located on one extremity) benefit less from the contributions of the others (they have one direct neighbor), while those inside the network have two direct neighbors. In star networks, one individual is central with more neighbors than the others. Thus, this individual can observe everyone but is also observed by everyone.

Example 1: Let us consider the following utility function:

$$U_i(a_i) = 4 \ln\left(1 + a_i + \sum_{j \in N_i} a_j\right) - a_i$$

According to the first-order condition, we have:

$$\frac{4}{1 + A^P} - 1 = 0$$

The Nash equilibrium level is such that $A^P = 3$. Note that we have:

$$f'(0) = 4 > 1 = c$$

In Fig. 1, we consider four individuals arranged around a circle. Two Nash equilibria are possible. First, there is a distributed equilibrium (left) in which each agent contributes a positive amount. There is also a specialized equilibrium (right) in which some contributes while others do not.

In Fig. 2, we consider four individuals arranged around a star. Contrary to the previous case, the individual in the center has three neighbors, while the others only

⁷We do not prove the existence of a Nash equilibrium. This relies on the use of Brouwer's Fixed Point Theorem.

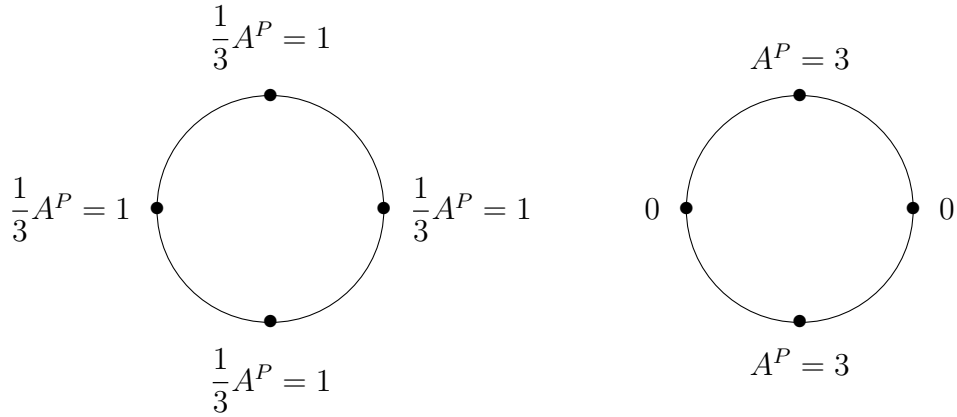


Figure 1: Equilibria in circle networks.

have the individual in the center as their neighbor. Two Nash equilibria are possible, both being specialized equilibria. In the first one, the individual in the center contributes nothing, whereas the others contribute to the level of the Nash equilibrium. The second case is the opposite of this first equilibrium.

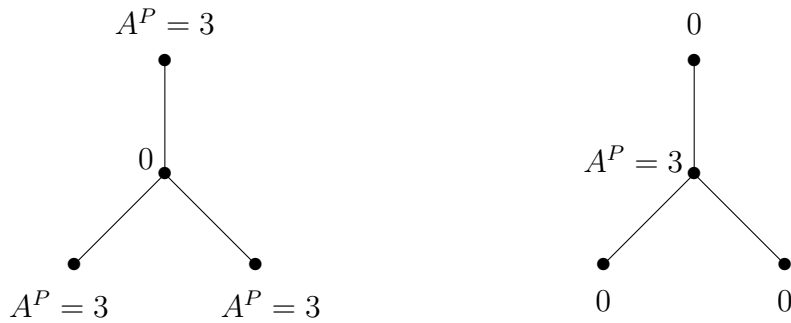


Figure 2: Equilibria in star networks.

In Fig. 3, we consider four individuals arranged along a complete network. A complete network is a special case in which each individual is linked to everyone. This situation corresponds to the classic public goods game. As for the circle network, all individuals have the same position in the network given that everyone has the same number of direct neighbors. Multiple equilibria exist (specialized, hybrid and distributed).⁸

Lastly, in Fig. 4 we consider four individuals in a line network. As emphasized below, the two individuals inside the network benefit from the contributions of two direct neighbors in addition to their contribution, while those on the extremities benefit from one direct neighbor only. Two specialized equilibria are thus possible.

■

The importance of an individual's position in the network is highlighted on the basis of these examples. In particular, it shows how Nash equilibria are shaped. The number of Nash equilibria might be a concern from a coordination point of view. This is due to

⁸Hybrid and distributed equilibria add complexity since it is more difficult for individuals to coordinate.

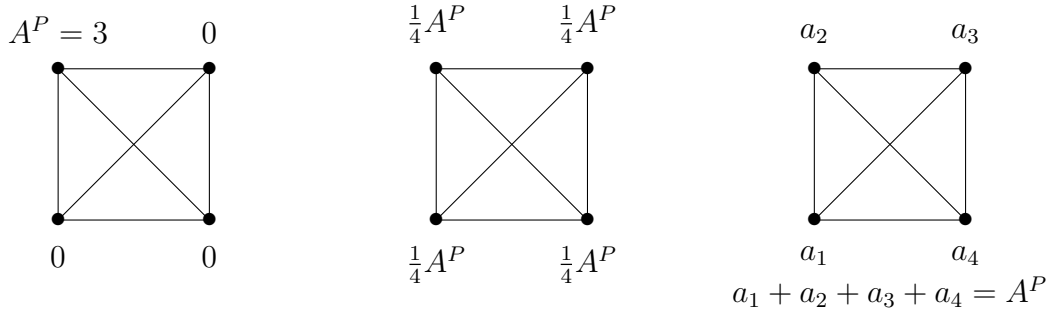


Figure 3: Equilibria in complete networks (specialized and distributed).

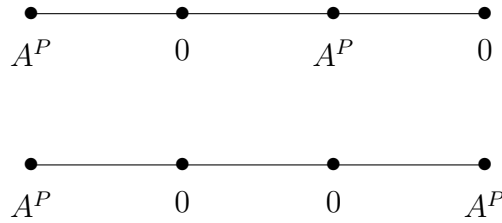


Figure 4: Equilibria in line networks.

strategic uncertainty (Van Huyck *et al.* 1990): economic agents may fail to coordinate their actions because they don't know which equilibrium to choose. In the next section, we will see if the implementation of the nudge may help to resolve this issue.

3.2 Social optima

We now study of social optima. It can be shown that the Nash equilibrium is lower than the social optimum. Indeed, let us consider the following utilitarian welfare function:

$$W(\mathbf{a}) = \sum_{i \in N} f \left(a_i + \sum_{j \in N_i} a_j \right) - \sum_{i \in N} ca_i \quad (3)$$

where $\mathbf{a} = (a_1, \dots, a_n)$ is the profile of contributions. The first-order condition is given by

$$f' \left(a_i + \sum_{j \in N_i} a_j \right) + \sum_{j \in N_i} f' \left(a_j + \sum_{l \in N_j} a_l \right) - c = 0 \quad (4)$$

By comparison with Equation (2), we note that individuals do not take into account the impact of their contributions on the benefit of their neighbors. This explains why the Nash equilibrium is lower than the social optimum.

