Optimal size of majoritarian committees under persuasion*

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Abstract

We analyze the 'optimal' size of non-deliberating majoritarian committees with no conflict of interest among its members when committees can be persuaded by a biased and informed expert. We find that when this bias is small, the optimal size is one; when it is intermediate, the optimal size increases monotonically in the precision of members' private information; when it is large this relation is non-monotonic. However the optimal committee-size never exceeds *five*. We also show that biased persuasion typically hurts a larger committee more severely. These results provide important implications on issues like universal enfranchisement, role of expert commentary in a democracy or size of governing boards in firms.

Keywords Persuasion · Committee size · Information aggregation

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1 Introduction

The Condorcet Jury Theorem (CJT) suggests that democratic societies are better off by delegating decision rights to larger committees as such committees make better collective judgements than smaller ones. As a result, if the cost of allowing an additional voter is insignificant, universal enfranchisement not only stands as a positive virtue of moral philosophy but also becomes an economically efficient practice. In today's age of media capture however, where expert opinion is abundantly available and experts typically do not represent the preferences of an average member of the society, committees – or empowered voters – can hardly operate in isolation as has been assumed in the classical CJT framework. The literature around the CJT does not have much to offer on whether biased expert opinions can be an important modifier of the theoretical conclusions on committee size. In this paper we ask if a larger committee necessarily enhances the quality of collective decisions when it is persuaded by an expert whose preferences are not always aligned with those of the committee members. Furthermore, we ask whether expert commentary, albeit informative, can hurt committee decisions even if voters understand that it emanates from a biased source and accordingly make the committee optimally sized.

We address this question in the *common interest voting* model of Austen-Smith and Banks (1996) with an odd number of voters (or committee members) to whom the society has delegated all decision rights.[1] We extend this classical framework to one where we allow for strategic information transmission by a single expert to a group of voters. While the expert's preferences are not aligned with those of the voters and he does not participate in the collective decision (an 'outsider'), he has free access to information. In particular, there are two possible alternatives that the voters must choose from collectively: X or Y. The expert always prefers X while the preference of the voters is state-dependent.[2] There is a state variable ω such that if ω is small enough then the voters also prefer X, but if ω is larger they instead prefer Y. Therefore, the 'magnitude' of ω determines the extent to which the voters' and the expert's preferences are *misaligned*. Any amount of information about the state ω is freely available to the expert, while the voters only receive limited information about ω. In particular, each voter receives privately an informative binary signal about the state ω with precision p. The expert chooses a *persuasion strategy* that generates a public message about ω that is *verifiable*. Based on their signal and the expert's message, a member votes in favor of the alternative that maximizes his expected utility and the committee is *a priori* unbiased so that the outcome voted for by the majority of voters is chosen. Our results are based on how voters behave in the most efficient Bayes-Nash equilibrium of the voting game.

The presence of expert persuasion is expected to provide additional public information and hence intuitively one would envisage that the usefulness of an additional committee member would get reduced when compared to the case without an expert. We find that while this is indeed the case, the effect is staggering: the presence of expert persuasion removes *all* benefits of large committees as we show that in no circumstance is it strictly beneficial for the society

[1] The literature on the CJT is large. Feddersen and Pesendorfer (1998) show that the unanimity rule can actually reverse the efficiency of large committees by increasing the probability of convicting an innocent. Nevertheless, even with strategic voting, certain environments yield equilibrium outcomes that converge to the efficient one as the number of voters goes to infinity. Feddersen and Pesendorfer (1997) show that in a setting with heterogeneous preferences where each voter receives a private signal about which alternative is best, there exists an equilibrium for each population size such that the outcome converges to the full information outcome as the number of voters goes to infinity. Myerson (1998) shows that asymptotic efficiency can be achieved even if there is population uncertainty.

[2] Thus we address the case of pure persuasion although all our results are qualitatively robust to state-dependent preferences of the expert as well.

to have a committee larger than *five*. When the expert's bias is small compared to that of the members so that the probability of preference alignment is greater than 1/2, we prove that the optimal committee size is *one*, that is, the society is equally well-off by delegating the decision rights to a single decision-maker (Theorem 5). When the expert's bias increases so that the probability of preference alignment is below 1/2, optimal committees become larger depending on the precision p of the members' private information (Theorem 4). In particular, if the probability of preference alignment is larger than a threshold of approx. $1/4$, then the optimal committee size *increases monotonically* in p with the largest required committee size being *three*. On the other hand, when the probability of preference alignment is lower than this threshold, the optimal committee size is *non-monotonic* in p: starting from a one-member committee with a very low p, as p increases, we obtain a three-member and then a five-member committee and as p rises further, the optimal size *falls* to three again.

As the presence of a biased expert restricts drastically the benefits from having large committees, we then ask if expert persuasion hurts or improves the quality of decision making of 'optimally-sized' committees, that is, committees that cannot improve by becoming larger in size. Among other things, we show that whenever society delegates all decision rights to a single voter (i.e. a one-member committee) the presence of a persuasive expert can never lower the quality of the decision (Theorems 6 and 7 part c). However, when the size of the optimal committee is larger, the results are nuanced. For each precision level p of private information, there exists a threshold value of the probability of preference alignment such that for all alignment probabilities below that threshold, expert persuasion enhances the probability of correct collective decision while for alignment probabilities above, expert persuasion is harmful. Moreover, this upper bound is non-increasing in p. In general, we also show that the presence of expert persuasion is more harmful the larger the committee size. In particular we show that with seven or more voters, persuasion unambiguously reduces the probability of a correct collective decision relative to a scenario without expert persuasion. Another robust conclusion from this study is that more informed committees obtain more information from expert persuasion.

Given these findings, what can we say about the desirability of universal enfranchisement and the usefulness of expert commentary in a democracy? From the CJT literature, we know that as the number of voters increases so does the probability of correct decisions (the exact conditions for this in our model is provided in Proposition 1). On the other hand we find that under expert persuasion, the optional committee-size is small (i.e. no lager than five) and in certain circumstances these committees perform better in the presence of the expert while in other circumstances expert persuasion is harmful. So whenever expert persuasion is harmful, it follows directly that the first-best scenario for the society is to impose universal voting rights and discourage expert commentary. On the other hand, when persuasion is useful, there will exist a threshold size of the society larger than the optimal committee size such that for all societies smaller than the threshold size, the first best outcome is to have expert persuasion but where decision rights are delegated to the optimal-sized committee, thereby violating universal enfranchisement. When the size of society is larger, the first best is to discourage expert persuasion and impose universal voting rights.

What if society considers delegating the decision rights to the informed expert? If the bias is small one may expect this to be desirable as this would minimize the variance of the action without distorting it too much from the optimal one. In our model with pure persuasion, the expert always takes the action X which is, a priori, the correct decision for society with probability $F(\omega_v)$. Hence such a delegation is equivalent to taking that action with certainty. Our analysis on equilibrium behavior (see Prop. 3 Part 2) shows that when the likelihood of preference alignment between the expert and the voters is high and the precision of private information of the individual voters is small, expert persuasion is fully uninformative and each committee member chooses alternative X. Hence in this case delegation to the expert is equivalent to having a committee of any size. However, in all other cases (see Prop. 2 and Prop. 3), expert persuasion is informative and influential but it is always the case that the committee chooses Y for some message that arrives with strictly positive probability and whenever this happens the decision is correct with probability 1. Hence in all other cases, delegation of decision rights to the expert is strictly worse for society.[3] To summarize, dictatorship of an informed and biased expert is never strictly desirable no matter how small is the bias.

1.1 Related literature

There is a literature concerning optimal size of committees that arise from a different problem faced by large committees. If individual members have to acquire private information through costly investment (unlike in our case), large committees may generate more stringent free rider problems: a single member has larger incentives to avoid this cost the larger is the committee size, particularly in a decision problem with no conflict of interest (like in our case). In view of this plausible source of inefficiency in large committees, Mukhopadhyay (2003) shows that with fixed cost of acquiring information, the optimal size of a committee is indeed bounded. Martinelli (2006) proves that with variable cost of acquiring private information where this cost is fully responsive to the 'quality' of information obtained, this problem can be avoided so that larger committees indeed perform better. Further, Koriyama and Szentes (2009) show that though the optimal size of committee remains bounded when this cost has a fixed and a variable component, the welfare loss from having a large committee is insignificant.

Austen-Smith and Banks (1996) also studies the impact of public information on committee decisions although the dissemination of public information is non-strategic in their framework. They show that with binary state-space where public information arrives non-strategically as a binary public signal, sincere voting (meaning votes are solely based upon private signals) cannot be informative (meaning votes mimic private signals) in equilibrium. However the study does not address the issue of optimal committee-size.

The social value of public information in general has been a well addressed subject since the work of Hirshleifer (1971). In a model with strategic complementarity, Morris and Shin (2002) show that public information can hurt social welfare only if agents also have access to independent sources of information. On the other hand, in the investment game of Angeletos and Pavan (2004) public information necessarily improves welfare. Also, Angeletos and Pavan (2007) show how welfare properties of public information depend not only on the form of strategic interaction but also on other external factors that determine the gap between equilibrium and efficient use of public information.[4] However in these papers, public information is non-strategic.

With this paper, we also add to the literature on strategic persuasion by complementing the findings of Kamenica and Gentzkow (2011). In their framework, the receiver of expert information is a single uninformed decision-maker, rather than a group of voters. Kamenica and Gentzkow (2011) characterize sender-optimal persuasion strategies and show that (w.l.o.g.) these strategies take the form of an action recommendation by the expert, which is duly followed by the decision-maker. However, they only briefly discuss the possibility of extending

[3] This continues to hold even as we move out of pure persuasion so that in some states the expert also prefers Y. See footnote 4.

[4] See also Bikchandani et al (1992) and Gersbach (2000), among others, for related works on impact of public information on social welfare.

their framework to either multiple receivers of information, or to a setting where the single decision-maker receives a private signal in addition to the expert's communication. Our setting combines both these scenarios – multiple receivers who are privately informed – and shows that the expert's equilibrium persuasion strategy is *no longer always simply a recommendation as to how voters should vote that is then followed in equilibrium.* Instead, in our model, there are states of the world in which the expert conveys a public signal so that voters vote in line with their private signals that are probabilistic. Ricardo and Câmara (2015) also study how an outside expert can persuade voters who may have *heterogeneous* preferences over two alternatives. In their model, however, voters do not receive private information and their focus is on comparative statics with respect to the voting rule. Like us, they find that when there is a single voter, persuasion is never harmful. However, contrary to our results, they find that if voters are homogeneous, persuasion has no impact on the probability of correct decisions irrespective of the voting rule used. This is driven by the fact that voters who are homogeneous in preferences remain 'informationally homogeneous' at the time the expert releases information. In contrast, in our model the interim preferences of voters always remain unpredictable as they depend on voters' private signals. This interim unpredictability of voters' preferences has deeper consequences in our model because Ricardo and Câmara (2015) find that when voters do not differ in their information but solely in their preferences, then a majority of voters is *always* weakly worse-off with persuasion when the collective decision is made using the simple majority rule. This stands in stark contrast with our findings whereby expert persuasion can hurt as well as help majoritarian committees. Therefore, there is a non-trivial difference between preference heterogeneity with informational homogeneity and preference homogeneity with informational heterogeneity. While both lead to 'interim preference heterogeneity', in our case it is incomplete information while in their case its full information.

The remainder of the paper is organized as follows. In Section 2 we describe the model. Section 3 contains the equilibrium characterization. In Section 4 we present comparative statics results with respect to the number of voters, allowing us to characterize the minimum number of voters required to achieve the highest possible ex ante probability of a correct collective decision (given the state of the world and the voters' preferences). In Section 5 we investigate whether biased expert persuasion is beneficial for voters, and if so for which model parameters. We conclude in Section 6 and the present all proofs in an appendix (Section 7).

2 The Model

Basic setting. A committe of n voters ($n \geq 1$ odd) must make a collective choice δ from a set of two alternatives $\{X,Y\}$. Voters have identical preferences over $\{X,Y\}$, and these preferences depend on an unknown state of the world $\omega \in \Omega \equiv [0,1]$. There is an expert with access to information about the true state of the world, but his preferences over $\{X,Y\}$ are not perfectly aligned with those of the voters.

Preferences. Voters all have the same state-dependent preferences over $\{X,Y\}$ such that (henceforth abbreviated by s.t.) for given $\omega_v \in (0,1)$, X is strictly preferred to Y for all $\omega \leq \omega_v$, and Y is strictly preferred to X for all $\omega > \omega_v$. These preferences are represented by a utility function $u : \{X,Y\} \times \Omega \to \mathbb{R}$ s.t. for $\underline{u}, \bar{u} \in \mathbb{R}, \underline{u} < \bar{u}$, we have:

$$u(X,\omega) = \begin{cases} \bar{u} & \text{if } \omega \leq \omega_v, \\ \underline{u} & \text{otherwise;} \end{cases} \quad \text{and} \quad u(Y,\omega) = \begin{cases} \underline{u} & \text{if } \omega \leq \omega_v, \\ \bar{u} & \text{otherwise.} \end{cases}$$

The expert strictly prefers X over Y in all states. His preferences are represented by a utility function $u_m : \{X,Y\} \times \Omega \to \mathbb{R}$ s.t. for $\underline{u}_m, \bar{u}_m \in \mathbb{R}$ with $\underline{u}_m < \bar{u}_m$ we have $u_m(X,\omega) = \bar{u}_m$ and $u_m(Y,\omega) = \underline{u}_m$ for all $\omega \in \Omega$.[5]

Information structure. The state of the world ω is modeled as a random variable with a common prior distribution. While the expert learns the realization of ω, voters only receive partial information. In particular, given an unknown state ω, each voter $i \in I$ observes privately and independently the realization of a binary signal $s_i \in S \equiv \{X,Y\}$ with *signal precision* $p \equiv \Pr[X|\omega \leq \omega_v] = \Pr[Y|\omega > \omega_v] \in (1/2,1)$. Note that these conditional i.i.d. signal distributions are derived from underlying joint densities $f(\omega,\boldsymbol{s})$ over the state space Ω and signal profiles $\boldsymbol{s} = (s_1,\ldots,s_n) \in S^n$, and we assume that these joint densities are continuous in ω.[6] From the joint densities $f(\omega,\boldsymbol{s})$ we can derive the prior probability that the expert's and the voters' preferences are aligned: $F(\omega_v) = \sum_{\boldsymbol{s}\in S^n} \int_0^{\omega_v} f(\omega,\boldsymbol{s})d\omega$. If $F(\omega_v) > 1/2$, we shall say there is *high likelihood of preference alignment* between voters and expert. If, instead, $F(\omega_v) < 1/2$, we say there is *low likelihood of preference alignment* between expert and voters.

Communication. The expert can costlessly disseminate information about ω through his choice of *persuasion strategy*, to which he commits, and which he announces before the state of the world ω is realized. Under such a strategy, voters receive coarse information about the state of the world in the form of a specific sub-interval of the state space containing the actual realization of ω. Formally, a *persuasion strategy* is a partition of the interval Ω into a finite number of closed sub-intervals whose union is Ω. For any integer $k \geq 1$, the finite sequence $(\omega_t)_{t=1,\ldots k}$ of real numbers s.t. $0 = \omega_0 < \omega_1 < \omega_2 < \ldots < \omega_k = 1$ gives rise to k sub-intervals $\Omega_t^k \equiv [\omega_{t-1},\omega_t]$ that form the partition $\Omega^k \equiv \{\Omega_1^k,\ldots\Omega_k^k\}$ of Ω. The sub-interval Ω_t^k (with $t \in \{1,\ldots,k\}$) is announced publicly to voters if and only if $\omega \in \Omega_t^k$.

