# To Migrate with or without ones' Children - the Chinese Case * 

Yiwen Chen ${ }^{\dagger}$ Vincent Fromentin ${ }^{\ddagger}$ Ioana Salagean ${ }^{\S}$ Benteng Zou ${ }^{〔}$


#### Abstract

What should Chinese internal migrant parents do with their children: taking them to migrate or leaving them behind? Empirical study based on the Rural-Urban Migration Survey in China data is inconclusive. So over-lapping-generation model is employed. Depending on the relative income level and the relocation cost of children, we provide necessary and sufficient conditions when migrant parents should take their children to migrate and how should they provide their children with private education. Furthermore, those optimal choices of migrant parents have impacts not only on their children's human capital accumulation, but also on the economic potential of their descendants in the future.


Keywords: Migrant children; left-behind children; hukou; relocation cost; relative income. JEL classification: ....

[^0]
## 1 Introduction

China has been witnessing a tremendous growing number of rural-to-urban migrants since the economic reform in 1978. The China Labor Bulletin ${ }^{1}$ reports that in 2015 there are total about 277.5 million internal Chinese migrant workers that is, $36 \%$ of the total workforce. Among all, 158.63 million (an increase of $3.4 \%$ compared to 2010) migrant workers who left their hometown and worked in other provinces and 94.15 million (an increase of $5.9 \%$ compared to 2010) migrant workers who worked within their home provinces. On the one hand, huge amount of migrant workers flowing into cities booms China's economic manufacturing; but on the other hand, it also poses different social issues. One of these issues which attract lots of attentions in recent years are that what should these migrants do with their children: leave them behind or take them to migrate?

Arguably, migrant workers face decision dilemma: If they leave their children behind, due to the lack of parental care and discipline, their children may suffer from mood swings, stress, depression, anxiety disorders and so on mental health problems (Lee, 2011; Hu et al., 2014; Zhao and Yu, 2016; Ye and Lu, 2011; Qin and Albin, 2010). However, if they bring their children to the city, migrant children cannot enjoy the same education opportunity(Liu et al., 2016; Li et al., 2010; Wang, 2008) and public health care (Milcent, 2010; Mou et al., 2013; Lu et al., 2016; Sun et al., 2016) as local urban children do due to the Chinese special household registration (hukou in Chinese) system, which we will present briefly in the next section. Poverty in some rural areas has forced millions of Chinese parents to make this impossible choice.

In this paper, we attempt to investigate what would be Chinese migrant workers' optimal choices: leaving their young children behind in the rural home village or taking them to the city? What could be their optimal educational choice for their children and

[^1]what would be the consequences of these choices in the short- and long-run?
According to the All-China Women's Federation's 2013 report, ${ }^{2}$ shown in Figure 1(a), in 2013, about 61 million Chinese children - one of every five in China - are left behind children ${ }^{3}$ who are left in the rural hometown and haven't seen one or both parents for at least three months in a year. They stay with either one parent or with relatives, usually grandparents ${ }^{4}$, friends or boarding schools. To make things even worse, nearly 3.4 percent of left behind children live alone. ${ }^{5}$ Due to the lack of parental supervision, some left-behind children felt into victims of tragedies, ${ }^{6}$ such as suicide, abused and human traffickers' targets; or they end up as street children and live on the edges of society.

[^2]
## (a)

## CHINA'S LEFT BEHIND CHILDREN

Some 61 million Chinese children grow up without one or both their parents - one in five nationwide.

HOW OLD ARE THEY?


HOW OFTEN DO THEY SEE THEIR PARENTS?


HOW DO THEIR PARENTS FEEL?
92\% said they believe raising children
is their responsibility.

## WHO CARES FOR THEM?


$5: \%$ to worry about their children.
(b)


Economist.com

Figure 1: (a) Left-behind Children (b) Children in China

Can whole family migration solve the left-behind children problem? Lots of migrant workers indeed bring their children into cities. Figure 1(b) displays percentage of all Chinese children in 2010. According to a survey reported by United Nations Children's Fund (Zhang, 2014) and All-China Women's Federation's 2013 report, in 2010, one out of every four children in China's urban areas was a migrant child who migrated together with their parents to the city. In 2013, that proportion rose to one out of three a total of 35.81 million migrant children in China. Arguably, Chinese migrant workers are different from the rural-urban migration of Lucas (2004), where the migrants can be part of the urbanization, while Chinese migrant workers still hold rural hukou and belong to the rural regions. Migrant children, who inherit the rural hukou from their parents, don't get the same rights to go to state public school (Liu et al., 2016; Li et al., 2010; Wang, 2008) as local urban children do. Therefore, the migrant children are either forced to go to makeshift, non-government schools, which some time short of adequate teachers and some time only profits making, or to pay higher fees in order to go to public schools(Liu et al., 2015; Lu and Zhang, 2004; Wong et al., 2007; Wang and Holland, 2011). Though the governments of some provinces are reforming the hukou system, migrant children were not treated equally. The urban schools prefer local urban children and set up extra obstacles for migrant children (Lu and Zhang, 2004; Liu et al., 2015). ${ }^{7}$ On top of the limitation of accessing to public schools, due to the hukou constraint, most of the migrant children do not have public health care in the city either (Lu et al., 2016; Sun et al., 2016). ${ }^{8}$

Recently, there have been a growing number of studies on the issue of Chinese internal migrant to explore the impact of parental absence on the outcome of left-behind and

[^3]migrant children, especially on educational outcome. The findings are mixed. Chen et al. (2009) points out that parental migration has no effect on school performance of their left-behind children. On the contrary, father out-migration improves the performance of left-behind children. Using data from north-Eastern provinces of Hebei and Liaoning, Meyerhoefer and Chen (2011) finds empirical evidence that parental migration is associated with a lag in grade-level attainment for left-behind children, especially for girls. Zhao et al. (2014), Lu (2012), Zhang et al. (2014), Meng and Yamauchi (2015) and Lu et al. (2016) conclude that parental migration could ruin the the scores of left-behind child compared to children without parental migration. Unfortunately, the outcomes from the studies on Chinese migrant children are not better. Chen and Feng (2013), Lu et al. (2016) and Sun et al. (2016), among others, report that a significant proportion of migrant children in China are not able to attend public schools for the lack of local hukou and turn to privately-operated migrant schools. These studies also suggest that access to public schools is the key factor determining the quality of education that migrant children receive.

To the best of our knowledge, even international migration studies, ${ }^{9}$ there has been no research comparing the differences between left-behind and migrant children. Thus, it is impossible to provide migrant parents with optimal choice of their decision on their children. Therefore, the current study is the first of its kind focusing on the optimal choice of rural-to-urban migrants with regard to the location of their children: taking children with them or leaving them behind. To do so, we first use the Rural Urban Migration in China data set to test whether there exists heterogeneity in school performances between left-behind and migrate children. According to the school perfor-

[^4]mance of children, keeping all other variables being equal, migrant workers' decision, whether taking them or not, may depend on the age of children. Advantages of studying in the city is stronger when the children are younger, but this advantage disappears among children who are in the junior high school.

Arguably, due to the limitation of data, we can not take into account the effects of unobserved heterogeneity and this prevents us from performing further empirical analysis. We thus have to account on theoretical model to obtain conclusive results. By employing an over-lapping generation (OLG) model, we demonstrate that the migrant workers' decision - taking their children to migrate and providing how much private education to their children - essentially depend on relocation-cost of children and relative-income. Here, relocation-cost refers to the extra fees paid to enroling children in urban school, extra health care due to lack of local urban hukou, extra living cost of their children being in the city and so on. The relative-income is measured by a share of migrant workers' lifetime aggregate income which is invested in children's education, where the share is calculated as the ratio of the importance of children's education in term of whole family's consumption.

Most importantly, the OLG model provides us with relocation-cost thresholds for different relative-income levels, which are the necessary and sufficient condition for migrant parents' decision of taking their children to the city or leaving them behind. Furthermore, these threshold depend not only on the migrants' life-time aggregate income, but also on the public educational gaps between the migrants' rural hometown and their destination working city. The larger the gap is, the higher is the threshold. In other words, the urban and rural educational imbalance is am implicit barrier for migrant workers taking their children with them.

In addition to the above migrating decision, the private education decision of migrant parents depends on the relationship between relative-income and the public education input. Not surprisingly, sufficiently high public education input discourages parents' incentive to invest in private education. In case of private education, the standard re-
sult in the literature is that high income parents provide more private education to their children than the low incomes ones. Our finding of private educational input depends on relative-income, which relies on how parents measure the importance of the education and life-time income. Thus, if all parents care their children's education equally, the standard result in the literature hold as well in the current setting. Nonetheless, it may happen that low income parents pay more private education to their children than the high income ones, because these poor parents value education more highly than the rich ones since that could be the only way for their children to change their lives. The data of Rural Urban Migration in China confirms this finding.

Additionally, migrant parents' migration and private education decision of their children influence not only on their children's human capital accumulation directly, but also forms the economic potential of their future offsprings.

The rest of paper is arranged as follows. Section 2 describes data and empirical methodologies. In Section 3, we employ an overlapping generation model to obtain the optimal choice of consumption and children's education investment. In Section 4, we provide answers to our original questions: When should migrant workers take their children to migrate and when should they leave their children behind. The Section 5 presents the dynamics and long-run outcome and we conclude in Section 6.

## 2 Empirical Analysis

Before presenting the data and empirical strategies, in the first subsection, we recall the Chinese hukou system given that is the most difficult barrier which the migrant parents can not avoid and that is the essential condition for our results. This subsection is based on Hao and Yu (2015).

### 2.1 Chinese hukou system and migrant children's access to urban public school

Hukou system, established in 1950s, is a strictly enforced household registration system in China and determines where citizens are supposed to live. Hukou's registration place is based on citizens' permanent residence in the 1950s. Each citizen can register one and only one place of residence. Children automatically inherit the status of hukou of one of their parents. Hukou defines one's rights for social and economic activities in a specified locality, i.e. citizens enjoy social benefits only in their registration place. Before 1970s, the rural population were restricted from moving to urban areas. Since 1980s, with the reformation of economy and development of urban areas, mobility from rural to urban areas have been gradually allowed by the government. Citizens with age equal to or above 16 could apply for the Certificate of Temporary Residence if they want to stay in urban areas for more than 3 month. It is valid for one year and is renewable. Thanks to loosing restrictions on hukou system, more and more rural residents work and stay in urban areas.

However, as mentioned before, hukou sets limitations on migrant workers' access to social benefit, as well as their children's. Due to the lack of local urban hukou, migrant workers' children face more obstacles in order to enrol in the public school in urban areas. The Provisional Regulations on Schooling for Children of Migrant Populations in Cities and Townships of 1996 and 1998 are the central government regulations targeting for the education of migrant children. According to these policies, the local government of origin should strictly control emigration of school-age children. If these children have custodians in their hukou registration place, they should receive the compulsory education ${ }^{10}$ in that place. If do not have custodians, they could register with extra fees in the public school of the destination city. These stumbling blocks stimulate the formation of privately-run, low-quality and profit-driven school, specialized only for

[^5]migrant children.
Some progress have been made since 2000. Article 12 of 2006 amendment says: "For school-aged children or adolescents, who have parents or other legal custodians working or living in places other than the hukou-registration places, who receive compulsory education in places other than the hukou-registration places, local government should provide them with equal conditions in receiving compulsory education. Specific policy is determined by province, autonomous region and municipality."

