

# Capitalist Spirit and the Markets: Why Income Inequality Matters

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## Abstract

I develop a simple, static general-equilibrium model with two classes of individuals, workers and entrepreneurs, and two goods. One good is in fixed supply, interpreted as status-good, and the other is a standard, producible and consumable commodity. All prices are set by firm owners (entrepreneurs) and labor-market competition is modelled ala Bertrand. Even though the model does not feature any type of price rigidity, asymmetric information or labor-market friction, its pure symmetric Nash equilibria produce markedly different results from the canonical competitive equilibrium models: (i) A positive output gap and involuntary unemployment may emerge in equilibrium. (ii) Income and wealth inequality matter for the determination of equilibrium prices and employment. (iii) An increase in income/wealth inequality or of productivity may reduce employment and increase the output gap. As a result, Say's Law may not hold in the economy and minimum-wage policies may have desirable effects in terms of employment and output. The model provides a justification for a number of arguments used in public debates.

**Keywords:** capitalist spirit, general equilibrium, income distribution, income inequality, minimum wage, output gap, unemployment, wealth distribution, wealth inequality

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# 1 Introduction

Many western economies, including US, UK and the European Union, suffer from weak growth, low inflation and extremely low, or even negative, interest rates in the aftermath of the Great Recession. Summers (2014a, 2014b) calls this phenomenon “secular stagnation” and many economists agree that it is a result of weak aggregate demand (Summers, 2014a). However, New-Keynesian models, which are the mainstream demand-side models used by academic economists, have often been criticised on the basis that it is hard to reconcile the persistence of demand weakness with existing evidence of the magnitude of price rigidities. More than eight years after the Great Recession one would expect prices and wages to have converged to their long-run equilibrium value, especially since empirical studies suggest that they adjust at least as frequently as once a year (Klenow and Malin, 2010; Nakamura and Steinsson, 2013). Moreover, income and wealth inequality, a prominent feature of recent public debates, play little role in the standard Keynesian framework. While non-academic economists often postulate that the increase in income inequality in the past two decades is a factor behind the weakness of aggregate demand (The Economist, 2014; Wolf, 2014), academic economists do not validate this argument on a theoretical level. On the contrary, models in this area tend to attribute a positive role to increasing inequalities (see Aghion et al. 1999, for example).

The purpose of this paper is twofold. First, to present a simple theoretical model where demand-side considerations matter for the determination of equilibrium employment and output without the presence of any price or wage rigidities. In doing so, it tries to provide a theoretical justification for the attention that demand-side models have received after the Great Recession, and to address the criticism launched against them, that they require ad-hoc assumptions on price rigidities. Second, to demonstrate how wealth and income inequality are non-neutral if the economy is demand-constrained, meaning that equilibrium employment and prices are implicit functions of the respective distributions of income and wealth.

To achieve these purposes, I develop a simple static general equilibrium model with two goods and two classes of citizens, workers and entrepreneurs. The first good is a standard commodity good that is produced by entrepreneurs’ firms through the workers’ (inelastic) labor supply, while the second good is the means of trade, a measure of wealth and status, and in fixed supply. A natural interpretation of the second good is money or gold, but it could also be any other good in fixed supply (e.g. land or classical

artwork). The key assumptions are that citizens derive utility from holding it and that it is in fixed supply. I allow prices and wages to be determined endogenously by entrepreneurs under an environment of perfect labor-market competition ala Bertrand, where the entrepreneur offering the highest wage rate fills his vacancies first, and a flexible modelling of commodity competition that encompasses both perfect competition and local monopolies as special cases.

Based on the above I analyse the pure symmetric Nash equilibria of the economy. The first main result that comes out of the analysis is that pure symmetric Nash equilibria exist, but they do not necessarily entail full employment. To the contrary, an equilibrium may feature a positive output gap and unemployment. The rest of the results comes out of comparative statics exercises, which demonstrate that the reaction of equilibrium prices and employment to economic shocks depend critically on whether the economy is demand or supply constrained. If the economy is supply constrained, that is if there is no output gap, then the conventional results hold: wealth and income inequality are output neutral and only the price level adjusts. But if the economy is demand constrained, neoclassical theory may fail to hold, i.e. increases in the inequality of either type may reduce employment and output. Say's Law may also fail to apply in a demand-constrained economy, namely an increase in worker productivity may reduce employment and increase the output gap. Moreover, minimum-wage policies, which force entrepreneurs to pay higher wages, may help to increase equilibrium employment and output.

From a theoretical perspective, these results are born by the interaction between the demand for the good in fixed supply and the equilibrium concept. The fact that one of the goods in the economy is in fixed supply means that, if citizens derive sufficiently high marginal utility from holding it, then their demand for the producible good is limited and so is labor demand as a result. This generates a tendency for equilibrium unemployment to arise endogenously. However, if one were to solve the model using the methodology of competitive equilibrium, this tendency, by the equilibrium's very definition, would not materialise. Thus, the use of Nash equilibrium removes the ad-hoc market-clearing condition and allows for the endogenous emergence of a positive output gap.

The comparative statics results, then, follow from the differences in the marginal utility of consumption of the status good between workers and entrepreneurs, which affect the composition of the aggregate demand for the producible good. If the marginal utility of the status good is sufficiently higher for entrepreneurs than workers then

income and wealth redistributions which favor the former weaken aggregate demand and, depending on the sensitivity of prices to demand changes, may decrease aggregate employment and output. The other side of the coin is that policies which favor workers, such as minimum-wage policies, may strengthen aggregate demand and increase employment. Similarly, increases in productivity have an ambiguous effect on output when the economy is demand constrained, since they increase entrepreneurs' profits and demand for the commodity, while they decrease labor demand and income for the working class. The overall effect depends on the differences in marginal utilities between the two classes and on the degree of commodity competition. While the model is too simple to provide a comprehensive policy analysis, it is easy to generalise and expand, and it demonstrates in a clear fashion the breath of results that can be obtained from an alternative modelling approach to demand-side models.

In terms of literature, the obvious point of comparison is New Keynesian models of demand-constrained economies (Christiano et al., 2005; Hall, 2005; Smets and Wouters, 2007; Hagedorn and Manovskii, 2008; Galí et al., 2011). The main difference between these papers and mine is, as indicated above, that they insert price or wage rigidities or both to achieve output gaps, while the model of this paper imposes no such restriction. Moreover, the output gap in these models is out-of-equilibrium, so to speak, while it is the 'long-run' equilibrium in mine. If given enough time to adjust, economies in New Keynesian models always revert back to full employment. This does not happen in the model of this paper. More recently, Christiano et al. (2016) combine the New Keynesian framework with search frictions to generate equilibrium unemployment and positive output gap. However, they assume both price rigidities ala Calvo and labor market frictions, while the purpose of my paper is to demonstrate that even in the absence of the above assumptions one can obtain a demand-constrained economy.

Other models of equilibrium unemployment involve search models, both random (Mortensen and Pissarides, 1994; Burdett and Mortensen, 1998; Herz and Van Rens, 2015; Mangin, 2015) and, more recently, directed search models (Burdett et al., 2001; Galenianos and Kircher, 2009; Menzio and Shi, 2011; Menzio et al., 2016), and asymmetric information models (Shapiro and Stiglitz, 1984). In this category one may also include Michailat and Saez (2014), who introduce search frictions in the labor market in a model with money in the utility function. These, however, are supply-side models of imperfect labor markets, where frictions impede labor demand from equalising with labor supply. This paper shows that, even when labor markets are frictionless, involuntary unemployment may arise as an equilibrium phenomenon, if aggregate demand

is sufficiently weak.

Thus, indirectly, the paper is also related to the theoretical literature on minimum-wage policies and unemployment (Stigler, 1946; Bhaskar and To, 1999; Bhaskar et al., 2002). These papers emphasize the role of local monopsonies or imperfect labor-market competition in creating unemployment and how minimum-wage policies may reduce it by encouraging labor force participation. However, these results are based on partial, not general, equilibrium models, where firms' profits do not generate demand for final products. On the contrary, this paper adopts a general equilibrium approach with perfect labor-market competition.

Another strand of relevant literature is the one on capitalist spirit (Zou, 1994, 1995; Bakshi and Chen, 1996) and the status effects of wealth (Hopkins and Kornienko, 2004; Becker et al., 2005). Capitalist-spirit preferences have been used in macroeconomic models in the context of growth (Zou, 1994; Doepke and Zilibotti, 2008), savings and investment decisions (Zou, 1998; Carroll, 2000; Gong and Zou, 2001; Luo and Young, 2009; Suen, 2014), and recently financial crises and business cycles (Karnizova, 2010; Kumhof et al., 2015). These models are dynamic, as opposed to mine, which is static, and they use competitive equilibrium as their solution concept. As a result they do not produce a long-run output gap. This paper, on the other hand, emphasizes the role of fixed-supply goods, like status goods, and the value of a game theoretic approach to model demand-constrained economies.

Last, but not least, is the topic of income and wealth inequality in relation to the weakness of aggregate demand. Several empirical studies have documented the rise in both types of inequality in western economies (Piketty and Saez, 2003; Blundell and Etheridge, 2010; Heathcote et al., 2010; Piketty, 2011; Attanasio et al., 2012; Kopczuk, 2015; Saez and Zucman, 2016). Many economists have also expressed their concern for its harmful consequences to politics (Acemoglu and Robinson, 2015) or to social cohesion (Piketty, 2014). However, far less attention has been paid by academic economists to its economic consequences (for notable exceptions see Rajan, 2011; Summers, 2014b), partially because there are very few models which link economic inequality to aggregate demand. One of the main purposes of this paper is to provide a theoretical link between the two.

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 presents the main analysis and the main features of the equilibrium. Section 4 provides the comparative statics exercises and discusses the wider implications of the model. Section 5 shows that the main results are robust to standard extensions,

namely the introduction of elastic labor supply and capital in the model. Finally section 6 concludes.

## 2 The Model

There are two classes of citizens, workers and entrepreneurs. Each class is homogeneous with respect to its characteristics. Workers provide labor supply, entrepreneurs set prices and wages, hire workers for their firm, and produce the final good. Both classes are consumers of output. The class of workers has  $H$  members in total, with  $h \in H$  being a generic worker. The class of entrepreneurs has  $E$  members in total, with  $e \in E$  being a generic member. On aggregate,  $E + H = I$ , with  $i \in I$  being a generic citizen.

There are two goods in the economy, one is a producible and consumable commodity, the other one is in fixed supply, it is used as a measure of wealth, and as means of payment. I refer to the former good as the ‘commodity’, and to the latter as the ‘status good’. One can think of the first as the standard good used in economic modelling and the second as ‘gold’ or ‘money’ in the economy. Citizens receive utility from both goods. The utility function of citizen  $i$  is  $u_i(c_i, w_i)$ , where  $c_i$  is  $i$ ’s consumption of the commodity and  $w_i$  is the final holding of wealth, i.e. the final stock holding of the status good. By definition,  $w_i = w_{i0} + y_i - pc_i$ , that is the final stock holding of wealth is equal to  $i$ ’s initial endowment of status good  $w_{i0}$ , plus his income  $y_i$  minus his expenditure  $pc_i$  where  $p$  is the price of the commodity in terms of the status good.  $\hat{w}_i$  denotes  $i$ ’s wealth before expenditure:  $\hat{w}_i = w_{i0} + y_i$ . Finally, the usual assumptions on  $u_i$  hold:  $u_{ic} > 0, u_{icc} < 0, u_{iw} > 0, u_{iww} < 0$ , where  $u_{ij}$  and  $u_{ijj}$  are the first and second partial derivatives of  $u_i$  with respect to  $j$  respectively.

