

Preemption contests between groups

Stefano Barbieri*, Kai A. Konrad† and David A. Malueg‡

February 25, 2017

Abstract

We consider a preemption game between groups, where one of the members of the group can take a costly action on behalf of his group. We describe the equilibrium solution of this problem if players differ in their own costs of action, and if these costs are the players' private information. The equilibrium is typically characterized by delay. The nature of the equilibrium depends on key parameters such as the number of groups and their size. More competition between groups reduces delay, whereas larger groups make members of a given cost type more reluctant to act but may lead to earlier resolution of the conflict between the groups.

Keywords: preemption, free-riding, dynamic conflict, inter-group conflict, dynamic conflict, incomplete information, waiting

JEL classification codes: D74, H41, L13

PRELIMINARY DRAFT

Comments welcome – do not post

1 Introduction

We study preemption between groups of players. One of several competing groups can preempt the other groups if one of its members volunteers to carry out an individually costly action. This situation is characterized by countervailing incentives. On the group level there is a strong incentive to preempt. However, the

*Department of Economics, 206 Tilton Hall, Tulane University, New Orleans, LA 70118, United States. E-mail address: sbarbier@tulane.edu

†Max Planck Institute for Tax Law and Public Finance, Marstallplatz 1, D-80539 Munich, Germany. E-mail: kai.konrad@tax.mpg.de.

‡Department of Economics, 3136 Sproul Hall, University of California, Riverside, CA 92521, United States. E-mail: david.malueg@ucr.edu.

members inside the group face a free-riding problem. Each player likes that his group acts first and preempts the other group. But each player prefers an member of his group to act first.

We describe the equilibrium solution of this problem when players differ in their own costs of action and these costs are the players' private information. We also show how the nature of the equilibrium depends on key parameters such as the number of groups and their size.

One could also see the free-riding problem of Bliss and Nalebuff (1984) as a possible starting point of our analysis. They study the waiting game among members of a single clan or group. All members of the clan benefit if one of its members takes an individually costly action. We use their framework as a building block but allow for two or more clans that want to preempt each other. The player who grabs first acquires the benefit for only himself and his clan. Hence, each player prefers that his own clan preempts the other clans and can individually achieve this goal by taking costly action early on. But each clan member also has an incentive to wait, hoping that another member of his own clan takes action.

An example of this general structure is given by competing firms consisting of various divisions. The firms engage in a preemption game, as it has been discussed in a number of business contexts including the timing of investment,¹ of patent efforts,² and of strategies to develop land.³ Each division may expend the effort needed to gain the benefit of preempting the other firms. All divisions within the firm may benefit from successful preemption. Of course, each division may hope that other divisions within the same firm expend the cost of this move while fearing being preempted by another firm.

Another example is firms' early adoption of a particular technology or product with consumer network externalities. In such a situation, early product innovation may set the industry standard, and a growing consumer base may make this standard successful, even if a competing standard may have been superior. This may lead to a race in which firms enter the market phase when the product is still immature. Suppose now that there are two groups of producers of a good for which a new standard is to be defined. Some producers prefer to produce according to a possible standard A, the other producers prefer a possible standard B. At an early stage, there is still considerable uncertainty about the relative merits of the two standards for consumers, but the network externalities would dominate these differences. Single firms must decide when to innovate and face the market. If a firm that prefers standard A innovates first, this increases the likelihood that standard A succeeds, and this benefits all firms in this group. But the innovating firm bears a considerable risk.

Regional agglomeration benefits, as discussed by Baldwin and Krugman (2004), may also cause a pre-

¹See, e.g., Boyarchenko and Levendorskii (2014), Bloch, Fabrizi and Lippert (2015), Nishide and Yagi (2016), and Ruiz-Aliseda (2016).

²See, e.g., Hopenhayn and Squintani (2011, 2016).

³See, e.g., Heubeck (2009).

emption problem between groups. Suppose there are two regions and a number of entrepreneurs who are attached to one or the other region and consider whether to found a company in the region. If the first firm that invests in a region bears a high cost but also triggers agglomeration benefits to follower firms in the same region, then the competition between two regions with regionally attached firms can generate a preemption problem between the two regions and a free-riding problem among the firms.

These examples already allude to the related literature. A large literature considers static problems of inter-group conflict, focussing on the trade-off between making a contribution to an outcome that favors the whole group, and the option to free ride and leave these costly activities to other members of the own group. This literature traces back to Olson and Zeckhauser (1966) and many aspects of this strategic problem have been studied since then.⁴ Much of this literature focuses on a technology that allows contributors' efforts to add linearly to determine the aggregate group effort. Hirshleifer (1983) introduced alternatives to this public good provision technology—particularly the “best-shot” technology according to which only the largest contribution of a member of the group matters. Barbieri and Malueg (2014) is based on this group-contribution technology.

More recently this effort-aggregation technology has been applied in inter-group contests. This includes work by Barbieri, Malueg and Topolyan (2014), Chowdhury, Lee and Sheremeta (2013), and Barbieri and Malueg (2016).⁵ A key problem emerging in static inter-group contests with a best-shot technology and simultaneous action by all players is coordination and this may lead to a major inefficiency: several players may expend effort, and all but the largest effort in each group is purely wasted. Information and coordination become crucial. In our dynamic framework this simultaneity problem is resolved, at least for “interior” equilibria: those in which agents do not bunch. Only one player expends effort, and it is the one with the lowest cost. The coordination device is “delay,” as in Bliss and Nalebuff (1984), and also in some of the games studied in the preemption literature.

Konrad (2012) approached the coordination problem inside a group that competes with other groups differently. He considers whether players would like to form a group that has the purpose of credible information exchange. He characterizes the conditions when players may form such information alliances. This voluntary exchange of information occurs in the equilibrium, reduces fighting activity, and overcomes the problem of wasteful parallel effort. If players do not have such a costless method to share hard evidence on their abilities to fight, then the players may rely on other devices to overcome the wasteful duplication of efforts that may emerge inside a group. In a dynamic framework, delay becomes such a device.⁶

⁴Seminal contributions in this field are Katz, Nitzan, and Rosenberg (1990) and Esteban and Ray (2001).

⁵This literature also considered asymmetric conflict, by which one group aggregates effort according to a best-shot technology and the other group aggregates according to a weakest-link technology (Clark and Konrad, 2007, Chowdhury and Topolyan, 2015).

⁶More recently a literature has addressed this problem and looked at outcomes in which several group members have to

We approach the problem in several steps. Section 2 provides the main analysis. First, we outline the key building blocks of the formal model, and then we study the equilibrium and its properties in the parameter range in which the equilibrium is interior. Then we turn to the case of corner equilibria, which exhibit partial “bunching.” Section 3 concludes.

2 The formal framework

We first describe the formal framework that combines the problem of preemption between groups with the problems of free-riding and coordination within each group. Then we turn to the characterization of equilibrium and study its properties.

2.1 Players, actions and payoffs

We define N the set of all players i and $\{N_1, N_2, \dots, N_K\}$ a partition of these players into K groups of identical size with n players in each group. A representative player is denoted by i . This player is further characterized by his cost of effort c_i . All players’ cost parameters are drawn independently from the same atomless cumulative distribution function F . We assume F is continuous and differentiable on its support $[\underline{c}, \bar{c}] \subset (0, \infty)$. We denote by f the density of F .

Each player i knows the value of his own c_i and knows the distribution from which all players’ valuations are drawn, but not the values of other players’ realized costs, neither for members of his own group nor for members of the other groups.

