Corruption and Monetary Policy in a Cash-in-Advance Economy

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Abstract

In this paper, we build an endogenous growth model describing a cash-in-advance economy to reassess the macroeconomic consequences of corruption in terms of monetary policy in the long run. To this end, we consider two main configurations: a model with exogenous corruption and a model with endogenous corruption. From this analysis, we can extract four main results. First, contrary to Al-Marhubi (2000) and Blackburn et al. (2011), the relation between corruption and inflation is not always negative but is characterized by a U-shaped relation. Second, corruption increases the growth-maximizing seigniorage rate for a lower economic growth rate for both exogenous and endogenous corruption. Third, unlike Paldam (2002) and Braun and Di Tella (2004), we show a negative impact of seigniorage on endogenous corruption. Fourth, we demonstrate that corruption is a key determinant which positively affects the aggregate demand for money.

Keywords: corruption, endogenous growth, monetary policy, seigniorage, inflation

1. Introduction

Corruption is commonly defined as the misuse or the abuse of public office for private gain (Rose-Ackerman, 1997; Bardhan, 1997; Amundsen, 1999). In all countries, rich and poor, this phenomenon has become an important public issue. “No region, and hardly any country has been immune from corruption” (Glynn et al., 1997). This is why international organizations, such as the World Bank, the International Monetary Fund and the United Nations Development Programme (UNDP) attempt to determine the optimal policies to fight against corruption since the second half of the 20th century.
From an academic standpoint, the first works on the issue of corruption focused on the link between corruption and efficiency, corruption and growth and corruption and development\(^1\). The studies about the corruption-monetary policy nexus have been much less prolific. This is only in the late 1990s that has emerged a literature dealing with the monetary policy implications of corruption.

Generally, the relation between corruption and monetary policy has been studied in order to determine the optimal inflation rate, the optimal seigniorage rate or the impact of seigniorage and inflation on the degree of corruption. Most papers conclude that the relation between corruption and seigniorage (or inflation\(^2\)) is positive. Blackburn and Powell (2011) show in a theoretical model that corruption adversely affects growth through the channel of inflation. Indeed, since corruption depletes the government of resources, it turns towards seigniorage to finance productive public expenditures\(^3\). Consequently, corruption indirectly affects growth (negatively) through the channel of inflation. Nevertheless, in their model, corruption is not endogenously determined and they fail to show the complexity of the relation between seigniorage and growth. Similarly, Al-Marhubi (2000) reaches the same conclusions and find a strong positive relation between corruption and inflation. These results are confirmed by many works (Abded and Davoodi, 2000; Myles and Yousefi, 2015). Furthermore, the literature always highlights a positive impact of inflation on corruption. Braun and Di Tella (2004) show that the higher the inflation variability\(^4\), the higher the level of corruption in equilibrium. The explanation lies in the fact that inflation variability increases the cost of investment caused by corruption (their main hypothesis stipulates that inflation variability increases the cost of auditing the agent’s behavior because of information problems). This leads to reduce the equilibrium number of entrepreneurs being able to invest and then to lower investment and

\(^{1}\)There are two main conflicting opinions about the impact of corruption on efficiency, growth and development. The first one is the greasing the wheels hypothesis which stresses that corruption has a beneficial effect on the economic activity because it improves efficiency by allowing firms to circumvent administrative delays and by providing incentives for bureaucrats to work harder (Leff, 1964; Leys, 1965; Nye, 1967; Huntington, 1968; Lui, 1985). The second opinion is the sanding the wheels hypothesis. According to the proponents of this theory, corruption undermines economic growth directly or indirectly through different channels among which public investment, political stability, human capital and institutional quality are the most important (Tanzi and Davoodi, 1997; Mo, 2001; Martinez-Vasquez et al., 2005; Aidt, 2009).

\(^{2}\)The majority of papers does not distinguish between seigniorage and inflation.

\(^{3}\)As in Stockman (1981), inflation acts as a tax on investment in their model because of a cash-in-advance constraint.

\(^{4}\)In the paper of Braun and Di Tella (2004), inflation variability should be understood as synonymous with inflation.
growth. Paldam (2002) and Goel and Nelson (2010) confirm these findings from an empirical perspective.

Thus, the purpose of this paper is to reassess the effects of seigniorage on corruption as well as the macroeconomic consequences of corruption on seigniorage and inflation. Thereafter, we will be able to identify the impact of corruption on economic growth through the channel of monetary policy. We follow two steps to address these issues. First, we consider that corruption is exogenous. In this configuration, corruption is just a parameter which increases the disposable income of households and reduces the tax revenues for the government. Second, corruption is endogenously determined by the presence of corrupt bureaucrats within the tax administration. Bureaucrats seek to maximize their profit by selling “bribery services” and households purchase them to reduce the amount of taxes that they must pay to the fiscal authority. In both cases, corruption generates tax evasion which undermines the government’s capacity to make productive public expenditures. Therefore, corruption requires from the government to rethink the way it should fight against corruption to sustain economic growth. To do so, we examine the optimal orientation of monetary policy for a growth-oriented government.

Our findings are the following. First, we find a U-shaped relation between corruption and inflation. Unlike most previous works, we exhibit a non-linearity between corruption and inflation which can be explained by the fact that corruption exerts a positive effect on consumption by increasing the disposable income of households and a negative effect on the government budget constraint. Second, our model highlights a negative relation between corruption and seigniorage and an inverted-U relation between seigniorage and growth. Below a certain threshold, seigniorage is beneficial for the economic activity. Above this threshold, the impact of seigniorage on growth becomes negative. In addition, the growth-maximizing seigniorage rate is always higher in the case of an economy with corruption for a lower economic growth rate. This result holds whatever corruption is exogenous or endogenous. Third, contrary to all previous studies, we highlight a negative effect of seigniorage on corruption. This result is explained by the fact that corruption is subject to transaction costs. Finally, we show that corruption is a key determinant of the demand for money: the higher the level of corruption in the economy, the higher the aggregate demand for money.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the long run effects of exogenous corruption on inflation, seigniorage and growth. In section 4, we endogenize corruption to study the impact of seigniorage of corruption as well as the growth-maximizing seigniorage rate when corruption is endogenously determined. Section 5 proposes a extension of the cash-
in-advance constraint to a generalized transaction cost function in order to study how corruption affects the demand for money. Section 6 concludes.

2. The model

We develop an endogenous growth model in continuous time describing a closed economy populated with a private sector and monetary and fiscal authorities.