Obviously, in a *regular*⁹ graph (like for the complete and circle networks), the social optimum is the same for everyone given that the number of direct neighbors is the same

⁹A graph is said to be regular if each individual has the same number of neighbors k_i .

for everyone. However, if individuals differ in the number of direct neighbors k_i , like in the star and line networks, then different social optima would be possible.¹⁰

To illustrate this, let us consider the example provided by Bramoullé and Kranton (2007; p.485) with the star network. Let us denote the contribution of an individual j in the periphery as $a_{per,j}$, and the one of the individual in the center as a_{center} . The first-order condition for an individual in the periphery is given by:

$$f'(a_{per,j} + a_{center}) + f' \left(a_{center} + a_{per,j} + \sum_{l \neq j} a_{per,l} \right) - c = 0 \quad (5)$$

That of an individual in the center is given by:

$$f' \left(a_{center} + \sum_{j \in N_{center}} a_{per,j} \right) + \sum_{j \in N_{center}} f'(a_{center} + a_{per,j}) - c = 0 \quad (6)$$

Let us consider individuals i and j , respectively, in the center and in the periphery. We denote the profile of the socially optimal contributions as A_1^* , where $a_{j,1}^* > 0$. Let us also consider A_2^* , the profile of socially optimal contributions such that $a_{j,2}^* = 0$ and $a_{i,2}^* = a_{i,1}^* + a_{j,1}^*$. Both profiles A_1^* and A_2^* yield the same level of contribution at the same cost. However, the higher contribution made by individual i in the second profile benefits more individuals. Thus, the second profile is more efficient than the first one.

A similar argument can be considered for line networks. Contributions of individuals on the extremities of the network benefit their direct neighbor and themselves. However, contributions of individuals inside the network benefit their two direct neighbors and themselves. As a consequence, individuals on the extremities should not contribute on the basis of the argument developed for star networks. Note that all individuals inside the network do not necessarily contribute since the contributions of the others may be enough. Some examples are provided in the Appendix.

To increase the aggregate level of contributions and to reduce strategic uncertainty, the implementation of a nudge is considered in the next section.

4 Implementation of a nudge under complete information

Let us consider the implementation of a nudge based on the announcement of the socially optimal contribution. In this section, we consider the case of a perfectly informed regulator who knows the structure of the network as well as each individual's position. Thus, the regulator can implement a personalized nudge for each individual, depending on her position, following the recommendations of Sunstein (2013).

As in Ouvrard and Spaeter (2016), the reaction to the nudge takes the form of a moral cost function that depends on the distance between the announcement and individual i 's contribution. The marginal disutility due to the nudge is higher for individuals with the highest environmental sensitivity. In this paper, we consider a more general setting. We

¹⁰Bramoullé and Kranton (2007) explain that it may be more efficient in a non-regular graph for some individuals to not contribute in an efficient profile (p. 485). In particular, if the neighborhood of some agent is a subset of the neighborhood of another agent, then it is more efficient if the agent with the largest neighborhood contributes because more agents benefit from her contributions. In that case, the agent with the smallest neighborhood should not contribute.

first describe how we model the reaction to the nudge, and then study the shape of the equilibrium profiles.

Formally, the moral cost cost is defined by the function $g(a_i - \hat{a}_i)$, where \hat{a}_i the announcement made by the regulator, $g(0) = 0$, $g' \lesseqgtr 0$ if $a_i - \hat{a}_i \lesseqgtr 0$, and $g'' > 0$. Thus, as long as contributions differ from the announcement, individuals incur the moral cost.¹¹ Note that individuals in circle or complete networks all have the same number of direct neighbors k_i . Thus, the aggregate social optimum is the same for everyone, and we can consider the implementation of a symmetric contribution. Let us denote the corresponding contribution, which is announced by the regulator, as \hat{a} .

Under the implementation of a nudge (announcement of the optimal contribution), individual i 's total utility is now given by:

$$U_i(a_i) = f(a_i + \sum_{j \in N_i} a_j) - ca_i - g(a_i - \hat{a}_i) \quad (7)$$

The first-order condition is given by

$$f'(a_i^N + \sum_{j \in N_i} a_j) - c - g'(a_i^N - \hat{a}_i) = 0 \quad (8)$$

where a_i^N is individual i 's level of contribution under the implementation of a nudge. Individuals now have to equalize the marginal benefit from contributing with the marginal cost of contributing and the marginal moral cost from deviating from the announcement.

The nature of the nudge depends on the network structure. In the circle and complete networks, we consider the announcement of the symmetric social optimal contribution. In the star and line networks, the nudge implemented is not the same across individuals. For those on the periphery, they are told not to contribute. For the individual(s) in the center, the optimal aggregate level of contribution is announced. We obtain the following proposition:

Proposition 1 *Assume that N individuals are members of a fixed network. Under complete information, if the regulator implements differentiated nudges according to individuals' positions in the network, then:*

- (i) *A symmetric distributed equilibrium exists in circle and complete networks.*
- (ii) *In circle networks, a specialized equilibrium may exist if, for individuals who do not contribute, the condition $-g'(0 - \hat{a}) \leq f'(A^P) - f'(\sum_{j \neq i} a_j^N)$ where $\sum_{j \neq i} a_j^N > A^P$ is satisfied.*
- (iii) *A distributed equilibrium may exist in star and line networks if,*
 $g'(a_i^N - 0) = f'(a_i^N + \sum_{j \in N_i} a_j^N) - f'(A^P)$, *where $a_i^N + \sum_{j \in N_i} a_j^N < A^P$, for an individual who should not contribute,*
and
 $-g'(a_i^N - \hat{a}) = f'(A^P) - f'(a_i^N + \sum_{j \in N_i} a_j^N)$ *where $a_i^N + \sum_{j \in N_i} a_j^N > A^P$, for an individual who should contribute.*
- (iv) *Specialized equilibria exist in star and line networks such that:*

¹¹In this setting, the formulation of the moral cost is similar to that of Figuères *et al.* (2013), who consider a moral cost function that depends on the distance to a moral ideal that individuals have. It is also similar to the model proposed by Brekke *et al.* (2003), in which the authors assume that individuals will incur a moral cost if they depart from their personal self-image.

(a) $a_i^N \in]A^P, A^*[,$ for those who should contribute, and the others do not contribute;
(b) Those who should contribute do not if $-g'(0 - \hat{a}) \leq f'(A^P) - f'(\sum_{j \neq i} a_j^N)$ where $\sum_{j \neq i} a_j^N > A^P$, and $g'(a_i^N - 0) = f'(a_i^N + \sum_{j \in N_i} a_j^N) - f'(A^P)$, where $a_i^N + \sum_{j \in N_i} a_j^N < A^P$, for individuals who should not contribute.