Unfortunately, not all local governments strictly follow the rules, for example, Bejing and Zhejiang province continue to ask the proof that "no custodians in the home village". Schools still impose some "special fees " for migrant children. Despite efforts have been made in recently years, it remains barriers for them studying in the urban areas.

With this special hukou system, though quality of education in urban areas are supposed to be higher than the rural village, migrant workers' children may not be better off even if they are taken to the city. Therefore, in next section, we use Rural Urban Migration in China (RUMiC) survey to investigate whether there are school performance (proxied by Chinese and Math test scores) differences between migrant children and left-behind children.

### 2.2 Data

The Rural Urban Migration in China survey was initiated by Australian National University, University of Queensland and Beijing Normal University and supported by IZA since 2008. It consisted of three independent surveys: the Urban Household Survey (UHS), the Rural Household Survey (RHS) and the Migrant Household Survey (MHS). UHS and RHS were conducted by China's National Bureau of Statistics. MHS was carried out by the RUMiC project team with collaboration with professional sur-
vey company (Kong, 2010). For our study, the second wave of MHS is extracted. The data were collected in early 2009, ${ }^{11}$ covering 15 cities in nine provinces. ${ }^{12}$

By definition, the respondents of MHS are migrant workers. The survey first randomly selected workplaces within defined city boundaries. Migrant workers in each workplaces were then randomly chosen based on their birth months. After the selection, the enumerators did face-to-face interview with the targeted workers and their households. ${ }^{13}$ The questionnaires of MHS thus provide detailed demographic and socioeconomic characteristics of migrant workers, their families members in the city as well as the spouses who live in the home village. Besides, parents or guardians were asked to answer the questions for the children who were no older than 16 years old and the children who were over 16 but still in school. We therefore have exact information of both migrant children and left-behind children. The second waves of MHS comprises of around 5000 households with 1219 pre-school age children and 1898 school-age children, including 1297 children in the period of the compulsory education. Considering few drop out in the period of the compulsory education, our analysis only focus on the children in that period. ${ }^{14}$ Our sample is restricted to 789 children with complete information, of which 415 are migrant children and 374 are left-behind children. ${ }^{15}$

[^6]
### 2.3 Descriptive Statistics

The descriptive statistics of variables are reported in Table 1. Standardized Chinese and Math test scores are obtained through dividing actual scores by full test scores. The average gaps between two groups are minor. Among the migrant workers' children, they are on average around 11 years old in the 5th grade and with more boys in both categories. Half of children in our sample are not the single child in the household, which is consistent with China's one-child policy. With rural hukou, if the first child is girl, the couple is allowed to have a second child.

The variable Regular fees include the cost of food and accommodation as well as the cost of remedial classes taken at school. ${ }^{16}$ Remedy cost outside of school measures the money spent on remedial classes after school. The difference in regular fees between two groups are not distinctive while migrant children spent more on the remedial classes outside school. Sponsorship is dummy variable equal to 1 if the children needs to pay for extra fees in order to be enrolled in the school and 0 otherwise. With regards to remittance, more money are sent back to the home village if children are left behind. In additions, it seems that both two groups of children are less likely to be accepted by the high-quality school.

Additional summary statistics of the parents' levels of education are also displayed. As expected, a small share of the parents (i.e migrant workers) obtain high level of education (senior high school or above). 65 percent of fathers and 77 percent of mothers receive no education or elementary school education.

[^7]Table 1: Summary statistics

|  | Migrant children | Left-behind children | All |
| :---: | :---: | :---: | :---: |
| Standardized test scores |  |  |  |
| Chinese | 0.86 | 0.836 | 0.849 |
|  | (0.106) | (0.121) | (0.114) |
| Mathematics | 0.868 | 0.855 | 0.862 |
|  | (0.118) | (0.12) | (0.119) |
| Age | 10.877 | 11.345 | 11.099 |
|  | (2.868) | (2.909) | (2.895) |
| Grade | 4.523 | 5.005 | 4.752 |
|  | (2.50) | (2.543) | (2.533) |
| Boys (d) ${ }^{a}$ | 0.576 | 0.588 | 0.582 |
| Single child (d) | 0.47 | 0.409 | 0.441 |
| Regular fees | 1.437 | 1.327 | 1.385 |
|  | (1.869) | (2.002) | (1.933) |
| Remedy cost outside of school | 0.112 | 0.037 | 0.077 |
|  | (0.438) | (0.223) | (0.357) |
| Sponorship (d) | 0.282 | 0.045 | 0.17 |
| Remittance | 3.825 | 5.312 | 4.53 |
|  | (6.162) | (6.492) | (6.36) |
| Quality of School |  |  |  |
| Worse than average | 0.029 | 0.029 | 0.029 |
| Average | 0.651 | 0.647 | 0.649 |
| Better than average | 0.282 | 0.259 | 0.271 |
| The best | 0.039 | 0.064 | 0.051 |
| Father's highest level of education |  |  |  |
| No education | 0.123 | 0.086 | 0.105 |
| Elementary school | 0.521 | 0.612 | 0.564 |
| Junior high school | 0.198 | 0.163 | 0.181 |
| Senior high school or above | 0.159 | 0.139 | 0.15 |
| Mother's highest level of education |  |  |  |
| No education | 0.178 | 0.214 | 0.195 |
| Elementary school | 0.61 | 0.575 | 0.593 |
| Junior high school | 0.108 | 0.123 | 0.115 |
| Senior high school or above | 0.104 | 0.088 | 0.096 |
| Observations | 415 | 374 | 789 |

Notes: Source of data: RUMiC data. MHS waves 2009. Standard deviations are reported in parenthesis. Variables
Regular fees, Remedy cost outside of school and Remittance are measured in thounsands of RMB.

[^8]
### 2.4 Empirical strategy

The empirical model can be written as:

$$
\begin{equation*}
S_{i h}=\alpha+\beta_{1} M_{h}+\beta_{2} A g e_{i h}+\beta_{3} M_{h} * A g e_{i h}+\beta_{4} X_{i h}+\epsilon_{i h}, \tag{1}
\end{equation*}
$$

where $S_{i h}$ stands for standardized Chinese or Math test scores of child $i$ in household $h$. According to National Curriculum Standard, designed by the Ministry of Education, starting from the first year of 9 -year compulsory education, Chinese and Math are main subjects. The contents of exam papers in each region are required to follow the National Curriculum Standard (Meng and Yamauchi, 2015), we thus use Chinese and Math scores to measure school performance of children (Chen et al., 2009; Zhao et al., 2014). The test scores of children are reported by their parents. In China, at the end of each semester, parents are asked to attend parents meeting. The final test scores are also sent to parents. Therefore, parents have good knowledge of their children's scores. ${ }^{17}$

Our key variable of interest ${ }^{18}$ is $M_{h} . M_{h}$ is equal to 1 if children in the household $h$ are migrant children; 0 if they are left behind children. In the analysis, children whose primary residence in 2008 was rural village are defined as left-behind children. ${ }^{19}$ Children living in the city in 2008 are defined as migrant children. Suggested by existing experimental and empirical studies, parental effects on children's school performance are likely to be stronger in primary than in secondary schooling (Entwisle and Hayduk, 1982, 1988; Topor et al., 2010). We introduce, therefore, interaction terms $M_{h} *$ Age $_{i h}$.
where $X_{i h}$ is a vector of control variables related to children, parents and household level characteristics which are considered as important determinants of children's Chinese or Math test scores, such as gender, age, quality of school, household expenditures on education, remittance and education levels of parents. A series of region dummy

[^9]variables are also included in the regression to account for regional differences which might have impact on children's test scores. Additionally, the coefficient $\alpha$ is the intercept and $\epsilon_{i h}$ is the error term. ${ }^{20}$

### 2.5 Results

The main objective of our analysis is to test whether there exists differences in school performances, captured by Chinese and Math scores, between two groups - migrant and left behind children. We display the regression results in Table 2. The first two columns show estimations with respect to Chinese scores, controlling for individual, household and region characteristics. Column 1 shows the results without interaction term. The coefficient of migrant children captures Chinese test scores gap. Chinese scores of migrant children are 1.9 percentage point higher than those of left-behind children. When the interaction term is included, as shown in Column 2, the result is changed. Positive coefficient of migrant children and negative coefficient of interaction term indicate that difference between two groups varies across age. To put it in more detail, we take school starting age - 6 years old - as an example. Keeping all other variables equal, Chinese test scores of migrant children is 6.7 percentage point higher than peers living in rural village (0.121-6*0.009). Notwithstanding, taking into account 13-year-old children, there is negligible distinction. The gap is narrowing and advantage of migrant children gets less and less pronounced as age increases. This may indicate that younger migrant children is more outstanding in Chinese score than peers left behind.

Regarding control variables, quality of school plays an important role in their test scores. The higher quality of school the children enrol, the higher test scores they get. Besides, father and mother's education also have significant effects.

Column 3 and column 4 display the results which are attained by using Math test

[^10]Table 2: OLS results for school performance in Chinese and Math


Notes: Standard errors in parentheses correct for clustering at the household level. ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$.
scores as dependent variables. As column 4 shows that, with age increasing, the advantage of migrant children in Math score weakens as well. This may suggest that migrant workers, considering their children's school performance, whether to take children to the city or to leave them behind, may depend on the age of the children. Advantages of studying in the city is stronger when children are younger; while they become older, the distinction are not very obvious.

### 2.5.1 Robustness

Table 3: Results for children in elementary school and in junior high school

|  |  | Dependent variable: |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Elementary School |  | Junior high School |  |
|  | Chinese | Math | Chinese | Math |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ |
| Migrant children | $0.033^{* * *}$ | $0.030^{* * *}$ | -0.014 | $-0.039^{* *}$ |
|  | $(0.010)$ | $(0.010)$ | $(0.018)$ | $(0.018)$ |
| Observations | 552 | 552 | 235 | 235 |
| Number of household clusters: | 504 | 504 | 219 | 219 |

Notes: Standard errors in parentheses correct for clustering at the household level. ${ }^{*} \mathrm{p}<0.1 ;{ }^{*} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$. Each regression includes a constant and children's, parents', households' and regions' characteristics variables.

To further gauge robustness of our result that the difference in school performance are age related, we repeat our analysis for children in elementary school and junior high school. The results in Table 3 indicate that migrant children in elementary school are outstanding in both Chinese and Math scores compared to the left behind ones in the same level. After controlling for other variables, there are about 3 percentage points higher in both two subjects. However, concerning the junior high school children, there is no significant difference in Chinese test scores while left-behind children's math scores are 3.9 percentage point higher than migrant children. That further confirms our findings that younger children may benefit more via migrating with their parents
to study in the city. According to their children's school performance, it may be better for migrant workers to take their children with them when their children are young.

Arguably, cross-sectional data prevent us from capturing the effects of unobserved heterogeneity. The OLS estimators may suffer from endogeneity if children's migration status is correlated with error term. Unfortunately, the limited data impede us to find a proper instrument satisfying exclusion condition and to perform further empirical analysis and investigation. In next section, we rely on theoretical model to provide more conclusive information on the optimal choices of migrant workers regarding whether taking children to the city or not.