### Workers

Each worker has fixed labor supply equal to  $\ell_0$  and an initial wealth endowment  $w_{\ell_0}$ . A worker  $h$  may be employed or unemployed. If he is employed, then he receives the wage income  $y_h = v_e \ell_0$ , where  $v_e$  is the wage rate that entrepreneur  $e$  pays to his workers. If  $h$  is unemployed then he has no income. The minimum possible wage rate is exogenously set to  $\underline{v} \geq 0$ .  $h$ ’s only essential decision is how many units of the commodity to purchase and consume. That is, a worker maximizes  $u_h(c_h, w_h)$  subject to  $pc_h \leq \hat{w}_h$ . For convenience,  $\hat{w}_\ell, \hat{w}_u, c_\ell^*(p)$  and  $c_u^*(p)$  denote the wealth before expenditure and the commodity demands of the employed and unemployed workers respectively.  $v_h(p, \hat{w}_h)$

denotes  $h$ 's indirect utility.

## Entrepreneurs

Entrepreneurs, apart from being consumers, are responsible for the production decisions in the economy. Specifically, each entrepreneur  $e$  owns a firm through which he hires workers to produce the commodity. All entrepreneurs have access to the same linear production technology but they do not have labor supply of their own. For every labor unit they employ they get  $\alpha$  commodity units. Thus,  $e$ 's output is equal to  $\alpha \ell_0 L_e$ , where  $L_e$  denotes  $e$ 's number of employees.  $e$  makes three decisions as producer: (i) He sets the price for his commodity  $p_e$ . (ii) He sets his wage rate  $v_e$ . (iii) He sets his labor demand  $\ell_e^d$  which is expressed in terms of worker numbers. Effectively,  $\ell_e^d$  denotes the number of vacancies  $e$  is posting.  $e$ 's final output is a joint-outcome of both the labor-market and the commodity-market competition which, in turn, depend on all other entrepreneurs' production decisions. For convenience, the list of vectors  $\{\mathbf{p}, \mathbf{v}, \boldsymbol{\ell}^d\}$  denotes the vector of prices, wage rates, and labor demands respectively set by all entrepreneurs. The list of vectors  $\{\mathbf{p}_{-e}, \mathbf{v}_{-e}, \boldsymbol{\ell}_{-e}^d\}$  denotes the vector of prices, wage rates, and labor demands respectively set by entrepreneurs excluding  $e$ . The modelling assumptions of labor-market and commodity-market competition are defined below.

## Labor-Market Competition

I assume perfect labor-market competition, which is modelled as follows. Given the vectors  $\mathbf{v}$  and  $\boldsymbol{\ell}^d$ , and the fixed aggregate labor supply  $H$ , entrepreneurs fill-up their vacancies by absolute priority based on the ranking of their wage rates from the highest to the lowest. That is, the set of entrepreneurs with the highest wage rate fill up their vacancies first, the set of entrepreneurs with the second highest wage rate fill up their vacancies second, and so on, until the pool of workers is exhausted. This implies that, if  $L^d \equiv \sum_e \ell_e^d < H$ , then all entrepreneurs fill their vacancies and a number of workers equal to  $H - L^d$  remain unemployed. On the other hand, if  $L^d > H$ , then all workers are employed and some entrepreneurs do not fill their vacancies, hence they do not produce. Specifically, in this case, there exists a wage rate  $\hat{v}$  and a number of entrepreneurs  $E_{\hat{v}} \equiv \#\{e \in E | v_e = \hat{v}\}$ , such that: (i)  $L_e = \ell_e^d$  for all  $e$  with  $v_e > \hat{v}$  (all entrepreneurs with higher wage rates than  $\hat{v}$  fill up their vacancies in full). (ii)  $L_e = 0$  for all  $e$  with  $v_e < \hat{v}$  (all entrepreneurs with lower wage rates than  $\hat{v}$  do not receive any worker). (iii)  $L_e = [E_{\hat{v}}]^{-1} \left( H - \sum_{\{e | v_e > \hat{v}\}} \ell_e^d \right)$  for all  $e$  with  $v_e = \hat{v}$ . In the last case I

assume that, if  $\sum_{\{e|v_e \geq \hat{v}\}} \ell_e^d > H > \sum_{\{e|v_e > \hat{v}\}} \ell_e^d$ , then entrepreneurs with  $v_e = \hat{v}$  share the remainder of the labor-supply pool equally among them.

Implicitly, the labor-market competition sets the cap  $\alpha \ell_0 L_e$  to the output of entrepreneurs. Also note that, because workers suffer no costs from moving from one firm to another, and because workers strictly prefer to work for the firm with the highest wage rate, the above formulation of the labor market is equivalent in terms of outcomes to a game where workers observe the wage-rate vector  $\mathbf{v}$  and they simultaneously apply for a vacancy. The only additional assumption needed in this case is that, whenever a worker is indifferent between two firms, he applies to both with equal probability.

### Commodity-Market Competition

Every entrepreneur has two sources of demand for his firm's commodity. The first source is the entrepreneur himself, that is  $e$  always consumes the commodity from the production of his firm.<sup>1</sup> The second source is the commodity demand by the worker class. I assume that, given the price vector  $\mathbf{p}_e = \{p_e, \mathbf{p}_{-e}\}$ , the fraction of workers that purchases the commodity from  $e$ 's firm is equal to  $\mu_e(p_e, \mathbf{p}_{-e})$  and satisfies the following assumptions:

$$(i) \quad 0 \leq \mu_e(p_e, \mathbf{p}_{-e}) \leq 1 \quad (ii) \quad \sum_{e \in E} \mu_e(p_e, \mathbf{p}_{-e}) = 1$$

$$(iii) \quad \text{if } p_e = \hat{p} \quad \forall e \in E \quad \text{then } \mu_e(\hat{p}) = E^{-1}$$

$$(iv) \quad -\infty < \frac{\partial \mu_e}{\partial p_e} < 0 \quad (v) \quad \lim_{p_e \rightarrow \infty} \frac{\partial \mu_e}{\partial p_e} = -\infty$$

Conditional on  $\mu_e(p_e, \mathbf{p}_{-e})$ , each of the workers who buy from  $e$ 's firm expresses a consumer demand  $c_h^*(p_e)$ , which is derived from the worker's maximization problem and depends on the worker's employment status. Generically,  $c_l^*(p_e) \neq c_u^*(p_e)$ . Thus, the total commodity demand for  $e$ 's firm is equal to:

$$D_e(\mathbf{p}) \equiv c_e + \mu_e(p_e, \mathbf{p}_{-e}) [L c_l^*(p_e) + (H - L) c_u^*(p_e)]$$

where  $L$  is the total number of employed workers. The aggregate commodity demand in the economy is equal to  $\sum_{e \in E} D_e(\mathbf{p})$ . Note that the above formulation assumes that the

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<sup>1</sup>This assumption is without loss of generality. One can easily relax it while obtaining the same main results.



same fraction of employed and unemployed workers are customers to  $e$ 's firm, i.e. the proportion of  $e$ 's customers is independent of their employment status. This assumption simplifies the proof of equilibrium existence (see also section 3).

Even though the above specification of commodity-market competition is ad hoc, it is flexible enough to account for both perfect competitive markets and local monopolies. Specifically, when  $\frac{\partial \mu_e}{\partial p_e} = -\infty$  for any  $p_e$ , then the model converges to one of perfect market competition, where the elasticity of demand with respect to price is infinite. On the other hand, when  $\frac{\partial \mu_e}{\partial p_e} = 0$  for all  $p_e$  then the model becomes one of local monopolies, with each entrepreneur's customer base unaffected by the pricing decisions of others. Finally, in the intermediate cases where  $-\infty < \frac{\partial \mu_e}{\partial p_e} < 0$ , the model is one of imperfect market competition. I focus on the analysis of the intermediate case for the time being, but I discuss the model's results in the two limit cases at the end of section 3.

### Firms' Profit Function and Entrepreneurs' Problem

Given the above specifications of labor and commodity-market competition,  $e$ 's firm total output is the minimum of its commodity demand and its production capacity:

$$\min\{D_e(\mathbf{p}), \alpha l_0 L_e\}$$

And  $e$ 's profit function is given by:

$$\pi_e \equiv (p_e - v_e \alpha^{-1}) \min\{D_e(\mathbf{p}) - c_e, \alpha l_0 L_e\} - v_e \alpha^{-1} c_e \quad (1)$$

Note that an entrepreneur's profits must be non-negative in equilibrium. Otherwise, he is always better off by closing down his firm and purchasing the commodity with his initial endowment  $w_{e0}$ . Thus, an entrepreneur's problem is to maximize his utility  $u_e(c_e, w_e)$  with respect to  $\{c_e, p_e, v_e, \ell_e^d\}$  subject to  $w_e = w_{e0} + \pi_e$ ,  $\pi_e \geq 0$ , (1), and the output constraint  $D_e(\mathbf{p}) \leq \alpha l_0 L_e$ .

## 3 Equilibrium

An entrepreneur's utility implicitly depends on the other entrepreneurs' decisions through their impact on labor-market competition, commodity-market competition and, thus, on his firm's profits. To make this clear in our notion of equilibrium, I adopt the notation  $v_e(c_e, p_e, v_e, \ell_e^d | \{\mathbf{p}_{-e}, \mathbf{v}_{-e}, \ell_{-e}^d\}) \equiv u_e(c_e, w_e(p_e, v_e, \ell_e^d | \{\mathbf{p}_{-e}, \mathbf{v}_{-e}, \ell_{-e}^d\}))$  in the

definitions below.

**Definition 1** A Nash equilibrium of the economy is a list of  $E$ -length vectors  $\{\mathbf{p}^*, \mathbf{v}^*, \boldsymbol{\ell}^{d*}\}$  and a  $H$ -length vector  $\{\mathbf{c}^*\}$  such that: (i)  $v_e(c_e^*, p_e^*, v_e^*, \ell_e^{d*} | \{\mathbf{p}_{-e}^*, \mathbf{v}_{-e}^*, \boldsymbol{\ell}_{-e}^{d*}\}) \geq v_e(c_e, p_e, v_e, \ell_e^d | \{\mathbf{p}_{-e}^*, \mathbf{v}_{-e}^*, \boldsymbol{\ell}_{-e}^{d*}\})$  for all  $\{c_e, p_e, v_e, \ell_e^d\}$  consistent with  $\pi_e \geq 0$  for all  $e \in E$ , (ii)  $u_h(c_h^*, w_h) \geq u_h(c_h, w_h)$  under  $p c_h \leq \hat{w}_h$  for all  $h \in H$ , (iii)  $\alpha \ell_0 \sum_{e \in E} L_e = \sum_{i \in I} c_i^*(p_i^*)$ .

The above definition is the standard definition of Nash equilibrium with the addition of the condition that total production must be consistent with aggregate demand for the commodity.<sup>2</sup>

**Definition 2** The vector  $\{p^*, v^*, \ell^{d*}, c_e^*, c_\ell^*(p^*), c_u^*(p^*)\}$  is a pure symmetric Nash equilibrium of the economy if it satisfies the conditions of definition (1).

In words, a pure symmetric Nash equilibrium is a Nash equilibrium of the economy where all entrepreneurs make the same decisions  $p_e = p^*, v_e = v^*, c_e = c_e^*$  and  $\ell_e^d = \ell^{d*}$ . The main analysis of the model focuses on this type of equilibria. However, the following result holds for all equilibria.