According to points (i) and (iv)(a), it seems possible that our nudge based on the disclosure of the socially optimal contribution helps individuals to coordinate their actions on the socially optimal profile of contributions, even if this profile is not implemented. Our nudge does not seem to be an optimal tool. However, the issue of strategic uncertainty still remains since other equilibria are possible according to points (ii), (iii) and (iv)(b).

The equilibrium condition $-g'(0 - \hat{a}_i) \leq f'(A^P) - f'(\sum_{j \neq i} a_j^N)$ states that an individual will not contribute if the marginal disutility from not contributing is lower than the gain in marginal benefit.

To illustrate our results, we propose another parametric example. In particular, we highlight the fact that it is also possible to interpret the conditions for the existence of equilibria that we obtained in the previous proposition as conditions for the sensitivity to the nudge (or "nudgeability", as in Farhi and Gabaix 2015), or the interest in the public good. More precisely, equilibria obtained in points (ii) and (iv)(b) are possible if the sensitivity to the nudge (or the interest in the public good) is low enough.

Example 2: Let us consider the following moral cost function:

$$g(a_i - \hat{a}_i) = \frac{m}{2} (a_i - \hat{a}_i)^2$$

where the parameter m is an individual's sensitivity to the nudge or her interest in the public good. Individual i 's utility is:

$$U_i(a_i) = 4 \ln \left(1 + a_i + \sum_{j \in N_i} a_j \right) - a_i - \frac{m}{2} (a_i - \hat{a}_i)^2$$

According to the first-order condition, we have:

$$\frac{4}{1 + A^N} - 1 - m(a_i - \hat{a}_i) = 0$$

Let us consider agents in a circle network and let us study the possibility of a specialized equilibrium as predicted by point (ii). Assume that each agent not contributing to the public good is separated by an agent who contributes to the equilibrium solution A^N . Thus, the first-order condition of an agent not contributing is:

$$\frac{4}{1 + 2A^N} - 1 - m(0 - \hat{a}) = \frac{4}{1 + 2A^N} - 1 + m\hat{a}$$

Compared with the first-order condition obtained in the previous example, we have:

$$\frac{4}{1 + 2A^N} - 1 + m\hat{a} \leq \frac{4}{1 + A^P} - 1 = 0$$

i.e. if:

$$m \leq \frac{\frac{4}{1+A^P} - \frac{4}{1+2A^N}}{\hat{a}} \quad (9)$$

In the previous example, we computed $A^P = 3$. According to the regulator objective given by Equation (3), the symmetric socially optimal contribution is $\frac{A^*}{3} = \hat{a} = 3.67$. With $m = 0.1$ (agents are not very sensitive to the nudge), we obtain that contributors invest $A^N = 3.20$ in the public good, and the expression on the left in (9) is equal to 0.13. We thus have $m = 0.10 < 0.13$, and a specialized equilibrium is possible. ■

On the basis of this second example, we can observe that the socially optimal profile of contributions could be implemented (provided that $m \neq 0$), if the regulator does not disclose the "true" value of the symmetric optimal contribution, but a higher value instead. However, such an approach by the regulator would not be an ethical solution. We discuss this issue in the last section.

Example 3: Let us consider the same functions as in the previous examples, with agents in a star network. We focus on point (iv)(b) of the previous proposition.

As explained in Section 3, in star networks, the socially optimal profile of contributions is such that the individual in the center contributes the optimal contribution, while those in the periphery should not contribute. In our case, we have $a_{center} = A^* = 15$ and $a_{per,j} = 0 \quad \forall j$.

Assume that individuals in the periphery contribute $a_{per,j}^N > 0$. If the individual in the center contributes a positive amount as well ($a_{center}^N > 0$), then individuals in the periphery are in equilibrium if their first-order condition is such that:

$$\frac{4}{1+a_{per,j}^N+a_{center}^N} - 1 - ma_{per,j}^N = 0$$

or, equivalently, if:

$$\frac{4}{1+a_{per,j}^N+a_{center}^N} - 1 - ma_{per,j}^N = \frac{4}{1+A^P} - 1 = 0$$

i.e. if:

$$m = \frac{\frac{4}{1+a_{per,j}^N+a_{center}^N} - \frac{4}{1+A^P}}{a_{per,j}^N} = \frac{\frac{4}{1+a_{per,j}^N+a_{center}^N} - 1}{a_{per,j}^N} \quad (10)$$

with $A^P = 3$ in this example. Notice that this condition is satisfied if $a_{per,j}^N + a_{center}^N \leq A^P$, to have $m \geq 0$. Similarly, we obtain that the condition for an equilibrium to be feasible for the individual in the center is:

$$m = \frac{\frac{4}{1+A^P} - \frac{4}{1+a_{center}^N + \sum_{j \in N_{center}} a_{per,j}^N}}{A^* - a_{center}^N} = \frac{1 - \frac{4}{1+a_{center}^N + \sum_{j \in N_{center}} a_{per,j}^N}}{A^* - a_{center}^N} \quad (11)$$

If $m = 0.01$ (individuals are not very sensitive to the nudge), we obtain $a_{per,j}^N = 0.29$ and $a_{center}^N = 2.70$. In that case, we have $a_{per,j}^N + a_{center}^N = 2.99 < A^P = 3$, and $a_{center}^N + \sum_{j \in N_{center}} a_{per,j}^N = a_{center}^N + 3a_{per,j}^N = 3.57 > A^P = 3$. Moreover, both conditions on m are satisfied. ■

This third example allows us to discuss the results we have obtained so far. In particular, we show that the existence of a unique equilibrium in both the circle and star networks remains on individuals' sensitivity to the nudge or their interest in the public good. In an environmental context (local biodiversity, protection of forests, etc.), the regulator could expect a higher environmental quality with a unique equilibrium when individuals are highly sensitive to the environment. This result differs from Ouvrard and Spaeter (2016) because, in their model, individuals are more or less sensitive to the environment and more or less optimistic regarding the risk of pollution. This last characteristic may interact with environmental sensitivity such that an individual highly sensitive to the environment but also highly optimistic concerning the risk of pollution may contribute less than an individual not very sensitive to the environment but also not very optimistic about the risk of pollution. In this paper, we do not consider attitudes toward risk and we focus on the content of the nudge depending on individuals' positions in the network and how the regulator may increase the total level of contributions by changing the content.

In the next section, we discuss the case of incomplete information from the regulator.

5 Implementation of a nudge under incomplete information

We now relax the assumption of complete information. In this section, the regulator only knows the structure of the network (complete, circle, star or line), but does not know each individual's position. The implementation of a nudge whose content depends on individuals' positions is no longer possible.