## 3 Theoretical model

### 3.1 The model

We consider an over-lapping generation model of rural migrants who have hukou in some rural area, but work in some other cities. Suppose each individual is one household and will live for two periods: young and old. The life time utility of generation $t$ is

$$
\begin{equation*}
U_{t}=u\left(c_{t}\right)+\beta u\left(d_{t+1}\right)+\gamma U_{t}^{f} \tag{2}
\end{equation*}
$$

where $c_{t}$ and $d_{t+1}$ are consumption of young and old age, respectively, with parameter $\beta(\in(0,1))$ denoting time preference. Following the altruism idea of Lucas and Stark (1985) and the case of Chinese tradition - children provide parents' old age support, we assume that individual also take care the other family members, which is denoted by $U_{t}^{f}$, with $\gamma(\in(0,1))$ altruism parameter. The other family members could be old age parents, children, siblings and so on. For simplicity, we take

$$
\begin{equation*}
U_{t}^{f}=a_{P} u_{P}\left(\frac{c_{P, t}}{N}\right)+a_{K} \sum_{k=1}^{K} u_{k}\left(c_{k, t}, h_{k, t+1}\right) \tag{3}
\end{equation*}
$$

where $c_{p}$ measures parents' consumption (if there are young siblings in the family, we consider that as part of parents' consumption), $N$ is number of siblings who share the cost of old age parents, $c_{k}$ and $h_{k, t+1}$ are children's consumption and human capital accumulation, namely schooling. Suppose there are $K$ children in each household ${ }^{21}$. We assume individual cares for her children equally, but cares for parents maybe differently, that is $a_{P}$ may differ from $a_{K}$. ${ }^{22}$

Denote human capital of migrant workers as $h_{t}$ which checks $h_{t} \geq h_{0}$ with $h_{0}$ measuring pure physical capital and no any education, training skills or experiences. Suppose unit human capital wage is $w_{t}$, which is exogenously given. Thus an individual with human capital $h_{t}$ earns income $w_{t} h_{t}$, which will be used for her own (and family) consumption when young and save $s_{t}$ for old age. At the same time, some amount $m_{t}$ will be sent back to home for supporting parents' old age and raising young children left behind. Additionally, there may be some extra cost for children's education, except the standard school books and other supply costs, which is not part of the standard consumption related remittances. These extra costs may come in two different ways: (1) due to the hukou system, it is not possible to take children to migrate, there are some kind of boarding schools or families, ${ }^{23}$ very often the teachers, keep the left-behind children with some fee; or (2) migrants parents can take children with them, but due to no hukou in the new resident city, they have to pay extra fee in order to enroll their children in the local schools. We denote this cost as $g\left(e_{t}\right)$, to be precise later.

[^11]As a conclusion, migrant worker faces the following financial budget constraint:

$$
\begin{equation*}
c_{t}+s_{t}+g\left(e_{t}\right)+m_{t}=w_{t} h_{t} . \tag{4}
\end{equation*}
$$

When she is old, her consumption would base on saving from young age with interest rate $r_{t+1}$, possible old age working income but with some discounted human capital $\phi h_{t}$ (parameter $0 \leq \phi \leq 1$ ) and maybe some exogenous transfer from her adult children or/and pubic pension, which we denote by $\widetilde{T_{t+1}}=\widetilde{m_{t+1}}+\widetilde{p_{t+1}}$. Thus old age budget constraint is

$$
\begin{equation*}
d_{t+1}=s_{t}\left(1+r_{t+1}\right)+w_{t+1} \phi h_{t}+\widetilde{T_{t+1}} . \tag{5}
\end{equation*}
$$

Furthermore, parents care not only for children's consumption, but also for their education and human capital accumulation, which crucially depend on where the education take places: Migrant in the city or left-behind in the rural area. We use $j$ denoting the location of the children's education: $j=m$ capturing the case that the children are migrants with their parents and study in the city where they live; while $j=l$ meaning the children are left-behind in the rural area and study there. For this special case of Chinese rural migrant workers, we modify the suggested formulation of de Brauw and Giles $(2012,2016)$ and de La Croix and Doepke (2003) about human capital accumulation in the following way:

$$
\begin{equation*}
h_{t+1}^{j}=h_{0}+B_{t}^{j}\left(\theta_{j}+e_{t}^{j}\right)^{\eta} h_{t}^{\alpha^{j}}\left(\overline{h_{t}^{j}}\right)^{\kappa}, \quad j=m, l, \tag{6}
\end{equation*}
$$

where, as mentioned above, $h_{0}$ is basic physical labor without any extra education or training, parameter $\eta \in(0,1)$ presents share of public and parents' contribution to the education outcome, $\alpha^{j}$ is parents' human capital impact, $\kappa(\in[0,1-\eta])$ can be interpreted as effect of the quality of schooling, and $\overline{h_{t}^{j}}$ is the average human capital of teachers. Positive parameter $\theta_{j}$ measures free public education, which states that even the parents chose not to make any contribution, it is still possible that children get some education if they make some efforts, that is, if $B_{t}^{j}>0$.

Here $B_{t}^{j}$ presents learning productivity parameter, reflecting children's ability and factors that affects motivation and efforts of children at time $t$. As reported about the lost
generation of those left-behind children ${ }^{24}, B_{t}^{j}$ could be age related and could be positive or zero. For all that, we are not clear at this stage how $B_{t}$ should related to the age of children. It may be that the young children suffer more from being left-behind; while it is also possible that the rebellion teenagers, who deserve more parental guide, suffer more from being left-behind. We leave this to the later empirical test.

Nevertheless, if $B_{t}^{l}$ is largely positive, it may state that the left-behind children are very much motivated to work hard in school- which is indeed some cases from the news and interviews. While more often $B^{l}$ may be just slightly larger than zero due to the fact that they are short of parental affection, survivance and direct care. More often those left-behind children need to take care of household, such as cooking their own meals, washing their own clothes, taking care the even young siblings and so on; or they need to work in the agriculture with grandparents. Those beyond their age burden may discourage their concentration and attitude in school. $B^{l}$ may be zero in the case that the children just do not make any efforts at all due to mentally ${ }^{25}$ hungry of parental affection, missing parents, bad influence from others or just running out of school and schools meaning nothing.

Nonetheless, that is no means saying that $B_{t}^{m}$ should be high. It may happen that the immigrant children do not like their new school, the city children make fun of them due to their poor family background or dressing, they may be short of homework time due to taking care household while immigrant parents work too long hours, the same as above mentioned the mentally hungry of their busy parental affection, or they are

[^12]short of previous knowledge and can not follow the new lectures, and so on.
In the above equation (6), the term $h_{t}^{\alpha^{j}}$ captures inter-generation human capital transfer. $\alpha^{j}=1$ would promise that the new generation automatically embodied with parents' human capital. However, in our special case of migrant parents and left-behind children, that may not be the case. In this situation, given the left-behind children do not grow up with parents and can not learn from parents directly the living skills, such as, agriculture work, some special hand-work, traditional skills and so on, it should be more accurately to assume that $\alpha^{l}=0$. Even the children migrate with parents together and study in the city, there is no reason the parents' human capital can be directly transferred to their children. Parents' relatively low human capital may even hinter children's human capital accumulation process due to no time, no energy, or not possible to help for homework, no extra money to send children to extra school as local city children do. Thus, we impose that $0 \leq \alpha^{j} \leq 1$.

Generally, we can impose the following assumptions.

Assumption 1 - The cities have better education infrastructure and better teachers than those poor rural regions: $\theta_{m}>\theta_{l}, \overline{h_{t}^{m}}>\overline{h_{t}^{l}}$;

- inter-generational parameter checks: $0 \leq \alpha^{j} \leq 1, j=m, l$;
- productivity parameter satisfies: $B_{t}^{j} \geq 0, j=m, l$.

Assumption 2 Education cost follows:

$$
g\left(e_{t}\right)=g^{j}\left(e_{t}^{j}\right)=\left\{\begin{aligned}
K\left(e_{t}^{l}+k_{l}\right), & \text { if children leftbehind } j=l, \\
K\left(e_{t}^{m}+k_{m}\right), & \text { if children immigrant } j=m
\end{aligned}\right.
$$

with per child extra and relocation cost $k_{m}>0$, while stay at original location it is free of charge, $k_{l}=0$.

Here $e_{t}^{j}$ is choice variable for migrant parents. Due to the hukou system, usually, rural children can not easily and directly enroll in city schools, though some time it is possible with a high fee, $k_{m}$. Alternatively, the migrants may organize together their own schools to educate their children as some reports mentioned and $k_{m}$ is part of the cost of running these schools, $k_{m}$ also includes the cost of children transportation, renting relatively bigger living place and so on costs. When parents make their choice to bring their children with them, these fees and extra stress are part of their decision as well.

Conclude the above statement, under Assumption 1 and 2, the migrant worker's optimization problem is:

$$
\max _{c_{t}, s_{t}, m_{t}, e_{t}} U_{t}=u\left(c_{t}\right)+\beta u\left(d_{t+1}\right)+\gamma\left[a_{P} u_{P}\left(\frac{c_{P, t}}{N}\right)+a_{K} \sum_{k=1}^{K} u_{k}\left(c_{k, t}, h_{k, t+1}\right)\right],
$$

subject to the two period budget constraints (4), (5), the children's human capital accumulation (6) and remittance constraint, which will be precise later.

We take logarithm utility function function to get explicit solutions, i.e.,

$$
U_{t}=\ln \left(c_{t}\right)+\beta \ln \left(d_{t+1}\right)+\gamma a_{P} \ln \left(\frac{c_{P, t}}{N}\right)+\gamma a_{K} K\left[\ln \left(c_{k, t}\right)+\widetilde{\beta}_{k} \ln \left(h_{k, t+1}-h_{0}\right)\right]
$$

where we assume that parents care their children equally, and parameter $\widetilde{\beta_{k}}$, with $0 \leq$ $\widetilde{\beta_{k}} \leq 1$, measures how much parents care about children's education compared to their consumption. Here, $h_{k, t+1}-h_{0}$, measures the result of schooling or education, given the physical care is included in the term of $c_{k}$. In other words, parents' care to their children are two-folds: consumption and education.

Additionally, it is clear from the above function form that if $B_{t}^{j}=0$, the last term is $\ln \left(h_{k, t+1}-h_{0}\right)=-\infty$. So parents do not have an optimal interior choice. Therefore, in the following, we distinguish two different cases: (1) normal case: $B_{t}^{j}>0$ and (2) $B_{t}^{j}=0$.

### 3.2 Theoretical results - normal case

Given the children in our study may be left-behind by their parents or migrate with parents, we have to treat children and parents' consumption separately, which differs from the classical over-lapping generation literature, such as de La Croix and Doepke (2003).

Given most of the left-behind children are living with their grandparents, we make no difference between the children and grandparents' consumption, that is, we assume and normalize to family consumption $c_{f}$ :

$$
\begin{equation*}
c_{k}^{l}=c_{P}:=c_{f} . \tag{7}
\end{equation*}
$$

Then the migrant's remittance checks

$$
\begin{equation*}
\frac{c_{P, t}}{N}+K c_{k, t}^{l} \leq m_{t}^{l}+\widetilde{y} \tag{8}
\end{equation*}
$$

which states that the consumption of left-behind children and grandparents depend on the remittances and other exogenous income, $\widetilde{y}$, which most probably is the agriculture income or land rented to someone else to plant. Here, the cost of parents is shared by total $N$ siblings of the migrant adults. Thus, migrants' utility can be rewritten as:

$$
\begin{equation*}
U_{t}=\ln \left(c_{t}^{l}\right)+\beta \ln \left(d_{t+1}^{l}\right)+\left(\gamma a_{P}+\Gamma_{K}\right) \ln \left(c_{f, t}\right)+\Gamma_{K} \widetilde{\beta}_{k} \ln \left(h_{k, t+1}^{l}-h_{0}\right), \tag{9}
\end{equation*}
$$

with $\Gamma_{K}=\gamma a_{K} K$ altruism factor for children.

