Tax revenue losses through cross-border loss offsets: an insuperable hurdle for implementing formula apportionment?∗

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Abstract
This paper analyzes the relevance of firm losses for tax revenues and welfare when switching from separate accounting (SA) to a system of tax base consolidation with formula apportionment (FA). We find that a system change unambiguously decreases tax revenues in the short run due to the cross-border loss offset inherent in FA. In the medium run, tax revenues are still larger under SA than FA if the probability of incurring losses and the costs of profit shifting are sufficiently small. However, this changes in the long run. Under the aforementioned conditions, a switch from SA to FA is beneficial if both the firms and governments are able to adjust their behavior after the system switch. Further, we show that a higher weight of input shares in the apportionment formula may mitigate tax competition because, contrary to output factors, input factors provide an insurance against tax revenue shortfalls due to loss-making affiliates.

Keywords: Separate accounting, Formula apportionment, Corporate losses, Cross-border loss offset, CCCTB

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1 Introduction

In October 2016, the European Commission (EC) proposed another set of Directives pushing forward unitary tax rules for business operations across the European Union (EU).\(^1\) If adopted, a common corporate tax base (CCTB) becomes mandatory, as of January 2019, for EU companies belonging to a group with a consolidated turnover exceeding EUR 750 million. In a second step, the cross-border consolidation of profits and losses will become mandatory from January 2021 onwards, transforming the CCTB into a common consolidated corporate tax base (CCCTB) with formula apportionment of taxable group profits to member states based on three equally weighted factors comprising labor (number and of employees and payroll costs), tangible fixed assets, and sales. The rationale for the two-step approach lies in the opposition of several EU member states against the consolidation matter as stipulated in the 2011 Directive. The consolidation of cross-border profits and losses is, on the one hand, seen as an enormous step towards the elimination of a major obstacle of cross-border business activities.\(^2\) On the other hand, the significance of loss-making affiliates suggests substantial negative tax revenue consequences (see Fuest et al., 2007; Cobham and Loretz, 2014) and thus a potentially insurmountable hurdle for the implementation of the CCCTB.

In this paper, we analyze the tax revenue and welfare consequences of potential losses on the governments’ decision to switch from a system of separate accounting (SA) to formula apportionment (FA). We set up a two-country framework with one representative multinational enterprise (MNE) having its headquarter (HQ) in one of the two countries and a subsidiary in the other. The MNE decides on the size of a risky investment in both entities (locations) and the transfer price for an intangible asset required for production. A failure of the risky investment results in zero output and thus in losses for that entity. Governments in each country maximize tax revenues by non-cooperatively setting their tax rates. By analyzing the implications of potential

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\(^1\)In 2001, the Commission presented a first communique proposing the switch from separate accounting to a system where multinational firms’ profits are consolidated and apportioned to each taxing country based on a formula; in 2011 a Directive on an EU-wide common consolidated corporate tax base was put forward (see European Commission 2001 and 2011). Council debates proved that the adoption of the latter Directive has become unlikely, forcing the launch of the 2016 Directives.

\(^2\)A fundamental motive for the European Commission to propose a move towards formula apportionment is based on the fact that ”the limited availability of cross-border loss relief is one of the most significant obstacles to cross-border business activity” (European Commission, 2006, p. 10).
losses for a government’s decision to favor a system of SA over FA and vice versa, the paper contributes to the understanding of the relevance of MNE losses for government behavior. Despite the widespread and empirically well-documented phenomena of loss-making affiliates, the implications of the latter have so far received only limited attention in theoretical papers.\footnote{Altshuler et al. (2011) document huge corporate losses in the U.S. context. Cooper and Knittel (2006) and Auerbach (2007) show that the problem of unused tax losses is quantitatively a highly important one. Ramb and Weichenrieder (2004) and Fuest et al. (2007) find a similar pattern for losses in the German context. For a more general discussion on corporate losses, see OECD (2011).}

In the analysis, we separate the behavioral adjustments of the MNE and the government in response to a switch from SA to FA from the mechanical base effect by analyzing three different scenarios. In the short-run analysis, we assume that neither the MNE nor the government are able to adjust their behavior following the system switch. In the medium run, only the MNE is able to alter its strategies while the government continues to impose the same tax rate as it did under SA. In the long run, both the MNE and the government respond to the change in the tax system. This approach proves helpful to disentangle the pure tax base effect due to the loss offset opportunity under FA from investment and tax competition issues.

We show that, in a symmetric equilibrium, tax revenues unambiguously decrease in the short run after a switch from SA to FA. Specifically, the tax base consolidation under FA mechanically reduces tax revenues due to the cross-border loss offsets inherent in FA. This finding provides a theoretical backing for the empirical studies by Fuest et al. (2007) and Cobham and Loretz (2014), which are dedicated to exactly those tax revenue consequences resulting from a cross-border loss offset when switching from SA to FA.

Taking the above studies as a benchmark when deciding on the introduction of a CCCTB, i.e., a FA system, in Europe is however delusive. Already in the medium run, when firms are able to adjust their investment decision to the new tax rules, the negative revenue consequences attested in the short run may dissipate under certain conditions, namely, if the probability of incurring losses and the MNE’s cost of profit shifting are both sufficiently large. Under these conditions, the level of investment is relatively low under SA and the boost of investment stimulated by the loss-offset provision when switching from SA to FA is large enough to ensure higher tax revenues under FA.
the more plausible (but polar) conditions, that is a relatively moderate loss probability and rather low costs of profit shifting, the investment response originating from the cross-border loss offset provision under FA is only limited and tax revenues remain larger under SA than under FA.

Yet, this changes in the long-run scenario. When both the MNE and the government respond to the switch from SA to FA, tax revenues are larger under FA if the probability of incurring losses and the MNE’s cost of profit shifting are both sufficiently small – requirements which most likely reflect real world conditions. Thus, our analysis proves relevant for policy makers by highlighting not only the already known short-run consequences but, importantly, also informing about the medium- and long-run consequences when switching from a system of SA to FA which have not yet been captured in the prior, purely empirical works. Especially, the fact that the conditions for ensuring larger tax revenues under FA are qualitatively polar in the medium versus the long run indicate that there may be a transitional period of tax revenue losses until the benefits of a system switch from SA to FA fully materialize.

Furthermore, our analysis provides important insights into the role of the apportionment factor weights for the intensity of tax competition once a government has chosen to switch from SA to FA. This issue is of major relevance because the potential of losses creates a qualitative difference between input and output factors in the apportionment formula. In case an affiliate fails, the output share in that country drops to zero and only the input shares remain relevant in the formula for apportioning the consolidated tax base. We show that a higher weight on input shares has two opposing effects on tax competition. First, tax competition is aggravated because the MNE may more easily manipulate input shares than output shares in the formula. Second, a high input share acts as an insurance against unsuccessful MNE affiliates. Despite the zero output of the failing MNE affiliate, governments still collect some tax revenues as long as consolidated corporate profits are positive. If the insurance effect is sufficiently large tax competition is mitigated. Against this background, the use of a high sales shares in the apportionment formula, as it is commonly applied, for example in some U.S. states, can prove counterproductive in reducing tax competition.

The paper is structured as follows. The next section provides a discussion of the related literature. Section 3 introduces the basic framework of our analysis. Section 4 assesses
the implications of losses under SA and FA in the symmetric case. Section 5 analyzes the conditions under which the switch from SA to FA would be beneficial. Section 6 focuses on the role of apportionment factors in tax competition under FA. Section 7 provides two extensions of the model and Section 8 concludes.

2 Related literature

Our analysis relates to two aspects within the same strand of literature. A quite substantial literature in public economics has by now theoretically investigated the implications of a switch from SA to FA. Starting with the work by Gordon and Wilson (1986) who show that a formula based on property, payroll and sales creates separate taxes on the factors used in the formula, the question whether to prefer SA or FA has been extended to many other contexts. Nielsen et al. (2003) undertake an examination of the implications of a switch from SA to FA for transfer pricing strategies when affiliates of a multinational firm face oligopolistic competition. They demonstrate that MNEs still have an incentive to use transfer prices under FA, albeit for strategic and not for tax reasons, and that the switch may intensify profit shifting activities. Kind et al. (2005) integrate this questions into a framework of economic integration. They show that welfare under FA is larger if trade costs are sufficiently low. In Riedel and Runkel (2007) only a subset of countries are forming an FA union but transfer prices are still needed for intra-firm transactions to affiliates at the water’s-edge, i.e. countries outside the FA union that still use SA. They find that despite remaining profit shifting opportunities to the water’s edge country, inefficiencies are likely to be larger under SA compared to FA both in the short run, where taxes remain constant, and in the long run, where governments engage in tax competition. A central issue in Nielsen et al. (2010) is whether spillovers are larger under SA or FA, and whether this results in inefficiently high or low rates. They show that whether a switch from SA to FA is beneficial depends on MNEs’ ability to shift profits and to generate pure profits. Gresik (2010) extends the basic question of whether SA or FA is beneficial by incorporating firm heterogeneity and private information about firm’s true cost of profit shifting. He shows that the decision to switch from SA to FA depends on the accuracy of tax authorities’ auditing technology. Eichner and Runkel (2011) contribute to the discus-
sion by embedding the commonly applied framework into a general equilibrium tax competition model in which the interest rate is endogenously determined. They find that FA is, in principle, superior to SA and this is independent of the MNEs’ ability to shift profits. All studies we referred to show that the decision between SA and FA in not unconditional. An exception is Eggert and Schjelderup (2003) who show that in a symmetric framework a switch from SA to FA is never beneficial if countries combine a property tax with a residence-based capital tax under SA but use an apportionment formula based on sales and property in conjunction with a residence-based capital tax under FA.