In a circle or a complete network, this loss of information is not, *a priori*, a concern, given that each individual has the same number of direct neighbors.¹² Considering a star or a line network, we saw in the previous sections that individuals' positions directly determine their level of contribution. Thus, two strategies may be implemented. In the first one, the regulator may announce to everyone that individuals in the periphery should not contribute. In the second one, the regulator may announce that the individual in the center should contribute the socially optimal contribution.

A third strategy may be considered: to announce the level of the socially optimal contribution to everyone, depending on their position in the star network.¹³ However, empirical evidence from laboratory experiments shows that individuals are "conditional cooperators" (Keser and van Winden 2000, Fischbacher *et al.* 2001): they contribute in the same proportions as the other individuals. In that case, individuals in the center of such networks could decide not to contribute regarding the level announced to those in the periphery. As a consequence, we do not consider this strategy in this paper.

The case of the line network is more specific as the number of individuals in the network, as well as the position inside determine the level of contribution in the socially optimal profile of contributions. We discuss it below.

5.1 Implementation of the nudge in circle and complete networks

Under incomplete information, the nudge implemented in a circle or complete network does not differ from the one implemented in these networks under complete information.

¹²"As if" individuals had the same position in the network.

¹³Everyone is told what the individual in the center should contribute, as well as what individuals in the periphery should contribute.

This is due to the fact that all individuals have the same number of direct neighbors (they have the "same" position).

As a consequence, individual i 's total utility is still given by (7), and the first-order condition is given by (8). We thus obtain Proposition 2:

Proposition 2 *Assume that N individuals are members of a fixed circle or complete network. The implementation of a nudge under incomplete information is equivalent to the one under complete information (points (i) and (ii) of Proposition 1).*

Regardless of the level of information that the regulator has concerning individuals arranged in a circle or complete network, the predicted equilibrium does not change. We could expect this result because individuals' positions in this type of network are not a direct determinant of their contributions in the public good. Given that everyone has the same position, it is the neighbors' behavior that determines the level of contribution.

Moreover, even if the regulator is in such a situation of incomplete information but has some knowledge about individuals' sensitivity to the nudge (or knows their interest in the public good), then strategic uncertainty may still be reduced provided this parameter is high enough.

5.2 Implementation of the nudge in star networks

We now consider the star network. As mentioned above, two strategies are possible: the individuals in the periphery are targeted, or the one in the center is targeted. In both cases, the content of the nudge is the same for all individuals (contrary to the previous section).

5.2.1 Targeting the individuals in the periphery

The regulator announces to the entire network that individuals in the periphery should not contribute. The total utility of the individual in the center does not change from (1) because this individual is not targeted by the nudge. However, the total utility of an individual j in the periphery becomes

$$U_{per}(a_{per}) = f(a_{per} + a_{centre}) - ca_{per} - g(a_{per} - 0) \quad (12)$$

The first-order condition is:

$$f'(a_{per}^N + a_{centre}) - c - g'(a_{per}^N - 0) = 0 \quad (13)$$

We obtain Proposition 3:

Proposition 3 *Assume that N individuals are members of a fixed star network. Under incomplete information, if the regulator implements a nudge that targets the individuals in the periphery, then:*

- (i) *A specialized equilibrium exists such that $a_{per,j}^N = 0 \quad \forall j$ and $a_{centre} = A^P$.*

(ii) A second specialized equilibrium may exist with $a_{per,j}^N > 0 \quad \forall j$ and $a_{center} = 0$, if $g'(a_{per}^N - 0) = f'(a_{per}^N + a_{center}) - f'(A^P)$ for individuals in the periphery.

(iii) A distributed equilibrium may exist if $g'(a_{per}^N - 0) = f'(a_{per}^N + a_{center}) - f'(A^P)$ for individuals in the periphery, and $a_{center} + \sum_{j \in N_{center}} a_{per,j}^N = A^P$ for the individual in the center.

Contrary to the results of Proposition 1 (under complete information), the implementation of the nudge does not necessarily lead to an increase in the aggregate level of contributions. Moreover, in Section 3 we saw that two specialized Nash equilibria were possible for a star network. Under the implementation of such a nudge, three equilibria are theoretically possible. Thus, this tool does not help to reduce strategic uncertainty in this particular context.

However, it is worth noticing that point (i) seems much more likely than points (ii) and (iii,) given the strong conditions these points require. Indeed, points (ii) and (iii) depend on individuals' sensitivity to the nudge.

We provide a numerical example to illustrate point (ii) of the previous proposition.

Example 4: Let us consider the same functions as in the previous examples.

If $a_{center} = 0$ and $m = 0.1$, we obtain $a_{per}^N = 2.262 < 3$. In that case, we have (for a star network with four individuals) $3 \times 2.262 > 3$, and the condition of point (ii) is satisfied:

$$g'(a_{per}^N - 0) = m(a_{per}^N - 0) = 0.1 \times 2.262 = 0.2262$$

and

$$f'(a_{per}^N + a_{center}) - f'(A^P) = \frac{4}{1 + 2.262} - \frac{4}{1 + 3} = 0.2262$$

■

5.2.2 Targeting the individual in the center

We now consider the case in which the regulator announces to everyone that the individual in the center should contribute the socially optimal contribution A^* .

The total utility of the individuals in the periphery is not affected by this announcement, and is given by (1). The one of the individual in the center becomes:

$$U_{center}(a_{center}) = f\left(a_{center} + \sum_{j \in N_{center}} a_{per,j}\right) - ca_{center} - g(a_{center} - A^*) \quad (14)$$

The first-order condition is:

$$f'\left(a_{center}^N + \sum_{j \in N_{center}} a_{per,j}\right) - c - g'(a_{center}^N - A^*) = 0 \quad (15)$$

We obtain Proposition 4:

Proposition 4 *Assume that N individuals are members of a fixed star network. Under incomplete information, if the regulator implements a nudge that targets the individual in the center, then:*

- (i) *A specialized equilibrium exists such that $a_{per,j} = 0 \quad \forall j$ and $a_{center}^N \in]A^P; A^*[$.*
- (ii) *A second specialized equilibrium may exist with $a_{per,j} = A^P \quad \forall j$ and $a_{center} = 0$, if $f'(A^P) - f'(\sum_{j \in N_{center}} A^P) = -g'(0 - A^*)$.*
- (iii) *A distributed equilibrium may exist if $a_{center}^N + a_{per,j} = A^P \quad \forall j$ in the periphery, and $f'(A^P) - f'(a_{center}^N + \sum_{j \in N_{center}} a_{per,j}) = -g'(a_{center}^N - A^*)$ with $a_{center}^N + \sum_{j \in N_{center}} a_{per,j} > A^P$ for the individual in the center.*

Again, the implementation of a targeted nudge does not help to reduce strategic uncertainty. However, as for Proposition 3, the equilibrium predicted by point (i) seems to be more likely than those predicted by points (ii) and (iii), given that they require more conditions.