2.1. The private sector

We consider a representative household who maximizes the present discounted sum of instantaneous utility functions based on consumption \( (c_t > 0) \)

\[
U = \int_0^{+\infty} \exp(-\rho t) u(c_t) \, dt
\]  

where \( U \) denotes the expected intertemporal welfare and \( \rho \) the discount rate of the representative household. In order to generate an endogenous growth path in the long run, we assume a constant-elasticity of substitution utility function

\[
u(c_t) = \begin{cases} 
\frac{S}{S-1} \left( (c_t)^{\frac{s-1}{s}} - 1 \right) & \text{if } S \neq 1 \\
\log(c_t) & \text{if } S = 1 
\end{cases}
\]  

(2)

with \( S \) the intertemporal elasticity of substitution defined as: \( S = \frac{1}{\sigma} \) where \( \sigma \) corresponds to the risk aversion coefficient.

In addition, for \( U \) to be bounded, we have to ensure that \( (S-1)\gamma_c < S\rho \) where \( \gamma_x \) denotes the growth rate of the variable \( x \). This condition corresponds to a no-Ponzi game constraint where \( \gamma_c < r_t \) and \( r_t \) is the real interest rate to be defined below.

The production function depends on private capital \( k_t \) and productive public expenditures \( h_t \). All variables are per capita. For the sake of simplicity, population is normalized to unity.

\[
y_t \equiv f(k_t, h_t) = Ak_t^\alpha h_t^{1-\alpha}
\]  

(3)
where $A$ is a strictly positive scale parameter which denotes the total factor productivity and $\alpha$ is the elasticity of output with respect to private capital such as $\frac{1}{2} < \alpha \leq 1$. Such relation is similar to that introduced by Barro (1990) who considered productive public expenditures as flows in his production function assuming that they provide “productive services” with an elasticity of $1 - \alpha$ (where $0 < 1 - \alpha \leq \frac{1}{2}$). However, following Futagami et al. (1993), we assume productive public expenditures to be a stock variable. At equilibrium, $h_t$ is endogenously determined, $y_t$ has constant returns to scale and a balanced-growth path arises in the long run.

The disposable income of households is noted $y^d_t$. Contrary to Barro (1990) and Futagami et al. (1993), we assume that households strive to reduce a share of their income taxes by purchasing bribery services $\theta_t$ at price $p_t$ from corrupt bureaucrats on behalf of the fiscal authority. Accordingly, the households’ disposable income is

$$y^d_t = (1 - \tilde{\tau})f(k_t, h_t)$$

where $\tilde{\tau} \equiv [1 - \mathcal{F}(\theta_t)]\tau$ is the tax rate actually paid by the household ($\mathcal{F}(\theta_t)$ will be defined below) which is lower than the income tax rate $\tau$ fixed by the government ($\tilde{\tau} < \tau$).

To motivate a demand for real balances and study how corruption modulates monetary policy, we suppose that all transactions included consumption, investment, public spending and corruption are subject to the following cash-in-advance constraint

$$m_t = \phi^y (c_t + \dot{k}_t + \delta_k k_t + \dot{h}_t + \delta_h h_t) + \phi^\theta p_t \theta_t$$

where $\phi^y$ and $\phi^\theta$ are parameters reflecting the efficiency of the transaction technology.

To the best of our knowledge, there is no study which previously assumed corruption to be subject to a cash-in-advance constraint. This specification is, however, very interesting since it allows to study how monetary policy affects corruption when corruption is defined as an endogenous variable. In addition, this is a more realistic representation of households’ behavior who usually have incentives to purchase “corruption services” by using money in order to remain undetected.

Thus, the representative household’s budget constraint in real variables is (we define $\dot{x}_t$ as the first derivative of the variable $x_t$ with respect to time: $\dot{x}_t \equiv \frac{\partial x_t}{\partial t}$ $\forall x_t$)

\[5\] In this configuration, $p_t$ is a relative price and $y_t$ is the numeraire.
\[
\dot{k}_t + \dot{m}_t = y_t^d - c_t - \delta_k k_t - \pi_t m_t - p_t \theta_t + tr_t
\]  
(6)

where \(\delta_k\) corresponds to the capital depreciation rate such as \(0 \leq \delta_k \leq 1\), \(\pi_t\) represents the inflation rate and \(\pi_t m_t\) “the inflation tax”. Assuming the Fisher relation, \(\pi_t = R_t - r_t\) where \(R_t\) is the nominal interest rate.

Thus, the representative household uses his disposable income to consume \(c_t\), to invest (we define investment by \(z_t\) where \(z_t = \dot{k}_t + \delta_k k_t\)), to purchase bribery services and to hold money \(m_t\). Finally, households receive a lump-sum transfer \(tr_t\) to close the model and to satisfy the Walras’ Law.

2.2. Monetary and fiscal authorities

The monetary authorities set a nominal stock of high-powered money \(M_t\) which is assumed to be exogenous. Since we ignore the existence of the banking and financial sectors, high-powered money is the unique form of money. It grows at a rate \(\dot{M}_t = \omega\) where \(\omega\) is the seigniorage rate. Thereafter, the monetary authorities transfer the seigniorage revenues to the government which uses them to finance productive public expenditures.

The government determines the tax rate and collects inflation taxes (seigniorage). In addition, a portion \(\eta\) of the output is used to combat corruption and more generally to improve institutional quality if corruption is endogenous. In the latter case, the parameter \(\eta\) can be considered as a parameter of semi-productive expenditures for the government since it allows to prevent the phenomenon of corruption but may also generate higher fiscal deficits. If corruption is exogenous, the parameter \(\eta\) should be understood as a parameter of unproductive public expenditures. As in Futagami et al. (1993), the government accumulates productive public expenditures. Thus, the government budget constraint in real terms is given by the following relation

\[
\dot{h}_t + \delta_h h_t = \tau y_t + \omega m_t - \eta y_t
\]  
(7)

where \(\delta_h\) is the depreciation rate of public capital.

This expression constitutes an extension of the government budget constraint of Futagami et al. (1993). Futagami et al. (1993) considered only balanced-budget rules \((\dot{h}_t = \tau y_t)\). In our model, productive public expenditures can either be higher or lower than the amount of taxes collected by the government. This depends on the
degree of corruption, the amount of resources invested to combat corruption and the seigniorage revenues.

In short, the phenomenon of corruption can be addressed in three different ways in our model: by modulating the tax rate, implementing anti-corruption policies or issuing money.

3. The simple case of exogenous corruption

To model exogenous corruption, we follow Huang and Wei (2006), Hefeker (2010), Minea and Villieu (2010) and Dimakou (2015) and assume that households save a constant portion of the taxes that they must pay to the fiscal authority. In other words, \( \theta_t = \bar{\theta} \) and \( \tilde{\tau} = (1 - \bar{\theta})\tau \) in the case of exogenous corruption.

