Family Background, After-school Tutoring, and Student Achievement: Theory and Experimental Evidence from Rural China[§]

Jere R. Behrman[a], C. Simon Fan[b], Xiangdong Wei[b],
Hongliang Zhang[c], and Junsen Zhang[d]

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[a] Department of Economics, University of Pennsylvania, McNeil 160, 3718 Locust Walk, Philadelphia, PA 19104-6297, United States

[b] Department of Economics, Lingnan University, Tuen Mun, Hong Kong

[c] Department of Economics, Hong Kong Baptist University, Kowloon Tong, Hong Kong

[d] Department of Economics, The Chinese University of Hong Kong, Shatin, Hong Kong

Abstract

The paramount importance of after-school tutoring has led some sociologists and educators to refer to it as the "shadow educational system," which exists alongside the formal educational system with increasing relative size, particularly in some East Asian countries. A number of empirical studies in the literature of economics and sociology examine the impact of after-school tutoring. However, most of these studies use non-experimental data, which often leads to problems of interpretation due to the issue of endogeneity. This paper conducts a theoretical and empirical investigation into the effect of after-school tutoring on the academic performances of some Chinese primary school students. Based on the data collected from a randomized controlled trial, our estimates show significant positive effect of after-school tutoring on the test score in mathematics. Moreover, this effect is significantly stronger for the children whose both parents are away from home, a phenomenon that is common in rural China.

1. Introduction

The substantial research on economics has led to a consensus that family is a pivotal determinant of children's educational attainment. In an influential survey article, Hanushek (1996, p.16) provokingly states that "schools seemed relatively unimportant in determining student achievement, while families were the key element of student success." While the first half of this statement appears to be an exaggeration, it does highlight the crucial role of family in shaping children's human capital.

What are the exact channels that family affects children's education? One may argue that better-educated parents may naturally and passively provide a better home environment for their children, which facilitates human capital accumulation. But it is only one aspect of the intergenerational transmission of human capital. Many parents do take active measures trying to enhance their children's educational attainment.

First, many parents spend much time and effort in tutoring and supervising their children in their study.[1] Second, private tutors are hired to provide individual help to the children. Each of these two types has its own advantage, and both serve to cater to the unique needs of individual students. On one hand, parents are more concerned about their children's achievement and welfare than private tutors. Hence, parents tend to be most motivated teachers of their own children. On the other hand, the opportunity cost of parents is often much higher than that of private tutors, who are often full-time students, senior than their tutees and often having much free time.

The global private after-school tutoring market has been booming, with East Asian countries leading the way. For example, in South Korea, total household expenditure on private tutoring is estimated to be 2.79% of GDP in 2006 (Nam, 2007), namely about 80% of government expenditure on public education for primary and secondary education (Kim and Lee, 2010). Also, according to the survey of PISA (Programme for International Student Assessment) in 2009, about half of the students used private after-school tutoring after school in a dozen western countries. The paramount importance of private tutoring has led some sociologists and educators to refer to it as the "shadow educational system," which exists alongside the formal educational system with increasing relative size.[2]

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[1] For example, see Spain and Bianchi (1996), Houtenville and Conway (2008), Bianchi and Milkie (2010), and Ramey and Ramey (2010).
[2] For example, see Stevenson and Baker (1992) and Bray (1999).

A number of empirical studies in the literature of economics and sociology examine the impact of private after-school tutoring, and generally find a positive effect on educational attainments. However, most of these studies use non-experimental data, which often leads to problems of interpretation due to the issue of endogeneity. A notable exception is Cook et al (2014), who use data from a randomized controlled trial of a two-pronged intervention and shows that the policy intervention of teaching disadvantaged youth some social-cognitive skills increased their test scores in mathematics.[3]

This paper conducts a theoretical and empirical investigation of the impacts of after-school tutoring, based on the data collected from a randomized experiment in a county of rural China. Moreover, by examining the "left-behind" children phenomenon caused by massive rural-to-urban migration in the county, it exploits the interactive effects between parental absence and the policy intervention of remedying education in children's school outcomes.

We first present a simple model that theoretically examines the impacts of after-class tutoring. Holding school quality constant, we consider that a child's educational outcome is influenced by three main factors: the tutoring she receives from her parents/guardian, the outside after-school tutoring she receives, and the time of her self-study. In this setup, while the outside after-school tutoring itself benefits the child, it is at a cost of reducing the child's time of self-study and independent thinking. Moreover, with an increase in the outside after-school tutoring, the parents/guardian may reduce their own tutoring and enjoy more leisure. Thus, the model implies that an increase in the outside tutoring will increase the child's human capital only if outside tutoring is effective and a child's self-study is relatively less effective in affecting the child's human capital formation. Furthermore, the model shows that the policy intervention of providing free outside tutoring will have a greater impact on the children from disadvantaged families (e.g. left-behind children) if the outside tutoring is close substitute to the tutoring of parents/guardian.

We then proceed to the empirical analysis of the impact of after-class tutoring, which is the core of this paper. The data used in this paper were collected by the authors from Longhui County in Hunan Province of China. The county was selected for this study to represent one of the country's poorest rural areas with a high prevalence of parental absence:

[3] Other related literature is surveyed in the next section.

per capita GDP is less than a quarter of the national average[4] and over two-thirds of children are left-behind by one or both parents. Working with the county's educational bureau, we conducted a randomized controlled experiment about after-school tutoring. In this experiment, we recruited high-achieving students in grades 4 and 5 to offer one-to-one tutoring to low-achieving students in grades 2 and 3 in 36 primary schools in Longhui County. In the sociology and educational psychology literature, one-to-one tutoring is often regarded as the most effective method of instruction (e.g. Bloom, 1984, Schofield & Katrina, 2005) The details of the experiment are described in Section 4.