**Lemma 1** In any Nash equilibrium of the economy  $v_e = \underline{v}$  for all  $e \in E$ .

**Proof:** First, the entrepreneur's profit constraint  $\pi_e \geq 0$  implies that  $p_e > v_e \alpha^{-1}$ . Second, if  $\ell_e^d > (\alpha \ell_0)^{-1} D_e(\mathbf{p})$ ,  $e$  suffers additional cost without additional benefit by setting his labor demand above what he needs for production, while if  $\ell_e^d < (\alpha \ell_0)^{-1} D_e(\mathbf{p})$ ,  $e$  loses the additional profit from potential customers who demand  $e$ 's commodity but can not buy it. Thus,  $\ell_e^d = (\alpha \ell_0)^{-1} D_e(\mathbf{p})$  for any  $e$ . Next, note that there can not exist a Nash equilibrium with  $L^d > H$ . Suppose this is the case. Then, there exists at least one entrepreneur who faces excess demand:  $L_e = [E_{\hat{v}}]^{-1} \left( H - \sum_{\{e | v_e > \hat{v}\}} \ell_e^d \right) < \ell_e^d = (\alpha \ell_0)^{-1} D_e(\mathbf{p}) \Rightarrow \alpha \ell_0 L_e < D_e(\mathbf{p})$ . But in this case  $\partial \pi_e / \partial p_e > 0$  so  $p_e$  can not be optimal. Therefore, an equilibrium is possible only if  $L^d \leq H$ . Finally,  $L^d \leq H \Rightarrow L_e = \ell_e^d \leq H - \sum_{\{f \in E, f \neq e\}} \ell_f^d$ . In this case,  $\partial \pi_e / \partial v_e < 0$ , so any  $v_e > \underline{v}$  is suboptimal. Hence,  $v_e = \underline{v}$  is the only optimal best-response in equilibrium. ■

<sup>2</sup>It is easy to check that the last requirement arises endogenously from entrepreneurs' optimal responses, i.e. any entrepreneur who faces excess supply for his commodity can strictly improve his utility by cutting back on production, while if he faces excess demand he can profitably deviate by increasing his price. In any Nash equilibrium of the game commodity demand must be equal to commodity production for each and every entrepreneur.

Lemma (1) contains two simple results. The first one is that, in equilibrium, there can not be excess demand for labor because this implies excess commodity demand, which the entrepreneurs can benefit from simply by raising prices slightly. Second, since equilibrium labor demand is at most equal to labor supply, entrepreneurs do not need to worry about labor market competition. This is because, if one of them reduces his wage rate slightly, even if he were to be the last one to fill his vacancies, he will still be able to get all the workers he demands. Thus labor market competition is mute in the presence of endogenous commodity demand and the only equilibrium wage rate is the minimum possible. Note that both these arguments are general and do not rely on any additional conditions. However, for the rest of the analysis I focus on pure symmetric equilibria, since they are easy to characterize. The following proposition provides the necessary conditions for a symmetric pure equilibrium.

**Proposition 1** *Let  $\tau_1$ ,  $\tau_2$ , and  $\tau_3$  be the Lagrange multipliers associated with the conditions  $w_e = w_{e0} + \pi_e$ ,  $\pi_e \geq 0$ , and  $D_e(\mathbf{p}) \leq \alpha \ell_0 L_e$  respectively. Then the necessary conditions for any symmetric pure Nash equilibrium where all entrepreneurs produce are:*

$$v^* = \underline{v} \tag{E1}$$

$$c_h^* : \quad u_{hc} = pu_{hw} \quad h \in \{\ell, u\} \tag{E2}$$

$$c_e^* : \quad u_{ec} = \underline{v} \alpha^{-1} (\tau_1 + \tau_2) + \tau_3 \tag{E3}$$

$$p^* : \quad (\tau_1 + \tau_2) \frac{\partial}{\partial p_e} \{ (p_e - \underline{v} \alpha^{-1}) (D_e - c_e) \} = \tau_3 \frac{\partial D_e}{\partial p_e} \tag{E4}$$

$$\tau_2 \pi_e^* = 0 \quad \forall e \in E \tag{E5}$$

$$\tau_3 (\alpha \ell_0 H E^{-1} - D_e) = 0 \tag{E6}$$

$$L^* = \frac{H c_u^* + E c_e^*}{\alpha \ell_0 - (c_\ell^* - c_u^*)} \tag{E7}$$

In all the above expressions the argument  $p$  is suppressed. (E1) comes from Lemma 1 directly. (E2) and (E3) come from the first order conditions of workers and entrepreneurs with respect to consumption. (E4) is the entrepreneurs' first order con-

dition with respect to price, while (E5) and (E6) are the Kuhn-Tucker conditions for non-negative profits and the output constraint respectively. Finally, (E7) provides the equilibrium employment and it comes directly from the commodity-clearing condition (iii) of Definition 1.

**Proposition 2** *A symmetric pure Nash equilibrium always exists.*

**Proof:** In accordance with Lemma 1, set  $v_e = \underline{v}$  for all  $e \in E$ . Suppose that all entrepreneurs set the same price  $p$  and the resulting demand functions  $c_\ell^*(p, \hat{w}_\ell)$ ,  $c_u^*(p, \hat{w}_u)$ , and  $c_e^*(p, \hat{w}_e, \underline{v})$  satisfy the respective conditions of Proposition 1. Let us separate two cases, one where the output constraint does not bind and one where it binds. When the output constraint does not bind,  $\tau_3 = 0$  and then (E4) rewrites as  $Lc_\ell^* + (H - L)c_u^* = -E \frac{\partial(D_e - c_e^*)}{\partial p_e} (p - \underline{v}\alpha^{-1})$ , which implicitly defines a correspondence between  $p$  and  $L$ . At the same time (E7) implicitly defines a correspondence between  $L$  and  $p$ . It is convenient to define both relations in terms of employment. Thus, define:

$$p_s(L) \equiv \left\{ p : Lc_\ell^* + (H - L)c_u^* = -E \frac{\partial(D_e - c_e^*)}{\partial p_e} (p - \underline{v}\alpha^{-1}) \right\}$$

$$p_d(L) \equiv \left\{ p : L = \frac{Hc_u^* + Ec_e^*}{\alpha\ell_0 - (c_\ell^* - c_u^*)} \right\}$$

The first result is that, in the limit, as  $L \rightarrow H$ ,  $p_s(H) \geq p_d(H)$ . Suppose the contrary, that is suppose  $p_s(H) < p_d(H)$ . In this case, the aggregate commodity demand that entrepreneurs face at  $p_d(H)$  is, by definition, equal to  $\alpha\ell_0 H$ , while the aggregate commodity demand that they face at the lower price  $p_s(H)$  exceeds  $\alpha\ell_0 H$ , since  $\partial c_h^* / \partial p < 0$ . But this violates the optimality of  $p_s(H)$  since firms can always increase their profits by increasing the price when faced with excess demand. Hence,  $p_s(H) \geq p_d(H)$ . Next, there exists a weakly positive level of  $L$ , say  $0 \leq \tilde{L} \leq H$ , such that  $\lim_{L \rightarrow \tilde{L}} p_d(L) = +\infty$ .

This is because  $\lim_{p \rightarrow +\infty} c_\ell^* = \lim_{p \rightarrow +\infty} c_u^* = 0$  and  $0 \leq \lim_{p \rightarrow +\infty} c_e^* \leq \alpha\underline{v}^{-1} \left( w_{e0} + E^{-1} \sum_h w_{h0} \right)$ . The last inequality holds because entrepreneurs' aggregate expenditure on wages can never exceed aggregate wealth and this limits their maximum consumption. Thus, if  $\lim_{p \rightarrow +\infty} c_e^* = 0$ , then  $\tilde{L} = \lim_{p \rightarrow +\infty} \frac{Hc_u^* + Ec_e^*}{\alpha\ell_0 - (c_\ell^* - c_u^*)} = 0$ , or, if  $\lim_{p \rightarrow +\infty} c_e^* > 0$ , then  $0 < \tilde{L} = \lim_{p \rightarrow +\infty} \frac{Hc_u^* + Ec_e^*}{\alpha\ell_0 - (c_\ell^* - c_u^*)} \leq H$ . In either case there exists  $\tilde{L}$  such that  $\lim_{L \rightarrow \tilde{L}} p_d(L) = +\infty$ . However,  $p_s(\tilde{L}) < +\infty$  since  $p_s(L)$  is always bounded for any value of  $L$  by merit of assumption (v) on  $\mu_e(\mathbf{p})$ . Thus,  $\lim_{L \rightarrow \tilde{L}} p_s(L) - p_d(L) < -\infty$  and  $p_s(H) - p_d(H) \geq 0$ . Hence, by

the fact that  $p_s(L)$  and  $p_d(L)$  are both continuous graphs of  $p$  with respect to  $L$ , there exists  $L^*$ ,  $\tilde{L} < L^* \leq H$ , such that  $p_s(L^*) = p_d(L^*) = p^*$ , and a symmetric pure Nash equilibrium exists.

The second possible case is when  $\tau_3 > 0$ . In this case the output constraint binds, so (E6) and (E7) together imply that  $\alpha\ell_0H = Hc_\ell^* + Ec_e^*$ , which implicitly depends on  $p^*$  but not  $L$ . The equilibrium price is then determined either by  $u_{ec} \left[ 1 - p^*(1 + \epsilon_{clp}^{-1}) + v\alpha^{-1} \right] = u_{ew}v\alpha^{-1}$ , when the profit constraint does not bind, where  $\epsilon_{clp}$  is the elasticity of the workers' demand with respect to the price level, or by  $p^*c_\ell^* = v\ell_0$ , when the profit constraint binds. In both cases, an equilibrium price  $p^*$  and employment level  $L^* = H$  exist, so a pure symmetric Nash equilibrium exists. ■

Although the equilibrium existence is a reassuring result, the interesting feature of the model is the fact that the equilibrium does not always entail full employment, or, to put it differently, it may entail a positive output gap. This is in stark contrast to competitive equilibrium models, which are usually used for macroeconomic modelling. Thus a pure symmetric equilibrium without full employment means that the production constraint is non-binding and  $\tau_3 = 0$ . In this case, the equilibrium is characterised by the pricing equation  $p_s(L)$  and the employment equation (E7). The former can be viewed as an aggregate supply equation that gives the firms optimal pricing choice conditional on employment conditions and the commodity demand that they generate. The latter is an effective aggregate demand equation, giving the employment level consistent with commodity demand for each price level. Figure 1 below demonstrates different possible shapes for the correspondences  $p_s(L)$  and  $p_d(L)$  and how they can generate economies with multiple or unique equilibria and with or without unemployment.

An equilibrium with positive output gap/unemployment may result in the case where both workers and entrepreneurs have relatively weak demand for the commodity in comparison to the status good, or, in other words, the marginal utility for the commodity declines at a substantially faster rate than the marginal utility of the status good. In this case, the profit constraint may or may not be binding, but the weak aggregate demand for the commodity translates into weak labor demand and unemployment. Thus, strong commodity competition does not automatically translate to full employment. The value of the status good relative to that of the commodity matters too.