While if children are also migrant, the remittance will be purely supporting left-behind parents and is given by:

$$
\begin{equation*}
\frac{c_{P}}{N} \leq m_{t}^{m}+\widetilde{y} \tag{10}
\end{equation*}
$$

and the migrants' utility is

$$
\begin{equation*}
U_{t}=\left(1+\Gamma_{K}\right) \ln \left(c_{t}^{m}\right)+\beta \ln \left(d_{t+1}^{m}\right)+\gamma a_{P} \ln \left(c_{p, t}\right)+\Gamma_{K} \widetilde{\beta_{k}} \ln \left(h_{k, t+1}^{m}-h_{0}\right) . \tag{11}
\end{equation*}
$$

Definition 1 We call $\left\{c_{t}^{j}, s_{t}^{j}, e_{t}^{j}, m_{t}^{j}\right\}(j=r, m)$ an optimal choice, if it maximizes utility (9) (or (11)) under budget constraints (4), (5), (8) (or(10)) and the children's human capital accumulation (6) with Assumptions 1 and 2.

The standard first order condition shows that

$$
d_{t+1}^{j}=\left\{\begin{array}{l}
\beta(1+r) c_{t}^{l}, \quad j=l  \tag{12}\\
\frac{\beta(1+r)}{1+\Gamma_{K}} c_{t}^{m}, \quad j=m
\end{array}\right.
$$

which presents the relationship between the two period's marginal utility. Combining with the old age consumption constraint, we obtain that the migrant's saving follows

$$
s_{t}^{j}=\left\{\begin{array}{l}
\beta c_{t}^{l}-\frac{\widetilde{T_{t+1}}+\phi h_{t} w_{t+1}}{1+r}, \quad j=l  \tag{13}\\
\frac{\beta}{1+\Gamma_{K}} c_{t}^{m}-\frac{\widetilde{T_{t+1}}+\phi h_{t} w_{t+1}}{1+r}, \quad j=m
\end{array}\right.
$$

The same calculation yields also that

$$
\begin{equation*}
c_{f}\left(\frac{1}{N}+K\right)=m_{t}^{l}+\widetilde{y}=\left(\gamma a_{p}+\Gamma_{K}\right) c_{t}^{l} \text { or } \frac{c_{P}^{m}}{N}=m_{t}^{m}+\widetilde{y}=\frac{\gamma a_{p}}{1+\Gamma_{K}} c_{t}^{m} \tag{14}
\end{equation*}
$$

whose intuition is straightforward. The left hand side presents the accumulate consumption of all left-behind children and parent, and it will be covered by the remittances and left behind potential incomes, $\widetilde{y}$. At the same time, this consumption is determined based on the migrant's own consumption corrected by the altruism factors of children and parents, $\gamma a_{p}$ and $\Gamma_{K}$.

Optimal choice of education $e_{t}^{j}$ must satisfies

$$
\frac{1}{c_{t}^{l}}\left(\text { or } \frac{\left(1+\Gamma_{K}\right)}{c_{t}^{m}}\right) K=\frac{\Gamma_{K} \widetilde{\beta}_{k}}{h_{k, t+1}^{j}-h_{0}} B_{t}^{j} h_{t}^{\alpha^{j}}\left(\overline{h^{j}}\right)^{\kappa}\left(\theta_{j}+e_{t}^{j}\right)^{\eta-1} \eta
$$

where the left hand side is the marginal lost of consumption due to education effort and the right hand side presents the marginal gain for children's human capital accumulation. Rearranging terms in the above equation, it yields that the optimal education per
child is:

$$
e_{t}^{j}=\left\{\begin{array}{l}
\frac{\Gamma_{K} \widetilde{\beta}_{k} \eta}{K} c_{t}^{l}-\theta_{l}, \quad j=l,  \tag{15}\\
\frac{\Gamma_{K} \widetilde{\beta}_{k} \eta}{\left(1+\Gamma_{K}\right) K} c_{t}^{m}-\theta_{m}, \quad j=m .
\end{array}\right.
$$

Substituting the above saving, remittance and education cost into the budget constraint, it follows, for $j=l, m$,
$c_{t}^{l}\left(\operatorname{or} \frac{c_{t}^{m}}{1+\Gamma_{K}}\right)\left(1+\beta+\gamma a_{P}+\Gamma_{K}+\Gamma_{K} \widetilde{\beta}_{k} \eta\right)=w_{t} h_{t}+\widetilde{y}+\frac{\widetilde{T_{t+1}}+\phi h_{t} w_{t+1}}{1+r}-K k_{j}+K \theta_{j}$, with $k_{l}=0$ and $k_{m}>0$. Denoted $W_{t}=w_{t} h_{t}+\widetilde{y}+\frac{\widetilde{T_{t+1}+\phi} h_{t} w_{t+1}}{1+r}$ as the life time earning, which includes labor incomes of two periods, potential income back home, discounted old age social transfer and children's remittances. Then, the left hand side is aggregate life time cost, which includes consumption by taking into account young and discounted old age, parents and children's consumptions, plus the education cost of children. The right hand side is life time potential income, which includes life time earning and public social transfer to education net of relocation cost of children's schooling.

Combining the above analysis together, we conclude that:

Proposition 1 Given Assumption 1 and 2. Assume that $B^{j}>0$ and $\widetilde{\beta_{k}}>0$, for migrant workers, there exists one and only one optimal choice, $c_{t}^{i n, j}$ which is given by,

$$
c_{t}^{i n, j}=\left\{\begin{array}{l}
\frac{\left(W_{t}+K \theta_{l}\right)}{\Lambda}, j=l  \tag{16}\\
\frac{\left[W_{t}+K\left(\theta_{m}-k_{m}\right)\right]\left(1+\Gamma_{K}\right)}{\Lambda}, j=m
\end{array}\right.
$$

$s_{t}^{i n, j}, e_{t}^{i n, j}$ are given by (13), (15) and

$$
\begin{equation*}
m_{t}^{i n, l}=\left(\gamma a_{P}+\Gamma_{K}\right) c_{t}^{i n, l}-\widetilde{y}, \quad m_{t}^{i n, m}=\frac{\gamma a_{P}}{1+\Gamma_{K}} c_{t}^{i n, m}-\widetilde{y}, \tag{17}
\end{equation*}
$$

where

$$
\Lambda=1+\beta+\gamma a_{p}+\Gamma_{K}\left(1+\widetilde{\beta}_{k} \eta\right)
$$

Furthermore, old age consumption, $d_{t+1}^{i n, j}$, is given by (12).

Noticing that migrant's consumption, hence everyone's consumption, increases in term of public education input, while private education cost decreases: $\frac{\partial e_{t}^{i n, j}}{\partial \theta_{j}}=\frac{\Gamma_{K} \widetilde{\beta_{k}} \eta}{\Lambda}-1<0$. High public education input induces parents to decrease their private educational input. Thus, instead of providing private education to their children, parents consume that part of income. This argument may lead to the case that no private investment in education is an optimal choice. Therefore, to guarantee that in Proposition $1, e_{t}^{i n, j}>0$, the following are needed.

Proposition 2 Given Assumption 1 and 2. Assume that $B^{j}>0$ and $\widetilde{\beta_{k}}>0$. The optimal education investment $e_{t}^{i n, j}>0$ if and only if

$$
\begin{equation*}
\frac{\Gamma_{K} \widetilde{\beta}_{k} \eta}{\lambda}\left(W_{t}-K k_{j}\right)>K \theta_{j} \tag{18}
\end{equation*}
$$

with $\lambda=1+\beta+\gamma a_{P}+\Gamma_{K}$.

Condition (18) plays the role of Tobin's-q in investment of education, whose intuition is straightforward. The right hand side is the total public education input of all children, while the left hand side measures the importance (or desired level) from education in term of consumption and income. Ratio $\frac{\Gamma_{K} \widetilde{\beta_{k}}}{\lambda}$ measures the relative importance of education compared to the net of relocation income, $\left(W_{t}-K k_{j}\right)$. Multiplied by the share, $\eta$, of education input, the left hand side is indeed total importance of education, or the optimal desired level of educational input. Proposition 2 states that there is private investment in children's education if and only if the public input in education is lower than parents' desired level of educational input for their children.

In the following, for simplicity, we shall call $\frac{\Gamma_{K} \widetilde{\beta_{k}} \eta}{\lambda} W_{t}$ as relative educational wage.
Additionally, given all other terms, education is relative more expensive for low-income parents than for high-income ones, which is a standard results. However, for migrant workers, the extra information we obtain is that school relocation cost plays an important role in the decision making of children's education. In the case of $j=l$, leaving
the children back home, there is no relocation cost, that is, $k_{l}=0$, it may happen that

$$
\begin{equation*}
\frac{\Gamma_{K} \widetilde{\beta}_{k} \eta}{\lambda} W_{t}>K \theta_{l} \quad \text { while } \frac{\Gamma_{K} \widetilde{\beta}_{k} \eta}{\lambda}\left(W_{t}-K k_{m}\right)<K \theta_{m} \tag{19}
\end{equation*}
$$

If that is the case, the following results hold:

Proposition 3 Suppose that the assumptions of Proposition 2 hold, especially, $B^{j}>0$. If furthermore, condition (19) holds, it is optimal to invest in children's private education back home, $e_{t}^{i n, l}>0$.

This proposition does not state that the migrant parents should take their children to migrate or leave them behind. It only states that if the parents leave their children behind and if condition (19) holds, then it is optimal to invest in their children's private education. Obviously, if they bring their children along, it is not optimal to invest in education.