A second aspect of this literature is the questions which formula should be used in order to mitigate tax competition. In Anand and Sansing (2000) and Pinto (2007) jurisdictions may non-cooperatively choose the apportionment weights of the formula. Both studies give an explanation for the observed heterogeneity of apportionment formulas across U.S. States. Whereas Anand and Sansing (2000) show that importing (exporting) states have an incentive to put more weight on the sales factor (input factors), although the deviation from an equal weights formula reduces aggregate social welfare, Pinto (2007) argues that capital cost deductions and the tax-elasticity of capital play a major role. Pethig and Wagener (2007) compare different, but centralized, methods of formula apportionment, neglecting the choice between SA and FA, and find that the incentives to engage in tax competition are generally minimized under a pure sales formula. Similarly, however taking into consideration the decision between SA and FA, Eichner and Runkel (2008) show that a formula based solely on sales shares may mitigate or even eliminate countries’ incentives to compete for firms’ profits. Eichner and Runkel (2009) enrich this question by incorporating involuntary unemployment into the model. They illustrate that unemployment creates an additional incentive to lower tax rates but this incentive can be minimized with a pure sales formula. Most of the above mentioned studies highlight the role of the sales factor in the apportionment formula. Exceptions to this literature are Wellisch (2004), Eichner and Runkel (2012) as well as Runkel and Schjelderup (2011). All three studies emphasize the role of input factor shares in reducing tax competition. Wellisch (2004) considers a decentralized formula choice ans shows that in equilibrium jurisdictions maximize the weight of immobile payroll to eliminate the competition for mobile capital. Eichner and Runkel
(2012) differs from the previous literature in that they assume a constant returns to scale production function, instead of a decreasing returns to scale technology, and find that, for the special case of a Cobb-Douglas production function, either a pure payroll formula or a formula in which formula weights equal the inputs’ production elasticities renders efficiency. Runkel and Schjelderup (2011) show that it is always optimal to include mobile as well as immobile factors in the apportionment formula irrespective of whether the formula is determined by a central planner or in a non-cooperative way by jurisdictions.

All the above mentioned studies are, however, silent about the implications of corporate losses. Only very few studies exist in this strand of the literature where corporate losses play a more prominent role. Haufler and Mardan (2014) analyze the implications of introducing cross-border loss offset provisions under the tax system of SA. They find that if a coordinated cross-border loss compensation were to be implemented the loss relief should be based on the tax rate of the loss-making subsidiary’s host country and not on the tax rate of the parent firm’s home country as the current international practice suggests. In a similar vein, Kalamov and Runkel (2016) investigate the implications of an uncoordinated introduction of cross-border loss-offset provisions when countries compete for capital and profits of MNEs. They show that the incentives to introduce cross-border loss offset provisions critically depends on whether countries compete for capital or profits. Closest to our analysis is Gérard and Weiner (2003, 2006) who analyze the effect of the introduction of cross border loss offset on the behaviour of governments engaged in interjurisdictional competition. They demonstrate that cross-border loss offset mitigates governments’ incentives to engage in harmful tax competition. Our analysis differs in at least three dimensions. First of all, Gérard and Weiner (2003, 2006) assume that firms’ investment is fixed and the only decision is how to distribute production between affiliates. Thus, they not draw conclusions about the allocative efficiency of cross-border loss offset. Second, the apportionment rule in their analysis only contains capital which makes it impossible for them to analyze the qualitative differences of formula weights inherent to losses. Third, the center of our analysis is the short-, medium-, and long-run consequences of a switch from SA to FA which they do not take into consideration.
3 The basic framework

We consider a simple one-period model of two small countries, labeled $a$ and $b$, which are both host countries and levy a corporate income tax $t_a$ and $t_b$, respectively. There is one representative MNE with the headquarter (HQ) in one country and a subsidiary in the other. Each entity produces a homogeneous output good which is sold at the world market at a price normalized to one and produced with the technology $f(k_i)$, with $f'(k_i) > 0 > f''(k_i)$. Mobile capital $k_i$ is the only input and for simplicity we assume that all investments are financed by external debt. Further, decreasing returns to scale in production imply the existence of a fixed factor which gives rise to positive pure profits. Production is successful with an exogenous and idiosyncratic probability $p_i$, but fails with probability $(1 - p_i)$ leading to an output of zero value.

The HQ owns the patent right for the intangible asset and claims license fees for the use of the patent. We assume that the subsidiary has to buy one unit of the intangible asset to enable production. For simplicity, the arm’s-length price of the intangible asset is normalized to zero. The MNE may minimize its overall tax payments by shifting profits from one affiliate to the other through overpricing or underpricing the license fee. We denote by $g$ the actual transfer price charged by the HQ. If the MNE overstates the transfer price, i.e. $g > 0$, profits are shifted from the subsidiary to the HQ and vice versa. However, any deviation from the true arm’s-length price is costly for the MNE because additional effort is needed to conceal the mispricing behavior. We model this by specifying a convex cost function $C(g)$, where $\text{sign}(C'(g)) = \text{sign}(g)$ and $C''(g) > 0$.

Given the model set-up, economic profit of affiliate $i$ is given by expected revenue from output sales less the user cost of capital and plus or minus the license cost for the intangible good

$$\pi_i^e = p_i[f(k_i) - k_i] - (1 - p_i)k_i + 1g.$$ (1)

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4 Allowing the MNE to endogenously determine the success probability substantially increase the complexity of the model without changing the results qualitatively. Indeed, the decision on the optimal success probability would suffer from a tax distortion under SA, similar to the one for capital investment, but the existence of the loss-offset provision under FA mitigates this distortion.

5 The assumption of a zero value output is a simplification which does not affect our results in a qualitative manner.
The operator \( \mathbb{1} \) is an indicator function that takes on the value of 1 for the HQ located in \( a \) and \(-1\) for the subsidiary located in \( b \).

4 Corporate tax systems

4.1 Separate accounting

4.1.1 Firms

Under SA, taxes are imposed by the source country where the investment takes place. Given the host country’s taxation of subsidiary profits, the parent country of the MNE usually exempts this income from taxation to avoid the double taxation of corporate profits. Nowadays, the exemption method is applied by the overwhelming majority of OECD countries, i.e., 28 out of 34 in 2012 (see Keen, 1993; PricewaterhouseCoopers, 2013). Taxable profits of affiliate \( i \) are given by

\[
\pi_i^t = p_i [f(k_i) - k_i + \mathbb{1} g].
\]  

(2)

Only if the investment is successful, the cost of capital and the expenses for the license fee can be deducted from sales revenue, i.e., the value of the output. Using (1) and (2), after-tax profits read

\[
\pi_i = p_i (1 - t_i) [f(k_i) - k_i] - (1 - p_i) k_i + \mathbb{1} (1 - p_i t_i) g.
\]  

(3)

The MNE maximizes the sum of affiliates’ profits minus costs for concealing profit shifting by choosing capital investment \( k_i \) and the transfer price \( g \). Total MNE profits read

\[
\pi = \pi_a + \pi_b - C(g),
\]  

(4)

\(^6\)For convenience, we assume that the income earned through profit shifting is insufficient to cover the costs when the income receiving affiliate is unsuccessful. It follows that the tax base of the income receiving affiliate is zero if the affiliate is unsuccessful.
and the optimal level of capital investment is given by

$$f'(k_i) = \frac{1 - p_i t_i}{p_i (1 - t_i)}. \quad (5)$$

In the absence of an unsuccessful investment ($p_i = 1$), the optimal investment level is determined by the usual condition that the marginal product of capital, $f_k$, equals the exogenous world interest rate (normalized to unity). In a world without taxation but with potential losses, the marginal productivity of capital must rise to $1/p_i$ in case of success to compensate a risk-neutral investor for the possibility of failure. This relation is distorted by a tax system that taxes positive profits but grants no tax relief for losses incurred. As a consequence, a situation of underinvestment emerges and this distortion is more severe the higher the host country’s tax rate.

From (5), the effects of the tax rates and the success probability on an affiliate’s optimal investment choice are

$$\frac{\partial k_i}{\partial t_i} = \frac{1 - p_i}{p_i (1 - t_i)^2 f''} < 0, \quad \frac{\partial k_i}{\partial t_j} = 0,$$

$$\frac{\partial k_i}{\partial p_i} = -\frac{1}{p_i^2 (1 - t_i) f''} > 0. \quad (6)$$

The first two derivatives in (6) show the standard tax effects in a model with small countries. A higher tax rate in country $i$ reduces capital investment in country $i$ but does not affect capital investment in country $j$. The third derivative implies that capital investments increase when the probability of success rises. The reason is that losses appear with a lower probability which effectively renders the tax system less distortive.

The optimal level of the transfer price for the intangible asset is determined by

$$C'(g) = p_t b - p_t a. \quad (7)$$

Importantly, it is the expected tax rate differential which is decisive for the magnitude of shifted profits. From (7), the effects of the tax rates and the success probability on the MNE’s optimal transfer price are

$$\frac{\partial g}{\partial t_a} = -\frac{p_a}{C''} < 0, \quad \frac{\partial g}{\partial t_b} = \frac{p_b}{C''} > 0.$$
\[
\frac{\partial g}{\partial p_a} = -\frac{t_a}{C''} < 0, \quad \frac{\partial g}{\partial p_b} = \frac{t_b}{C''} > 0,
\]
\[
\frac{\partial^2 g}{\partial p_a \partial t_a} = \frac{\partial^2 g}{\partial t_a \partial p_a} = -\frac{\partial^2 g}{\partial p_b \partial t_b} = \frac{\partial^2 g}{\partial t_b \partial p_b} = -\frac{1}{C''} < 0
\]

The effects emerging from (8) are straightforward. A higher tax rate or a higher probability of success in one country increases the incentives to shift profits to the other country because expected tax payments of the MNE in the respective country increase.

4.1.2 Governments

Turning to government behavior, we postulate that governments set tax rates to maximize corporate tax revenues. This objective captures the concern about the erosion of tax revenue that features prominently in both policy debates and court decisions on cross-border loss offset and is most relevant in the context of FA.\(^7\)\(^8\) A more rigorous welfare analysis is additionally provided in Section 7.1 of the paper.

Tax revenues in country \(i\) are thus given by

\[
T_i = t_i \pi_i^i. \tag{9}
\]

Maximizing (9) with respect to \(t_i\) yields an implicit expression for country \(i\)'s optimal tax rate

\[
\frac{\partial T_i}{\partial t_i} = p_i [f(k_i) - k_i + 1 g] + t_i \left[ \frac{(1 - p_i)^2}{p_i(1 - t_i)^3 f''} - \frac{p_i^2}{C''} \right] = 0. \tag{10}
\]

When analyzing the symmetric equilibrium, we are interested in the effect of a global change in the probability of success on optimal tax rates. Totally differentiating (10) with respect to \(t_i\) and \(p\), applying the symmetry condition and the simplifying assump-

\(^7\) The list of EU member states which had rejected the 2011 CCCTB Directive widely coincided with the list of countries identified as losers in terms of tax revenues (except for the U.K.), following a mandatory CCCTB among the EU 27 member states (Ernst&Young, 2011, Commissioned by the Irish Department of Finance).