Moreover, the levels of contributions in this proposition are higher than those obtained in the previous proposition and in Section 3 as well (in the absence of outside incentives). This result seems to indicate that, under incomplete information, a policy based on informational nudges should target individuals who should contribute.¹⁴

5.3 Implementation of the nudge in line networks

As emphasized below, socially optimal profiles of contributions in line networks depend on the number of individuals in the network. Moreover, we explained that individuals on the extremities of the network should not contribute, and some inside the network should not contribute as well. Lastly, the level of contribution also depends on individuals' positions (see the Appendix). Thus, socially optimal profiles of contributions in line networks may be more complex than in the other networks we focused on in this paper. As a consequence, a drawback of this nudge is that it is not necessarily suited for line networks under incomplete information.

We propose the implementation of a "second best" nudge, based on the announcement of the highest contribution of the individuals in the line network considered. More precisely, under incomplete information, the regulator announces to the entire network what the highest contributors should contribute. For instance, we propose the case of a line network with eight individuals in the appendix. In this example, the regulator could announce to the entire network that individuals in position 2 and 7 should contribute 9.75.¹⁵ Individuals 4 and 5 in the network should contribute 4.88 each. However, announcing their contribution level in the same message could raise fairness issues from their point of view. We thus exclude this possibility. As a consequence, a drawback of this "second best" nudge is that a non-perfect maximizing utility individual in position 4 (or 5) could consider contributing at a level close to the one suggested by the nudge. Given that we did not consider such a bias in this paper, we also exclude this possibility.¹⁶

¹⁴This point may appear obvious. Notwithstanding, efficiency in this type of network requires some individuals to not contribute. This motivated the study of these two strategies.

¹⁵This level of contribution has been computed using the same utility function as in the rest of the paper.

¹⁶This last issue could be considered as an extension of this paper.

Under the implementation of the "second best" nudge, the total utility of individuals who are not targeted is still given by (1) since they are not concerned by the nudge. For the individuals targeted, the total utility is now given by (7).

We obtain Proposition 5:

Proposition 5 *Assume that N individuals are members of a fixed line network. Under incomplete information, if the regulator implements a "second best" nudge that targets the individuals who should contribute the most, then:*

(i) *A specialized equilibrium exists such that individuals who are targeted contribute $a_i^N \in]A^P; \hat{a}_i[$, while the other individuals do not contribute if they are a direct neighbor of individuals who are targeted, or $a_j > 0$ otherwise, with $\sum_{j \neq i} a_j = A^P$.*

(ii) *A second specialized equilibrium may exist with individuals targeted not contributing if their direct neighbors contribute A^P , and $f'(A^P) - f'(2A^P) \geq -g'(0 - \hat{a}_i)$.*

(iii) *A distributed equilibrium may exist if, for an individual i who is targeted (8) is satisfied, i.e., $f'(A^P) - f'(a_i^N + \sum_{j \in N_i} a_j) = -g'(a_i^N - \hat{a}_i)$. For the other individuals, (2) must also be satisfied.*

As for the previous propositions, the sensitivity to the nudge (or the interest in the public good) is the key to the efficiency of this incentive tool with individuals in the network.

We provide a numerical example to illustrate point (iii) of the previous proposition.

Example 5: Let us consider the same functions as in the previous examples, with a line network with eight individuals (see the Appendix).

Consider the contribution profile illustrated in Fig. 5 obtained with $m = 0.03$ (very low sensitivity to the nudge):

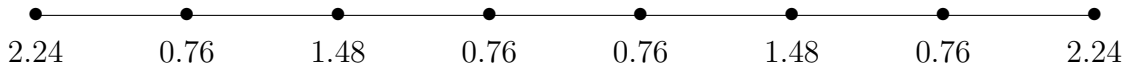


Figure 5: Distributed equilibrium in a line network with eight agents under the implementation of a second best nudge.

The conditions of point (iii) are satisfied:

$$-g'(a_i^N - 9.75) = -m(a_{per}^N - 9.75) = -0.03(0.76 - 9.75) = 0.27$$

and

$$f'(A^P) - f'\left(a_i^N + \sum_{j \in N_i} a_j\right) = \frac{4}{1+3} - \frac{4}{1+2.24+0.76+1.48} = 1 - 0.73 = 0.27$$

and the sum of contributions for individuals who are not targeted is always equal to 3 (A^P). ■

For this network under incomplete information, we do not consider the strategy that consists of targeting the individuals who should not contribute. Indeed, following the

same steps as for the star network (still under incomplete information), we can show that this strategy does not help to increase the level of contributions, or to reduce strategic uncertainty.

6 Discussion

In this section, we propose a discussion on the different results we have obtained so far. We first compare our conclusions to those of Bramoullé and Kranton (2007), and then discuss some implications of our results in terms of ethics.

In their paper, Bramoullé and Kranton (2007) focus on the existence and on the shape of Nash equilibria. As previously explained equilibria may be specialized, hybrid or distributed depending on the structure. In particular, the authors show that a specialized equilibrium always exists.

In this paper, we are concerned with socially optimal profiles of contributions and with strategic uncertainty. In the next subsections, we explain that it is possible to both increase the levels of contributions and reduce strategic uncertainty if nudge implementation targets the individuals most sensitive to environmental matters (or those with the highest interest in the public good).

On the basis of Proposition 1, we saw that, under the implementation of the informational nudge we consider, it is not possible to achieve the socially optimal outcome if the regulator announces the "true" contribution levels. However, it could be possible to achieve the socially optimal outcome if the regulator would announce a higher level of contributions than the socially optimal ones (provided that the sensitivity to the nudge is positive).

This solution would raise some ethical concerns since it would manipulate individuals. Hausman and Welch (2010), Goodwin (2012) and Vallgarda (2012) have proposed a discussion on the manipulation induced by the implementation of nudges. More recently, Sunstein (2016) argued that nudges do not manipulate individuals in the same way so it would be necessary to consider "shades of manipulation". In particular, an action is said to be manipulative if the agent cannot sufficiently carry out her capacity for reflection. Hansen and Jespersen (2013) propose to distinguish nudges according to their *transparency* (if individuals can detect their implementation) and their impact on *System 1* or *System 2* (Kahneman (2003)).¹⁷ Nudges that cannot be detected by individuals (*non-transparent*) and that impact *System 1* are those that manipulate the most individuals. Those that are transparent and that impact *System 2* are those that manipulate the least individuals.

In this paper, the nudge we consider is based on the disclosure of a piece of information. Given that individuals receive this piece of information that they would not receive otherwise, we can consider that our nudge is transparent (like the one proposed in Allcott 2011, Ayres *et al.* 2013, Costa and Kahn 2013, Ferraro and Price 2013). Moreover, this piece of information needs to be analyzed by individuals, thus requiring an action by *System 2*. Finally, we can consider our nudge as one that manipulates the least individuals.