Voting. Voters cast their votes simultaneously, and abstention is not allowed. Given the expert's public signal Ω_t^k and the private signal s_i, each voter i forms a posterior belief $f(\omega,\boldsymbol{s}_{-i}|\Omega_t^k,s_i)$ about the state of the world. A *pure voting strategy* is a function $v_i : \Omega^k \times S \to \{X,Y\}$, $(\Omega_t^k,s_i) \mapsto v_i(\Omega_t^k,s_i)$. Due to the fact that voters are ex ante symmetric in our model, we restrict attention to *symmetric* voting strategies: $v_i(\Omega_t^k,s) = v(\Omega_t^k,s)$ for all $\Omega_t^k \in \Omega^k$ and all $i \in I$. Let $\boldsymbol{v}(\Omega_t^k,\boldsymbol{s}) = (v(\Omega_t^k,s_1),\ldots,v(\Omega_t^k,s_n)) \in \{X,Y\}^n$ denote the profile of n votes. Votes are aggregated into a collective decision δ by *simple majority rule*. I.e. $\delta : \{X,Y\}^n \to \{X,Y\}$, $\boldsymbol{v}(\Omega_t^k,\boldsymbol{s}) \mapsto \delta(\boldsymbol{v}(\Omega_t^k,\boldsymbol{s}))$ s.t. $\delta(\boldsymbol{v}(\Omega_t^k,\boldsymbol{s})) = X$ iff $|i \in I : v(\Omega_t^k,s_i) = X| \geq \frac{n+1}{2}$.

3 Equilibrium characterization

In this section, we characterize by backward induction the perfect Bayesian equilibria of the two-stage game in which the expert first chooses and commits to a persuasion strategy, and voters subsequently vote after observing their respective private signals as well as the public signal generated by the expert's persuasion strategy. We start with the second stage of the game in order to derive the Bayes Nash *equilibrium of the voting subgame*. As mentioned in the model description above, in doing so we will focus on symmetric pure strategy equilibria.

[5]Our results remain qualitatively intact in a more general environment where in some states the expert prefers alternative Y.

[6]E.g. $\Pr[X|\omega \leq \omega_v] = \sum_{\boldsymbol{s}_{-i}\in S^{n-1}} \int_0^{\omega_v} f(\omega,X,\boldsymbol{s}_{-i})d\omega / \sum_{\boldsymbol{s}\in S^n} \int_0^{\omega_v} f(\omega,\boldsymbol{s})d\omega$

3.1 Voting equilibrium under persuasion

Suppose the expert's persuasion strategy has generated the public signal Ω_t^k. A voting strategy v is a symmetric Bayes Nash equilibrium of the voting subgame if for every voter $i \in I$ and every signal-pair (Ω_t^k, s_i) it is a best response to use v given that all other voters use v. That is:

$$v(s_i) = \arg \max_{v_i \in \{X,Y\}} \int_{\Omega_t^k} \left(\sum_{\boldsymbol{s}_{-i} \in S^{n-1}} \mathbb{P}[\boldsymbol{s}_{-i}|\omega] u(\delta(v_i, \boldsymbol{v}_{-i}(\Omega_t^k, \boldsymbol{s}_{-i})), \omega) \right) f(\omega|\Omega_t^k, s_i) d\omega$$

where the maximand is voter i's interim expected utility given signals (Ω_t^k, s_i), and $\boldsymbol{v}_{-i}(\Omega_t^k, \boldsymbol{s}_{-i})$ denotes the vote-profile across all voters other than i. In computing his best response, each voter takes account of the fact that his vote affects the outcome only if he is pivotal. Note that there are multiple symmetric pure strategy equilibria of the voting subgame: there is always one where all voters vote X (Y resp.) regardless of their signal. The reason is that given these vote-profiles, no voter is pivotal. This means that no voter has a strict incentive to change his vote as it does not affect the collective decision. As a way of selecting a particular symmetric equilibrium for the voting subgame, we focus on the equilibrium that maximizes the probability of a correct decision. Note that this equilibrium exists and is unique for every persuasion strategy Ω^k.

Proposition 1 (voting equilibrium). *In the voting subgame that commences with a public signal $\Omega_t^k = [\omega_{t-1}, \omega_t] \subseteq \Omega$ from the expert, the unique symmetric pure strategy Bayes Nash equilibrium that maximizes the probability of a correct decision is as follows: for every voter $i \in I$ and all $s_i \in S$:*

$$v(\Omega_t^k, s_i) = \begin{cases} Y & \text{if } F(\omega_v) < pF(\omega_{t-1}) + (1-p)F(\omega_t) \\ s_i & \text{if } pF(\omega_{t-1}) + (1-p)F(\omega_t) \leq F(\omega_v) < (1-p)F(\omega_{t-1}) + pF(\omega_t) \\ X & \text{if } F(\omega_v) \geq (1-p)F(\omega_{t-1}) + pF(\omega_t) \end{cases}$$

In the remainder of this paper, we will follow Austen-Smith and Banks (1996) by using the term *informative voting* to describe the strategy $v(\Omega_t^k, s_i) = s_i$ under which voters vote in line with their respective private signals. Note that as a corollary of Prop. 1, whose proof can be found in Sec. 7.1 of the appendix, we obtain the voting equilibrium for the setting without expert communication (which is equivalent to the expert choosing the uninformative persuasion strategy Ω^1 where $\omega_0 = 0$ and $\omega_1 = 1$): if $F(\omega_v) \geq p$, the equilibrium is for every voter to vote for X regardless of his private signal. Similarly, if $F(\omega_v) < 1-p$, the equilibrium is for every voter to vote for Y regardless of his signal. Finally, if $1-p \leq F(\omega_v) < p$, the equilibrium involves informative voting. It is only in this case that the Condorcet Jury Theorem prevails in our setting where voters use pure strategies.

3.2 Equilibrium persuasion

We now go on to analyse the *equilibrium of the full game*. As in Kamenica and Gentzkow (2011), a persuasion strategy constitutes an equilibrium if it maximizes the expert's ex-ante expected payoff. Take a strategy-pair (Ω^k, v) s.t. the symmetric voting-strategy v is the symmetric pure strategy equilibrium of the voting subgame given the persuasion strategy Ω^k. Then (Ω^k, v) is an equilibrium of the full game if for all other strategy-pairs $(\Omega^{k'}, v')$ we have:

$$\int_{\Omega} \left(\sum_{\boldsymbol{s} \in S^n} \mathbb{P}[\boldsymbol{s}|\omega][u_m(\delta(\boldsymbol{v}(\Omega^k(\omega), \boldsymbol{s})), \omega) - u_m(\delta(v'(\Omega^{k'}(\omega), \boldsymbol{s})), \omega)] \right) f(\omega) d\omega \geq 0$$

There may be multiple equilibria (Ω^k, v) of the full game. But since all of them are payoff-equivalent for the expert and the voters, we shall consider only the *coarsest equilibrium persuasion strategy*. Such a strategy features the minimum number of partitions k that elicit the same voting behavior in every state of the world when compared to any other persuasion strategy. Note carefully that in our setting, where voters classify states of the world according to a binary criterion (namely those for which the correct decision is X, and those for which it is Y), the coarsest persuasion strategy features *at most* three partitions: $\Omega^3 = \{\Omega_1^3, \Omega_2^3, \Omega_3^3\}$ with $\Omega_1^3 = [0, \omega_1]$, $\Omega_2^3 = [\omega_1, \omega_2]$, $\Omega_3^3 = [\omega_2, 1]$ where $\omega_1 \leq \omega_v \leq \omega_2$.

3.2.1 Equilibrium persuasion when $F(\omega_v) < 1/2$

We begin our characterization of the expert's equilibrium persuasion strategy with the case of low likelihood of preference alignment between the expert and the voters. In order to state our result, we begin by introducing some additional notation and terminology that will be used throughout the remainder of this paper. First, denote by $J_n(p)$ the probability that more than half of the voters receive the *correct* private signal given the true state of the world:

$$J_n(p) \equiv \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

Note that in any voting subgame where voters vote in line with their private signals, $J_n(p)$ captures the probability that the *correct* outcome is chosen (in the sense of voters' favorite outcome). In the following definition, we give a precise formal meaning to the statement that with informative voting there is a high chance of voters making the *correct* collective choice

Definition 1. *The odds $J_n(p)/(1-J_n(p))$ of a correct collective decision under informative voting are said to be **high** if:*

$$\frac{J_n(p)}{1-J_n(p)} > \frac{(1-F(\omega_v))p - (F(\omega_v)(1-p) + (1-F(\omega_v))(1-p))}{F(\omega_v)(1-p)} \tag{1}$$

Otherwise, we say that these odds are low.[7]

Before we provide some intuition for the condition in (1), we present the expert's equilibrium persuasion strategy, along with the induced voting equilibrium according to Prop. 1.

Proposition 2 (equilibrium persuasion for $F(\omega_v) < 1/2$). *The unique coarsest equilibrium persuasion strategy features a binary partition of the state space. In particular:*

1. *if $F(\omega_v) > 1-p$ and:*
   - (a) ***if the odds of a correct collective decision under informative voting are high**, then the expert's persuasion strategy is $\hat{\Omega}^2 = \{[0, \hat{\omega}_1], [\hat{\omega}_1, 1]\}$ with a threshold $\hat{\omega}_1$ that $F(\hat{\omega}_1) = F(\omega_v)/p$. This implies $\hat{\omega}_1 > \omega_v$. By Prop. 1, this persuasion strategy induces the following equilibrium voting behavior: $v_i([0, \hat{\omega}_1], s_i) = X$ and $v_i([\hat{\omega}_1, 1], s_i) = Y$ for all $s_i \in S$;*

[7] We show in Lemma 1 in the Appendix that (1) holds when the number of voters is sufficiently large (i.e. for $n \geq 5$). It also holds if the electorate consists of $n = 3$ voters and $F(\omega_v) \geq (1-p)(2p-1)(2p+1)/p(4p(1-p)+1)$, and if the electorate consists of a single decision-maker and $F(\omega_v) \geq (2p-1)/2p$.

(b) ***if the odds of a correct collective decision under informative voting are low****, then the expert's persuasion strategy is $\tilde{\Omega}^2 = \{[0, \tilde{\omega}_1], [\tilde{\omega}_1, 1]\}$ with a threshold $\tilde{\omega}_1$ that solves $F(\tilde{\omega}_1) = (F(\omega_v) - (1-p))/p$. This implies $\tilde{\omega}_1 < \omega_v$. By Prop. 1, this persuasion strategy induces the following equilibrium voting behavior: $v_i([0, \tilde{\omega}_1], s_i) = X$ and $v_i([\tilde{\omega}_1, 1], s_i) = s_i$ for all $s_i \in S$;*

2. *if $F(\omega_v) < 1 - p$ and:*

(a) *$n \geq 5$, then the equilibrium persuasion strategy and induced voting behavior is as in 1.(a);*

(b) *$n = 3$ and $p < \bar{p}_3 \approx 0.76069$, then the equilibrium persuasion strategy and induced voting behavior is as in 1.(a).[8] If, instead, $p > \bar{p}_3$, then the expert's persuasion strategy is $\breve{\Omega}^2 = \{[0, \breve{\omega}_1], [\breve{\omega}_1, 1]\}$ with threshold $\breve{\omega}_1$ that solves $F(\breve{\omega}_1) = F(\omega_v)/(1-p)$. This implies $\breve{\omega}_1 > \omega_v$. By Prop. 1, this persuasion strategy induces the following voting behavior: $v([0, \breve{\omega}_1], s_i) = s_i$ and $v([\breve{\omega}_1, 1], s_i) = Y$ for all $s_i \in S$;*

(c) *$n = 1$ and $p < \bar{p}_1 = \sqrt{2}/2 \approx 0.70711$, then the persuasion strategy and induced voting behavior is as in 1.(a). If, instead, $p > \bar{p}_1$, then the equilibrium persuasion strategy and induced voting behavior is as in case 2.(b).*

The proof is in Section 7.2 in the appendix. We now provide some intuitive understanding for the results in Prop. 2 by focusing on the case where $F(\omega_v) > 1 - p$. Without expert persuasion (i.e. when the uninformative persuasion strategy Ω^1 is used), Prop. 1 tells us that the equilibrium of the voting subgame involves informative voting. This means that in every state of the world, there is a positive chance that voters collectively choose Y instead of the expert's favorite alternative X. In particular, the *ex ante* probability of Y is $F(\omega_v)(1 - J_n(p)) + (1 - F(\omega_v))J_n(p)$, and the expert's goal is to devise a persuasion strategy that brings this probability down. One way of reducing the expert's exposure to Y is to make voters vote X in as many states of the world as possible. This gives rise to the persuasion strategy $\hat{\Omega}^2$ in item 1.(a) of Prop. 2, which offers the largest possible sub-interval s.t. voters vote X regardless of their private signals. It is easy to see from Prop. 1 that no sub-interval $\Omega_t^k = [\omega_{t-1}, \omega_t]$ for which voters always vote X can stretch all the way to the upper bound of the state space. The reason is that if Ω_t^k features an upper bound of $\omega_t = 1$, then there is no feasible lower bound ω_{t-1} so that voters would be willing to ignore their signals and vote for X when the expert sends the public signal Ω_t^k.[9] If, instead, Ω_t^k features a lower bound of $\omega_{t-1} = 0$, then the expert can stretch the upper bound all the way up to $\omega_t = F(\omega_v)/p$.[10] Note, however, that there is a 'price' to be paid for inducing a guaranteed vote for X over such a large stretch of the state space: voters will ignore their signals and vote for Y in all remaining states $\omega > F(\omega_v)/p$. This generates an *ex ante* probability of Y equal to $1 - (F(\omega_v)/p)$.

As an alternative to the persuasion strategy $\hat{\Omega}^2$ in item 1.(a) of Prop. 2, consider the strategy $\tilde{\Omega}^2$ in item 1.(b). It stretches the sub-interval for which voters vote for X regardless of their private signals only as far as is compatible with informative voting in all remaining states.[11] As

[8] The precise expression for $\bar{p}_3$ can be found at the end of Sec. 7.2.2 of the proof of Prop. 2.

[9] The third branch of the equilibrium voting strategy in Prop. 1 implies a lower bound ω_{t-1} s.t. $F(\omega_{t-1}) \leq (F(\omega_v) - p)/(1-p)$, which cannot hold as $F(\omega_v) < p$.

[10] This also follows from the third branch of the voting strategy in Prop. 1, which now requires $F(\omega_t) \leq F(\omega_v)/p$.

[11] The second and third branches of the equilibrium voting strategy in Prop. 1 dictate where the upper bound $\tilde{\omega}_1$ of this sub-interval is.

a result, the *ex ante* probability of Y being chosen is $((1-p)(1-F(\omega_v))/p)(1-J_n(p))+(1-F(\omega_v))J_n(p)$, which shows that $\tilde{\Omega}^2$ is always superior to the uninformative persuasion strategy Ω^1. In the proof of Prop. 2, we show formally that the expert's choice of equilibrium persuasion strategy boils down to a comparison of $\hat{\Omega}^2$ and $\tilde{\Omega}^2$. The outcome of this comparison depends on the model parameters n, p, and $F(\omega_v)$ because:

- for states $\omega \in [\hat{\omega}_1, 1]$, the persuasion strategy $\tilde{\Omega}^2$ leads to a collective choice of Y with probability $J_n(p)$, while strategy $\hat{\Omega}^2$ leads to Y for sure;
- for states $\omega \in [\omega_v, \hat{\omega}_1]$, strategy $\tilde{\Omega}^2$ again leads to Y with probability $J_n(p)$, while strategy $\hat{\Omega}^2$ leads to X for sure;
- for states $\omega \in [\tilde{\omega}_1, \omega_v]$, strategy $\tilde{\Omega}^2$ leads to Y with probability $1-J_n(p)$, while strategy $\hat{\Omega}^2$ leads to X for sure.