Regardless of the relocation cost, $k_{m}$, given the differences in regional development, $\theta_{m}$ could be largely above $\theta_{l}$ such that, condition (19) holds. In other words, changing hukou system itself to reduce the relocation cost of children's migration may not enough to solve the left-behind children problem. From this point of view, left-behind children may continue to exist for a long time. Of course, condition (18) may fail in any case, which we call as a corner solution, denoted as $e_{t}^{c o}=0$. If so, the following results can be obtained:

Proposition 4 Suppose Assumption 1 and 2 hold and $B^{j}>0$. If no private education investment, $e_{t}^{c o, j}=0$, the optimal consumption is

$$
\begin{equation*}
c_{t}^{c o, l}=\frac{W_{t}}{\lambda} \text { and } c_{t}^{c o, m}=\frac{\left(W_{t}-K k_{m}\right)\left(1+\Gamma_{K}\right)}{\lambda} \tag{20}
\end{equation*}
$$

$S_{t}^{c o, j}, m_{t}^{c o, j}$ and $d_{t}^{c o, j}$ with $j=l, m$ are given by (13), (14) and (12), respectively. Moreover, $e_{t}^{c o, j}=0$ is an optimal choice if and only if

$$
\begin{equation*}
\frac{\Gamma_{K} \widetilde{\beta}_{k} \eta}{\lambda}\left(W_{t}-K k_{j}\right) \leq K \theta_{j}, j=l, m \tag{21}
\end{equation*}
$$

### 3.3 Lost generation

To close this section, we briefly show the results of migrant parents' choices if their children are not motivated to school study. In other words, in the human capital accumulation equation, $B_{t}^{j}=0$, and we have $h_{k, t+1}=h_{0}$. There is no education affect and the children are left only with their physical capital, thus, this generation of children are often called the lost generation in the newspaper. In this case, the migrant parents do not have an optimal educational choice for their children. Thus, $e_{t}^{j}=0, j=l, m$ and the following results can be obtained:

Proposition 5 Given Assumption 1 and 2. Assume that $B^{j}=0$ and $\widetilde{\beta_{k}}=0$, the unique optimal choice of migrant workers is $e_{t}^{L, j}=0$,

$$
c_{t}^{L, j}=\left\{\begin{array}{l}
\frac{W_{t}}{\lambda}, j=l, \\
\frac{\left(W_{t}-K k_{m}\right)\left(1+\Gamma_{K}\right)}{\lambda}, j=m,
\end{array} \quad m_{t}^{L, j}=\left\{\begin{array}{l}
\left(\gamma a_{P}+\Gamma_{K}\right) c_{t}^{L, l}-\widetilde{y}, \quad j=l, \\
\frac{\gamma a_{P}}{1+\Gamma_{K}} c_{t}^{L, m}-\widetilde{y}, \quad j=m .
\end{array}\right.\right.
$$

The utility is

$$
U_{t}^{L, j}=\lambda \ln \left(c_{t}^{L, j}\right)+\beta \ln (\beta(1+r))+\left(\gamma a_{P}+\Gamma_{K}\right) \ln \left(\frac{\gamma a_{P}+\Gamma_{K}}{K+1}\right)
$$

Obviously, both private and public education costs are no longer migrant parents' concern, though the relocation cost of children, $k_{m}$ still decreases migrant parents' consumption.

The difference between the corner solution of last subsection and the current situation is the following. In the former, the children will make an effort to study, $B_{t}^{j}>0$, though parents may optimally choose not to pay extra after school intuition fee, $e^{c o, j}=0$. While in the later case, parents does not have a choice for children's education.

It is straightforward that in both cases without extra education investment, $c_{t}^{L, j}=c_{t}^{c o, j}$. Nonetheless, the children's human capital accumulation differs: $h_{t+1}^{L, j}=h_{0}$ due to $B_{t}^{j}=$ 0 ; while $h_{t+1}^{c o, j}>h_{0}$ given $B_{t}^{j}>0$.

## 4 Answers to the original questions

After Chinese internal migrants making their optimal choices of consumption, saving and investment in their children's education, now we are ready to answer the fundamental question: Should these migrant parents take their children with them? When should they, or what are the conditions for them to, take their children? In this section, we will give precise answers to these questions based on relative educational income and relocation cost, which will be defined later on.

From migrant parents' optimal choices, four possible combinations appear: (I) invest in private education wherever their children are: $e_{t}^{j}>0$, for both $j=l$ and $j=m$; (II) no private education in any case: $e_{t}^{j}=0$, for $j=l, m$; (III) left-behind children have private education while not the migrant ones: $e_{t}^{l}>0, e_{t}^{m}=0$ and (IV) migrant children have private education, but not the left-behind ones: $e_{t}^{l}=0, e_{t}^{m}>0$.

Actually, case (IV) can not happen. Intuitively, migrant parents may notice that their migrant children have difficulties to follow city school or the original good students back to rural hometown is no longer that good. In order not to discourage their migrant children, parents may pay extra intuition fee for their children to attend after school courses to catch-up with the city students. If these children are left-behind, this cost is unnecessary. Mathematically, it means:

$$
e_{t}^{l}=0, \quad e_{t}^{m}>0
$$

However, from Proposition 2, that implies $\theta_{l}>\theta_{m}$, which contradicts to Assumption 1. Hence, this case can not happen ${ }^{26}$. In other words, it is never optimal for Chinese internal migrant parents to take their children to migrate and provide them with private education in the city, but not in the rural region.

Thus, in the following, we only need to study the other three cases.
In order to eliminate the effects from siblings left-behind who may be able to help to

[^13]take care the left-behind children, which is the case sometimes, in the following, we take $N=1$. Arguably, the human capital of Chinese internal migrants transferring to their children's human capital accumulation may be very limited regardless their children are left-behind or migrant, thus, we also assume that
$$
h_{t}^{\alpha^{m}}=h_{t}^{\alpha^{l}} .
$$

### 4.1 Case (I): $e_{t}^{j}>0$, for $j=l, m$

Parents would like to offer their children optimal private education no matter where their children are living. By Proposition 2, parents' willingness to invest in their children's private education means that migrant parents' desired level of educational input should satisfies condition (18), which can be rewritten as (recall $k_{l}=0$ )

$$
\begin{equation*}
0<k_{m}<\widehat{K_{m}^{(I)}} \triangleq \frac{W_{t}}{K}-\frac{\lambda}{\Gamma_{K} \widetilde{\beta}_{k} \eta} \theta_{m} \tag{22}
\end{equation*}
$$

This condition implies that migrants' relative educational income ${ }^{27}$ must check

$$
\begin{equation*}
\frac{\Gamma_{K} \widetilde{\beta}_{k} \eta}{\lambda} W_{t}>K \theta_{m} \tag{23}
\end{equation*}
$$

If so, the utility of migrant parents are:

$$
\begin{equation*}
U_{t}^{j}=\Lambda \ln \left(c_{t}^{i n, j}\right)+\Gamma_{K} \widetilde{\beta_{k}} \ln \left[B_{t}^{j} h_{t}^{\alpha^{j}}\left(\overline{h_{t}^{j}}\right)^{\kappa}\right]+\varepsilon^{j}+\delta^{j}, \quad j=l, m \tag{24}
\end{equation*}
$$

with

$$
\begin{gathered}
\varepsilon^{l}=\beta \ln (\beta(1+r))+\left(\gamma a_{f}+\Gamma_{K}\right) \ln \left(\frac{\gamma a_{f}+\Gamma_{K}}{1+K}\right) \\
\varepsilon^{m}=\beta \ln \left(\frac{\beta(1+r)}{1+\Gamma_{K}}\right)+\gamma a_{f} \ln \left(\frac{\gamma a_{f}}{1+\Gamma_{K}}\right) \\
\delta^{l}=\Gamma_{K} \widetilde{\beta}_{k} \eta \ln \left(\frac{\Gamma_{K} \widetilde{\beta}_{k} \eta}{K}\right)
\end{gathered}
$$

[^14]and
$$
\delta^{m}=\Gamma_{K} \widetilde{\beta}_{k} \eta \ln \left(\frac{\Gamma_{K} \widetilde{\beta}_{k} \eta}{\left(1+\Gamma_{K}\right) K}\right)
$$

To see what would be the different between leaving the children behind and migrating, we can easily check that:

$$
\begin{align*}
U^{i n, m}-U^{i n, l} & =\Lambda \ln \left[\frac{W_{t}+K\left(\theta_{m}-k_{m}\right)}{W_{t}+K \theta_{l}}\right]+I\left(a_{k}\right) \\
& +\Gamma_{K} \widetilde{\beta}_{k}\left[\ln \left(\frac{B_{t}^{m}}{B_{t}^{l}}\right)+\kappa \ln \left(\frac{\overline{h^{m}}}{\overline{h^{l}}}\right)+\ln \left(\frac{h_{t}^{\alpha^{m}}}{h_{t}^{\alpha}}\right)\right], \tag{25}
\end{align*}
$$

where $I\left(a_{k}\right)$ stands for the gains from whole family being together which is deduced from difference in altruism term:

$$
I\left(a_{k}\right)=\left(1+\Gamma_{K}\right) \ln \left(1+\Gamma_{K}\right)+\gamma a_{f} \ln \left(\gamma a_{f}\right)+\left(\gamma a_{f}+\Gamma_{K}\right) \ln \left(\frac{1+K}{\gamma a_{f}+\Gamma_{K}}\right) .
$$

Given $0<\gamma a_{f}<1$, it can be shown that as long as parents caring for their children no less than caring for their parents, that is, as long as $a_{k} \geq a_{f}$, taking their children to migrate is beneficial for parents:

$$
\begin{equation*}
I\left(a_{k}\right)>0 . \tag{26}
\end{equation*}
$$

It is trivial to see that if the relocation cost is sufficiently high, such as using up migrant parents' lifetime income: $K k_{m}>W_{t}+K \theta_{m}$, we have $U^{i n, m}-U^{i n, l}=-\infty$. Obviously, the only choice for migrants is leaving their children behind. At the same time, it is also easy to check that if there is no relocation cost, that is, $k_{m}=0$, we have $U^{i n, m}-U^{i n, l}>0$ provided $B^{m}$ is not much smaller than $B^{l}$. Thus, the whole family migrating are better off than leaving the children behind. By continuity, there exists positive constant
$K_{m}^{(I)}=K_{m}^{(I)}\left(W_{t}, \theta_{m}, \theta_{l}, \frac{B^{m}}{B^{l}}, \frac{\overline{h^{m}}}{\overline{h^{l}}}\right) \triangleq \frac{1}{K}\left[W_{t}+K \theta_{m}-\frac{W_{t}+K \theta_{l}}{e^{I\left(a_{k}\right) / \Lambda}}\left(\frac{B^{l} \overline{h^{l}} h^{\alpha^{l}}}{B^{m} \overline{h^{m}} h^{\alpha^{m}}}\right)^{\Gamma_{K} \widetilde{\beta_{k}} / \Lambda}\right]$,
such that,

$$
U^{i n, m}-U^{i n, l} \begin{cases}>0 & \text { if } 0 \leq k_{m}<K_{m}^{(I)} \\ <0 & \text { if } k_{m}>K_{m}^{(I)}\end{cases}
$$

In other words, the threshold of relocation $\operatorname{cost}, K_{m}^{(I)}$, determines the gains from migrating children. This threshold depends on the differences in education level in different regions: $\theta_{j}$ and $\overline{h^{j}}$, and the children's motivation $B^{j}$ for $j=l, m$, given parents' human capital, number of children in the family and other altruism parameters.

Additionally, the relocation cost threshold increase with the destination's public input in education, $\frac{\partial K_{m}^{(I)}}{\partial \theta_{m}}>0$, and decrease with the rural's public eduction input, $\frac{\partial K_{m}^{(I)}}{\partial \theta_{l}}<0$, similarly, we also have that $\frac{\partial K_{m}^{(I)}}{\partial\left(\frac{h^{m}}{h^{l}}\right)}>0$. In other words, the smaller the educational gap between the original rural region and the destination city, the lower is the relocation cost threshold; thus, it is easier and more beneficial for parents to bring their children to migrate together. The larger the gap is, the higher is the relocation cost threshold and the more difficult for parents to take their children along. This last statement is some counterintuitive. The reason lies on the the fact that given it is more beneficial to bring the children to the destination to enjoy better education, there is higher price to pay, the price here is the relocation cost.

Therefore, for the policy maker in order to reduce the left-behind children problem, it is essential to increase the educational input in the poor rural region, such that, the educational gap between the rural and city is reduced.