Player i ’s action is denoted by T_i and is chosen from the interval $[0, \infty]$. The action is the time which player i waits until the player provides the public good to his own group (“grabs”), provided that none of the other $Kn - 1$ players grabbed prior to T_i . All players choose their T_i independently and simultaneously, based on the information of their own cost, about the distribution F and the rules of the game. Players cannot observe the actual choices of grabbing times T_i , but as time goes on, they observe whether one of the other players has grabbed. If a player did not observe any of the other players grabbing prior to time T_i , then i takes action at this point of time and the game ends.

The gross benefit for each member of a group is V if a member of the group grabs first. We assume throughout that $\bar{c} \leq V$. The payoff of player i with cost c is equal to $(V - c)e^{-\rho T}$ if he is the player who grabs first and at time T , equal to $Ve^{-\rho T}$ if a different player from the same group as i grabs first and at

take action, where some of the actions are more expensive than others, and the less expensive tasks may be awarded first. In this framework the players have countervailing incentives. They prefer not to take any of the costly actions, but conditional on taking up one of the costly tasks, they prefer to assume a task with a lower cost (see, e.g., Bonatti and Hörner, 2011).

time T , and equal to zero if a player from another group grabs first.⁷ If several players grab at the same time, we assume that then one of them is chosen at random and only that player incurs his cost.

It is important to note that the description of the cost and benefits allows for a number of interpretations that are more general than the narrow notation suggests. In particular, the analysis includes an interpretation in which V is the gross benefit for all players of the winning group who do not grab themselves, and $\hat{V} < V = V - c$ is the net benefit of the player who grabs first. This allows for a larger or smaller gross benefit for the grabbing player himself, if c is simply defined as the difference $c = V - \hat{V}$ between the player's net benefit if another member inside his group grabs and the net benefit if the player himself grabs. — **KAI, WE ARE NOT SURE WHAT YOU INTEND WITH THIS PARAGRAPH. If the point is to allow $c < 0$, won't everyone grab at $t = 0$?**

2.2 Properties of an interior equilibrium

Throughout we focus on symmetric equilibria. We describe a strategy of player by a function $T : [\underline{c}, \bar{c}] \rightarrow [0, \infty]$, where the function $T(\cdot)$ maps the player's own cost c to the his conditional time of own grabbing, $T(c)$. In equilibrium, this choice is, indeed, dynamically consistent, for the same reasons described by Bliss and Nalebuff (1984).

Standard incentive compatibility arguments imply that a player's optimal strategy is weakly increasing in c . We define a (symmetric) equilibrium as *interior* if the equilibrium strategy T is strictly increasing on $[\underline{c}, \bar{c}]$.⁸ Because there are no ties when T is strictly increasing, delay that does not change the probability of grabbing first is wasteful—it follows that $T(\underline{c}) = 0$ and T is continuous. Because T is nondecreasing, it follows that T is differentiable almost everywhere. Our first proposition characterizes the interior symmetric equilibrium.

Proposition 1. *If $\underline{c} \geq c_0 \equiv \frac{(K-1)n}{Kn-1}V$, then the unique interior symmetric equilibrium strategy T satisfies*

$$T(\underline{c}) = 0 \quad \text{and} \quad T'(c) = \frac{f(c)}{(1 - F(c)) \rho(V - c)} \left(c - \frac{(K-1)n}{Kn-1}V \right) \quad \forall c \in (\underline{c}, \bar{c}). \quad (1)$$

Proof. If the strategy T is strictly increasing, then a player with value \underline{c} knows he will grab first, preempting all others. Therefore, he will choose $T(\underline{c}) = 0$ because if $T(\underline{c})$ were larger than zero, then grabbing the prize an instant sooner would be profitable deviation as it would reduce wasteful delay. Similarly, given that T is strictly increasing it must be continuous, for otherwise some types just above the point of discontinuity

⁷The choice of a zero as the payoff of members of a non-winning group is simply a normalization. The analysis can easily be modified to assume that members of non-winning groups receive a non-zero loser prize and that the benefit of being a non-grabbing member of the winning group is some positive.

⁸It will follow from Lemma 1 below that an equilibrium strategy is interior if and only if it is strictly positive for all $c > \underline{c}$.

could increase their payoffs by reducing their grabbing time to avoid wasteful delay.

Now suppose $T(\cdot)$ is continuous and strictly increasing. Consider the calculus of a single player i contemplating grabbing at $T(c^*)$. We first determine the probability that someone *else* will take action before date t . This probability depends on the minimum realized cost among the other $Kn - 1$ players. The cumulative distribution function of the minimum cost among them, denoted by c_{\min} , is given by

$$\Pr(c_{\min} \leq x) = 1 - \Pr(\text{all other } Kn - 1 \text{ costs exceed } x) = 1 - (1 - F(x))^{Kn-1},$$

for which the associated density function is

$$(Kn - 1)(1 - F(x))^{Kn-2} f(x). \quad (2) \quad \text{density}$$

Moreover, if the game stops before $T(c^*)$, then the probability that i 's team has won is $\frac{n-1}{Kn-1}$ because players are acting symmetrically. Following Bliss and Nalebuff (1984), we can now write the payoff to a player with cost c (a “type- c player”) acting as if his cost were c^* as

$$\begin{aligned} U(c^*, c) &= (V - c)e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-1} + \frac{n-1}{Kn-1} V \int_{\underline{c}}^{c^*} e^{-\rho T(x)} (Kn - 1)(1 - F(x))^{Kn-2} f(x) dx \\ &= (V - c)e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-1} + (n-1)V \int_{\underline{c}}^{c^*} e^{-\rho T(x)} (1 - F(x))^{Kn-2} f(x) dx. \end{aligned} \quad (3) \quad \text{utility gene}$$

The first addendum of the payoff displayed in (3) captures the possibility that this player carries his group to victory, while the second corresponds to a teammate carrying the group to victory. The type- c player's first-order condition for choosing c^* is

$$\begin{aligned} 0 &= \frac{\partial U(c^*, c)}{\partial c^*} = -\rho T'(c^*)(V - c)e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-1} \\ &\quad - (Kn - 1)(V - c)e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-2} f(c^*) \\ &\quad + (n - 1)V e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-2} f(c^*) \\ &= e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-2} \\ &\quad \times \left\{ V(n - 1)f(c^*) - (Kn - 1)(V - c)f(c^*) - \rho(V - c)(1 - F(c^*))T'(c^*) \right\} \\ &= e^{-\rho T(c^*)}(1 - F(c^*))^{Kn-2} \\ &\quad \times \left\{ [(Kn - 1)c - (K - 1)nV]f(c^*) - \rho(V - c)(1 - F(c^*))T'(c^*) \right\}. \end{aligned} \quad (4) \quad \text{foc}$$

Where $T(c) > 0$, it must be that the first-order condition holds at $c^* = c$, so (4) implies

$$T'(c) = \frac{f(c)}{1 - F(c)} \times \frac{(Kn - 1)c - (K - 1)nV}{\rho(V - c)}. \quad (5) \quad \boxed{\text{diffeq}}$$

(6)