It is eqn. (1) in Definition 1 that determines whether the expected gain from the persuasion strategy $\tilde{\Omega}^2$ (in terms of a lower probability of Y) outweighs the expected losses in comparison with persuasion strategy $\hat{\Omega}^2$. Intuitively, when the preferences of the expert and the voters are aligned, the expert would like the probability of a correct collective decision $J_n(p)$ to be high so as to avoid a 'wrong' choice of Y. However, if their preferences are misaligned, the expert would like $J_n(p)$ to be low so as to avoid a 'correct' choice of Y. The precise trade-off between these counterveiling tendencies means that the persuasion strategy $\tilde{\Omega}^2$ is an equilibrium only if the odds $J_n(p)/(1-J_n(p))$ of a correct collective decision under informative voting are *low*. A *necessary* condition for this is that voters' *ex ante* probability of a correct Y-signal (given by $(1-F(\omega_v))p$) exceeds the *ex ante* probability of an incorrect signal (given by $F(\omega_v)(1-p)+(1-F(\omega_v)(1-p)))$.[12]

Remark 1. *The threshold $\hat{\omega}_1$ in item 1.(a) of Prop. 2 is decreasing in signal precision p, while the threshold $\tilde{\omega}_1$ in item 1.(b) of Prop. 2 is increasing in p. Therefore, it is the case under both persuasion strategies that more informed voters receive more accurate public information from the expert.*

3.2.2 Equilibrium persuasion when $F(\omega_v) > 1/2$

We present here the results for the case of a high likelihood of preference alignment between the expert and the voters. The expert's equilibrium persuasion strategy and the corresponding equilibrium voting strategy according to Prop. 1 is as follows:

Proposition 3 (equilibrium persuasion for $F(\omega_v) > 1/2$). *The unique coarsest equilibrium persuasion strategy features a binary partition of the state space. In particular:*

1. *if $F(\omega_v) > p$, then the expert's persuasion strategy is $\bar{\Omega}^2 = \{[0, \breve{\omega}_1], [\breve{\omega}_1, 1]\}$ with threshold $\breve{\omega}_1 = 1$ (i.e. persuasion yields no information). By Prop. 1, this persuasion strategy induces the following equilibrium voting behavior: $v([0, \breve{\omega}_1], s_i) = X$ for all $s_i \in S$;*
2. *if $F(\omega_v) < p$, then the expert's persuasion strategy is the same as in item 1.(a) of Prop. 2: $\hat{\Omega}^2 = \{[0, \hat{\omega}_1], [\hat{\omega}_1, 1]\}$ with threshold $\hat{\omega}_1$ s.t. $F(\hat{\omega}_1) = F(\omega_v)/p$, which induces the following equilibrium voting behavior: $v([0, \hat{\omega}_1], s_i) = X$ and $v([\hat{\omega}_1, 1], s_i) = Y$ for all $s_i \in S$;*

[12] This is evident from the numerator of the right-hand side ratio in equation (1).

The proof is in Section 7.4 in the appendix. The intuition for the results in Prop. 3 is straightforward. When $F(\omega_v) > p$ it follows from Prop. 1 that in the absence of persuasion, every voter votes for X irrespective of his private signal. This is the ideal scenario for the expert and it is therefore optimal for him to choose the uninformative persuasion strategy Ω^1. If, instead, the voters' signals are sufficiently informative (i.e. $1/2 < F(\omega_v) < p$), then Prop. 1 states that in the absence of expert persuasion, voters vote in line with their private signals. The inuition for item 2. of Prop. 3 is therefore the same as for Prop. 2 above, except that the present parameter values p and $F(\omega_v)$ are such that the odds of a correct collective decision under informative voting are always high, so that it is optimal for the expert to induce voting behavior in which voters disregard their private signals.

4 Optimal committee size and information aggregation

In this section, we present our insights into the optimal number of voters in the committee when they are subject to expert persuasion. We use the term 'optimal' in the sense of the *minimum* number of voters required to achieve the *highest possible ex ante probability of a correct collective decision* from the voters' point of view. We denote this 'number' by $n_{\min}$.

4.1 Optimal committee size when $F(\omega_v) < 1/2$

We begin with the case of low likelihood of preference alignment between the expert and the voters. Fig. 1 provides a graphical illustration of the parameter-pairs $(p, F(\omega_v))$ for which the *minimum* committee size needed to achieve the *maximum ex ante* probability of a correct collective decision is $n_{\min} = 1$ (the white unshaded area of Fig. 1), $n_{\min} = 3$ (the gray shaded area), or $n_{\min} = 5$ (the black shaded area).

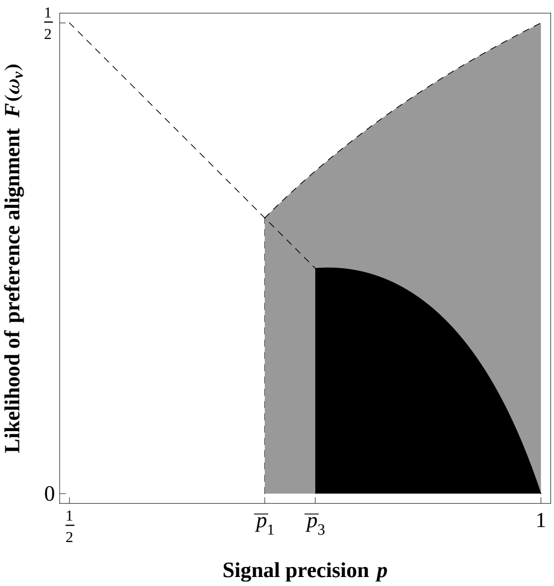

Figure 1: Optimal committee size with persuasion

The following result states formally what is illustrated graphically in Fig. 1:

Proposition 4 (**optimal committee size for** $F(\omega_v) < 1/2$)**.** *The minimum number of voters required to achieve the maximum* ex ante *probability of a correct collective decision varies as follows with the model parameters p and* $F(\omega_v)$*:*

1. *if* $F(\omega_v) > 1-p$*:*

   (a) *and* $F(\omega_v) \in [\frac{2p-1}{2p}, 1/2)$*, then* $n_{\min} = 1$*;*

   (b) *and* $F(\omega_v) \in [\frac{(1-p)(2p-1)(2p+1)}{p(4p(1-p)+1)}, \frac{2p-1}{2p})$*, then* $n_{\min} = 3$*;*

   (c) *and* $F(\omega_v) < \frac{(1-p)(2p-1)(2p+1)}{p(4p(1-p)+1)}$*, then* $n_{\min} = 5$.

2. *if* $F(\omega_v) < 1-p$*:*

   (a) *for* $p < \bar{p}_1 = \sqrt{2}/2 \approx 0.707\,11$*, then* $n_{\min} = 1$*;*

   (b) *for* $\bar{p}_1 < p < \bar{p}_3 \approx 0.760\,69$*, then* $n_{\min} = 3$*;*

   (c) *if* $p > \bar{p}_3$*, then* $n_{\min} = 5$.

The proof of Prop. 4 is in Sec. 7.5 of the appendix. They key idea behind this result is to adjust the committee size n so as to ensure that the expert uses the signal-invariant persuasion strategy $\hat{\Omega}^2$ for all parameter-pairs $(p, F(\omega_v))$ with $1-p < F(\omega_v) < 1/2$. To see why this is optimal from the perspective of maximizing the *ex ante* probability of a correct collective decision, recall that for five or more voters, the odds of a correct collective decision under informative voting $(J_n(p)/(1-J_n(p)))$ are high for all values of p. This makes informative voting 'too accurate' from the expert's point of view, who instead induces signal-invariant voting through the persuasion strategy $\hat{\Omega}^2$ in item 1.(a) of Prop. 2. Only for those model parameters p, $F(\omega_v)$, and n for which the odds $J_n(p)/(1-J_n(p))$ are low can the expert further reduce the accuracy of collective decision-making (i.e. achieve a lower *ex ante* probability of outcome Y than under $\hat{\Omega}^2$) by resorting to the persuasion strategy $\tilde{\Omega}^2$ in item 1.(b) of Prop. 2. This latter strategy involves informative voting for all states $\omega > \tilde{\omega}_1$. As our goal of maximizing the *ex ante* probability of a correct decision is diametrically opposed to what the expert wants, we simply have to find for all parameters p and $F(\omega_v)$ the minimum committe size n that makes the odds $J_n(p)/(1-J_n(p))$ high in the sense of Definition 1 so as to ensure that the expert uses the signal-invariant persuasion strategy $\hat{\Omega}^2$ instead of $\tilde{\Omega}^2$.

Note that the optimal committee size $n_{\min}$ pictured in Fig. 1 is increasing with the precision p of voters' private signals if $F(\omega_v) > 1-\bar{p}_3 \approx 0.24$. If, instead, $F(\omega_v) < 1-\bar{p}_3$ the optimal committee size is non-monotonic and drops from five to three as the signal precision p grows. The basic reason for this non-monotonicity is that when private signals become perfectly informative (i.e. $p \to 1$), a single-decision maker's odds of a correct 'collective' decision under informative voting, $J_1(p)/(1-J_1(p))$, become *low* in the sense of Definition 1 for *all* values of $F(\omega_v)$ in $[0, 1/2]$. In contrast, for any $n \geq 3$, the odds $J_n(p)/(1-J_n(p))$ are *high* for all $F(\omega_v)$ as p becomes perfectly informative. The reason for this stark difference is that with just one voter, the probability of a wrong signal, $1-p$, equals the probability of a wrong 'collective' decision under informative voting, $1-J_1(p)$. In contrast, for three or more voters, the chances that the collective makes a wrong decision when voting informatively are always strictly lower than the probability that any one of them gets the wrong private signal. To see all this formally, note first that the right-hand side ratio in eqn. (1) is *decreasing* in $F(\omega_v)$. This is due to the fact that in the numerator, the *ex ante* probability of a correct Y-signal, $(1-F(\omega_v))p$, falls and the *ex ante* probability of a wrong signal remains unchanged (this is given by $F(\omega_v)(1-p)+(1-F(\omega_v))(1-p)$). Furthermore, in the denominator the *ex ante* probability of a wrong Y-signal, $F(\omega_v)(1-p)$, goes up. We can therefore compute for given n and p the maximum value of $F(\omega_v)$ for which the odds $J_n(p)/(1-J_n(p))$ are low: $\bar{F}_n(p) \equiv (1-J_n(p))(p-(1-p))/(p(1-J_n(p))+J_n(p)(1-p))$. Using L'Hôpital's Rule to obtain the limit as p becomes perfectly informative, we find that $\bar{F}_n(1) = J_n'(1)/(1+J_n'(1))$. Thus, with a single decision-maker (for which $J_1(p) = p$), we obtain $\bar{F}_1(1) = 1$, while with a three-voter committee (for which $J_3(p) = p^2(3-2p)$) we obtain $\bar{F}_3(1) = 0$. Furthermore, $\bar{F}_1(p)$ is an increasing function (which demarcates the gray area in Fig. 1), while $\bar{F}_3(p)$ is a decreasing one (which demarcates the black area in Fig. 1). Thus, there exist values $p < 1$ and $1-p < F(\omega_v) \leq \bar{F}_3(p)$ for which the odds $J_3(p)/(1-J_3(p))$ are low, so that the expert adopts persuasion strategy $\tilde{\Omega}^2$. It is for all these values that we raise the committee size to $n_{\min} = 5$, which gives rise to the aforementioned non-monotonicity.

4.2 Optimal committee size when $F(\omega_v) > 1/2$

Here we consider briefly the case where the likelihood of preference alignment between the expert and the voters is high:

Proposition 5 (optimal committee size for $F(\omega_v) > 1/2$). *The probability of a correct collective decision is constant for all n, and so $n_{\min} = 1$.*

Thus, when preference misalignment between expert and voters is unlikely, committee size is irrelevant. This is because the expert's equilibrium persuasion strategies in Prop. 3 are signal-invariant, which means there is no information aggregation and benefit from larger committees.

5 Desirability of expert persuasion

In this section, we show that the expert's communication can adversely affect the *ex ante* probability of a correct decision in both large *and* small committees.

5.1 Benefits from persuasion when $F(\omega_v) < 1/2$

We begin with a graphical illustration of our result regarding the desirability of expert persuasion when the likelihood of preference alignment is low. Fig. 2 shows (in the gray shaded areas) for which parameter-pairs $(p, F(\omega_v))$ the presence of the expert helps generate a higher *ex ante* probability of a correct decision in a committee of three (left-hand panel of Fig. 2) or in a committee of five voters (right-hand panel of Fig. 2) than each committee could achieve without the expert.

Proposition 6 (Desirability of expert persuasion for $F(\omega_v) < 1/2$). *In the following, we state for which model parameters p, $F(\omega_v)$ and n biased expert persuasion benefits voters.*

1. *Let $F(\omega_v) > 1-p$.*

   (a) ***When the odds of a correct collective decision under informative voting are high,*** *then the* ex ante *probability of a correct collective decision is higher* **with** *biased expert persuasion than without if $F(\omega_v) < G_n(p)$, where:*

$$G_n(p) \equiv \frac{p}{1-p}(1-J_n(p)). \tag{2}$$

If, instead, $F(\omega_v) > G_n(p)$, then expert persuasion adversely affects the ex ante *probability of a correct decision.*

(b) ***When the odds of a correct collective decision under informative voting are low****, then the* ex ante *probability of a correct collective decision is higher* **with** *biased expert persuasion than without it.*

2. *Let $F(\omega_v) < 1 - p$. The* ex ante *probability of a correct collective decision is higher* **with** *biased expert persuasion than without it.*

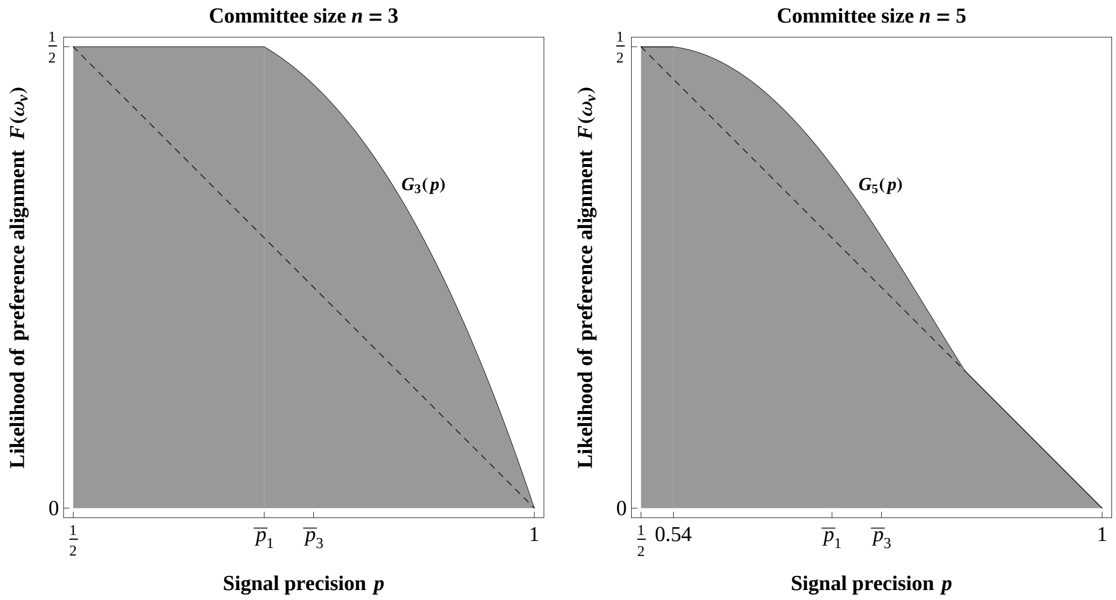

Figure 2: Desirability of expert persuasion in committees of three and five voters

The proof of Prop. 6 is in Sec. 7.7 of the appendix. Whether expert persuasion is beneficial for voters or not depends on the equilibrium persuasion strategy used by the expert. If the odds $J_n(p)/(1-J_n(p))$ are high, then strategy $\hat{\Omega}^2$ in item 1.(a) of Prop. 2 induces a signal-invariant voting strategy for which the ex ante probability of a correct decision is independent of the number of voters. In the expert's absence, voters vote according to their private signals, which implies that in large committees a large amount of information is being aggregated into a collective decision. However, this improves on the the outcome induced by persuasion strategy $\hat{\Omega}^2$ only if private signals are sufficiently precise (i.e. if $F(\omega_v) < G_n(p)$).