Another possible case where a positive output gap may arise in equilibrium is when

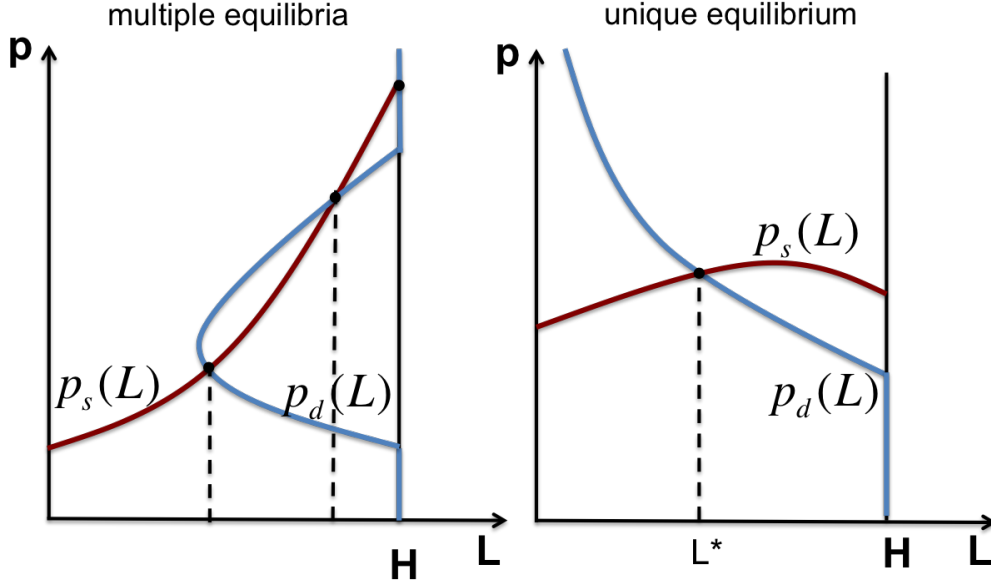


Figure 1: Positive Equilibrium Output Gap and Unemployment

workers have strong demand for the commodity, but entrepreneurs have strong preference for the status good and commodity competition is relatively weak so that the profit constraint is non-binding. In this case, even though a sufficiently high level of commodity competition across entrepreneurs may be enough to lead to full employment, the lack of it, along with weak commodity demand by entrepreneurs, leads to low aggregate labor demand. If this holds then an increase in wealth or income inequality may widen the output gap, as the loss of demand by workers is only partially compensated by the increased demand by entrepreneurs. This effect is demonstrated even more clearly in the comparative statics section.

To present the above arguments in a more concrete manner consider the following example with explicit functional forms. First, suppose that the utility function of an agent  $i$  is given by  $u_i(c_i, w_i) = \epsilon_i \ln(c_i) + \ln(w_i)$ , for  $i \in \{h, e\}$ , where  $\ln(\cdot)$  is the natural logarithm function. Second, suppose that  $\mu_e(p_e, \mathbf{p}_{-e})$  is given by the expression below:

$$\mu_e(p_e, \mathbf{p}_{-e}) = \begin{cases} E^{-1} + \mu \frac{p^{min} - p_e}{p^{max} - p^{min} + \delta} & \text{if } p_e > p^{min} \\ [N(p^{min})]^{-1} \left( 1 - \sum_{p_f \neq p^{min}} \mu_f(p_f, \mathbf{p}_{-f}) \right) & \text{if } p_e = p^{min} \end{cases} \quad (2)$$

where  $p^{max} = \max\{p_e : p_e \in \mathbf{p}\}$ ,  $p^{min} = \min\{p_e : p_e \in \mathbf{p}\}$ ,  $N(p^{min})$  is the number of entrepreneurs with the minimum price in  $\mathbf{p}$ , and  $\mu, \delta$  are positive constants.

Under these functional forms, the demand function is  $c_h^* = \frac{\epsilon_h \hat{w}_h}{1 + \epsilon_h} \frac{1}{p}$  for workers and  $c_e^* = \frac{\epsilon_e \hat{w}_e}{1 + \epsilon_e} \frac{1}{\underline{v}\alpha^{-1}}$  for entrepreneurs. Next, postulating a pure symmetric equilibrium with positive output gap and positive profits for entrepreneurs, which implies zero values for the Lagrangian multipliers  $\tau_2$  and  $\tau_3$ , and computing the optimal best-responses with respect to the price level, gives the unique optimal price  $p^* = 2^{-1} \left( x_2 + \sqrt{x_2^2 + 4E^{-1}x_1^{-1}x_2} \right)$ , with  $x_1 \equiv \mu\delta^{-1}$  and  $x_2 \equiv \underline{v}\alpha^{-1}$ .  $p^*$  is independent of the employment level  $L$  of the economy. Combining the above with (E7) one obtains the unique employment level given by:

$$L^* = \frac{(1 + \epsilon_h) [E\epsilon_e w_{e0} + (\epsilon_e + \underline{v}(\alpha p^*)^{-1}) H\epsilon_h (1 + \epsilon_h)^{-1} w_{h0}]}{(1 + \epsilon_e + \epsilon_h - \epsilon_h \underline{v}(\alpha p^*)^{-1}) \underline{v} \ell_0} \quad (3)$$

The conjecture about the equilibrium featuring both positive output gap and profits is correct if  $L^* < H$  and  $\pi_e(p^*) > 0$ . The proposition below gives the necessary conditions for this conjecture to be correct and figure 2 demonstrates the functions  $p_d(L)$ ,  $p_s(L)$ , and the equilibrium in this case.

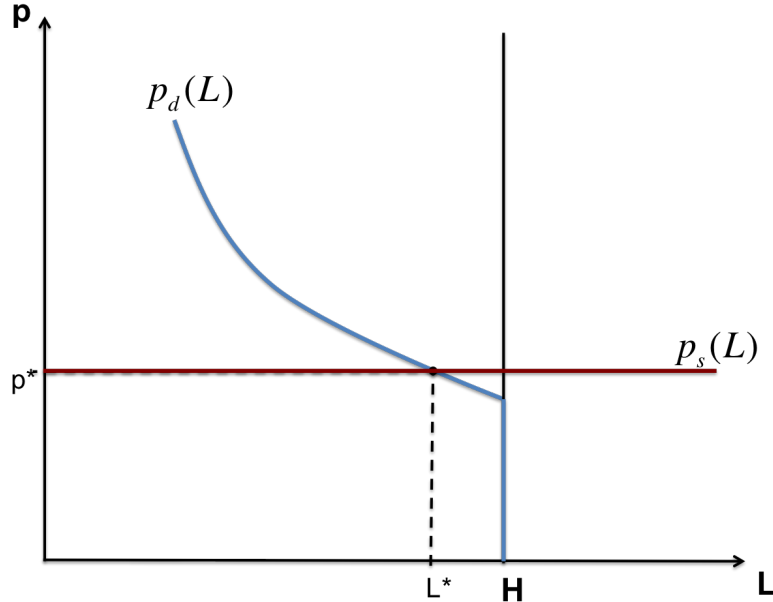


Figure 2: Equilibrium With Explicit Functional Forms

**Proposition 3** Consider the economy with the explicit functional forms for  $u_i$  and  $\mu_e$  given by  $u_i(c_i, w_i) = \epsilon_i \ln(c_i) + \ln(w_i)$  and (2) respectively. Then a pure symmetric

Nash equilibrium exists with  $p^* = 2^{-1} \left( x_2 + \sqrt{x_2^2 + 4E^{-1}x_1^{-1}x_2} \right)$ ,  $L^*$  given by (3), and positive output gap, only if the following inequalities hold:

$$(1 - \underline{v}(\alpha p^*)^{-1}) (H\epsilon_h w_{h0} + L^* \epsilon_h \underline{v} \ell_0) > E(1 + \epsilon_h)(1 + \epsilon_e)^{-1} \epsilon_e w_{e0} \quad (4)$$

$$(1 + \epsilon_e + \epsilon_h) \underline{v} \ell_0 > \epsilon_h \underline{v} (\alpha p^*)^{-1} (w_{h0} + \underline{v} \ell_0) + \epsilon_e [(1 + \epsilon_h) E H^{-1} w_{e0} + \epsilon_h w_{h0}] \quad (5)$$

Condition (4) is necessary for entrepreneurs to gain positive profits from production, while condition (5) is necessary for obtaining a positive output gap in equilibrium. Note that these conditions are not mutually incompatible. For example, as  $\epsilon_e$  and  $w_{h0}$  both approach zero, then both conditions hold by the fact that  $\alpha p^* > \underline{v}$ . In fact, in this limit case (3) shows that  $L^*$  approaches zero. This is intuitive since when  $\epsilon_e = 0$  entrepreneurs get no utility from the commodity, so their demand is zero, and when  $w_{h0} = 0$  workers have no endowment of the status good which entrepreneurs want to acquire in exchange for the commodity. In general, (3) shows that equilibrium employment is increasing in the initial endowments of status goods for both types of citizens and in the  $\epsilon_i$  parameter, which indicates the relative utility gain of the commodity vis-à-vis the status good. In the next section I discuss in more detail the effects of redistributive policies for equilibrium employment, as well as the ambiguous role of the productivity parameter and the minimum wage.

Before the conclusion of this section, however, note that, as the minimum wage approaches zero the economy converges to full employment. This can be easily demonstrated in the general form of the model by looking at the first order condition with respect to commodity demand of entrepreneurs, which is:  $u_{ec} - \underline{v} \alpha^{-1} (\tau_1 + \tau_2) - \tau_3$ . As  $\underline{v}$  approaches zero the condition converges to  $u_{ec} = \tau_3$ . Since the marginal utility of the commodity is always positive, then  $\tau_3 > 0$ , and, therefore, the production constraint is binding. Intuitively, as the cost of production becomes infinitesimally small, the opportunity cost of entrepreneurs from consuming the commodity vanishes and their demand for it increases. In the limit the economy must approach full employment. Nonetheless, this result depends crucially on the assumption that  $\underline{v}$  can be made arbitrarily small. If households have outside options, like home production, then  $\underline{v}$  may remain bounded from below strictly above zero and in this case a full employment equilibrium is not guaranteed to exist (see section 5.1 for more details). The following proposition summarises the above.



**Proposition 4** *A pure symmetric Nash equilibrium may involve a strictly positive output gap and unemployment:  $L^* < H$ . However,  $\lim_{\underline{v} \rightarrow 0} L^*(\underline{v}) = H$ .*

Finally, it is worth examining what happens in the two limit cases with respect to commodity competition. The first case is when commodity competition is non-existent, which corresponds to the assumption that  $\partial\mu_e(p_e, \mathbf{p}_{-e})/\partial p_e = 0$  for all  $p_e$  and  $\mathbf{p}_{-e}$ . Then the model becomes one of local monopolies where each entrepreneur has a fraction  $1/E$  of the workers' as customers (but not necessarily his own workers). In this case  $\partial(D_e - c_e^*)/\partial p_e = E^{-1}(L^*c_l^* + (H - L^*)c_u^*)$  which implies that the  $p_s(L)$  curve becomes steeper, i.e. the optimal price for the firm is higher than the case of imperfect commodity competition, since the entrepreneur does not lose customers to other firms. Thus, if there exists a pure symmetric equilibrium with positive output gap in the case of imperfect commodity competition, the output gap is even larger in the limit case of local monopolies, as expected. In other words, the set of parameter values where a symmetric equilibrium with positive output gap exists weakly increases with the decrease in the intensity of competition.

The other limit case is the one of perfect commodity competition, namely the case where  $\partial\mu_e(p_e, \mathbf{p}_{-e})/\partial p_e = -\infty$  for all  $p_e$  and  $\mathbf{p}_{-e}$ . In this case the entrepreneurs' participation constraint is always binding:  $\pi_e = 0$ . However, the labor supply constraint may or may not be binding, depending on the strength of commodity demand. If the marginal utility for the commodity is sufficiently low then equilibrium unemployment may emerge even in the presence of perfect competition. For example, consider the special case of a quasi-linear utility function of the form  $u_h(c_h, w_h) = \epsilon_h \ln(c_h) + (w_h)^\beta$ , with  $\epsilon_h > 0$  and  $0 < \beta < 1$ . Then it is easy to check that as  $\epsilon_h \rightarrow 0$  then  $c_h^*(p) \rightarrow 0$  and the demand for the producible commodity becomes zero. If both classes have similar preferences then the aggregate demand for the commodity is zero and, hence, it is not produced in equilibrium. This implies that one can always find an  $\epsilon_h$  small enough but strictly positive, such that the economy features a positive equilibrium gap. On the other hand, if both the profit and the labor supply constraint are binding, then the model gives the canonical result of a perfectly competitive economy with full employment and zero profits.