Our estimates show significant positive effect of after-school tutoring on the test score in mathematics. It also demonstrates that the intervention has no significant impact on the test score in Chinese. These findings are consistent with our theoretical predications. Through the patient help of her tutor, a student (particularly a "slow" student) may learn a number of efficient ways of solving mathematical questions, which she did not fully understand in the lectures of the school teachers. Thus, in mathematics, the tutoring from senior students substantially enhanced their abilities in this subject. In contrast, the study of Chinese, particularly in Grades 2 and 3, depends mainly on memorization and simple repeated practices. Thus, in this subject, tutors' help may matter little.

Moreover, our empirical analysis reveals that the treatment effect of the tutoring program is significantly larger for those students with both parents absent. It thus suggests an important role of family background as measured by parental absence status in affecting the effectiveness of the remedying educational program in the form of after-school tutoring.

In this experiment, the tutors are high-achieving students in grades 4 and 5, who are 2 to 3 years older than their tutees. We believe that such tutors are highly suitable in providing detailed mentoring to grades 2 and 3 primary schools students. However, if they are not the most efficient tutors, then our empirical study provides an estimation of the low bound of the effect of after-school tutoring.

Finally, we examine the impacts of the intervention on the test scores of tutors. In theory, the effect is ambiguous. On one hand, teaching junior students may reduce the tutors' time for self-study. On the other hand, the practice of teaching may lead to more independent thinking and a deeper understanding of basic knowledge. Our estimates indicate that there is

[4] In 2010, the county's per capita GDP was RMB 6,992, less than a quarter of the national average of RMB 29,748.

no significant impact on the tutors of this program, which implies that these two effects likely cancelled out each other.

The remainder of this paper is organized as follows. Section 2 provides a further review of the related literature. Section 3 presents a simple model, which analyzes the theoretical implications of after-school tutoring and guides our empirical analysis. Section 4 describes our randomized trials and the data set it generates. Section 5 introduces our empirical framework, and presents our main estimation results. Section 6 provides some concluding remarks.

2. Related Literature

There are many previous studies of peer or cross-age tutoring programs carried out by researchers in education and educational psychology disciplinary. Cohen, Kulik & Kulik (1982), Rohrbeck et al. (2003) and Robinson, Schofield & Katrina (2005) provide good reviews of the early studies on this topic. In general, many studies have found that tutoring programs can improve tutees' academic achievements (87% of the studies summarized by Cohen, Kulik & Kulik 1982). Studies also found that there are academic gains for tutors as well as for tutees (e.g. Topping et al. 2004). Tutoring programs also appear to have other positive effects on tutees and tutors, such as classroom behavior, attitudes about school, self-esteem and academic efficacy (see Fantuzzo, King & Heller 1992 and Roswal et al. 1995). However, most of the past studies either used non-experimental design or had very small sample size. The results are therefore prone to sample selection and low statistical power problems.

In a recent survey and meta-analysis, Shenderovich, Thurston & Miller (2015) use the following criteria for inclusion of papers in their study: (1) use of randomized controlled trials (RCTs) design; (2) more than one classroom per treatment; (3) reliable measure of academic outcomes and (4) intervention length at least 12 weeks. This has significantly reduced qualified studies from the original sample of 10,727 to only 15. Their meta-analysis found only small significant effects on tutees' composite measure of reading and insignificant effects on mathematics. The latter results may be a result of small sample as only two studies selected by them involved tutoring in mathematics. Moreover, in their survey, 7 out of 15 studies have the sample size of less than 100 tutees, and only 3 out of 15 studies cover more than 500 tutees. Hence, despite the existence of large number of studies in the literature, there are only very few of them using RCTs design that have achieved large enough sample size.

So far, only few related studies have been carried out by economists using RCTs. Banerjee et al (2007) studied the impact of a remedial program in India that hired young women as additional teachers to teach 3rd and 4th graders identified as lagging behind their peers in basic literacy and numeracy skills during school hours (2 hours per school day). Students in this program were tutored in a group of approximately 15-20 and the program lasted for two years. They found the tutees' math (language) test scores were increased by 0.18 σ (standard deviation) (0.08 σ) in the first year, and 0.35 σ (0.19 σ) in the second year, respectively. Cabezas, Cuesta and Gallego (2011) implemented a 3-month program of small group tutoring (a total of fifteen 90-minute sessions) to fourth graders in Chile using college student volunteers as tutors. The only significant gains were found in language scores in the lowest performing schools in one of the two regions and the gains were between 0.15 and 0.20 σ. The program also improved students' attitudes towards reading with respect to self-perception as reader, enjoyableness for reading and perception of reading at school. Cook et al. (2014) evaluated an intervention program that provided 106 disadvantaged male 9th and 10th graders in Chicago who are lagging behind in both academic and non-academic achievements with both a behavioral therapy and remedial academic tutoring on math for one academic year (9 months). The 27 one-hour behavioral therapy sessions took place once a week with an average youth-to-adult ratio of 8:1, whereas the one-hour math tutorials were conducted every day during the school day with a student-to-adult ratio of 2:1. Participants increased math test scores by 0.65 σ.

The only related study carried out in China is by Li et al (2014). They looked at a combined program of group cash incentive and peer tutoring that involves pairing high and low achieving classmates as benchmates. They found that the program improved low achievers' test scores by 0.265 σ without hurting the high achievers' scores. On the other hand, it had no effect by providing cash incentive alone to the low achievers. However, the study cannot separate the peer effect from incentive effect. It is also not clear whether the exact effect is due to peer tutoring as there is no structured tutoring program.