\(^8\) A case in point is the Marks and Spencer ruling. The European Court of Justice permitted the U.K. government to deny the parent company of Marks and Spencer to deduct the losses incurred by its subsidiaries in Belgium, France and Germany from its positive taxable profits in the U.K. The main arguments put forward by the U.K. government emphasized that the cross-border loss offset jeopardizes a balanced allocation of taxing power across countries and that it might result in additional opportunities for tax avoidance.
tions \( f''(k) = C'''(g) = 0 \) gives

\[
\frac{\partial t}{\partial p} = \frac{(1-p)(1+t)}{p^2(1-t)^2f''} + \frac{pt}{C''} - \frac{(1-p)^2(2+t)}{p(1-t)^4f''}.
\]  

(11)

The above expression is ambiguous in sign due to the numerator. Specifically, the sign of \( \frac{\partial t}{\partial p} \) depends on the MNE’s ability to shift profits. If profit shifting is very costly, i.e., \( C'' \) is very large, (11) reduces to

\[
\frac{\partial t}{\partial p} = \frac{(1-t)(1+t)}{p(1-p)(2+t)} > 0.
\]  

(12)

In this situation a global increase in the likelihood of success exerts an upward pressure on optimal tax rates. However, if MNEs can shift profits easily, then \( C'' \) is very small and (11) boils down to

\[
\frac{\partial t}{\partial p} = -\frac{t}{2p} < 0.
\]  

(13)

Equation (13) shows that a global increase in the probability of success decreases optimal tax rates if profits are very shifty. The intuition for the ambiguous effect is straightforward. An increase in \( p \) has a positive effect on the MNE’s investments and hence the tax base of the host country. This, ceteris paribus, raises the government’s incentive to increase its tax rate. At the same time, an increase in \( p \) also raises the tax sensitivity of profit shifting and, ceteris paribus, induces governments to reduce their tax rate. Whether tax rates increase or decrease when \( p \) changes thus depends on the relative magnitude of the two effects.

4.2 Formula apportionment

4.2.1 Firms

Under FA, all MNE affiliates’ tax bases are consolidated and assigned to the members of the FA union for taxation. The assignment follows according to a certain formula which usually contains capital and sales shares. In a setting without potential losses the formula reads

\[
A_i = \lambda \frac{k_i}{k_i + k_j} + (1 - \lambda) \frac{f(k_i)}{f(k_i) + f(k_j)}.
\]  

(14)

11
λ denotes the weight of the capital share and \((1 - \lambda)\) the weight of the sales share. The occurrence of potential losses, however, changes the weighting scheme in a qualitative manner since the sales share varies depending on whether the investment is successful or not. For instance, if the investment in country \(i\) fails, the sales share drops to zero but the capital share is still positive.\(^9\) Hence, after rearranging terms, country \(i\)’s expected share of the consolidated tax base reads\(^10\)

\[
\Gamma_i = A_i \{ p_i f(k_i) + p_j f(k_j) - [1 - (1 - p_i)(1 - p_j)](k_i + k_j) \} + (1 - \lambda) \frac{f(k_j)}{f(k_i) + f(k_j)} p_i (1 - p_j) [f(k_i) - k_i - k_j] \\
- (1 - \lambda) \frac{f(k_i)}{f(k_i) + f(k_j)} p_j (1 - p_i) [f(k_j) - k_i - k_j].
\] (15)

The first term in (15) is similar to the case without losses and says that the share \(A_i\) of the MNE’s expected consolidated tax base is allocated to country \(i\). However, the occurrence of potential losses introduces two additional effects. If affiliate \(i\) is successful but affiliate \(j\) is unsuccessful, a larger share of the consolidated tax base is allocated to country \(i\) and vice versa.

The MNE’s after-tax profits can be written as

\[
\pi = \pi_a^e + \pi_b^e - t_a \Gamma_a - t_b \Gamma_b - C(g),
\] (16)

and the MNE maximizes (16) with respect to capital investments and the transfer price. The first-order condition for investment reads

\[
\frac{\partial \pi_i^e}{\partial k_i} - t_i \frac{\partial \Gamma_i}{\partial k_i} - t_j \frac{\partial \Gamma_j}{\partial k_i} = 0.
\] (17)

Applying the symmetry assumption yields

\[
f'(k) = \frac{1 - pt - pt(1 - p)}{p(1 - t)}.
\] (18)

\(^9\)Our results remain qualitatively the same if we assume an additional third factor, e.g. labor, in the formula. The distinction lies in the fact that input shares remain unaffected ex post, i.e. after the realization of whether the affiliate is successful or not.

\(^10\)We assume that the tax base of a successful affiliate is large enough to cover the losses of an unsuccessful affiliate, i.e. \(f - 2k > 0\).
Comparing (18) with the first-order condition under SA, see (5), we infer that an additional positive effect on investment exists under FA (last term in the numerator). The origin of this additional effect is the MNE’s possibility to offset losses of one affiliate against profits of the other if tax bases of the affiliates are consolidated. This loss offset opportunity reduces the asymmetric treatment of profits and losses induced by the tax system under SA and therefore increases the MNE’s incentive to invest in both affiliates.

Under symmetry, the effects of a change in tax rates and in the success probability on affiliates’ optimal investment choices are

\[
\frac{\partial k_i}{\partial t_i} = \frac{\partial \Gamma_i/\partial k_i}{p(1-t)f''} < 0, \quad \frac{\partial k_i}{\partial t_j} = \frac{\partial \Gamma_j/\partial k_i}{p(1-t)f''} > 0,
\]

\[
\frac{\partial k}{\partial p} = -\frac{1-p^2t}{p^2(1-t)f''} > 0.
\]

(19)

In Appendix A.1 we show that \(\partial \Gamma_i/\partial k_i > 0\) and \(\partial \Gamma_j/\partial k_i < 0\). In contrast to SA, (19) implies that country \(j\)’s tax rate has a positive effect on the capital investment in country \(i\). A higher tax rate in \(j\) induces the MNE to invest more in country \(i\) in order to reduce its overall tax burden. Specifically, a higher capital investment in country \(i\) implies that, according to the formula, a larger fraction of the tax base is attributed to country \(i\) and thereby reduces the MNE’s overall tax burden. Moreover, a higher probability of success increases capital investments. This effect is, however, smaller under FA than under SA because the possibility of tax base consolidation under FA reduces the tax penalty associated with losses, i.e. the asymmetric treatment of losses under SA.

The first-order condition for the transfer price under FA is given by

\[
C'(g) = 0,
\]

(20)
yielding an optimal transfer price of \(g = 0\). Given that the tax bases of the two affiliates are consolidated under FA, the MNE has no benefit from manipulating the transfer price. Thus, FA eliminates profit shifting in the FA union.\(^{11}\)

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\(^{11}\)The result hinges on the fact that we abstract from profit shifting to countries outside the FA union (Riedel and Runkel, 2007) or the strategic use of transfer prices under imperfect competition.
4.2.2 Governments

As before, we assume tax revenue maximizing governments. Under FA, tax revenues of country $i$ are given by

$$T_i = t_i \Gamma_i. \quad (21)$$

Differentiating (21) with respect to the tax rate $t_i$ implicitly determines country $i$’s optimal tax rate

$$\frac{\partial T_i}{\partial t_i} = \Gamma_i + t_i \left[ \frac{\partial \Gamma_i}{\partial k_i} \frac{\partial k_i}{\partial t_i} + \frac{\partial \Gamma_i}{\partial k_j} \frac{\partial k_j}{\partial t_i} \right] = 0. \quad (22)$$

Applying the implicit function theorem and using the symmetry condition and (19), we establish (under the sufficient and mild condition that $f''$ is not too large) that\textsuperscript{12}

$$\frac{\partial t_i}{\partial p} = -\frac{\partial^2 T_i/\partial t_i \partial p}{\partial^2 T_i/\partial (t_i)^2} > 0, \quad (23)$$

that is, an increase in $p$ exerts a positive impact on equilibrium tax rates. The intuition for this finding is twofold. First, a higher $p$ increases the tax base due to the positive impact of $p$ on MNE investment. Second, the tax elasticity of capital declines and hence the competition for capital decreases. Both effects increase the government’s incentive to raise tax rates.

5 Comparing corporate tax systems

In this section, we analyze how a change in the probability of success affects the decision to choose one over the other corporate tax system. Tax revenues in a symmetric Nash-equilibrium under SA and FA can be written as

$$T^F_{FA} = T^F_{FA} \left(p, k^F_{FA}(p), k^F_{FA}(p), t^F_{FA} \left(p, k^F_{FA}(p), k^F_{FA}(p)\right)\right), \quad (24)$$

$$T^S_{SA} = T^S_{SA} \left(p, k^S_{SA}(p), t^S_{SA} \left(p, k^S_{SA}(p)\right)\right). \quad (25)$$

\textsuperscript{12}The derivation is relegated to Appendix A.2.
We define $\Delta = T^{FA} - T^{SA}$ as the difference in tax revenues under FA and SA. A positive value of $\Delta$ implies that tax revenues under FA are larger than under SA and vice versa.

In the following, we consider the effects of a switch from SA to FA on tax revenues as well as the implications resulting from a change in $p$. In order to disentangle the various effects resulting from alterations in the tax base, changes in investment incentives and adjustments in tax rates, we consider three different scenarios, i.e., a short-run, a medium-run and a long-run scenario. The different scenarios are distinguished in the following way: in the short run, neither the MNE nor the government can adjust their behavior upon the system switch. In the medium run, only the MNE is able to adjust its strategies while governments continue to levy the same tax rates under FA as they did under SA. In the long run, both the MNE and the governments react to the change in the tax system.

