How to Tax Polluting Firms? A Bargaining Solution

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Abstract

This paper studies taxation on firms by examining profits tax and emission tax in a unified framework. We develop a model in which the government and the polluting firms bargain over the rates of profits tax and emission tax. We demonstrate that the bargaining position of firms is a key determinant of profits tax, but it does not affect emission tax. Emission tax may not be imposed, even if it has a twofold merit of increasing public revenue and regulating pollution. Moreover, public environmental expenditure will facilitate a cut in emission tax and therefore boost the production of firms. With the increased market demand for polluting goods, public environmental expenditure may be a better regulatory instrument than emission tax.

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1 Introduction

In his influential book *Capital in the Twenty-First Century*, Thomas Piketty demonstrates that the rate of capital return has been persistently greater than the rate of economic growth over the past 250 years (Piketty 2014). With the ever-rising concentration of social wealth toward capitalists, corporate profits become an increasingly significant source of government revenue (Zucman 2014, Piketty 2015). Imposing higher tax rates on wealthy capitalists can finance public expenditure and lessen income inequality, which improves social welfare.

The advances of globalization characterized by the growing cross-border capital flows complicate the design of profits tax confronted by policymakers (e.g., Wilson and Wildasin 2004, Haufler and Lülfesmann 2015). To attract inward investments in support of economic growth, many governments are tempted to undercut each other in tax competition, which gives rise to the possibility of “race to the bottom”.1 Firms may hold the option of “moving abroad” as a bargaining chip to press the local government to lower the profits tax rate, although their mobility is often constrained by the sunk cost of fixed investments and the risk of overseas operations.2 It is hence necessary for governments to strike a balance between raising public revenue and encouraging business investment when levying a profits tax.

The tax design becomes more challenging in case that firms emit pollution in their production (Fullerton, Leicester, Smith 2010). People enjoy the economic growth boosted by new investments, but they never want the adverse environmental consequences (Brock and Taylor 2010). Addressing the public environmental concern requires governmental efforts in regulating polluting activities. Governments have a broad range of tools at their disposal among which emission tax is pervasively recognized as a central pillar (e.g., Requate 2005, Williams 2016). Yet a new emission tax on top of profits tax will further dampen production and raise the stake of losing business investment. Where economic growth is given a higher priority than environment protection, the inter-governmental competition is likely to result in the “race to the bottom” in setting emission tax.3

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1 There is extensive and growing literature investigating the international competition in corporate taxation. See Wilson and Wildasin (2004) for a review and the more recent contributions by Devereux, Lockwood, and Redano (2008), Slemrod and Wilson (2009), Becker, Egger, and Merlo (2012), and Haufler and Stöhler (2013).
2 See Dixit and Pindyck (1994) and Busse and Hefeker (2007).
3 See, for example, Revesz (1992), Markusen, Morey, and Olewiler (1995), and Rudra (2008).
How to tax profitable firms whose production generates environmental externalities? One popular viewpoint in the arena of public policy posits that profits tax should be replaced by emission tax. This view has found audience among practitioners in some developed countries. For example, the Province of British Columbia, Canada, has collected the carbon tax revenues since the year 2008 and used the funds to lower taxes on firms. A similar “tax swap” was proposed in the U.S. by Congressman John Delaney, who filed a new legislation *Tax Pollution, Not Profits Act* in 2015 to establish the system of carbon tax while cut the rate of profits tax by 7%. Nevertheless, many developing countries still pursue a weak policy of environmental protection by setting a very low or even zero emission tax (Greenstone and Jack 2015).

This paper provides a theoretical analysis of taxation on firms with a lens that combines profits tax and emission tax. Taxing firms raises the public revenue but also raises the possibility of firm exodus. In addition, the two taxes generate different impacts on the economy and the environment. Profits tax, which is proportional to the gross profits of firms, has no distorting effect on the production, but it can hardly correct the environmental externalities generated by firms. Emission tax is a useful instrument to increase the incentive for pollution abatement, but it heightens the tax-inclusive price of polluting goods and hence reduces consumer surplus. The tax design therefore involves the determination of the overall tax collection but also the tradeoff between profits tax and emission tax, which is in aligned with the argument that emission tax reform is not a free lunch (e.g., Buchanan 1969, Fullerton and Metcalf 1998).

We set up the analytical framework based on Haufler and Wooton (1999) – a general-equilibrium model of profits tax – and Copeland and Taylor (1999) – a two-sector model investigating environmental externalities of manufacturing production. In our model, there are pollution-free perfectly-competitive farms producing agricultural goods and pollution-intensive oligopolistic firms producing manufactured goods. The government, who cares about people’s welfare, levies on both profits and emissions of manufacturing firms and then rebates the tax revenues to people. To determine the two tax rates, the government engages in a Nash bargaining with firms, which have an outside

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4See Repetto, Dower, Jenkins, and Geoghegan (1992) and Hammond and Dunkiel (1998). For instance, Hammond and Dunkiel (1998) argue that “it makes perfect sense to tax good things less heavily and bad things heavily ... we ought to reduce taxes on work, investment, and entrepreneurship, and increase them on pollution, waste, and greenhouse gas emissions”.


6See the proposal at [https://www.govtrack.us/congress/bills/114/hr2202/text](https://www.govtrack.us/congress/bills/114/hr2202/text).
option of operating their business elsewhere.

Our analysis yields several results. First, the bargaining position of manufacturing firms will affect the rate of profits tax but not the rate of emission tax. In effect, the government and firms can first address the environmental externalities arising from the production and then negotiate how to split and share the pie of firm profits. Second, the rate of emission tax is determined by a number of factors such as market structure, market demand, production and abatement technologies, and social damage of emissions. Under some circumstances, the government may prefer not to impose an emission tax, although the emission tax can create benefits accruing to both environment quality and public revenue. In that case, the lump-sum transfer of the profits tax revenue to people can be viewed as a compensation for their suffering in a polluted environment.

Moreover, the market-based approaches that spur abatement activities at the private sector is not the singular way of curbing pollution. Earlier studies have recognized the attraction of environmental initiatives led by the government (Bovenberg and Smulders 1995, Preuss 2007). Nowadays governments are increasingly called upon to partner with the private sector to invest in pollution abatement. Many countries have integrated environmental programs into the expenditure frameworks of public administration (OECD 2011). For example, among the 28 member states of European Union, the public sector contributed 87.2 billion euros for environmental protection in 2013, which accounted for 31% of the total environmental protection expenditure; the public environmental expenditure was concentrated in waste management and wastewater treatment. López and Palacios (2014) find that a reallocation of government spending composition towards public goods reduces the concentrations of air pollutants in Europe.

In accordance with the stylized fact regarding public environmental expenditure, we extend our baseline model to study the case that the government allocates a part of tax revenues to finance the pollution abatement. We show that public environmental expenditure will lead to a lower emission tax, which in turn facilitates the expansion of industrial production. Also, as people have greater willingness to pay for polluting goods, the government tends to devote more resources to public abatement while cut the emission tax to save the abatement efforts at the private sector.

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7For instance, see “Creating an American Infrastructure Investment Strategy” by Henry M. Goldberg at OurEnergyPolicy.org.