So for all the economic studies mentioned above none of them involve cross-age tutoring using older students serving as tutors for younger students in the same school. The latter is more cost-effective than programs involving outside adult tutors and can be easily replicated by many schools. Furthermore, no existing studies have investigated the potential substitution effects of school run tutoring programs from tutoring at home by family members. Recent studies by Houtenville and Conway (2008) and Das et al. (2013) show that there can be significant substitution effects between school inputs and family inputs and hence without taking into account this potential substitution the estimated impacts of increased school input program can be biased downwards.

3. Theoretical Background and Hypotheses

This section discusses the theoretical background of the study and provides a simple heuristic model that derives our hypotheses for empirical analysis.

We consider a household that consists of a child and a parent/guardian. For simplicity, we assume that the parent/guardian is the sole decision maker of a household. A child's human capital formation function is defined as follows:

$$q = F(h,s) \qquad (1)$$

Where q is the child's educational attainment, h is the tutoring that the child receives after class, s is the amount of time that the child's self-study after class.[5] Tutoring can be conducted by potentially two parties: (1) the parents or the guardian of the child; (2) an outside tutoring program.

The outside tutoring program is free of charge.[6] But it is offered to the school children as a lottery, and hence it may differ across children. We denote the amount of tutoring program that a child receives by λ, which is treated as a parameter by a household. Although a household does not pay for the outside tutoring program, the child needs to spend time on it. The bigger the program, the more time is needed. We choose the measurement unit in such a way that the amount of time attending the program is exactly λ.

Parents/guardian can also provide tutoring to the child. Its effectiveness depends on the human capital and motivation of the parents/guardian. Thus, we define the amount of tutoring provided by the parents/guardian as follows:

$$p = \pi t \qquad (2)$$

where p is the amount of tutoring provided by the parents/guardian; t is the amount of time provided by the parents/guardian, π is an indicator of the education and the motivation of the

[5] We assume that school quality is exogenous in the model, and hence is not included explicitly in (1).
[6] This assumption is in line with the experimental design of the paper, which is described in Section 4.

parents/guardian in tutoring the child. In this type of tutoring, the child also needs to spend "*t*" amount of time.[7] Then, we define the tutoring that a child receives is as follows:

$$h = H(\lambda, \pi t) \qquad (3)$$

Suppose that the child is endowed with one unit of time for study after class, which can be allocated to three aspects: (1) self-study, (2) being tutored by an outside tutor, and (3) by the parent/guardian. Then, the child's time constraint is as follows:

$$s = 1 - \lambda - t \qquad (4)$$

Inserting (3) and (4) into (1), we get

$$q = F[H(\lambda, \pi t), 1 - \lambda - t] \qquad (5)$$

Next, we consider that parents/guardians get a disutility from teaching the child, and the disutility function is

$$-v(t, \pi) \qquad (6)$$

We assume that $v(t,\pi)$ may increase with π as well as t, and hence this specification takes into account that those guardians who are more productive tutoring the child may also have a greater opportunity cost of tutoring. For example, parents are usually much better tutors than grandparents in rural China, but parents tend to be much busier at work than grandparents, namely parents get a higher disutility from tutoring children than grandparents.

Finally, *u(), v(), F(,)* and *H(,)* are all increasing with respect to its variables, and all satisfy the neoclassical properties. (The mathematical expressions are stated in details in the Appendix.)

In sum of the above, we can write the utility function of the parents/guardian as follows:

$$F[H(\lambda, \pi t), 1 - \lambda - t] - v(t, \pi) \qquad (7)$$

The parents/guardian, being the sole decision maker of the family, aims to maximize (7) by choosing an optimal "*t*".

First, we consider the case of guardian tutoring given any level of outside tutoring, which is characterized by the following claim.

[7] Alternatively, we may assume that the time that the child needs to spend is proportional to the time of parental tutoring. But the result will be materially the same.

Claim 1. (i) $t = 0$ *if π is sufficiently small.*

(ii) *When* $t > 0$ *, then the sign of* $\frac{dt}{d\pi}$ *is ambiguous.*

Proof: See Appendix.

Two comments are in order as for Claim 1. First, a large fraction of the children in the dataset employed in our empirical study are left-behind children. Their parents were away from home, and their guardians are usually their little-educated grandparents, who often do not have enough academic qualifications or incentives to tutor them. In relation to the model, π is very small for the guardians of a large fraction of left behind children, and hence the tutoring from their guardians (usually grandparents) is little.

Of course, parents can choose also choose $t = 0$ even if their "π" is sufficiently high. From the proof in the Appendix, we can see that parents will choose $t = 0$ if their opportunity cost of time is very high so that $v_1(t, \pi)$ is very large. However, such extremely busy parents are rare (if they stay in rural China with their children). Thus, we expect that the proportion of left behind children who receive little tutoring at home is much greater than the proportion of the other children who receive little tutoring at home.

Second, when $t > 0$, the familiar "income effect" and "substitution effect" are at work in response to an increase in the efficiency of parental/guardian tutoring. On one hand, the "substitution effect" induces abler parents/guardians to spend more time tutoring the child. On the other hand, the "income effect" (the "income" here is the children's human capital) induces less able parent/guardian to spend more time tutoring the child to make up his lesser ability in teaching the child. Besides, less able guardian may have a smaller opportunity cost of engaging in tutoring. Thus, these opposite effects result in that sign of $\frac{dt}{d\pi}$ is ambiguous.

The next claim analyzes the impacts of outside tutoring on parents/guardians' time of tutoring children.

Claim 2. (i) *In the case that* $t = 0$*,* $\frac{dt}{d\lambda} = 0$.

(ii) *In the case that* $t > 0$*, we have* $\frac{dt}{d\lambda} < 0$ *if and only if*

$$\pi F_1 H_{12} < (H_1 + \pi H_2) F_{12} - \pi F_{11} H_1 H_2 - F_{22} \quad (8)$$

Proof: See Appendix.