¹⁷According to Kahneman (2003), individuals act according to two systems of thinking. The first one, System 1, is quick and automatic. The second one, System 2, is slow and rational (this is the system that is used when individuals need to think before acting).

Notwithstanding, were the regulator to announce a higher value in order to achieve the socially optimal outcome, our nudge would then be much less ethical.

According to Proposition 2, the loss of information from the regulator's point of view does not seem to be a concern with the circle and complete networks, given that we obtained the same results as those under Proposition 1. For the star (Propositions 3 and 4) and line (Proposition 5) networks, we saw that strategic uncertainty is increased by comparison with the situation in the absence of outside incentives. This may be due to the fact that the regulator can no longer target individuals and send a nudge whose content is not adapted to certain individuals' positions in the network.

Lastly, the "second best" nudge we propose loses in precision compared to the differentiated nudges we started with. Moreover, as for the nudge implemented in star networks under incomplete information, this kind of nudge that targets some individuals in the network may be seen as unfair for targeted individuals since it suggests that some individuals should contribute, without giving any precision on what the others in the network should do. This is why such nudges should be tested in the laboratory.

7 Conclusion

In this paper, we considered the implementation of an informational nudge to increase contributions to a local public good such as environmental quality or local biodiversity. Our objective was also to implement these instruments to observe the possibility to reduce the number of possible equilibria. Indeed, we highlighted that in the model proposed by Bramoullé and Kranton (2007), individuals may have difficulties to choose their level of contribution due to multiple equilibria.

We show that in complete and circle networks, regardless of the level of information the regulator has on individuals' position, the aggregate level of contributions may increase compared to the Nash equilibrium, and strategic uncertainty may be reduced (the predicted equilibrium is unique) as long as individuals' sensitivity to the nudge (or their interest in the public good) is high enough. However, in line and star networks, the results are mixed. Under complete information, two equilibria are still possible (one specialized and one distributed), but the specialized equilibrium seems to be more likely as it requires less conditions. Under incomplete information, two equilibria are once again possible, regardless of the type of individuals targeted by the content of the nudge. Notwithstanding, if the individuals in the periphery are targeted, the aggregate level of contributions does not necessarily increase compared to the Nash equilibrium.

In sum, these results seem to indicate that the implementation of a nudge in networks has to target some individuals in order to be as efficient as possible. More importantly, these results seem to indicate that it may be possible to induce a higher level of contributions.

However, this result is highly dependent on the properties linked to the moral cost function. Like in Ouyard and Spaeter (2016), we considered very general properties. Future research and especially experimental research should try to characterize the way individuals react to such an informational nudge. Moreover, the results of this paper should also be tested.

Another extension of this paper could be to consider the implementation (and the modelization) of other nudges. Indeed, we highlighted some drawbacks of our nudge

for line networks under incomplete information. Finally, heterogeneity in individuals' sensitivity to the nudge could be considered as well.

APPENDIX

Proof of Proposition 1

(i) We first consider the case of a circle network (each individual having two direct neighbors). Let us assume that an equilibrium exists, and let us denote the corresponding aggregate level of contributions as A^N . We begin by showing that this equilibrium is a symmetric one. Under the implementation of the nudge, the first-order condition for an individual in the circle network is:

$$f'(a_i^N + a_{i-1}^N + a_{i+1}^N) - c - g'(a_i^N - \hat{a}) = 0$$

where a_{i-1}^N and a_{i+1}^N are individual i 's direct neighbors (to the left and to the right).

Let us consider the symmetric contribution $\frac{A^N}{3}$. Moreover, let us assume that individual $i - 1$ contributes $\frac{A^N}{3} - \epsilon$, $\epsilon > 0$, while individual $i + 1$ contributes $\frac{A^N}{3} + \epsilon$.

The first-order condition of individual $i + 1$ is

$$f'(A^N) - c - g'\left(\frac{A^N}{3} + \epsilon - \frac{A^*}{3}\right) < 0$$

while the one for individual $i - 1$ is

$$f'(A^N) - c - g'\left(\frac{A^N}{3} - \epsilon - \frac{A^*}{3}\right) > 0$$

These two individuals are not in equilibrium given that their first-order conditions are not satisfied with equality. Individual $i + 1$ has an interest in reducing her level of contribution to reduce her marginal moral benefit, while individual $i - 1$ has an interest in increasing her level of contribution to reduce her marginal moral cost. Thus, if an equilibrium exists, it is symmetric.

We now show that an equilibrium exists. Let us consider the case in which $\frac{A^*}{3} \leq A^P$. If all individuals contribute $\frac{A^*}{3}$, then their first-order condition is such that

$$f'(A^*) - c - g'(0) < 0$$

All individuals have an interest in reducing the level of their contributions to increase the marginal benefit of contributions.

If all individuals contribute $\frac{A^P}{3}$, then their first-order condition is such that

$$f'(A^P) - c - g'\left(\frac{A^P}{3} - \frac{A^*}{3}\right) > 0$$

All individuals have an interest in increasing the level of their contributions. Thus, an equilibrium exists such that $\frac{A^P}{3} < a_i^N < \frac{A^*}{3}$ or, equivalently, $A^P < A^N < A^*$.

Following the same steps for the case in which $\frac{A^*}{3} > A^P$, we show that the only existing equilibrium is such that $\frac{A^P}{3} < a_i^N < \frac{A^*}{3}$.

The proof for the complete network follows the same steps.

(ii) Let us consider individuals in a circle network, and let us assume that individuals $i + 1$ and $i - 1$, individual i 's two direct neighbors (to the right and to the left), do not contribute. Let us denote individual i 's equilibrium contribution when individuals $i + 1$ and $i - 1$ do not contribute as a_i^N , i.e., a_i^N is such that:

$$f'(a_i^N) - c - g'(a_i^N - \hat{a}) = 0$$

In turn, individual $i + 1$ is in equilibrium if her first-order condition is such that:

$$f' \left(\sum_{j \in N_{i+1}} a_j^N \right) - c - g'(0 - \hat{a}) \leq 0$$

or, equivalently,

$$f' \left(\sum_{j \in N_{i+1}} a_j^N \right) - c - g'(0 - \hat{a}) \leq f'(A^P) - c = 0$$

i.e., if

$$-g'(0 - \hat{a}) \leq f'(A^P) - f' \left(\sum_{j \in N_{i+1}} a_j^N \right)$$

provided that $\sum_{j \in N_{i+1}} a_j^N > A^P$. Then individual $i + 2$ is in equilibrium if

$$f'(a_{i+2}^N) - c - g'(a_{i+2}^N - \hat{a}) = 0$$

if the number of individuals in the network is even, or if:

$$f'(a_{i+2}^N + a_{i+3}^N) - c - g'(a_{i+2}^N - \hat{a}) = 0$$

if the number of individuals is odd.