8Data Source: Database of Environmental Protection Expenditure in eurostat.
2 The Model Setup

We develop the analytical framework based on Haufler and Wooton (1999), who construct a two-sector general equilibrium model to study the international competition of profits tax. We limit our attention to the policy decision of one country and attempt to make two extensions. First, we follow Copeland and Taylor (1999) to consider that one of the two sectors generates adverse environmental externalities. Second, in addition to profits tax, the government levies an emission tax to regulate the pollution, and both tax rates are determined through the Nash bargaining between the government and the firms of the polluting sector.

Consider an economy with a single household who consumes agricultural goods and manufactured goods. The household has the same utility as in Haufler and Wooton (1999), with an addition that its concern about the environmental damage enters the utility function as an additively separable term. Specifically, the preferences of the household are given by

\[ u = C_A + \alpha C_M - \frac{\beta}{2} C_M^2 - \gamma E, \]  

(2.1)

where \( C_A \) denotes the consumption of agricultural goods, \( C_M \) denotes the consumption of manufactured goods, \( E \) denotes the total pollution emission, and \( \alpha, \beta, \) and \( \gamma \) are positive parameters.

The household is endowed with one unit of labor, and supplies the labor inelastically for which it receives a wage income of \( w \). Suppose that the government collects a total tax revenue of \( T \geq 0 \) from the production of goods and then redistributes it in a lump-sum fashion to the household.\(^9\) Let the agricultural goods be numeraire goods and \( p \) represent the relative price of manufactured goods. The household’s budget constraint amounts to

\[ w + T = C_A + p C_M. \]  

(2.2)

Maximization of (2.1) subject to the budget constraint (2.2) yields the inverse demand function of

\(^9\)It is possible to have \( T < 0 \), which can be interpreted as “government subsidy to production” or “payroll tax on the household”. With a focus on the policy of taxation on firms, we only discuss the case \( T \geq 0 \) in this paper.
manufactured goods:

\[ p = \alpha - \beta C_M, \] (2.3)

which shows that \( \alpha \) measures the household’s highest willingness to pay for manufactured goods and \( \beta \) is the slope of the demand curve. It is worth noting that the transfer payment does not enter the demand function of manufactured goods since changes in income will only affect the demand of agricultural goods (numeraire goods) at the margin.

There are two sectors, denoted by \( A \) and \( M \), which hire labor as the only input in their production. Sector \( A \), or agricultural sector, consists of a large number of perfectly-competitive farms. Each farm converts one unit of labor into one unit of agricultural goods without generating pollution. The zero profit condition implies that the equilibrium wage rate in agricultural sector equals one (as measured by numeraire goods). Given that labor is freely mobile in a competitive market, it will be reallocated across sectors instantaneously to ensure that wage rates are equalized when both sectors are active, i.e., \( w = 1 \) for the whole economy in equilibrium.

Sector \( M \), or Smokestack manufacturing, is composed of \( N > 1 \) identical firms that produce manufactured goods under Cournot competition and create environmental externalities. Following Copeland and Taylor (1999), we assume that one unit of labor can produce one unit of manufactured goods and generate \( \theta \) units of pollution as a by-product. Moreover, each firm will invest in abatement technology to mitigate its pollution emission. If a representative firm produces \( m \) units of output and abates \( x \) units of pollution, its emission can be expressed by

\[ e = \theta m - x. \] (2.4)

Pollution abatement incurs a cost at firms, which is increasing and strictly convex in the abatement level. Define the cost of pollution abatement as

\[ c(x) = \frac{x^2}{\eta}, \] (2.5)

where \( \eta \) measures the cost-effectiveness of pollution abatement.

Market clearing constraint for manufactured goods requires the household’s demand equals the
aggregate supply of the manufacturing sector:

\[ C_M = Nm. \]  \hspace{1cm} (2.6)

The total pollution emission is the sum of emissions across all the manufacturing firms:

\[ E := Ne = N\theta m - Nx. \]  \hspace{1cm} (2.7)

The government serves to maximize the household’s utility.\(^{10}\) There are two policy instruments available for the government: an emission tax \( \tau_e \geq 0 \) per unit of emission and a profits tax by a proportion \( \tau_p \in [0, 1) \) of the profit. The agricultural sector will have no tax burden because it is free of pollution and earns zero profit in equilibrium. The manufacturing sector, which consists of pollution-intensive oligopolistic firms, will be the only potential source of taxes.

The government and the manufacturing sector enter a Nash bargaining process to determine the rates of emission tax and profits tax. The agreement reached by the collective bargaining will be uniformly applied to each firm. The manufacturing sector has an outside option of fleeing to another country where each firm can always earn a profit \( \Omega > 0 \), which is exogenously given.\(^{11}\) Assume that international trade incurs excessively high transportation costs to rule out the possibility of importing manufactured goods from abroad. If the manufacturing sector moves elsewhere, the household has to consume agricultural goods only but can enjoy a pollution-free environment (i.e., \( C_A = 1, C_M = E = 0 \)), thereby achieving a utility level of \( u = 1 \). Let \( \delta \in (0, 1) \) indicate the bargaining power of the manufacturing sector versus the government. The Nash bargaining equation is

\[
\max_{\tau_p, \tau_e} (\pi - \Omega)^\delta (u - 1)^{1-\delta}.
\]  \hspace{1cm} (2.8)

Nash bargaining implies that the two tax rates are set to maximize the match surplus.

\(^{10}\) We assume that the government concerns only about the welfare of the household but does not concern about the net profit of firms. For instance, we might as well assume that the government is only responsible for its citizens while all the manufacturing firms are owned by foreigners.

\(^{11}\) Firms will expect their relocation to incur a large amount of investments such as plant establishment, employee recruitment and training, and product quality control, and also they will forfeit the “sunk” domestic fixed investments. These considerable costs tend to erode the profitability of new businesses (e.g., Dixit and Pindyck 1994), which may lead to a small \( \Omega \).
3 Bargaining over Taxation

This section demonstrates the Nash bargaining solution to the rates of profits tax and emission tax. We start by analyzing the individual manufacturing firm’s decisions of output and pollution abatement to determine its equilibrium profit. Given the wage rate and tax rates, each firm earns a net profit as its total sales less the labor cost of labor, the cost of pollution abatement, and the total tax payments, namely,

\[ \pi = (1 - \tau_p) \left[ pm - m - c(x) - \tau_e e \right] \]
\[ = (1 - \tau_p) \left[ (p - 1 - \theta \tau_e) m + \tau_e x - \frac{x^2}{\eta} \right]. \quad (3.1) \]

Facing the demand curve (2.3) and the two tax rates \((\tau_p, \tau_e)\), each firm seeks profit maximization by choosing the levels of output and pollution abatement simultaneously. The first order conditions of (3.1) with respect to \(m\) and \(x\) obtain

\[ \frac{\partial \pi}{\partial m} = (1 - \tau_p) \left[ \alpha - 1 - \theta \tau_e - (N + 1) \beta m \right] = 0 \quad \Rightarrow \quad m = \frac{\alpha - 1 - \theta \tau_e}{(N + 1) \beta}, \quad (3.2) \]

\[ \frac{\partial \pi}{\partial x} = (1 - \tau_p) \left( \tau_e - \frac{2x}{\eta} \right) = 0 \quad \Rightarrow \quad x = \frac{\eta \tau_e}{2}. \quad (3.3) \]

Equation (3.2) suggests that emission tax tends to distort the manufacturing production: a higher emission tax results in a lower level of output of manufactured goods, i.e., \(\frac{\partial m}{\partial \tau_e} < 0\). The marginal distortion effect will be more pronounced if firms generate pollution at a higher rate (larger \(\theta\)), the market is more concentrated (smaller \(N\)), and the change in quantity demanded is more responsive to price change (smaller \(\beta\)). Yet the profits tax, \(\tau_p\), does not affect a firm’s output level. Equation (3.3) indicates that firms will choose the optimal level of pollution abatement at which the marginal abatement cost equals the emission tax. Firms tend to abate more pollution if they adopt the more cost-effective abatement technology (larger \(\eta\)) and if the government imposes a higher emission tax (larger \(\tau_e\)).