Note further that $T(c)$ as described in (1) identifies the global best response: equations (4) and (5) imply

$$\begin{aligned} \frac{\partial U(c^*, c)}{\partial c^*} &= e^{-\rho T(c^*)} (1 - F(c^*))^{Kn-2} f(c^*) \\ &\times \underbrace{\left\{ (Kn - 1)c - (K - 1)nV - \left(\frac{V - c}{V - c^*} \right) [(Kn - 1)c^* - (K - 1)nV] \right\}}_{\equiv \varphi(c^*; c)}. \end{aligned}$$

Now observe that $\varphi(c; c) = 0$ and

$$\frac{\partial \varphi(c^*; c)}{\partial c^*} = -(V - c) \frac{(n - 1)V}{(V - c^*)^2} < 0,$$

so $U(c^*, c)$ is strictly quasi-concave in c^* with a maximum at $c^* = c$. Thus, $\varphi(c; c) = 0$ yields the best response in (1). \square

The equilibrium strategy in Proposition 1 follows from balancing the marginal cost of delay with its marginal benefit. As noted, a player with the lowest possible cost will grab immediately, i.e., $T(\underline{c}) = 0$. Now consider a player i with cost c who plans to grab at date $T(c)$. The marginal cost of delaying slightly is

$$\rho(V - c)T'(c),$$

i.e., the loss in the present (net) value of the prize, and the marginal benefit of delaying is

$$h(c)(Kn - 1) \left(c - \frac{(K - 1)n}{Kn - 1} V \right),$$

where $h(\cdot)$ is the hazard rate function for F , i.e., $h(c) = f(c)/(1 - F(c))$. The term $h(\tilde{c})(Kn - 1)$ is the hazard rate of the minimum cost of all other agents at \tilde{c} : if the cdf of the minimum cost of all other agents is $G(\tilde{c}) \equiv 1 - (1 - F(\tilde{c}))^{Kn-1}$, then

$$g(\tilde{c}) \equiv G'(\tilde{c}) = (Kn - 1)(1 - F(\tilde{c}))^{Kn-2} f(\tilde{c}),$$

so

$$\frac{g(\tilde{c})}{1 - G(\tilde{c})} = \frac{(Kn - 1)(1 - F(\tilde{c}))^{Kn-2} f(\tilde{c})}{(1 - F(\tilde{c}))^{Kn-1}} = (Kn - 1) \frac{f(\tilde{c})}{1 - F(\tilde{c})} = (Kn - 1) h(\tilde{c}).$$

Therefore, $h(c)(Kn - 1)$ is the probability that by delaying slightly beyond $T(c)$ player i is no longer the first to grab. And the term $c - \frac{(K-1)n}{Kn-1}V$ is the change in player i 's payoff arising when, because of this delay, he is not the first to grab—it is the saving in the cost of grabbing minus the value of being preempted by another group. Now setting the marginal cost equal to the marginal benefit of delay yields

$$T'(c) = \frac{h(c)(Kn - 1)}{\rho(V - c)}(c - c_0),$$

which is equivalent to (1).

Note that, in the absence of possible preemption by another group ($K = 1$), equation (1) reduces to equation (4) in Bliss and Nalebuff (1984) if we take into consideration that they assume $\rho = 1$ and derive equilibrium for the game with $n + 1$ agents.

Proposition 1 deals with a case of “high costs,” where $\underline{c} \geq c_0$. This condition is necessary and sufficient to ensure $T' \geq 0$ in (1). Section 2.4 deals with the possibility of “low costs,” where $\underline{c} < c_0$. When low costs are possible, some types of players will grab immediately, that is, there is partial bunching at $T = 0$ and an interior equilibrium does not exist. We turn to the case of bunching in Section 2.4, but we note now that arguments there developed to handle bunching will establish that the equilibrium described in Proposition 1 is the unique symmetric equilibrium if $c \geq c_0$, without restricting attention to strictly increasing strategies. In the remainder of this section we maintain the assumption that $\underline{c} \geq c_0$ and explore the properties of the interior equilibrium.

Because $T(\underline{c}) = 0$, we have

$$T(c) = \int_{\underline{c}}^c T'(y) dy, \text{ for all } c \in [\underline{c}, \bar{c}].$$

Recalling from the proof of Proposition 1 that $U(c^*, c)$ is the payoff to a player with cost c acting as if his cost were c^* , by the envelope theorem we can write the equilibrium utility $U^E(c) \equiv U(c, c)$ as

$$U^E(c) = V - \underline{c} - \int_{\underline{c}}^c e^{-\rho T(y)} (1 - F(y))^{Kn-1} dy. \tag{7}$$

eq utility

For comparative statics we include the dependence of T and U^E on K and n .⁹

Proposition 2. *For a player of a given type c , the grabbing time $T(c; K, n)$ is (linearly) decreasing in the number K of groups and it is (linearly) increasing in the size n of the groups.*

⁹The next proposition extends the linearity property of $T(c; K, n)$ with respect to n found by Bliss and Nalebuff (1984).

Proof. Consider first an increase in K . We have, using (1),

$$\frac{\partial T'(c; K, n)}{\partial K} = \frac{h(c)n}{\rho(V-c)}(c-V) = -\frac{h(c)n}{\rho} < 0. \quad (8)$$

partial tp K

We now have

$$\begin{aligned} T(c; K, n) - T(c; K+1, n) &= \int_{K+1}^K \frac{\partial T(c; k, n)}{\partial k} dk \\ &= \int_{K+1}^K \left(\int_{\underline{c}}^c \frac{\partial T'(y; k, n)}{\partial k} dy \right) dk \\ &= \int_{K+1}^K \left(\int_{\underline{c}}^c -\frac{h(y)n}{\rho} dy \right) dk \\ &= \frac{n}{\rho} \int_{K+1}^K \log(1 - F(c)) dk \\ &= -\frac{n}{\rho} \log(1 - F(c)) \\ &\equiv \Delta_n(c), \end{aligned}$$

where it is clear that $\Delta_n(c) > 0$ for all $c \in (\underline{c}, \bar{c})$ and $\Delta_n(c)$ is independent of K . By induction, $T(c; K+1, n) = T(c; 2, n) - (K-1)\Delta_n(c)$.

Consider next an increase in n . We have

$$\begin{aligned} T(c; K, n+1) - T(c; K, n) &= \int_{\underline{c}}^c \frac{h(z)}{\rho(V-z)} [(K(n+1)-1)z - (K-1)(n+1)V - ((Kn-1)z - (K-1)nV)] dz \\ &= \int_{\underline{c}}^c \frac{h(z)}{\rho(V-z)} [Kz - (K-1)V] dz \equiv \Delta_K(c). \end{aligned}$$

Then, for $z > \underline{c}$,

$$z > \underline{c} \geq c_0 = \frac{(K-1)n}{Kn-1}V \geq \frac{K-1}{K}V,$$

implying $\Delta_K(c) > 0$ for all $c > \underline{c}$. Consequently, $T(c; n+1) > T(c; n)$ for all $c \in (\underline{c}, \bar{c}]$. That is, as the number of players per team increases, free-riding is more pervasive, with all types above \underline{c} grabbing at a later dates. Further, since $\Delta_K(c)$ is independent of n , we have $T(c; K, n+1) = T(c; K, 2) + (n-1)\Delta_K(c)$. \square

One should note well that the comparative statics results reported in the foregoing proposition and subsequently in this subsection implicitly assume changes in parameters continue to yield interior equilibria. Recall that the condition for the interior equilibrium described by Proposition 1 is $\underline{c} \geq c_0 \equiv \frac{(K-1)n}{Kn-1}V$. Because c_0 is decreasing in n , it is clear that if one starts from an interior equilibrium then increasing n continues to yield the interior equilibrium. In contrast, c_0 increases in K , with limit V . Consequently,

cet. par., as K becomes sufficiently large the symmetric equilibrium will not be interior. We consider such equilibria in Section 2.4.