First, when the tutoring from guardians, such as grandparents, is little in the first place, it can reduce any further in response to outside tutoring. Second, (8) will be satisfied if H_{12} is small so that in magnitude, $F_1 H_{12}$ is less than the sum of the other three items of the right-hand side of (8), which are all positive. In our experimental study, the dosage of tutoring is helping students to do their homework, which is similar to that of parental tutoring. In other words, the contents of outside tutoring are similar to those of parental tutoring, suggesting that the two types of tutoring are substitutes and hence H_{12} should be small. Thus, we hypothesize that for the non-left-behind-children, their parental tutoring will decrease with the implementation of our program.

Part (ii) of Claim 1 implies that it may substitute parental tutoring. Usually parents are much more concerned about their children's achievement than private tutors, which implies that parents would often tutor their children most efficiently. However, many parents have long hours of working every week, and hence their opportunity cost of tutoring their children is very high. The working hours in many East Asian countries are particularly long,[8] which may be an important reason for why private tutoring is a large market in those countries. Therefore, the availability of others' tutoring of their children will induce them to spend less time tutoring the children.

In relation to Claims 1 and 2, we hypothesize that the proportion of grandparents or other guardians of left behind children who provide little tutoring is significantly higher than the proportion of parents who provide little tutoring (when they are with their children). Thus, we have the following hypothesis.

Hypothesis 1: (1) For left-behind-children, their guardian tutoring are more likely to responds little to outside tutoring. (2) For non-left-behind-children, their parental tutoring tends to decrease when there is outside tutoring.

The major negative impact of outside after-school tutoring is the reduced time of self-study. The primary schools in China, including rural China, usually have long hours of teaching in both mornings and afternoons on weekdays. Under such a scenario, adding one

[8] For example, in Hong Kong, the median hours of work in recent years are about 50 hours per week, and many people work overtime (e.g. Fan, 2007).

more hour's tutoring may substantially increase the study load of the children. Thus, children's self-study time may be significantly reduced if they participate in the program of after-school tutoring. Moreover, outside tutoring may substitute parental tutoring. In sum, there are both positive and negative impacts of after-school tutoring on children's education. Under what conditions will the net effect be positive? The answer to this question is provided in the following claim.

Claim 3. (i) If $t = 0$, then $\frac{dq}{d\lambda} > 0$ if and only if

$$F_1 H_1 > F_2 \tag{9}$$

(ii) If $t > 0$, then $\frac{dq}{d\lambda} > 0$ if and only if

$$F_1 H_1 - F_2 > (-\frac{dt}{d\lambda})(\pi F_1 H_2 - F_2) \tag{10}$$

Proof: See Appendix.

Note that $F_1 H_1$ can be interpreted as the effectiveness of outside tutoring, and F_2 can be interpreted as the effectiveness of self-study. Thus, Condition (9) will be satisfied if and only if outside tutoring is relatively effective and a child's self-study is relatively less effective.

Also, πH_2 is clearly an indicator of the effectiveness of parental/guardian tutoring. Thus, Condition (10) will be satisfied with the additional assumption that parental/guardian tutoring is relatively ineffective and a child's self-study is relatively less effective. Thus, in sum, more outside tutoring will increase the child's human capital only if outside tutoring is relatively effective and a child's self-study is relatively less effective.

However, that if outside tutoring is relatively ineffective relative to a child's self-study, Conditions (9) and (10) will not be satisfied. In this case, $\frac{dq}{d\lambda}$ may be small in magnitude and we may have $\frac{dq}{d\lambda} \leq 0$.

A necessary condition for an after-school tutoring program to be beneficial for children's education is that its effect is greater than that of children's self-study. This condition is not always satisfied. For example, if the tutor mostly repeats what the teachers have taught in regular classes, the marginal benefit of after-school tutoring will be low, and may not compensate for the children's lost time for self-study and independent thinking.

Some educators put forward the concept, "active learning," which challenges the traditional educational system in which students sit in class listening to teachers and highlights the importance of students' independent thinking (e.g. Bonwell and Eison, 1991). Thus, particularly in many East Asian countries in which there is a heavy regular curriculum, an after-school tutoring program needs to be carefully designed to make it effective. Moreover, if the children already have long hours of regular classes, they will sacrifice much of the time of their self-study in attending the after-school tutoring program, which may lead to an ambiguous effect on their educational outcomes.

In relation to our experimental study, the value of outside tutoring may differ across subjects. For example, in mathematics, there are many detailed "tricks" in solving questions, which can be best leant through individual tutoring. In Chinese, on the other hand, a student can learn more simply spending more effort memorizing the newly learned words. While study skills are important for all subjects, it may be more important in mathematics than Chinese. In relation to our model, the relative effectiveness between outside tutoring and self-study may differ across subjects. Thus, we have the following hypothesis.

Hypothesis 2: A student's attending a program of after-school tutoring on her academic achievement may differ across subjects. The program is more beneficial in the subjects that emphasize more on the skills of solving problems.

Recall that left behind children are more likely to receive little tutoring at home. Thus, the effect of an after-school tutoring program on children's educational attainment depends on their family background, which is characterized by the following claim.

Claim 4. *If Condition (8) holds,* $\frac{dq}{d\lambda}$ *is larger in the case when* $t = 0$ *than the case when* $t > 0$.

Proof: See Appendix.

The intuition of this claim is straightforward. A child may benefit from individual tutoring, which is tailored to her/his individual need. Such a tutoring can be easily provided by well-educated parents but may not be provided by poorly educated parents or guardians. In this case, if an after-school tutoring program can provide the children with a service that largely substitutes the roles of the parents/guardians, the children with a disadvantaged family background will tend to benefit more from this program.

Thus, we have the following hypothesis.

Hypothesis 3: Outside tutoring tend to benefit more left behind children, who are more likely to receive little tutoring at home.