## 4 Comparative Statics and Wider Implications

The main purpose of this paper is to provide a very simple model where demand-side considerations become crucial in determining the economy's equilibrium behavior. To demonstrate why demand-side considerations are important beyond the short-run, I provide some comparative-statics results which are absent in other theoretical models. Nonetheless, they make a lot of sense in a framework where equilibrium demand is weak and there is a positive long-run output gap.

### Non-Neutral Effects of Income and Wealth Inequality

Income and wealth inequality have drawn a lot of attention in both public debates and in academic research. Some economists have spoken openly against the dangers of a widening income and wealth gap. But the main dangers cited are either political (Acemoglu and Robinson, 2015) or based on considerations of fairness and social unrest (Piketty, 2014). So far, academic economists have refrained from linking rising income and wealth inequality to the poor economic performance after the Great Recession, with few exceptions (Rajan, 2011; Summers, 2014a). This subsection provides an economic rationale on why increasing levels of income or wealth inequality may be undesirable.

To be more specific, I examine how equilibrium employment responds to small changes in either the minimum wage, which links income inequality to the output gap, or a small redistribution of the entrepreneurs' status-good endowment to workers. General comparative statics results are discussed first, but to make the arguments more concrete, results with the explicit functional forms from the previous section are also presented.

For comparative statics with respect to income inequality, consider a small exogenous positive change to the minimum wage  $d\underline{v} > 0$ . By the envelope theorem, this change leads to a net negative effect on the entrepreneurs' profit ( $d\pi_e/d\underline{v} < 0$ ), but it increases employed worker's income by  $\ell_0 d\underline{v} > 0$ . Thus, it reduces income inequality between entrepreneurs and employed workers. If the economy is output constrained then, clearly, employment can not increase any further and, therefore, the change in the minimum wage has no positive effect on employment. On the contrary, in some cases, it may even be negative. However, if the economy is demand constrained, then, by a direct application of the implicit function theorem, the change in equilibrium employment is given by:

$$\frac{dL^*}{d\underline{v}} = \frac{\Delta_p \Sigma_{\underline{v}} - \Delta_{\underline{v}} \Sigma_p}{\Delta_L \Sigma_p - \Delta_p \Sigma_L}$$

where

$$\Delta \equiv L^* - \frac{Hc_u^* + Ec_e^*}{\alpha\ell_0 - (c_\ell^* - c_u^*)}$$

$$\Sigma \equiv L^*c_\ell^* + (H - L^*)c_u^* + E \frac{\partial(D_e - c_e^*)}{\partial p_e} (p^* - \underline{v}\alpha^{-1})$$

and  $\Delta_x$  ( $\Sigma_x$ ) denotes the partial derivative of  $\Delta$  ( $\Sigma$ ) with respect to  $x$ .

Even if one focuses attention to cases where the direct effects dominate the indirect effects and the denominator of  $dL^*/d\underline{v}$  can be signed (in this case  $\Delta_p > 0, \Delta_L > 0, \Sigma_p < 0, \Sigma_L > 0$ , so  $\Delta_L \Sigma_p - \Delta_p \Sigma_L < 0$ ), the signs of  $\Delta_{\underline{v}}$  and  $\Sigma_{\underline{v}}$  are both ambiguous and the sign of the numerator can be either positive or negative. Thus, the effects of the increase of the minimum wage are ambiguous and it depends on the relative strength between the demand-side and the supply-side channel. If commodity competition is sufficiently strong, so that the increase in wage costs is not fully passed on to consumers ( $\Sigma_{\underline{v}}$  is relatively small in absolute value), and if the workers' marginal utility of the commodity is sufficiently higher than the one of entrepreneurs, so that the redistribution of income leads to increased aggregate demand ( $\Delta_{\underline{v}}$  is negative and relatively high in absolute value), then the increase of the minimum wage leads to increased employment. The opposite holds if commodity competition is weak ( $\Sigma_{\underline{v}}$  is positive and relatively high) and the effect of redistribution on aggregate demand is weak or negative ( $\Delta_{\underline{v}}$  is negative or positive and relatively low in absolute value).

To see this more clearly consider the model under the explicit functional forms of section 3. Recall that in this case the equilibrium employment and price level are given by the expressions:

$$L^* = \frac{(1 + \epsilon_h) [E\epsilon_e w_{e0} + (\epsilon_e + \underline{v}(\alpha p^*)^{-1}) H\epsilon_h (1 + \epsilon_h)^{-1} w_{h0}]}{(1 + \epsilon_e + \epsilon_h - \epsilon_h \underline{v}(\alpha p^*)^{-1}) \underline{v}\ell_0}$$

$$p^* = 2^{-1} \left( x_2 + \sqrt{x_2^2 + 4E^{-1}x_1^{-1}x_2} \right) \quad \text{where} \quad x_1 \equiv \mu\delta^{-1}, x_2 \equiv \underline{v}\alpha^{-1}$$

From the above it is obvious that the direct effect of  $\underline{v}$  on  $L^*$  is ambiguous as an increase in the former increases the numerator and has an ambiguous effect on the

denominator. However, as  $\epsilon_h \rightarrow 0$  the direct effect becomes strictly negative. Thus the direction of the direct effect depends on the relative magnitude of the worker's propensity to consume. The higher the marginal utility of workers from the commodity, the more they tend to consume and hence the higher the equilibrium employment as the minimum wage increases.

On top of this there is the indirect effect through  $p^*$ , which is always negative in this example, since  $dp^*/d\underline{v} > 0$  and  $dL^*/dp^* < 0$ . Thus, the overall effect is ambiguous. Note, however, that as customer competition intensifies (higher values of  $\mu$ ), the size of the indirect effect is reduced ( $d^2p^*/(d\underline{v}d\mu) < 0$ ) so that the direct effect becomes more important. This means that an increase in the minimum wage is more likely to generate an increase in employment in economies with strong customer competition (high values of  $\mu$ ) and with strong commodity demand by workers (high values of  $\epsilon_h$ ). Analytically, this conclusion is best demonstrated in the limit case  $w_{h0} \rightarrow 0$ , where  $L^* = E(1 + \epsilon_h)\epsilon_e w_{e0} [(1 + \epsilon_e + \epsilon_h(1 - \underline{v}(\alpha p^*)^{-1})) \underline{v} \ell_0]^{-1}$ . In this case  $dL^*/d\underline{v} > 0$  only if  $\epsilon_h \left[ \left(1 - \underline{v}(p^*)^{-1} \frac{dp^*}{d\underline{v}}\right) \underline{v}(\alpha p^*)^{-1} - (1 - \underline{v}(\alpha p^*)^{-1}) \right] > 1 + \epsilon_e$ . Note that as  $\mu$  increases  $\frac{dp^*}{d\underline{v}}$  falls and approaches one in the limit. Therefore, as long as the term in the brackets of the left-hand side of the previous inequality is positive, higher values of  $\mu$  and  $\epsilon_h$  are more likely to imply that  $dL^*/d\underline{v} > 0$ .

Overall, income distribution is non-neutral when the economy has a positive output gap and increases of the minimum wage can lead to increased employment and output. Therefore, the model has an interesting implication on minimum-wage policies. Contrary to what conventional economic theorists predict, minimum wage policies can have positive economic effects even in the long run.

Next, consider a balanced redistribution of wealth from entrepreneurs to workers. In particular, suppose that before economic activity takes place every entrepreneur loses  $dw_{0e}$  units of wealth which are equally distributed across workers. So, each workers gains  $EH^{-1}dw_{0e}$  units of the status good. By the implicit function theorem one obtains:

$$\frac{dL^*}{dw_{0e}} = \frac{\Delta_p \Sigma_w - \Delta_w \Sigma_p}{\Delta_L \Sigma_p - \Delta_p \Sigma_L}$$

Similarly to above, the effects of the wealth redistribution are non-neutral when the economy is demand constrained and, conditional on direct effects dominating indirect ones, they depend on the sign of  $\Delta_w$  and  $\Sigma_w$ , which are ambiguous. When commodity competition is strong ( $\Sigma_w$  small in absolute value) and the marginal propensity to

consume of workers is higher than entrepreneurs' ( $\Delta_w$  is negative and big in absolute value), then wealth redistribution increases employment and output. The other side of this effect is that an increase in wealth inequality, through a redistribution of wealth from workers to entrepreneurs, has the opposite effect, i.e. it decreases employment and output. However, in the case of weak commodity competition ( $\Sigma_w$  positive and large in absolute value) and small differences in marginal propensities of consumption between the two groups ( $\Delta_w$  small in absolute value) employment decreases and small changes to wealth inequality have positive effects on economic activity.

Once again, the example from before is elucidating. Since  $p^*$  does not depend on individual wealth in this case, there are no indirect effects and, hence, only the change of the numerator matters. In this case the impact of wealth redistribution on equilibrium employment is proportional to  $(\epsilon_e + \underline{v}(\alpha p^*)^{-1})\epsilon_h(1 + \epsilon_h)^{-1} - \epsilon_e$  and the effect of the policy is non-negative only if  $\epsilon_h/\epsilon_e \geq \alpha p^* \underline{v}^{-1}$ , which is interpreted as the ratio of the marginal utility between workers and entrepreneurs being higher than the entrepreneurs' gross profit margin per hour worked. Clearly, then, the higher the marginal utility of workers the more likely is that redistribution of wealth increases the efficiency of the economy. Again, higher intensity of commodity competition (higher values of  $\mu$ ) lowers  $p^*$  and, thus, lowers the required threshold for the redistribution to have positive effects. This is consistent with the interpretation of the role of competition on  $\Sigma_w$ .

## Productivity Growth

Conventional economic intuition suggests that an increase in productivity translates to higher employment and output, and modern theoretical models are consistent with it. This can be viewed as a special application of Say's Law, that aggregate supply generates its own demand. Yet, as I show below, this statement relies crucially on the underlying assumption that the economy is output constrained. The same reasoning does not necessarily apply when the economy is demand constrained. In particular, consider the comparative statics implications of a small increase in the productivity parameter  $\alpha$ , denoted by  $d\alpha$ . Similarly to the previous subsection, the change in equilibrium employment is given by:

$$\frac{dL^*}{d\alpha} = \frac{\Delta_p \Sigma_\alpha - \Delta_\alpha \Sigma_p}{\Delta_L \Sigma_p - \Delta_p \Sigma_L}$$

Once again attention is paid to the case where direct effects dominate indirect ones,

and this implies that  $\Delta_p > 0, \Delta_L > 0, \Sigma_p < 0, \Sigma_L > 0$  and  $\Delta_L \Sigma_p - \Delta_p \Sigma_L < 0$ . Because  $\Delta_\alpha$  and  $\Sigma_\alpha$  have ambiguous signs, the effect of productivity on equilibrium employment is also ambiguous. This is because, on the one hand, an increase in productivity raises entrepreneurial profits and consumption, which feeds back to an increase in aggregate demand and employment. On the other hand, a productivity increase reduces labor demand, which reduces employment and workers' commodity demand, and feeds back to the initial reduction of employment. Which channel dominates depends on the relative sizes of  $\Delta_\alpha$  and  $\Sigma_\alpha$ . As long as  $\Delta_\alpha$  is negative or small in absolute value and  $\Sigma_\alpha$  is negative, employment is positively affected by productivity changes. But if  $\Sigma_\alpha$  is positive (i.e. prices are positively affected by productivity growth), and  $\Delta_\alpha$  is positive (i.e. aggregate demand falls with productivity increases), then employment and output suffer a negative effect.