3.1. Equilibrium

The resolution of the model is provided in Appendix. It leads to the following two relations

\[
\gamma_c \equiv \frac{\dot{c}_t}{c_t} = S \left[ r_t - \rho - \frac{\phi^y \dot{R}_t}{1 + \phi^y \dot{R}_t} \right] \tag{8}
\]

\[
\frac{\phi^y \dot{R}_t}{1 + \phi^y \dot{R}_t} = r_t + \delta_k - \frac{(1 - \tilde{\tau})\alpha Ah^{1-\alpha}}{1 + \phi^y \dot{R}_t} \tag{9}
\]

Equation (8) corresponds to the usual Keynes-Ramsey rule which gives the optimal consumption path. In this relation, we can observe that the path of consumption is related to the path of the nominal interest rate provided in (9). This is explained by the presence of a cash-in-advance constraint on consumption goods (\( \phi^y > 0 \)). If \( \phi^y = 0 \), then \( \gamma_c = S(r_t - \rho) \). In addition, since the cash-in-advance constraint also affects investment, the real interest rate must be deflated by the financing cost \( (1 + \phi^y \dot{R}_t) \).

To find the endogenous growth equilibrium of the model, we should define intensive variables by deflating all growing variables by the stock of private capital \( (x_k \equiv x_t/k_t) \). Hence
\[
\frac{\dot{k}_t}{c_t} = S \left[ r_t - \rho - \frac{\phi_y \bar{R}_t}{1 + \phi_y \bar{R}_t} \right] - \gamma_k
\]  

(10)

The IS equilibrium gives the expression of the growth rate of capital

\[
\gamma_k \equiv \frac{\dot{k}_t}{k_t} = Ah^{1-\alpha} - c_k - \delta_k - \left( \frac{\dot{h}_t}{h_t} + \delta_h h_k \right)
\]  

(11)

We get the expression \(\frac{\dot{h}_t}{h_t} + \delta_h h_k\) from the government budget constraint

\[
\frac{\dot{h}_t}{h_t} + \delta_h h_k = (\bar{\tau} - \eta)Ah^{1-\alpha} + \omega m_k
\]  

(12)

In a configuration of perpetual growth, \(y_t\) grows at a rate \(\gamma\). The condition for which corruption does not disappear is to put the ratio of the value of corruption to GDP equal to a certain constant noted \(\upsilon\). Thus, for \(p_t \bar{\theta}\) to grow at a rate \(\gamma\), \(p_t\) should be defined as \(p_t \equiv \upsilon y_t\).

\[
m_k = \left( \phi^y + \phi^\theta \upsilon \bar{\theta} \right) Ah^{1-\alpha}
\]  

(13)

The money equilibrium is such that

\[
\frac{\dot{m}_k}{m_k} = \omega - \pi_t - \gamma_k = \omega + r_t - R_t - \gamma_k
\]  

(14)

Equations (13) and (14) provides the dynamics of productives public expenditures over time

\[
\frac{\dot{h}_k}{h_k} = \frac{1}{1 - \alpha} \left[ \omega + r_t - R_t - \gamma_k \right]
\]  

(15)

Finally, by using (15) and the government budget constraint, we get the real interest rate
The system composed by equations (8) - (16) fully characterizes the equilibrium of the model.

3.2. The steady state

In the steady state, $\dot{c}_k = \dot{h}_k = \dot{m}_k = R = 0$ and all growing variables grow at the same rate along the balanced growth path ($\gamma_k = \gamma_h = \gamma_c = \gamma_m = \gamma^*$).

Thus, the Keynes-Ramsey rule in the steady state becomes

$$\gamma^* = S[r^* - \rho]$$

(17)

In addition, the real interest rate in the long run is defined as the marginal productivity of capital

$$r^* = \frac{(1 - \tilde{\tau})\alpha Ah_k^{1-\alpha}}{1 + \phi^y R^*} - \delta_k$$

(18)

and the steady state nominal interest rate is a function of the real interest rate and the long run growth rate

$$R^* = \omega + r^* - \gamma^*$$

(19)

The combination of equation (17) and equation (19) provides the following implicit relation

$$R^* = \omega + \mathcal{K}(\gamma^*)$$

(20)

where $\mathcal{K}(\gamma^*) \equiv \rho - \gamma^* \left( \frac{S-1}{S} \right)$. We can notice that when $S = 1$, $\mathcal{K}(\gamma^*) = \rho$ and the expression of the steady state nominal interest rate becomes positive.

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6A star exponent denotes the steady-state solution.
Hence, we find a first implicit relation between the economic growth rate and productive public expenditures in the steady state

\[
h_k^*(\gamma) = \left\{ \frac{1}{A \alpha (1 - \tau)} \left( \frac{\gamma}{S} + \rho + \delta_k \right) \left[ 1 + \phi y \left( \omega + \rho - \gamma^* \left( \frac{S - 1}{S} \right) \right) \right] \right\}^{\frac{1}{1 - \alpha}} \tag{21}
\]

To obtain an another relation between \(h_k^*\) and \(\gamma\) in the long run, we use the government budget constraint. Hence, we can extract the following expression

\[
h_k^*(\gamma) = \left\{ \frac{A}{(\gamma + \delta_h)} \left[ \tilde{\tau} + \omega \left( \phi^y + \phi^\theta \nu \tilde{\theta} \right) - \eta \right] \right\}^{\frac{1}{\alpha}} \tag{22}
\]

Thus, the steady state equilibrium is obtained at the intersection of equation (21) and equation (22). Equation (21) is a positive function of \(\gamma\) while equation (22) is a negative function of \(\gamma\). Figure 1 provides an numerical illustration of the steady state.

\[\text{Figure 1: The steady state equilibrium of the model with exogenous corruption}\]

\[\text{\(S = \phi^y = \phi^\theta = 1.\) In addition, \(\rho = \delta_k = \delta_h = 0.05.\) As estimated by Aschauer (1989), we consider } \alpha = 0.6. \text{ For the sake of realism, we set } A = 0.75. \text{ Since corruption generates tax evasion, we consider that } \tau = 0.5 \text{ (} \tau > 1 - \alpha\). \text{ In addition, we fix } \theta = \nu = 0.5.\]
In this general case, there is a unique solution for the steady-state growth rate $\gamma^*$. If $S < 1$, there is only one solution for $\gamma^*$. For $S > 1$, there are two solutions $\gamma_1^*$ and $\gamma_2^*$ but the highest solution is always excluded because of the solvency condition (see Minea and Villieu, 2010).