4. The Peer Tutoring Experiment

4.1 Background and Context

A driving force of China's phenomenal economic growth in the past three decades is the large-scale and persistent rural-to-urban migration: half of the country's 1.3 billion people now live in cities as compared to only one-fifth in the early 1980s. However, due to the household registration (*Hukou*) system, nearly one-third of the country's urban population – or an estimated 210 million – do not possess a *Hukou* in their residence city (National Bureau of Statistics, 2012), and hence are excluded from full access to city welfare including free public education for children. As a consequence, the majority of migrant parents choose to leave their children behind in the countryside, leading to a huge left-behind children phenomenon in the countryside. According to the All-China Women's Federation's (ACWF, 2013) report based on the 2010 Population Census, over 61 million children under age 17 – or over one-third of all children in the countryside – are left-behind by one or both parents, almost half of whom are left-behind by both parents. Applying dynamic panel methods controlling for both unobserved individual heterogeneity and endogeneity in parental absence, Zhang et al. (2014) find significant negative impacts of being left-behind by both parents on children's cognitive achievements, but much smaller insignificant impacts of being left-behind by one parent. Moreover, they also find that only the absence of both parents is associated with substantially lower family inputs on after-school tutoring whereas the absence of a single parent is not, suggesting a potentially critical role of family inputs on after-school tutoring in determining children's cognitive achievements.[9]

This remedying education experiment was conducted in the Longhui County in Hunan Province of China. This county has been designated as a national poverty county since 1994. With a per capita GDP less than a quarter of the national average[10] and over two-thirds of children left-behind by one or both parents was selected to represent one of the country's poorest rural areas with a high prevalence of parental absence. In 2011, the county has a population about 1.2 million, of which 90% are rural residents.

4.2 The Randomized Trial

In this experiment, we recruited high-achieving students in grades 4 and 5 (hereafter senior grades) to offer one-to-one peer tutoring to low-achieving students in grades 2 and 3 (hereafter junior grades) in 36 primary schools in the study county, Longhui County of Hunan Province. In this study, we define a student as "high-achieving" ("low-achieving") if his/her cumulative scores in Chinese and math in the baseline score were above (below) the median of his/her class.

The tutees participating in this peer tutoring program were selected in two steps. First, 76 (junior) experiment classes were selected from a total of 133 junior classes in these 36 schools as follows: (i) from the 48 junior school-grades with multiple classes in each grade, 60 classes were randomly selected as the experiment classes and the remaining 59 classes were randomly selected as the (junior) control classes; (ii) from the remaining 24 junior school-grades with only a single class in each grade, 16 classes were designated as the (junior) experiment classes. Second, for each experiment class, we randomly selected 10 such low-achieving students (about half of all low-achieving students) to participate in the tutoring experiment as tutees, whereas the remaining unselected students were designated as within-class controls. In addition, for the subset of 60 experiment classes with control classes in the

[9] In another study focusing on migrant children who stay and attend schools in Shanghai, Chen and Feng (2013) find that a significant proportion of them are excluded from the public education system and have to turn to privately operated "migrant schools" that are serve exclusively for migrant children and viewed as inferior to public schools. They also find significant disadvantages in the learning outcomes for children enrolled in these "migrant schools" compared to their fortunate counterparts enrolled in public schools.

[10] In 2010, the county's per capita GDP was RMB 6,992, less than a quarter of the national average of RMB 29,748.

same school-grade, all low-achieving students in the control classes were used (within-school-grade) between-class controls. In the empirical analysis on the treatment effect of the tutoring program on tutees below, we define the full sample as consisting of all low-achieving students in the 76 experimental classes, including both tutees and their within-class controls, and the two-tier random assignment subsample as consisting of all low-achieving students in school-grades with multiple classes, including tutees from the 60 randomly selected experiment classes, their within-class controls in the same experiment class, and their between-class controls in the control class(es) in the same school-grade.

The tutors participating in this peer tutoring program were recruited from high-achieving students in senior grades within the same school. With the cooperation of the school and the approval of students' parents, we received more applicants than the quota needed in all participating schools and randomly selected tutors from the pool of applicants.

This tutoring experiment lasted for 7 months, from November 2012 to June 2013, with a one-month winter break in February 2013. During this experiment period, the randomly paired tutors and tutees met in a designated tutorial room (usually the tutee's classroom) for a 45-minute tutorial Monday through Thursday every week. Each tutorial room hosted 10 randomly assigned tutor-tutee pairs. A teacher was recruited to a grade different to both the tutees' and tutors' grade to be the supervisor in a tutorial room. But the teacher's role was rather passive: he/she only helped to keep the discipline and answered questions on request, and was not supposed to involve in any classroom teaching directly. During the tutorial session, each tutor helped his/her assigned tutee to finish the homework and also answer any study questions raised by the tutee.

While this experiment started with 760 assigned tutor-tutee pairs, only 90% of such pairs lasted till the end. While most of the terminations were caused by the school switching of the tutees or tutors, in some cases either tutees or tutors decided to withdraw from the tutoring experiment though still enrolled in the same school. Whenever a tutee switched a school or simply withdrew from the experiment, we suspended the pair. However, if a tutor switched a school or simply withdrew from the experiment, we replaced him/her with another tutor applicant not selected in the first round. Nonetheless, throughout all empirical analysis, we only used the assigned tutee or tutor status in the first round.

4.3 Data

We conducted two rounds of surveys: a baseline survey in October 2012 and a follow-up survey in June 2013. The baseline survey conducted in October 2012 consists of a student questionnaire asking each student's age, gender, time allocations after school, family inputs on study (including after-school tutoring), subjective emotional feelings, and a household questionnaire asking information on family composition, parents' ages, schooling attainment, and migration status. When at least one parent stayed at home, the household questionnaire was filled out by a parent; otherwise, it was filled out by the primary caregiver, who was asked to verify the information by phone with the student's parents. In the latter case, information of the primary caregiver was also collected. About two weeks before the end of the 2012-2013 school year, a follow-up survey was conducted in June 2013.