In the analytical example presented earlier (see expressions for  $L^*$  and  $p^*$  on page 19) the parameter  $\alpha$  affects directly  $L^*$  by decreasing the numerator and increasing the denominator. Thus, the direct effect of  $\alpha$  on  $L^*$  is always negative. The indirect effect, however, is always positive, since  $dp^*/d\alpha < 0$  and  $dL^*/dp^* < 0$ . The overall effect depends on the impact of  $d\alpha$  on the term  $\alpha p^*$  and so on the magnitude of the elasticity of the price level to productivity:  $dL^*/d\alpha \leq 0 \Rightarrow -\frac{dp^*}{d\alpha} \frac{\alpha}{p^*} \leq 1$  or  $E_{p|\alpha} \leq 1$ . To sum up, if the price level is inelastic to productivity increases then increases in the latter reduce equilibrium employment, as the relative purchasing power of workers falls and so do aggregate expenditure and production. On the other hand, if prices are elastic to productivity increases, then the opposite holds. Increases in productivity increase the relative purchasing power of workers and production and employment increase. This is an interesting finding because it is often invoked by non-academic economists, especially when discussing the impact of technological innovations on the future of labor (Rifkin, 1996), but academics often dismiss it as theoretically unfounded.<sup>3</sup> In this example one can see that the distinction between a demand or a supply constrained economy is crucial for the validity of the statement.

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<sup>3</sup>For more extensive discussions on this topic see that papers by Autor (2015), Mokyr et al. (2015), and Pratt (2015). Sachs et al. (2015) provide a good literature review of the area and a different theoretical argument on why technological improvements may reduce social welfare.

## 5 Model Extensions and Robustness

The previous two sections presented the main model and its implications. The model's simplicity makes it amenable to several extensions. Some of them are straightforward in terms of implementation and implications. For example, one can easily extend the model to include individual heterogeneity with regards to productivity or to initial endowments or to introduce some form of bargaining between workers and entrepreneurs. It is also easy to check that such extensions do not overturn the model's main results, on the contrary, they enrich them. Thus, this section focuses on the robustness of the model to less obvious extensions. In particular, I consider two important cases, one where there is elastic labor supply and workers have outside options (home production), as is commonly assumed in macroeconomic literature,<sup>4</sup> and one where capital and capital production are introduced.

### 5.1 Elastic Labor Supply and Outside Options for Workers

One may argue that one of the model's limitations is the fact that the minimum feasible wage,  $\underline{v}$ , is exogenous and that this may drive the results. Indeed, as proposition 4 shows, in the case where the minimum wage is set to zero, which seems as the natural lower bound, the economy features full employment and income or wealth inequality have no impact on aggregate demand. Moreover, the inelastic labor supply assumption may seem unrealistically stringent and is commonly relaxed in both RBC and search models.<sup>5</sup> In this subsection I demonstrate that the results go through in the more realistic economic setting where workers provide labor time elastically but they have the option to produce the commodity at home. Thus, on the one hand  $\underline{v}$  is determined endogenously and remains strictly above zero as long as some home production is feasible, and on the other hand labor supply is elastic and depends on  $\underline{v}$ .

Formally, consider the following extension of the model of section 2. The main elements of the economy remain as before, namely there are two goods, one producible and one status, and two classes, workers and entrepreneurs. However, workers have now access to an inferior technology that allows them to produce the commodity at home. For every hour they work at home they produce  $\alpha_h$  units of the commodity,

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<sup>4</sup>Recent examples in this area are Baxter and Jermann (1999), McGrattan et al. (1997), Attanasio and Weber (2010), Burda and Hamermesh (2010), and Aguiar et al. (2012)

<sup>5</sup>See Angeletos and La'O (2013) and Jaimovich et al. (2015) for examples of the former and Head et al. (2012) for an example of the latter literature.

with  $\alpha_h < \alpha_e$ , where  $\alpha_e$  is the productivity when working at a firm. Furthermore, labour supply is elastic but costly. An employed worker  $\ell$  who works  $\lambda_e$  hours in a firm and  $\lambda_m$  hours at home provides  $\lambda_\ell = \lambda_e + \lambda_m$  hours of work in total and thus his utility is expressed as  $u_\ell(c_\ell, w_\ell, \lambda_\ell)$  with  $u_{\ell\lambda} < 0$  and  $u_{\ell\lambda\lambda} < 0$ . The last two assumptions indicate the increasing cost of labour supply. For an unemployed worker,  $\lambda_e = 0$  by definition and his utility is expressed as  $u_u(c_u, w_u, \lambda_u)$ , where  $\lambda_u$  is the number of hours spent in home production. Both types of workers have the opportunity to produce the commodity at home, but at the same time they have access to the commodity market where they can purchase it. Thus, the total consumption  $c_h$  of worker  $h \in \{\ell, u\}$  is the sum of his home production  $c_{hm}$  and his purchase  $c_{he}$ . For employed workers  $c_{\ell m} = \alpha_h \lambda_m$  and for unemployed workers  $c_{um} = \alpha_h \lambda_u$ .

Workers' extended problem therefore is choosing how much to work at home and how many units of the commodity to buy from firms subject to their budget constraint and the non-negativity constraints for the choice variables. In addition, employed workers, or more precisely workers with a job offer have an option whether to accept it or not. This generates an employee participation constraint for entrepreneurs so that the terms of the work contract can not make the former worse-off than unemployed workers. Thus, in terms of the labor market, the main modification from section 2 is that each entrepreneur now offers a number of work hours per position  $\lambda_e$  in addition to the number of positions  $\ell_e^d$  and the wage rate  $v_e$ . Moreover, the entrepreneur's labor contract is now subject to the worker participation constraint  $u_\ell(c_\ell^*, w_\ell^*, \lambda_\ell^*) \geq u_u(c_u^*, w_u^*, \lambda_u^*)$ , each worker costs an arbitrarily small but positive cost  $\psi > 0$  to hire, and the modified labor-market-competition condition is that the entrepreneur whose contract offers the greatest utility to workers fills his vacancies first. The modified choice vector of an entrepreneur is  $\{p_e, v_e, \ell_e^d, \lambda_e, c_e\}$ . Finally, in order to make the model tractable, I adopt the assumption that a firm's customer base is disjoint from its workforce, so that when an entrepreneur sets his commodity price he does not worry about its impact on labor costs. Any other element of the model, such as commodity market competition, remain the same as before.

First, from the optimization problem of the unemployed worker one finds threshold values  $\underline{p}_u, \bar{p}_u$  such that: (i)  $0 \leq \underline{p}_u < \bar{p}_u$ , (ii)  $\lambda_u^* = 0, c_{ue}^* > 0$  for  $p \leq \underline{p}_u$ , (iii)  $\lambda_u^* > 0, c_{ue}^* > 0$  for  $\underline{p}_u < p < \bar{p}_u$ , (iv)  $\lambda_u^* > 0, c_{ue}^* = 0$  for  $p \geq \bar{p}_u$ . Intuitively, if the commodity price at the marketplace is sufficiently small unemployed workers have no incentive to work at home, while if it is sufficiently large they have no incentive to buy from the marketplace. For intermediate values unemployed workers do both but home



production is a substitute of market purchases and the former increases as the latter decreases. This defines the implicit function  $\lambda_u^*(p)$ , with  $d\lambda_u^*/dp > 0$  for  $\underline{p}_u < p < \bar{p}_u$ , which will be later used for the distinction between full-employment and unemployment equilibria. Conditional on the participation constrained being satisfied, similar results apply to the employed worker's problem. However, the thresholds  $\underline{p}_\ell$  and  $\bar{p}_\ell$  are implicit functions of  $v_e$  and  $\lambda_e$ .

Second, consider an entrepreneur  $e$  who does not take into account the labor-market competition rule because he expects that his labor contract offers worse conditions than his competitors. Thus, his only constraint is the workers' participation constraint. Suppose that his price and the demand for his commodity are  $p_e$  and  $D_e(\mathbf{p})$  respectively and that the potential pool of his workers, i.e. those who do not have a job offer from any other entrepreneur, is equal to  $\ell_e^s \geq 1$ . Finally, suppose that  $\lambda_u^*(p) > 0$  for all those workers. Then  $e$ 's optimal labor contract is split into two distinct cases. (i) If  $\alpha_e^{-1}D_e(\mathbf{p}) \leq \ell_e^s \lambda_u^*(p)$  then  $e$ 's labor demand can be covered by utilising only a fraction of the pool of unemployed workers and the optimal choice for the entrepreneur is to set  $\ell_e^d = \alpha_e^{-1}D_e(\mathbf{p})/\lambda_u^*(p)$ ,  $\lambda_e = \lambda_u^*(p)$ ,  $v_e = \alpha_h p$ . The labor contract  $\{\lambda_u^*(p), \alpha_h p\}$  substitutes the workers' labor supply to the firm for home production and pays the wage rate that allows them to purchase it back from the commodity market. Thus, the participation constraint is satisfied exactly and the hiring cost is minimized. (ii) If  $\alpha_e^{-1}D_e(\mathbf{p}) > \ell_e^s \lambda_u^*(p)$  then  $e$  needs to hire all the remaining workers to cover his demand. In this case  $\ell_e^d = \ell_e^s$ ,  $\lambda_e = \alpha_e^{-1}D_e(\mathbf{p})/\ell_e^s$ ,  $v_e(p, \lambda_e) : u_\ell(c_\ell^*, w_\ell^*, \lambda_\ell^*) = u_u(c_u^*, w_u^*, \lambda_u^*)$ . Clearly, in this case  $v_e > \alpha_h p$  and  $dv_e/d\lambda_e > 0$ . The above imply that, conditional on a level of  $p$ , the wage rate that satisfies the workers' participation constraint is constant for low working hours ( $\lambda_e \leq \lambda_u^*$ ) and increasing for higher levels ( $\lambda_e > \lambda_u^*$ ). This is diagrammatically shown in figure 3.

If  $p$  is such that  $\lambda_u^*(p) = 0$ , then the flat-part of the above graph disappears the  $v_e$  is increasing for every level of  $\lambda_e$ . Intuitively, if the commodity price is sufficiently low, time devoted to home production is zero even if a worker is unemployed and therefore any employed worker must be compensated with an increasing wage rate for every additional unit of time spent at the workplace.

Next, a modified version of Lemma 1 applies to the extended model, namely that in any symmetric Nash equilibrium the wage rate of entrepreneurs is such that the participation constraint of workers is exactly satisfied for any level of employment. The proof is very similar to the one of Lemma 1, so it is omitted here. The line of reasoning is straightforward. In any Nash equilibrium, aggregate commodity demand

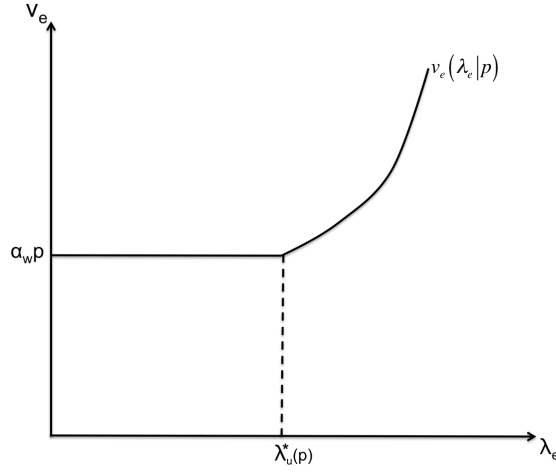


Figure 3: The Wage Rate as an Implicit Function of Labor Time

must be equal to aggregate commodity supply, otherwise at least one entrepreneur can profitably deviate by either increasing his price slightly (case of excess demand) or by decreasing production inputs (case of excess supply). This then rules out cases where there is excess aggregate labor demand in equilibrium, because this would imply excess commodity demand. The only viable equilibria involve cases where aggregate labor demand is less or equal to aggregate labor supply. In these cases each entrepreneur can unilaterally reduce his wage rate without losing workers to other entrepreneurs. The only wage rate that survives this profitable deviations is the one where the participation constraint of each worker binds given the demanded labor hours by the entrepreneur. Formally then, the result is the following.