Inserting (3.2) and (3.3) into (2.4) solves each firm’s emission level:

\[ e = \frac{(\alpha - 1) \theta}{(N + 1) \beta} - \left[ \frac{\theta^2}{(N + 1) \beta} + \frac{\eta}{2} \right] \tau_e. \quad (3.4) \]
Since the emission level cannot be less than zero, we have an upper bound of the emission tax rate:

$$\tau_e \leq \frac{2\theta(\alpha - 1)}{(N + 1)\beta\eta + 2\theta^2}. \quad (3.5)$$

We assume that condition (3.5) holds throughout the paper.

Plugging (2.3), (3.2), and (3.3) into (3.1) obtains the equilibrium profit of each firm:

$$\pi = (1 - \tau_p) \left[ (\alpha - 1 - \theta\tau_e)m - N\beta m^2 + \tau_e x - \frac{x^2}{\eta} \right]$$

$$= (1 - \tau_p) \left[ \frac{(\alpha - 1 - \theta\tau_e)^2}{(N + 1)\beta} - N\beta \left( \frac{\alpha - 1 - \theta\tau_e}{(N + 1)\beta} \right)^2 + \frac{\eta\tau_e^2}{2} - \frac{\eta\tau_e^2}{4} \right]$$

$$= (1 - \tau_p) \left[ \frac{(\alpha - 1 - \theta\tau_e)^2}{(N + 1)^2\beta} + \frac{\eta\tau_e^2}{4} \right]. \quad (3.6)$$

Equation (3.6) provides two policy implications. First, a higher rate of profits tax inevitably erodes a firm’s net profit, i.e., \( \frac{\partial \pi}{\partial \tau_p} < 0 \). Second, a higher rate of emission tax may not result in a loss at firms.\(^{12}\) The simple intuition is that the increased pollution tax heightens the price of manufactured goods, which tends to create revenues for firms to offset their increased pollution costs.

The government revenue can be expressed as the sum of profits taxes and emission taxes being collected from the \( N \) firms of the manufacturing sector:

$$T = N \left[ \tau_p (pm - m - c(x) - \tau_e e) + \tau_e e \right]$$

$$= N \left( -\pi + pm - m - \frac{x^2}{\eta} \right). \quad (3.7)$$

Substituting (2.6) and (3.7) into the household’s budget constraint (2.2) and rearranging derives the consumption of agricultural goods:

$$C_A = 1 - N\pi - N \left( m + \frac{x^2}{\eta} \right). \quad (3.8)$$

\(^{12}\)It can be inferred from (3.5) and (3.6) that given \( N > 1 \), \( \pi \) decreases with \( \tau_e \) if and only if \( \tau_e \leq \frac{2\theta(\alpha - 1)}{(N + 1)^2\beta\eta + 2\theta^2} \).
By (2.6), (2.7), and (3.8), we can rewrite the household’s utility function (2.1) as

\[
u = \left[ 1 - N\pi - N \left( m + \frac{x^2}{\eta} \right) \right] + \alpha(Nm) - \frac{\beta}{2} (Nm)^2 - \gamma(N\theta m - Nx)
\]

\[= 1 - N\pi + N \left[ (\alpha - 1 - \gamma\theta)m - \frac{N\beta}{2} m^2 + \gamma x - \frac{x^2}{\eta} \right], \quad (3.9)
\]

which depends on \((m, x, \pi)\). Equation (3.9) implicitly defines the household utility as a function of the two tax rates \((\tau_p, \tau_e)\).

We proceed to examine the bargaining process between the government and the manufacturing firms. Making a monotonic transformation of the bargaining function (2.8) as

\[
\Phi \equiv \delta \ln(\pi - \Omega) + (1 - \delta) \ln(u - 1).
\]

(3.10)

To pin down the rates of emission tax and profits tax, we take the first order conditions of (3.10) with respect to \(\tau_p\) and \(\tau_e\) as follows:

\[
\frac{\partial \Phi}{\partial \tau_p} = \frac{\delta}{\pi - \Omega} \frac{\partial \pi}{\partial \tau_p} + \frac{1 - \delta}{u - 1} \frac{\partial u}{\partial \tau_p} = 0,
\]

(3.11)

\[
\frac{\partial \Phi}{\partial \tau_e} = \frac{\delta}{\pi - \Omega} \frac{\partial \pi}{\partial \tau_e} + \frac{1 - \delta}{u - 1} \frac{\partial u}{\partial \tau_e} = 0.
\]

(3.12)

Given that \(m\) and \(x\) are independent of \(\tau_p\), rewrite (3.11) by using (3.9):

\[
\frac{\delta}{\pi - \Omega} \frac{\partial \pi}{\partial \tau_p} + \frac{1 - \delta}{u - 1} \left( -N \frac{\partial \pi}{\partial \tau_p} \right) = 0 \quad \Rightarrow \quad \frac{\delta}{\pi - \Omega} = \frac{N(1 - \delta)}{u - 1}.
\]

(3.13)

Substituting (3.13) into (3.12) and then using (3.2) and (3.3) characterizes the closed-form interior solution to the emission tax rate:

\[
\frac{N(1 - \delta)}{u - 1} \frac{\partial \pi}{\partial \tau_e} + \frac{1 - \delta}{u - 1} \left[ -N \frac{\partial \pi}{\partial \tau_e} + N(\alpha - 1 - \gamma\theta - N\beta m) \frac{\partial m}{\partial \tau_e} + N \left( \gamma - \frac{2x}{\eta} \right) \frac{\partial x}{\partial \tau_e} \right] = 0
\]

\[
\Rightarrow \quad - \left[ \alpha - 1 - \gamma\theta - \frac{N(\alpha - 1 - \theta \tau_e)}{N + 1} \right] \frac{\theta}{(N + 1)\beta} + (\gamma - \tau_e) \frac{\eta}{2} = 0
\]

\[
\Rightarrow \quad \tau_e = \gamma - \frac{2\theta(\alpha - 1 - \gamma\theta)}{(N + 1)^2 \beta \eta + 2N\theta^2}.
\]

(3.14)

The next proposition follows directly from (3.14):
**Proposition 1** The government will not impose an emission tax ($\tau_e = 0$) if and only if
\[
\gamma \leq \frac{2(\alpha - 1)\theta}{(N + 1)[2\theta^2 + (N + 1)\beta\eta]}.
\] (3.15)

**Proof.** See Appendix. ■

Proposition 1 demonstrates that the government may not levy on emissions. The government is likely to do so when the household is insensitive to environmental damage (small $\gamma$) but has a sizable demand for manufactured goods (large $\alpha$). This result parallels Greenstone and Jack (2015), who find that many low-income economies adopt weak environmental policies: poor people spend their income for immediate consumption needs without much consideration of environment protection, because the benefits of higher consumption level outweigh the benefits from the improved environmental quality. Moreover, the emission tax is more likely to be set as zero in a highly concentrated market (small $N$). Given that manufactured goods are supplied at a restricted level by a small number of firms, the government would choose to weaken environmental regulation to avoid further distortion on the production. This intuition bears a resemblance to Buchanan (1969) and Barnett (1980), who suggest that the imposition of a Pigouvian tax on the imperfectly-competitive polluters will further restrict the supply of goods and aggravate the losses of consumer surplus.