However, in the remainder of the paper, we will just consider the simple case $S = \frac{1}{8}$. If $S = 1$, then $K(\gamma^*) = \rho$ and we obtain simplest relations to describe the steady state of the economy.

$$F(\gamma, \tilde{\theta}, \omega) = \left\{ \begin{array}{c} \frac{(\gamma + \rho + \delta_k)}{A\alpha(1-\tau)} [1 + \phi^y(\omega + \rho)] \right\}^{\frac{1}{1-\alpha}} - \left\{ \begin{array}{c} A \left[ \tilde{\tau} + \omega \left( \phi^y + \phi^\theta \nu \tilde{\theta} \right) - \eta \right] \gamma + \delta_h \right\}^{\frac{1}{\alpha}} = 0 \right. \tag{23}$$

3.3. Corruption and inflation

Since the seminal work of Al-Marhubi (2000), it is widely agreed in the empirical literature that corruption stimulates inflation (see Abed and Gupta, 2002 and Samimi et al., 2012 among others). However, this issue has been less addressed from a theoretical standpoint. Therefore, we aim to provide some new insights about the corruption-inflation nexus.

In our model, the inflation rate is defined as the difference between the seigniorage rate and the economic growth rate ($\pi^* = \omega - \gamma^*$). Accordingly, studying the impact of corruption on the inflation rate amounts to studying the impact of corruption on the economic growth rate.

**Proposition 1.** (The relation between corruption and inflation)

There is a U-shaped relation between corruption and inflation. At low levels, corruption reduces inflation. Conversely, high levels of corruption lead to an increase in the inflation rate.

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This assumption that we consider for a sake simplicity does not lead to a loss of generality or any qualitative change in the results.
Proof.

We determine the first order condition of the inflation rate with respect to corruption:

$$\frac{\partial \pi^*}{\partial \theta} = -\frac{\partial \gamma^*}{\partial \theta} = 0$$ (24)

From the Implicit Function Theorem, we know that

$$\frac{\partial \gamma^*}{\partial \theta} = \frac{\partial F(\gamma, \bar{\theta}, \omega)}{\partial \gamma} \frac{\partial \gamma}{\partial \gamma} + \frac{\partial F(\gamma, \bar{\theta}, \omega)}{\partial \bar{\theta}} \frac{\partial \bar{\theta}}{\partial \theta} = 0 \iff \frac{\partial \gamma^*}{\partial \theta} = -\left(\frac{\partial F(\gamma, \bar{\theta}, \omega)}{\partial \bar{\theta}}\right) / \left(\frac{\partial F(\gamma, \bar{\theta}, \omega)}{\partial \gamma}\right).$$

Hence, we can extract a threshold (noted $\hat{\theta}$) below (beyond) which corruption decreases (increases) the inflation rate. When $\bar{\theta} < \hat{\theta}$, then $\frac{\partial \pi^*}{\partial \theta} < 0$ and when $\bar{\theta} > \hat{\theta}$, then $\frac{\partial \pi^*}{\partial \theta} > 0$.

$$\hat{\theta} = \frac{\tau + \phi^b \omega - \eta}{\tau - \phi^b \nu \omega}$$ (25)

This proposition calls into question the majority view concerning the corruption-inflation nexus. While most existing works claim that corruption always increases the level of inflation, we show the existence of a threshold effect in this relation. The proponents of a positive effect of corruption on inflation advance two main reasons to justify this result. First, seigniorage revenues should be higher to compensate for the losses caused by corruption. Second, corruption increases public deficits (and sometimes unproductive public expenditures) which generates a crowding-out effect leading to inflationary pressures.

Despite these well-founded arguments, we should take into account another effect of corruption on economic growth. Indeed, the arguments mentioned above focus just on the effect of corruption on the government budget constraint. Nonetheless, although corruption forces the government to finance public expenditures by using another instrument (seigniorage), it also leads to an increase in aggregate consumption. The latter effect stimulates growth and then decreases inflation (since $\frac{\partial \pi^*}{\partial \theta} = -\frac{\partial \gamma^*}{\partial \theta}$).

3.4. The growth-maximizing seigniorage rate with exogenous corruption

The objective of this subsection is twofold. It consists to analyze how corruption affects the optimal seigniorage rate of a growth-oriented government as well as the
impact of this change in the optimal seigniorage rate on economic growth.

**Proposition 2.** *(Exogenous Corruption, seigniorage and growth)*  
(i) There is an inverted U-shaped curve between seigniorage and long run growth.  
(ii) Corruption increases the growth-maximizing seigniorage rate for a lower growth rate.

**Proof.**

Simulation-based proof

![Graph](image1.png)  
(a) Without corruption ($\tilde{\theta} = 0$)  
(b) With full corruption ($\tilde{\theta} = 1$)

Figure 2: Seigniorage rate and growth with exogenous corruption

At least two reasons can be given to justify this result. First, corruption undermines the quality of the tax collection system and leads to a flight of tax revenues. Consequently, the government has no choice but to resort to other instruments to finance productive public expenditures. Since seigniorage and tax income are substitutes in terms of government finance, the government may have incentives to generate seigniorage, so as to collect the inflation tax. Thus, by financing public spending using seigniorage, the government is protected from corruption in its conduct of public policy.

Second, corruption generates unproductive public expenditures for the government that must be financed. Effectively, in an economy without corruption, the
parameter $\eta$ would be equal to zero. Thus, corruption contributes to more substantial budget deficits through the anti-corruption policies. This increase in fiscal deficits generates a crowding-out effect and can give rise to inflationary pressures, especially in countries in which the level of financial development is low. Figure 2 provides an illustration of this proposition.

4. The model with endogenous corruption

4.1. Microfoundations of endogenous corruption: the role of bureaucrats

We now introduce a corruption sector in the economy. In this sector, households and bureaucrats are engaged in a “bribery market”. In other words, households purchase “bribery services” $\theta_t$ ($\theta_t$ is no longer a constant parameter but is varying over time) at a price $p_t$ from corrupt bureaucrats who produce them. We also suppose the corruption sector as a sector of monopolistic competition since each bureaucrat provides specific services to each household and should therefore be specialized. This specificity allows bureaucrats to extract monopoly rents from their “business”. Thus, assuming an isoelastic function, the production of bribery services is described by the following technology:

$$\kappa$$ is a strictly positive scale-parameter and $0 < \beta < 1$.