5.1 Short-run analysis

Suppose governments enact a switch in the tax system from SA to FA. Given the assumption that tax rates and capital investments remain unchanged in the short run, tax revenues under FA and SA are expressed as

$$T^{FA} = pt^{SA} [f(k^{SA}) - (2 - p)k^{SA}], \quad (26)$$

$$T^{SA} = pt^{SA} [f(k^{SA}) - k^{SA}], \quad (27)$$

and the difference in tax revenues states

$$\Delta^{SR} = -pt^{SA}(1 - p)k^{SA} < 0. \quad (28)$$

The superscript $SR$ indicates the change in tax revenues in the short run. In this scenario, tax revenues unambiguously fall if governments decide to switch from SA to FA. The obvious reason is that the tax base consolidation under FA mechanically reduces tax revenues given the loss offset possibility inherent to FA. We summarize our findings in:
**Proposition 1** *In the short run, where neither the MNE nor the government can adjust their behavior, a switch from separate accounting to formula apportionment always reduces tax revenues.*

Proposition 1 mirrors the findings of Fuest et al. (2007) and Cobham and Loretz (2014). Both studies estimate the tax revenue effects associated with a switch from SA to a system of FA with a cross-border loss offset. Fuest et al. (2007) resort to the German MiDi database for the time period 1996-2001 which contains the universe of German multinational firms. They find that a switch in the tax system from SA to FA results in a decline of the corporate tax base by 20%. Cobham and Loretz (2014), who make use of the Orbis database over the period 2003-2011 and which comprises data of all registered companies worldwide, estimate a drop in the corporate tax base by more than 10% in response to the system switch. Importantly, these estimates solely reflect the decline in the corporate tax base associated with the possibility of making use of the cross-border loss offset. Behavioral responses of firms are neglected and tax rates remain unchanged in both studies. Therefore, the insights of these studies seem to be only limited for policy advice. In the next two sections we overcome this drawback and assess the tax revenues consequences associated with the behavioral adjustments by the MNE and the government after the system switch.

### 5.2 Medium-run analysis

In this section we assume that the MNE is able to adjusts its investment behavior but governments still keep the same tax rates as under SA. In detail, we show that the positive investment effect stimulated by the cross-border loss offset provision under FA can have a qualitative effect on the results established in the previous subsection. In the medium run, tax revenues under the two systems are given by

\[
T^{FA} = pt^{SA} \left[ f(k^{FA}) - (2 - p)k^{FA} \right], \tag{29}
\]

\[
T^{SA} = pt^{SA} \left[ f(k^{SA}) - k^{SA} \right], \tag{30}
\]
and the difference in medium-run tax revenues states

\[ \Delta^{MR} = pt^{SA} \left[ f(k^{FA}) - (2 - p)k^{FA} - f(k^{SA}) + k^{SA} \right] \equiv pt^{SA} \Phi. \quad (31) \]

The superscript \( MR \) indicates the change in tax revenues in the medium run. We note that the sign of the tax revenue differential \( \Delta^{MR} \) only depends on the sign of \( \Phi \). Further, we know that tax revenues under SA and FA are identical, i.e. \( \Delta^{MR} = 0 \), when the probability of success either approaches zero or one. Therefore, it is sufficient to analyze the change in \( \Phi \) due to a change of the probability of success \( p \) to draw conclusions about the effect of losses on the medium-run tax revenue differential. Differentiating \( \Phi \) with respect to \( p \), using (5), (6), (11), (18) and (19), yields

\[ \frac{\partial \Phi}{\partial p} = k^{FA} + \frac{1 - p}{p^2 (1 - t^{SA})^3} \left\{ \left( 1 - t^{SA} \right) \left[ 1 + p(1 - p)t^{SA} \right] - p(1 - p)(2 - p) \frac{\partial t^{SA}}{\partial p} \right\}. \quad (32) \]

Equation (32) consists of two terms. The first term is the direct effect of an increase in the probability of success and it is positive. This means that the tax base under FA increases by more than the tax base under SA because the probability of cross-border loss offset decreases. The second term comprises the indirect effects via an adjustment of investments either directly through the change in \( p \) or indirectly through the change in the tax rate. This term is ambiguous since the change in the tax rate is ambiguous.

We first highlight that for very high probabilities of success \( (p \to 1) \) investment adjustments of the MNE are negligible and the sign of equation (32) is positive. Thus, \( \Phi \) and in turn \( \Delta^{MR} \) approach zero if the success probability \( p \) approaches 1 and for probabilities of success tax revenues under SA are always larger than under FA. The reason is that for high probabilities of success, the investment level of the MNE is already relatively high under SA and thus, the additional opportunity to offset losses across the borders under FA has only a small impact on investments. In case of an unsuccessful investment the government’s tax revenues, however, decrease by a large portion given the substantial size of the investment.

In order to evaluate whether tax revenues under FA are smaller or larger compared to SA for lower probabilities of success, we distinguish two cases. In the first one we assume

\[ ^{13}\text{The derivation can be found in Appendix A.3.} \]
that profit shifting is rather strong under SA, that is, the cost of profit shifting are sufficiently low ($C''(g)$ is sufficiently small). In contrast, in the second case we assume that profit shifting is only mild because the cost of profit shifting are sufficiently high (i.e., $C''(g)$ is sufficiently large).

For sufficiently small costs of profit shifting Appendix A.3 shows that $\frac{\partial^2 \Phi}{\partial p^2}$ is unambiguously positive. Moreover, the tax rate under SA decreases with the probability of success. Therefore, the second term in equation (32) is negative and overcompensates the positive first term if the probability of success $p$ is sufficiently low. Since $\Phi$ and in turn $\Delta MR$ approach zero if the success probability $p$ approaches 0, tax revenues in the medium run are always smaller under FA if the MNE’s costs of profit shifting are sufficiently low.

If instead the cost of profit shifting is sufficiently large, equation (12) reveals that equilibrium tax rates are increasing with $p$. Thus, the second term in curly brackets in (32) is positive and counteracts the first term. Appendix A.3 shows that the second term overcompensates the first term for sufficiently large costs of profit shifting which renders an overall positive sign of the terms in curly brackets. Hence, because $\Phi$ approaches zero if the success probability $p$ approaches 0, tax revenues in the medium run are larger under FA for low probabilities of success and if the MNE’s cost of profit shifting is sufficiently high. Unfortunately, we are not able to show what happens for intermediate probabilities of success. However, since the second term in (32) starts to diminish with an increase in $p$, it is highly likely that there exists a level for $p$ below (above) which medium-run tax revenues are larger (smaller) under FA than under SA.

To show that there is indeed a cut-off level, we complement the theoretical analysis with numerical simulations. For this purpose, we specify the affiliates’ production function as $f(k_i) = \alpha k_i^\beta$, where $\alpha = 5$ and $\beta = 0.3$, and the concealment cost function as $C(g) = \delta(g)^2/2$. Moreover, we assume that, under FA, sales and property shares have the same weight in the formula, i.e., $\lambda = 0.5$. Under SA, three cases are considered including low, intermediate and high costs of profit shifting represented by $\delta = 0.01$, $\delta = 0.05$ and $\delta = 0.15$, respectively. The solid curve in Figure 1 illustrates the case of low costs of profit shifting ($\delta = 0.01$) and confirms our analytical result in that tax revenues are always lower under FA compared to SA. The dashed line displays the case of high costs of profit shifting which is in accordance with our theoretical results. For
high success probabilities tax revenues are larger under SA whereas they are larger under FA for low probabilities of success. Moreover, it also shows that there exists a cut-off level for $p$ for which tax revenues are equivalent under SA and FA. The dotted line illustrates the case of intermediate costs of profit shifting. It shows that this case is a convex combination of the other two cases discussed above and highlights that lower costs of profit shifting reduce the range of values for $p$ below which medium-run tax revenues are larger under FA. We summarize our result in:

**Proposition 2** In the medium run, where only the multinational firm can adjust its behavior, a switch from separate accounting to formula apportionment is never beneficial if (i) either the probability of success is sufficiently high or (ii) the costs related to profit shifting are sufficiently low.

The intuition for Proposition 2 is straightforward. Two effects are at work when countries switch from SA to FA. First, the opportunity of having cross-border loss offset mechanically reduces the tax base. We have already analyzed this effect in the short-run scenario. Second, the cross-border loss offset additionally reduces the asymmetric treatment of profits and losses. This induces the MNE to increase its investment and thus leads to a expansion of the tax base. The mechanical negative effect dominates the investment effect if either the probability of success is sufficiently large or costs related
to profit shifting and hence tax rates under SA are low. The explanation, for both cases, is that the level of investment is already relatively high under SA and therefore the additional investment stimulated by the switch to FA is only modest and insufficient to compensate for the tax revenue losses due the loss-offset.

Whether, from an empirical point of view, the switch from SA to FA is beneficial in the medium run depends crucially on the magnitude of the success probability of firms. Studies which analyze firm losses in greater detail find that the average success probability, proxied by the share of profitable affiliates relative to the total number of affiliates, lies within the range of $p \in [0.5; 0.8]$ (see, e.g., Altschuler et al., 2011; De Simone et al., forthcoming; Hopland et al., 2016). For the upper bound of success probabilities the government most likely favors SA over FA. For intermediate probabilities of success the decision whether to switch to FA or to remain with SA becomes much more delicate.

5.3 Long-run analysis

In this subsection, we analyze the long-run equilibrium. In the long run, governments can adjust their tax rates in response to the change in the tax system. Tax revenues under the two systems now read

$$T^{FA} = t^{FA} p \left[ f(k^{FA}) - (2 - p)k^{FA} \right],$$

$$T^{SA} = t^{SA} p \left[ f(k^{SA}) - k^{SA} \right].$$ (33) (34)

Similar to the medium-run analysis, capital investment is not distorted by taxes and hence tax bases are equivalent under both systems if the likelihood of making losses is zero. In this case, the difference in long-run tax revenues is only determined by the tax rate differential between the two tax systems. From (23) we know that the tax rate under FA is highest at $p = 1$ whereas it is lowest under SA if the costs of profit shifting are sufficiently low. Moreover, the lower the costs of profit shifting for the MNE the lower is the equilibrium tax rate under SA. Hence, if the MNE has a sufficiently great potential to shift profits, the tax rate and therefore also tax revenues will be larger under FA than under SA if the success probability is sufficiently large.
Moreover, in Appendix A.4 we show that a reduction in the probability of success, when starting from a high level, reduces the benefits of FA relative to SA irrespective of the costs of profit shifting. Due to the complexity inherent to the model, it is however not possible to derive more general analytical results at this stage. Therefore, we complement our analysis with numerical simulations using the same functional forms and parameter values as in the previous subsection. Our analytical results derived hitherto are confirmed by the simulation results. For high probabilities of success, tax revenues are larger under FA than under SA if profit shifting is sufficiently severe. Further, a reduction in $p$ when starting at high levels reduces the difference in tax revenues, i.e. $\Delta^{LR} = T^{FA} - T^{SA}$, irrespective of the costs of profit shifting. In addition, our analysis shows that for intermediate costs of profit shifting a cut-off value for $p$ exists for which the decision to prefer one over the other tax system is reverted. For success probabilities smaller than the cut-off level, the government prefers SA whereas the opposite is true for larger probabilities of success. The stronger the engagement in profit shifting under SA, the lower is this cut-off level. We summarize our results in:

**Proposition 3** In the long run, where both the MNE and the government can adjust their behavior, a switch from separate accounting to formula apportionment is only beneficial if the probability of success is sufficiently high and the costs related to profit shifting are sufficiently low.
Interestingly, the long-run conditions for a beneficial switch from SA to FA are qualitatively polar to the medium-run conditions. Since we have shown that tax revenues are unambiguously smaller under FA in the short-run, governments, which decide to switch to FA based on the long-run benefits, may likely face a transitional period of tax revenue losses until the benefits of system switch from SA to FA fully materialize.