Proposition 2 highlights a major difference between the preemption game between groups and the between-groups contest problem with a best-shot contest technology in Barbieri and Malueg (2016). The qualitative comparative static properties of the grabbing time requires no assumptions regarding the shape of F (e.g., nothing is invoked about elasticity of F). Intuitively, for given n and K , the equilibrium function $T(c; K, n)$ describes the optimal timing of players. This timing is a function of the cumulative distribution function F and sorts grabbing times according to the players' grabbing costs. If there are more groups or smaller groups, the strict order of grabbing times is maintained but the whole function $T(\cdot)$ shifts—for given behavior of other players, each player has a higher preemption motive and a smaller free-riding incentive. This direct effect also dominates in the new equilibrium.

Now consider reconstituting a given number of players into a smaller number of larger symmetric groups. By Proposition 2 we know that decreasing the number of groups (without changing team size) would increase grabbing times. And then increasing group size would further increase grabbing times. Thus, we have the following corollary.

corT

Corollary 1. *Suppose $Kn = \tilde{K}\tilde{n}$ and $n < \tilde{n}$ (or, equivalently, $K > \tilde{K}$). Then for any $c > \underline{c}$, $T(c; \tilde{K}, \tilde{n}) > T(c; K, n)$.*

Corollary 1 says that $T(c; K, n)$ is higher for a player with a given cost if the group size increases and the number of groups decreases in a way that just keeps the size of the population constant. If groups can be re-organized in a way that merges groups or re-groups players in a way that leads to a smaller number of larger groups, this will make each player less eager to grab. Put differently, if existing groups can be split into a larger number of smaller groups, this increases competition between groups and changes the free-riding incentives inside the group in a way such that, overall, each player type grabs earlier.

We now calculate the expected time at which the game stops. Let $T^E(K, n)$ denote the equilibrium random time at which the game stops when there are K groups of n players each, each player following the strategy in Proposition 1. The expected time at which the game stops equals

$$\begin{aligned} ET^E &\equiv \int_{\underline{c}}^{\bar{c}} T(c) d(1 - (1 - F(c))^{Kn}) \\ &= \int_{\underline{c}}^{\bar{c}} \left(\int_{\underline{c}}^c T'(y) dy \right) d(1 - (1 - F(c))^{Kn}) \\ &= \int_{\underline{c}}^{\bar{c}} T'(y) \left(\int_y^{\bar{c}} d(1 - (1 - F(c))^{Kn}) \right) dy \end{aligned}$$

$$\begin{aligned}
&= \int_{\underline{c}}^{\bar{c}} T'(y)(1 - F(y))^{Kn} dy \\
&= \int_{\underline{c}}^{\bar{c}} \frac{f(y)}{1 - F(y)} \times \frac{(Kn - 1)y - (K - 1)nV}{\rho(V - y)} (1 - F(y))^{Kn} dy \\
&= \int_{\underline{c}}^{\bar{c}} \frac{1}{K\rho} \left(\frac{n - 1}{n} \frac{y}{V - y} - (K - 1) \right) d(1 - (1 - F(y))^{Kn}), \tag{9} \quad \boxed{\text{ET}}
\end{aligned}$$

where the third equality follows from interchanging the order of integration, and the fifth uses the strategy in (5). Equation (9) can be rewritten as

$$K\rho ET^E + (K - 1) = \int_{\underline{c}}^{\bar{c}} \frac{n - 1}{n} \frac{y}{V - y} d(1 - (1 - F(y))^{Kn}). \tag{10} \quad \boxed{\text{ETr}}$$

The next proposition provides comparative statics properties for the expected duration of the game.

Proposition 3. *The following comparative static results hold about the expected time at which the game ends: The expected end occurs sooner (i) if the discount rate ρ is higher or if the gross benefit V for each group member is higher or (ii) if the number K of groups is higher. (iii) If $\frac{y(1-F(y))^K}{V-y}$ is decreasing (increasing) in y , then ET^E increases (decreases) in n . (iv) Let $F(c; z)$ represent the cdf for c with risk shift parameter z . For a mean-preserving spread (in the sense of Rothschild and Stiglitz, 1970) ET^E increases.*

Proof. (i) From (10) it is immediate that an increase in the discount rate ρ or in the value of the gross benefit V decrease ET^E .

(ii) The result on an increase in the number of groups follows immediately from the comparative statics of $T(\cdot)$ in Proposition 2 by which increasing K leads each type to grab sooner, which, even without there being more players would result in the expected stopping time decreasing.

(iii) Consider an increase in n . We can rewrite the right-hand side of (10) as

$$\int_{\underline{c}}^{\bar{c}} \frac{y(1 - F(y))^K}{V - y} d\left(1 - (1 - F(y))^{K(n-1)}\right).$$

We see that the probability distribution in this equation is that of the minimum cost out of $K(n - 1)$ independent realizations. As n increases, this probability distribution decreases in the sense of first-order stochastic dominance; therefore, if $\frac{y(1-F(y))^K}{V-y}$ is decreasing (increasing) in y , then ET^E increases (decreases) in n .

(iv) We consider the same increase in risk considered in Bliss and Nalebuff (1984): let $F(c; z)$ represent the cdf for c with a risk shift parameter z . Increasing z corresponds to a mean preserving increase in risk in the sense defined by Rothschild and Stiglitz (1970) if, for some \tilde{c} , $F_z(c; z) \geq 0$ if $c \leq \tilde{c}$, while $F_z(c; z) \leq$

0 if $c \geq \tilde{c}$, and

$$\int_{\underline{c}}^{\tilde{c}} [1 - F(c; z)]^{Kn-1} F_z(c; z) dc = 0,$$

where the last condition keeps constant the mean of the minimum cost across all $K \times n$ agents. We can rewrite (10) as

$$\begin{aligned} \frac{n}{n-1} (K\rho ET^E + (K-1)) &= - \int_{\underline{c}}^{\tilde{c}} \frac{y}{V-y} d((1-F(y))^{Kn}) \\ &= V \int_{\underline{c}}^{\tilde{c}} \frac{[(1-F(y))^{Kn}]}{(V-y)^2} dy + \frac{\underline{c}}{V-\underline{c}}, \end{aligned}$$

after integrating by parts, and this expression is entirely analogous to equation (18ii) in Bliss and Nalebuff (1984), so their proof carries through. \square

The unambiguous comparative static results (i) and (ii) in Proposition 3 are intuitively plausible. The possibility described in part (iii) that n may increase or decrease ET^E is also intuitive. An increase in n has two countervailing effects: each player with a given type c grabs later, but as the number of players increases, the probability distribution of the lowest cost type shifts. Indeed, the probability that the lowest realized cost, among those of Kn players, is higher than a given c becomes less and less as n increases, for all possible c inside the support. Our identifying a sufficient condition for either effect to dominate also improves on Bliss and Nalebuff's Theorem 5 that characterizes only the behavior at the tails.

The next proposition describes the effects parameter changes have on the expected utility.

Proposition 4. (i) *The expected payoff of a player with a given cost type c is higher if the size of all groups is larger.* (ii) *The expected payoff is constant with respect to changes in the number of groups.*

Proof. (i) Considering (7), we see that both $e^{-\rho T(y;n)}$ and $(1-F(y))^{Kn-1}$ decrease with n , so $U^E(c)$ strictly increases for $c > \underline{c}$, and the larger c , the larger the increase.