An emission tax is expected to not only stimulate pollution abatement of firms ("environmental benefit") but also increase the public revenue and hence the household’s income budget ("revenue-generating benefit"). Despite such “double benefits”, we show in Proposition 1 that it is possible for the government not to impose an emission tax. This possibility will occur in particular when both benefits induced by the emission tax are of small magnitude. Specifically, the induced benefits are slim if (i) the improved environmental quality is not much valued (small $\gamma$) and (ii) the tax causes a heavy distorting effect on the production (small $\beta$ and small $N$), which hurts consumer welfare. Meanwhile, firms have to pay a large amount of emission taxes if they adopt inefficient abatement technology (small $\eta$). To the extent that the cost exceeds the two benefits, the government will not pursue a direct environmental regulation. This result is consistent with Buchanan (1969) and Fullerton and Metcalf (1998), who emphasize that emission tax revenue is not free money.

We proceed to analyze the case that the government imposes a positive emission tax on firms ($\tau_e > 0$). As shown in equation (3.14), $\tau_e$ is determined by six parameters, namely, ($N, \alpha, \beta, \theta, \eta$).
\( \eta, \gamma \). The first three parameters capture the characteristics of the market of manufactured goods (i.e., market structure and market demand), while the later three parameters pertain to pollution emission from firms. We investigate their impacts on the bargaining outcome of the emission tax rate in the next two propositions:

**Proposition 2** *The emission tax rate*

(a) increases with the number of manufacturing firms, i.e., \( \frac{\partial \tau_e}{\partial N} > 0 \);

(b) decreases with the highest willingness to pay for manufactured goods, i.e., \( \frac{\partial \tau_e}{\partial \alpha} < 0 \);

(c) increases with the slope of the demand curve of manufactured goods, i.e., \( \frac{\partial \tau_e}{\partial \beta} > 0 \).

**Proof.** See Appendix.

Proposition 2(a) implies that the government will impose a higher emission tax rate if the manufacturing sector is more competitive (larger \( N \)). This is because the intensified market competition tends to relieve the distortive effect of emission tax on the production of manufactured goods. We can see from (3.14) that \( \tau_e \) approaches \( \gamma \) as \( N \) goes to infinity. Interpreted intuitively, if the manufacturing sector is perfectly competitive, then the emission tax rate will exactly be the Pigouvian rate at which the private cost of pollution abatement equals the marginal social damage of emission. For a relatively concentrated market (\( N \) is finite), however, the emission tax rate falls short of the Pigouvian rate: the government will agree to weaken the environmental regulation to avert further distortion on firm production.

Proposition 2(b) and 2(c) suggest that the emission tax rate hinges on the household’s demand for manufactured goods. All else equal, a high \( \alpha \) or a low \( \beta \) indicates that the household places a high value on the consumption of manufactured goods, which confers firms an advantage to obtain a lower emission tax. If the household has greater willingness to buy manufactured goods (higher \( \alpha \)), then it is desirable for the government to foster their supply by relieving environmental regulation. With the demand for manufactured goods being more responsive to the price change (lower \( \beta \)), a fall in the price of manufactured goods resulting from a tax cut will lead to a sharper increase in the quantity demanded, which can be translated into the increased consumer surplus. In that case, the government tends to concede at a lower emission tax during the bargaining.

Next, we examine the comparative statics of the emission tax rate with respect to the three pollution-related parameters.
Proposition 3  The emission tax rate
(a) increases with the pollution generation rate, i.e., \( \frac{\partial \tau_e}{\partial \theta} > 0 \) if and only if
\[
\frac{\gamma \theta}{\alpha - 1} + \frac{N \theta^2}{(N + 1)^2 \beta \eta} > \frac{1}{2};
\]
(b) increases with the efficiency of pollution abatement, i.e., \( \frac{\partial \tau_e}{\partial \eta} > 0 \); 
(c) increases with the marginal damage of emission, i.e., \( \frac{\partial \tau_e}{\partial \gamma} > 0 \).

Proof. See Appendix.

In Proposition 3(a), we show that the impact of pollution generation rate on emission tax seems ambiguous but will be positive under some configuration of parameters. All else equal, an initially dirtier production technology (higher \( \theta \)) tends to result in a higher emission level, which consequently generates two opposing effects. On one hand, the government needs to raise the emission tax rate to protect the household away from the increased emissions ("environmental concern effect"). On the other hand, the increased emissions aggravate firms' burden of emission taxes, which prompts the government to adjust the emission tax rate downward to prevent their exodus ("tax constraint effect"). The environmental concern effect will outweigh the tax constraint effect if and only if condition (3.16) is satisfied. In other words, a higher pollution generation rate will lead to a higher rate of emission tax when the household is environmentally aware (large \( \gamma \)) and has a small demand for manufactured goods (small \( \alpha \)).

Proposition 3(b) demonstrates that the abatement technology adopted by firms is a vital ingredient in the determination of the emission tax rate. If firms abate pollution in a more cost-effective manner (larger \( \eta \)), the government will encourage their abatement activities by setting a higher rate of emission tax (higher \( \tau_e \)). One immediate inference is that a stringent environmental policy is well suited where polluting firms are equipped with efficient abatement technologies. This finding is consonant with Lovely and Popp (2011), who present empirical evidence that the current state of pollution-control technology tends to drive the stringency of regulatory standards.

According to Proposition 3(c), the emission tax rate will be designed to reflect the society’s value of environmental damage. When people become more concerned about pollution (higher \( \gamma \)), the government should levy a higher emission tax to address their concern. This prediction is in
line with the consensus that high-income countries with environmentally-aware residents usually enact stringent environmental regulations (e.g., Markusen, Morey, and Olewiler 1995, Copeland and Taylor 2003, Greenstone ad Jack 2015).