6 Formula dependent tax competition under FA

In this section, we focus on tax competition within the tax system of FA. Given that governments decide to switch from SA to FA, the question remains which weights to use in the apportionment formula. As the possibility of incurring a loss creates a qualitative difference between output and input shares in the apportionment formula, it is interesting to know how a change of the apportionment factor weights affects tax competition under FA. To answer this question, we first derive the tax externality, i.e. the effect of one country’s tax rate on the other country’s tax revenues which is given by

\[
\frac{\partial T_i}{\partial t_j} = t_i \frac{\partial \Gamma_i}{\partial t_j} = t_i \left[ \frac{\partial \Gamma_i}{\partial k_i} \frac{\partial k_i}{\partial t_j} + \frac{\partial \Gamma_i}{\partial k_j} \frac{\partial k_j}{\partial t_j} \right].
\] (35)

Using the comparative static results in (19) and the symmetry condition, the tax externality simplifies to

\[
\frac{\partial T_i}{\partial t_j} = \frac{2t}{SOC_k} \frac{\partial \Gamma_i}{\partial k_i} \frac{\partial \Gamma_j}{\partial k_j} > 0.
\] (36)

The positive externality gives rise to inefficiently low tax rates under FA. The reason is that an increase in country j’s tax rate induces the MNE to reduce its investment in j and to increase it in i to minimize tax payments. Consequently, country i’s expected share of the consolidated tax base increases due to a higher capital share and a higher expected sales share.

To answer the question how a change in the factor weights affects tax competition, we differentiate the tax externality in (35) with respect to \( \lambda \), the weight of capital shares,
and get\textsuperscript{14}

\[
\frac{\partial^2 T_i}{\partial t_j \partial \lambda} = \frac{2\Gamma(1-p)^4}{\Omega^2(1-t)^2} \left( \frac{\partial \Gamma_i}{\partial k_i} - \frac{\partial \Gamma_j}{\partial k_i} \right) \frac{\partial^2 \Gamma_i}{\partial k_i \partial \lambda}.
\] (37)

The first two factors in (37) are non-negative. Hence, the sign is determined by the last factor which is given by

\[
\frac{\partial^2 \Gamma_i}{\partial k_i \partial \lambda} = \frac{1}{2} (1 - \varepsilon) \left[ p \frac{f'}{k} - [1 - (1 - p)^2] \right] - p(1-p)\varepsilon,
\] (38)

where \(\varepsilon = kf'(k)/f(k)\) is the elasticity of the production function with respect to capital and can be interpreted as the share of capital in the production function. The first term in (38) is positive. This implies that tax competition increases if the weight of capital shares \(\lambda\) increases. Specifically, a high weight of capital shares allows the MNEs to manipulate the formula more directly via changing investments (see, e.g., Eichner and Runkel, 2008; Pethig and Wagener, 2007). The second term in (38), which we label 'insurance effect', is negative whenever \(0 < p < 1\). Thus, tax competition is mitigated if the weight of capital shares increases. Intuitively, a higher share of property in the formula increases the expected tax base of a country hosting an unsuccessful affiliate given the affiliate in the other country is successful. This effect is stronger the larger the share of capital in the production function \(\varepsilon\). Hence, under certain conditions the insurance effect may dominate the positive first factor in (38) resulting in a mitigation of tax competition whenever the weight of capital shares is increased. With a pure sales formula (\(\lambda = 0\)), the tax base of the country hosting an unsuccessful affiliate is always zero. We summarize our findings in:

**Proposition 4** A higher weight of capital shares reduces tax competition under FA for intermediate probabilities of success and a sufficiently large share of capital in the production function.

The possibility that a system of FA insures governments against a shortfall in tax revenues in case of unsuccessful affiliates has also been mentioned by Gérard and Weiner (2003, 2006). Their analysis is, however, silent about the different roles apportionment factors play in the presence of firm losses. Indeed, our analysis shows that such an

\textsuperscript{14}The full derivation is relegated to Appendix A.5.
insurance effect is never present if apportionment is solely based on sales, but it only emerges for input related apportionment factors.

In some countries where FA is used, the sales factor has become increasingly important. In the U.S., for example, many states have turned to double-weighted sales formula, and some even apply a sales-only formula, while payroll and property have identical weights. This seems to be a reasonable practice given that the formula can only be manipulated in an indirect way which reduces tax competition (see Eichner and Runkel, 2008). However, it is also true that in certain countries payroll plays a more important role in the formula than property (and sometimes sales). In Canada, for example, payroll and sales enter the formula with equal weights, whereas the German system only uses payroll as an allocation factor. An obvious justification why payroll is attributed a higher weight than capital is that it is harder to manipulate payroll shares.

Although our model does not explicitly model the payroll factor, we can interpret the fixed factor in our model, which gives rise to pure profits, as labor. Empirical estimates suggest that the production share of labor is about two thirds (see, e.g., Jones and Williams, 2000; Eicher and Turnovsky, 2001; and Steger, 2005). Following our analysis it is therefore more likely that an increase in the weight of payroll shares (instead of an increase in the weight of capital shares) reduces tax competition. Recalling that the probability of incurring a loss can be as high as 0.5, our model provides a novel justification for having a high weight of payroll shares. It is the insurance against the shortfall of tax revenues in case affiliates are unsuccessful. These consideration should be taken into account if the EU decides to switch to a tax system of FA.

7 Extensions

In this section we extend our analysis in two directions. In Section 7.1 we modify the governments’ objective function to account for the profits of the MNE. In Section 7.2

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15See Martens-Weiner (2005) for an extensive discussion on common consolidated base taxation with formula apportionment.

16It is straightforward to extend our model by endogenizing the MNE’s labor input decision and thus to also capture payroll as an apportionment factor.
we introduce asymmetries between countries and compare the two tax systems in a setting where redistributive effects between the two countries are present.

7.1 Welfare considerations

In the first extension, we analyze how the comparison between SA and FA is affected when each government’s objective function includes a fraction $\gamma < 1$ of the MNE’s profits. The welfare discount on MNE’s profits either reflects the fact that raising corporate tax revenue is important for society (either for redistributive reasons or to reduce other distortive taxes), or that the MNE is partly owned by foreign investors who are not considered for national welfare. In this setting, national welfare in country $i$ equals the weighted sum of the MNE’s net-of-tax profits and country $i$’s tax revenues. The different effects resulting from a switch from a system of SA to a system of FA are isolated by again analyzing three separate scenarios to which we refer as short, medium and long run.

In the short run, tax rates and capital investments remain unchanged. Thus, welfare under FA and SA takes the form

$$
W^{FA} = \gamma \left[ pf(k^{SA}) - k^{SA} \right] + (1 - \gamma) pt^{SA} \left[ f(k^{SA}) - (2 - p)k^{SA} \right],
$$

(39)

$$
W^{SA} = \gamma \left[ pf(k^{SA}) - k^{SA} \right] + (1 - \gamma) pt^{SA} \left[ f(k^{SA}) - k^{SA} \right],
$$

(40)

and the difference in welfare is

$$
\Delta^{SR} = -(1 - \gamma) pt^{SA} (1 - p) k^{SA} \leq 0.
$$

(41)

Since economic profits are identical in the short run, equation (41) is almost identical to the one under tax revenue maximization, see (28), except for the multiplicative weighting factor $(1 - \gamma)$. Thus, if governments care to a great extent about the MNE’s profits, $(\gamma \rightarrow 1)$, welfare under SA and FA becomes identical.

In the medium run, behavioral responses of the MNE are considered while the governments continues to levy the same tax rate as under SA. Under these assumptions,
welfare in the symmetric equilibrium is determined by

\[ W^{FA} = \gamma \left[ pf(k^{FA}) - k^{FA} \right] + (1 - \gamma) pt^{SA} \left[ f(k^{FA}) - (2 - p)k^{FA} \right], \quad (42) \]
\[ W^{SA} = \gamma \left[ pf(k^{SA}) - k^{SA} \right] + (1 - \gamma) pt^{SA} \left[ f(k^{SA}) - k^{SA} \right], \quad (43) \]

The difference between revenue versus welfare maximizing governments culminates in each of the first terms in (42) and (43), respectively, which represent the MNE’s economic profits under each tax systems. For a given tax rate, what confines the medium run scenario in our analysis, economic profits are always larger under FA than under SA due to the possibility to offset losses under FA. If the government is mostly interested in maximizing the MNE’s profits \((\gamma \rightarrow 1)\), welfare is strictly larger under FA than under SA. Thus, the inclusion of the MNE’s profits into the government’s objective function increases the benefits associated switching from SA to FA, and this relaxes the conditions summarized in Proposition 2 which render a switch from SA to FA beneficial in the medium run.

In the long run, both the MNE and the government respond to the system switch form SA to FA and thus welfare is given by

\[ W^{FA} = \gamma \left[ pf(k^{FA}) - k^{FA} \right] + (1 - \gamma) pt^{FA} \left[ f(k^{FA}) - (2 - p)k^{FA} \right], \quad (44) \]
\[ W^{SA} = \gamma \left[ pf(k^{SA}) - k^{SA} \right] + (1 - \gamma) pt^{SA} \left[ f(k^{SA}) - k^{SA} \right]. \quad (45) \]

The difference compared to the medium run scenario originate from the fact that the government now adjusts its tax rate under FA.