Remark. We are not talking about decrease the development gap among different rural and urban regions, which is a much harder task. Instead, we focus only on public education input and training of qualified teachers, which the policy maker in China are quite possible to pursue.

We conclude the above analysis in the following:

Proposition 6 Suppose migrant parents are able to, that is, condition (23) is true, invest in their children's private education regardless where their children are living. There exits a relocation cost threshold which is given by (27), which is decreasing in terms of educational gap between city and rural region. Moreover,

- it is optimal for migrant parents to take their children with them and pay private education in the destination (additional to the public education), if and only if,

$$
k_{m}<\min \left\{\widehat{K_{m}^{(I)}}, \quad K_{m}^{(I)}\right\}
$$

- otherwise, it is optimal for parents to leave their children behind and offer them with private education (additional to public education) in the rural hometown.

The first part of this proposition is what we demonstrated above. And it is easy to check that the migrant parents leave their children behind with private education if and only if

$$
k_{m}>\max \left\{\widehat{K_{m}^{(I)}}, \quad K_{m}^{(I)}\right\} .
$$

Between the above two polar cases, given both $\widehat{K_{m}^{(I)}} \lessgtr K_{m}^{(I)}$ are possible, the conclusion is not straightforward. Nonetheless, if relocation cost checks $\widehat{K_{m}^{(I)}}<k_{m}<K_{m}^{(I)}$, though parents are better off by taking their children to migrate, the private investment in education can not reach to its optimal level. While if relocation cost checks $K_{m}^{(I)}<$ $k_{m}<\widehat{K_{m}^{(I)}}$, migrant parents offer optimal city private education to their children in the city, but they are worse off than leaving their children behind at least in the shortrun. Therefore, the last two cases both should belong to the second statement in the Proposition 6.

For relatively high income parents (or parents who care more for their children's education than consumption), this proposition provides necessary and sufficient condition on which parents should take their children to migrate and provide them with private education in the destination city. Violating this condition means either parents will be worse off by taking their children to migrate than leaving their children behind, or it is not an optimal educational choice. Thus, leaving them behind with private education is the optimal choice.

### 4.2 Case (II): $e_{t}^{j}=0$, for $j=l, m$

The other symmetric case is that regardless where their children are living, private education is too costly for migrant parents considering their income. By Proposition 2, that means the migrants' relative-educational-income checks

$$
\begin{equation*}
\frac{\Gamma_{K} \widetilde{\beta}_{k} \eta}{\lambda} W_{t}<K \theta_{l}\left(<K \theta_{m}\right) \tag{29}
\end{equation*}
$$

In other words, either parents' relative educational income are too low comparing to public educational input or the public educational input is sufficiently high and no need for extra investment in education. Either way, with no private education cost, the migrant workers' utility can be easily rewritten as

$$
\begin{equation*}
U_{t}^{j}=\lambda \ln \left(c_{t}^{c o, j}\right)+\Gamma_{K} \widetilde{\beta_{k}} \ln \left[B_{t}^{j} \theta_{j}^{\eta} h_{t}^{\alpha^{j}}\left(\overline{h_{t}^{j}}\right)^{\kappa}\right]+\varepsilon^{j}, \quad j=l, m . \tag{30}
\end{equation*}
$$

Thus, higher utility essentially depends on higher consumption, higher human capital accumulation of children or both. Direct calculation yields that the difference in consumption between taking children to migrate and leaving them behind is:

$$
\begin{equation*}
c_{t}^{c o, m}-c_{t}^{c o, l}=\frac{W_{t} \Gamma_{K}-K k_{m}\left(1+\Gamma_{K}\right)}{\lambda}(\geqq 0) . \tag{31}
\end{equation*}
$$

Obviously, taking the children to migrate does not automatically increase the consumption and utility. The difference in consumptions essentially lies on the relationship between altruism gain, $W_{t} \Gamma_{K}$, and relation cost of children, $K k_{m}\left(1+\Gamma_{K}\right)$. If the gain is more than covering the relocation cost, everyone in the family would have higher consumption with children migration than leaving them behind. However, when the gain is less than the cost, all consumptions, including the migrant workers, their children and parents, are less than the ones leaving the children behind. In this scenario, the only possible improvement in migrant worker's utility is children's human capital accumulation, measured by school performance, $h_{t+1}^{m}-h_{0}$. Notwithstanding, there is no guarantee that the migrant children's school performance is better than those of being left-behind, as we mentioned in the empirical test.

More precisely, similar to the previous case, direct calculation yields

$$
\begin{align*}
U^{c o, m}-U^{c o, l} & =\lambda \ln \left[\frac{W_{t}-K k_{m}}{W_{t}}\right]+I\left(a_{k}\right) \\
& +\Gamma_{K} \widetilde{\beta_{k}}\left[\ln \left(\frac{B_{t}^{m}}{B_{t}^{l}}\right)+\kappa \ln \left(\frac{\overline{h^{m}}}{\overline{h^{l}}}\right)+\ln \left(\frac{h_{t}^{\alpha^{m}}}{h_{t}^{\alpha^{l}}}\right)+\eta \ln \left(\frac{\theta_{m}}{\theta_{l}}\right)\right], \tag{32}
\end{align*}
$$

in which the first term is always negative given $k_{m}>0$ and the second and last terms are always nonnegative, provided migrant children do not decrease too much their motivation and efforts compared to being left-behind.

Similar to Case (I), there exists positive relocation cost threshold

$$
\begin{equation*}
K_{m}^{(I I)}=\frac{W_{t}}{K}\left[1-\left(\frac{B^{l} \bar{h}^{\kappa} h^{\alpha^{l}} \theta_{l}^{\eta}}{B^{m}{\overline{h^{m}}}^{\kappa} h^{\alpha^{m}} \theta_{m}^{\eta}}\right)^{\Gamma_{K} \widetilde{\beta_{k}} / \lambda} e^{-\frac{I\left(a_{k}\right)}{\lambda}}\right] \tag{33}
\end{equation*}
$$

such that,

$$
U^{c o, m}-U^{c o, l} \begin{cases}>0 & \text { if } 0 \leq k_{m}<K_{m}^{(I I)}  \tag{34}\\ <0 & \text { if } k_{m}>K_{m}^{(I I)}\end{cases}
$$

Thus, for this group of migrants, the following conclusion can be drawn:

Proposition 7 Suppose migrant's income checks (29), that is, parents can not (or no need to) afford any private education to their children regardless where their children are living. Then, there is relocation cost threshold, which is defined by (33), such that,

- if $k_{m}<K_{m}^{(I I)}$, parents would be better off by taking their children to migrate;
- if $k_{m}>K_{m}^{(I I)}$, it is optimal for parents to leave their children behind.


### 4.3 Case (III): $e_{t}^{l}>0, \quad e_{t}^{m}=0$

Parents may realize that the education level in the city is higher than back home rural area and education is essentially important for their children's future. In order to give
their children a better chance to get better education, the migrant parents may invest in private education for their children when they leave them behind, while no needs when these children migrate to the city. That is, via Proposition 2, migrant parents' relative educational income checks

$$
\begin{equation*}
K \theta_{l}<\frac{\Gamma_{K} \widetilde{\beta}_{k} \eta}{\lambda} W_{t}<K \theta_{m} \tag{35}
\end{equation*}
$$

In this case, parents' utilities are:

$$
U_{t}^{i n, l}=\Lambda \ln \left(c_{t}^{i n, l}\right)+\Gamma_{K} \widetilde{\beta}_{k} \ln \left[B_{t}^{l} h_{t}^{\alpha^{l}}\left(\overline{h_{t}^{l}}\right)^{\kappa}\right]+\varepsilon^{l}+\delta^{l}
$$

and

$$
U_{t}^{c o, m}=\lambda \ln \left(c_{t}^{c o, m}\right)+\Gamma_{K} \widetilde{\beta_{k}} \ln \left[B_{t}^{m} \theta_{m}^{\eta} h_{t}^{\alpha^{m}}\left(\overline{h_{t}^{m}}\right)^{\kappa}\right]+\varepsilon^{m}
$$

Thus, the difference is:

$$
\begin{align*}
U^{c o, m}-U^{i n, l} & =\lambda \ln \left[\frac{W_{t}-K k_{m}}{W_{t}+K \theta_{l}}\right]+\Gamma_{K} \widetilde{\beta_{k}} \eta \ln \left(\frac{\theta_{m}}{e^{i n, l}+\theta_{l}}\right) \\
& +J\left(a_{k}\right)+\Gamma_{K} \widetilde{\beta_{k}}\left[\ln \left(\frac{B_{t}^{m}}{B_{t}^{l}}\right)+\kappa \ln \left(\frac{\overline{h^{m}}}{\bar{h}^{i}}\right)+\ln \left(\frac{h_{t}^{\alpha^{m}}}{h_{t}^{\alpha^{l}}}\right)\right], \tag{36}
\end{align*}
$$

where $J\left(a_{k}\right)=\lambda \ln \left(\frac{\Lambda}{\lambda}\right)+\left(1+\Gamma_{K}\right) \ln \left(1+\Gamma_{K}\right)>0$ and $e^{i n, l}+\theta_{l}=\frac{\Gamma_{K} \widetilde{\beta}_{k} \eta\left(W_{t}+K \theta_{l}\right)}{K \Lambda}$ by (15). It is easy to see that the terms in the second line are nonnegative given the assumptions and if the migrant children's motivation do not decrease too much compared to being left-behind. The second term on the right hand side could be positive or negative depending on the aggregate educational input ratio, $\frac{\theta_{m}}{e^{i n, l}+\theta_{l}}$. If the destination's education input is sufficiently high, such that, $\theta_{m}>e^{i n, l}+\theta_{l}$, then children migration should benefit from the public school in the destination. Nevertheless, the first term on the right hand side is always negative due to relocation cost. More precisely, the relocation threshold in this case is given by

$$
\begin{equation*}
K_{m}^{(I I I)}=\frac{1}{K}\left[W_{t}-\frac{W_{t}+K \theta_{l}}{e^{J\left(a_{k}\right) / \lambda}}\left(\frac{B^{l} \bar{h}^{\kappa} h^{\alpha^{l}}}{B^{m} \overline{h^{m}} h^{\alpha^{m}}}\left(\frac{e^{i n, l}+\theta_{l}}{\theta_{m}}\right)^{\eta}\right)^{\Gamma_{K} \widetilde{\beta_{k}} / \lambda}\right] \tag{37}
\end{equation*}
$$

which can be positive or negative. The following conclusion can be made in this case:

Proposition 8 Suppose condition (35) holds and relocation cost threshold $K_{m}^{(I I I)}$ is defined by (37),

- if $K_{m}^{(I I I)}>0$ and if $0<k_{m}<K_{m}^{(I I I)}$, migrant parents are better off by taking their children to migrate, though without private education in the destination;
- otherwise, if $k_{m}>K_{m}^{(I I I)}$, it is optimal for migrant parents to leave their children behind but invest in their private education.

The information from the second part of this proposition is two-fold: (1) $k_{m}>K_{m}^{(I I I)}>$ 0 and (2) $k_{m} \geq 0>K_{m}^{(I I I)}$. In the first case, it is still possible that reducing the real relocation cost, $k_{m}$, to such an extend that parents are better off by taking the children to migrate, for example, hukou reforming such that registration in the destination school is free. However, the second case makes that impossible. If $K_{m}^{(I I I)}<0$, the optimal choice for parents is leaving their children behind and invest in their private education - too costly to take their children to migrate.