(ii) Constancy of $U^E(c)$ with respect to changes in K follows from the fact that the integrand in (7) is constant with respect to K . To see this, we first take the natural log of the integrand to get

$$-\rho T(c; K) + (Kn-1) \log(1-F(c))$$

and then differentiate with respect to K to get

$$-\rho \frac{\partial T(c; K)}{\partial K} + n \log(1-F(c)).$$

Now use $\frac{\partial T(c;K)}{\partial K} = \int_{\underline{c}}^c \frac{\partial T'(y;K)}{\partial K} dy$ and substitute for $\frac{\partial T'(y;K)}{\partial K}$ from (7) to obtain

$$\begin{aligned} -\rho \int_{\underline{c}}^c \frac{-h(y)n}{\rho} dy + n \log(1 - F(c)) &= n \left(\log(1 - F(c)) + \int_{\underline{c}}^c h(c) dc \right) \\ &= n \left(\log(1 - F(c)) + \int_{\underline{c}}^c \frac{f(y)}{1 - F(y)} dy \right) \\ &= 0. \end{aligned}$$

Because the log of the integrand is constant with respect to K , so too is the integrand itself. \square

An increase in n gives each player the same expected gross benefit V/K , but it dilutes the cost burden of grabbing among a larger number of players. The proposition shows that this holds strictly, not only *ex ante*, but also for all players, irrespective of their cost type, except for the type with the lowest possible type who does not gain or lose. If there is only one group (as in Bliss and Nalebuff, 1984), this is easy to grasp: each player has a higher probability that another player from his group grabs first, which preserves the benefit, but probabilistically shifts the cost burden of grabbing. Proposition 4 shows that this effect carries over to a multi-group framework with preemption.

The payoff neutrality with respect to the number of groups is less intuitive. A larger number of groups makes it more likely that a single group is preempted. This reduces all players' expected payoffs. However, the increase in preemption pressure induces players of given types to grab earlier. This reduces wasteful delay. The proposition shows that the two effects just offset each other.¹⁰ Therefore, the overall effect of reconstituting a given number of players into a smaller number of larger symmetric groups is to raise equilibrium payoffs:

Corollary 2. *Suppose $Kn = \tilde{K}\tilde{n}$ and $n < \tilde{n}$ (or, equivalently, $K > \tilde{K}$). Then for any $c > \underline{c}$, $U^E(c; \tilde{K}, \tilde{n}) > U^E(c; K, n)$.*

2.3 A numerical example

We illustrate some of the comparative-statics results by way of a numerical example. In particular, the example illustrates a non-monotonicity result suggested by part (iii) of Proposition 3.

Example 1 (The effect of n on expected stopping time). *Let $V = 2$ and assume costs are distributed according to $F(c) = 2^t (c - \frac{3}{2})^t$, for $c \in [\frac{3}{2}, 2]$.*

¹⁰It is worth pointing out that we work under the assumption that c_0 remains smaller than \underline{c} so that an interior equilibrium exists. If K is allowed to grow without bound, then $c_0 \rightarrow V > \underline{c}$, so an interior equilibrium does not exist and Proposition 4 does not apply. Indeed, if $K \rightarrow \infty$, then the symmetric-equilibrium gross benefit V/K converges to zero, and so do payoffs.

First note that

$$\text{sign} \left[\frac{d}{dy} \left(\frac{y(1-F(y))^K}{V-y} \right) \right] = \text{sign} [V - Ky(V-y)h(y)]. \quad (11)$$

dersign

We begin with $K = 2$, in which case $c_0 = \frac{n}{2n-1}$. Thus, the symmetric equilibrium is interior for all $n \geq 2$. Now consider $t = 1$. Here, we see that

$$V - Ky(V-y)h(y) = 2(1-y),$$

which is always negative for the relevant range. Therefore, (11) and part (iii) of Proposition 3 imply ET^E is increasing in n ; Figure 1 depicts the relationship between ρET^E and n for $n = 2, \dots, 30$.

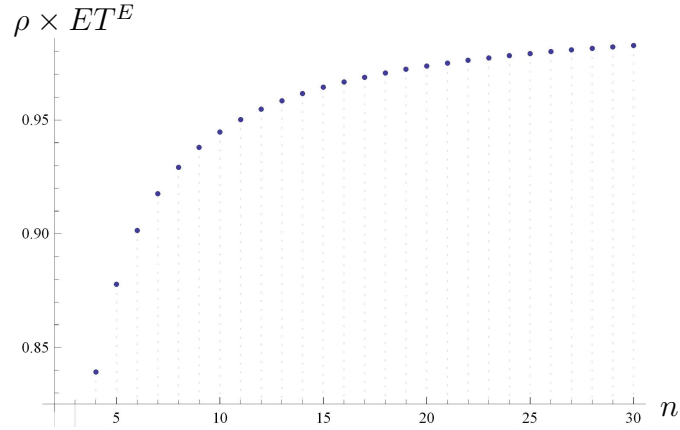


Figure 1: Effects of increasing n on ET^E : $V = K = 2$ and $F(c) = 2(c - \frac{3}{2}) \quad \forall c \in [\frac{3}{2}, 2]$.

fig:ETincreas

A pattern similar to that in Figure 1 holds for any $t \leq 1$. Thus, for $t \leq 1$ we see that the free-riding effect is very strong and it overwhelms the presence of additional agents on each team, which would otherwise lead to a lower expected stopping time. In contrast, if $t = 2$, for example, we find that ET^E first increases and then decreases with n , as depicted in Figure 2.

Here we see that ET^E increases for n going from 2 to 4, but further increases in team size make ET^E smaller because the “order-statistic” effect of having a better distribution of the minimum cost eventually prevails. Proposition 3 implies that $\frac{y(1-F(y))^K}{V-y}$ must be increasing for at least a range of y , as it is indeed depicted in Figure 3.

The fact that $\frac{y(1-F(y))^K}{V-y}$ is first increasing and then decreasing in y helps rationalize why ET^E is first increasing with n , then it turns decreasing and it stays so. As n grows, the distribution of the minimum costs is more and more concentrated towards lower values of c . Therefore, for n sufficiently large, the relevant part of $\frac{y(1-F(y))^K}{V-y}$ is increasing and the result follows as for Proposition 3. A pattern similar to that in Figure 2 is

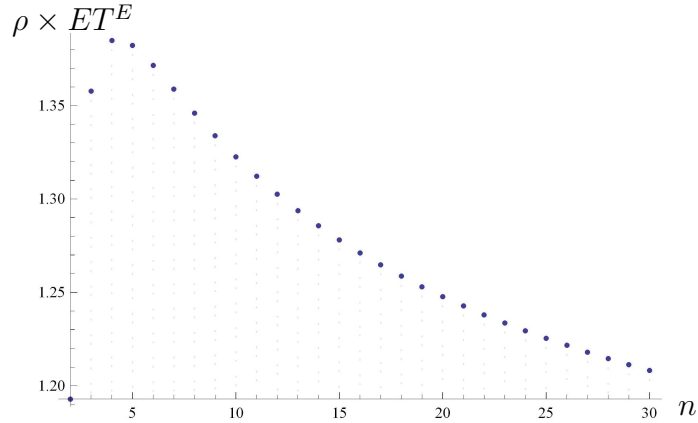


Figure 2: Effects of increasing n on ET^E : $V = K = 2$ and $F(c) = 4\left(c - \frac{3}{2}\right)^2 \quad \forall c \in \left[\frac{3}{2}, 2\right]$.