(iii) Let us consider that $a_i^N > 0$ for individuals who should not contribute. The first-order condition for such an individual is given by:

$$f' \left(a_i^N + \sum_{j \in N_i} a_j^N \right) - c - g'(a_i^N - 0) = 0 \quad (16)$$

which may be satisfied if $\sum_{j \in N_i} a_j^N < A^P$.

Let us consider the case of the star network. The first-order condition for the individual in the center is

$$f' \left(a_{centre}^N + \sum_{j \in N_{centre}} a_{per,j}^N \right) - c - g'(a_{centre}^N - A^*) \quad (17)$$

If $a_{centre}^N > 0$, Equation (19) is satisfied if it is equal to Equation (2), i.e., if $-g'(a_{centre}^N - A^*) = f'(A^P) - f'(a_{centre}^N + \sum_{j \in N_{centre}} a_{per,j}^N)$, which also requires $a_{centre}^N + \sum_{j \in N_{centre}} a_{per,j}^N > A^P$.

The proof follows the same steps as for a line network.

(iv) (a) Let us consider that $a_i^N = 0$ for individuals who should not contribute (following the optimal profile of contributions). The first-order condition of such an individual is

$$f' \left(a_i^N + \sum_{j \in N_i} a_j^N \right) - c - g'(a_i^N - 0) = 0 \quad (18)$$

Equation (18) is satisfied if $\sum_{j \in N_i} a_j^N \geq A^P$. In a star network, if $a_{centre}^N = A^*$, then the first-order condition for the individual in the center is such that:

$$f'(a_{centre}^N) - c - g'(0) < 0$$

The individual in the center is not in equilibrium and has an interest in reducing her level of contributions.

If $a_{centre}^N = A^P$, then the first-order condition for the individual in the center is such that:

$$f'(a_{centre}^N) - c - g'(a_{centre}^N - A^N) > 0$$

Again, the individual in the center is not in equilibrium and has an interest in increasing her level of contributions. Thus, a contribution exists $a_{centre}^N \in]A^P; A^*[$ such that the first-order condition of the individual in the center is satisfied.

The proof follows the same steps for a line network.

(b) Let us consider that $a_i^N > 0$ for individuals who should not contribute. The first-order condition for such an individual is given by (18), which may be satisfied if $\sum_{j \in N_i} a_j^N < A^P$.

Let us consider the case of the star network. The first-order condition for the individual in the center is

$$f' \left(a_{centre}^N + \sum_{j \in N_{centre}} a_{per,j}^N \right) - c - g'(a_{centre}^N - A^*) \quad (19)$$

If $a_{centre}^N = 0$, Equation (19) is satisfied if it is less or equal to Equation (2), i.e., if $-g'(0 - A^*) \leq f'(A^P) - f'(\sum_{j \in N_{centre}} a_{per,j}^N)$, which also requires $\sum_{j \in N_{centre}} a_{per,j}^N > A^P$.

The proof follows the same steps as for a line network.

◆

Proof of Proposition 2

The proof follows the same steps as the proof of points (i) and (ii) of Proposition 1. ◆

Proof of Proposition 3

(i) Let us consider that $a_{per,j}^N = 0 \quad \forall j$. The first-order condition for an individual in the periphery receiving the nudge is given by (13). The one of the individual in the center is given by (2).

Knowing that $a_{per,j}^N = 0 \quad \forall j$, and considering Equations (13) and (2), all individuals are in equilibrium if $a_{centre} = A^P$.

(ii) Let us consider that $a_{per,j}^N > 0 \quad \forall j$. According to the first-order condition for an individual in the periphery given by (13), these individuals are in equilibrium if:

$$f'(a_{per,j}^N + a_{centre}) - c - g'(a_{per,j}^N - 0) = 0$$

i.e., if:

$$f'(a_{per,j}^N + a_{centre}) - c - g'(a_{per,j}^N - 0) = f'(A^P) - c = 0$$

or, equivalently, if

$$g'(a_{per,j}^N - 0) = f'(A^P) - f'(a_{per,j}^N + a_{centre})$$

A condition for this last equation to hold is that $a_{per,j}^N + a_{centre} < A^P$.

In turn, the individual in the center is in equilibrium while not contributing if $\sum_j a_{per,j}^N \geq A^P$.

(iii) Following the same steps as the previous point with $a_{per,j}^N > 0 \quad \forall j$ and $a_{centre} > 0$, Equation (19) may be satisfied if $a_{per,j}^N + a_{centre} < A^P$, and Equation (2) may be satisfied if $a_{centre} + \sum_j a_{per,j}^N = A^P$. ♦

Proof of Proposition 4

The proof of this proposition follows the same steps as the previous one, with Equation (2) that has to be satisfied for individuals in the periphery, and Equation (8) that has to hold for the individual in the center. ♦

Proof of Proposition 5

(i) Let us consider a line network. For individuals who are targeted by the second best nudge, their first-order condition is given by Equation (8). As shown in the previous proof, if the direct neighbors of targeted individuals do not contribute, then targeted individuals contribute $a_i^N \in]A^P; \hat{a}_i[$ to be in equilibrium.

If a direct neighbor j of a targeted individual has a direct neighbor who is not a direct neighbor of another targeted individual (for line networks with $n \geq 7$), then her first-order condition given by Equation (2) is satisfied if she contributes $a_j = A^P$ or if $a_j + \sum_{l \in N_j} a_l = A^P$.

(ii) Let us assume that the targeted individuals' direct neighbors contribute the Nash equilibrium solution A^P . Then those who are targeted will not contribute if their first-order condition is such that:

$$f'(2A^P) - c - g'(0 - \hat{a}_i) \leq 0$$

i.e., if

$$f'(2A^P) - c - g'(0 - \hat{a}_i) \leq f'(A^P) - c = 0$$

or, equivalently, if

$$-g'(0 - \hat{a}_i) \leq f'(A^P) - f'(2A^P)$$

Note that this is true for line networks with $n > 4$ (two targeted individuals have to be separated by at least one individual).

The proof of point (iii) follows the same steps as the previous one, with the direct neighbors of targeted individuals contributing $0 < a_i < A^P$.



Examples of socially optimal profiles of contributions in line networks

Below, we provide some examples of socially optimal profiles of contributions in different line networks. We considered the same utility function as in the paper. Note that for individuals not contributing, their first-order condition (obtained from the program of an utilitarian regulator) is always negative.

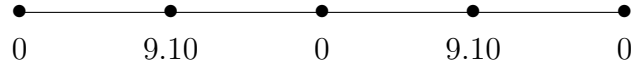


Figure 6: Socially optimal contributions in a line network with five agents.