### 4.4 Summary of theoretical findings and data revisit

Combining the above three cases, we summarize the findings in the following Figure 2, which gives precise idea what Chinese internal migrant parents should do with their children depending on their income, concerns of human capital accumulation and relocation cost threshold.

## (The figure needs to be updated in term of relative income.)

In Figure 2, the horizontal axis is the relocation cost for children migration and the vertical axis presents the educational relative income of migrant parents. Recall the relative income is determined by mainly two parts: the absolute lifetime income, $W_{t}$, and how parents value children's education, $\widetilde{\beta_{k}}$. If all parents value their children's education equally, that is, $\widetilde{\beta_{k}}$ is the same for everyone, then the relative income is equivalent

| Wage |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Migrant children with private education |  | Left-behind children with private education |
| $\frac{\lambda}{\Gamma_{K} \beta_{k} \eta} K \theta_{m}$ | Migrant children with no private education | $K_{m}^{(I)}$ | Left-behind children with private education |
| $\frac{\lambda}{\Gamma_{K} \beta_{k} \eta} K \theta_{l}$ | Migrant children with no private education | $K_{m}^{(I I I)}$ | Left-behind children with no private education |
| 0 |  |  | Relocation co |

Figure 2: Choice of migrant parents.
to the absolute income. Thus, parents' decision of taking their children to migrate or leave them behind, and how to invest in their education, will only depend on absolute income. Nonetheless, parents may value their children's education differently. It could happen that some high income parents do not care about their children's education due to the fact education is not rewarded as it should be or they are just too busy. If so, the relative income, $\frac{\Gamma_{K} \widetilde{\beta_{k} \eta}}{\lambda} W_{t}$ is low compared to public education input. On the other side, it may be the case that low income parents realize how important education is to their children and consider it as the only way to change their children's lives. And thus they value $\widetilde{\beta_{k}}$ highly, such that, $\frac{\Gamma_{K} \widetilde{\beta_{k}} \eta}{\lambda} W_{t}$ is high related to public education input, though their absolute income, $W_{t}$, is low. In other words, these parents sacrifice their consumption, and the whole family's consumption, in order to give good education to their children.

In the rest of this section, we exploit RUMiC data on how Chinese internal migrants value their children's education and how that reflects in their decision making.

From the RUMiC data, in Figure 3(a), we plot the educational input in 2008 (horizontal axis) and number of children benefits from it (vertical axis). it is obvious that regardless of their children being left-behind or migrant, parents invest in their children's education. Anyhow, from the direct data, it is hard to see the theoretical patterns shown in


Figure 3: (a) number of left-behind and migrant children and (b) education cost.

Figure 2. In order to obtain clearer information, in Figure 3(b), we plot the yearly (2008) education investment in term of monthly household income. Given the students' registration is usually semestral and yearly, so for education cost, we choose the total year of 2008. In order to make the plots readable, the horizontal line is monthly income instead of yearly income. It is clear that on the far right of (b), the high income parents take their children to migrant, but their investment in their children's education may be high (upper-right corner) or low (lower-right corner ), which depends on how the parents value their children's education (that is, $\widetilde{\beta_{k}}$ in the theoretical part) compared to other consumption. As well, on the lower-left corner, where parents' income are low (monthly income is between 1000-2000 yuan), parents invest about 2500-5000 yuan per year on their left-behind children's education, which are no less than the higher incomes parents who earn 5000-6000 yuan per month. Thus, education input of Chinese migrant parents are not linearly related to their absolute income level, rather it is the relative income which determines their educational decision of their children and this confirms the theoretical conditions.

Likewise, Figure 4 presents the relative income-cost information. The vertical axis is the education-consumption ratio and the horizontal axis is migrants' monthly income. In Figure 4(a), the upper-left corner presents the left-behind children from low income


Figure 4: (a) education-consumption ratio to income and (b) education-consumption ratio to income.
family but with relatively high investment in education, while the lower-right corner are migrant children from high income family but with relatively low education input. However, both (a) and (b) in Figure 3 clearly state that in absolute term, the high income parents do not invest less in their children's education than the low incomes ones. Both high and low income parents invest in their children's private education and the difference mainly lies on taking their children to migrate or leaving them behind.

Figure 4 (a) on the one hand shows that the education is relatively expensive for low income parents, and on the other hand confirms the results in Figure 2 that low income parents optimally choose to leave their children behind but invest in their education, while it is optimal for high income parents to take their children and invest in their education as well. Nevertheless, from Figure 4 (a), it is harder to draw clear conclusion for the majority of the migrant workers, that is, income level between 2000 to

5000 yuan per month. To see clearly this part, in Figure 4(b), we only plot the income group of income between 1000-6000 yuan per month. It is clear that compared to the middle income migrants ( monthly income between 300o-5000yuan), lower income migrants (monthly income less than 3000yuan) leave more of their children behind- the red points, and less private education investment, though there are some invest quite highly in the children's education- the upper-left corner red points. Additionally, we also notice that for the education-consumption ratio is above 0.4 , there are more leftbehind children than migrant ones, which may confirms the theoretical finding that parents' optimal choice by considering that the relocation cost is high.


Figure 5: China Labor Bulletin: Employment of migrant workers by sector 2015.

Given the work most of Chinese internal migrant undertake (see Figure 5), the relative low income and high relocation cost may be the reason that there are so many Chinese young children being left-behind. Though the decision of leaving their children behind is difficult for migrant parents, that may be the rational choice.

## 5 The dynamics and the long-run outcomes

In this section, we will investigate what would be the long-run consequences from the decision of Chinese internal migrant parents as to where to locate their children and how to educate their children.

For rural Chinese children, except working as farmer or as migrant worker just like their parents, due to the hukou constraint, they have only two ways to change their lives: (1) individual efforts to succeed in the National College Entrance Examination (Gaokao in Chinese) and become skilled labor after graduation and; (2) to be lucky falling into the urbanization process.

### 5.1 Gaokao and urbanization

If the Chinese rural children succeedingly enter and finish university study, their high education will enable them to find a better paid job and live in the city, so will their future children and descendants. Thus, succeeding in Gaokao is essential for the family's short- and long-run welfare. Usually, Gaokao is a prerequisite for entrance into almost all Chinese universities at the undergraduate level and it is taken by students in their last year of senior high school. Depending on province, the entrance exams last about nine hours over a period of two to three days (currently in June). Most provinces require all students to take Chinese, Mathematics and English language. But the subjects may change across provinces. Obviously, some extra private classes to complement the public schools are important for the preparation of Gaokao. Generally, the students need to take their exam in the region where their hukou belongs and, therefore, most of the migrant children have to return to their hometown before the exam.

Besides the chance of entering university, due to the current urbanization process in China, there are some opportunities to obtain city hukou (so do their descendants in the future) and earn city income without university degree, which is still better than
being migrant workers.

Therefore, migrant parents' decisions are not only important for their own welfare, but also essential for their children's future (the short-run effects) as well as their future descendent's economic potential (the long-run effects).

Before the detail study of the long-run consequence from the migrant decision, it is worth to notice that the current setting of human capital and wealth accumulation dynamics of Chinese internal migrants are similar to the seminal contribution of Galor and Zaira (1993) and Galor and Moav (2004), where parents' wealth and bequest play one of the central rules in determining the long-run equilibrium of the economy. In their studies, the parents' bequest may limit children's ability to borrow from the credit market and hence constraint their chances of educational investment. The current study differs from their contribution in the following two aspects: (1) Chinese internal migrants usually do not rely on the financial credit market, due to the limitation and imperfection of credit market, rather they rely on their own income to invest in their children's education; (2) in the current study, we do not investigate the macroeconomy, rather we focus only on the descent of the current migrant workers by assuming that the Chinese macroeconomic environment, especially the hukou system, will not change in the short- and long-run. Of course, that does not mean the long-run Chinese macroeconomic study is not interesting, rather, it is a very important topic and deserve some separated and more serious study. We will discuss this in the conclusion again.

### 5.2 Long-run consequences

We start with the children who are motivated to study, that is, $B^{j}>0, j=l, m$. Following the theoretical finding in Figure 2, there are four possible outcomes from parents' decisions on where to locate their children and how to educate them: children are living in (a) city with private education, (b) city with no private education, (c) rural with
private education, or (d) rural with no private education.
We denote that children with private education have probabilities $p^{j} \in(0,1)(j=l, m)$ to enter university. Without private education, the probabilities of entering university are $q^{j} \in(0,1), j=l, m$, depending on their are left-behind or migrant children. Mathematically, for $j=l, m$, the entry probability, which is measured only on final entry exam, checks

$$
\left.P\left(h_{t+1}^{j}-h_{0}=B_{t}^{j}\left(\theta_{j}+e_{t}^{j}\right)^{\eta} h_{t}^{\alpha^{j}} \overline{\left(h_{t}^{j}\right.}\right)^{\kappa} \geq h^{*}\right)= \begin{cases}p^{j}, & \text { if } e_{t}^{j}>0 \\ q^{j}, & \text { if } e_{t}^{j}=0\end{cases}
$$

where $h^{*}$ is the lowest level to enter university. ${ }^{28}$
By assumption that in the city the public schools are better than the rural ones, then with private complementary investment in education, we can easily impose that

$$
p^{m}>p^{l}, q^{m}>q^{l}
$$

However, it is harder to justify the relationship between $p^{l}$ and $q^{m}$, given the children need to go back to their hometown to take the exam. If the migrant destination is in a different province than their home town, the exam may not be the same, thus the migrant children may face some disadvantage in comparison to the left-behind ones. Of course, for the migrant students with private education, the private classes may compensate this difference.

As well, with the urbanization process, migrant children have probability $\tau \in(0,1)$ to get city hukou during their childhood, regardless their education situation.

Combining the two channels of changing lives together, migrant children with private education have probability $\tau+p^{m}$ to get city hukou or become high skilled worker. And the rest $1-\tau-p^{m}$ migrant children will stay with rural hukou. While for migrant children without private education, the chances to remain rural hukou is $1-\tau-q^{m}$.

[^15]Given these children grow up in the city instead of their original rural villages, they do not know the farm work and will remain as migrant just like their parents.

For the left-behind children, private eduction increases their chances $\left(p^{l}-q^{l}>0\right)$ to enter the university. And the rest will remain rural hukou and grow up in the rural hometown. So they will face exactly the same decision as their parents: to migrate to the city to look for better paid jobs or stay in the country side and work as farmers. And if they decide to migrate, they will face the same dilemma as their parents: What will they do with their children - leave them behind or take them, provide them with private education or not?

For those children who lost motivation to study, that is $B^{j}=0(j=l, m)$, as demonstrated before, the parents do not have much choices on their education and entering university is impossible. Nevertheless, if they migrate with their parents to the city, they have the same chances obtain city hukou as the other migrant children via urbanization. The rest of them will remain rural hukou and work as their migrant parents. For the left-behind non-motivated children, they hold always rural hukou and repeat their parents' decisions and choices.

We use the tree in Figure 6 to illustrates the above dynamics of hukou/skills changing over generations.