fig:ETincdec

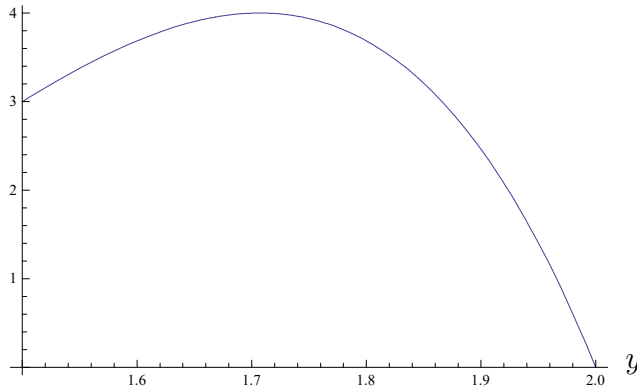


Figure 3: Plot of $\frac{y(1-F(y))^K}{V-y}$: $V = K = 2$.

fig:t=2

displayed by all parameterizations with $t > 1$. It is interesting to note that, with more competing teams, the switch in the direction of the relationship between ET^E and n occurs later. Indeed, by (11), if K increases, then $\frac{y(1-F(y))^K}{V-y}$ becomes decreasing for a larger set of y . For example, if $t = 2$ and $K = 3$, then ET^E remains increasing in n up to $n = 7$.¹¹

2.4 “Corner” solutions

Proposition 1 provided the symmetric equilibrium strategy under the assumption that $\underline{c} \geq c_0 \equiv \frac{(K-1)n}{Kn-1}V$. The strategy there fails to be weakly increasing if $\underline{c} < c_0$ because (1) shows T to be strictly decreasing for $c \in [\underline{c}, c_0)$. To maintain weak monotonicity of the equilibrium strategy, we now investigate the possibility

¹¹One needs to be a little careful here because for $K = 3$ and $n = 2$, $c_0 = 8/5 > 3/2$, which implies we need to analyze the corner solution. But as explained in the next section, the comparative statics work out here, too. It is the case that for $K = 3$ and $n \geq 3$, we have $c_0 \leq 3/2$.

that an equilibrium strategy has a “flat spot,” that is, an interval over which it is constant. The following lemma shows that if a symmetric equilibrium strategy T has a flat spot, then it must occur at 0, which implies that, for some $\hat{c} \geq \underline{c}$, $T(c) = 0$ on $[\underline{c}, \hat{c}]$ and T is strictly increasing for $c > \hat{c}$. Moreover, the strategy T must be continuous.

l:flats

Lemma 1. *Suppose T is a symmetric equilibrium strategy. Then T is continuous and if T is constant on $[\tilde{c}_l, \tilde{c}_h]$ and $\underline{c} \leq \tilde{c}_l < \tilde{c}_h \leq \bar{c}$, then $T(c) = 0$ for all $c \in [\tilde{c}_l, \tilde{c}_h]$. Further, $\tilde{c}_h \leq c_0$.*

From Lemma 1 we now see that if an equilibrium strategy has a flat spot it must be over an interval of the form $[\underline{c}, \tilde{c}_h]$, where T takes on value 0, and $\tilde{c}_h \leq c_0$. Furthermore, for any c where T is strictly increasing, the equilibrium analysis is precisely as for an interior equilibrium, thus requiring $c \geq c_0$. This reasoning has two implications. First, the equilibrium in Proposition 1 is unique among all symmetric ones, without focusing only on strictly increasing strategies. Second, we have the following:

p:corner

Proposition 5. *If $\underline{c} < c_0$, then the unique symmetric equilibrium strategy T satisfies $T(\underline{c}) = 0$ for $c \in [\underline{c}, c_0]$ and*

$$T'(c) = \frac{f(c)}{(1-F(c))} \frac{(Kn-1)}{\rho(V-c)} \left(c - \frac{(K-1)n}{Kn-1} V \right) \quad \forall c \in (c_0, \bar{c}). \quad (12)$$

Tprimecorner

Once c_0 is established as the point at which T begins increasing, the comparative statics described in Section 2.2 can be seen to hold generally.

p:cornerCST

Proposition 6. *For each $c \in [\underline{c}, \bar{c}]$, the equilibrium strategy $T(c; K, n)$ is weakly decreasing in K and weakly increasing in n . Furthermore, $c_0(K, n+1) < c_0(K, n) < c_0(K+1, n)$, and if $\underline{c} < c_0(K, n) \equiv \frac{(K-1)n}{Kn-1} V$, then*

$$T(c; K, n+1) > T(c; K, n) \quad \forall c \in (c_0(K, n+1), \bar{c})$$

and

$$T(c; K, n) > T(c; K+1, n) \quad \forall c \in (c_0(K, n), \bar{c}).$$

Moreover, increasing K also decreases ET^E .

From Proposition 6 we see that an increase in the number of teams, by reducing individual player’s grabbing times has the effect of decreasing the expected time at which the game ends. Indeed, because $c_0(K, n) \rightarrow V$ as $K \rightarrow \infty$, everyone grabs almost instantly and $ET^E \rightarrow 0$. Not surprisingly, as the effect of increasing n on the expected duration of the contest was ambiguous for interior equilibria, so too is it ambiguous for corner equilibria. Surprisingly, however, while increasing n had the effect of increasing interim

payoffs at interior equilibria, the effect is ambiguous for corner equilibria. This is most easily illustrated in an example.

eg:ambign

Example 2 (The effect of n on interim utility at corner equilibria). Suppose $K = 2$, $V = 1$, and $F(c) = c^t$ on $[0, 1]$, where $t > 0$.

Then $c_0(K, n) = n/(2n-1)$ and $p = F(c_0(K, n))$ is the probability an individual player grabs immediately. Consider the payoff of player i with cost $c = 0$. For this player, who is sure to grab at date 0,

$$\begin{aligned} U^E(0; K, n) &= \Pr(\text{someone on player } i\text{'s team is the (randomly) selected grabber at date 0}) \\ &= \sum_{j=0}^{n-1} \sum_{k=0}^{(K-1)n} \binom{n-1}{j} \binom{(K-1)n}{k} p^j (1-p)^{n-j-1} p^k (1-p)^{(K-1)n-k} \frac{j+1}{1+j+k}, \\ &= \frac{2n-2 + \left(\frac{n}{-1+2n}\right)^{-t} \left(1 - \left(1 - \left(\frac{n}{-1+2n}\right)^t\right)^{2n}\right)}{2(-1+2n)}, \end{aligned}$$

where the last equality uses properties of the binomial distribution¹² and substitutes for the value of p .