We proceed to investigate the bargaining process between the government and firms over the profits tax rate \( \tau_p \). Rewrite equation (3.13) as

\[
\pi = \Omega + \delta \left( \frac{u + N\pi - 1}{N} - \Omega \right).
\]  

(3.17)

Plugging (3.2), (3.3), and (3.6) into the above equation yields the closed-form interior solution to the profits tax rate:

\[
(1 - \tau_p) \left[ \frac{(\alpha - 1 - \theta \tau_e)^2}{(N + 1)^2} \beta + \frac{\eta \tau_e^2}{4} \right] = \Omega + \delta \left[ \frac{(\alpha - 1 - \gamma \theta)m - \frac{N\beta m^2}{2} + \gamma x - \frac{x^2}{\eta} - \Omega}{(N + 1)^2} \beta \right]
\]

\[
\Rightarrow 1 - \tau_p = \frac{\Omega + \delta \left[ \frac{(\alpha - 1 - \gamma \theta)(\alpha - 1 - \theta \tau_e)}{(N + 1)^2} \beta - \frac{(N/2 - 1)(\alpha - 1 - \theta \tau_e)^2}{(N + 1)^2} \beta \right] + \frac{\gamma \eta \tau_e - \frac{\eta \tau_e^2}{4} - \Omega}{(N + 1)^2} \beta}{\frac{(\alpha - 1 - \theta \tau_e)^2}{(N + 1)^2} \beta + \frac{\eta \tau_e^2}{4}}
\]

\[
\Rightarrow \tau_p = 1 + \delta - \frac{\Omega + \delta \left[ \frac{(\alpha - 1 - \gamma \theta)(\alpha - 1 - \theta \tau_e)}{(N + 1)^2} \beta - \frac{(N/2 - 1)(\alpha - 1 - \theta \tau_e)^2}{(N + 1)^2} \beta + \gamma \eta \tau_e - \frac{\eta \tau_e^2}{4} - \Omega}{\frac{(\alpha - 1 - \theta \tau_e)^2}{(N + 1)^2} \beta + \frac{\eta \tau_e^2}{4}}\right],
\]

(3.18)

with \( \tau_e \) being solved in (3.14). We develop the following proposition based on (3.14) and (3.18):

**Proposition 4**

(a) The emission tax rate is independent of the firms’ bargaining power and value of outside option, i.e., \( \frac{\partial \tau_e}{\partial \delta} = 0 \) and \( \frac{\partial \tau_e}{\partial \Omega} = 0 \);

(b) The profits tax rate decreases with the firms’ bargaining power and value of outside option, i.e., \( \frac{\partial \tau_p}{\partial \delta} < 0 \) and \( \frac{\partial \tau_p}{\partial \Omega} < 0 \).

**Proof.** See Appendix. ■

Proposition 4(a) demonstrates that the bargaining position of firms does not affect the rate of emission tax. During the bargaining process, the singular objective of each firm is to determine its overall tax burden, but the “mix” of taxes (i.e., the specific type of tax) is far less important. In contrast, the government has multiple considerations in its fiscal planning, including not only the total tax revenue (and hence the transfer payment) but also the alignment of taxation with environmental goals. In absence of public environmental concern, profits tax will be strictly preferred owing to
its nature of non-distortionary. But if the environmental concern is nontrivial (large $\gamma$) so that an environmental regulation is an imperative ($\tau_e > 0$), the government can first set the emission tax rate by evaluating its influence on the environment and consumer surplus, and then lay down the profits tax rate to generate adequate public revenues.

We turn next to Proposition 4(b): concerned about their overall tax burden, firms will drive a hard bargaining over the rate of profits tax given the rate of emission tax. The bargaining power $\delta$ reflects the patience of firms in the bargaining process (e.g., Rubinstein 1982, Binmore, Rubinstein, and Wolinsky 1986). If firms are anxious to reach an agreement with the government – for example, under the financial pressure from the banks where they have made loans – they will be put in a disadvantageous position in the bargaining and have to bear a higher profits tax. On the contrary, if the government is eager to settle down the bargaining – for example, in face of the strike that calls for increasing jobs in the manufacturing sector – it will compromise to set a favorable profits tax for firms.

It is also conceivable that a better outside option – although firms may not actually take it – enables firms to attain a lower rate of profits tax. With the trends of globalization, firms can choose between different locations of production. However, firms are not perfectly footloose: if moving elsewhere, they would have to make new investments, such as plant establishments and employee recruitment, and also forfeit the “sunk” investments in their old plants. Firms will have a strong tendency to migrate only if their new operations abroad generate a large net profit (large $\Omega$). To prevent firms from relocating their plants, the government has to keep the profits tax low.

Although equation (3.18) presents a closed-form solution to the profits tax rate, the effects of most parameters seem complex and indeterminate. Nevertheless, we find that profits tax can be used as a policy instrument to address the household’s concern over pollution under some reasonable condition, as summarized in the next proposition:

**Proposition 5** Given that condition (3.15) holds, the profits tax rate increases with the marginal damage of emission, i.e., $\frac{\partial \tau_p}{\partial \gamma} > 0$.

**Proof.** See Appendix. ■

Even if the emission tax is set as zero when condition (3.15) holds (Proposition 1), it does not mean that the government will undertake no corrective measure against environmental damage. As
the household is hurt by the emissions from the manufacturing firms, it can rightfully make a claim for compensation. The compensation will be paid through the government’s lump-sum transfer, which is financed by the profits taxes levied on the firms. Clearly, the household deserves a larger amount of compensation if it experiences a greater suffering (larger \( \gamma \)).

Proposition 5 essentially echoes the Coase theorem, which states that free bargaining will lead to a Pareto-efficient outcome (Coase 1960). If the initial legal framework gives people the right to breathe clean air and drink clear water, they can sell the right to polluters and put up with an acceptable level of pollution. The bargaining provides a feasible solution to the dispute between polluters and pollutees by translating the price of pollution into a fraction of polluters’ profits. In other words, profits tax can be viewed as a potential substitute for direct intervention to address the environmental concern under some circumstances. A proper, non-distortionary profits tax alone is expected to yield Pareto efficiency: it ensures pollutees to be compensated for their suffering in a dirty environment without hampering the usual business of polluters (i.e., there is no need for firms to cut their output or commit to pollution abatement).

4 An Extension: Public Environmental Expenditure

We have so far made an assumption that tax revenues are fully rebated to the household, which is a useful simplification to highlight the essential idea of this paper. Yet the effects of emission tax and profits tax on the economy and the environment also depend on the use of tax revenues (e.g., Gupta, Miranda, and Parry 1995, Halkos and Paizanos 2013, U.S. CBO 2013). Government spending may have multiple functions, including the provision of public environmental infrastructure and services. Public environmental expenditure may cover a variety of domains, ranging from establishment of municipal sewage treatment plants to research and development of carbon capture and storage, which exhibit economies of scale and suffer underprovision by the private sector (World Bank 2003, Pink 2013).

This section aims to enrich our baseline model to consider public environmental expenditure. Suppose that the government collects a total revenue of \( T \) and then allocates an amount of \( \tilde{T} \) as transfer payment and an amount of \( G > 0 \) as public environmental expenditure. The government’s
budget constraint can be expressed by

\[ T = G + \tilde{T}. \]  

(4.1)

In accordance with the practices in Europe that waste management and wastewater treatment accounts for the largest proportion of environmental protection expenditure by the public sector, we model \( G \) as to subsidize *end-of-pipe solutions* to emissions. The total emission that matters to the household can be specified as

\[ \tilde{E} := \frac{E}{1 + \lambda G} = \frac{N\theta m - Nx}{1 + \lambda G}, \]  

(4.2)

where the parameter \( \lambda > 0 \) measures the abatement efficiency of public environmental expenditure. Note that if the government does not spend money on environmental protection \((G = 0)\), the total emission will be the same as in (2.7), i.e., \( \tilde{E} = E \), and the model can hence be simplified as in Section 3.