If, instead, the odds $J_n(p)/(1-J_n(p))$ are low, then persuasion strategy $\tilde{\Omega}^2$ in item 1.(b) of Prop. 2 is used by the expert. This induces voters to vote for X regardless of their respective private signals in all states $\omega \in [0, \tilde{\omega}_1]$, while without expert persuasion they would cast their vote according to their private signals. In all remaining states, they vote in line with their private signals both with and without the expert. It is therefore immediately obvious that expert persuasion enhances the chances of the committee making the correct decision. Note that $G_n(p) > G_{n+1}(p)$ for all p, which implies that the zone of benefit from persuasion shrinks with committee size, as illustrated by Fig. 2.

Finally, in item 2. of Prop. 6 the signal precision p is sufficiently low so that in the absence of expert communication voters vote for Y irrespective of their private signals. In this case, the expert's help in raising the probability of a correct collective decision when the state is in $[0, \omega_v]$

outweighs any losses from manipuating voters into voting for X when the state is in $[\omega_v, \hat{\omega}_1]$ or in $[\omega_v, \breve{\omega}_1]$ (see item 2. of Prop. 2 for the different persuasion strategies that can arise in equilibrium).

5.2 Benefits from persuasion when $F(\omega_v) > 1/2$

Our final result shows that even when the expert and the voters are highly likely to agree on what is the correct decision, the expert's presence can still adversely affect the probability of it being taken. In general, whether expert persuasion harms information aggregation or not depends on the size n of the electorate. We will see that with seven or more voters, persuasion unambiguously reduces the probability of a correct collective decision relative to a scenario without expert persuasion. However, with three and with five voters, the situation is more nuanced in that the result will depend on the interplay of signal precision p and the likelihood of preference misalignment. In particular, for an *intermediate* level of signal precision, the probability of a correct decision will be higher *with* persuasion, while for *low and high* levels of signal precision it will be higher *without* persuasion.

Proposition 7. *In the following, we state for which model parameters p, $F(\omega_v)$ and n biased expert persuasion benefits voters.*

1. If $p < F(\omega_v)$, the *ex ante* probability that voters collectively choose the correct alternative is unaffected by the expert's presence.
2. If $p > F(\omega_v)$ and furthermore:
   (a) if $n \geq 7$, the *ex ante* probability of choosing the correct alternative is higher ***without*** expert persuasion;
   (b) if $n = 3, 5$, the *ex ante* probability of a correct collective decision is higher ***with*** biased persuasion if $F(\omega_v) < G_n(p)$, where $G_n(p)$ is defined in (2). If, instead, $F(\omega_v) > G_n(p)$, then expert persuasion adversely affects the probability of a correct decision.
   (c) if $n = 1$, the *ex ante* probability of choosing the correct alternative is always higher ***with*** persuasion.

The proof is in Sec. 7.9 of the appendix. In order to get a better sense for the results in Prop. 7, note that when signal precision is low (i.e. $p < F(\omega_v)$) each voter votes for X in all states irrespective of his private signal and irrespective of the number of voters (see Prop. 3 above). As a result, the probability of making the correct decision is the same whether or not an expert is present. But when the signal strength is high (i.e. $p > F(\omega_v)$), the probability of making the correct decision is higher without persuasion for a sufficiently large committee (i.e. $n \geq 7$). This is because in the presence of the expert, voters opt wrongly for X when $\omega \in (\omega_v, \hat{\omega}_1]$, regardless of how many voters there are. Without an expert, voters instead vote in line with their private signals so that the chance of a wrong decision in states $\omega \in (\omega_v, \hat{\omega}_1]$ diminishes as the size of the electorate grows.

Now consider the case of $n = 3$ or $n = 5$ voters. In these cases, Prop. 3 above implies that the chance of voters making the right choice when voting informatively is low. It is here that the analysis gets interesting due to the non-monotonicity in the ranking of the probabilities of a correct decision with and without the expert: for $p > F(\omega_v)$, the length of the interval $(\omega_v, \hat{\omega}_1]$

over which the expert can manipulate voters into choosing the wrong decision *decreases* with p. I.e. for low p there is a large range of states for which voters are being manipulated, meaning that the probability of a correct decision is higher without the expert. If, instead, the signal precision p is very high, the chance that the collective will the correct alternative without persuasion is high in all states - even in such small committees. Only in an intermediate range of p is it worthwhile to suffer the expert's manipulation in exchange for his help in choosing the correct alternative in all states except $\omega \in (\omega_v, \hat{\omega}_1])$.

6 Conclusion

In this paper we have studied the effect of expert persuasion on the general conclusion of the Condorcet Jury Theorem that larger committees take better collective actions. Our main finding is that in a common-interest non-deliberating voting model where collective decisions are reached via the simple-majority rule, a small committee (of size ≤ 5) is 'good enough'. We also find that persuasion never limits information aggregation if the precision of voters' private signals is low. Otherwise, persuasion will hurt information aggregation in large committees. This is because the information conveyed through the equilibrium persuasion strategy overpowers voters' private information and invariably makes them vote for a particular alternative. In contrast, without persuasion voters will vote according to their private signals so that the probability of the correct decision increases with the size of the electorate. Thus, absence of expert advice can actually improve information aggregation in large constituencies. Another key insight of this paper is that a similar issue arises even in small constituencies, even though not for all constellations of the model parameters. We also find that in general the amount of information transmitted through expert persuasion increases when each member gets more precise private information.

We have used these results to draw conclusions regarding voting rights and the desirability of expert commentary in a democracy. Our results can also enhance one's understanding on how firms may take their business decisions.Yermack (1996) studies the Fortune 500 firms and finds a negative correlation between firm value and the size of a firm's board of directors who take decisions on behalf of the firm. This finding is not only confined to large firms. By studying small and mid-sized Finnish firms, Eisenberg et al (1998) et al find a negative relation between board size and profitability. The same negative relation is confirmed in other contexts by Bhagat and Black (2002), Mak and Kusnadi (2005) and Conyon and Peck (1998). On the other hand, there are also studies which support the opposite idea that group size and group performance are positively linked. Guo and Schick (2003) surveyed 294 chairpersons and 223 members from 334 ethics committees in the USA in 2000, and found that larger committees are perceived to be more successful. Hence the empirical evidence regarding group size and group performance goes in both directions, and a theoretical framework is warranted to understand the relationship better. Our results suggest the following in this regard. If decisions are majoritarian, very large firms with many shareholders should take decisions by allowing all shareholders to vote and by disallowing independent expert recommendations while smaller firms should form small committees and invite outside experts for advice.

In our analysis we have assumed that the committee does not deliberate and this assumption is common to most of the literature around CJT. If it did deliberate, then, given there is no conflict of interest amongst voters, the problem would be akin to the one with a partially informed single decision maker (or receiver) who receives n independent private signals in addition to any information transmitted by the expert. A single decision-maker with many binary signals may be harder to influence than one with a single binary signal because with more signals there might be a chance that the expert's information is drowned out by the private information of the decision-maker. In any event, as private information does not hurt voters in our model, and as expert persuasion cannot be detrimental to the social objective under a single-member committee, we expect that larger deliberating committees will make (weakly) better judgements and expert commentary will also make a (weakly) positive impact.

Possible extensions to our model include alternative voting rules (such as approval voting or cumulative voting), and three or more alternatives. It would also be quite natural to introduce multiple experts with either similar or conflicting biases, and examine the effect of their communication on the electorate's chances of making the correct decision. We reserve these for future research. Another aspect of our model that one could think of relaxing is the assumption that voters receive their private signals without having to pay for them. In relation to this possible extension, it is worth highlighting that Gershkov and Szentes (2009) study the design of an optimal collective decision mechanism in a scenario without conflict of interest among voters (like in the present paper) *and* without expert persuasion (unlike the present paper). In their model, voters need to pay in order to obtain partially informative signals, which gives rise to free-rider problems - particularly when voters are homogeneous. In order to mitigate this source of inefficiency, Gershkov and Szentes (2009) show that the optimal mechanism must involve an interesting protocol: voters are selected one-by-one at random and asked to acquire a costly signal whose realization they must subsequently report to the mechanism designer. Voters are neither informed about their position in this sequence nor of the other voters' reports. As this is a very general property of an optimal mechanism (beyond the environments studied by Gershkov and Szentes (2009)), we conjecture that it would remain optimal even if voters were subject to persuasion by a biased expert.

7 Appendix

7.1 Proof of Proposition 1

Suppose all voters other than i use the same voting strategy v. As the collective decision is made according to the simple majority rule, voter i is *pivotal* if and only if the remaining $n-1$ votes are split equally across the two alternatives. Let $n_X(\boldsymbol{v}(\Omega_t^k, \boldsymbol{s}_{-i}))$ be the number of votes cast for alternative X among voters other than i. The event that voter i is pivotal given the other voters' strategies can then be captured by the set:

$$\Pi_i(\Omega_t^k, v) \equiv \{\boldsymbol{s}_{-i} \in S^{n-1} : n_X(\boldsymbol{v}(\Omega_t^k, \boldsymbol{s}_{-i})) = (n-1)/2\}$$

As voter i faces a binary choice when computing best reponses (namely to either vote X or Y), his best response can be characterized easily on the basis of the interim expectation of the utility-difference $u(\delta(X, \boldsymbol{v}_{-i}(\Omega_t^k, \boldsymbol{s}_{-i})), \omega) - u(\delta(Y, \boldsymbol{v}_{-i}(\Omega_t^k, \boldsymbol{s}_{-i})), \omega)$ from voting for X versus voting for Y given signals (Ω_t^k, s_i). This expectation reduces to the following expression:

$$\sum_{\boldsymbol{s}_{-i} \in \Pi_i(\Omega_t^k, v)} (\bar{u} - \underline{u}) \mathbb{P}[\boldsymbol{s}_{-i} | s_i] \left(2F(\omega_v | \Omega_t^k, s_i, \boldsymbol{s}_{-i}) - 1 \right)$$

Voter i's best response is to vote X if this expected utility-difference is positive, and to vote Y if it is negative. In order to further characterize equilibrium voting behavior in terms of the model parameters p and $F(\omega_v)$, as well as the expert's public signal Ω_t^k, suppose first that the voting strategy v used by the $n-1$ voters other than i is s.t. voter i is not pivotal: $\Pi_i(\Omega_t^k, v) = \emptyset$. In this case, our focus on the equilibrium that maximizes the probability of a correct decision requires that voter i votes on the basis of his private signal s_i as if his vote alone determined the outcome. Computing voter i's interim utility-difference from voting for X versus Y given signals (Ω_t^k, s_i) yields:

$$\begin{aligned} & \int_\Omega (u(X,\omega) - u(Y,\omega)) f(\omega|\Omega_t^k, s_i) d\omega \\ = \quad & (\bar{u} - \underline{u}) \left(2F(\omega_v|\Omega_t^k, s_i) - 1 \right). \end{aligned}$$

Therefore, if $F(\omega_v|, \Omega_t^k, s_i) > 1/2$, it is optimal for voter i to vote for X. Instead, if $F(\omega_v|, \Omega_t^k, s_i) < 1/2$, it is optimal for voter i to vote for Y. If $F(\omega_v|, \Omega_t^k, s_i) = 1/2$, voter i is indifferent between voting for X and Y. For simplicity, we assume in this case that any voter i acts in line with the expert's preference, which is to vote for alternative X. We can further characterize voter i's equilibrium voting strategy by computing the posterior $F(\omega_v|\Omega_t^k, s_i)$ using Bayes' Rule. In particular, if $s_i = X$:

$$\begin{aligned} F(\omega_v|\Omega_t^k, X) &= \frac{\mathbb{P}[X|\omega \leq \omega_v] F(\omega_v|\Omega_t^k)}{\mathbb{P}[X|\omega \leq \omega_v] F(\omega_v|\Omega_t^k) + \mathbb{P}[X|\omega > \omega_v](1 - F(\omega_v|\Omega_t^k))} \\ &= \frac{p\,(F(\omega_v) - F(\omega_{t-1}))}{p\,(F(\omega_v) - F(\omega_{t-1})) + (1-p)(F(\omega_t) - F(\omega_v))}. \end{aligned}$$

Thus, voter i with private signal $s_i = X$ votes for X if $F(\omega_v) \geq pF(\omega_{t-1}) + (1-p)F(\omega_t)$, and he votes for Y if $F(\omega_v) < pF(\omega_{t-1}) + (1-p)F(\omega_t)$. If, instead, voter i's private signal is $s_i = Y$, we obtain by Bayes' Rule:

$$\begin{aligned} F(\omega_v|\Omega_t^k, Y) &= \frac{\mathbb{P}[Y|\omega \leq \omega_v] F(\omega_v|\Omega_t^k)}{\mathbb{P}[Y|\omega \leq \omega_v] F(\omega_v|\Omega_t^k) + \mathbb{P}[Y|\omega > \omega_v](1 - F(\omega_v|\Omega_t^k))} \\ &= \frac{(1-p)\,(F(\omega_v) - F(\omega_{t-1}))}{(1-p)\,(F(\omega_v) - F(\omega_{t-1})) + p(F(\omega_t) - F(\omega_v))}. \end{aligned}$$

Thus, if $s_i = Y$ voter i votes for X if $F(\omega_v) \geq (1-p)F(\omega_{t-1}) + pF(\omega_t)$, and he votes for Y if $F(\omega_v) < (1-p)F(\omega_{t-1}) + pF(\omega_t)$.

Next, suppose voter i is pivotal with positive probability: $\Pi_i(\Omega_t^k, v) \neq \emptyset$. Note that with our focus on pure strategies, this is the case if and only if the $n-1$ voters other than i either all vote according to signal ($v(s_j) = s_j$ for all $j \neq i$), or contrary to signal. In either case, for every $\boldsymbol{s}_{-i} \in \Pi_i(\Omega^1, v)$:

$$\mathbb{P}[\boldsymbol{s}_{-i}|\omega \leq \omega_v] = \mathbb{P}[\boldsymbol{s}_{-i}|\omega > \omega_v] = p^{\frac{n-1}{2}}(1-p)^{n-1}$$

This implies that being pivotal reveals no additional information to voter i over and above the information contained in his private signal:

$$\begin{aligned} F(\omega_v|\Omega_t^k, X, \boldsymbol{s}_{-i}) &= \frac{p\mathbb{P}[\boldsymbol{s}_{-i}|\omega \leq \omega_v] F(\omega_v|\Omega_t^k)}{p\mathbb{P}[\boldsymbol{s}_{-i}|\omega \leq \omega_v] F(\omega_v|\Omega_t^k) + (1-p)\mathbb{P}[\boldsymbol{s}_{-i}|\omega > \omega_v](1 - F(\omega_v|\Omega_t^k))} \\ &= F(\omega_v|\Omega_t^k, X) \end{aligned}$$

Similarly: $F(\omega_v|\Omega_t^k, Y, \boldsymbol{s}_{-i}) = F(\omega_v|\Omega_t^k, Y)$. It follows immediately that voter i's equilibrium voting behavior is the same as in the first part of this proof where $\Pi_i(\Omega_t^k, v) = \emptyset$. □

7.2 Proof of Proposition 2

Recall from Sec. 3.2 that the maximum number of partitions we have to consider as candidate for the expert's coarsest equilibrium persuasion strategy is *three*. We therefore commence the proof by looking for the expert's optimal persuasion strategy within the class of *ternary* partitions $\Omega^3 = \{\Omega^3_1, \Omega^3_2, \Omega^3_3\}$ with $\Omega^3_1 = [0, \omega_1]$, $\Omega^3_2 = [\omega_1, \omega_2]$, $\Omega^3_3 = [\omega_2, 1]$ where $\omega_1 \leq \omega_v < \omega_2$. Note that binary persuasion strategies Ω^2 can be viewed as special cases of ternary partitions Ω^3 (where either $\omega_1 \in (0, \omega_v]$ and $\omega_2 = 1$, or $\omega_1 = 0$ and $\omega_2 \in (\omega_v, 1]$). Likewise, the uninformative persuasion strategy Ω^1 is a special case of Ω^3 with $\omega_1 = 0$ and $\omega_2 = 1$.