Table 1: Calculations of $U^E(0; K, n)$ for Example 2: $K = 2$ and $F(c) = c^t$ on $[0, 1]$.

t:ambign

$n \rightarrow$	2	3	4	5	6	7	8	9	10
$t = 1$	0.580247	0.565984	0.553429	0.544414	0.537872	0.532966	0.529166	0.526144	0.523684
$t = 8$	0.961987	0.975363	0.977773	0.977854	0.977097	0.975951	0.974598	0.973124	0.971574

Table 1 shows that for the uniform distribution ($t = 1$), interim utility of the player with cost 0 decreases as n increases from 2 to 10; however, for $t = 8$ this interim utility increases as n increases from 2 to 5 and then decreases. □

Appendix

Proof of Lemma 1. Suppose T is a symmetric equilibrium strategy for which types $c \in [\tilde{c}_l, \tilde{c}_h]$ grab at $\tilde{T} > 0$, i.e. a strategy with a strictly positive flat spot: $T(c) = \tilde{T}$ for all $c \in [\tilde{c}_l, \tilde{c}_h]$. We establish two facts to show this cannot be an equilibrium strategy. First, under the equilibrium conjecture, type \tilde{c}_h must prefer a contribution of \tilde{T} to one of $\tilde{T} + \varepsilon$. As $\varepsilon \downarrow 0$, this will imply $\tilde{c}_h \leq c_0$. Second, type \tilde{c}_l must prefer a contribution of \tilde{T} to one of $\tilde{T} - \varepsilon$. As $\varepsilon \downarrow 0$, this will imply $\tilde{c}_l \geq c_0$. Therefore, a flat spot at $\tilde{T} > 0$ cannot exist in equilibrium.

¹²This is the same procedure used in the proof of Lemma 1 to reach (16) and (17), which then need to be added up.

Consider first \tilde{c}_h . The utility of one agent in group 1 with cost realization equal to \tilde{c}_h that contributes \tilde{T} is

$$\frac{n-1}{Kn-1}V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)} d(1 - (1 - F(x))^{Kn-1}) + (1 - F(\tilde{c}_l))^{Kn-1} e^{-\rho \tilde{T}} U_I(\tilde{c}_h), \quad (13) \quad \text{ULN}$$

where $U_I(c)$ represents the payoff of a type c agent if the minimum cost of all other agents is above \tilde{c}_l , i.e., conditional on all other agents having cost above \tilde{c}_l .

Now the logic behind (13) is this. The first addendum is the expected payoff if the minimum cost of all other agents is below \tilde{c}_l : the average present value of V multiplied by the probability that one of the other group-1 agents wins, calculated under symmetry. The second addendum is the product of the probability that the minimum cost of all other agents is above \tilde{c}_l , multiplied by the present value of $U_I(\tilde{c}_h)$, with $U_I(c)$ defined as

$$U_I(c) = VS_1(n, K) + (V - c)S_2(n, K),$$

where

$$S_1(n, K) = \sum_{j=1}^{n-1} \sum_{k=0}^{(K-1)n} \binom{n-1}{j} \binom{(K-1)n}{k} p^j (1-p)^{n-j-1} p^k (1-p)^{(K-1)n-k} \frac{j}{1+j+k},$$

$$S_2(n, K) = \sum_{j=0}^{n-1} \sum_{k=0}^{(K-1)n} \binom{n-1}{j} \binom{(K-1)n}{k} p^j (1-p)^{n-j-1} p^k (1-p)^{(K-1)n-k} \frac{1}{1+j+k},$$

and

$$p \equiv \frac{F(\tilde{c}_h) - F(\tilde{c}_l)}{1 - F(\tilde{c}_l)}. \quad (14) \quad \text{pdef}$$

The payoff U_I can be understood as follows. Here j indexes other group 1 players and k indexes group 2 players. Beginning with S_2 , if there are j other players in group 1 bidding \tilde{T} and k players in the other $K-1$ groups bidding \tilde{T} (as well as this player of interest in group 1), then the player of interest in group 1 is selected with probability $\frac{1}{1+j+k}$, in which case he earns payoff $V - c$; but (moving to S_1) if one of the other group-1 players is selected, which happens with probability $\frac{j}{1+j+k}$, then he gets the benefit V without incurring any cost. And, of course, the probability of this configuration of other players bidding \tilde{T} is

$$\binom{n-1}{j} p^j (1-p)^{n-j-1} \binom{(K-1)n}{k} p^k (1-p)^{(K-1)n-k}.$$

With a similar logic, the limit for $\varepsilon \downarrow 0$ of the utility of one agent in group 1 with cost \tilde{c}_h that contributes $\tilde{T} + \varepsilon$ is

$$\frac{n-1}{Kn-1}V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)} d(1 - (1 - F(x))^{Kn-1}) + (1 - F(\tilde{c}_l))^{Kn-1} e^{-\rho \tilde{T}} U_R(\tilde{c}_h), \quad (15) \quad \text{URN}$$

where $U_R(\tilde{c}_h)$ is this player's payoff "from the right":

$$U_R(\hat{c}) = VS_3(n, K) + (V - \hat{c})(1 - p)^{Kn-1},$$

where

$$S_3(n, K) = \sum_{j=1}^{n-1} \sum_{k=0}^{(K-1)n} \binom{n-1}{j} \binom{(K-1)n}{k} p^j (1-p)^{n-j-1} p^k (1-p)^{(K-1)n-k} \frac{j}{j+k}$$

and p is again given by (14).

Using properties of a binomial distribution, one can establish

$$S_1(n, K) = \frac{n-1}{Kn-1} \left(1 - \frac{1 - (1-p)^{Kn}}{pKn} \right) \quad (16) \quad \text{S1simple}$$

$$S_2(n, K) = \frac{1 - (1-p)^{Kn}}{pKn}, \quad (17) \quad \text{S2simple}$$

and

$$S_3(n, K) = \frac{n-1}{Kn-1} (1 - (1-p)^{Kn-1}). \quad (18) \quad \text{S3simple}$$

Since utility in (13) must be at least as large as the one in (15), we have

$$VS_1 + (V - \tilde{c}_h) S_2 = U_I(\tilde{c}_h) \geq U_R(\tilde{c}_h) = VS_3 + (V - \tilde{c}_h) (1-p)^{Kn-1}, \quad (19) \quad \text{c0boundabove}$$

and the extremes of (19) imply $V(S_1 + S_2 - S_3 - (1-p)^{Kn-1}) \geq \tilde{c}_h(S_2 - (1-p)^{Kn-1})$, and therefore, substituting from (16)–(18), we have $\tilde{c}_h \leq c_0$.¹³

Moving now to \tilde{c}_l , the utility of one agent in group 1 with cost realization equal to \tilde{c}_l that contributes \tilde{T} is

$$\frac{n-1}{Kn-1} V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)} d(1 - (1 - F(x))^{Kn-1}) + (1 - F(\tilde{c}_l))^{Kn-1} e^{-\rho \tilde{T}} U_I(\tilde{c}_l), \quad (20) \quad \text{URNN}$$

while the limit for $\varepsilon \downarrow 0$ of the utility of one agent in group 1 with cost \tilde{c}_l that contributes $\tilde{T} - \varepsilon$ is

$$\frac{n-1}{Kn-1} V \times \int_{\underline{c}}^{\tilde{c}_l} e^{-\rho T(x)} d(1 - (1 - F(x))^{Kn-1}) + (1 - F(\tilde{c}_l))^{Kn-1} e^{-\rho \tilde{T}} (V - \tilde{c}_l). \quad (21) \quad \text{ULNN}$$

¹³We note that $S_2 - (1-p)^{Kn-1} > 0$ is equivalent to

$$1 > (1-p)^{Kn-1}(1-p+pKn) \equiv \psi(p).$$

This latter inequality is satisfied because $\psi(0) = 1$ and $\psi'(p) < 0$. Therefore, if a flat spot exists, then $p > 0$ and $S_2 - (1-p)^{Kn-1} > 0$.