In light of the above two refinements, we need to modify the household’s budget constraint and utility in equilibrium. Specifically, the budget constraint (2.2) can be modified as

\[ w + \tilde{T} = C_A + pC_M. \]  

(4.3)

The household’s utility in (3.9) can be amended as

\[
\begin{align*}
    u &= C_A + \alpha C_M - \frac{\beta}{2} C_M^2 - \gamma \tilde{E} \\
    &= 1 - N\pi + N \left[ \alpha - 1 - \frac{\gamma\theta}{1 + \lambda G} \right] m - \frac{N\beta}{2} m^2 + \frac{\gamma x}{1 + \lambda G} - \frac{x^2}{\eta} - G.
\end{align*}
\]  

(4.4)

It is also noteworthy that since the public environmental expenditure does not directly affect firm production, the presence of \( G \) will not alter the expressions of the individual firm’s output \((m)\) abatement \((x)\), emission \((e)\), and net profit \((\pi)\) in Section 3.

The government and the manufacturing firms enter a Nash bargaining process to determine the
two tax rates and the public environmental expenditure. We take the first order conditions of (3.10) with respect to \( \tau_p, \tau_e, \) and \( G \) as follows:

\[
\frac{\partial \Phi}{\partial \tau_p} = \frac{\delta}{\pi - \Omega} \frac{\partial \pi}{\partial \tau_p} + \frac{1 - \delta}{u - 1} \frac{\partial u}{\partial \tau_p} = 0, \tag{4.5}
\]

\[
\frac{\partial \Phi}{\partial \tau_e} = \frac{\delta}{\pi - \Omega} \frac{\partial \pi}{\partial \tau_e} + \frac{1 - \delta}{u - 1} \frac{\partial u}{\partial \tau_e} = 0, \tag{4.6}
\]

\[
\frac{\partial \Phi}{\partial G} = \frac{\delta}{\pi - \Omega} \frac{\partial \pi}{\partial G} + \frac{1 - \delta}{u - 1} \frac{\partial u}{\partial G} = 0. \tag{4.7}
\]

It is straightforward to derive from (4.4) and (4.5) that

\[
\frac{\delta}{\pi - \Omega} = \frac{N(1 - \delta)}{u - 1}, \tag{4.8}
\]

which turns out to be a representation of (3.13). We insert (4.8) into (4.6) to derive the emission tax rate, which is analogous to (3.14):

\[
\tau_e = \frac{\gamma}{1 + \lambda G} - \frac{2\theta}{(N + 1)^2 \beta \eta + 2N \theta^2} \left( \alpha - 1 - \frac{\gamma \theta}{1 + \lambda G} \right). \tag{4.9}
\]

A comparison between (3.14) and (4.9) leads to the following proposition:

**Proposition 6** Compared with the policy that tax revenues are all returned to the household \((G = 0)\), the policy that allows for public environmental expenditure \((G > 0)\) leads to a lower rate of emission tax \((\text{lower } \tau_e)\) and a higher output of manufactured goods \((\text{higher } m)\).

**Proof.** See Appendix. \textit{□}

Both emission tax and public environmental expenditure are regulatory instruments to mitigate the environmental damage. With a part of its budget being allocated to pollution abatement, the government can adjust the emission tax rate downward, which echoes the argument of Kleinbard (2010) that tax expenditure can be presented to taxpayers as “targeted tax cuts”. We can therefore infer that public environmental expenditure helps to save the pollution abatement efforts dedicated by the private sector. This result accords with Bennett (2012), who presents a case study showing that the improved water supply infrastructure led to lower private sanitation investments in the Philippines. The savings in pollution costs enable firms to expand the scale of their production and
increase the supply of manufactured goods, which in turn improves consumer surplus. It seems a pro-growth policy to spend the public revenue on emission mitigation.

To solve the public environmental expenditure, we can simplify (4.7) by using (2.4) and (4.8):

\[
N \left(1 - \delta \right) u - 1 \frac{\partial \pi}{\partial G} + \frac{\gamma \theta m}{\left(1 + \lambda G\right)^2} + \gamma x \left(1 - \frac{\lambda}{\left(1 + \lambda G\right)^2}\right) - 1 = 0
\]

\[
\Rightarrow \quad N \left[\frac{\gamma \lambda \theta m}{\left(1 + \lambda G\right)^2} - \frac{\gamma \lambda x}{\left(1 + \lambda G\right)^2}\right] - 1 = 0
\]

\[
\Rightarrow \quad \frac{\gamma \lambda (Ne)}{\left(1 + \lambda G\right)^2} = 1.
\] (4.10)

The decision problem regarding the use of tax revenues involves a trade-off between the public environmental expenditure \( G \) and the transfer payment \( \tilde{T} \). Equation (4.10) indicates that the marginal dollar of public environmental expenditure should achieve a social benefit of reduced emission equal to the (opportunity) cost associated with the foregone transfer payment. Inserting (3.4) into (4.10) and rearranging yields the public environmental expenditure:

\[
G = \sqrt{\frac{N \gamma \lambda}{\left(1 + \lambda G\right)^2}} \left[\frac{(\alpha - 1)\theta}{N} - \frac{\theta^2}{\left(N + 1\right)\beta} + \frac{\eta}{2}\right] \tau_e - \frac{1}{\lambda},
\] (4.11)

which characterizes another inverse relationship between \( G \) and \( \tau_e \).

Combining (4.9) and (4.11) derives the public environmental expenditure:

\[
\frac{(1 + \lambda G)^2}{N \gamma \lambda} = \frac{(\alpha - 1)\theta}{\left(N + 1\right)\beta} - \left[\frac{\theta^2}{\left(N + 1\right)\beta} + \frac{\eta}{2}\right] \tau_e
\]

\[
\Rightarrow \quad G = \sqrt{\frac{N \gamma}{\left(1 + \lambda G\right)^2}} \left[\frac{(\alpha - 1)\theta}{\left(N + 1\right)\beta} - \left(\frac{\theta^2}{\left(N + 1\right)\beta} + \frac{\eta}{2}\right) \tau_e\right] - \frac{1}{\lambda},
\] (4.12)

Equation (4.12) is a cubic function of \( G \), which gives rise to the possibility of multiple solutions.

We can find the “best” public environmental expenditure under the assumption \( G > 0 \) by evaluating the match surplus (\( \Phi \)) in different equilibria. The next proposition follows directly from (4.12):

**Proposition 7** The public environmental expenditure
(a) is independent of firms’ bargaining power and value of outside option, i.e., \( \frac{\partial G}{\partial \delta} = 0 \) and \( \frac{\partial G}{\partial \Omega} = 0 \);

(b) increases with the highest willingness to pay for manufactured goods, i.e., \( \frac{\partial G}{\partial \alpha} > 0 \);

(c) increases with the marginal damage of emission, i.e., \( \frac{\partial G}{\partial \gamma} > 0 \) if and only if

\[
\frac{\gamma}{1 + \lambda G} < \frac{(\alpha - 1)\theta([(N + 2)\beta \eta + 2\theta^2])}{((N + 1)\beta \eta + 2\theta^2)^2}. \tag{4.13}
\]

**Proof.** See Appendix. ■

Proposition 7(a) suggests that the bargaining position of firms does not influence the amount of public environmental expenditure. All else equal, public environmental expenditure seems to benefit firms by lowering their pollution costs (Proposition 6). However, this benefit tends to be countervailed as firms may have to pay a greater amount of profits taxes to finance the increased government expenditure. Since firms are concerned only about the overall tax burden (Proposition 4), they will be indifferent to how the government spends the collected tax revenues.