Similar to tax revenue maximization, under welfare maximization capital investment are not distorted by taxes if the probability of making losses is zero, i.e., \(p = 1\), and thus economic profits are the same under SA and FA. As a consequence, a sufficient statistic, indicating whether a switch from SA to FA is beneficial in the long run, is given by the relative size of tax rates, i.e., whether the tax rate under FA is larger than under SA. This condition is identical to the one under tax revenue maximization, and thus we conclude that our results derived under tax revenue maximizing governments are not qualitatively different from those of welfare maximizing governments as long as the weight governments assign to MNE profits is not too large.
7.2 Asymmetric countries

Our analysis has so far focused on the case of symmetric countries. This is a suitable benchmark for analyzing allocative inefficiencies. However, in practice a switch from SA to FA will almost always involve redistributive tax revenue effects. This more realistic setting is, however, overly complex and analytical solutions for the tax competition game cannot be established. We therefore rely on numerical simulations to draw conclusions about the redistributive issues. A simple way to introduce asymmetries in the model is to assume varying probabilities of success for the affiliates in the two countries while keeping identical production functions for the affiliates.

Figure 3: Differences in tax revenue under FA and SA for asymmetric countries and low costs of profit shifting.

From the previous analysis we learned that in the short run, where capital investments and tax rates are fixed, tax revenues unambiguously decline after a switch form SA to FA and this holds irrespective of the success probability of the affiliates. Thus, we focus the analysis on the medium- and long-run consequences resulting from the system switch for each country. Similar to the symmetric case, we again analyze three scenarios under SA including low, intermediate and high costs of profit shifting. In all scenarios, the probability of success in country $a$ is kept constant at $p_a = 0.5$ and we simulate the tax revenue consequences for both countries for varying levels of $p_b$. The functional form of the concealment costs and the factor weights in the apportionment formula are defined as before.

Figure 3 presents the simulation results in the case of low costs of profit shifting under SA. The left panel displays the medium-run results while the right panel shows the
results for the long run. For the medium run we find that, in contrast to the symmetric case where a switch from SA to FA always reduces tax revenues when profit shifting is substantial, a switch from SA to FA can be beneficial for either country. This, however, critically depends on the probability of success in country $b$. Only for low levels of $p_b$ the switch is beneficial for country $a$ while it is beneficial for country $b$ only for large levels of $p_b$. The reasoning is that hosting the affiliate with a higher probability of success increases tax revenues because a larger share of the consolidated tax base is apportioned to that country (cf. equation (15)). If countries are sufficiently asymmetric, this effect dominates the loss arising from the tax base consolidation in the country which hosts the more successful affiliate. For the long run scenario, the simulations show that, similar to the symmetric case, a switch from SA to FA is, in principle, beneficial for both countries. However, the country hosting the affiliate with the higher success probability benefits to a greater extent from the switch. Again, the reasoning is based on the finding that a larger fraction of the consolidated corporate tax base is allocated to this country.

Figure 4: Differences in tax revenue under FA and SA for asymmetric countries and low costs of profit shifting.

Figure 4 presents the simulation results for the case of high costs of profit shifting under SA. Again, the left panel displays the medium-run results while the right panel shows the results for the long run. The graph for the medium run is basically the same as in the case with low costs of profit shifting. The only difference is that for both countries the range of values increases for which the switch from SA to FA is beneficial. The explanation for this finding is the same as in the symmetric case. If tax rates are high under SA the investment distortion is large and hence a switch from SA
to FA brings about a substantial positive impact on investments. Additionally, in the medium run there exists a small range of probabilities for which the system switch is beneficial for both countries. The graph representing the long-run scenario (right panel of Figure 4) indicates that a switch from SA to FA is basically detrimental for both countries. Similar to the case with low costs of profit shifting, the country hosting the more successful affiliate experiences the smaller loss in tax revenues accompanying the system switch. The reasoning is again similar to the one in the symmetric case. A low potential of profit shifting leaves the equilibrium tax rate high and hence ensures high tax revenues.

Figure 5: Differences in tax revenue under FA and SA for asymmetric countries and intermediate costs of profit shifting.

Finally, Figure 5 displays the results for the case of intermediate costs of profit shifting. The medium-run and the long-run results are a convex combination of the two scenarios discussed above. The main takeaway from Figure 5 is that, in the long-run, the country hosting the more successful affiliate may also benefit from the system switch if countries are sufficiently asymmetric.

The analysis of country asymmetries has shown that distributional aspects may play an important role, especially in the case where the costs of concealing profit shifting are of intermediate size. This finding raises the question whether countries are willing to form a FA union, knowing that a system switch is most likely not mutually beneficial. A possible solution to this problem could be the installment of side payments between the countries. The countries benefiting from the system switch could "buy in" the loosing countries by granting them a compensation payment which ensures that the tax revenues of the loosing countries under FA are at least as high as they were under
SA. The existence of such a compensation scheme increase efficiency whenever the sum of the tax revenues under FA is larger than under SA. Figure 6 displays the difference of the sum of tax revenues under FA and SA. Both graphs in Figure 6 have qualitatively strong similarities with the symmetric case. In the medium run a switch from SA to FA is more likely to be beneficial if the costs of profit shifting are sufficiently high and/or the success probability in country $b$ is sufficiently low. In the long run, the system switch is only beneficial under qualitatively polar conditions to the ones that render the switch beneficial in the medium run. The only difference to the symmetric case is that the switch from SA to FA is, irrespective of the concealment cost function, mutually beneficial in the medium and long run if the success probability in country $b$ is sufficiently low. From an empirical point of view, this case might be less relevant.

8 Conclusion

In its recent directives, the European Commission proposes the introduction of a CCTB and later on a CCCTB within Europe (European Commission, 2016). Thus, the directives imply a switch from the current system of separate accounting to a system where multinationals first have to consolidate all entities’ profits which are then apportioned to countries based on a certain formula. The existing literature has extensively analyzed the properties and implications emerging from a switch between the two systems. One drawback associated with the previous studies is that the investment decisions of firms is usually assumed to be free of risk and therefore corporate losses never occur.
In this paper, we analyze the implications of losses for the decision to choose one tax system over the other. In the short run, where neither the MNE nor the government can react to the system switch, tax revenues unambiguously fall following a switch from SA to FA. In the medium run, where only the MNE but not the government can adjust its strategy, tax revenues decline as well if the affiliates’ probability of success is sufficiently large and the MNE can shift profits sufficiently easy under SA. Under these conditions, the switch, however, turns out beneficial in the long run, i.e., in the situation where the MNE as well as the government can re-optimize after a change in the tax system. The polar difference of the medium- and long-run conditions which render a switch from SA to FA beneficial, implies that it may take some time until the benefits of the switch actually materialize.

In a situation where the switch of the tax system towards FA is beneficial, we find that the existence of potential losses creates a qualitative difference between input and output factors in the apportionment formula and that this difference is relevant for tax competition incentives. With losses, output factors tend to be more volatile. Therefore, a higher weight of input factors, such as capital or payroll, might be preferable but it also has two competing effects on tax competition. On the one hand, tax competition is aggravated because the MNE’s incentives to distort investment, i.e., to locate a larger share of the tax base in the low tax country, increases. On the other hand, a high input factor share serves as an insurance for tax revenue collection and this effect is stronger the larger the input share in the production function. Therefore, our model provides a novel argument why a high weight of input factors in the apportionment formula might be desirable.
A Appendix

A.1 Deriving the tax effects under formula apportionment

To determine the tax effects under FA, we need to determine the signs of \( \frac{\partial \Gamma_i}{\partial k_i} \) and \( \frac{\partial \Gamma_j}{\partial k_j} \).

Differentiating \( \Gamma_i \) with respect to \( k_i \) and \( k_j \) yields

\[
\frac{\partial \Gamma_i}{\partial k_i} = \left[ \lambda \frac{k_j}{k_i + k_j} + (1 - \lambda) \frac{f(k_j) f'(k_i)}{f(k_i) + f(k_j)} \right] \\
\times \left\{ p_i f(k_i) + p_j f(k_j) - [1 - (1 - p_i)(1 - p_j)](k_i + k_j) \right\} \\
+ A_i \{ p_i f'(k_i) - [1 - (1 - p_i)(1 - p_j)] \}
\]

\[
- (1 - \lambda) \frac{f(k_j) f'(k_i)}{f(k_i) + f(k_j)^2} p_i (1 - p_j) [f(k_i) - k_i - k_j]
\]

\[
- (1 - \lambda) \frac{f(k_j)}{f(k_i) + f(k_j)} p_j (1 - p_i) [f'(k_i) - 1]
\]

\[
+ (1 - \lambda) \frac{f(k_i)}{f(k_i) + f(k_j)} p_j (1 - p_i)
\]  

(A.1)

\[
\frac{\partial \Gamma_i}{\partial k_j} = - \left[ \lambda \frac{k_i}{k_i + k_j} + (1 - \lambda) \frac{f(k_i) f'(k_j)}{f(k_i) + f(k_j)} \right] \\
\times \left\{ p_i f(k_i) + p_j f(k_j) - [1 - (1 - p_i)(1 - p_j)](k_i + k_j) \right\} \\
+ A_i \{ p_j f'(k_j) - [1 - (1 - p_i)(1 - p_j)] \}
\]

\[
+ (1 - \lambda) \frac{f(k_i) f'(k_j)}{f(k_i) + f(k_j)} p_j (1 - p_j) [f(k_j) - k_i - k_j]
\]

\[
+ (1 - \lambda) \frac{f(k_i)}{f(k_i) + f(k_j)} p_i (1 - p_j) [f'(k_j) - 1]
\]

\[
- (1 - \lambda) \frac{f(k_j)}{f(k_i) + f(k_j)} p_i (1 - p_j)
\]  

(A.2)
Using the symmetry condition, the two derivatives simplify to

\[
\frac{\partial \Gamma_i}{\partial k_i} = \frac{1}{2} \left[ \lambda \frac{1}{k} + (1 - \lambda) \frac{f_k}{f} \right] \{ p f - [1 - (1 - p)^2] k \}
+ \frac{1}{2} \{ p f_k - [1 - (1 - p)^2] \}
- \frac{1}{2} (1 - \lambda) \frac{f_k}{f} p(1 - p)[f - 2k]
+ \frac{1}{2} (1 - \lambda) p(1 - p) f_k
\]

(A.3)

\[
\frac{\partial \Gamma_i}{\partial k_j} = -\frac{1}{2} \left[ \lambda \frac{1}{k} + (1 - \lambda) \frac{f_k}{f} \right] \{ p f - [1 - (1 - p)^2] k \}
+ \frac{1}{2} \{ p f_k - [1 - (1 - p)^2] \}
+ \frac{1}{2} (1 - \lambda) \frac{f_k}{f} p(1 - p)[f - 2k]
- \frac{1}{2} (1 - \lambda) p(1 - p) f_k
\]

(A.4)

By rearranging terms, the two expressions reduce to

\[
\frac{\partial \Gamma_i}{\partial k_i} = \frac{1}{2} \left[ \lambda \frac{1}{k} + (1 - \lambda) \frac{f_k}{f} \right] \{ p f - [1 - (1 - p)^2] k \}
+ \frac{1}{2} \{ p f_k - [1 - (1 - p)^2] \}
+ (1 - \lambda) p(1 - p) \frac{f_k}{f} k > 0,
\]

(A.5)

which is unambiguously greater than zero and

\[
\frac{\partial \Gamma_j}{\partial k_j} = -\frac{1}{2} \left[ \lambda \frac{1}{k} + (1 - \lambda) \frac{f_k}{f} \right] \{ p f - [1 - (1 - p)^2] k \}
+ \frac{1}{2} \{ p f_k - [1 - (1 - p)^2] \}
- (1 - \lambda) p(1 - p) \frac{f_k}{f} k.
\]

(A.6)
By rearranging terms, $\frac{\partial \Gamma_i}{\partial k_j}$ can be written as

$$\frac{\partial \Gamma_i}{\partial k_j} = -\frac{\lambda pk}{f} (1 - \varepsilon) - (1 - \lambda)p \left[ 1 - \frac{p}{2} (1 + \varepsilon) \right] < 0,$$

(A.7)

where $\varepsilon = k f / f < 1$. The derivative is unambiguously smaller than zero as the term in the first squared brackets is greater than zero and the term in the second squared brackets is smaller than 2 due to the concavity of $f$.