**Lemma 2** *In any symmetric Nash equilibrium of the economy the wage rate is given by:*

$$v(\lambda_e, p) = \begin{cases} \alpha_h p & \text{if } \alpha_e^{-1} ED_e \leq H \lambda_u^*(p) \\ \text{such that } u_\ell(c_\ell^*, w_\ell^*, \lambda_e) = u_u(c_u^*, w_u^*, \lambda_u^*) & \text{if } \alpha_e^{-1} ED_e > H \lambda_u^*(p) \end{cases}$$

Lemma 2 gives us one of the necessary features of the symmetric equilibria in the extended model. The rest come from the optimal-price correspondence as a best-response to the entrepreneurs' utility maximisation problem and the commodity-clearing condition, which, as shown by the lemma, necessarily holds in equilibrium. The proposition below provides all the necessary conditions.

**Proposition 5** *Let  $c_{\ell_e}^*$ ,  $c_{ue}^*$  and  $c_e^*$  denote the commodity demand correspondences of employed, unemployed workers and entrepreneurs respectively, which are derived by the respective optimisation problems and which implicitly depend on the endogenous variables  $\{p, v, \lambda_e\}$ . Then, any symmetric Nash equilibrium of the extended model with positive aggregate commodity production satisfies the following conditions.*

$$v(\lambda_e, p) = \begin{cases} \alpha_h p & \text{if } \alpha_e^{-1} E D_e \leq H \lambda_u^*(p) \\ \text{such that } u_\ell(c_{\ell_e}^*, w_\ell^*, \lambda_e) = u_u(c_{ue}^* + \alpha_h \lambda_u^*, w_u^*, \lambda_u^*) & \text{if } \alpha_e^{-1} E D_e > H \lambda_u^*(p) \end{cases} \quad (\text{EX1})$$

$$p : \frac{\partial}{\partial p} \{(p - v \alpha_e^{-1}) (D_e - c_e^*)\} = 0 \quad (\text{EX2})$$

$$\alpha_e [IH + (1 - I)L] [I \lambda_e + (1 - I) \lambda_u^*] = E c_e^* + H [I c_{\ell_e}^* + (1 - I) c_{ue}^*] + L(1 - I)(c_{\ell_e}^* - c_{ue}^*) \quad (\text{EX3})$$

$$I = \begin{cases} 0 & \text{if } \alpha_e^{-1} E D_e \leq H \lambda_u^*(p) \\ 1 & \text{if } \alpha_e^{-1} E D_e > H \lambda_u^*(p) \end{cases} \quad (\text{EX4})$$

In the above proposition (EX1) is given by lemma 2 and by the results that an employed worker does not produce at home in equilibrium, hence  $c_\ell^* = c_{\ell_e}^*$ , and that the unemployed workers' total consumption of the commodity is given by the sum of home production and market purchases, that is  $c_u^* = \alpha_h \lambda_u^* + c_{ue}^*$ . (EX2) implicitly defines the optimal-price correspondence in terms of  $v$  and  $\lambda_e$ , and, as in the previous sections, this can be interpreted as the aggregate supply correspondence of the economy. Similarly, (EX3) gives the aggregate labor demand of the economy consistent with the aggregate commodity demand for given values of  $p$  and  $v$ . The indicator function  $I$  in (EX4) separates the economy into two cases, the case where aggregate commodity demand exceeds aggregate home-production hours ( $\alpha_e^{-1} E D_e > H \lambda_u^*(p)$ ), and the case where it does not ( $\alpha_e^{-1} E D_e \leq H \lambda_u^*(p)$ ). The first case corresponds to a full employment economy, while the second case to an economy with positive output gap and unemployment.

The impact of these two cases on aggregate labor demand are incorporated in (EX3). In a full employment equilibrium aggregate labor demand is equal to  $H\lambda_e$  while in an equilibrium with unemployment it is equal to  $L\lambda_u^*$ .

The key result from the above extension is that the main features of the baseline model carry forward to this one too. This means that one can find examples where the aggregate demand is sufficiently weak so that an equilibrium with unemployment exist. Moreover, conducting comparative statics exercises on this type of equilibrium yields ambiguous results just as in the previous section. Namely, increases in income or wealth inequality or increases in firm productivity may increase unemployment and the output gap. Consider for example the special case of explicit functional forms, with  $u_h(c_h, w_h, \lambda_h) = \epsilon_h \ln(c_h) + \ln(w_h) - (\lambda_h)^2$ , for  $h \in \{\ell, u\}$  (working class),  $u_e(c_e, w_e) = \epsilon_e \ln(c_e) + \ln(w_e)$  (entrepreneurial class), and  $\mu(p_e, \mathbf{p}_{-e})$  given by equation 2. Although one cannot solve the extended model analytically, the demand and supply correspondences and the model's equilibrium can easily be shown for specific parameter values. Figure 4 has been constructed in one such way, and it features two different equilibria, one of full employment and of no output gap (on the right-hand side of the graph), and one of positive unemployment and output gap (on the left-hand side). The  $DD$  ( $SS$ ) curves represent the demand (supply) correspondences, and are derived by solving EX3 (EX4) with respect to  $p$  for every level of  $\lambda_e$ . The  $\lambda_u(p)$  curve gives the number of hours in home production that unemployed workers put in at any given price level. Thus, combinations of  $p$  and  $\lambda_e$  to the left of the curve represent cases where firm-labor time per worker is less than home- production time per worker and the economy is below full capacity. Combinations to the right of the curve, on the other hand, represent cases where the economy is at full capacity.

Figure 4 shows that the extended model can generate the same type of main results as the baseline one. It also demonstrates the main difference between the two. While the inelastic supply model features the same maximum capacity constraint at every price level, in the elastic supply model the maximum capacity constraint is increasing in the price level. Graphically,  $\lambda_u(p) = \ell_0$  for the inelastic supply model, while  $d\lambda_u/dp > 0$  for the elastic supply model. The difference, of course, emerges as workers adjust their available time for home production according to the incentives to produce the commodity at home, even if this happens off the equilibrium. In other words, the  $\lambda_u$  curve represents the endogenous outside option of workers, which does not exist in the inelastic labor supply variant. Although this differences enriches the model's results and makes the framework more realistic, the important point is that it does not take

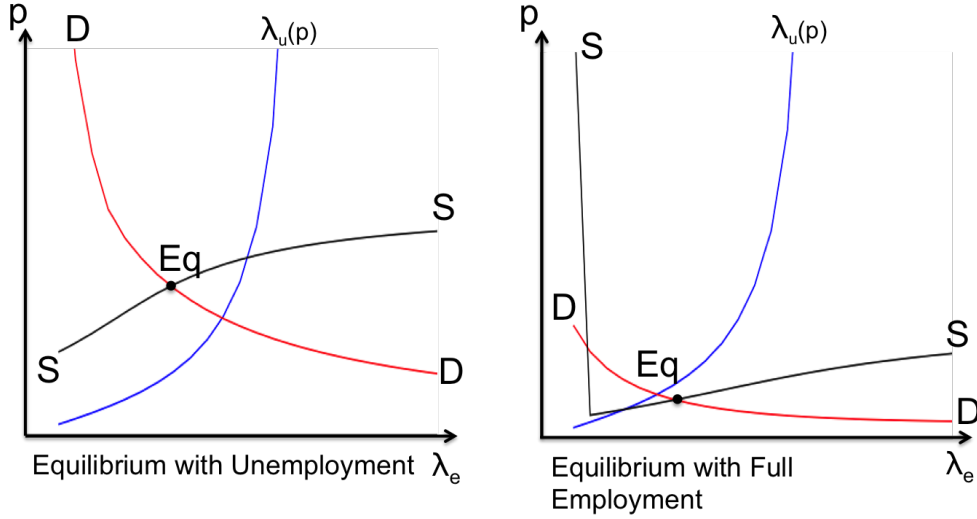


Figure 4: Examples of Equilibria in the Extended Model

anything away from the main focus of the paper.

## 5.2 Model with Capital

Another issue with the baseline model is that there is no capital in the economy. At first sight, this may seem as an implausible assumption. One may go as far as thinking that the absence of capital is what drives the results because it does not allow the excess capacity to be absorbed in investment. This subsection shows that the model results are robust to this criticism. In particular, I maintain all the elements of the baseline model with the exception of the production technology, which is modified to include both labor and capital as inputs. Specifically, the output  $y_e$  of an entrepreneur's firm is given by the function  $g(\lambda_e, \kappa_e)$ , where  $\lambda_e$  is the number of workers and  $\kappa_e$  is the units of capital in the firm.

In addition, capital is not endowed to entrepreneurs directly but produced by capital producers, which I will call '*engineers*'. The entrepreneurs place orders to the engineers and buy the capital they need for commodity production, so that the expenditure on capital becomes the entrepreneurs' investment decision. There are  $J$  engineers in total, with  $j$  denoting a generic engineer. Each one of them have an endowment  $w_{j0}$  of the status good and utility function  $u_j(c_j, w_j)$ , where  $c_j$  is an engineer's consumption of the commodity and  $w_j$  is his final wealth in terms of the status good. Engineers need to purchase the commodity from entrepreneurs just like the workers do, while they earn

income from the profits of their capital manufacturing firms, just as entrepreneurs do. Thus, their final wealth is equal to  $w_j = w_{j0} + \pi_j - pc_j$ , where  $\pi_j$  is the profits from manufacturing capital goods.

To produce capital, an engineer hires workers in his firm. The final output in terms of the capital good is linear in labor:  $\kappa_j = \alpha_j \lambda_j$ , where  $\kappa_j$  is the units of capital produced by  $j$ 's firm,  $\alpha_j$  is workers' productivity in capital manufacturing, and  $\lambda_j$  is the total number of workers employed by  $j$ .  $j$ 's output is sold in the capital market, where he faces competition from other engineers. To keep the model structure as similar as possible to the baseline model, I assume of flexible structure for capital-good competition. Thus, the fraction of entrepreneurs who buy their capital from  $j$  depends both on  $j$ 's price for each unit of capital ( $p_j$ ) and on the rest of engineers pricing decisions  $\mathbf{p}_{-j}$ . This is given by the capital-good market share function  $\mu_j(p_j, \mathbf{p}_{-j})$ , for which the same assumptions hold as for the function  $\mu_e(p_e, \mathbf{p}_{-e})$  of the baseline model (see page 8).

Overall, the engineers' problem is very similar to the one of entrepreneurs, namely they choose the vector  $\{c_j, p_j, v_j, \lambda_j\}$  in order to maximise their utility subject to their budget constraint, the non-negative profit constraint, and the output constraint. Only the last constraint is different in formulation from section 2 and this is presented formally later on. As mentioned earlier, all other elements of the model remain as in the baseline.

In terms of analysis, the analogue of Lemma 1 holds for this extension as well. Even though there are two labor markets operating, one for workers in the capital-good manufacturing and one in the commodity-good production, it still remains that in any Nash equilibrium of the game aggregate labor demand from both markets does not exceed labor supply and wage competition is void. The equilibrium wage for both sector is the minimum, namely  $\underline{v}$ . Since this result is built in the same way as in the baseline model, its formal presentation is omitted here.