Consequently, corruption becomes a growing variable and the disposable income of households henceforth is:

$$y^d_t = [1 - (1 - F(\theta_t))\tau]f(k_t, h_t) \quad (26)$$

From the first order condition of the Hamiltonian with respect to corruption (the resolution of the model with endogenous corruption is provided in Appendix), we obtain an inverse demand function for bribery services.

$$p_t = \frac{F'(\theta_t)\tau y_t}{1 + \phi^\theta R_t} \quad (27)$$
Notice that the price of bribery services is a positive function of the income, the tax rate and the aggregate level of corruption (since $\mathcal{F}'(\theta_t)>0$) and a negative function of the interest rate.

Hence, we can determine the demand function of bribery services. To do so, we suppose that bureaucrats have incentives to pursue their production activity of bribery services if and only if the profit (noted $\Pi_t$) released is positive.

The supply of bribery services has a cost $CT_t$ for bureaucrats which we assume to be proportional to the intensity of the corruption activity. We also consider that this cost depends on a constant proportion of the revenue (noted $\eta$) invested by the government to fight against corruption. In other words, this cost corresponds to the risk related to corruption, or the probability of detection. Formally speaking, the “cost of corruption” is defined as $CT_t = \eta \theta_t$.

The latter relation makes intuitive sense: the higher the level of corruption, the higher the probability of detection. Thus, bureaucrats maximize their profit $\Pi_t$ subject to the inverse demand function determined in (27):

$$\begin{align*}
\max \quad & \Pi_t = p_t \theta_t - CT_t \\
\text{s.t.} \quad & p_t = \frac{\mathcal{F}'(\theta_t) \tau y_t}{1 + \phi^\theta R_t}
\end{align*}$$

Hence, the first order condition of the bureaucrats’ program allows us to determine the demand function of bribery services:

$$\theta_t = \left[\frac{\kappa \beta^2 \tau}{\eta (1 + \phi^\theta R_t)}\right]^{\frac{1}{1-\beta}} \tag{29}$$

With endogenous corruption, the monetary equilibrium becomes

$$m_k = \left[\phi^\theta + \phi^\theta \eta \xi (1 + \phi^\theta R_t)^{-\frac{1}{1-\beta}}\right] Ah_k^{1-\alpha} \tag{30}$$

where $\xi = \left(\frac{\kappa \beta^2 \tau}{\eta}\right)^{\frac{1}{1-\beta}}$.

By differentiating (30), we get

$$\frac{\dot{m}_k}{m_k} = (1-\alpha) \frac{\dot{h}_k}{h_k} - g(R) \dot{R} \tag{31}$$
Combining (31) with (14), we obtain the new dynamics of productive public expenditures

\[
\frac{\dot{h}_k}{h_k} = \frac{1}{1-\alpha} \left[ \omega + r_t - R_t - \frac{\dot{k}_t}{k_t} + g(R)\dot{R} \right]
\]  

(32)

where

\[
\gamma_k \equiv \frac{\dot{k}_t}{k_t} = Ah_k^{1-\alpha} - c_k - \delta_k - \left( \frac{\dot{h}_t}{h_k} + \delta_h h_k \right)
\]  

(33)

From the government budget constraint, we find

\[
\frac{\dot{h}_t}{k_t} + \delta_h k_t = f(R)Ah_k^{1-\alpha}
\]  

(34)

where \( f(R) \equiv \left[ \tilde{\tau} - \eta + \omega \phi^\eta + \omega \phi^\eta \frac{\eta}{\beta} (1 + \phi^\eta R_t)^{-\frac{1}{1-\eta}} \right] \) and \( \tilde{\tau} = \left[ 1 - \kappa \left( \frac{\kappa^2 \eta}{\eta(1+R_t)} \right)^{\frac{\beta}{1-\eta}} \right] \tau \).

Hence, we finally get

\[
\gamma_k \equiv \frac{\dot{k}_t}{k_t} = Ah_k^{1-\alpha} - f(R)Ah_k^{1-\alpha} - c_k - \delta_k
\]  

(35)

and the real interest rate when corruption is endogenous is

\[
r_t = \frac{(1-\alpha)[f(R)Ah_k^{1-\alpha} - \delta_h] + \alpha \gamma_k + R_t - \omega + \left( \frac{1+\phi^\eta R_t}{\phi^\eta} \right) \left[ (1-\tau)\alpha Ah_k^{1-\alpha} \right] - \delta_k} {1 + \left( \frac{1+\phi^\eta R_t}{\phi^\eta} \right) g(R)}
\]

(36)

4.2. The steady-state impact of seigniorage on corruption

Most works generally exhibit a strong positive impact of corruption on seigniorage as well as a strong positive impact of seigniorage on corruption. Gosh and Neanidis
(2010) have modelled corruption using three different ways. All of them conclude that corruption leads to an increase in seigniorage and a decrease in growth.

At steady-state, we know that $R^* = \rho + \omega$ when $S = 1$. Therefore, the demand function of bribery services in the long run is expressed as:

$$\theta^* = \left\{ \left[ \frac{\kappa \beta^2 \tau}{\eta [1 + \phi^\theta (\rho + \omega)]} \right] \right\}^{\frac{1}{1-\beta}}$$  \hspace{1cm} (37)

This expression shows that corruption depends on three policy parameters: the tax rate, the seigniorage rate and the percentage of GDP invested in anti-corruption policies. In the following proposition, we focus on the impact of one of them, namely the money-growth rate, on the level of corruption in the long run.

**Proposition 3.** (The impact of seigniorage on the aggregate level of corruption)

(i) Any increase in the seigniorage rate reduces the long-term level of corruption.

(ii) This reduction depends on the transaction cost related to corruption $\phi^\theta$.

**Proof.**

We can easily show that the first derivatives of $F(\theta_t)$ around its steady-state value with respect to both the money growth rate $\omega$ and the parameter describing the transaction cost related to corruption $\phi^\theta$ are negative:

$$\frac{\partial F(\theta_t)}{\partial \omega} \bigg|_{\theta^*} = -\left[ \frac{\kappa^2 \beta^3}{\eta (1 - \beta)} \right] \left[ \frac{\phi^\theta \tau (\theta^*)^{2\beta - 1}}{(1 + \phi^\theta (\rho + \omega))^2} \right] < 0$$  \hspace{1cm} (38)

9Three forms of corruption are modelled and studied. First, corruption decreases the tax revenues raised from households. Second, corruption inflates public expenditures. Third, corruption undermines the productivity of effective public expenditures. These three configurations lead to similar results.

10For more details about the impact of corruption on seigniorage in the literature, see also Paldam, 2002; Braun and Di Tella, 2004 and Goel and Nelson, 2010 among others.