7.2.1 Optimal ternary persuasion strategy

We characterize the optimal partition Ω^3 by choosing thresholds ω_1 and ω_2 so as to maximize the expert's ex ante expected utility, which can be expressed as follows:

$$\int_{\Omega} \left(\sum_{\boldsymbol{s} \in S^n} \mathbb{P}[\boldsymbol{s}|\omega] u_m(\delta(\boldsymbol{v}(\Omega^3(\omega), \boldsymbol{s})), \omega) \right) f(\omega) d\omega \tag{3}$$

We have to take into account how the expert's choice of ω_1 and ω_2 affects voters' equilibrium voting strategy, which is detailed in Prop. 1:

- for any $\omega_1 \leq \omega_v$, announcement of the sub-interval $\Omega^3_1 = [0, \omega_1]$ induces voters to vote for X regardless of their private signals as they know with certainty that X is the correct choice given their preferences (this can also be seen formally from Prop. 1 as $F(\omega_v) \geq (1-p) \cdot 0 + pF(\omega_1)$).
- for any $\omega_2 > \omega_v$, announcement of the sub-interval $\Omega^3_3 = [\omega_2, 1]$ induces voters to vote for Y regardless of their private signals as they know with certainty that Y is the correct decision given their preferences (this can also be seen formally from Prop. 1 as $F(\omega_v) < pF(\omega_2) + (1-p) \cdot 1$).
- if the sub-interval $\Omega^3_2 = [\omega_1, \omega_2]$ is announced, the voting strategy depends on the specific values of the thresholds ω_1 and ω_2 as described in Prop. 1. I.e. the expert can choose which one of the following three voting behaviors he wants to induce: either a vote for Y regardless of voters' private signals, or a vote for X regardless of voters' private signals, or a vote in line with each voter's private signal.

We now compute explicitly the expert's ex ante expected utility in (3) under each of the three voting behaviors that he can induce through his choice of sub-interval Ω^3_2. We then compute the corresponding utility-maximizing values of ω_1 and ω_2:

1. Suppose $v(\Omega^3_2, s_i) = Y$ for all s_i. By Prop. 1, this means that ω_1 and ω_2 are s.t. $F(\omega_v) < pF(\omega_1) + (1-p)F(\omega_2)$. The expert's expected utility in (3) can then be written as follows:
$$\int_0^{\omega_1} \bar{u}_m f(\omega) d\omega + \int_{\omega_1}^1 \underline{u}_m f(\omega) d\omega = \underline{u}_m + (\bar{u}_m - \underline{u}_m) F(\omega_1)$$
As this function is increasing in ω_1, it is optimal to make its value as large as possible, subject to the constraint that $F(\omega_v) < pF(\omega_1) + (1-p)F(\omega_2)$. Thus, we set $\omega_1 = \omega_v$ and can choose any $\omega_2 > \omega_v$.

2. Suppose $v(\Omega_2^3, s_i) = X$ for all s_i. By Prop. 1, this means that ω_1 and ω_2 are s.t. $F(\omega_v) \geq (1-p)F(\omega_1) + pF(\omega_2)$. The expert's expected utility in (3) can then be written as follows:

$$\int_0^{\omega_2} \bar{u}_m f(\omega) d\omega + \int_{\omega_2}^1 \underline{u}_m f(\omega) d\omega = \underline{u}_m + (\bar{u}_m - \underline{u}_m) F(\omega_2)$$

As this function is increasing in ω_2, it is optimal to make its value as large as possible, subject to the constraint that $v(\Omega_2^3, s_i) = X$ for all s_i. This constraint can be re-arranged as follows: $F(\omega_2) \leq (F(\omega_v) - (1-p)F(\omega_1))/p$. Thus, we set $\omega_1 = 0$ and choose the value of ω_2 that solves $F(\omega_2) = F(\omega_v)/p$. It is immediate that no persuasion strategy Ω^3 that maximizes the expert's expected utility can involve the voting behavior detailed in item 1. above, because a strictly higher expected utility can be guaranteed by inducing the voting behavior detailed here in item 2.

3. Suppose $v(\Omega_2^3, s_i) = s_i$. By Prop. 1, this means that ω_1 and ω_2 are s.t. $pF(\omega_1) + (1-p)F(\omega_2) \leq F(\omega_v) < (1-p)F(\omega_1) + pF(\omega_2)$. The expert's expected utility in (3) can then be written as follows:

$$\begin{aligned} &\int_0^{\omega_1} \bar{u}_m f(\omega) d\omega + \int_{\omega_1}^{\omega_v} (J_n(p)(\bar{u}_m - \underline{u}_m) + \underline{u}_m) f(\omega) d\omega \\ &+ \int_{\omega_v}^{\omega_2} (\bar{u}_m - J_n(p)(\bar{u}_m - \underline{u}_m)) f(\omega) d\omega + \int_{\omega_2}^1 \underline{u}_m f(\omega) d\omega \\ &= \underline{u}_m + (\bar{u}_m - \underline{u}_m) \left[(F(\omega_1) + F(\omega_2))(1 - J_n(p)) - F(\omega_v)(1 - 2J_n(p)) \right] \end{aligned}$$

As this function is increasing in $F(\omega_1) + F(\omega_2)$, it is optimal to make the value of this sum as large as possible, subject to the following constraints: (i) $F(\omega_2) > (F(\omega_v) - (1-p)F(\omega_1))/p$ and (ii) $F(\omega_2) \leq (F(\omega_v) - pF(\omega_1))/(1-p)$. There are two candidate solutions to this optimization problem:

   (a) Set $F(\omega_2)$ equal to its upper bound $(F(\omega_v) - pF(\omega_1))/(1-p)$ given in constraint (ii). This makes the expert's expected utility a *decreasing* function of ω_1 as $F(\omega_1) + F(\omega_2) = (F(\omega_v) - (2p-1)F(\omega_1))/(1-p)$. It is therefore optimal to set $\omega_1 = 0$, which implies that the optimal value of ω_2 solves $F(\omega_2) = F(\omega_v)/(1-p)$. Note that this solution is feasible only if the model parameters are s.t. $F(\omega_v) \leq 1-p$.

   (b) Set $F(\omega_1)$ equal to the upper bound $(F(\omega_v) - (1-p)F(\omega_2))/p$, which can be obtained by re-arranging constraint (ii) above. This makes the expert's expected utility an *increasing* function of ω_2 as $F(\omega_1) + F(\omega_2) = (F(\omega_v) + (2p-1)F(\omega_2))/p$. It is therefore optimal to set $\omega_2 = 1$, which implies that the optimal value of ω_1 solves $F(\omega_1) = (F(\omega_v) - (1-p))/p$. Note that this solution is feasible only if the model parameters are s.t. $F(\omega_v) \geq 1-p$.

7.2.2 Equilibrium persuasion strategy

Given the optimal thresholds ω_1 and ω_2, we now compute the expert's equilibrium persuasion strategy (which is a binary partition, as shown in items 2. and 3. of Sec. 7.2.1 above).

Suppose $F(\omega_v) > 1-p$**.** The difference in the expert's expected utility from the persuasion strategies in items 3.(b) and 2. of Sec. 7.2.1 is:

$$\frac{(\bar{u}_m - \underline{u}_m)}{p(1-p)F(\omega_v)(1-J_n(p))}\left(\frac{(1-F(\omega_v))(p-(1-p))}{F(\omega_v)(1-p)} - 1 - \frac{J_n(p)}{1-J_n(p)}\right) \tag{4}$$

If this difference is negative, then the persuasion strategy in item 2. of Sec. 7.2.1 is optimal for the expert. If it is positive, then the persuasion strategy in item 3.(b) is optimal. The sign of the utility difference in (4) depends on whether equation (1) in Definition 1 holds or fails. The following result shows for which parameter values this is the case:

Lemma 1. *Let* $F(\omega_v) > 1-p$*:*

1. *if* $F(\omega_v) \in ((\sqrt{2}-1)/\sqrt{2}, 1/2)$ *and:*
   (a) $n=1$*, then eqn. (1) in Definition 1 holds for all* $F(\omega_v) \geq (2p-1)/2p$ *and fails otherwise;*
   (b) $n \geq 3$*, then eqn. (1) holds for all* $F(\omega_v) > 1-p$*;*
2. *if* $F(\omega_v) \in ((7\sqrt{7}-10)/(7\sqrt{7}+17), (\sqrt{2}-1)/\sqrt{2}]$ *and:*
   (a) $n=1$*, then eqn. (1) fails for all* $F(\omega_v) > 1-p$*;*
   (b) $n \geq 3$*, then eqn. (1) holds for all* $F(\omega_v) > 1-p$*;*
3. *if* $F(\omega_v) < (7\sqrt{7}-10)/(7\sqrt{7}+17)$ *and:*
   (a) $n=1$*, eqn. (1) fails for all* $F(\omega_v) > 1-p$*;*
   (b) $n=3$*, eqn. (1) fails for* $p < \hat{p}_3^F$ *and holds otherwise, where* $\hat{p}_3^F$ *is the biggest real root in* $(1/2,1)$ *of* $4p^3-4p^2-p+1+F(\omega_v)/(1-F(\omega_v))$*. Note that this can be expressed equivalently by saying that for given* p *s.t.* $F(\omega_v) > 1-p$*, eqn. (1) fails for* $F(\omega_v) < (1-p)(2p-1)(2p+1)/p(4p(1-p)+1)$*;*
   (c) $n \geq 5$*, eqn. (1) holds for all* $F(\omega_v) > 1-p$*.*

The proof of Lemma 1 is given below in Sec. 7.3. With the results from Lemma 1, item 1. of Prop. 2 follows immediately.

Suppose $F(\omega_v) \leq 1-p$**.** The difference in the expert's expected utility from the persuasion strategies in items 3.(a) and 2. of Sec. 7.2.1 is:

$$-F(\omega_v)(\bar{u}_m - \underline{u}_m)\left[J_n(p)p(2p-1) - p^2 - p + 1\right]/p(1-p)$$

Label as $\eta(n,p)$ the term in square brackets: $\eta(n,p) \equiv J_n(p)p(2p-1) - p^2 - p + 1$. Note that $\eta(5,1/2) = 1/4$, $\eta(5,1) = 0$, and that the equation $\eta(5,p) = 0$ has no solution in $p \in (1/2,1)$. Thus, $\eta(5,p) > 0$ for all $p \in (1/2,1)$. As $p > 1/2$ it follows that $\eta(n,p)$ is increasing in $J_n(p)$. We now make recourse to the following result that is derived in the proof of Lemma 2 in Karotkin and Paroush (2003):

Lemma (Karotkin and Paroush, 2003). $J_n(p)$ is increasing in n. In particular: $J_{n+2}(p) - J_n(p) = p(2p-1)\binom{n}{\frac{n-1}{2}}p^{\frac{n-1}{2}}(1-p)^{\frac{n+1}{2}}$.

It follows that $\eta(n,p)$ is increasing in n. This implies that $\eta(n,p) > 0$ for all $n \geq 5$, which establishes the result in item 2.(a) of Prop. 2.

To prove part 2.(b), set $n = 3$. Note that $\eta(3,1/2) = 1/4$, $\eta(3,1) = 0$, and that $\eta(3,p) = 0$ has a unique solution in $p \in (1/2,1)$ given by $\bar{p}_3 = ((27-3\sqrt{78})^{1/3} + (3\sqrt{78}+27)^{1/3})/6 \approx 0.76$. It follows that for all $p \in (1/2, \bar{p}_3)$, we have $\eta(3,p) > 0$, while for all $p \in (\bar{p}_3, 1)$, we have $\eta(3,p) < 0$. This proves item 2.(b) of Prop. 2.

Finally, to prove part 2.(c), set $n = 1$. Note that $\eta(1,1/2) = 1/4$, $\eta(3,1) = 0$, and that $\eta(3,p) = 0$ has a unique solution in $p \in (1/2,1)$ given by $\bar{p}_1 = \sqrt{2}/6 \approx 0.71$. It follows that for all $p \in (1/2, \bar{p}_1)$, we have $\eta(1,p) > 0$, while for all $p \in (\bar{p}_1, 1)$, we have $\eta(1,p) < 0$. This proves item 2.(c) of Prop. 2. and completes the proof. □

7.3 Proof of Lemma 1

To prove the result, we start by re-arranging eqn. (1) as follows:

$$\frac{F(\omega_v)}{1-F(\omega_v)} > (1 - J_n(p))\,\frac{2p-1}{1-p}$$

For ease of notation, we define as $H_n(p)$ the function of p on the right-hand side of this inequality. The proof idea is best gleaned from Fig. 3, which illustrates eqn. (1) for five different committee sizes. The upward-sloping line and the curved single-peaked shapes in the figure represent the respective functions $H_n(p)$ which capture the components of eqn. (1) that vary with signal precision p. The horizontal line in Fig. 3 represents the odds in favor of agreement. It is obvious that the graph of $H_1(p) = 2p - 1$ has a unique intersection point

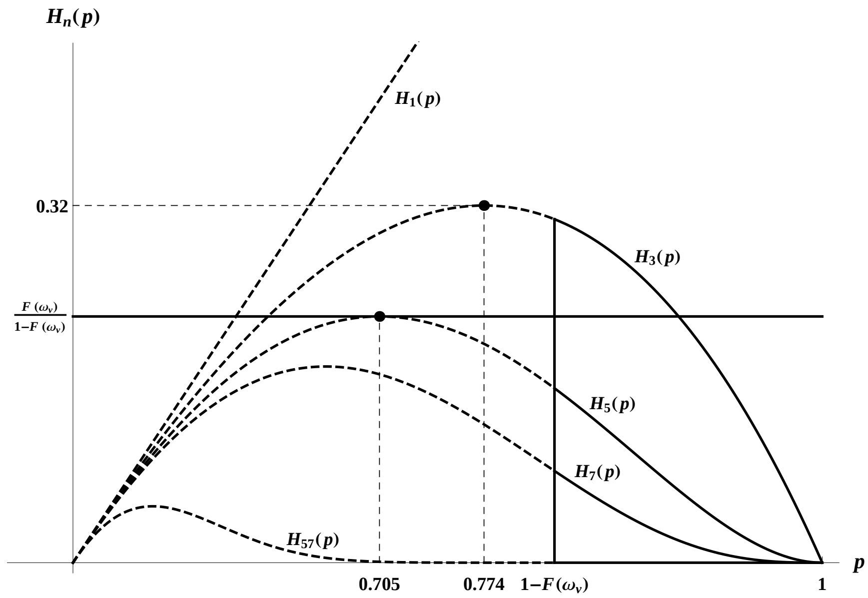

Figure 3: Graphical illustration of eqn. (1)

$p_1 = 1/2(1-F(\omega_v))$ with the horizontal line at $F(\omega_v)/(1-F(\omega_v))$ for any $F(\omega_v) \in (0,1/2)$. Furthermore, we have $p_1 \geq 1 - F(\omega_v)$ if $F(\omega_v) \geq (\sqrt{2}-1)/\sqrt{2}$. Thus, for $n = 1$ eqn. (1) fails for all $p > p_1$ if $F(\omega_v) \geq (\sqrt{2}-1)/\sqrt{2}$, and otherwise it fails for all $p > 1 - F(\omega_v)$.

In the remainder of this proof, we show first that for *any* committee size $n \geq 3$, the function $H_n(p)$ features a single-peaked graph which, when intersected by a horizontal line, yields exactly *two* strictly ordered intersection points p_n^1 and p_n^2. We then show for a five member committee that when signal precision p exceeds the likelihood of disagreement $1-F(\omega_v)$, both intersection points with a horizontal line at any $F(\omega_v)/(1-F(\omega_v))$ below the maximum value $H_5(p_5^*) \approx 0.217549$ lie outside the range of admissible values $p \in (1-F(\omega_v),1)$. Therefore, eqn. (1) holds for $n=5$ and all $p > 1-F(\omega_v)$. Finally, by virtue of the fact that $H_{n+2}(p) < H_n(p)$ for all $p \in (1/2,1)$ due to the aforementioned lemma by Karotkin and Paroush (2003), it follows immediately that eqn. (1) also holds for all $p > 1-F(\omega_v)$ and all $n > 5$. Thus, only in one-member and in three-member committees can eqn. (1) fail for some $p \in (1-F(\omega_v),1)$.