### A.2 The effect of $p$ on the equilibrium tax rate under FA

Starting from equation (23) a change in $p$ affects the equilibrium tax rate in the following way:

$$\frac{\partial t_i}{\partial p} = -\frac{\partial^2 T_i}{\partial t_i \partial p} \frac{\partial^2 T_i}{\partial (t_i)^2} > 0.$$  

(A.8)

In the following, we will explicitly derive and sign the numerator and the denominator.

Starting with the denominator we get

$$\frac{\partial^2 T_i}{\partial (t_i)^2} = 2 \left( \frac{\partial \Gamma_i}{\partial k_i} + \frac{\partial \Gamma_i}{\partial k_j} \frac{\partial k_j}{\partial t_i} \right) + t_i \left[ \frac{\partial^2 \Gamma_i}{\partial (k_i)^2} \left( \frac{\partial k_j}{\partial t_i} \right)^2 + \left( \frac{\partial^2 \Gamma_i}{\partial k_j \partial k_i} \right) \frac{\partial k_j}{\partial t_i} \frac{\partial k_i}{\partial t_i} + \frac{\partial^2 \Gamma_i}{\partial (k_j)^2} \left( \frac{\partial k_j}{\partial t_i} \right)^2 \right].$$

(A.9)

Using the symmetry condition in conjunction with equation (19) and the fact that under symmetry $\frac{\partial^2 \Gamma_i}{\partial k_i \partial k_j} = 0$, we can rewrite the denominator to

$$\frac{\partial^2 T_i}{\partial (t_i)^2} = \frac{2}{p(1-t)f^n} \left[ \left( \frac{\partial \Gamma_i}{\partial k_i} \right)^2 + \left( \frac{\partial \Gamma_i}{\partial k_j} \right)^2 \right] + \frac{t}{p(1-t)f^n} \left[ \frac{\partial^2 \Gamma_i}{\partial (k_i)^2} \left( \frac{\partial \Gamma_i}{\partial k_i} \right)^2 + \frac{\partial^2 \Gamma_i}{\partial (k_j)^2} \left( \frac{\partial \Gamma_i}{\partial k_j} \right)^2 \right].$$

(A.10)

Because $\frac{\partial^2 \Gamma_i}{\partial (k_i)^2} < 0$ and $\left| \frac{\partial^2 \Gamma_i}{\partial (k_i)^2} \right| > \left| \frac{\partial^2 \Gamma_i}{\partial (k_j)^2} \right|$ as well as $\left| \frac{\partial \Gamma_i}{\partial k_i} \right| > \left| \frac{\partial \Gamma_i}{\partial k_j} \right|$ it must be that the denominator is negative.
The derivation of the numerator is given by
\[
\frac{\partial^2 T_i}{\partial t \partial p} = \frac{\partial \Gamma_i}{\partial p} + t_i \left( \frac{\partial^2 \Gamma_i}{\partial k_i \partial p} \frac{\partial}{\partial t_i} + \frac{\partial \Gamma_i}{\partial k_i} \frac{\partial^2}{\partial t_i \partial p} + \frac{\partial^2 \Gamma_i}{\partial k_j \partial p} \frac{\partial}{\partial t_i} + \frac{\partial \Gamma_i}{\partial k_j} \frac{\partial^2}{\partial t_i \partial p} \right)
\]  

(A.11)

Using equations (18), (19) and the symmetry condition, we get
\[
\frac{\partial^2 T_i}{\partial t \partial p} = \frac{\partial \Gamma_i}{\partial p} + \frac{t}{p(1-t)f''} \left\{ 2 \left( \frac{\partial^2 \Gamma_i}{\partial k_i \partial p} \frac{\partial}{\partial k_i} + \frac{\partial \Gamma_i}{\partial k_j} \frac{\partial^2}{\partial k_j \partial p} \right) - \frac{1}{p} \left[ \left( \frac{\partial \Gamma_i}{\partial k_i} \right)^2 + \left( \frac{\partial \Gamma_i}{\partial k_j} \right)^2 \right] \right\}
\]  

(A.12)

where \( \frac{\partial \Gamma_i}{\partial p} = f(k) - p(2-p)k + \frac{1-p(2-p)}{1-t} > 0 \). Since the second term in curly brackets is unambiguously negative, a sufficient condition for the sign of the derivation to be positive is that the first term in curly brackets is negative. Moreover, we know that \( \left| \frac{\partial \Gamma_i}{\partial k_i} \right| > \left| \frac{\partial \Gamma_i}{\partial k_j} \right| \) and by equation (18) that \( \frac{\partial \Gamma_i}{\partial k_i} = -\frac{\partial \Gamma_i}{\partial k_j} - \frac{(1-p)^2}{1-t} \). Deriving with respect to \( p \) yields that \( \frac{\partial^2 \Gamma_i}{\partial k_j \partial p} = -\frac{\partial^2 \Gamma_i}{\partial k_j \partial p} - \frac{2(1-p)(1-t)}{f''} \) and thus \( \left| \frac{\partial^2 \Gamma_i}{\partial k_j \partial p} \right| > \left| \frac{\partial^2 \Gamma_i}{\partial k_j \partial p} \right| \). It follows that the sufficient condition making \( \frac{\partial^2 \Gamma_i}{\partial k_i \partial p} \) negative reduces to \( \frac{\partial \Gamma_i}{\partial k_i} < 0 \). Deriving \( \frac{\partial \Gamma_i}{\partial k_i} \) with respect to \( p \) yields
\[
\frac{\partial^2 \Gamma_i}{\partial k_i \partial p} = \frac{1}{2} \left[ \frac{\lambda}{k} + (1-\lambda) \frac{f'}{f} \right] \left( f - 2(1-p)k \right)
\]  

+ \[\frac{1}{2} \left[ \frac{\lambda}{k} + (1-\lambda) \frac{f'(k)}{f} \right] \left( pf' - p(2-p) \right) \frac{\partial k}{\partial p} \]
\[ + \frac{1}{2} \left[ -\lambda \frac{1}{k^2} + (1-\lambda) \frac{f''f - (f')^2}{f^2} \right] \left( pf - p(2-p)k \right) \frac{\partial k}{\partial p} \]
\[ + \frac{1}{2} \left[ f' - 2(1-p) + pf'' \frac{\partial k}{\partial p} \right] \]
\[ + (1-\lambda)(2-p) \frac{kf'}{f} \]
\[ + (1-\lambda)p(1-p) \left( \frac{f'}{f} + \frac{k[f' - (f')^2]}{f^2} \right) \frac{\partial k}{\partial p}. \]  

(A.13)

Heavily rearranging terms leads to
\[
\frac{\partial^2 \Gamma_i}{\partial k_i \partial p} = \frac{\lambda}{2k} \frac{1-p^2 t}{p(1-t)f''} \left[ \frac{f}{k} - f' + p(1-t)f f'' \right] - \frac{1}{2} \frac{p k f'}{f} - (1+\lambda)(1-p)
\]  

\[ - \frac{1}{1-t} \frac{f - pk(1-p)t}{f} - (1-\lambda) \frac{p^2 f'[f - k]}{2} \frac{\partial k}{\partial p} - \frac{(1-p)t}{1-t}. \]  

(A.14)
Only the first term has an ambiguous sign, because the combination of the first two terms in the squared brackets is positive due to the concavity of the production function while the third term is negative. The third term will be overcompensated by the first two terms if $f''$ is not too large.

### A.3 Comparison of medium-run tax revenues

Starting from (31), differentiating $\Phi$ with respect to $p$ yields

\[
\frac{\partial \Phi}{\partial p} = k^{FA} + \left[ f'(k^{FA}) - (2-p) \right] \left[ \frac{\partial k^{FA}}{\partial p} + \frac{\partial k^{FA}}{\partial t^{SA}} \frac{\partial t^{SA}}{\partial p} \right]
\]

\[\] - \left[ f'(k^{SA}) - 1 \right] \left[ \frac{\partial k^{SA}}{\partial p} + \frac{\partial k^{SA}}{\partial t^{SA}} \frac{\partial t^{SA}}{\partial p} \right]. \tag{A.15}

Using (6) and (19) we get

\[
\frac{\partial \Phi}{\partial p} = k^{FA} + \frac{1-p}{p^2 (1-t^{SA})^3 f''} \left[ (1-t^{SA}) \left[ 1 + p(1-p)t^{SA} \right] - p(1-p)(2-p) \frac{\partial t^{SA}}{\partial p} \right], \tag{A.16}
\]

as stated in equation (32) in the main text.