The above analysis demonstrates that the migrant children with private education have much bigger chances to obtain city hukou or higher paid jobs than the rest of the children. Therefore, if current migrant parents take into account not only their children's human capital accumulation (short-run), but also their future descendants' economic potential (long-run), the optimal choice should take the children to migrate and provide them with private education. Alter all, Figure 2 shows that the decision making depends on the relatively income. The ones, at the upper-left-corner of Figure 2, who have relatively high educational income and can afford to take their children to migrate and offer them with private education, their children will be better off than the rest, given Figure 4 (b) confirms that the low income parents pays less private


Figure 6: Dynamics of Migrants' hukou change
education to their children. Thus, the inequality among different migrant families are increased over a few generations. But of course, if low income parents cares very much for their children's education, the inequality situation may change over time.

Nevertheless, to study the macroeconomic environment and the long-run distribution of the Chinese economy is beyond the current study.

## 6 Conclusion Remarks

The aim of this paper is to provide answers to the following questions: What should Chinese internal migrant parents do with their children - leave them behind or take them to migrate? And how to invest in their children's education?
"Take your child with you" (to migrate) is one of the suggestions from scholars to the
migrant job seeking parents. However, the reality is far more complicated than this simple slogan. Emotionally, most, if not all, migrant parents, Chinese internal and international migrants, would like to take their children with them. But financially, among some other difficulties, sometime this good wish become impossible. Thus, leaving their children behind in the home town becomes a rational choice.

To give precise answers, we rely not only on empirical studies but also on theoretical analysis. The originality of our study is that we compare the differences of school performance between the migrant and left-behind children of different ages. However, the empirical test based on RUMiC data is inconclusive, though it indeed suggests that younger children may benefit more (in their school performance) from living with their migrant parents than teenagers. Theoretical OLG model demonstrates that taking their children to migrate or not rely on the relocation cost of children migration. The relocation costs includes extra school registration fee and possible extra health insurance due to the constraint of hukou and depends on the educational development gaps between rural regions and cities. The larger the educational gap is, the higher is the relocation cost threshold. Thus, to make it possible for children to migrate with their parents, some basic child-related policies and infrastructure are needed. These policy includes reducing the educational gap between differential regions, diminishing school registration fee, providing public health care for migrants children and so on to remove the barriers of children migration and decrease the children relocation cost.

Furthermore, providing children with extra private education to complement the public school not only affects children's human capital accumulation but also influences the economic potential of their descendants in the future. The provision of private education relies on the comparison between relative income of migrant parents and the public educational input. The educational relative income is defined as lifetime income multiplied by the education-consumption ratio, and the lifetime income includes also potential remittances from children in the future. Thus, with the promise from their children of old age support, the inequality can be decreased if the low income migrant parents cares more for their children's education than the high income ones. Nonethe-
less, if all parents care equally for their children's education, inequality increases over generations.

It is worth noting that our theoretical results are based on a tractable OLG model that ignores many economic and non-economic effects of Chinese internal migration. Nevertheless, omitting these features allows us to focus on the main concerns of the migrants workers. Future work should account for the extensions which include the macroeconomic impacts of migrant workers. Especially, we should forecast and estimate the gain and lost in GDP when these left-behind and migrant children enter the job market. One possible further study is in line with the framework of Galor and Zeira (1993), but including migrant worker and original city residents together, to study the long-run distribution of wealth and inequality among all population, not only among the migrant workers as this current study. Furthermore, with more panel data available, tracing down the development differences of those left-behind and migrant children would provide new insight into the understanding of the wellbeing of these children.

To conclude, the present paper demonstrates conditions under which Chinese internal migrant parents should take their children with them and how to provide them with private education. This insight can contribute to explaining the enormous left-behind children in China and may be helpful in designing immigration policies.

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    ${ }^{\dagger}$ Yiwen Chen: CREA, University of Luxembourg. 162a, avenue de la Faiencerie, L-1511, Luxembourg. E-mail: yiwen.chen@uni.lu
    ${ }^{\ddagger}$ Vincent Fromentin: University of Lorraine, Nancy, CEREFIGE and CREA, France. E-mail: vincent.fromentin@univ-lorraine.fr
    ${ }^{\S}$ Ioana Salagean: STATEC, 13, rue Erasme L-1468 Luxembourg. E-mail: Ioanacristina.salagean@statec.etat.lu
    ${ }^{\top}$ Benteng Zou: CREA, University of Luxembourg. E-mail: benteng.zou@uni.lu. This author acknowledges the financial support from NSFC project 71573230.

[^1]:    ${ }^{1}$ The report is based on the annual survey of migrant workers conducted by the National Bureau of Statistics of China. See the report "Migrant workers and their children" on line at http://www.clb.org.hk/content/migrant-workers-and-their-children.

[^2]:    ${ }^{2}$ Early November 2016, China's Ministry of Civil Affairs released a survey of the number of leftbehind children in rural China. According to this survey, a total of 9.02 million Chinese children under the age of 16 were not under the direct care of their parents - both parents are absent. Of those 9.02 million, 360,000 were not under the direct care of anyone at all. It also found that $62 \%$ of them were aged between 6-13 years old - school age children. The difference between this new survey and the All-China Women's Federation's 2013 report is the following: In the new survey, it only considers the children whose both parents are migrant and children's ages are between 0-16 years old; while the AllChina Women's Federation's 2013 report, following the guideline of United Nations Children's Fund (UNICEF), includes both parents and only one parent are migrants and the children are age 0-18 years old.
    ${ }^{3}$ See also The Economist, October 17th 2015, page 29-30.
    ${ }^{4}$ Currently, in China's rural area, most of the left-behind children's grandparents are illiterate and can not help for school work.
    ${ }^{5}$ The left-behinds are not only children, but also parents. The Economist August 29th 2015 reported that: In 2009-11 people over 65 accounted for just under half of all suicides, and more in rural area: living alone in old age can be harsh anywhere, but in China it may be particularly isolating, given that so many young Chinese have left their villages, and parents, in search of work. The government has tried to enforce filial piety, passing a law in 2013 that threaten fines or jail if people fail to visit parents and feed their 'spiritual needs'.
    ${ }^{6}$ For example, The Economist October 17th 2015 Page 32: "In May a teacher in one such" boarding "school in Gansu province in the north-west was executed for abusing 26 primary-school students. In Ningxia province in June, a teacher got life in prison for raping 12 of his pupils, 11 of whom had been left behind". We can read more similar kind of tragedies from Chinese news than what is reported by The Economist.

[^3]:    ${ }^{7}$ As mentioned by Aris Chan (2009), Chinese educational system is highly competitive and examination oriented. Schools are desperate to maintain their academic standards because prestigious schools can demand higher fees and donations. Migrants are usually seen as academically inferior and are usually assigned to mediocre or poor quality schools, shunned by those of a higher standard. Even migrant workers who have been living in their host cities for many years face problems in getting their children into decent state schools.
    ${ }^{8}$ A 2012 survey in Cixi, Zhejiang province, for example found that 57 percent of migrant children did not have any medical insurance, see China Labor Bulletin at http:/ /www.clb.org.hk.

[^4]:    ${ }^{9}$ UNICEF provide systematic studies about left-behind and migrant children. However, almost all the studies are either focus on migrant children in developed economies: UK (Crawley et al., 2009), France (Kirszbaum et al., 2009), Germany (Clauss and Nauck, 2009), Australia (Katz and Redmond, 2009), Netherland (De Valk et al., 2009), Switzerland (Fibbi et al., 2009) and so on or left-behind children in the developing counties: Indonesia, Thailand and Philippines (Bryant et al., 2005), Argentina, Chile and South Africa (Yaqub et al., 2009), Mexican and Salvador (De La Garza, 2010). Rossi (2008) provides a general unpublished survey of the children migration from developing countries.

[^5]:    ${ }^{10}$ China implements the 9-year compulsory education, including six years elementary school and three years junior high school.

[^6]:    ${ }^{11}$ Currently, only two waves of survey are accessible, while Chinese and Math test scores of children are not available in the first wave.
    ${ }^{12}$ Nine provinces are Shanghai, Guangdong, Jiangsu, Zhejiang, Anhui, Huibei, Sichuan, Choongqing and Henan. According to 2000 Census, two-thirds of migrant workers in China are migrated to the provinces of Shanghai, Guangdong, Jiangu and Zhejiang. There are $47 \%$ of migrants coming from Sichuan, Chongqing, Anhui, Hubei and Henan provinces (Akgüç et al., 2014). The distribution of sample is loosely associated with the overall population size of the city, with larger cities like Shanghai and Guangzhou (Kong, 2010).
    ${ }^{13}$ More details about methodologies of selecting migrant workers are described in (Kong, 2010).
    ${ }^{14}$ Dropout rate in this sample is $1.6 \%$. After compulsory education, there is a selection process that only children with good test scores can be enrolled in senior high school.
    ${ }^{15}$ Observations with missing information on variables are excluded. Besides, $7 \%$ children already got local city hukou are not included.

[^7]:    ${ }^{16}$ In general, no tuition fee may be charged in the implementation of compulsory education.

[^8]:    ${ }^{a} \mathrm{~d}$ represents dummy variable.

[^9]:    ${ }^{17}$ See Meng and Yamauchi (2015) for more detail.
    ${ }^{18}$ Migration status of children in the sample vary across household.
    ${ }^{19}$ Since the second wave of MHS was surveyed in early 2009, we use main residence place in 2008 to define children's migration status.

[^10]:    ${ }^{20} \mathrm{We}$ assume the error term is independent among households.

[^11]:    ${ }^{21}$ There is one child policy in China. However, there are lots of exceptional cases especially in the rural region. Furthermore, from the data set, it is easy to see that there are quite some families who have more than one child.
    ${ }^{22}$ Usually we should assume $a_{P} \leq a_{K}$, meaning individual cares parents less than children. But this assumption will be justified and clear later.
    ${ }^{23}$ In some mountain regions, while young adults migrate to work, grandmothers rent a room nearby to school where their school age grandchildren are registered. In this way, it grantees that young children can present in school safely, instead of wasting time on the dangerous roads to school and at the same time grandmothers can take care of their food and clothing. Very often, those grand mothers walk back to their villages to take care their farm and husbands after their grand children going to school; and come back to their renting places before their grandchildren finishing school.

[^12]:    ${ }^{24}$ See for example, the BBC news, on Sep1. 2011 at: http://www.bbc.com/news/world-asia-pacific14743222 and 2 Oct. 2012 at http://www.bbc.com/news/magazine-19787240. The Wall Street Journal article at: http://www.wsj.com/articles/SB10001424052702304173704579260900849637692.
    ${ }^{25}$ As stated by The Guardian report on August 30, 2014, that "Almost 50\% of these 'leftbehind' children suffer depression and anxiety, compared with $30 \%$ of their urban peers, according to a new study funded by the Heilongjiang provincial government." See detail at http:/ /www.theguardian.com/world/2014/aug/30/china-left-behind-children-mental-health. See also Lee (2011); Hu et al. (2014); Zhao and Yu (2016); Ye and Lu (2011); Qin and Albin (2010) for systematic study about the left-behind children's mental health problem.

[^13]:    ${ }^{26}$ Obviously, that may not be the case for some international migrants.

[^14]:    ${ }^{27}$ Recall Proposition 2 for more detail intuition.

[^15]:    ${ }^{28}$ Different universities have different entry requirement.