Proposition 7(b) identifies the crucial role of market demand in determining the public environmental expenditure. To accommodate the household’s growing demand for manufactured goods (larger \( \alpha \)), firms will expand their production scale, which tends to emit more pollution. The government has two fiscal policies available to curb emissions: \( (i) \) impose an emission tax to induce abatement efforts at firms, \( (ii) \) provide public pollution abatement. Because the provision of public abatement does not directly dampen the supply of manufactured goods, it is a more desirable policy choice compared with the emission tax.

Proposition 7(c) shows that the government will raise public environmental expenditure in response to the increased social damage of emissions under some configuration of parameters. To mitigate the increased emissions, the government needs to allocate more budget to public abatement ("direct intervention effect"). It is also necessary to impose a higher rate of emission tax to motivate the pollution abatement activities of the private sector (Proposition 3(c)), which in turn allows the government to devote fewer resources to the provision of public abatement ("crowding-out effect"). The direct intervention effect dominates the crowding out effect if and only if condition (4.13) is satisfied, namely, the marginal emission damage is mild (small \( \gamma \)), public environmental
expenditure is efficient in pollution abatement (large $\lambda$), and the household’s willingness to pay for polluting goods is great (large $\alpha$).

5 Conclusion and Further Discussion

Economics literature has a sustained interest in the important role that taxation on firms plays in fiscal consolidation (e.g., Fullerton, King, Shoven, and Whalley 1981, Kopczuk and Slemrod 2006, Kleven, Kreiner, and Saez 2016). A high profits tax can enhance social welfare by raising public revenue and lessening income inequality. In case that profitable firms generate pollution as a by-product in production, the emission tax – a hybrid tool with both revenue-raising and regulatory properties – will be placed on policy agenda. Moreover, in the globalized world where firms are at least to some extent footloose across borders, policymakers will have discretion on the tax design, taking into account the mobility of firms. This paper follows this direction of research by analyzing the determination of profits tax and emission tax in a unified framework.

We develop a model in which the government enacts the rates of profits tax and emission tax by bargaining with the polluting firms and then returns the public revenue to the household. Despite its incentive to impose high tax rates to increase household welfare, the government finds its overall taxation capability restrained in the context that firms can choose to flee abroad. In addition, there is a tradeoff between the two taxes: compared with profits tax, emission tax has a disadvantage in distorting the production but has a merit of correcting environmental externalities.

We derive the analytical solutions to both tax rates, which helps to offer some policy implications. First, the bargaining position of firms, as measured by the bargaining power and the value of outside option, is a key determinant of profits tax but does not affect emission tax. In effect, the government and firms can first set the emission tax by evaluating its economic and environmental impacts and then set the profits tax to share the profit pie. Second, our model derives a consistent result with Buchanan (1969) and Barnett (1980), who suggest that the emission tax should lie below the Pigouvian rate in an imperfectly competitive market to prevent a greater contraction of output. Third, our analysis helps to better understand why some countries become a “pollution haven”. The “pollution haven hypothesis” posits that a country with less strict environmental regulations will lure investments in the production of dirty goods (Copeland and Taylor 2003, Taylor
We show that a weak regulatory design tends to result from a strong market demand for polluting goods, dirty production technology, inferior abatement technology, and poor environmental awareness.

In an extension to our baseline model, we consider the scenario in which a part of tax revenues are spent on emission mitigation. Given that pollution abatement contributed by the government and the manufacturing firms are substitutable, public environmental expenditure enables a cut in emission tax, which lightens the distortions caused to industrial production and hence enhances consumer surplus. Also, when people have an increased demand for polluting goods, public environmental expenditure serves as a better regulatory instrument than emission tax, in view of their impacts on the economy and the environment.

Finally, this paper complements the “double dividend hypothesis” literature in addressing the role of emission tax in the tax reform. The double dividend hypothesis posits that emission tax can not only help to correct the negative environmental externalities of industrial production but also foster economic efficiency by financing the cut in other taxes such as income tax that distorts decisions of labor supply and savings (e.g., Bovenberg and De Mooij 1994, Goulder 1995, Bovenberg and Goulder 1996, Bovenberg 1999, Parry, Williams, and Goulder 1999, Parry and Bento 2000, Fullerton and Metcalf 2001, Williams 2003, Williams 2016). While these studies investigate the substitution between emission tax with the other distortionary taxes, we focus on the interaction of emission tax and the non-distortionary profits tax, both of which target on firms.
Appendix

Proof of Proposition 1

Given \( \tau_e \geq 0 \), we can infer from (3.14) that the emission tax equals 0 if and only if

\[
\gamma - \frac{2(\alpha - 1 - \gamma \theta)\theta}{(N + 1)^2 \beta \eta + 2N \theta^2} \leq 0
\]

\[
\Rightarrow \frac{(N + 1)^2 \beta \eta + 2N \theta^2 + 2\theta^2}{(N + 1)^2 \beta \eta + 2N \theta^2} \cdot \gamma \leq \frac{2(\alpha - 1)\theta}{(N + 1)^2 \beta \eta + 2N \theta^2}
\]

\[
\Rightarrow \gamma \leq \frac{2(\alpha - 1)\theta}{(N + 1)[2\theta^2 + (N + 1)\beta \eta]}.
\] (A.1)

Proof of Proposition 2

(a) If \( \gamma \leq \tau_e \), we can infer from (3.14) that \( \alpha - 1 - \gamma \theta \leq 0 \). On the other hand, equation (3.2) implies that \( \alpha - 1 - \theta \tau_e > 0 \). Taken together, we have \( \tau_e < \frac{\alpha - 1}{\theta} \leq \gamma \), which contradicts the premise \( \gamma \leq \tau_e \). Therefore, it must be true that \( \gamma > \tau_e \). By (3.14), we have

\[
\alpha - 1 - \gamma \theta > 0.
\] (A.2)

Differentiating (3.14) with respect to \( N \) yields

\[
\frac{\partial \tau_e}{\partial N} = \frac{2\theta(\alpha - 1 - \gamma \theta)[2(N + 1)\beta \eta + 2\theta^2]}{[(N + 1)^2 \beta \eta + 2N \theta^2]^2} > 0.
\] (A.3)

(b) Differentiating (3.14) with respect to \( \alpha \) obtains

\[
\frac{\partial \tau_e}{\partial \alpha} = -\frac{2\theta}{2N \theta^2 + (N + 1)^2 \beta \eta} < 0.
\] (A.4)

(c) Given (A.2), differentiating (3.14) with respect to \( \beta \) obtains

\[
\frac{\partial \tau_e}{\partial \beta} = \frac{2\theta \eta(\alpha - 1 - \gamma \theta)(N + 1)^2}{[(N + 1)^2 \beta \eta + 2N \theta^2]^2} > 0.
\] (A.5)
Proof of Proposition 3

(a) Differentiating (3.14) with respect to \( \theta \) yields
\[
\frac{\partial \tau_e}{\partial \theta} = \frac{[4 \gamma \theta - 2(\alpha - 1)][(N + 1)^2 \beta \eta + 2 N \theta^2] - [2 \gamma \theta^2 - 2(\alpha - 1) \theta] 4 N \theta}{[(N + 1)^2 \beta \eta + 2 N \theta^2]^2}
\]
\[
= \frac{4(N + 1)^2 \beta \eta \gamma \theta + 4 N (\alpha - 1) \theta^2 - 2(\alpha - 1) \beta \eta}{[(N + 1)^2 \beta \eta + 2 N \theta^2]^2}
\]
\[
= \frac{4(N + 1)^2 (\alpha - 1) \beta \eta}{[(N + 1)^2 \beta \eta + 2 N \theta^2]^2} \left[ \frac{\gamma \theta}{\alpha - 1} + \frac{N \theta^2}{(N + 1)^2 \beta \eta} - \frac{1}{2} \right], \quad (A.6)
\]
which is positive if and only if the condition (3.16) holds.