In the following, we will show that if profit shifting is sufficiently strong, medium tax revenues will always be lower under FA compared to SA. To do this, we first rewrite $\partial \Phi/\partial p = k^{FA} + \Psi(1-p)/[p^2(1-t^{SA})^3 f'']$ and differentiate with respect to $p$ which yields

\[
\frac{\partial^2 \Phi}{\partial p^2} = \frac{\partial k^{FA}}{\partial p} + \frac{\partial k^{FA}}{\partial t^{FA}} \frac{\partial t^{FA}}{\partial p} \]

\[\] - \frac{p^2(1-t^{SA})^3 + (1-p)}{p^4 (1-t^{SA})^6 f''} \left[ 2p(1-t^{SA})^3 - 3p^2(1-t^{SA})^2 \frac{\partial t^{SA}}{\partial p} \right]

\[\] - \frac{1-p}{p^2 (1-t^{SA})^3 f''} \left[ 1 + (1-2p)(2-p) - p(1-p)(1-2t^{SA}) \right] \frac{\partial t^{SA}}{\partial p}

\[\] + \frac{1-p}{p^2 (1-t^{SA})^3 f''} \left[ (1-2p)t^{SA}(1-t^{SA}) - p(1-p) \left( (2-p) \frac{\partial^2 t^{SA}}{\partial p^2} - \frac{\partial t^{SA}}{\partial p} \right) \right] \tag{A.17}
Using (13) and heavily rearranging terms delivers

$$\frac{\partial^2 \Phi}{\partial p^2} = \chi \left\{ (1-p)(1-t^{SA})^2[1-t^{SA}(1-t^{SA})] + p(1-p)^2t^{SA}(1-t^{SA})[3-2(1-t^{SA})] \right\}$$

$$+ \chi \left\{ 2p^2(1-p)t^{SA}(1-t^{SA})^3 + 2p(1-p)^2t^{SA}(1-t^{SA})^4 + \frac{2-p}{2}(1-p)^2t^{SA}(1-t^{SA})^3 \right\}$$

$$+ \chi \left\{ p(1-p)(2-p)t^{SA}(1-t^{SA})^3 + 2p(1-p^2t^{SA})(1-t^{SA})^3 + p(1-p)^2t^{SA}(1-t^{SA})^3 \right\}$$

$$+ \chi \left\{ 2p(1-t^{SA})^2 + 2p^2(1-p)t^{SA}(1-t^{SA})^2 + p(1-p)(2-p)t^{SA}(1-t^{SA}) \right\}$$

$$+ \left\{ 3(1-p)(1-t^{SA}) + p(1-p^2t^{SA}(1-t^{SA})^2 + \frac{3}{2}(1-p)^2(2-p)t^{SA} \right\}$$

$$+ \chi \frac{1}{2}(1-p)^2(2-p)t^{SA}(1-t^{SA}). \quad (A.18)$$

Since $\chi = -[2p^3(1-t^{SA})^4 f'']^{-1} > 0$, $\partial^2 \Phi/\partial p^2$ is unambiguously positive. This means that for low costs of profit shifting there exists one global minimum for the tax revenue differential.

Moreover, and as already noted in the main text, because tax revenues are smaller under FA for high and low probabilities of success it must be that tax revenues under FA are globally smaller than under SA due to the unambiguous positive sign of $\partial^2 \Phi/\partial p^2$ for low costs of profit shifting.

If instead profit shifting is very costly for firms $\partial t^{SA}/\partial p$ is positive and the sign of the second term in $\partial \Phi/\partial p$ becomes ambiguous. Plugging in equation (12) into (32) gives

$$\frac{\partial \Phi}{\partial p} = k^{FA} + \frac{(1-p)(1-t^{SA})^2 f''}{p^2(1-t^{SA})^3 f''} \left[ 1 + p(1-p)t^{SA} - (2-p)\frac{1+t^{SA}}{2+t^{SA}} \right]. \quad (A.19)$$

Obviously, the term in squared brackets is positive for high probabilities of success. However, for low values of $p$ the term in squared brackets is negative. As the multiplier in front of the squared brackets is also negative and very large for low $p$, $\partial \Phi/\partial p$ is positive for low probabilities of success. Since tax revenues under FA and SA converge when $p \to 0$, it must be that for small $p$ tax revenues under FA are larger than under SA in the medium run if costs of profit shifting are sufficiently large.
### A.4 Comparison of long-run tax revenues

To analyze the implications of losses in the long-run, we define the difference in long-run tax revenues as

\[
\Delta^{LR} = p\{t^{FA}[f(k^{FA}) - (2 - p)k^{FA}] - t^{SA}[f(k^{SA}) - k^{SA}]\} \equiv p\Theta. \tag{A.20}
\]

Because \(\Delta^{LR} = 0\) if the probability of making a loss is zero, it is sufficient to analyze the sign of \(\Theta\) to draw conclusions about the sign of \(\Delta^{LR}\) around \(p = 1\). Differentiating \(\Theta\) with respect to \(p\) yields

\[
\begin{align*}
\frac{\partial \Theta}{\partial p} &= t^{FA}[f'(k^{FA}) - (2 - p)] \left( \frac{\partial k^{FA}}{\partial p} + \frac{\partial k^{FA}}{\partial t^{FA}} \frac{\partial t^{FA}}{\partial p} \right) \\
&+ [f(k^{FA}) - (2 - p)k^{FA}] \frac{\partial t^{FA}}{\partial p} - t^{SA}[f'(k^{SA}) - 1] \left( \frac{\partial k^{SA}}{\partial p} + \frac{\partial k^{SA}}{\partial t^{SA}} \frac{\partial t^{SA}}{\partial p} \right) \\
&- [f(k^{SA}) - k^{SA}] \frac{\partial t^{SA}}{\partial p} \\
&= t^{FA} \left( 1 - p \right)^2 \left( \frac{\partial k^{FA}}{\partial p} + \frac{\partial k^{FA}}{\partial t^{FA}} \frac{\partial t^{FA}}{\partial p} \right) \\
&+ [f(k^{FA}) - (2 - p)k^{FA}] \frac{\partial t^{FA}}{\partial p} - t^{SA} \frac{1 - p}{p(1 - t^{SA})} \left( \frac{\partial k^{SA}}{\partial p} + \frac{\partial k^{SA}}{\partial t^{SA}} \frac{\partial t^{SA}}{\partial p} \right) \\
&- [f(k^{SA}) - k^{SA}] \frac{\partial t^{SA}}{\partial p}. \tag{A.21}
\end{align*}
\]

Evaluating at \(p = 1\) delivers

\[
\left. \frac{\partial \Theta}{\partial p} \right|_{p=1} = [f(k) - k] \left( \frac{\partial t^{FA}}{\partial p} - \frac{\partial t^{SA}}{\partial p} \right), \tag{A.22}
\]

where we suppressed the superscript for \(k\) because capital investments are identical at \(p = 1\). If the extent of profit shifting is sufficiently large under SA, then \(\frac{\partial t^{SA}}{\partial p} < 0\). Since \(\frac{\partial t^{FA}}{\partial p} > 0\) the derivation is positive at \(p = 1\). Moreover, because \(t^{FA} > t^{SA}\) and, in turn, \(\Delta^{LR} > 0\) when profit shifting is sufficiently severe, this means that reducing \(p\) from a high level decreases the benefits of switching from SA to FA.

Instead, if profit shifting is sufficiently costly for the MNE, then \(\frac{\partial t^{SA}}{\partial p} > 0\). Moreover, for very high probabilities of success \(\frac{\partial t^{SA}}{\partial p}\) gets very large such that \(\left. \frac{\partial \Delta^{LR}}{\partial p} \right|_{p=1} < 0\). Because \(t^{SA} > t^{FA}\) and, in turn, \(\Delta^{LR} < 0\) if costs of profit shifting are sufficiently low,
\[ \Delta^{LR} \text{ will further decrease meaning that the costs of switching to FA increase for high probabilities of success. Thus, irrespective of the costs of profit shifting, } \Delta^{LR} \text{ decreases for very high probabilities of success.} \]

### A.5 Signing the effect of \( \lambda \) on the tax externality under formula apportionment

Differentiating (36) with respect to \( \lambda \) yields

\[
\frac{\partial^2 T_i}{\partial t_j \partial \lambda} = -\frac{2\Gamma_i}{\Omega} \left[ \frac{\partial \Gamma_i}{\partial k_i} \frac{\partial^2 \Gamma_j}{\partial k_i \partial \lambda} + \frac{\partial \Gamma_i}{\partial k_i} \frac{\partial^2 \Gamma_i}{\partial k_i \partial \lambda} \right] + \frac{2}{\Omega} \frac{\partial \Gamma_i}{\partial k_i} \frac{\partial \Gamma_j}{\partial k_i} \frac{\partial \Omega}{\partial \lambda}, \quad (A.23)
\]

where we make use of \( \frac{\partial \Gamma_i}{\partial \lambda} = 0 \) in a symmetric equilibrium. Using \( \frac{\partial \Gamma_i}{\partial k_j} = -\frac{\partial \Gamma_i}{\partial k_i} + \frac{(1-p)^2}{1-\varepsilon} \), the derivation with respect to \( \lambda \) reads

\[
\frac{\partial^2 \Gamma_i}{\partial k_j \partial \lambda} = -\frac{\partial^2 \Gamma_i}{\partial k_i \partial \lambda}. \quad (A.24)
\]

Differentiating \( \Omega \) with respect to \( \lambda \) yields

\[
\frac{\partial \Omega}{\partial \lambda} = 2 \left[ \frac{\partial \Gamma_i}{\partial k_i} - \frac{\partial \Gamma_j}{\partial k_i} \right] \frac{\partial^2 \Gamma_i}{\partial k_i \partial \lambda}. \quad (A.25)
\]

Collecting terms in (A.23) the effect of \( \lambda \) on the tax externality reads

\[
\frac{\partial^2 T_i}{\partial t_j \partial \lambda} = \frac{2\Gamma_i}{\Omega^2} \left[ \frac{\partial \Gamma_i}{\partial k_i} - \frac{\partial \Gamma_j}{\partial k_i} \right] \left[ \frac{\partial \Gamma_i}{\partial k_i} + \frac{\partial \Gamma_j}{\partial k_i} \right]^2 \frac{\partial^2 \Gamma_i}{\partial k_i \partial \lambda}. \quad (A.26)
\]

Hence,

\[
sign \left( \frac{\partial^2 T_i}{\partial t_j \partial \lambda} \right) = sign \left( \frac{\partial^2 \Gamma_i}{\partial k_i \partial \lambda} \right), \quad (A.27)
\]

where

\[
\frac{\partial^2 \Gamma_i}{\partial k_i \partial \lambda} = \frac{1}{2} (1 - \varepsilon) \left[ p \frac{f}{k} - [1 - (1 - p)^2] \right] - p(1 - p) \varepsilon. \quad (A.28)
\]
References