11Notice that the steady-state level of corruption is also a positive function of the taxe rate and a negative function of the amount of ressources invested by the government to implement anti-corruption policies.
This proposition makes intuitive sense. Contrary to the prevailing opinion, we find a negative impact of seigniorage on the level of corruption in the long run. This negative effect is caused by the fact that corruption is subject to a transaction cost. This explains why previous works do not find such an impact. Even when they assume that households are subject to cash-in-advance constraints, corruption is not endogenous and does not enter into the cash-in-advance constraint. In other words, increasing the seigniorage rate can be considered as a mean to fight corruption. This is particularly true in the developing countries where the level of financial development is rather low (and then, the amount of seigniorage revenues retrieved by the government is high).\footnote{For the government to resort to seigniorage to combat corruption, the Central Bank should also be non-independent. This is generally the case in the developing countries in which corruption is largely widespread.}

We also remark that generating seigniorage to fight against corruption is more effective when $\phi^{\theta}$ is high, i.e. when the transaction technology related to corruption is efficient. When corruption is no longer subject to a transaction cost ($\phi^{\theta} = 0$), then...
the government can no longer combat corruption by using seigniorage.

4.3. The growth-maximizing seigniorage rate with endogenous corruption

When corruption is endogenously determined, both corruption and growth depend on the money growth rate. With endogenous corruption, the steady-state growth rate is got at the intersection of these following two relations.

\[
h^*_k(\gamma) = \left\{ \frac{1}{A \alpha (1 - \tau)} \left( \frac{\gamma}{S} + \rho + \delta_k \right) \left[ 1 + \phi^y \left( \omega + \rho - \gamma^* \left( \frac{S - 1}{S} \right) \right) \right] \right\}^{\frac{1}{1 - \alpha}} (40)
\]

\[
h^*_k(\gamma) = \left\{ \frac{A}{(\gamma + \delta_k)} \left[ \tilde{\tau} + \omega \left( \phi^y + \phi^y \xi \eta \beta \left( 1 + \phi^y \left( \omega + \rho - \gamma^* \left( \frac{S - 1}{S} \right) \right) \right)^{-\frac{1}{1 - \beta}} \right) - \eta \right] \right\}^{\frac{1}{1 - \beta}} (41)
\]

Figure 4: The steady state equilibrium of the model with endogenous corruption

Figure 4 illustrates the steady state equilibrium of the model with endogenous corruption.\(^{13}\)

\(^{13}\)We suppose that \(\kappa < A\) such as \(\kappa = 0.5\) and \(\beta = 0.7\)
Proposition 4. (Endogenous corruption, seigniorage and growth)

(i) Corruption is a channel causing non-superneutrality of money in the long run.

(ii) There is an inverted U-shaped curve between seigniorage and long run growth.

(iii) Corruption increases the growth-maximizing seigniorage rate for a lower growth rate.

Proof.

Simulation-based proof

Using numerical simulations, we reproduce the non-linear effect of seigniorage on growth.

![Figure 5: Seigniorage rate and growth with endogenous corruption](image)

(a) Without corruption ($\kappa = 0$)   
(b) With corruption ($\kappa = 0.5$)

The concept of superneutrality of money describes a situation in which the real variables (among which the economic growth rate) are not affected by the money-growth rate in the long run. Generally, investment (Stockman, 1981) and capital accumulation (Cooley and Hansen, 1989) are considered as the main channels causing non-superneutrality of money.

We reach a similar conclusion in our model. We can actually notice in (41) that the money growth rate actually affects economic growth ($\gamma^* \equiv \gamma(\omega)$). In other words, money is not supernerual in the long run. Specifically, there are three “effects” causing non-superneutrality of money in the steady-state in our framework. There is
a “Stockman effect” linked to fact that investment is subject to the cash-in-advance constraint, a “corruption effect” through the parameter $\phi^6$ and a “seigniorage effect” through the parameter $\phi^9$. The latter corresponds to the fact that seigniorage can be used to finance productive public expenditures.

Thus, the “corruption effect” deepen the idea of Stockman (1981) and Palivos and Yip (1995)\textsuperscript{14} by highlighting the fact that corruption is an another cause of non-superneutrality of money in the steady-state. Indeed, the money growth rate affects the level of corruption and the function describing the level of corruption affects the long run economic growth rate. Therefore, the money growth rate affects the long run growth rate through the channel of corruption.

As previously, there is a threshold effect in the relation between seigniorage and growth. The arguments mentioned above remain true when corruption is endogenous. Corruption leads to a need to resort to other policy instruments to face the losses caused by a flight of tax revenues and then generates inflationary pressures which increase the growth-maximizing seigniorage rate. Moreover, we could also advance an another explanation related to the fact that corruption causes a non-superneutrality of money. In this section, we have shown that seigniorage is an instrument which can be used to fight corruption and to improve the effectiveness of the fiscal administration. The higher the seigniorage rate, the lower the level of corruption in the economy and then the higher the taxes collected to finance productive public expenditures. In other words, increasing the seigniorage rate is a strategy leading to collect more tax revenues. Therefore, seigniorage allows to collect more tax revenues on the one hand and to reduce the need to finance government expenditures by using the inflation tax on the other hand.

5. Extension: the model with a generalized transaction cost function

In this section, we generalize the cash-in-advance constraint to a transaction cost function. Henceforth, we assume that households are no longer subject to a CIA constraint but to a transaction cost constraint. The relevance of this specification lies in the fact that it allows to derive a more realistic money demand function. This transaction cost function is defined over the same variables as in the CIA function (consumption, investment, government expenditures and corruption).

\textsuperscript{14}Both of them argue that investment is one of the main channels through which the money growth rate affects the growth rate in the long run.
\[ T_t = \frac{\zeta}{\mu} \left[ \phi^y (c_t + \dot{k}_t + \delta_k k_t + h_t + \delta_h h_t) + \phi^\theta p_t \theta_t \right]^{1+\mu} m^{-\mu}_t \] (42)

where \( \zeta \) is a strictly positive scale-parameter which ensures low transaction costs, \( \phi^y > 0 \) and \( \phi^\theta > 0 \) parameters highlighting the efficiency of the transaction technology, and \( \mu \) (such as \( \mu \geq -1 \)) a proxy for the elasticity of the real aggregate money demand with respect to the nominal interest rate \( R_t \) to be defined below. This specification of the transaction cost function is more general and more realistic than the usual cash-in-advance specification and allows to study different particular cases: the no-money case when \( \zeta = 0 \) and the CIA case when \( \mu \to \infty \).\(^{15}\)

**Proposition 5.** *(Corruption and money demand function)*

The money demand positively depends on income and the level of aggregate corruption and negatively on the interest rate.