In fact, it is easy to compute that for $n=3$, the maximum value of $H_3(p) = (1-p)(2p-1)(1+2p)$ is reached at the point $p_3^* = (\sqrt{7}+2)/6 \approx 0.77429$, with $H_3(p_3^*) = (7\sqrt{7}-10)/27$. Therefore, if $F(\omega_v)/(1-F(\omega_v)) \geq H_3(p_3^*)$ (which is equivalent to $F(\omega_v) > (7\sqrt{7}-10)/(7\sqrt{7}+17)$), then eqn. (1) cannot fail for any $n \geq 3$. Observe that for $F(\omega_v) = (7\sqrt{7}-10)/(7\sqrt{7}+17)$, the corresponding lower bound on p is $\underline{p}(F(\omega_v)) \equiv 1-F(\omega_v) = (7\sqrt{7}-17)/2 \approx 0.76013$. Therefore: $p_3^* > \underline{p}(F(\omega_v)) = (7\sqrt{7}-17)/2$. To see that for any $F(\omega_v) < (7\sqrt{7}-10)/(7\sqrt{7}+17)$ the corresponding lower bound $\underline{p}(F(\omega_v))$ is smaller than the intersection point $p_3^2(F(\omega_v))$ of the function $H_3(p)$ with the horizontal line at $F(\omega_v)/(1-F(\omega_v))$, note that a change in the value of $F(\omega_v)$ affects these two benchmarks in the following way: $d\underline{p}(F(\omega_v))/dF(\omega_v) = -1$ and $dp_3^2(F(\omega_v))/dF(\omega_v) = 1/H_3'(p_3^2(F(\omega_v))(1-F(\omega_v))^2 < 0$ for all $p_3^2(F(\omega_v)) > p_3^*$. In particular, it is straightforward to verify that a drop in the value of $F(\omega_v)$ raises the value of the lower bound $\underline{p}(F(\omega_v))$ by 1, while it raises the value of the intersection point $p_3^2(F(\omega_v))$ by more than 1. Thus, for any $F(\omega_v) < (7\sqrt{7}-10)/(7\sqrt{7}+17)$ we have $p_3^2(F(\omega_v) > \underline{p}(F(\omega_v))$, and therefore eqn. (1) holds for all $p \in (\underline{p}(F(\omega_v)), p_3^2(F(\omega_v)))$.

General shape of H_n. We now characterize the shape of the function H_n for arbitrary $n \geq 3$. First, note that for all n: $H_n(1/2) = 0$ and $H_n(1) = 0$. Next, consider the monotonicity and curvature of H_n. To this end, we define for ease of notation $L_{n,j}(p) \equiv \binom{n}{j} p^j (1-p)^{n-j}$. Also let:

$$K_n(p) \equiv \sum_{j=0}^{\frac{n-1}{2}} \binom{n}{j} p^j (1-p)^{n-j} = \sum_{j=0}^{\frac{n-1}{2}} L_{n,j}(p).$$

As n is odd, the integer $(n-1)/2$ is even. We can therefore express equivalently the function H_n as: $H_n(p) = ((2p-1)/(1-p))K_n(p)$. In order to obtain the derivatives of this function, it is useful to note that:

$$L_{n,j}'(p) = n\left(L_{n-1,j-1}(p) - L_{n-1,j}(p)\right). \tag{5}$$

Thus, $K_n'(p) = \sum_{j=0}^{\frac{n-1}{2}} n\left(L_{n-1,j-1}(p) - L_{n-1,j}(p)\right) = -nL_{n-1,\frac{n-1}{2}}(p)$. We therefore obtain our desired derivative:

$$\begin{aligned} H_n'(p) &= \frac{1}{(1-p)^2} K_n(p) - \frac{n(2p-1)}{1-p} L_{n-1,\frac{n-1}{2}}(p) \\ &= \sum_{j=0}^{\frac{n-1}{2}} \binom{n}{j} p^j (1-p)^{n-j-2} - n(2p-1)\binom{n-1}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n-3}{2}}. \end{aligned}$$

We first evaluate the derivative at $p = 1/2$:

$$H_n'(1/2) = 4K_n(p) = \frac{4}{2^n}\sum_{j=0}^{\frac{n-1}{2}}\binom{n}{j}.$$

Note that $\sum_{j=0}^{n}\binom{n}{j} = 2^n$, and since n is odd, we also have $\sum_{j=0}^{\frac{n-1}{2}}\binom{n}{j} = (1/2)\sum_{j=0}^{n}\binom{n}{j}$. Thus, $H_n'(1/2) = 2$, which shows that H_n is strictly increasing at $p = 1/2$. Next, we evaluate the derivative H_n' at $p = 1$:

$$H_n'(1) = \sum_{j=0}^{\frac{n-1}{2}}\binom{n}{j}1\cdot 0^{n-j-2} - n\binom{n-1}{\frac{n-1}{2}}1\cdot 0^{\frac{n-3}{2}}$$

First consider the case $n = 3$:

$$H_3'(1) = \binom{3}{0}1\cdot 0^1 + \binom{3}{1}1\cdot 0^0 - 3\binom{2}{1}1\cdot 0^0.$$

As $0^0 = 1$, this expression reduces to: $H_3'(1) = 3 - 6 = -3$. Next, consider $n \geq 5$. In these cases, the expression for $H_n'(1)$ no longer features any expressions involving 0^0, but only expressions where 0 is raised to a strictly positive power, which are all equal to zero. Thus, $H_n'(1) = 0$ for all $n \geq 5$. We can therefore infer from the fact that H_n is strictly increasing at $p = 1/2$ and non-increasing at $p = 1$, that H_n must have a global maximum somewhere in $(1/2, 1)$. In fact, we now argue that H_n has a unique critical point in $(1/2, 1)$, which must therefore be the unique global maximum.

Uniqueness of critical point of H_n. For $n \geq 3$, at any critical point p_n^*, the following condition holds:

$$\frac{1}{(1-p_n^*)^2}K_n(p_n^*) - \frac{n(2p_n^*-1)}{1-p_n^*}L_{n-1,\frac{n-1}{2}}(p_n^*) = 0. \tag{6}$$

Multiplying through (6) by $(1-p_n^*)^2$ yields:

$$K_n(p_n^*) = n(1-p_n^*)(2p_n^*-1)L_{n-1,\frac{n-1}{2}}(p_n^*).$$

To establish that there is a unique $p_n^* \in (1/2, 1)$ for which this equality holds, we study separately the functions $K_n(p)$ and $n(1-p)(2p-1)L_{n-1,\frac{n-1}{2}}(p) \equiv \Lambda_n(p)$. In fact, it will be more useful for our purposes to study the behavior of the functions $K_n(1-x)$ and $\Lambda_n(1-x)$ for values $x \in [0, 1/2]$, as illustrated in Fig. 4 for the case of $n = 5$. In the remainder of this section of the proof, we establish that for any $n \geq 3$, the functions $K_n(1-x)$ and $\Lambda_n(1-x)$ behave qualitatively in the way depicted in Fig. 4, so that we can conclude there exists a unique intersection point in $(0, 1/2)$ of these two functions.

The reader can easily verify that $K_n(1) = \Lambda_n(1) = \Lambda_n(1/2) = 0$. Furthermore, it is straightfor-

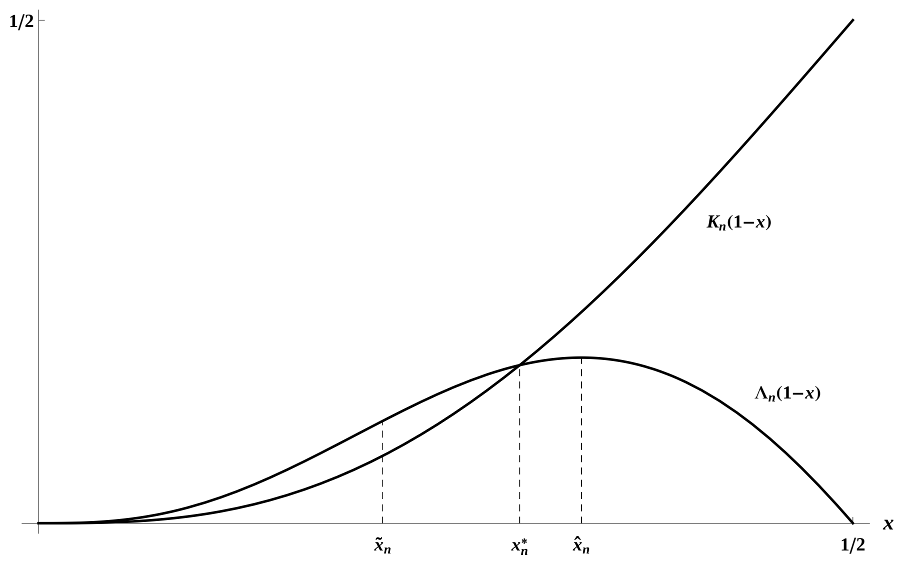

Figure 4: Uniqueness of critical point of H_n.

ward to obtain the first two derivatives of K_n and their signs:[13]

$$\begin{aligned} K_n'(p) &= -\frac{n\Gamma(n)}{\Gamma^2\left(\frac{n+1}{2}\right)} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2}} < 0 \text{ for all } p \in [1/2, 1), \\ K_n''(p) &= \frac{n\Gamma(n)}{2\Gamma^2\left(\frac{n+1}{2}\right)} p^{\frac{n-3}{2}} (1-p)^{\frac{n-3}{2}} (n-1)(2p-1) > 0 \text{ for all } p \in (1/2, 1). \end{aligned}$$

This implies that, as illustrated in Fig. 4, the function $K_n(1-x)$ is strictly increasing and convex for all $x \in (0, 1/2)$. We can also easily compute the first derivative of Λ_n:

$$\Lambda_n'(p) = -\frac{n\Gamma(n)}{2\Gamma^2\left(\frac{n+1}{2}\right)} p^{\frac{n-3}{2}} (1-p)^{\frac{n-1}{2}} \left(4(1+n)p^2 - 2(1+2n)p + (n-1)\right).$$

Note that:

$$\Lambda_n'(p) > 0 \Leftrightarrow p > \hat{p}_n \equiv \frac{2n + \sqrt{4n+5} + 1}{4(n+1)}, \tag{7}$$

and that $\Lambda_n'(p) < 0 \Leftrightarrow p < \hat{p}_n$. Thus, $\Lambda_n(p)$ has a critical point at $\hat{p}_n$. We can therefore state that the function $\Lambda_n(1-x)$ is strictly monotonically increasing in x for $x < \hat{x}_n \equiv 1 - \hat{p}_n$, and that it is strictly monotonically decreasing for $x > \hat{x}_n$. Next, note that:

$$\begin{aligned} K_n'(p) > \Lambda_n'(p) &\Leftrightarrow 0 < p^2 - p + \frac{n-1}{4(1+n)} \\ &\Leftrightarrow p > \tilde{p}_n \equiv \frac{1}{2}\left(1 + \sqrt{\frac{2}{n+1}}\right), \end{aligned} \tag{8}$$

and that $K_n'(p) < \Lambda_n'(p) \Leftrightarrow p < \tilde{p}_n$. At the point $\tilde{p}_n$ both derivatives are equal. By comparing the binomial expressions in (7) and (8) that define $\hat{p}_n$ and $\tilde{p}_n$, resp., it is easy to verify that

[13]In the following expressions, we use the Gamma function to express more compactly the familiar factorial function: $\Gamma(n) \equiv (n-1)!$.

$\hat{p}_n < \tilde{p}_n$. We can therefore state that while both functions $K_n(1-x)$ and $\Lambda_n(1-x)$ start at $x=0$ with a value of 0, for values $x < \tilde{x}_n \equiv 1-\tilde{p}_n$, the function $\Lambda_n(1-x)$ increases faster than the function $K_n(1-x)$. For any $x \in (\tilde{x}_n, \hat{x}_n)$, the function $K_n(1-x)$ grows at increasing rate, while the function $\Lambda_n(1-x)$ now grows at a rate below that of $K_n(1-x)$. Finally, for all $x > \hat{x}_n$, the function $K_n(1-x)$ continues to grow at increasing rate, while the function $\Lambda_n(1-x)$ smoothly falls back to a value of 0. In other words, for any $n \geq 3$, the behavior of the functions $K_n(1-x)$ and $\Lambda_n(1-x)$ is as depicted in Fig. 4, which proves that there exists a unique intersection point $x_n^* \equiv 1-p_n^*$ of these two functions in $(0,1/2)$. This point of intersection constitutes the unique critical point of the function H_n, which therefore is the unique global maximum of H_n in $(1/2,1)$.

Having argued that for any $n \geq 3$ the function H_n starts and ends with a value of zero, and has a unique interior maximum, we now turn to the question of how this function varies with committee size n. By the above lemma due to Karotkin and Paroush (2003) (see proof of Prop. 2), we know that $J_{n+2}(p) > J_n(p)$ for all $p \in (1/2,1)$. It therefore follows immediately that $H_n(p) > H_{n+2}(p)$ for all $p \in (1/2,1)$.

Evaluating eqn. (1) for $n=5$. For committee size $n=5$ we can explicitly write $H_5(p) = (1-p)^2(2p-1)\left(3p+6p^2+1\right)$. Now fix a value $F(\omega_v) \in (0,1/2)$. This implies the following lower bound on the admissible values of p: $p \geq \underline{p} = 1-F(\omega_v)$. We now ask if the function H_5, when evaluated at the lower bound $\underline{p}$, satisfies or violates eqn. (1). I.e. we check if:

$$H_5(\underline{p}) \lesseqqgtr \frac{F(\omega_v)}{1-F(\omega_v)} = \frac{1-\underline{p}}{\underline{p}} \Leftrightarrow \underline{p}\left(1-\underline{p}\right)\left(2\underline{p}-1\right) \lesseqqgtr \frac{1}{1+3\underline{p}(1+2\underline{p})}, \tag{9}$$

where $\underline{p} \in [1/2,1]$.

First consider the function $\underline{p}(1-\underline{p})(2\underline{p}-1)$, and note that both its limits as $\underline{p} \to 1/2$ and $\underline{p} \to 1$, resp., are equal to zero. Furthermore, the derivative of this function w.r.t. $\underline{p}$ is $6\underline{p}(1-\underline{p})-1$. Thus, for $\underline{p} \in [1/2, 1/2+\sqrt{3}/6)$, the function $\underline{p}(1-\underline{p})(2\underline{p}-1)$ is strictly increasing, while for $\underline{p} \in (1/2+\sqrt{3}/6, 1]$ it is strictly decreasing. At $\underline{p} = 1/2+\sqrt{3}/6$, the function has a critical point, which constitutes its global maximum as the second derivative, when evaluated at $\underline{p} = 1/2+\sqrt{3}/6$, takes the value $-2\sqrt{3}$. We can therefore conclude that the function $\underline{p}(1-\underline{p})(2\underline{p}-1)$ reaches its maximum value of $\sqrt{3}/18$ at $\underline{p} = 1/2+\sqrt{3}/6$, while taking strictly lower values for all other $\underline{p}$.

Next, consider the denominator of the fraction on the right-hand side of (9). The quadratic polynomial $1+3\underline{p}(1+2\underline{p})$ is strictly increasing (its derivative is $12\underline{p}+3$) and takes values in $[4,10]$. This implies that the right-hand side fraction in (9) is a strictly decreasing function that reaches its global minimum at $\underline{p}=1$, where it takes the value $1/10$. Now observe that the maximum value of $\sqrt{3}/18$ of the function $\underline{p}(1-\underline{p})(2\underline{p}-1)$ is strictly lower than the minimum value of the fraction $1/(1+3\underline{p}(1+2\underline{p}))$. We can therefore conclude that eqn. (1) is satisfied when p takes its lowest possible value of $\underline{p}$, regardless of what this value $\underline{p}$ is.