(b) Given (A.2), differentiating (3.14) with respect to \( \eta \) obtains
\[
\frac{\partial \tau_e}{\partial \eta} = \frac{2 \beta \theta (\alpha - 1 - \gamma \theta)(N + 1)^2}{[(N + 1)^2 \beta \eta + 2 N \theta^2]^2} > 0. \quad (A.7)
\]

(c) Differentiating (3.14) with respect to \( \gamma \) obtains
\[
\frac{\partial \tau_e}{\partial \gamma} = 1 + \frac{2 \theta^2}{(N + 1)^2 \beta \eta + 2 N \theta^2} > 0. \quad (A.8)
\]

Proof of Proposition 4

(a) It holds because \( \tau_e \) is not a function of \( (\delta, \Omega) \).

(b) Because \( \tau_e \) is independent of \( \delta \), we can infer from (3.2) and (3.3) that both \( m \) and \( x \) are independent of \( \delta \). Totally differentiating (3.18) with respect to \( \delta \) and \( \tau_p \) and then inserting (3.9) and (3.17) obtains
\[
- \left[ \frac{(\alpha - 1 - \theta \tau_e)^2}{(N + 1) \beta} + \frac{\eta \tau_e^2}{4} \right] d\tau_p = \left[ (\alpha - 1 - \gamma \theta) m - \frac{N \beta m^2}{2} + \gamma x - \frac{x^2}{\eta} - \Omega \right] d\delta
\]
\[
\Rightarrow \frac{d\tau_p}{d\delta} = - \left( \frac{(\alpha - 1 - \theta \tau_e)^2}{(N + 1) \beta} + \frac{\eta \tau_e^2}{4} \right) \delta \left[ \frac{(\alpha - 1 - \theta \tau_e)^2}{(N + 1) \beta} + \frac{\eta \tau_e^2}{4} \right], \quad (A.9)
\]
which is negative because \( \pi > \Omega \).

Since \( \tau_e \) is independent of \( \Omega \), we can infer from (3.2) and (3.3) that both \( m \) and \( x \) are independent.
of Ω. Taking the first order condition of (3.18) with respect to Ω obtains

\[
\frac{d\tau_p}{d\Omega} = -\frac{1 - \delta}{(N+1)^2\beta} + \frac{\eta^2}{4} < 0. \tag{A.10}
\]

**Proof of Proposition 5**

If condition (3.15) holds, we have τ_e = 0 (Proposition 1). Rewrite (3.18) as

\[
\tau_p = 1 + \delta - \frac{\Omega + \delta}{\frac{(\alpha - 1 - \gamma\theta)(\alpha - 1)}{(N+1)^2\beta} - \frac{(N/2-1)(\alpha - 1)^2}{(N+1)^2\beta} - \Omega}. \tag{A.11}
\]

Differentiating (A.11) with respect to γ yields

\[
\frac{d\tau_p}{d\gamma} = -\frac{\delta\theta(\alpha - 1)}{(N+1)^2\beta} = \frac{(N + 1)\delta\theta}{\alpha - 1} > 0. \tag{A.12}
\]

**Proof of Proposition 6**

Rearranging equation (3.14) obtains the emission tax (given G = 0) as

\[
\tau_e = \frac{(N + 1)^2\beta\eta + 2(N + 1)^2\theta^2}{(N + 1)^2\beta\eta + 2N\theta^2} \cdot \gamma - \frac{2\theta(\alpha - 1)}{(N + 1)^2\beta\eta + 2N\theta^2}. \tag{A.13}
\]

Rearranging equation (4.9) obtains the emission tax (given G > 0) as

\[
\tau_e = \frac{(N + 1)^2\beta\eta + 2(N + 1)^2\theta^2}{(N + 1)^2\beta\eta + 2N\theta^2} \cdot \gamma \frac{\gamma}{1 + \lambda G} - \frac{2\theta(\alpha - 1)}{(N + 1)^2\beta\eta + 2N\theta^2}. \tag{A.14}
\]

It is straightforward to derive \(\frac{\gamma}{1 + \lambda G} < \gamma\). Given other parameters constant, \(\tau_e\) in (A.14) is lower than \(\tau_e\) in (A.13). In addition, we can infer from (3.2) that all else equal, \(m\) is strictly decreasing in \(\tau_e\), and hence, the presence of \(G\) leads to an increase in \(m\).

**Proof of Proposition 7**

(a) It is clear from (4.11) that \(G\) is not a function of \((\delta, \Omega)\).
(b) We can represent (4.12) as the following implicit form:

\[ f(G, \alpha, \beta, \gamma, \theta, \eta, N, \lambda) = 0. \]  
(A.15)

Taking the total differentiation with respect to \( G \) and \( \alpha \) obtains \( f_G dG + f_\alpha d\alpha = 0 \), namely,

\[ \frac{dG}{d\alpha} = -\frac{f_\alpha}{f_G} = -\frac{2\theta \gamma [(N + 2)\beta \eta + 2\theta^2](1 + \lambda G)}{[(N + 1)\beta \eta + 2\theta^2]^2} \frac{1}{f_G}, \]  
(A.16)

where \( f_G \) reflects the second order condition of \( \Phi \) with respect to \( G \), which must be negative at the optimum. Hence, we have \( \frac{dG}{d\alpha} > 0 \) in equilibrium.

(c) Taking the total differentiation of (A.15) with respect to \( G \) and \( \gamma \) obtains \( f_G dG + f_\gamma d\gamma = 0 \), namely,

\[ \frac{dG}{d\gamma} = -\frac{f_\gamma}{f_G} = -\left\{ \frac{(\alpha - 1)\theta [(N + 2)\beta \eta + 2\theta^2](1 + \lambda G)}{[(N + 1)\beta \eta + 2\theta^2]^2} - \gamma \right\} \frac{1}{f_G}, \]  
(A.17)

where \( f_G < 0 \) at the optimum. Hence, we have \( \frac{dG}{d\gamma} > 0 \) in equilibrium if and only if

\[ \frac{(\alpha - 1)\theta [(N + 2)\beta \eta + 2\theta^2](1 + \lambda G)}{[(N + 1)\beta \eta + 2\theta^2]^2} > \gamma, \]  
(A.18)

namely, condition (4.13) holds.
References