A baseline cognitive test on Chinese and math was conducted in September 2012, a month before the baseline survey. Students' cumulative scores in both Chinese and math in this baseline test were used to determine their eligibility for participating in the peer tutoring program in the role of tutees for second and third graders or tutors for the fourth and fifth graders. At the end of June 2013, a post test on Chinese and math was conducted to evaluate the achievement effect of the peer tutoring program. Both rounds of the tests were graded centrally. For the post test, we also recruited teachers from different schools as enumerators to proctor the exam in each classroom.

Table 1 checks the balance of four pre-experiment variables on parental absence status and baseline test scores between tutees and controls. Parental absence is indeed a pervasive phenomenon in our sample of low-achieving junior grade students: Column 1 shows that among the 760 tutees, 45% have both parents absent from home and 29% have one parent absent from home, leaving only 26% have both parents present at home. These tutees also scored about 0.6 σ below the mean of all students in both Chinese and math. Column 2 compares tutees and their within-class controls and finds no evidence of any significant differences. Columns 3-5 perform the balance checks to the subsample of 600 tutees from 60 experiment classes selected from school-grades with multiple classes. Columns 4 and 5 show that both the within-class controls and between-class (within school-grade) controls have similar parental migration status and initial achievement in the baseline achievements as the selected tutees themselves.

5. Empirical Results

5.1 Evaluation Design

The main empirical strategy to assess the effect of the peer tutoring program on the tutees is captured by the following class fixed-effect regression applied to both tutees and their within-class controls in the experiment classes:

$$y_{ij,1} = \lambda y_{ij,0} + \rho T_{ij} + \delta_j + \varepsilon_{ij}, \tag{19}$$

where $y_{ij,1}$ denotes the post test scores of student *i* from experiment class j, $y_{ij,0}$ denotes the pre-test scores of student *i* from experiment class j, T_{ij} is a dummy indicator that equals 1 if student *i* was assigned to be a tutee to participate in the peer tutoring program and 0 if otherwise, δ_j is a class fixed effect that captures the unobserved determinates of learning shared in common at the regular classroom among all students from class j, and ε_{ij} is an error term, consisting of both an individual-level component and a class-level component. For both $y_{ij,0}$ and $y_{ij,1}$, we use the standardized test scores throughout the paper.

Alternatively, we can also assess the effect of the peer tutoring program on the tutees by running a school-grade fixed-effect regression comparing tutees with other low-achieving students from other classes within the same school-grades but not selected to be experiment classes (i.e., between-class controls) as follows:

$$y_{ijg,1} = \lambda y_{ijg,0} + \rho T_{ijg} + \pi_g + \mu_{ijg}, \tag{20}$$

where $y_{ijg,1}$ denotes the post test scores of student *i* from class j of school-grade g, $y_{ijg,0}$ denotes the pre-test scores of student *i* from class j of school-grade g, T_{ijg} is a dummy indicator that equals 1 if student *i* was assigned to be a tutee to participate in the peer tutoring program and 0 if otherwise, π_g is a school-grade fixed effect that captures the unobserved determinates of learning shared in common at the school-grade among all students from school-grade g, and μ_{ijg} consisting of both an individual-level component and a class-level component.

Because both ε_{ij} and μ_{ijg} consists a class-level component, we always cluster the standard errors at the class level when estimating Equations (19) and (20). When tutees and their controls from the same experiment class are used to estimate Equation (19), the variation in T_{ij} is at the individual level, clustering at the class-level does not significantly increase the standard error of the estimated treatment effect $\hat{\rho}_w$. However, when only tutees from the experiment classes and low-achieving students from the control classes are used to estimate Equation (20), T_{ijg} only varies at the class level. As a result, clustering at the class-level significantly increases the standard error of the estimated treatment effect $\hat{\rho}_b$. Therefore, the estimated $\hat{\rho}_b$ from Equation (20) are usually less precise compared to the estimated $\hat{\rho}_w$ from Equation (19).

5.2 Main Empirical Results

Table 2 reports estimates of the treatment effect of the tutoring program on tutees' test scores in Chinese (in columns 1-3) and math (in columns 4-6). The estimates for Chinese show no evidence that the tutoring program improves tutees' Chinese test scores. For math, the within-class estimate of the treatment effect for the full sample is 0.135σ, significant at 1% level (column 4), the same estimate for the subsample is 0.092σ, significant at 5% level (column 5), whereas the between-class estimate for a subsample of tutees with between-class (within-school-grade) controls is 0.086σ, marginally significant at 15% level.

Table 3 further estimates specifications that allow the treatment effect of the tutoring program on tutees to vary by tutees' absence status. The results suggest a substantially larger treatment effect on math scores for tutees with both parents absent as compared to those with one or both parents at home. Specifically, the point estimates in column 5 shows the treatment effect to be 0.092σ for tutees with both parents present at home, 0.077σ for tutees with one parent absent from home, and 0.205σ for tutees with both parents absent from home. Since the first two estimates are very close to each other, we pool these two categories together as having at least one parent present at home in columns 6-8 and include only the tutee dummy and its interaction with both parents absent dummy. The coefficients in column 6 suggest that the tutoring program increase the math score of tutees with at least one parent at home by 0.084σ, significant at the 10% level. The additional effect on tutees with both parents absent (compared with the base effect on other tutees) is estimated to be 0.122σ, significant at the 5% level. Because of the reduction in the precision of the between-class estimates for reasons discussed in Section 5.1, the coefficients on the tutee dummy and its interaction with the both parents absent dummy are not always significant for the between-estimations using the subsample of tutees in columns 7-8, we can always reject that the overall effect on tutees with both parents absent (i.e., the sum of the two coefficients) is 0 at the 10% level. Taken together, the results in Table 3 indicate larger treatment effects for tutees with both parents absent from home, suggesting an important role of family background as measured by parental absence status in affecting the effectiveness of the remedying educational program in the form of peer tutoring implemented here.