Next, it is useful to think of the entrepreneurs' maximisation conditions first (the workers remain the same as in section 3 and are omitted). The main difference from the baseline section is that there are two inputs of production now and the demand for each is optimised. In equilibrium, the demand for  $e$ 's commodity,  $D_e(\mathbf{p})$ , is equal to output  $g(\lambda_e, \kappa_e)$  and the the input optimality condition  $g_\lambda/g_\kappa = \bar{v}/p_j$  holds, where  $g_x$  is the partial derivative of  $g$  with respect to input  $x$ . These two conditions pin-down the optimal demands for labor and capital as functions of the rest of the endogenous variables, and the commodity price, in particular. Hence, let  $\lambda^*(p_e)$  and  $\kappa^*(p_e)$  denote the optimal input demands as functions of  $e$ 's choice of commodity price.

It is also useful to define the function  $\hat{\kappa}_e(\lambda_e)$  which gives the optimal level of  $e$ 's capital-good demand for every level of his labor demand. In a symmetric equilibrium, then, the aggregate capital-good demand is  $E\hat{\kappa}_e(\lambda_e)$  and the require labor force for its production is  $\alpha_j^{-1}E\hat{\kappa}_e(\lambda_e)$ . The maximum number of workers that  $e$  can hire is  $\bar{\lambda}_e$  and is given as the implicit solution to the equation:  $E^{-1}H - \alpha_j^{-1}\hat{\kappa}(\bar{\lambda}_e) = \bar{\lambda}_e$ . This means that in a symmetric equilibrium the workforce for an entrepreneur can not exceed the available number of workers divided equally among entrepreneurs minus those required for the production of the capital that the said workforce will use in commodity production. Thus, this equation defines the maximum production capacity of the economy given its number of workers and this is equal to  $Eg(\bar{\lambda}_e, \hat{\kappa}(\bar{\lambda}_e))$ , and an entrepreneur's output constraint is  $y_e = \min\{D_e(\mathbf{p}), g(\bar{\lambda}_e, \hat{\kappa}(\bar{\lambda}_e))\}$ . Combining all the above,  $e$ 's problem can be formulated through the following Langrangian:

$$\begin{aligned} \mathcal{L}_e = & u_e(c_e, w_e) + \tau_{e1} \left[ w_{e0} + p_e \mu_e(p_e, \mathbf{p}_{-e}) \sum_{i \notin E} c_i^*(p_e) - \underline{v} \lambda_e^*(p_e) - p_j \kappa_e^*(p_e) - w_e \right] \\ & + \tau_{e2} \left[ p_e \mu_e(p_e, \mathbf{p}_{-e}) \sum_{i \notin E} c_i^*(p_e) - \underline{v} \lambda_e^*(p_e) - p_j \kappa_e^*(p_e) \right] + \tau_{e3} \left[ g(\bar{\lambda}_e, \hat{\kappa}(\bar{\lambda}_e)) - \mu_e(p_e, \mathbf{p}_{-e}) \sum_{i \notin E} c_i^*(p_e) - c_e \right] \end{aligned}$$

The necessary first order conditions for its solution are given by the following system of equations, where the arguments of the functions are suppressed for compactness:

$$u_{ec} = (\tau_{e1} + \tau_{e2}) \left( \underline{v} \frac{\partial \lambda_e^*}{\partial c_e} + p_j \frac{\partial \kappa_e^*}{\partial c_e} \right) + \tau_{e3} \quad (\text{EN1})$$

$$(\tau_{e1} + \tau_{e2}) \frac{\partial [p_e(D_e - c_e) - \underline{v} \lambda_e^* - p_j \kappa_e^*]}{\partial p_e} = \tau_{e3} \frac{\partial (D_e - c_e)}{\partial p_e} \quad (\text{EN2})$$

$$u_{ew} = \tau_{e1} \quad (\text{EN3})$$

$$\tau_{e2} \pi_e = 0 \quad (\text{EN4})$$

$$\tau_{e3} [g(\bar{\lambda}_e, \hat{\kappa}(\bar{\lambda}_e)) - D_e] = 0 \quad (\text{EN5})$$

The problem of an engineer is very similar to that of an entrepreneur. The only noteworthy differences are that the engineer's demand comes from the entrepreneurs' optimal use of capital,  $D_j(p_j, \mathbf{p}_{-j}) = \mu_j(p_j, \mathbf{p}_{-j})E\kappa_e^*(p_j)$ , and that the output constraint is expressed in terms of a limit in the aggregate demand of capital,  $\alpha_j\lambda_j \leq EJ^{-1}\hat{\kappa}_e(\bar{\lambda}_e)$ . Thus, we have the following Lagrangian and necessary conditions:

$$\begin{aligned} \mathcal{L}_j = & u_j(c_j, w_j) + \tau_{j1} [w_{j0} + (p_j - \underline{v}\alpha_j^{-1})\mu_j(p_j, \mathbf{p}_{-j})E\kappa_e^*(p_j) - p_e c_j - w_j] \\ & + \tau_{j2} [(p_j - \underline{v}\alpha_j^{-1})\mu_j(p_j, \mathbf{p}_{-j})E\kappa_e^*(p_j)] + \tau_{j3} [J^{-1}\hat{\kappa}_e(\bar{\lambda}_e) - \mu_j(p_j, \mathbf{p}_{-j})\kappa_e^*(p_j)] \end{aligned}$$

$$u_{jc} = p_e u_{jw} \quad (\text{J1})$$

$$(\tau_{j1} + \tau_{j2}) \frac{\partial [(p_j - \underline{v}\alpha_j^{-1})D_j]}{\partial p_j} = \tau_{j3} \frac{\partial D_j}{\partial p_e} \quad (\text{J2})$$

$$\tau_{j2}\pi_j = 0 \quad (\text{J3})$$

$$\tau_{j3} [EJ^{-1}\hat{\kappa}_e(\bar{\lambda}_e) - D_j] = 0 \quad (\text{J4})$$

The model closes with the necessary condition for workers (E1) from proposition 1, and the equilibrium unemployment condition:

$$L^* = \min\{E(\lambda_e^* + \alpha_j^{-1}\kappa_e^*), H\} \quad (\text{EL})$$

**Proposition 6** *Any symmetric Nash equilibrium of the extended model with capital production satisfies conditions (E1), (EN1) to (EN5), (J1) to (J4) and (EL).*

The necessary conditions of the extended model are very similar to the ones of the baseline model. (EL) also makes clear that equilibrium unemployment is still a possible outcome. If product demand is relatively weak, because individuals do not value commodity consumption as much as the status good, and the productivity of workers



in producing capital is sufficiently high, then the aggregate demand for labor is low and unemployment emerges in equilibrium. Furthermore, the comparative statics results of section 4 remain relatively unaffected by the introduction of capital.

To demonstrate this clearly, it is useful to adopt once again explicit functional forms. As before, utility functions are given by  $u_i(c_i, w_i) = \epsilon_i \ln(c_i) + \ln(w_i)$  for all classes of individuals.  $\mu_e(p_e, \mathbf{p}_{-e})$  maintains the same functional form as in equation (2). The production function for the commodity good is given by a Cobb-Douglas function,  $g(\lambda_e, \kappa_e) = \alpha \lambda_e^\beta \kappa_e^{1-\beta}$ . For simplicity, I will consider the limit case where there is perfect competition between engineers, so  $\partial \mu_j / \partial p_j \rightarrow -\infty$ ,  $p_j = \underline{v} \alpha_j^{-1}$  and  $\pi_j = 0$ . The commodity-demand functions are  $c_h = \epsilon_h \hat{w}_h ((1 + \epsilon_h) p_e)^{-1}$  for workers,  $c_j = \epsilon_j \hat{w}_j ((1 + \epsilon_j) p_e)^{-1}$  for engineers, and  $c_e = \epsilon_e \hat{w}_e ((1 + \epsilon_e) X_c)^{-1}$ , where  $X_c \equiv \underline{v} \alpha^{-1} \beta^{-\beta} [a_j (1 - \beta)]^{-(1-\beta)}$ . Note that  $X_c$  is a generalised form of the term  $x_2$  that appears in proposition 3. The entrepreneurs' optimal input demand functions are  $\lambda_e^*(p_e) = \alpha^{-1} (\beta p_j (1 - \beta)^{-1} \underline{v}^{-1})^{1-\beta} D_e(p_e)$  and  $\kappa_e^*(p_e) = \alpha^{-1} (\beta^{-1} p_j^{-1} (1 - \beta) \underline{v})^\beta D_e(p_e)$ . Finally, the equilibrium price level and employment are given by the following equations:

$$p^* = 2^{-1} \left( X_c + \sqrt{X_c^2 + 4E^{-1} x_1^{-1} X_c} \right) \quad (6)$$

$$L^* = \frac{(1 + \epsilon_h) \left[ E \epsilon_e w_{e0} + (\epsilon_e + X_c / p^*) \left( J \frac{\epsilon_j}{1 + \epsilon_j} w_{j0} + H \frac{\epsilon_h}{1 + \epsilon_h} w_{h0} \right) \right]}{(1 + \epsilon_e + \epsilon_h - \epsilon_h X_c / p^*) \underline{v} \ell_0} \quad (7)$$

where  $x_1 \equiv \mu \delta^{-1}$  is the same term as in proposition 3.

The above equations are very similar to their counterparts in the baseline model. The main difference is the replacement of the term  $\underline{v} \alpha^{-1}$  by its generalized form  $X_c$ . As in the baseline model,  $p^*$  remains independent of  $L^*$  and equilibrium employment depends positive on initial endowments and  $X_c$  but negatively on the price level. Furthermore, it depends positively on engineers' propensity to consume, which is a function of  $\epsilon_j$ , and negatively on the capital-good productivity  $\alpha_j$ . Thus, if  $\epsilon_j$ ,  $w_{j0}$  are sufficiently low and  $\alpha_j$  is sufficiently high, then the introduction of capital production into the model has a small impact on equilibrium employment. A similar point can be made for the comparative statics exercises. Since equations (6) and (7) are almost identical to their baseline counterparts, the main findings of section 4 apply with minor modifications to this extension of the model as well.

## 6 Conclusion

This paper presents a simple model of a demand-constrained economy without any price rigidities, where a positive output gap and unemployment emerge as equilibria phenomena, and where the distribution of income and wealth matter. The main elements that give rise to these results is the presence of a good in fixed supply, which I justify in terms of capitalist-spirit preferences, and the adoption of a game-theoretic approach in solving for the equilibrium. In particular, the focus of the analysis is on pure symmetric Nash equilibria, where agents of the same type take the same actions.

The main intention of the model is to provide a theoretical justification for the attention that demand-side models have received after the Great Recession, and to address the criticism launched against them, that they require ad-hoc assumptions on price rigidities. Furthermore, now that public opinion in many western countries has turned its attention to income inequities, and minimum-wage regulations have been proposed both in US and UK, it is important to provide both theoretical and empirical justifications for their implementation. This paper is a small step in this direction.

Of course, the model of this paper has several simplified features that make it unsuitable for direct policy analysis. It is static, and thus ignores many important dynamic macroeconomic forces such as savings and growth, it lacks a properly micro-structured financial sector, and it is concerned with a closed economy. But its simplicity is a deliberate choice in order to convey a clear message, namely that price rigidities and labor-market frictions are not necessary elements of a demand-side based model. Neither do demand-side considerations enter economic modelling exclusively through intertemporal decision making. A static framework can serve this purpose equally well.

Finally, the simple structure of the benchmark model makes it amenable to extensions that address the above shortcomings. I think that it is worth embedding it into more elaborate models and exploring its implications to issues such as financial crises or the negative side-effects of trade openness. For the time being this remains in the plans for future work.

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