**Proof.**

The first order condition of the hamiltonian with respect to \( m_t \) becomes as follows

\[
\frac{\dot{\lambda}_{1t}}{\lambda_{1t}} = \rho - r_t = \rho + \pi_t - \zeta \left( \frac{\phi^y y_t + \phi^\theta p_t \theta_t}{m_t} \right)^{1+\mu}
\] (43)

Hence, we can also extract from (43) the expression of the nominal interest rate (i.e. the marginal cost of money) and we observe that it is equal to its marginal return:

\[
R_t = \zeta \left( \frac{\phi^y y_t + \phi^\theta p_t \theta_t}{m_t} \right)^{1+\mu}
\] (44)

\(^{15}\)From (42), we can write: \( m_t = \left( \frac{\zeta \left[ \phi^y (c_t + \dot{k}_t + \delta_k k_t + h_t + \delta_h h_t) + \phi^\theta p_t \theta_t \right]^{1+\mu}}{\phi(\cdot)} \right)^\frac{1}{\mu} \). Hence, we can determine: \( \lim_{\mu \to +\infty} \left( \frac{1}{\mu} \right)^\frac{1}{\mu} = \lim_{\mu \to +\infty} \left[ \exp \left( \left( \frac{1}{\mu} \right) \log \left( \frac{1}{\mu} \right) \right) \right] = 1 \). Therefore, when \( \mu \to +\infty \), we have: \( m_t = \phi^y (c_t + \dot{k}_t + \delta_k k_t + h_t + \delta_h h_t) + \phi^\theta p_t \theta_t \).
The latter expression gives rise to a very interesting demand function for money:

\[ m_t = \varsigma^{\frac{1}{1+\rho}} (\phi^\rho y_t + \phi^\theta p_t \theta_t) R_t^{\frac{\rho}{1+\rho}} \]  

(45)

This expression constitutes an extension of the Baumol-Tobin model. As in the Baumol-Tobin model, the demand for money depends positively on income and negatively on the interest rate. Nevertheless, contrary to Baumol-Tobin, the originality of our model comes from the fact that the demand for money depends positively on the level of corruption. The intuition behind this result is the following. Households may have incentives to pay in cash when they enter in the “corruption market” in order to be undetected by the fiscal authority. As a matter of fact, cash is one of the less traceable means of payment. More generally, this is why we can consider that any increase in illegal practices would lead to increase the demand for money.

6. Conclusion

In this paper, we have built an endogenous growth model to provide some key insights for the governments which are experiencing the phenomenon of corruption within the bureaucratic administration. Corruption has been specified through two types of configurations: an exogenous corruption and an endogenous corruption. Specifically, we have focused on how corruption affects the orientation of monetary policy. To do so, we have assumed that households are subject to a cash-in-advance constraint on all transactions, included corruption. Then, we have extended our CIA constraint into a transaction cost function in order to identify a more realistic money demand function.

This analysis provides several interesting results. First, contrary to most previous works (Al-Marhubi, 2000; Abed and Davoodi, 2002; Smith-Hillman, 2007; Samimi et al., 2012; Myles and Yousefi, 2015), our model exhibits a U-shaped relation between inflation and corruption. Since corruption allows households to increase their disposable income and leads to decrease the tax revenues at the same time, two antagonistic effects come into conflict. When the first dimension dominates, corruption stimulates growth and then reduces inflation. When the second dimension dominates, the opposite effect occurs. Secondly, we show that corruption always increases the growth-maximizing seigniorage rate. This result is in line with a lot of theoretical and empirical studies explaining that governments should increase the seigniorage rate to compensate for the losses caused by corruption (see Al Marhubi,
However, unlike all previous works (Paldam, 2002; Braun and Di Tella, 2004; Goel and Nelson, 2010), we show that seigniorage negatively affects the level of corruption in the long run. This can be explained by the fact that corruption is subject to transaction costs in our model. Finally, we derive a realistic money demand function which is positively affected by corruption.

This paper can be extended in several different directions. First, the conclusions of our model requires more empirical investigations, in particular about the link between corruption and inflation. For instance, we could resort to nonlinear panel data models (like the PSTR or the PTR models) to examine the threshold effects between corruption and inflation. Secondly, it might be interesting to study the impact of corruption on fiscal policy within the framework of this model. This would allow to study the interactions between corruption, monetary and fiscal policies and growth, and then to derive an optimal policy mix in countries which are facing the phenomenon of corruption. Finally, we could study the interactions between corruption, financial development and monetary policies by introducing a parameter of financial development in the government budget constraint in order to analyze the relation between monetary policies and financial repression when corruption is widespread in the economy.
References


Appendix A: Resolution of the model with endogenous corruption

In equilibrium, the representative household maximizes his intertemporal utility function described in (1) subject to the constraints (2), (3), (4), (5) and (6). The initial value $k_0$ is given and the transversality condition is standard:

$$\lim_{t \to +\infty} \left( \exp \left( - \int_0^{+\infty} r_s ds \right) (k_t + m_t) \right) = 0$$  \hspace{1cm} (A.1)

There are two state variables: $m_t$ which represents the monetary wealth and $k_t$, using the definition of net investment: $\dot{k}_t = z_t - \delta k_t$. Thus, the current hamiltonian associated with the household’s maximization program is:

$$H_c = u(c_t) + \lambda_{1,t} [(1 - \tilde{\tau}) y_t - c_t - \pi_t m_t - z_t - p_t \theta_t + tr_t] + \lambda_{2,t} [z_t - \delta k_t] + \chi_t [m_t - \phi^y y_t - \phi^\theta p_t \theta_t]$$  \hspace{1cm} (A.2)

where $\lambda_{1,t}$ and $\lambda_{2,t}$ are the co-state variables respectively associated with $m_t$ and $k_t$ and $\chi_t$ is the co-state variable associated with the cash-in-advance constraint.