5.3 Robustness Analysis

Besides the differences in the extent substitution between family inputs and exogenous increases in tutoring inputs by tutees' parental absence status, another possible reason for larger treatment effects for tutees with both parents absent from home is the complementarity between tutoring inputs and children's innate learning ability. Because we selected the low-achieving student sample by truncating students' scores in the baseline test, it is possible that low-achieving students with both parents absent from home as a group have higher innate ability than their counterparts with at least one parent present at home. That is, they scored below the class median b/c they received fewer family inputs rather than b/c they lagged behind in innate learning ability. To the extent that this innate ability-tutoring input complementarity explanation is true, one would expect the tutoring program yields larger benefits to the relatively higher-achieving students. Table 4 tests such hypothesis by adding an interaction term between the tutee dummy and the baseline score. The coefficient on this interaction term is either negative or small and insignificant in all the six specifications in Table 4, showing no support for differential treatment effects by baseline scores (or initial ability) of tutees.

5.4 Behavioral Responses

In this subsection, we consider three potential behavioral responses to the treatment of participating in the tutoring program as a tutee, taking in the form of parental absence status, home tutoring inputs, and teacher efforts respectively.

Table 5 conducts the first check of behavioral responses to the treatment by examining whether being assigned to treatment changes the parental absence status of tutees at the end of the experiment compared with immediately before the (unanticipated) experiment relative to control students. The results show no evidence that tutees' parental absence status was affected by the tutoring treatment, suggesting that the experiment did not lead to *extensive margin* changes in family inputs in terms of parental presence status at home.

Table 6 examines the *intensive margin* behavioral responses in family inputs to the treatment taking in the form of home tutoring inputs, and more importantly whether such responses vary by their initial parental absence status. Columns 1-2 first examine the pre-experiment relationship between parental absence status and home tutoring input. The negative and highly significant coefficient on the both parents absent dummy indicates that *a priori* students with both parents absent were about 20 percentage points (or 40%) less likely to receive any tutoring help outside school. However, there is virtually no difference in the probability of receiving outside school tutoring help between students with one parent absent and those with both present. In columns 3 and 5, we examine the differential treatment effect on the change in the *reported* home tutoring indicator *for tutees only*. Taking the results in Column 3 as an example, the estimates suggest that tutees with both parents present were 17 percentage points less likely to report to have received other outside school tutoring help after experiment. The reduction in the reported home tutoring status is similar to tutees with only one parent absent from home. However, there seems to be little reduction on the reported home tutoring status for tutees with both parents absent from home.

It is important to note that the reduction in the reported home tutoring status combines both the actual substitution effect and behavioral underreporting effect in response to the treatment. The latter exists when students, both in the treatment and control groups, think they would be more likely to continue or start to receive free one-to-one tutoring if they reported that they got no help elsewhere. Moreover, the extent of such underreporting may be adversely correlated with the initial level so that the exercise in columns 3 and 5 may overestimate the extent of differential substitution effect since students with both parents absent from home (and thus received much fewer family tutoring help) had fewer room to underreport their home tutoring inputs. In columns 4, 6, and 7 of Table 6, we pool tutees and their controls and run a fully-saturated regression interacting tutee dummy with both the dummy indicator for having both parents absent and class fixed effect. Under the assumption that differences in the extent of underreporting by parental absence status be the same between tutees and controls, the coefficient on the interaction term between the tutee dummy and having both parents absent dummy will estimate the differential substitution effect. The coefficients on this interaction term are always positive, significant in 2 out of 3 specifications, but much smaller in magnitude than those in columns 3 and 5 not accounting for differential underreporting effect by parental absence status.

In Table 7, the dependent variable is the change in a teacher's annual evaluation score (assigned by the school principal) in the year when the experiment was carried out and the year prior to the experiment. The regression analysis compares this change for subject/head teachers in the experiment classes and that in the control classes. In a few cases, the subject/head teacher changed during the experiment, we excluded those cases from the regression. Also, teachers with missing or zero scores were also excluded. At least based on teacher's annual evaluation scores, we cannot see any evidence of performance responses to the treatment.[11]

5.5 Treatment Effects on Tutors

In Table 8, we estimate the treatment effect of this remedying education program on the tutors by using the unselected high-achieving senior grade volunteers as controls. The coefficient on the tutor dummy is positive (though very small) for both subjects, suggesting no evidence of any adverse effect of participating in this remedying education program on tutors' achievements.

6. Conclusion

The global private after-school tutoring market is large in size and is rapidly increasing, which led some sociologists and educators to refer to it as the "shadow educational system" existing alongside the formal educational system. However, there is little rigorous academic research in this area. This paper aims to help fill this gap by conducting a theoretical and empirical investigation into the impacts of after-school tutoring on children's educational outcomes.