Since utility in (20) must be at least as large as the one in (21), we have

$$VS_1 + (V - \tilde{c}_l)S_2 = U_I(\tilde{c}_l) \geq V - \tilde{c}_l,$$

and the extremes of the above-displayed equation imply $\tilde{c}_l(1 - S_2) \geq V(1 - S_1 - S_2)$, or $\tilde{c}_l \geq c_0$. Thus, we obtain $\tilde{c}_l = \tilde{c}_h$, so a flat spot at $\tilde{T} > 0$ is impossible in equilibrium.

To see that T is continuous note that any discontinuity must be a jump discontinuity. If such a jump occurs at c' , then for sufficiently small $\delta > 0$ the types in $(c', c' + \delta)$ will find it strictly profitable to decrease their grabbing times discretely to avoid wasteful delay (there is no chance of a tie since there are no flat spots at positive times).

Finally, note that the logic leading to (19) remains valid even if $\tilde{T} = 0$. Therefore, even if T is flat at zero for $c \in [\underline{c}, \tilde{c}_h]$, we obtain $\tilde{c}_h \leq c_0$. \square

Proof of Proposition 6. First consider the effect of increasing n . Because $c_0(K, n + 1) < c_0(K, n)$, it follows that $T(c; K, n + 1) = T(c; K, n) = 0$ for $c \leq c_0(K, n)$ and $T(c; K, n + 1) > T(c; K, n)$ for $c \in (c_0(K, n + 1), c_0(K, n)]$. Finally, $T(c; K, n + 1) > T(c; K, n) = 0$ for $c \in (c_0(K, n), \bar{c}]$ because $T(c_0(K, n); K, n + 1) > T(c_0(K, n); K, n)$ and T' increases with n on $(c_0(K, n), \bar{c}]$. To see this latter property, note that

$$\begin{aligned} \frac{\partial T'(c; K, \hat{n})}{\partial \hat{n}} &= \frac{h(c)K}{\rho(V - c)} \left(c - \frac{K - 1}{K}V \right) \\ &> \frac{h(c)K}{\rho(V - c)} \left(c - \frac{(K - 1)\hat{n}}{K\hat{n} - 1}V \right) && \text{(because } K \geq 2) \\ &> 0, \end{aligned}$$

where the second inequality follows because $T(c; K, \hat{n}) > 0$ implies $c > c_0(K, \hat{n}) \equiv \frac{(K-1)\hat{n}}{K\hat{n}-1}V$.

One similarly shows that an increase in K reduces T . Analogously to the proof of the effect of increasing n , here we use the fact that increasing K increases $c_0(K, n)$ and decreases T' (see (8)). Moreover, because individual grabbing strategies decrease with an increase in K and because increasing K increases the number of players, it follows immediately that ET^E also decreases with K . \square

References

- \square [1] Baik, Kyung Hwan, In-Gyu Kim, and Sunghyun Na, 2001, Bidding for a group-specific public-good prize, *Journal of Public Economics*, 82(3), 415-429.

- [2] Baik, Kyung Hwan, and Sanghack Lee, 1998, Group rent seeking with sharing, in: Michael R. Baye (ed.), *Advances in Applied Microeconomics*, Vol. 7, JAI Press, Stamford, Connecticut, 75-85.
- [3] Baldwin, Richard E. and Paul Krugman, 2004, Agglomeration, integration and tax harmonization, *European Economic Review*, 48, 1-23.
- [4] Barbieri, Stefano, and David A. Malueg, 2014, Group efforts when performance is determined by the “best shot,” *Economic Theory*, 56, 333-373.
- [5] Barbieri, Stefano, and David A. Malueg, 2016, Private-information group contests: Best-shot competition, *Games and Economic Behavior*, 98, issue C, 219-234.
- [6] Barbieri, Stefano, David Malueg, and Iryna Topolyan, 2014, The best-shot all-pay (group) auction with complete information, *Economic Theory* 57, 603-640.
- [7] Bliss, Christopher, and Barry Nalebuff, 1984, Dragon slaying and ballroom dancing: the private supply of a public good, *Journal of Public Economics*, 25, 1-12.
- [8] Bloch, Francis; Fabrizi, Simona; Lippert, Steffen, 2015, Learning and collusion in new markets with uncertain entry costs, *Economic Theory*, 58(2), 287-303.
- [9] Bonatti, Alessandro, and Johannes Hörner, 2011, Collaborating, *American Economic Review*, 101(2), 632-663.
- [10] Boyarchenko, Svetlana and Sergei Levendorskii, 2014, Preemption games under Levy uncertainty, *Games and Economic Behavior*, 88, 354-380.
- [11] Chowdhury, Subhasish M, Dongryul Lee, and Roman M. Sheremeta, 2013, Top guns may not fire: Best-shot group contests with group-specific public good prizes. *Journal of Economic Behavior & Organization*, 92, 94–103.
- [12] Clark, Derek J., and Kai A. Konrad, 2007, Asymmetric conflict: weakest link against best shot, *Journal of Conflict Resolution*, 51(3), 2007, 457-469.
- [13] Davis, Douglas D., and Robert J. Reilly, 1999, Rent-seeking with non-identical sharing rules: an equilibrium rescued, *Public Choice*, 100(1-2), 31-38.
- [14] Esteban, Joan M., and Debraj Ray, 2001, Collective action and the group size paradox, *American Political Science Review*, 95(3), 663-672.
- [15] Heubeck, Steven, 2009, Competitive sprawl, *Economic Theory*, 39(3), 443-460.
- [16] Hirshleifer, Jack, 1983, From weakest link to best shot: the voluntary provision of public goods, *Public Choice*, 41, 371-386

- [17] Hopenhayn, Hugo A., and Francesco Squintani, 2011, Preemption Games with Private Information, *Review of Economic Studies*, 78(2), 667-692.
- [18] Hopenhayn, Hugo A., and Francesco Squintani, 2016, Patent Rights and Innovation Disclosure, *Review of Economic Studies*, 83(1), 199-230.
- [19] Katz, Eliakim, Shmuel Nitzan, and Jacob Rosenberg, 1990, Rent-seeking for pure public goods, *Public Choice*, 65(1), 49-60.
- [20] Konrad, Kai A., 2012, Information alliances in contests with budget limits, *Public Choice*, 151(3-4), 679-693.
- [21] Lee, Sanghack, 1995, Endogenous sharing rules in collective-group rent-seeking, *Public Choice*, 85, 31-44.
- [22] Nishide, Katsumasa, and Kyoko Yagi, 2016, Investment under regime uncertainty: Impact of competition and preemption, *International Journal of Industrial Organization*, 45, 47-58.
- [23] Nitzan, Shmuel, and Kaoru Ueda, 2008, Collective contests for commons and club goods, *Journal of Public Economics*, 93(1-2), 48-55.
- [24] Olson, Mancur, and Richard Zeckhauser, 1966, An economic theory of alliances, *Review of Economics and Statistics*, 73, 200-207.
- [25] Rothschild, Michael, and Joseph E. Stiglitz, 1970, Increasing risk: I. A definition, *Journal of Economic Theory*, 2(3), 225-243.
- [26] Ruiz-Aliseda, Francisco, 2016, Preemptive investments under uncertainty, credibility and first mover advantages, *International Journal of Industrial Organization*, 44, 123-137.
- [27] Sandler, Todd, 1993, The economic theory of alliances, *Journal of Conflict Resolution*, 37(3), 446-483.