Figure 7: Socially optimal contributions in a line network with six agents.

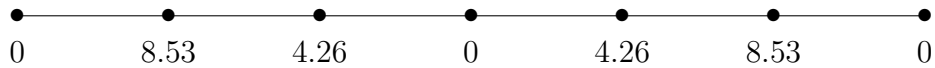


Figure 8: Socially optimal contributions in a line network with seven agents.

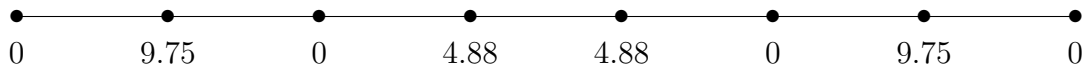


Figure 9: Socially optimal contributions in a line network with eight agents.

References

- [1] Allcott, H., 2011. "Social Norms and Energy Conservation." *Journal of Public Economics*, 95: 1082–1095.
- [2] Allouch, N., 2015. "On the private provision of public goods on networks." *Journal of Economic Theory*, 157: 527–552.
- [3] Ayres, I., S. Raseman and A. Shih, 2013. "Evidence from two large field experiments that peer comparison feedback can reduce residential energy usage." *Journal of Law, Economics and Organization*, 29: 992–1022.
- [4] Banerjee, S., F.P. de Vries, N. Hanley and D.P. van Soest, 2014. "The Impact of Information Provision on Agglomeration Bonus Performance: An Experimental Study on Local Networks." *American Journal of Agricultural Economics*, 96: 1009–1029.
- [5] Berglas, E. and D. Pines, 1981. "Clubs, local public goods and transportation models." *Journal of Public Economics*, 15: 141–162.
- [6] Bloch, F. and U. Zenginobuz, 2007. "The effect of spillovers on the provision of local public goods." *Review of Economic Design*, 11: 199–216.
- [7] Bramoullé, Y. and R. Kranton, 2007. "Public goods in networks." *Journal of Economic Theory*, 135: 478–494.
- [8] Bramoullé, Y., R. Kranton and M. D'Amours, 2014. "Strategic Interaction and Networks." *American Economic Review*, 104: 898–930.
- [9] Brekke, K.A., S. Kverndokk and K. Nyborg, 2003. "An economic model of moral motivation." *Journal of Public Economics*, 87: 1967–1983.
- [10] Costa, D.L. and M.E. Kahn, 2013. "Energy conservation "nudges" and environmentalist ideology: evidence from a randomized electricity field experiment." *Journal of the European Economic Association*, 11: 680–702.
- [11] Farhi, E. and X. Gabaix, 2015. "Optimal Taxation with Behavioral Agents." Mimeo.
- [12] Ferraro, P. and M.K. Price, 2013. "Using Non-Pecuniary Strategies to Influence Behavior: Evidence from a Large Scale Field Experiment." *The Review of Economics and Statistics*, 95: 64–73.
- [13] Figuières, C., D. Masclet and M. Willinger, 2013. "Weak Moral Motivation Leads to the Decline of Voluntary Contributions." *Journal of Public Economic Theory*, 15: 745–772.
- [14] Fischbacher, U., S. Gächter and E. Fehr, 2001. "Are people conditionally cooperative ? Evidence from a public goods experiment." *Economics Letters*, 71: 397–404.
- [15] Goodwin, T., 2012. "Why We Should Reject "Nudge"." *Politics*, 32: 85–92.
- [16] Hansen, P.G. and A.M. Jespersen, 2013. "Nudge and the Manipulation of Choice - A Framework for the Responsible Use of the Nudge Approach to Behaviour Change in Public Policy." *The European Journal of Risk Regulation*, 4: 3–28.

- [17] Harding, M. and A. Hsiaw, 2014. "Goal Setting and Energy Conservation." *Journal of Economic Behavior and Organization*, 107: 209–227.
- [18] Hausman, D.M and B. Welch, 2010. "To Nudge or Not to Nudge." *The Journal of Political Philosophy*, 18: 123–136.
- [19] Johnson, E.J. and D. Goldstein, 2003. "Do Defaults Save Lives?" *Science*, 302: 1338–1339.
- [20] Kahneman, D., 2003. "Maps of Bounded Rationality : Psychology for Behavioral Economics." *American Economic Review*, 93: 1449–1475.
- [21] Keser, C. and F. van Winden, 2000. "Conditional Cooperation and Voluntary Contributions to Public Goods." *The Scandinavian Journal of Economics*, 102: 23–39.
- [22] Kurz, T., M. Linden and N. Sheedy, 2007. "Attitudinal and Community Influences on Participation in New Curbside Recycling Initiatives in Northern Ireland." *Environmental and Behavior*, 39: 367–391.
- [23] López-Pintado, D., 2013. "Public goods in directed networks." *Economics Letters*, 121: 160–162.
- [24] Madrian, B.C. and D.F. Shea, 2001. "The Power of Suggestion: Inertia in 401(k) Participation and Savings Behavior." *The Quarterly Journal of Economics*, 116: 1149–1187.
- [25] McCallum, W., K.F.D. Hughey and S.S. Rixecker, 2007. "Community Environmental Management in New Zealand: Exploring the Realities in the Metaphor." *Society and Natural Resources*, 20: 323–336.
- [26] Ouvrard, B. and S. Spaeter, 2016. "Environmental incentives: nudge or tax?" BETA Working Paper 2016-23.
- [27] Sanditov, B. and S. Arora, 2016. "Social network and private provision of public goods." *Journal of Evolutionary Economics*, 26: 195–218.
- [28] Scotchmer, S., 2002. "Local public goods and clubs." In A. J. Auerbach and M. Feldstein (eds.). *The Handbook of Public Economics*, 4, 1997–2042, Elsevier.
- [29] Sunstein, C.R., 2013. "The Storrs Lectures : Behavioral Economics and Paternalism." *Yale Law Journal*, 122: 1826–1899.
- [30] Sunstein, C.R., 2016. "Fifty Shades of Manipulation." *Journal of Marketing Behavior*, 1: 213–244.
- [31] Thaler, R. H. and Sunstein C. R., 2009. *Nudge: Improving Decisions about Health, Wealth, and Happiness*. Penguin Books.
- [32] Tiebout, C.M., 1956. "A Pure Theory of Local Expenditures." *The Journal of Political Economy*, 64: 416–424.
- [33] Vallgarda, S., 2012. "Nudge - A new and better way to improve health ?" *Health Policy*, 104: 200–203.

- [34] Van Huyck, J.B., R.C. Battalio and R.O. Beil, 1990. "Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure." *American Economic Review*, 80: 234–248.
- [35] Videras, J., A.L. Owen, E. Conover and S. Wu, 2012. "The influence of social relationships on pro-environment behaviors." *Journal of Environmental Economics and Management*, 63: 35–50.