Now suppose that $F(\omega_v) \leq H_5(p_5^*)/(1+H_5(p_5^*))$, where p_5^* is the unique global maximum point of the function H_5, at which it takes the value $H_5(p_5^*)$. For any such $F(\omega_v)$, the horizontal line at $F(\omega_v)/(1-F(\omega_v))$ will intersect the graph of the function H_5 at two points: $p_5^1(F(\omega_v))$ and $p_5^2(F(\omega_v))$ (with $p_5^1(F(\omega_v)) < p_5^2(F(\omega_v))$), where $H_5(p_5^i(q)) = F(\omega_v)/(1-F(\omega_v))$ for all $i=1,2$. Fig. 5 provides an illustration. Also indicated in Fig. 5 is the lower bound $\underline{p} = 1-F(\omega_v)$ of admissible values p, which illustrates our finding above that $H_5(1-F(\omega_v)) <$

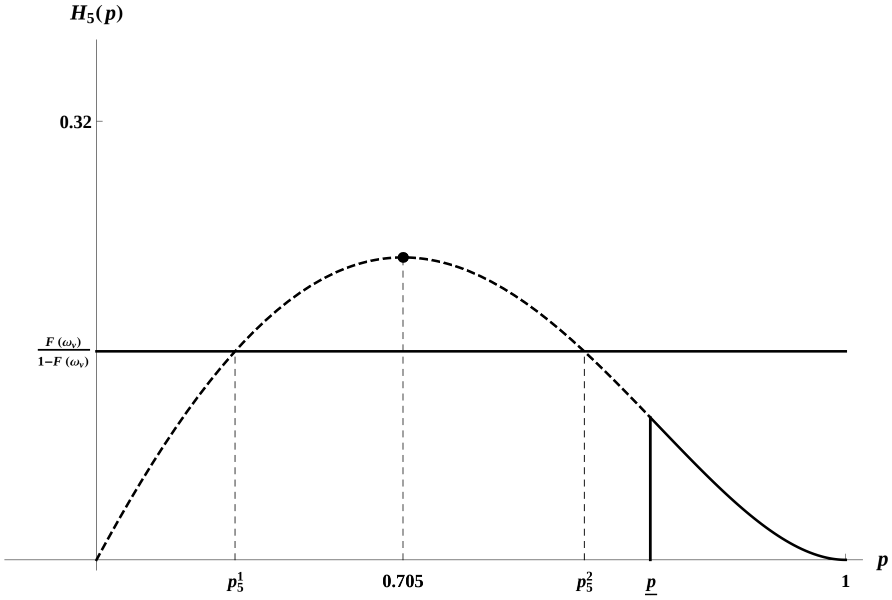

Figure 5: Eqn. (1) for $n = 5$

$F(\omega_v)/(1-F(\omega_v))$. In the remainder of this section of the proof of Lemma 1, we focus on showing that for all $F(\omega_v) \in (0, 1/2)$, it holds that $\underline{p} = 1 - F(\omega_v) > p_5^2$. To see this, note that:

$$H_5(1 - F(\omega_v)) < \frac{F(\omega_v)}{1 - F(\omega_v)} = H_5(p_5^i(F(\omega_v))),$$

so that we have either $1 - F(\omega_v) < p_5^1(q)$ (due to the fact that H_5 is strictly increasing for all $p < p_5^1(F(\omega_v))$), or $1 - F(\omega_v) > p_5^2(F(\omega_v))$ (due to the fact that H_5 is strictly decreasing for all $p > p_5^2(F(\omega_v))$).

As the function H_5 is a polynomial of order 5, we can neither compute analytically its global maximum p_5^*, nor the two points $p_5^1(F(\omega_v))$ and $p_5^2(F(\omega_v))$ for given $F(\omega_v) \leq H_5(p_5^*)/(1 + H_5(p_5^*))$. Therefore, we take an indirect approach to establishing that $1 - F(\omega_v) > p_5^2(F(\omega_v))$ by verifying that $H_5'(1 - F(\omega_v)) < 0$ for all $F(\omega_v) \leq H_5(p_5^*)/(1 + H_5(p_5^*))$. To do this, we recall that the function H_3 has a global maximum at $p_3^* = (\sqrt{7} + 2)/6$, with associated maximum value $H_3(p_3^*) = (7\sqrt{7} - 10)/27$. Thus, if $F(\omega_v) = F_3^* \equiv (7\sqrt{7} - 10)/(7\sqrt{7} + 17)$, we have $H_3(p_3^*) = F_3^*/(1 - F_3^*)$. Note that p_3^* is in the admissible range because $1 - F_3^* = (7\sqrt{7} - 17)/2 < p_3^* = (\sqrt{7} + 2)/6$.

We now evaluate the derivative H_5' at the point $1 - F_3^*$, which yields $H_5'(1 - F_3^*) = (6517\,630 - 2463\,433\sqrt{7})/2$. It is easy to verify analytically that this number is strictly negative (and is approx. equal to $-0.544\,73$). This shows that the maximum of H_5 must occur at some point $p_5^* < 1 - F_3^* < p_3^*$. Thus, $H_3(p_3^*) > H_3(p_5^*) > H_5(p_5^*)$, where the latter inequality follows from the aforementioned Lemma by Karotkin and Paroush (2003). Therefore, we know that:

$$F_3^* = \frac{H_3(p_3^*)}{1 + H_3(p_3^*)} > \frac{H_5(p_5^*)}{1 + H_5(p_5^*)},$$

which implies that for any $F(\omega_v) \leq H_5(p_5^*)/(1+H_5(p_5^*))$ we have:

$$p_5^* < 1 - F_3^* < 1 - \frac{H_5(p_5^*)}{1+H_5(p_5^*)} \leq 1 - F(\omega_v).$$

Due to the fact that $H_5'(p) < 0$ for all $p > p_5^*$, we can finally state that $H_5'(1-F(\omega_v)) < 0$. This is what we needed in order to prove that for any $F(\omega_v) \leq H_5(p_5^*)/(1+H_5(p_5^*))$, eqn. (1) is sastisfied for all $p \in (1-F(\omega_v), 1)$. This, finally, establishes that eqn. (1) is satisfied for $n = 5$ and all $F(\omega_v) \in (0, 1/2)$ and $p \in (1-F(\omega_v), 1)$ as claimed in Lemma 1. □

7.4 Proof of Proposition 3

The details of the proof mirror those in the proof of Prop. 2 in Sec. 7.2: we characterize the optimal partition Ω^3 by choosing thresholds ω_1 and ω_2 so as to maximize the expert's ex ante expected utility. As explained in Sec. 7.2.1, we only have to consider those threshold-values for which either $v(\Omega_2^3, s_i) = X$ for all s_i, or $v(\Omega_2^3, s_i) = s_i$.

1. Suppose $v(\Omega_2^3, s_i) = X$ for all s_i. From item 2. in Sec. 7.2.1 it follows immediately that for $1/2 < F(\omega_v) < p$ the optimal thresholds are $\omega_1 = 0$ and ω_2 s.t. $F(\omega_2) = F(\omega_v)/p$. If, instead, $F(\omega_v) > p$, then it is easy to see that the optimal thresholds are $\omega_1 = 0$ and $\omega_2 = 1$.
2. Suppose $v(\Omega_2^3, s_i) = s_i$. From item 3. in Sec. 7.2.1 it is easy to see that the threshold ω_2 in item 3.(a) is not feasible as $F(\omega_v) > 1/2 > 1-p$. Thus, the optimal thresholds for any $F(\omega_v) > 1/2$ are those given in item 3.(b) of Sec. 7.2.1: ω_1 s.t. $F(\omega_1) = (F(\omega_v) - (1-p))/p$ and $\omega_2 = 1$.

Given these optimal thresholds ω_1 and ω_2, we can compute the expert's equilibrium persuasion strategy. First suppose that $F(\omega_v) > p$. In this case, the expert can induce voters to vote for his preferred alternative regardless of their private signals by setting $\Omega_2^3 = [0, 1]$. No other persuasion strategy can give the expert a higher utility. This established item 1. of Prop. 3.

Now suppose that $1/2 < F(\omega_v) < p$. In this case, the difference in the expert's expected utility from the persuasion strategies in items 1. and 2. of the present proof is given by (4). If this difference is negative, then the persuasion strategy in item 1. is optimal for the expert. If it is positive, then the persuasion strategy in item 2. is optimal. The sign of the utility difference in (4) depends on whether eqn. (1) holds or fails. Note that the left-hand side of eqn. (1) is increasing in the value of $J_n(p)$. By Karotkin and Paroush's lemma stated above in the proof of Prop. 2, it follows that *if* eqn. (1) is holds for $n = 1$, *then* it will also hold for all odd $n > 1$. It is easy to verify that eqn. (1) holds for $n = 1$. Note that $J_1(p) = p$. Suppose now that eqn. (1) is violated for $n = 1$. I.e.:

$$2p \geq 1 + \frac{F(\omega_v)}{1-F(\omega_v)}$$

This latter inequality cannot hold: as $F(\omega_v) > 1/2$, the expression on the right-hand side of the inequality is strictly larger than 2, while the value of $2p$ on the left-hand side is at most 2. We can therefore conclude by contradiction that the expert's utility-difference in (4) is negative for all $n \geq 1$. This establishes item 2. of Prop. 3. □

7.5 Proof of Proposition 4

First consider the case $F(\omega_v) > 1-p$. If eqn. (1) holds, then by item 1.(a) of Prop. 2, the probability of making the correct collective choice is constant and will therefore not rise if additional voters are added: $1-F(\omega_v)(1-p)/p$. Now contrast this with the case where eqn. (1) fails. In this latter case (which corresponds to item 1.(b) of Prop. 2), the probability of making the correct collective choice is $1-(1-J_n(p))(1-F(\omega_v))/p$. Note that the former probability of correct decision-making is *higher* than the latter iff:

$$1-J_n(p) > \frac{F(\omega_v)(1-p)}{1-F(\omega_v)}. \tag{10}$$

Note that this inequality holds whenever eqn. (1) in Definition 1 *fails* (i.e. the odds $J_n(p)/(1-J_n(p))$ are low). We can express equivalently as follows: $(1-J_n(p))(2p-1) > F(\omega_v)(1-p)/(1-F(\omega_v))$. If this in equality holds, it is obvious that the inequality in (10) also holds. This implies that the probability of correct decision-making is lower under the voting equilibrium induced by the expert's persuasion strategy $\tilde{\Omega}^2$ in item 1.(b) of Prop. 2 than it would be under the persuasion strategy $\hat{\Omega}^2$ in item 1.(a). In order to make the strategy $\hat{\Omega}^2$ the expert's equilibrium choice, the number of voters must be raised to the minimum for which eqn. (1) in Definition 1 *holds*. Lemma 1 above allows us to establish this minimum number for every parameter-pair $(p, F(\omega_v))$ s.t. $F(\omega_v) > 1-p$.

Now consider the case of $F(\omega_v) < 1-p$. By item 2. of Prop. 2 the probability of correct decision-making is $1-F(\omega_v)(1-p)/p$ when either $n \geq 5$, or $n = 3$ and $p < \bar{p}_3$, or $n = 1$ and $p < \bar{p}_1$. Thus, for $p < \bar{p}_1 < \bar{p}_3$ committee size is irrelevant, and a single voter generates the same probability of correct decision-making as any larger committee. Next consider $p \in (\bar{p}_1, \bar{p}_3)$ and compare a committee with three or more voters to one with just a single decision-maker, for whom the probability of a correct decision is $1-F(\omega_v)$. Comparing these two probabilities of correct decision-making, we find that a 'large' committee (with $n \geq 3$ voters) is superior to a single decision-maker. Finally, consider $p > \bar{p}_3$ and compare a committee with five or more voters to one with just three voters, for which the probability of a correct decision is $1-F(\omega_v)(1-J_3(p))/(1-p)$. Comparing the probabilities of correct decision-making across these two committee sizes, we find that a 'large' committee (with $n \geq 5$ voters) is superior to the three-member one iff:

$$1-J_3(p) > \frac{(1-p)^2}{p} \Leftrightarrow \frac{(2p-1)(1+p)(1-p)^2}{p} > 0. \tag{11}$$

As $p \in (1/2, 1)$, the inequality on the right-hand side of (11) always holds. Thus, any 'large' committee with five or more voters maximizes the probability of correct decision-making when $p \in (\bar{p}_3, 1-F(\omega_v))$. □

7.6 Proof of Proposition 5

By Prop. 3, we know that all voters use the same signal-independent voting strategy. This results in a constant probability of making the correct choice (namely $F(\omega_v)$ in the scenario where $p < F(\omega_v)$, and $1-F(\omega_v)(1-p)/p$ if $p > F(\omega_v)$) regardless of the size of the electorate. □

7.7 Proof of Proposition 6.

First consider the case where $F(\omega_v) > 1-p$, and suppose that eqn. (1) holds. Rewriting eqn. (1), it is easy to verify that this is the case iff $F(\omega_v) > (1-J_n(p))(2p-1)/(p-J_n(p)(2p-1))$. Now recall that Prop. 1 implies that the ex ante probability of a correct collective decision in the absence of persuasion is $J_n(p)$ as voters vote according to their private signals. With persuasion, item 1.(a) of Prop. 2 implies that the correct collective decision is made with ex ante probability $1-F(\omega_v)(1-p)/p$. Thus, expert persuasion aides information aggregation iff $F(\omega_v) < G_n(p)$. Observe that these two boundaries on the value of $F(\omega_v)$ are mutually compatible iff $J_n(p) < (1-3p(1-p))/p(2p-1)$, which is true for all $p \in (1/2,1)$.

Now note that $G_1(p) = p$ so that $1-p < F(\omega_v) < 1/2 < G_1(p)$ for all $p \in (1/2,1)$. This implies that persuasion *always* enhances the likelihood that a single decision-maker takes the correct decision. Next, note that for all $n \geq 3$, the function G_n is continuously differentiable, that $G_n(0) = G_n(1) = 0$, and that $G_n(1/2) = 1/2$. The behavior of G_n is further characterized by the following lemma:

Lemma 2. *For all $n \geq 7$, the function $G_n(p)$ is strictly decreasing for all $p \in [1/2,1)$.*

The proof of this lemma is in Section 7.8 below, and Fig. 6 provides a graphical illustration of G_n for select $n \geq 7$. Furthermore, Fig. 7 shows that for committee sizes $n = 3,5$ the functions G_3 and G_5 have a unique maximum in the interior of the range $(1/2,1)$ at p_3 and p_5, resp. Thus, as $F(\omega_v) < 1/2$, it is obvious that for every n there exists an interval whose boundaries arise from the intersection of the horizontal line at $F(\omega_v)$ with the graph of G_n. For signal precision p within this interval, expert persuasion - despite its bias - enhances the probability of a correct collective decision relative to the benchmark of no persuasion.

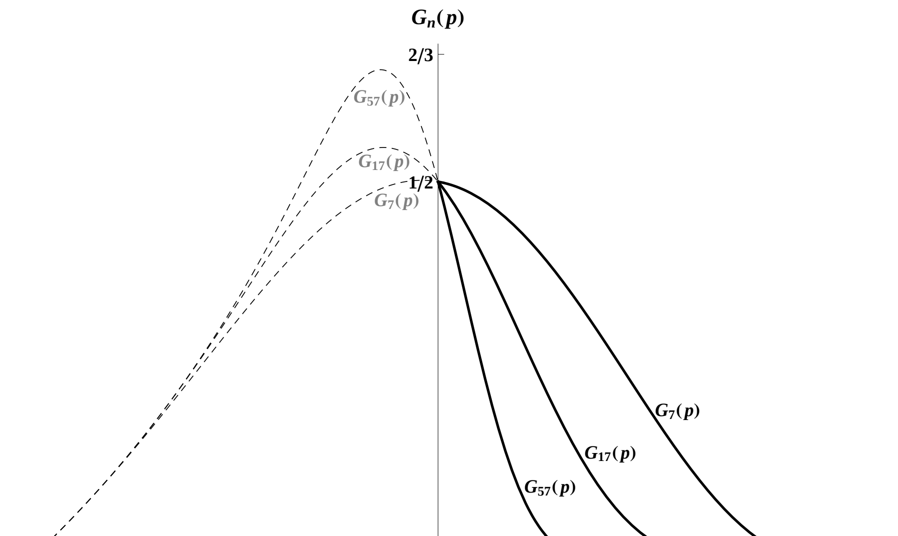

Figure 6: Three illustrations of the function G_n defined in (2)

Now let $F(\omega_v) > 1-p$ and suppose that eqn. (1) fails. In this case, item 1.(b) of Prop. 2 implies that the probability of a correct collective decision is $1-(1-J_n(p))(1-F(\omega_v))/p$. It is straightforward to verify that this probability exceeds the probability $J_n(p)$ that voters will make a correct decision in the absence of persuasion.