Our theoretical analysis is based on a model of time allocation in the spirit of Becker (1965). We consider that a child's educational outcome is influenced by three main factors: the tutoring she receives from her parents/guardian, the outside after-school tutoring she receives, and the time of her self-study. In this setup, while the outside after-school tutoring itself benefits the child, it is at a cost of reducing the child's time of self-study and independent thinking. Moreover, with an increase in the outside after-school tutoring, the parents/guardian

[11] Nonetheless, the power of this test is likely to be very weak. Over 85% of the teachers in the sample received a score of 90 or above (out of a total score of 100) and the within-school standard deviation in this assessment is less than 3. Thus, even if there were some moderate behavioral responses from the teachers, such responses may not be detected by this test of the change in performance score.

may reduce their own tutoring and enjoy more leisure. Thus, the model implies that an increase in the outside tutoring will increase the child's human capital only if outside tutoring is effective and a child's self-study is relatively less effective in affecting the child's human capital formation. Furthermore, the model shows that the policy intervention of providing free outside tutoring will have a greater impact on the children from disadvantaged families if the outside tutoring is close substitute to the tutoring of parents/guardian.

Our empirical investigation is based on a randomized experiment, which is the first randomized experimental study examines a within school cross-age tutoring program in China. We use high achieving 4th and 5th grade students as tutors to help low achieving 2nd and 3rd grade students in the same school. The program covers in total 36 schools and over 700 tutees and lasts for 7 months with four 45-minutes sessions per week in a rural area of China. The results show that such a program helps tutees to improve their math scores but not in Chinese scores. It also shows there is no adverse effects on tutors' academic performance.

Another major contribution of the current paper is that due to the large number of students in our tutee sample (about 40%) have both of their parents absent from home we are able to better observe the potential substitution effects of our program. We find that the program does trigger a decreased use of home-based tutoring, which is more prevalent for students with at least one parent present at home. It is also the main reason that we find the program has a larger effect on students with both parents absent from home.

The cost of running such a within school cross-age tutoring program is relatively low. The fact that it does generate positive effects on tutees and no adverse effects on tutors means that the government can encourage more schools to adopt this type of programs, especially for schools in less developed countries/regions, where they may face more resource constraint to implement other forms of remedying programs. Furthermore, due to the poor family support for students in those regions, the potential substitution effect for the program may be low. This further increases the effectiveness of such programs.

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Appendix

In the text, we state that *u(), v(), F(,)* and *H(,)* are all increasing with respect to its variables, and all satisfy the neoclassical properties. These assumptions are expressed as follows:

$$\begin{aligned} & F_1 > 0, F_{11} < 0 \\ & F_2 > 0, F_{22} < 0 \\ & H_1 > 0, H_{11} < 0 \\ & H_2 > 0, H_{22} < 0 \\ & v_1 > 0, v_{11} > 0 \end{aligned} \tag{A1}$$

Proof of Claim 1.

The first order condition of (7) is:

$$\pi F_1 H_2 - F_2 \leq v_1 \tag{A2}$$

with (A2) holds with strict equality if $t > 0$.

When π=0, we know that (A2) must hold with strict inequality since

$$-F_2 < v_1 \tag{A3}$$

This means that we must have $t = 0$. By continuity, (A2) will continue to hold with strict inequality when π is sufficiently small. In other words, we have $t = 0$ if π is sufficiently small.

(ii) When $t > 0$, (A2) holds with strict equality. Totally differentiating (A2) with respect to t and π, we get

$$\frac{dt}{d\pi} = -\frac{\pi t F_{11}(H_2)^2 + F_1 H_2 + \pi t F_1 H_{22} - t F_{12} H_2 - v_{12}}{\pi^2 F_1 H_{22} + \pi^2 (H_2)^2 F_{11} - 2\pi H_2 F_{12} + F_{22} - v_{22}} \tag{A4}$$

Again, note that the denominator of the right hand side of (10) is exactly the second order condition, which must be negative at optimum. However, the items of the nominator have mixed signs, which means that the sign of $\frac{dt}{d\pi}$ is ambiguous.

Proof of Claim 2.

(i) When (A2) holds with strict inequality (implying $t = 0$), it will continue to hold with strict inequality with a marginal increase in λ, which means that we continue to have $t = 0$. In this case, obviously $\frac{dt}{d\lambda} = 0$.

(ii) When $t > 0$, (A2) holds with strict equality. Totally differentiating (A2) with respect to t and λ, we get

$$\frac{dt}{d\lambda} = -\frac{\pi F_{11} H_1 H_2 + \pi F_1 H_{12} - (H_1 + \pi H_2) F_{12} + F_{22}}{\pi^2 F_1 H_{22} + \pi^2 (H_2)^2 F_{11} - 2\pi H_2 F_{12} + F_{22} - v_{11}} \quad \text{(A5)}$$

Note that the denominator of the right hand side of (A4) is exactly the second order condition, which must be negative at optimum. Then, when (8) holds, we have $\frac{dt}{d\lambda} < 0$.

Proof of Claim 3.

From (5), we have

$$\frac{dq}{d\lambda} = F_1 H_1 - F_2 + (\pi F_1 H_2 - F_2)\frac{dt}{d\lambda} \quad \text{(A6)}$$

If $t = 0$, then we will have $\frac{dq}{d\lambda} > 0$ if and only if

$$\frac{dq}{d\lambda} = F_1 H_1 - F_2 > 0 \quad \text{(A7)}$$

If $t > 0$, then we will have $\frac{dq}{d\lambda} > 0$ if and only if

$$F_1 H_1 - F_2 > (-\frac{dt}{d\lambda})(\pi F_1 H_2 - F_2) \quad \text{(A8)}$$

Proof of Claim 4.

If $t = 0$, $\frac{dq}{d\lambda}$ is given by (A7).

If $t > 0$, then from (A2) we have

$$\pi F_1 H_2 - F_2 = v_1 \quad \text{(A9)}$$

If (8) holds, we know $\frac{dt}{d\lambda} < 0$. Inserting (A9) into (A6) we have

$$
\begin{aligned}
\frac{dq}{d\lambda} &= F_1 H_1 - F_2 + (\pi F_1 H_2 - F_2)\frac{dt}{d\lambda} \\
&< F_1 H_1 - F_2
\end{aligned}
\tag{A10}
$$