The first-order conditions of this maximization problem are

$$u'(c_t) = c_t^{-\frac{\gamma}{\gamma - 1}} = \lambda_{1,t} + \phi^y \chi_t = \lambda_{1,t} (1 + \phi^y R_t)$$  \hspace{1cm} (A.3)

$$\lambda_{2,t} = \lambda_{1,t} + \phi^y \chi_t \Rightarrow \frac{\lambda_{2,t}}{\lambda_{1,t}} = 1 + \phi^y R_t$$  \hspace{1cm} (A.4)

$$\phi^\theta p_t \theta_t = (\lambda_{1,t} + \phi^\theta \chi_t) p_t$$  \hspace{1cm} (A.5)

$$\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}} = \rho - r_t = \rho + \pi_t - \frac{\chi_t}{\lambda_{1,t}} = \rho + \pi_t - R_t$$  \hspace{1cm} (A.6)
The first order conditions can be easily interpreted. \(\lambda_{1,t}\) represents the shadow price (i.e. the opportunity cost) of financial wealth while \(\lambda_{2,t}\) corresponds to the shadow price of capital. The shadow price of financial wealth \(\lambda_{1,t}\) differs from the shadow price of capital \(\lambda_{2,t}\) because investment expenditures are subject to a transaction cost (\(\lambda_{1,t} = \lambda_{2,t}\) if \(\phi^y = 0\) or if \(m_t \neq m_\left(\hat{k}_t + \delta_k k_t\right)\)). Indeed, in our specification, capital cannot be acquired without money. This is why the opportunity cost of capital is higher than the opportunity cost of financial wealth. Moreover, the dynamics of the shadow prices of financial wealth and capital are given in (A.6) and (A.7), respectively.

**Appendix B: Local stability of the steady-state**

We present here the local stability of the steady state in the case of endogenous corruption. When corruption is exogenous, we reach the same conclusions. The reduced form of the model is given by (8), (9) and (32).

\[
\begin{align*}
\dot{c}_k &= S \left[ \frac{(1-\bar{\tau})\alpha A h_k^{1-\alpha}}{1+\phi^y R_t} - \delta_k - \rho \right] c_k - \gamma_k c_k \\
\dot{R}_t &= \left( \frac{(1+\phi^y R_t)}{\phi^y} \right) \left[ r_t + \delta_k - \frac{(1-\bar{\tau})\alpha A h_k^{1-\alpha}}{1+\phi^y R_t} \right] \\
\dot{h}_k &= \left( \frac{1}{1-\alpha} \right) \left[ \omega + r_t - R_t - \gamma_k + g(R) \dot{R} \right] h_k
\end{align*}
\]

where

\[
\gamma_k \equiv \frac{\dot{k}_t}{k_t} = Ah_k^{1-\alpha} - f(R) A h_k^{1-\alpha} - c_k - \delta_k
\]

The linearization of the model in the neighborhood of the steady state can be expressed by
\[
\begin{pmatrix}
\dot{c}_k \\
\dot{R} \\
\dot{h}_k
\end{pmatrix} =
J
\begin{pmatrix}
c_k - c^*_k \\
R - R^* \\
h_k - h^*_k
\end{pmatrix}
\] (B.3)

where \( J \) is the Jacobian matrix defined as
\[
J =
\begin{pmatrix}
c^*_k & CR & CH \\
RC & RR & RH \\
HC & HR & HH
\end{pmatrix}
\] (B.4)

First, let us define the first derivatives of the real interest rate with respect to \( c_k, h_k \) and \( R \).

\[
r'(c^*_k) = -\frac{\alpha}{1 + \left(1 + \phi y R^* \right) g(R^*)}
\] (B.5)

\[
r'(h^*_k) = \frac{-\alpha(1 - \alpha) f(R^*) \frac{y_k^*}{h^*_k} + \alpha \gamma'(h^*_k) + \left(\frac{1 + \phi y R^*}{\phi y} \right) \left(\frac{1 - \tilde{\tau}(1 - \alpha) \alpha y_k^*}{1 + \phi y R^* h^*_k} \right) g(R^*)}{1 + \left(\frac{1 + \phi y R^*}{\phi y} \right) g(R^*)}
\] (B.6)

\[
r'(R^*) = \frac{u'(R^*)v(R^*) - u(R^*)v'(R^*)}{\left[1 + \left(\frac{1 + \phi y R^*}{\phi y} \right) g(R^*) \right]^2 g(R^*)^2}
\] (B.7)

with

\[
u(R^*) = \left[1 + \left(\frac{1 + \phi y R^*}{\phi y} \right) \right] r^*
\] (B.8)

\[
w'(R^*) = 1 + (1 - \alpha) f'(R^*) \frac{y_k^*}{h^*_k} + \alpha \gamma'_k(R^*) + \left[ g(R^*) + \left(\frac{1 + \phi y R^*}{\phi y} \right) g'(R^*) \right] r^* - \left(\frac{1 + \phi y R^*}{\phi y} \right) \left[\frac{R'(R^*)(1 + \phi y R^*) + \phi y'(1 - \tilde{\tau}(R^*))}{(1 + \phi y R^*)^2} \right] u y g(R^*)
\] (B.9)
\[ v(R^*) = 1 + \left( \frac{1 + \phi y R^*}{\phi y} \right) g(R^*) \]  
(B.10)

\[ v'(R^*) = g(R^*) + \left( \frac{1 + \phi y R^*}{\phi y} \right) g'(R^*) \]  
(B.11)

In addition, to keep the expressions in the Jacobian matrix simple, we define the following relations

\[ \tilde{\tau}(R^*) = \left[ 1 - \kappa \left( \frac{\kappa \beta^2 \tau}{\eta(1 + \phi^y R^*)} \right)^{\frac{\beta}{1 - \beta}} \right] \tau \]  
(B.12)

\[ \tilde{\tau}'(R^*) = \frac{\beta^3 \kappa^2 \tau^2 \phi^y}{(1 - \beta) \eta(1 + \phi^y R^*)^2} \left( \frac{\kappa \beta^2 \tau}{\eta(1 + \phi^y R^*)} \right)^{-\frac{\beta}{1 - \beta}} \]  
(B.13)

\[ \gamma'_k(h^*_k) = (1 - \alpha)(1 - f(R^*)) \frac{y_k^*}{h^*_k} \]  
(B.14)

\[ \gamma'_k(R^*) = -f'(R^*) y_k^* \]  
(B.15)

\[ f'(R^*) = \tilde{\tau}'(R^*) - \frac{\omega \phi \beta^2 \eta \xi}{\beta(1 - \beta)} (1 + \phi^y R^*)^{\frac{\beta - 2}{1 - \beta}} \]  
(B.16)

\[ g'(R^*) = -\frac{\phi^{y2} \eta \xi}{\beta(1 - \beta)} \frac{\beta \phi^y}{\beta - 1} \left[ \phi^y (\beta - 1) \eta \xi + \phi^y (\beta - 2) (1 + \phi^y R^*)^{\frac{1}{1 - \beta}} \right] \]  
(B.17)

Hence, we can easily determine the elements of the Jacobian matrix
All our numerical simulations lead to the same conclusion. There are one negative eigenvalue and two positive eigenvalues. Since the reduced form contains one predetermined variable ($h_k^*$) and two jump variables ($c_k$ and $R$), the Blanchard-Kahn conditions are fulfilled and the unique BGP is saddle-path stable.