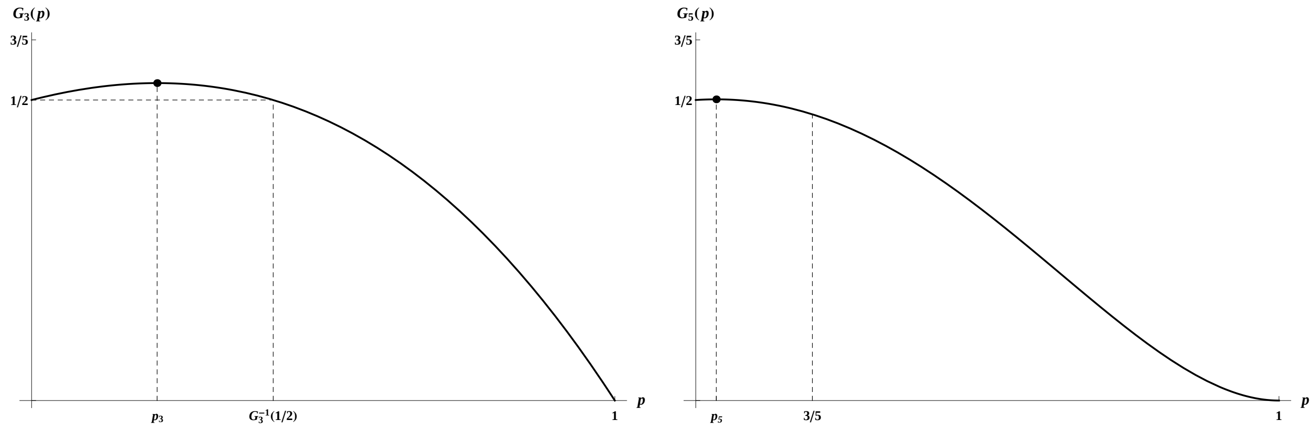

Figure 7: Illustration of the functions G_3 and G_5 for values $p \in [1/2, 1]$

Finally, consider the case where $p < 1 - F(\omega_v)$. By Prop. 1 the ex ante probability of a correct decision in the absence of persuasion is $1 - F(\omega_v)$, regardless of the number of voters, as they disregard their private signals and vote for Y . In the presence of the expert, item 2.(a) of Prop. 2 implies that for $n \geq 5$ voters, the ex ante probability of a correct collective decision is $1 - F(\omega_v)(1-p)/p$. Thus, expert persuasion always harms information aggregation. The same results follows by item 2.(b) of Prop. 2 for $n = 3$ voters and signal precision $p < \bar{p}_3$, and for $n = 1$ voters and signal precision $p < \bar{p}_1$. If, instead, $n = 3$ and $p > \bar{p}_3$, or $n = 1$ and $p > \bar{p}_1$, then item 2.(b) of Prop. 2 implies that the ex ante probability of a correct collective decision is $1 - F(\omega_v)(1 - J_n(p))/(1-p)$. It is straightforward to verify that this probability exceeds the probability $J_n(p)$ with which voters make a correct decision in the absence of persuasion. □

7.8 Proof of Lemma 2

Recall the function G_n which was introduced in (2) in the main text. Using the notation introduced above in the proof of Lemma 1, we can write: $G_n(p) = (p/(1-p))K_n(p)$. Furthermore, recalling the expression for $K_n'(p)$ given there, we obtain:

$$\begin{aligned} G_n'(p) &= \frac{1}{(1-p)^2}K_n(p) + \frac{p}{1-p}K_n'(p) \\ &= \frac{1}{(1-p)^2}K_n(p) - \frac{np}{1-p}L_{n-1,(n-1)/2}(p). \end{aligned}$$

Observe that we have $G_n'(p) < 0 \Leftrightarrow K_n(p) < np(1-p)L_{n-1,(n-1)/2}(p) \equiv \hat{\Lambda}_n(p)$. To see that this inequality holds for all $p \in [1/2, 1)$ and any $n \geq 7$, we study the behavior of the two functions K_n and $\hat{\Lambda}_n$. In the proof of Lemma 1, we have already obtained the first two derivatives of K_n, and so we know that it is a strictly decreasing and convex function that starts at $p = 1/2$ with a value of $K_n(1/2) = 1/2$, and ends at $p = 1$ with a value of $K_n(1) = 0$. Next consider the function $\hat{\Lambda}_n$. Its first derivative is:

$$\hat{\Lambda}_n'(p) = -\frac{n(n+1)\Gamma(n)}{2\Gamma^2\left(\frac{n+1}{2}\right)}(2p-1)p^{\frac{n-1}{2}}(1-p)^{\frac{n-1}{2}},$$

which is negative for all $p \in (1/2,1)$, with $\hat{\Lambda}_n'(1/2) = 0$. The second derivative is:

$$\hat{\Lambda}_n''(p) = \frac{n^2(n+1)\Gamma(n)}{\Gamma^2\left(\frac{n+1}{2}\right)} \left(p(1-p)\right)^{\frac{n-3}{2}} \left(p^2 - p + \frac{n-1}{4n}\right),$$

which is negative if $p < (\sqrt{n}+1)/2\sqrt{n}$. Thus, $\hat{\Lambda}_n$ is decreasing and concave for $p < (\sqrt{n}+1)/2\sqrt{n}$, and decreasing and convex for $p > (\sqrt{n}+1)/2\sqrt{n}$. Furthermore:

$$\hat{\Lambda}_n(1/2) = \frac{n\Gamma(n)}{2^{n+1}\Gamma^2\left(\frac{n+1}{2}\right)} \text{ and } \hat{\Lambda}_n(1) = 0.$$

Note that for $n = 7$, we have $\hat{\Lambda}_7(1/2) = 35/64 \approx 0.54688$. Also, an increase in committee size raises the value of $\hat{\Lambda}_n(1/2)$ as $\hat{\Lambda}_{n+2}(1/2) - \hat{\Lambda}_n(1/2) = \hat{\Lambda}_n(1/2)/(n+1)$. We can therefore conclude that for any odd $n \geq 7$, the value $\hat{\Lambda}_n(1/2)$ strictly exceeds $1/2$. This means that at $p = 1/2$, the function K_n starts at a lower value than the function $\hat{\Lambda}_n$.

To complete the proof, we now show by contradiction that $K_n(p) < \hat{\Lambda}_n(p)$ for all $p \in [1/2,1)$. For this purpose, we assume that the two functions intersect at some point p_n^* (given that both functions are strictly decreasing, there can be at most one intersection point), as illustrated in Fig. 8. As the figure illustrates, this implies that there exist two points $\tilde{p}_n$ and $\hat{p}_n$

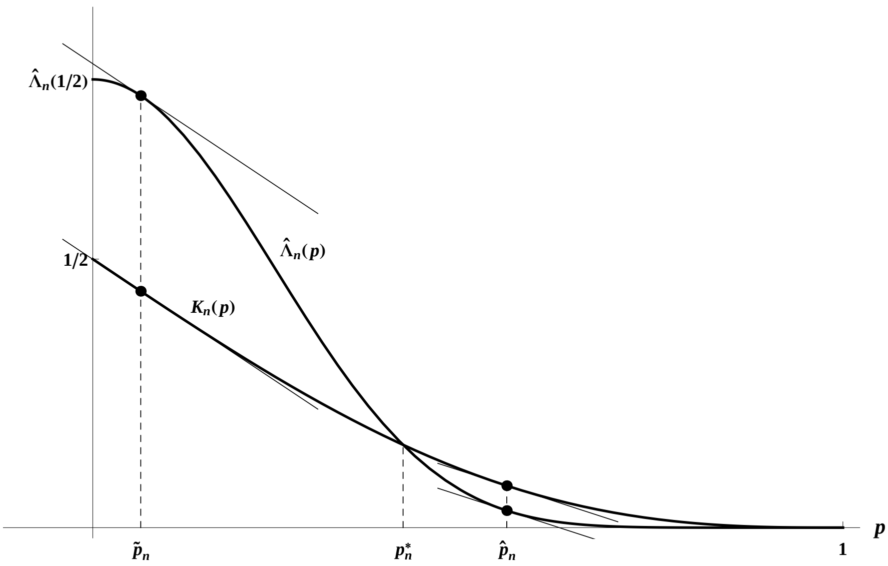

Figure 8: Monotonicity of G_n for $n \geq 7$.

(with $\tilde{p}_n < p_n^* < \hat{p}_n$) at which the first derivatives of the two functions are identical, so that for any $p \in (\tilde{p}_n, \hat{p}_n)$ the function $\hat{\Lambda}_n$ falls more steeply than K_n. However, a comparison of the two derivatives shows:

$$\hat{\Lambda}_n'(p) < K_n'(p) \Leftrightarrow p > \frac{1}{2} + \frac{1}{n+1}.$$

Thus, there is only the single point $1/2 + 1/(n+1)$ at which the two derivatives are identical. Note that there cannot be an intersection point to the left of this value, as K_n starts below $\hat{\Lambda}_n$ and initially declines more sharply than $\hat{\Lambda}_n$. Furthermore, there cannot be an intersection to the right of $1/2 + 1/(n+1)$ as $\hat{\Lambda}_n$ falls more steeply than K_n and would have to end up taking a negative value at $p = 1$, in contradiction to the fact that $\hat{\Lambda}_n(1) = 0$. We can therefore conclude that there exists no intersection point of K_n and $\hat{\Lambda}_n$. $\square$

7.9 Proof of Proposition 7

First consider the case $p < F(\omega_v)$. Item 1 follows immediately from Prop. 1 and item 1. of Prop. 3. Next, consider the case $p > F(\omega_v)$. From Prop. 1 it follows that, in the absence of an expert, voters cast their votes in line with their respective private signals. As a result, they choose the correct alternative with probability $J_n(p)$, which converges to 1 as the size of the electorate gets large. By item 2. of Prop. 3. we know that, in the presence of an expert, the equilibrium $(\hat{\Omega}^2, v)$ induces the following probability of making the correct decision: $1 - F(\omega_v)(1-p)/p$. Thus, expert persuasion harms information aggregation iff $F(\omega_v) > G_n(p)$, where the function $G_n(p)$ is defined in (2). As we have $F(\omega_v) > 1/2$, it is immediate that the condition $F(\omega_v) > G_n(p)$ holds for all $p \in (1/2, 1)$ if $n \geq 7$. This establishes that expert persuasion impedes information aggregation in electorates with seven or more voters. In the case of a single decision-maker (item 2.(c) of Prop. 7), the result is obvious as $G_1(p) = p$ and therefore $F(\omega_v) < G_1(p) = p$ for all p under consideration.

We now turn to the proof of item 2.(b) of Theorem 7. The idea behind the proof can be readily understood by looking at Fig. 7, which shows that the functions G_3 and G_5 each have a unique maximum in the interior of the range $(1/2, 1)$. Thus, if $F(\omega_v)$ exceeds this maximum, then expert persuasion harms information aggregation. If, instead, $F(\omega_v)$ is below this maximum, then there exists an interval whose boundaries arise from the intersection of the horizontal line at $F(\omega_v)$ with the graph of G_n ($n = 3, 5$). For signal precision p within this interval, expert persuasion - despite its bias - enhances the probability of a correct collective decision relative to the benchmark of no persuasion.

To show this formally, we start by considering the case of $n = 3$ voters. The left-hand panel of Fig. 7 shows the graph of the function $G_3(p) = p + p^2 - 2p^3$. Note that $G_3'(1/2) = 1/2$, and that the first-order condition $G_3'(p) = -6p^2 + 2p + 1 = 0$ yields a unique critical point $p_3 = (1+\sqrt{7})/6 \approx 0.60763$. As $G_3''(p) < 0$ for all $p \in [1/2, 1]$, the function G_3 is strictly concave everywhere in this range, and so the point p_3 is the unique global maximum of G_3. The corresponding functional value is $G_3(p_3) = (7\sqrt{7}+10)/54$, which we label as q_3. Note that since $q_3 \in (1/2, 1)$, it follows immediately that in settings where $F(\omega_v) > q_3$, expert persuasion is detrimental to information aggregation. Now suppose instead that $1/2 < F(\omega_v) < q_3$. As G_3 is strictly concave and $G_3'(p) < 1$ for all $p \in [1/2, 1]$, the equation $p + p^2 - 2p^3 = F(\omega_v)$ has two real-valued solutions $p_3^{F,1}$ and $p_3^{F,2}$ with $F(\omega_v) < p_3^{F,1} < p_3^{F,2} < 1$. For all $p \in (p_3^{F,1}, p_3^{F,2})$ we have $G_3(p) > F(\omega_v)$, and for all $p \notin [p_3^{F,1}, p_3^{F,2}]$ we have $G_3(p) < F(\omega_v)$.

Next, consider the case of $n = 5$ voters. The right-hand panel of Fig. 7 shows the graph of the function $G_5(p) = p(1-p)^2(6p^2+3p+1)$. Note that $G_5'(1/2) = 1/8$, and that the first-order condition $G_5'(p) = (1-p)(1+3p+6p^2-30p^3) = 0$ yields a unique critical point $p_5 = (2 + (548 - 30\sqrt{290})^{1/3} + (548 + 30\sqrt{290})^{1/3})/30 \approx 0.51761$. Note that whilst it is tedious to compute the roots of the cubic function $G_5''(p) = 2((1+3p) - (2p^2(27-30p)))$ in order to ascertain the curvature of G_5, it is easy to verify that $G_5''(p) < 0$ for all $p \in [1/2, 3/5]$. To see this, note that both the functions $1 + 3p$ and $2p^2(27-30p)$ are strictly increasing for $p \in [1/2, 3/5)$, and that the maximum value of the former function (i.e. 2.8 when $p = 3/5$) is strictly lower than the minimum value of the latter function (i.e. 6 when $p = 1/2$). Therefore, G_5 is strictly concave for all $p \in [1/2, 3/5]$. From this, we can conclude that the point p_5 is the unique global maximum of G_5.

Rather than incur the tedium of verifying directly that the corresponding functional value $q_5 \equiv G_5(p_5)$ lies strictly in the range $(1/2, 1)$, we simply argue that for *any* n it holds that $G_n(p) < 1$ for all $p \in [1/2, 2/3)$. The condition $G_n(p) < 1$ is equivalent to $(1 - J_n(p)) < (1-p)/p$. Recall that $J_n(1/2) = 1/2$ and $J_n(1) = 1$. Thus, $(1 - J_n(p)) \in [0, 1/2]$ for all $p \in [1/2, 1]$.

Now note that $(1-p)/p$ strictly exceeds $1/2$ for all $p \in [1/2, 2/3)$, which immediately implies that $G_5(p_5) < 1$. We can therefore state that in settings where $F(\omega_v) > q_5$, expert persuasion is detrimental to information aggregation.

Finally, the fact that G_5 is strictly concave for all $p \in [1/2, 3/5]$, that $G_5'(p) < 1$ for all $p \in [1/2, 1]$, and that $G_5(3/5) = 1488/3125 \approx 0.47616 < 1/2$ implies immediately that in settings where $1/2 < F(\omega_v) < q_5$, the equation $G_5(p) = F(\omega_v)$ has two real-valued solutions $p_5^{F,1}$ and $p_5^{F,2}$ with $F(\omega_v) < p_5^{F,1} < p_5^{F,2} < 1$. For all $p \in (p_5^{F,1}, p_5^{F,2})$ we have $G_5(p) > F(\omega_v)$, and for all $p \notin [p_5^{F,1}, p_5^{F,2}]$ we have $G_5(p) < F(\omega_v)$. □

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